Subleading power corrections to the color-singlet transverse momentum spectra

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Why the subleading power corrections?

- The **computation of the non-singular contributions** to the spectrum of the given resolution variable (i.e. the transverse momentum of the color singlet q_{T}) is often the main **computational bottleneck** in state-of-the-art QCD NNLO calculations
- The analytic knowledge of the spectrum beyond leading power would allow us to
  - $\rightarrow$ **Approximate the non-singular contributions** in the small q_{T} limit **eliminating the need for a numeric subtraction** up to very low q_{T} values
  - $\rightarrow$ Get an insight into the **factorization structure** of the next-to-leading power corrections

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Subleading power corrections

- We consider the process of production of a **color singlet** (inclusive Drell-Yan for now) from a **hadronic scattering**

$$h_a\, h_b \to \mathrm{CS} + X$$

- The differential cross section for the process with respect to the color-singlet **minus and plus components** q^- and q^+ and **transverse momentum** q_T can be written as

$$\frac{d\sigma_{h_a h_b \to \mathrm{CS}+X}}{dq^- \, dq^+ \, dq_\mathrm{T}^2} = K_\delta\left(q^-, q^+\right) \delta\left(q_\mathrm{T}^2\right) + \frac{K\left(q^-, q^+, q_\mathrm{T}^2\right)}{q_\mathrm{T}^2}$$

- The QCD **logarithmic structure** of K is well known and given by

$$K\left(q^-, q^+, q_\mathrm{T}^2\right) = \sum_{n=1}^{\infty} \left(\frac{\alpha_\mathrm{S}}{2\pi}\right)^n \sum_{m=0}^{2n-1} \log^m\left(\frac{q_\mathrm{T}^2}{q^- q^+}\right) K_{nm}\left(q^-, q^+, q_\mathrm{T}^2\right)$$

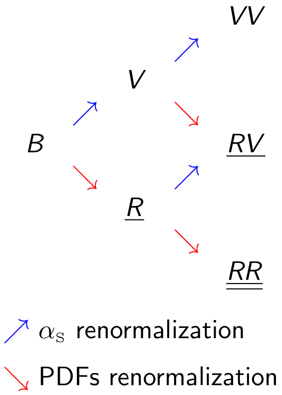
Subleading power corrections

- The **power structure** of the K_{nm} terms is obtained by expanding them with respect to q_{T}^2 and is given by

$$K_{nm}(q^-, q^+, q_{\mathrm{T}}^2) = \sum_{p=0}^{\infty} K_{nmp}(q^-, q^+) \, (q_{\mathrm{T}}^2)^p$$

- At **NLO**, several results are available (in some cases up to the **next-to-next-to-leading power**) using both $\mathcal{T}_0$ and q_{T} as resolution variables [Boughezal, Isgrò, Petriello '18] [Ebert, Moult, Stewart, Tackmann, Vita, Zhu '18] [Ebert, Moult, Stewart, Tackmann, Vita, Zhu '18] [Cieri, Oleari, Rocco '19] [Ferrera, Ju, Schönherr '24]
- Only partial results are available at **NNLO** and **N^{3}LO** [Moult, Rothen, Stewart, Tackmann, Zhu '16] [Boughezal, Liu, Petriello '16] [Oleari, Rocco '19] [Vita '24]
- I will present a systematic way of computing the **subleading power corrections** at **NNLO**, i.e. the K_{2mp} terms for $0 \leq m \leq 3$ and $p \geq 1$

General picture



- B, V and VV: No partons in the final state, proportional to $\delta\left(q_T^2\right)$
  - → They **do not contribute at subleading power**
- R, RV: One parton in the final state, 4 degrees of freedom: q^-, q^+, q_T^2 and z
  - → One PDF convolution, **no additional integrals to be done**
- RR: Two partons in the final state, 7 degrees of freedom
  - → **Up to three integrals to be done**
  - → Subject of the talk

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The SCET power expansion

- We know from the **SCET** that the contributions to the spectrum will come from the regions of the phase space where the partons are either **soft** or **collinear to the beam**
- Given the two **strongly separated scales**

$$Q \sim \sqrt{q^- q^+} \qquad q_\mathrm{T} \sim \lambda Q$$

we will get a non-zero contribution from the regions where the partons of momentum k have a

