Two-loop mixed QCD-EW corrections to charged-current Drell-Yan

In collaboration with R. Bonciani, S. Devoto, N. Rana, A. Vicini

Tommaso Armadillo - UCLouvain & UNIMI

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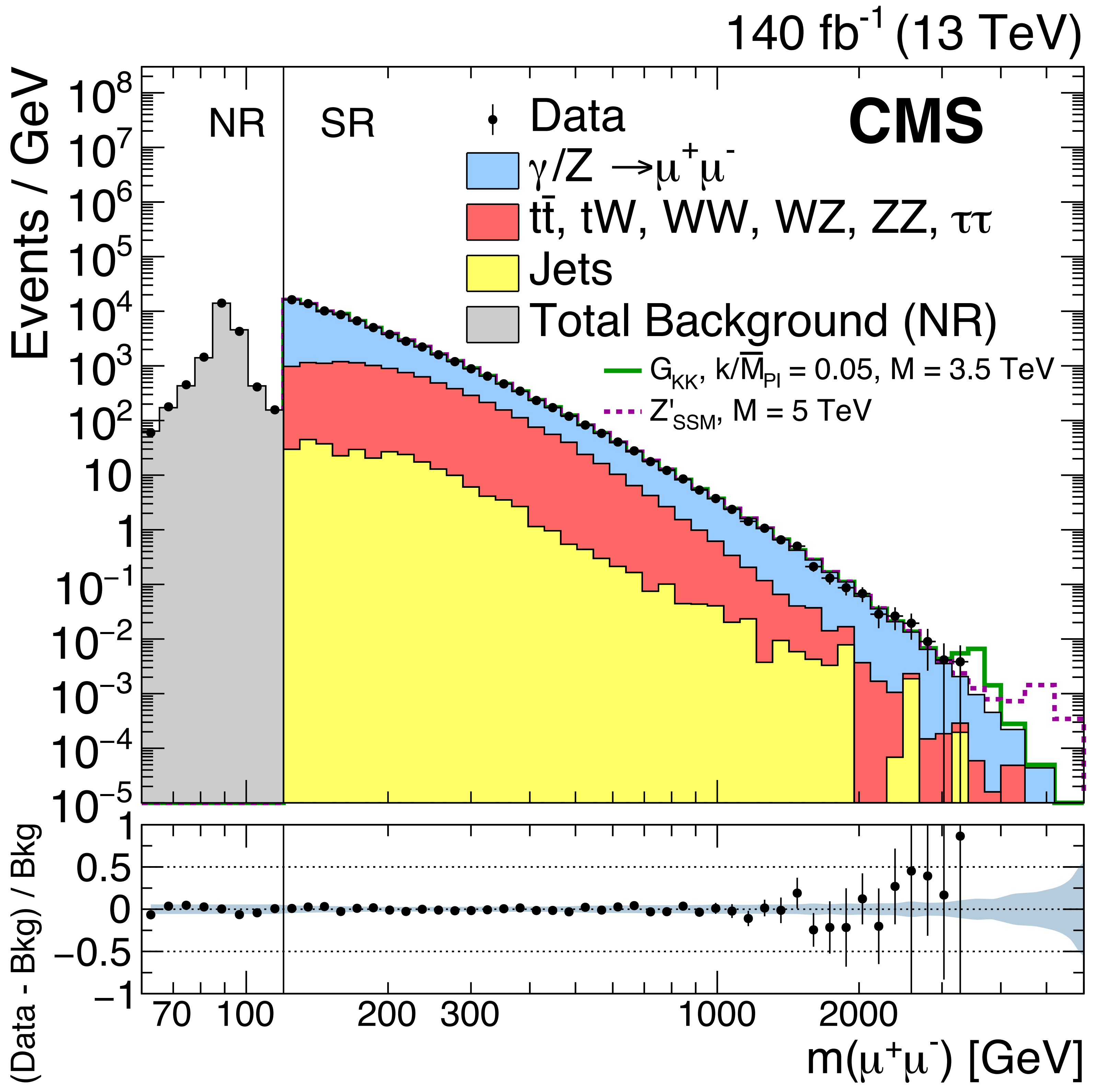
Motivations

- The **inclusive production of a fermion pair** is a standard candle process at LHC;
- At HL-LHC the **statistical errors** will reach $\mathcal{O}(0.5\%)$ across the entire invariant mass range

Bin range (GeV)	% error 140 fb^{-1}	% error 3 ab^{-1}
91-92	0.03	6 x 10^{-3}
120-400	0.1	0.02
400-600	0.6	0.13
600-900	1.4	0.30
900-1300	3.2	0.69

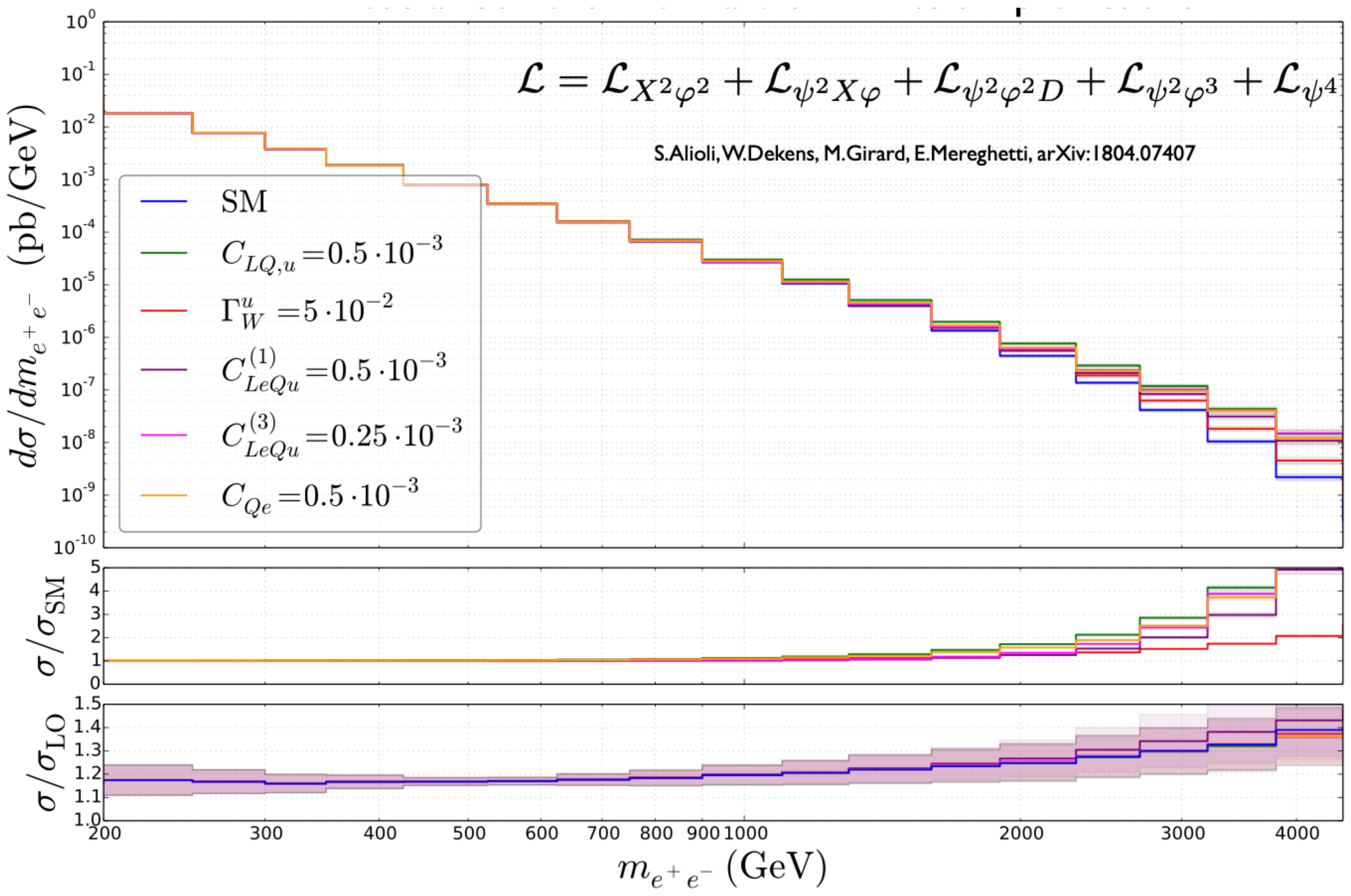
arXiv:2106.11953

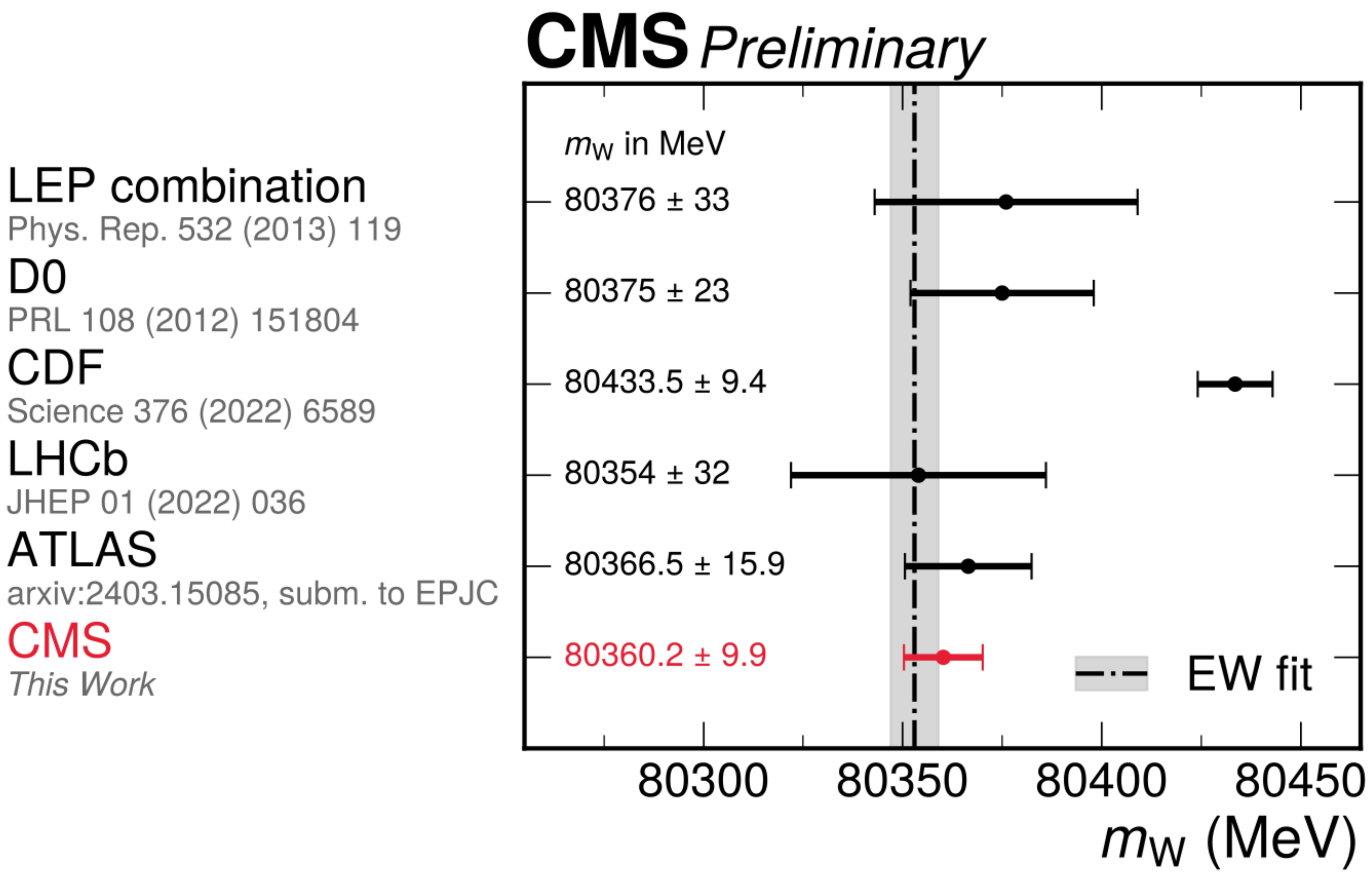


- The **theory systematics** (e.g. input scheme, PDFs, EW corrections, …) must be kept under control at this level of precision.

Motivations

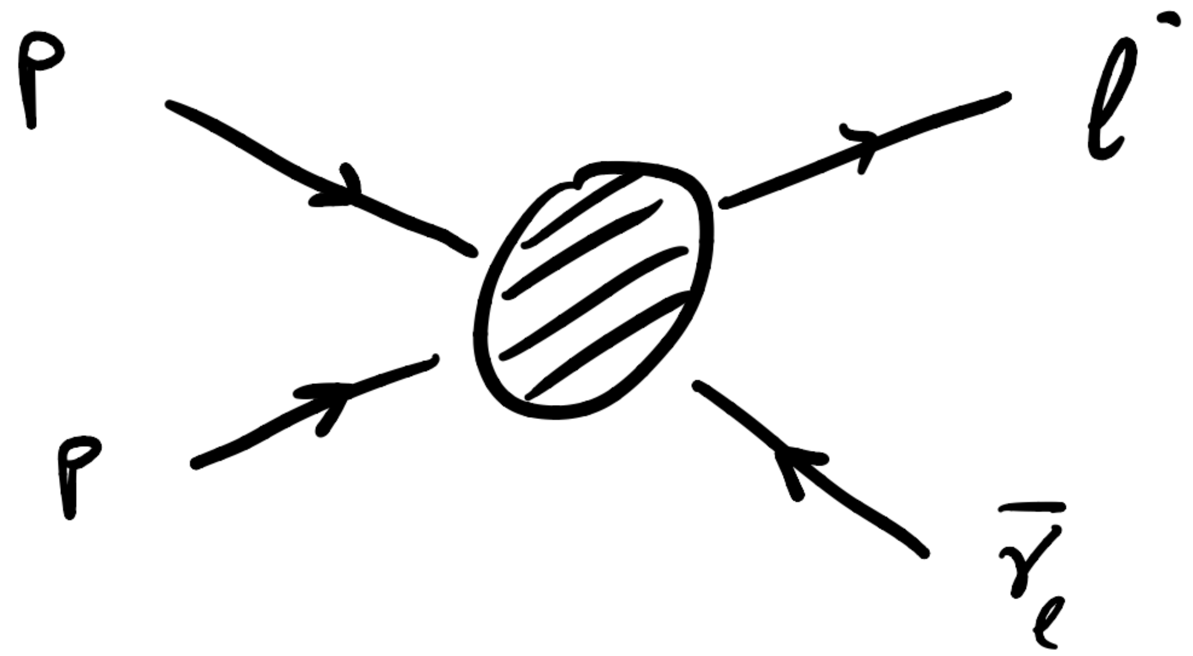
- Precision calculations are important for **constraining higher dimensional operators** in the SMEFT language;
- Predictions for the charged-current Drell-Yan are important for the m_W **measurement**;





- The **computational challenges** are similar to the ones for FCC-ee.

Higher orders



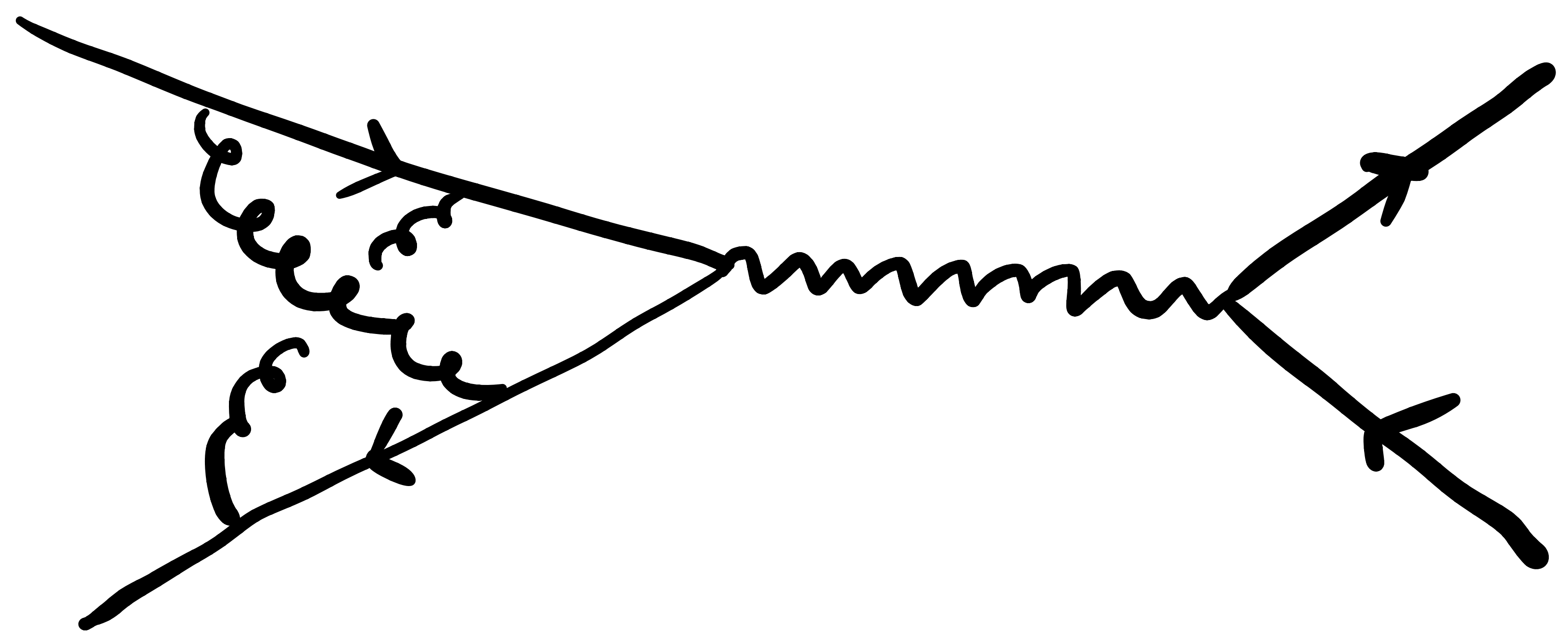
$$\sigma_{ij} = \sigma_{ij}^{(0,0)} + \alpha_s\, \sigma_{ij}^{(1,0)} + \alpha\, \sigma_{ij}^{(0,1)} + \alpha_s^2\, \sigma_{ij}^{(2,0)} + \alpha_s \alpha\, \sigma_{ij}^{(1,1)} + \alpha^2\, \sigma_{ij}^{(0,2)} + \alpha_s^3\, \sigma_{ij}^{(3,0)} + \dots$$