→ Soft scaling: $k \sim (\lambda, \lambda, \lambda)\, Q$

→ n-collinear scaling: $k \sim \left(1, \lambda^2, \lambda\right) Q$

→ $\bar{n}$-collinear scaling: $k \sim \left(\lambda^2, 1, \lambda\right) Q$

The SCET regions

The **9 relevant regions predicted by the SCET** are those where the two partons have a

→ Double soft scaling

$$k_1^- \sim k_1^+ \sim k_2^- \sim k_2^+ \sim \lambda\, Q$$

→ Mixed soft and (n- or $\bar{n}$-) collinear scaling (4 possibilities)

$$k_1^- \sim k_1^+ \sim \lambda\, Q \qquad k_2^- \sim Q \qquad k_2^+ \sim \lambda^2\, Q$$

→ Double (n- or $\bar{n}$-) collinear scaling (2 possibilities)

$$k_1^- \sim k_2^- \sim Q \qquad k_1^+ \sim k_2^+ \sim \lambda^2\, Q$$

→ Mixed n-collinear and $\bar{n}$-collinear scaling (2 possibilities)

$$k_1^- \sim k_2^+ \sim Q \qquad k_1^+ \sim k_2^- \sim \lambda^2\, Q$$

Phase space parametrization

A useful way to parametrize the phase space with **two partons in the final state** in terms of plus and minus components is

$$d\Phi_{\mathrm{CS}+2j} = \frac{\left(4\pi\mu^2\right)^{2\epsilon}}{\Gamma(1-2\epsilon)} \frac{dq^- \, dq^+ \, dq_{\mathrm{T}}^2}{\pi S} \frac{dk_1^- \, dk_1^+}{(4\pi)^2} \frac{dk_2^- \, dk_2^+}{(4\pi)^2} \, d\Phi_{\mathrm{CS}}$$

$$\times \left\{ \left[q_{\mathrm{T}}^2 - \left(\sqrt{k_1^- k_1^+} - \sqrt{k_2^- k_2^+} \right)^2 \right] \left[\left(\sqrt{k_1^- k_1^+} + \sqrt{k_2^- k_2^+} \right)^2 - q_{\mathrm{T}}^2 \right] \right\}^{-\frac{1}{2}-\epsilon}$$

$$\times \, \theta\left(\left[q_{\mathrm{T}}^2 - \left(\sqrt{k_1^- k_1^+} - \sqrt{k_2^- k_2^+} \right)^2 \right] \left[\left(\sqrt{k_1^- k_1^+} + \sqrt{k_2^- k_2^+} \right)^2 - q_{\mathrm{T}}^2 \right] \right)$$

$$\times \, \theta\left(\sqrt{S} - q^- - k_1^- - k_2^- \right) \theta\left(\sqrt{S} - q^+ - k_1^+ - k_2^+ \right)$$

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The analytic regulator

- The rapidity divergences are regulated through a **pure rapidity regulator** [Ebert, Moult, Stewart, Tackmann, Vita, Zhu '18] that multiplies the phase space whenever there is at least one parton in the final state
- If k is the **total momentum of the final-state partons**, the regulator is defined as

$$R_Y = \left(\frac{k^-}{k^+}\right)^{\alpha}$$

- This regulator has **two main advantages**
  - → It makes the **soft contributions zero** beyond leading order
  - → If there are two partons in the final state, it **does not depend separately on k_1 and k_2 but only on their sum $k = k_1 + k_2$**

The soft regions

- In the region where **one parton with rapidity y is soft**, the differential cross section can be written as a sum over n of terms proportional to $e^{(n+2\alpha)y}$
- Since the integral

$$\int_{-\infty}^{\infty} dy\, e^{Ay} = \int_{0}^{\infty} de^{y}\, \left(e^{y}\right)^{A-1} = 0$$

all **the contributions from single-soft regions integrate to 0**

- In the region where **both the partons are soft**, after trading the rapidities y_1 and y_2 of the two partons with the new integration variables

$$\tilde{y} = \frac{y_1 - y_2}{2} \qquad y = \frac{y_1 + y_2}{2}$$

the **integral over y vanishes** for the same reason

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The double collinear region

In the region where **the two partons are n_a-collinear** it is convenient to introduce

$$y = \frac{q_T^2}{k^- k^+} \qquad z = \frac{q^-}{q^- + k^-}$$

and parametrize the phase space as

$$d\Phi_{\mathrm{CS}+2j} = \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \left(\frac{\mu^2}{q_T^2}\right)^\epsilon \frac{dq^-\, dq^+\, dq_T^2\, q_T^2}{(4\pi)^2\, S} \frac{dz}{z\,(1-z)} \frac{dy}{y^2} \frac{d\Phi_{2j}}{2\pi} d\Phi_{\mathrm{CS}}$$