Partonic cross-section

$$\sigma_{tot} = \sum_{i,j \in q,\bar{q},g,\gamma} \int_0^1 dx_1\, dx_2\, f_i(x_1, \mu_F)\, f_j(x_1, \mu_F)\, \sigma_{ij}(\mu_F, \mu_R)$$

Parton Distribution Functions

Higher orders



$$\sigma_{ij} = \sigma_{ij}^{(0,0)}$$
$$+ \alpha_s\, \sigma_{ij}^{(1,0)} + \alpha\, \sigma_{ij}^{(0,1)} +$$
$$+ \alpha_s^2\, \sigma_{ij}^{(2,0)} + \alpha_s \alpha\, \sigma_{ij}^{(1,1)} + \alpha^2\, \sigma_{ij}^{(0,2)} +$$
$$+ \alpha_s^3\, \sigma_{ij}^{(3,0)} + \ldots$$

QCD Corrections

NLO:

[G.Altarelli, R.Ellis, G.Martinelli Nucl.Phys.B 157 (1979)];

NNLO:

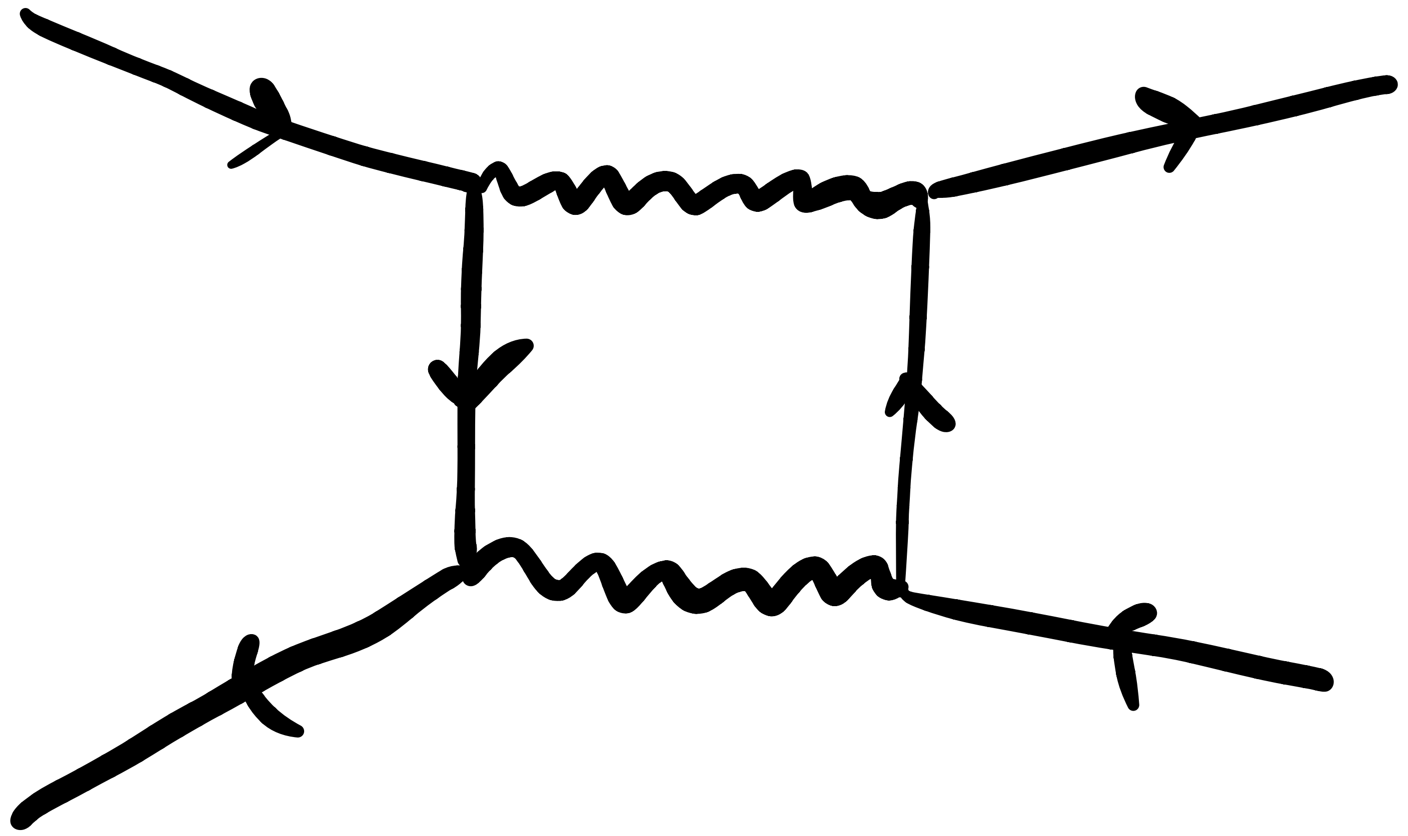
[R.Hamberg, T.Matsuura, W.van Nerveen, Nucl. Phys. B 359 (1991)];
[C.Anastasiou, L.J.Dixon, K.Melnikov, F.Petriello, hep-ph:0306192];
[S.Catani, L.Cieri, G.Ferrera, D.de Florian, M.Grazzini arXiv:0903.2120];

N3LO:

[C.Duhr, F.Dulat, B.Mistlberger arXiv:2007.13313];
[X.Chen, T.Gehrmann, N.Glover, A.Huss, T.Yang, and H.Zhu arXiv:2107.09085];
[S.Camarda, L.Cieri, G.Ferrera arXiv:2103.04974];
[X.Chen, T.Gehrmann, N.Glover, A.Huss, P.Monni, E.Re, L.Rottoli, P.Torrielli arXiv:2203.01565];
[T.Neumann, J.Campbell arXiv:2207.07056]

Higher orders

$$\sigma_{ij} = \sigma_{ij}^{(0,0)} + \alpha_s\, \sigma_{ij}^{(1,0)} + \alpha\, \sigma_{ij}^{(0,1)} + \alpha_s^2\, \sigma_{ij}^{(2,0)} + \alpha_s \alpha\, \sigma_{ij}^{(1,1)} + \alpha^2\, \sigma_{ij}^{(0,2)} + \alpha_s^3\, \sigma_{ij}^{(3,0)} + \ldots$$



EW Corrections

NLO:

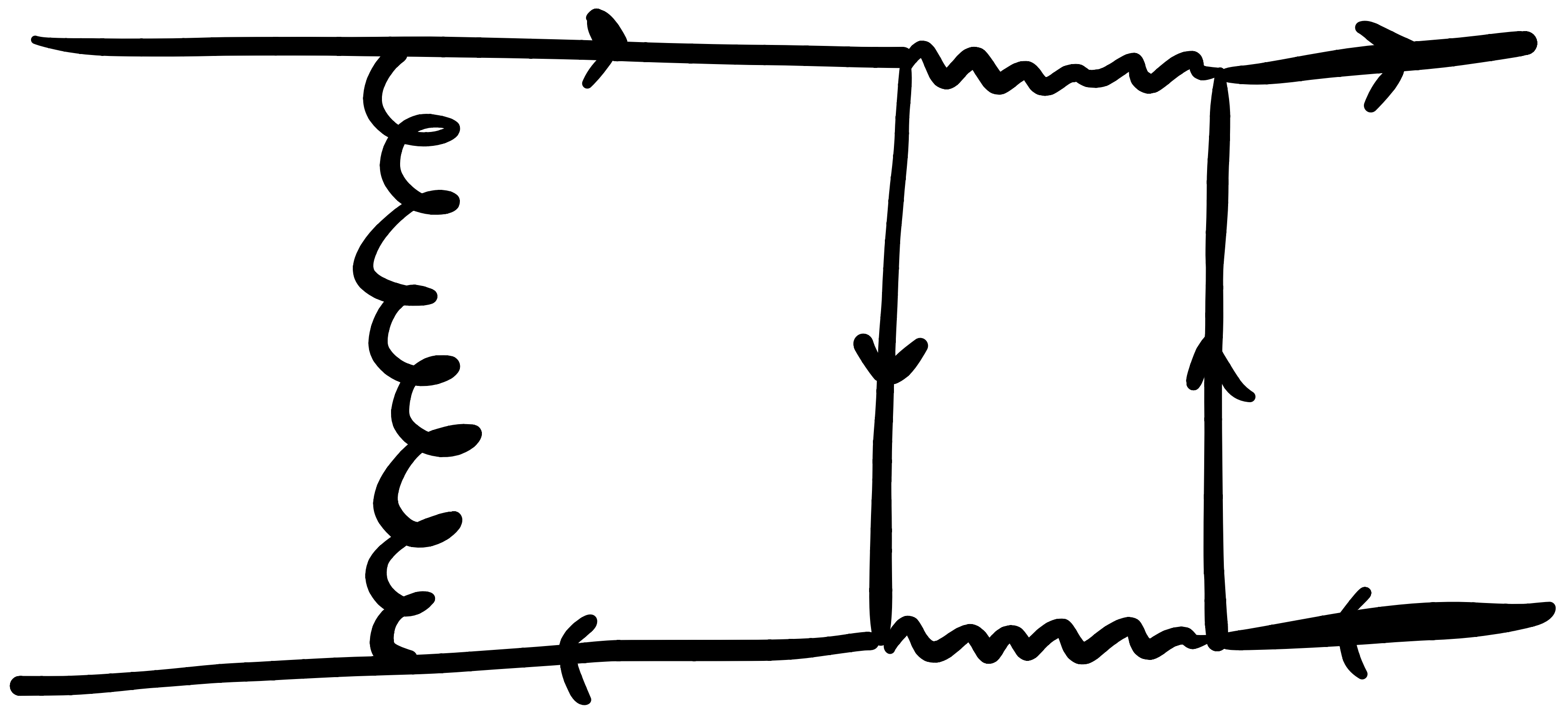
[U.Baur, O.Brein, W.Hollik, C.Schappacher, D.Wackeroth, hep-ph:0108274];
[S.Dittmaier, M.Kramer, hep-ph:0109062];
[U.Baur, D.Wackeroth, hep-ph:0405191];

NNLO (Sudakov approximation):

[B. Jantzen, J.H.Kühn. A.A.Penin, V.A.Smirnov, hep-ph:0509157];

Higher orders

$$\sigma_{ij} = \sigma_{ij}^{(0,0)} + \alpha_s\,\sigma_{ij}^{(1,0)} + \alpha\,\sigma_{ij}^{(0,1)} + \alpha_s^2\,\sigma_{ij}^{(2,0)} + \alpha_s\alpha\,\sigma_{ij}^{(1,1)} + \alpha^2\,\sigma_{ij}^{(0,2)} + \alpha_s^3\,\sigma_{ij}^{(3,0)} + \dots$$



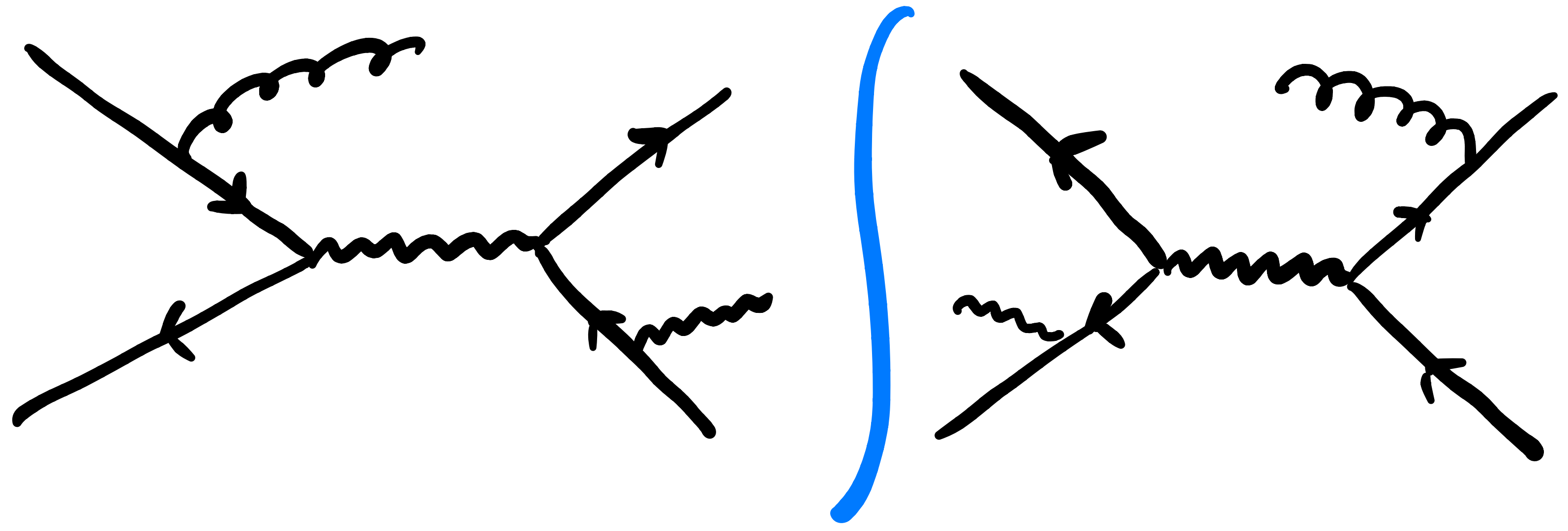
Mixed corrections

- Naively they have **similar magnitude of N3LO QCD**: $\alpha_s^3 \simeq \alpha_s\alpha$;
- In specific phase-space points, fixed order EW corrections can become **very large because of logarithmic enhancement** (weak and QED Sudakov type);
- They reduce the **input scheme dependence**.

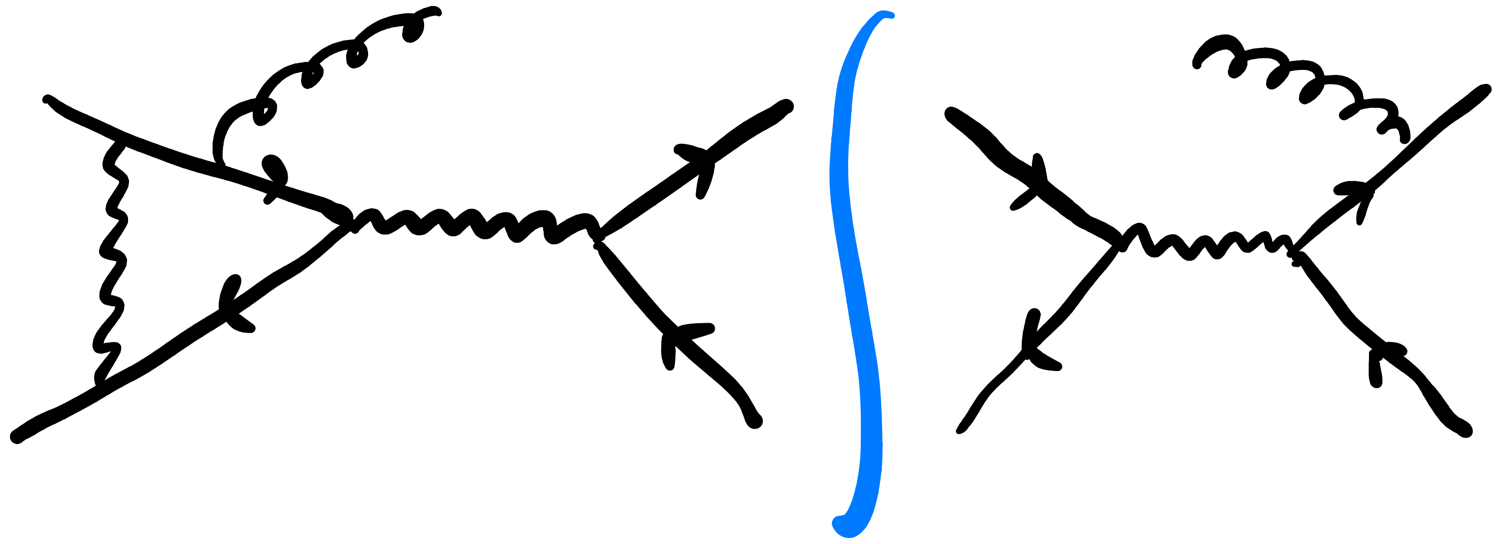
Extremely important for high precision phenomenology (per-cent and sub per-cent level)

Mixed QCD-EW corrections

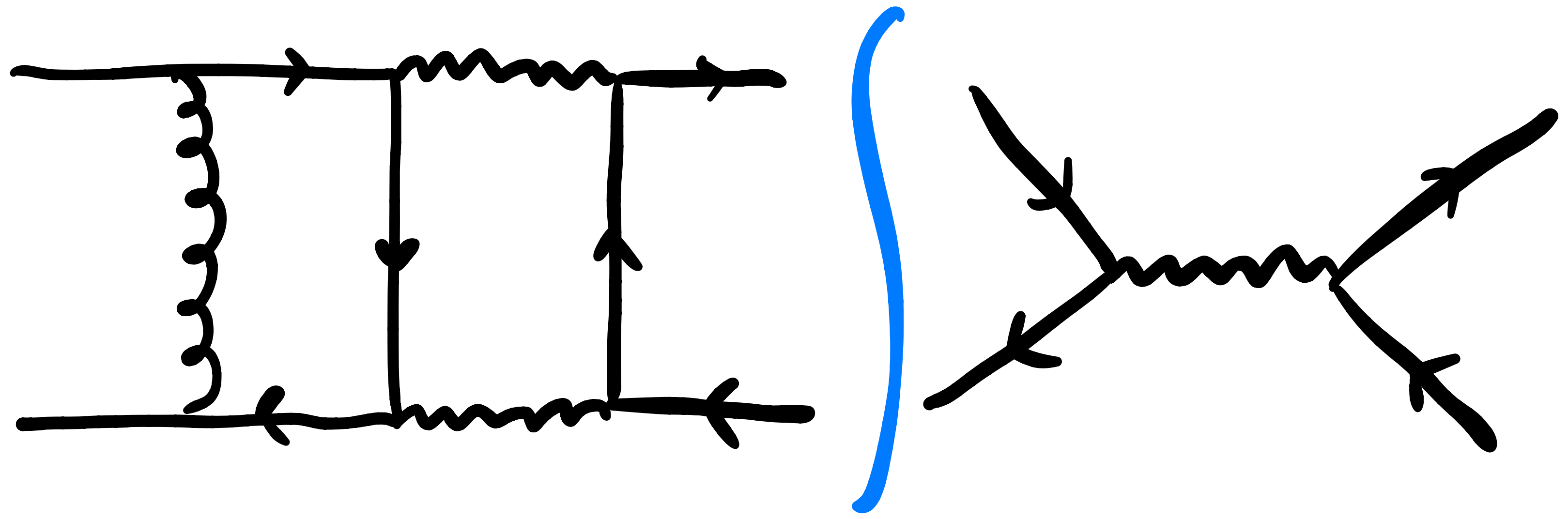
Double Real



Real-Virtual



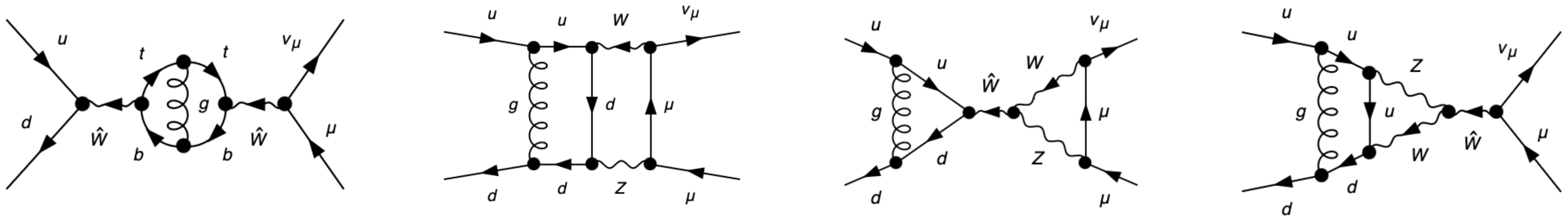
Pure Virtual



- Each of the three pieces carries its own challenges;
- The **pure virtual contributions are usually the main bottleneck**;
- Each individual contribution is divergent in the dimensional regulator ϵ.