Integration variables

- Analytic integration over Φ_{2j} and y
- Numeric convolution over z

Leading ϵ poles at leading power

- The angular integration over Φ_{2j} exposes the **collinear singularity** of the 2nd emission
  $\rightarrow \frac{1}{\epsilon}$ pole
- The y integration exposes the **soft singularity** of the 2nd emission
  $\rightarrow \frac{1}{\epsilon}$ pole
- The q_{T}^2 distribution exposes the **collinear singularity** of the 1st emission
  $\rightarrow \left(q_{\mathrm{T}}^2\right)^{-1-\epsilon-\alpha} \sim \frac{1}{\epsilon+\alpha}\, \delta\left(q_{\mathrm{T}}^2\right)$ pole
- The z distribution exposes the **soft singularity** of the 1st emission
  $\rightarrow (1-z)^{-1+2\alpha} \sim \frac{1}{\alpha}\, \delta(1-z)$ pole

$$\frac{1}{\epsilon^2\,\alpha}\,\frac{1}{\epsilon+\alpha} = \frac{1}{\epsilon^3\,\alpha}\left(1-\frac{\alpha}{\epsilon}\right) = \frac{1}{\epsilon^3\,\alpha} - \frac{1}{\epsilon^4} + \mathcal{O}(\alpha)$$

Leading ϵ poles at next-to-leading power

- The angular integration over Φ_{2j} exposes the **collinear singularity** of the 2nd emission
  $\rightarrow \frac{1}{\epsilon}$ pole
- The y integration exposes the **soft singularity** of the 2nd emission
  $\rightarrow \frac{1}{\epsilon+\alpha}$ pole
- The q_{T}^2 distribution is now regular
- The z distribution exposes the **soft singularity** of the 1st emission
  $\rightarrow (1-z)^{-1+2\alpha} \sim \frac{1}{\alpha}\,\delta(1-z)$ pole

$$\frac{1}{\epsilon\,\alpha}\,\frac{1}{\epsilon+\alpha} = \frac{1}{\epsilon^2\,\alpha}\left(1-\frac{\alpha}{\epsilon}\right) = \frac{1}{\epsilon^2\,\alpha} - \frac{1}{\epsilon^3} + \mathcal{O}(\alpha)$$

The mixed collinear and anti-collinear region

In the region where **one parton is n_a-collinear and the other is n_b-collinear** it is convenient to introduce

$$z_a = \frac{q^-}{q^- + k_1^-} \qquad z_b = \frac{q^+}{q^+ + k_2^+}$$

and parametrize the phase space as

$$d\Phi_{\mathrm{CS}+2j} = \frac{dq^- \, dq^+ \, dq_{\mathrm{T}}^2}{S} \, d\Phi_{\mathrm{CS}} \, \mu^{2\epsilon} \frac{d^{2-2\epsilon} k_{1\mathrm{T}}}{2\,(2\pi)^{3-2\epsilon}} \, \frac{d z_a}{z_a \,(1 - z_a)} \times \mu^{2\epsilon} \frac{d^{2-2\epsilon} k_{2\mathrm{T}}}{2\,(2\pi)^{3-2\epsilon}} \, \frac{d z_b}{z_b \,(1 - z_b)} \, \delta\left(q_{\mathrm{T}}^2 - \left(\vec{k}_{1\mathrm{T}} + \vec{k}_{2\mathrm{T}}\right)^2\right)$$

Integration variables

- Analytic integration over $k_{1\mathrm{T}}$ and $k_{2\mathrm{T}}$
- Numeric convolution over z_a and z_b