The 2L amplitude

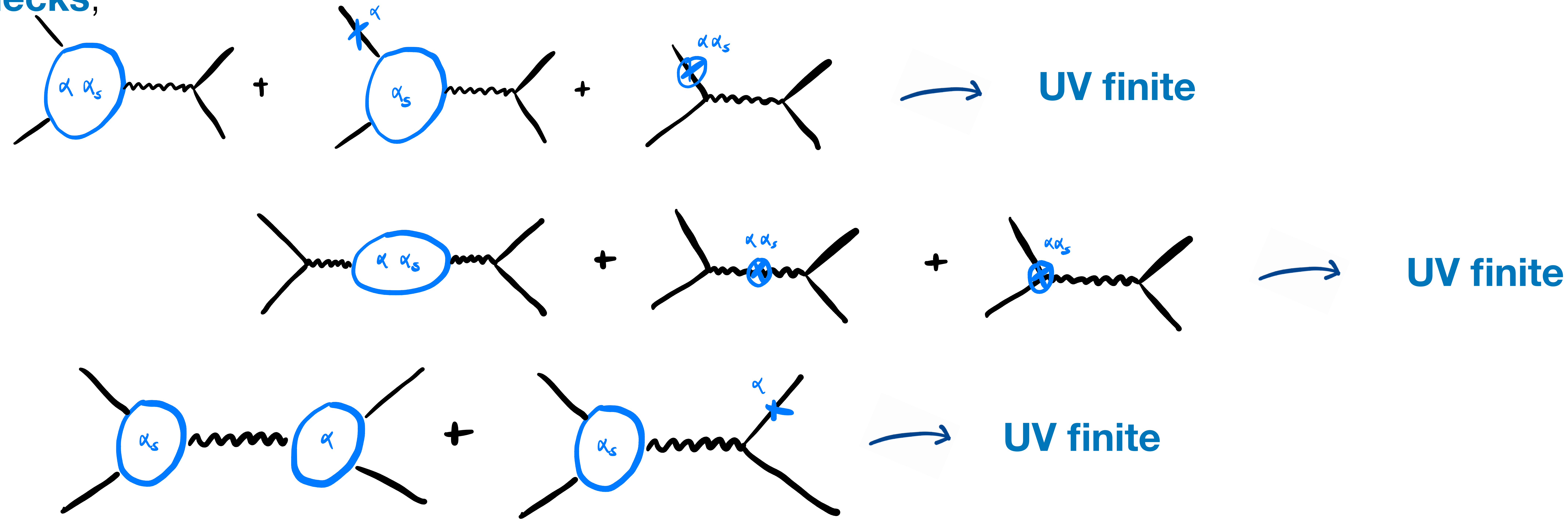
- The diagrams are generated using **FeynArts**;



- The computation of the interference terms between the 2L diagrams and the born has been done with in-house **Mathematica** routines;
- We treated γ^5 in d dimensions using the **naive anti commuting scheme**;

UV renormalisation

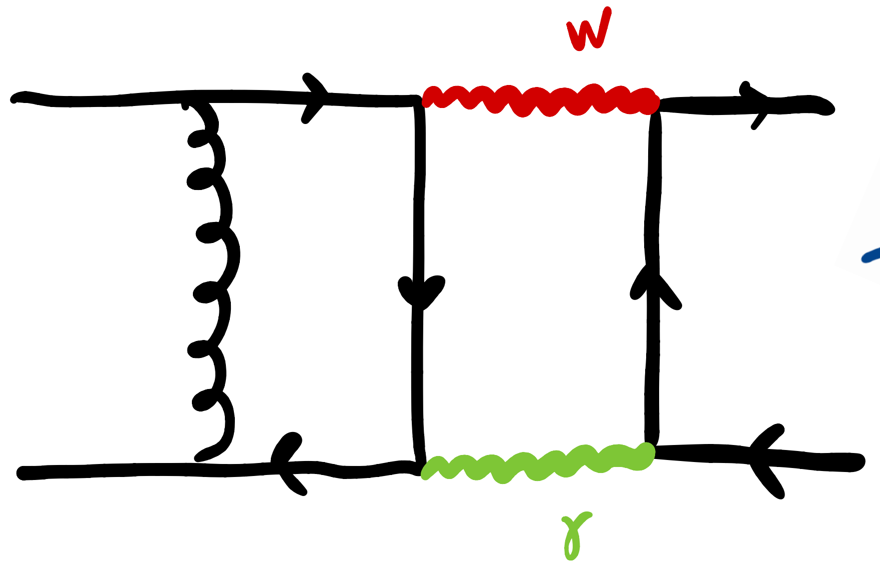
- In the computation we employed the **Background Field Method**. This let us identify some subsets of diagrams which are **UV finite**, which is useful for performing intermediate non trivial **cross-checks**;



- All the counter-terms were computed in the **on-shell scheme**.

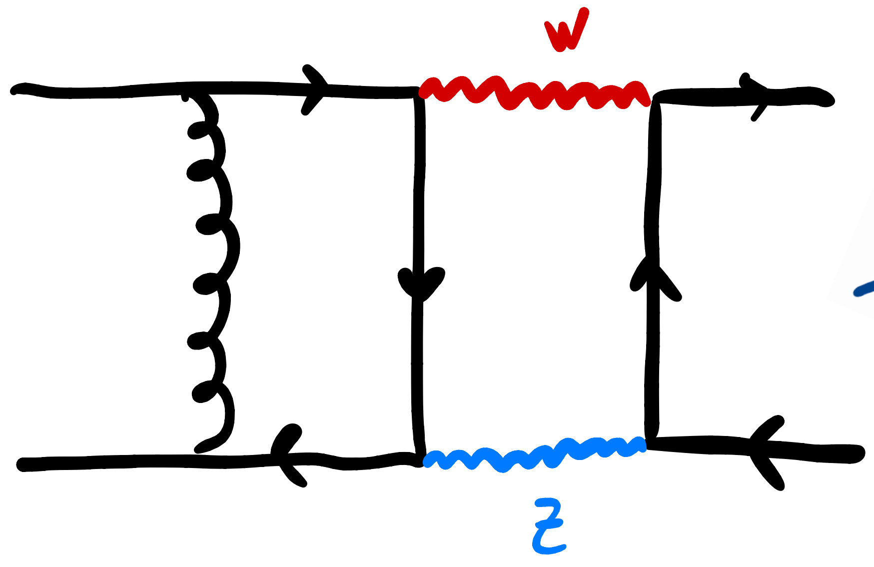
IR subtraction

- IR singularities are handled by the **qT-subtraction formalism**;
- The qT-subtraction requires the final state emitters (leptons) to be **massive**! I.e. that the final state collinear divergences are regularised by $\log(m_\ell^2/s)$;
- However, retaining the exact dependance on the lepton mass is extremely challenging. For this reason, we kept the lepton mass only when the photon originates IR divergences;



Exact dependence on m_ℓ

We are introducing a **mismatch** $\mathcal{O}(m_\ell^2/s)$



Massless lepton
Missing terms $\mathcal{O}(m_\ell^2/s)$

IR subtraction

- The UV renormalised and IR subtracted scattering amplitude is given by:

UV renormalised amplitude

$$\left|\mathcal{M}^{(1,1)}_{fin}\right\rangle = \left|\mathcal{M}^{(1,1)}\right\rangle - \mathcal{I}^{(1,1)}\left|\mathcal{M}^{(0,0)}\right\rangle - \tilde{\mathcal{I}}^{(0,1)}\left|\mathcal{M}^{(1,0)}_{fin}\right\rangle - \tilde{\mathcal{I}}^{(1,0)}\left|\mathcal{M}^{(0,1)}_{fin}\right\rangle$$

Subtraction operators:

$$\mathcal{I}^{(1,1)} = C_F\left[\frac{Q_u^2+Q_d^2}{2}\left(\frac{4}{\epsilon^4}+\frac{1}{\epsilon^3}(12+8i\pi)+\frac{1}{\epsilon^2}(9-28\zeta_2+12i\pi)+\frac{1}{\epsilon}\left(-\frac{3}{2}+6\zeta_2-24\zeta_3-4i\pi\zeta_2\right)\right)+\left(-\frac{2}{\epsilon^2}-\frac{1}{\epsilon}(3+2i\pi)+\zeta_2\right)\frac{4}{\epsilon}\Gamma_l^{(0,1)}\right]$$

$$\Gamma_l^{(0,1)} = -\frac{1}{4}\left[Q_l^2(1-i\pi)+Q_l^2\log\left(\frac{m_l^2}{s}\right)+2Q_uQ_l\log\left(\frac{2p_1\cdot p_4}{s}\right)-2Q_dQ_l\log\left(\frac{2p_2\cdot p_4}{s}\right)\right]$$

- We verified **analytically** the cancellation of the poles $1/\epsilon^4$, $1/\epsilon^3$ and $1/\epsilon^2$;
- We verified **numerically** the cancellation of the $1/\epsilon$ pole up to the 6th significant digit.

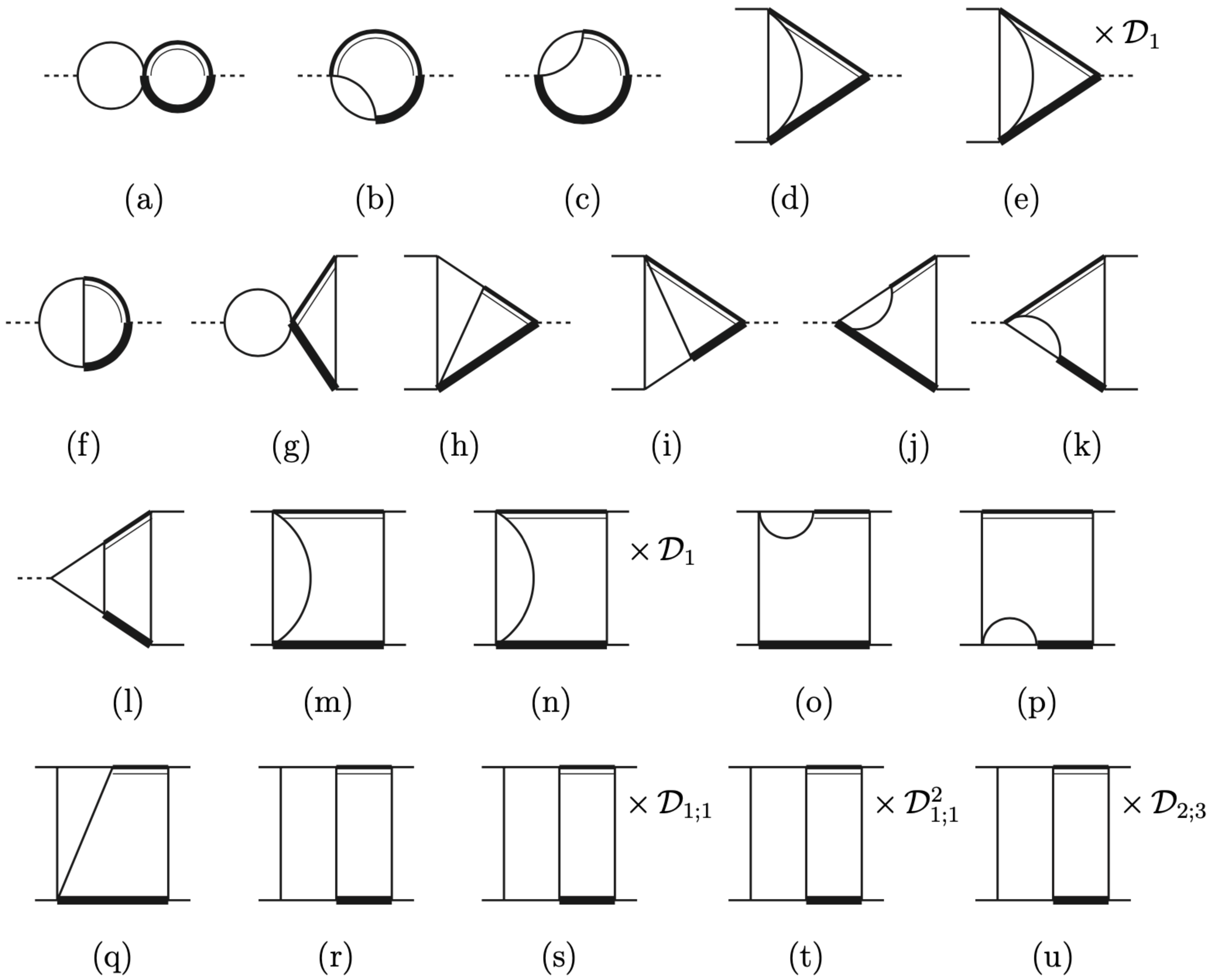
Reduction to Master Integrals

- We identified **11 integral families** with either 0, 1 or 2 masses. We reduced them to Master Integrals using **Kira** in combination with **Firefly**. The complete reduction took $\mathcal{O}(16h)$.

- We ended up with **274 masters integrals** to evaluate.

- The most complicated topology was a **two-loop box with two internal different masses**;

- We evaluated all the masters using the method of **differential equations**, using a **semi-analytical approach**.



SeaSyde

[**TA**, R. Bonciani, S. Devoto, N.Rana, A.Vicini, arXiv:2205.03345]

https: //github.com/TommasoArmadillo/SeaSyde

SEASYDE

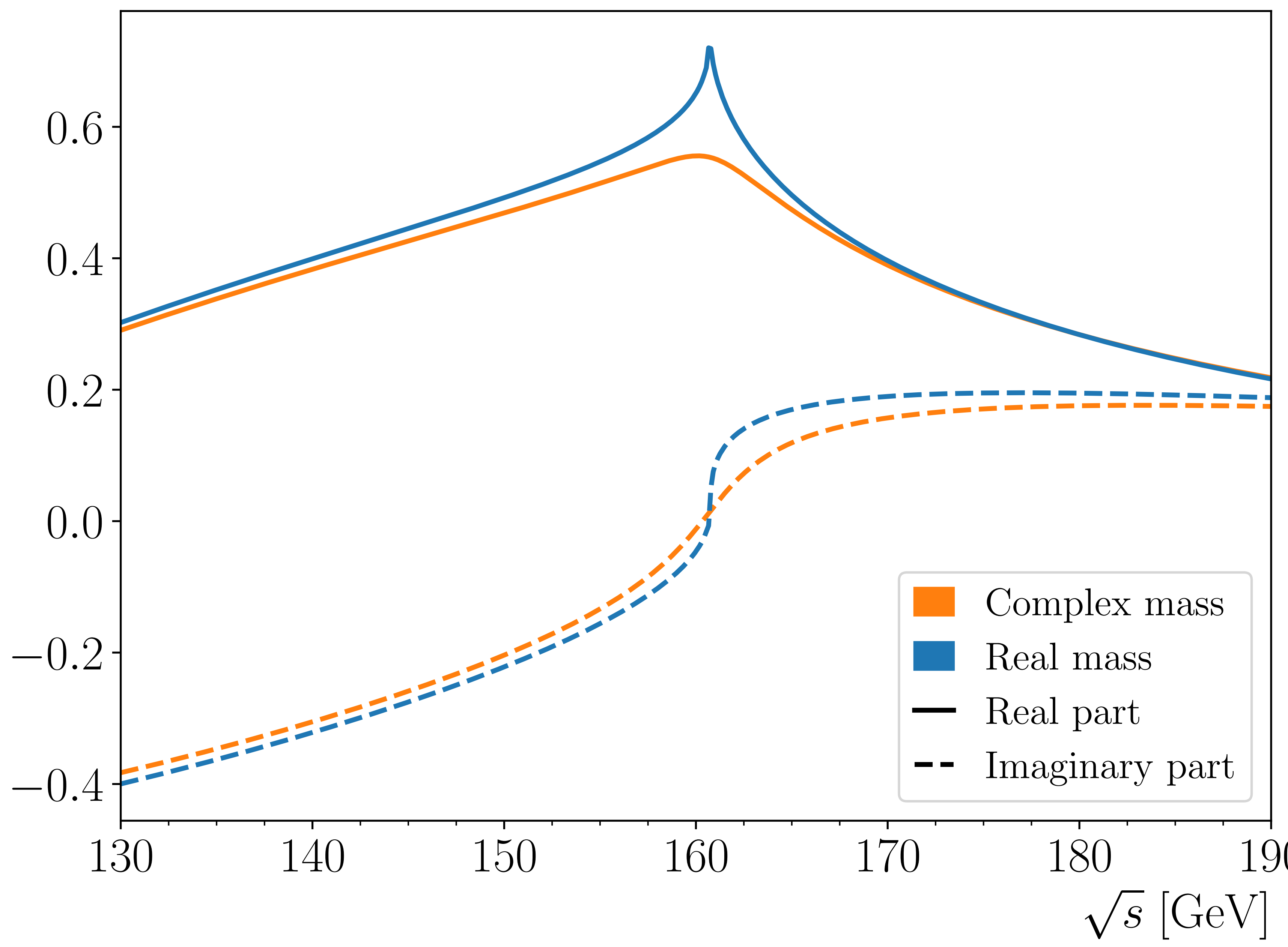
- Our goal in the end is to fit the W mass to the data, hence, we need to employ a gauge invariant definition of the mass. For this reason, it is important to perform the calculations in the **complex-mass scheme**.

$$\mu_V^2 = m_V^2 - i\Gamma_V m_V$$

- The complex mass scheme **regularises** the behaviour at the resonance: $\dfrac{1}{s - \mu_V^2 + i\delta}$