The mixed collinear and anti-collinear region

The analytic regulator in the mixed collinear and anti-collinear region reads

$$R_Y = \left(\frac{q^- \left(1 - z_a\right) z_b}{q^+ \, z_a \left(1 - z_b\right)} \right)^\alpha \left[1 + \alpha \, \frac{z_a z_b \left({k_{2\mathrm{T}}}^2 - {k_{1\mathrm{T}}}^2 \right)}{q^- q^+ \left(1 - z_a\right) \left(1 - z_b\right)} + \mathcal{O}\left(\alpha^2\right) \right]$$

NO ϵ^3 POLES

The leading poles are proportional to

$$\left(1 - z_a\right)^{-1+\alpha} \left(1 - z_b\right)^{-1-\alpha} \frac{1}{\epsilon^2} \sim \frac{1}{\alpha^2 \, \epsilon^2}$$

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The need for an additional region

After combining the contributions from the SCET regions

- → All the α poles cancel
- → The ϵ poles **DO NOT** cancel

SOMETHING IS MISSING

We need to study the phase space more carefully

The phase space constraint

$$\theta\Big(\Big[q_{\mathrm{T}}^2-(k_{1\mathrm{T}}-k_{2\mathrm{T}})^2\Big]\Big[(k_{1\mathrm{T}}+k_{2\mathrm{T}})^2-q_{\mathrm{T}}^2\Big]\Big)$$

also allows for a **back-to-back region** where

$$k_{\mathrm{T}}^+=k_{1\mathrm{T}}+k_{2\mathrm{T}}\sim Q \qquad k_{\mathrm{T}}^-=k_{1\mathrm{T}}-k_{2\mathrm{T}}\sim\lambda\, Q$$

The back-to-back region

In the **back-to-back region**, it is convenient to write the phase space as

$$d\Phi_{\mathrm{CS}+2j} = \frac{(4\pi)^{2\epsilon}}{\Gamma(1-2\epsilon)} \frac{dq^- \, dq^+ \, dq_{\mathrm{T}}^2}{\pi S} \left(\frac{\mu^2}{q^- q^+}\right)^{\epsilon} \left(\frac{\mu^2}{q_{\mathrm{T}}^2}\right)^{\epsilon} \frac{q^- q^+}{(4\pi)^4} \, d\Phi_{\mathrm{CS}}$$
$$\times \frac{dz_a}{{z_a}^2} \frac{dz_b}{{z_b}^2} \left(\frac{z_a}{1-z_a}\right)^{\epsilon} \left(\frac{z_b}{1-z_b}\right)^{\epsilon}$$
$$\times \frac{dv}{2} \left(1-v^2\right)^{-\epsilon} dw \left(1-w^2\right)^{-\frac{1}{2}-\epsilon} + \mathcal{O}(\lambda^4)$$

where we defined

$$z_a = \frac{q^-}{q^- + \frac{k_{\mathrm{T}}^+}{2}\left(e^{y_1} + e^{y_2}\right)} \qquad z_b = \frac{q^+}{q^+ + \frac{k_{\mathrm{T}}^+}{2}\left(e^{-y_1} + e^{-y_2}\right)}$$

$$v = \sqrt{1 - \frac{\left(k_{\mathrm{T}}^+\right)^2}{q^- q^+} \frac{z_a}{1-z_a} \frac{z_b}{1-z_b}} \qquad w = \frac{k_{\mathrm{T}}^-}{\sqrt{q_{\mathrm{T}}^2}}$$

Properties of the back-to-back region

- The region requires at least two partons in the final state
  $\rightarrow$ It does not contribute at **NLO**
- The region is naturally power-suppressed
  $\rightarrow$ It does not contribute at **leading power**
- The region does not produce any α pole
- The terms of the power expansion proportional to an **odd power** of q_{T} are also proportional to an **odd power** of w, whose integration range is $(-1, 1)$
  $\rightarrow$ They integrate to 0
- After combining the results from this region with those from the other (collinear) regions, **all the ϵ poles properly cancel**

Conclusions

- We presented a framework for computing the **subleading power corrections** to the differential cross section for the production of an electroweak boson
- We derived the soft and collinear contributions expanding the cross section in the regions predicted by the SCET
- In the case where there are two partons in the final state, the **SCET regions are NOT enough** to correctly cancel all the ϵ poles, but we need an **additional region**, where the two partons are hard but almost **back-to-back in the transverse plane**
- After adding the extra region, **all the ϵ poles properly cancel**

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