- If we utilise **adimensional variables**, they become complex-valued:

$$\tilde{s} = \frac{s}{m_V^2} \rightarrow \frac{s}{\mu_V^2}$$



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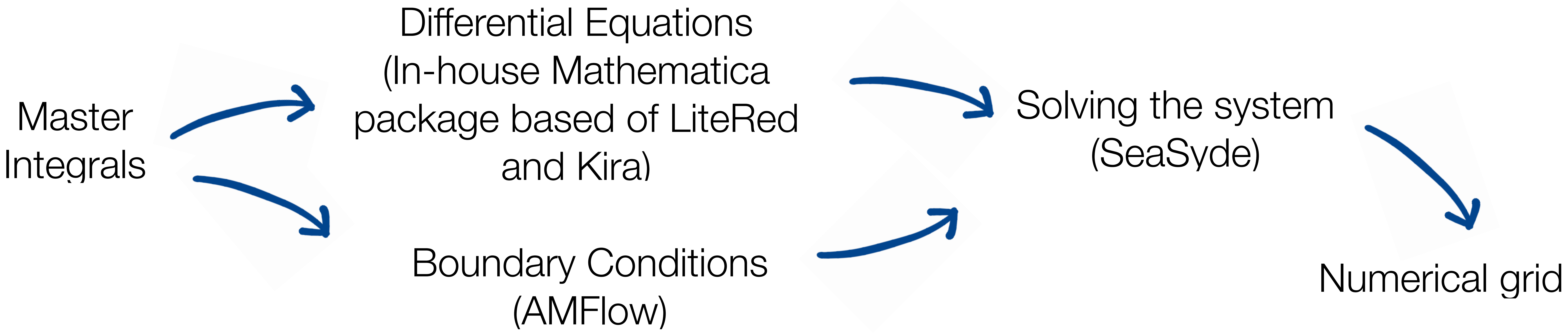
SEASYDE

- **SeaSyde** (**S**eries **E**xpansion **A**pproach for **SY**stems of **D**ifferential **E**quations) is a general package for solving a system of differential equations using the series expansion approach;

- Seasyde can handle **complex kinematic variables** by introducing an original algorithm for the analytic continuation in the complex plane, thus being able to handle **complex internal masses**;

- **SeaSyde** can deal with arbitrary system of differential equations, covering also the case of **elliptic integrals**.

Creating a grid



- The computation of a **grid with 3250 points** for the two-loop box with two internal and different masses (56 equations) required ∼3 weeks on 26 cores.
- This approach is **completely general and easy to automate**, and can be applied, in principle, to **any integral family**.

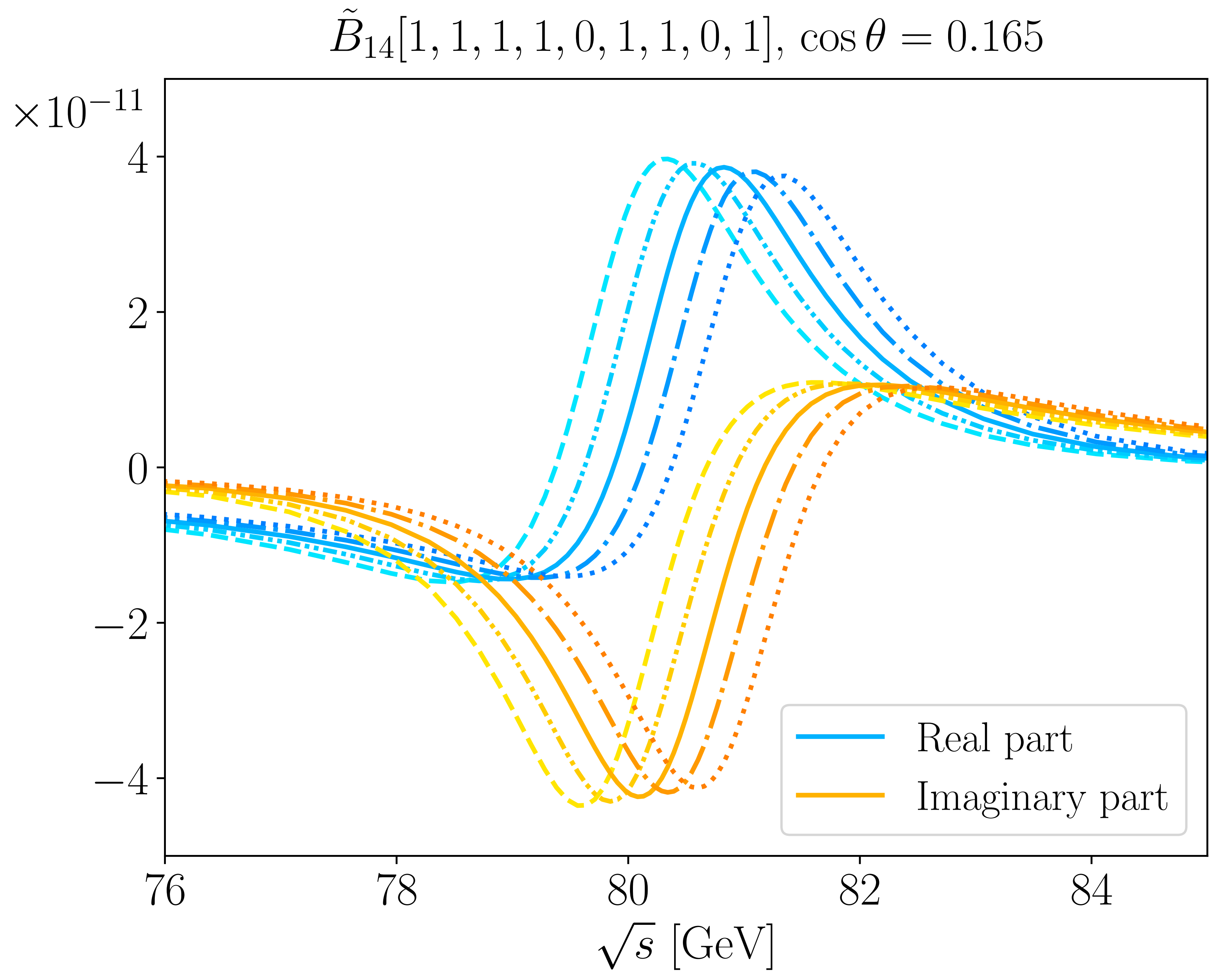
The expansion in $\delta\mu_W$

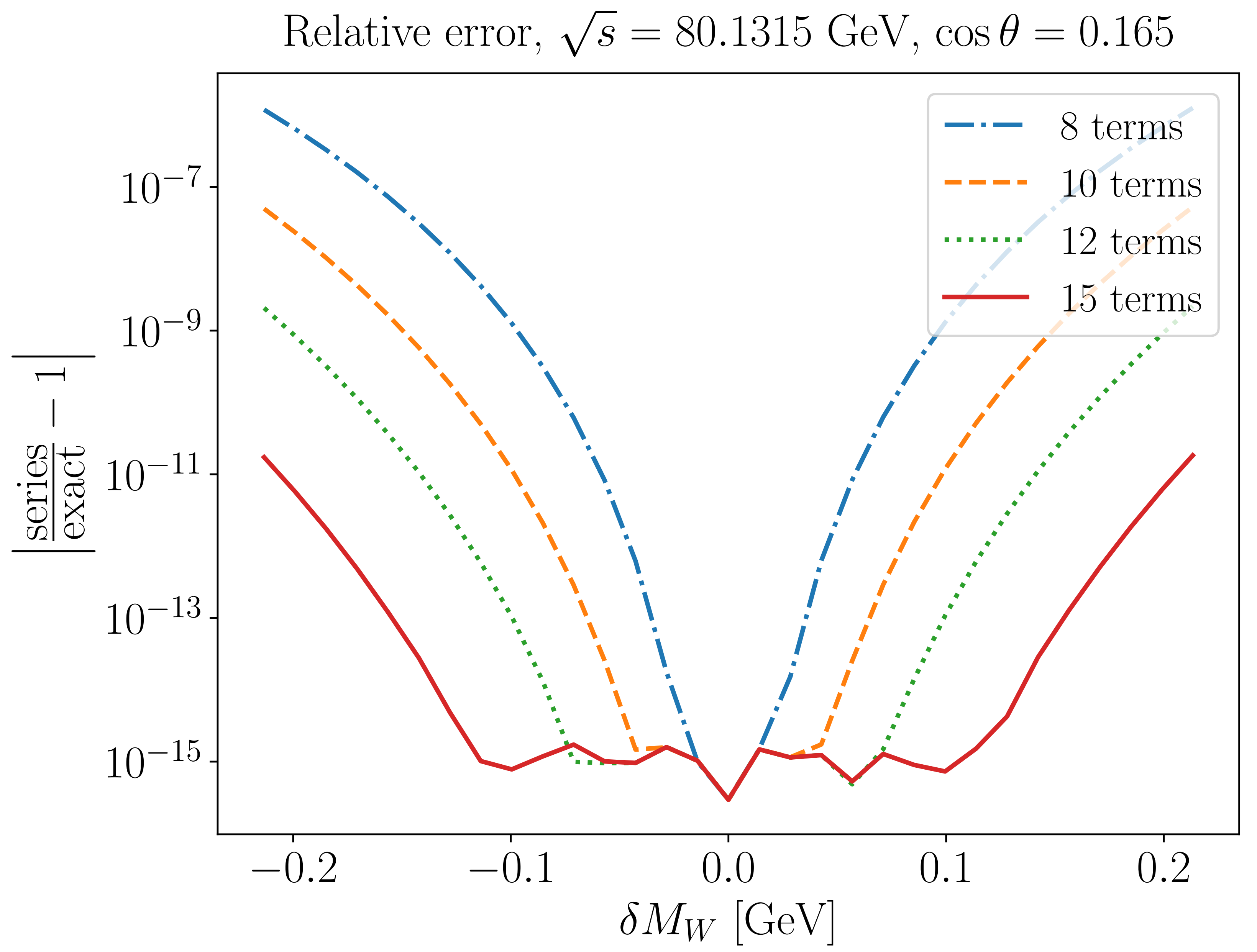
- $\mathcal{O}(10^2)$ templates would require an amount of time which is **non feasible**;

Master Integral which depends on μ_W → Differential equation w.r.t μ_W → **Taylor series** in $\delta\mu_W = \mu_W - \bar{\mu}_W$ for each MI

Point of the grid $s = s_0$, $t = t_0$ used as **BC** → Differential equation w.r.t μ_W

- For each point we reconstruct **a posteriori** the μ_W dependence;
- The calculation of the $\delta\mu_W$ expansion for the entire grid took ~ 1.5 days.



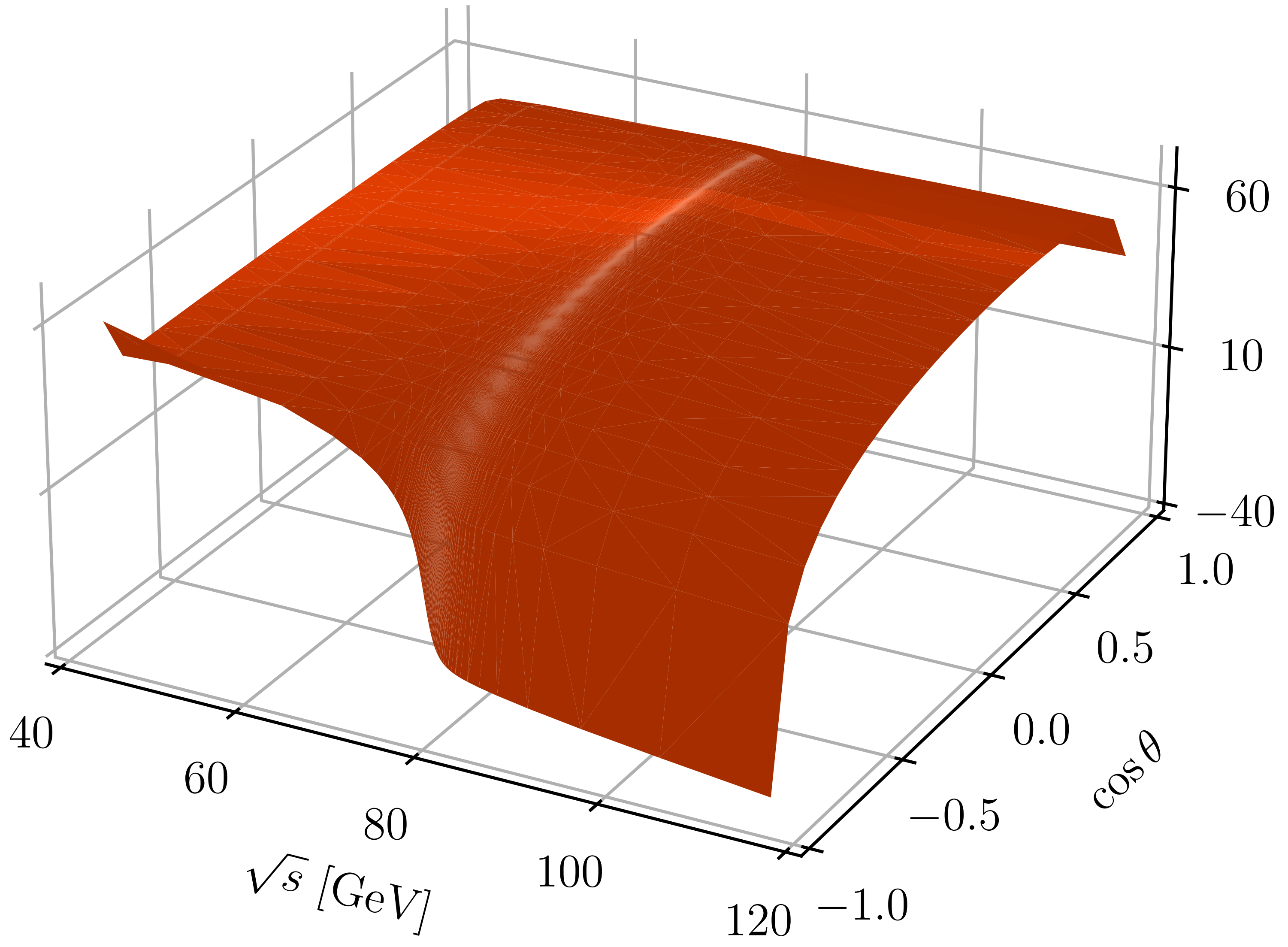


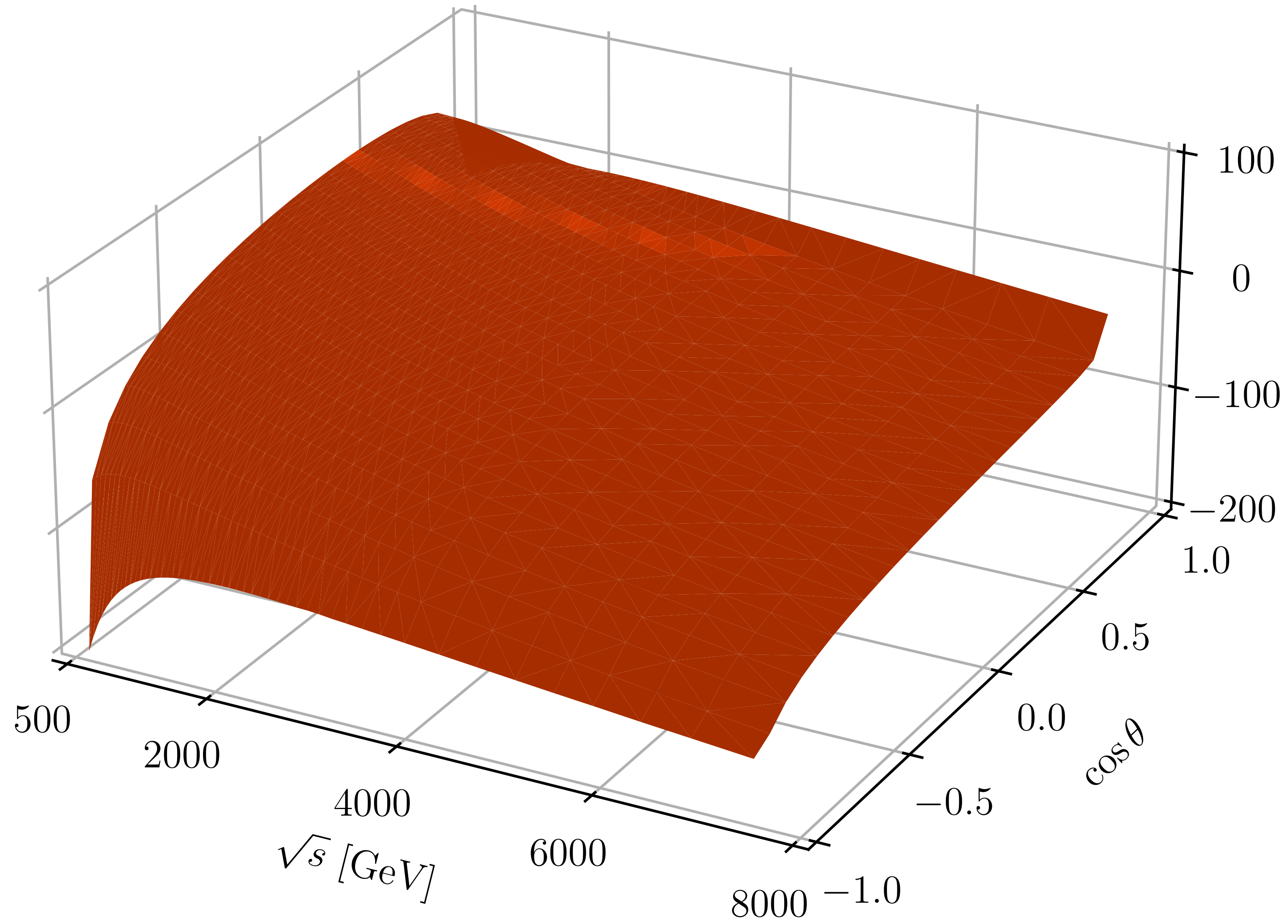
The hard function

- We present our final result in the form of the **hard function** $H^{(1,1)}$, which can be passed to a Monte-Carlo generator, e.g. **MATRIX**

$$H^{(1,1)} = \frac{1}{16}\left[2\mathrm{Re}\left(\frac{\langle \mathcal{M}^{(0,0)} | \mathcal{M}^{(1,1)}_{fin}\rangle}{\langle \mathcal{M}^{(0,0)} | \mathcal{M}^{(0,0)}\rangle}\right)\right]$$

- We can interpolate the value of $H^{(1,1)}$ in the entire phase-space. Thanks to its smoothness the error is, at worst, at the 10^{-3} level.

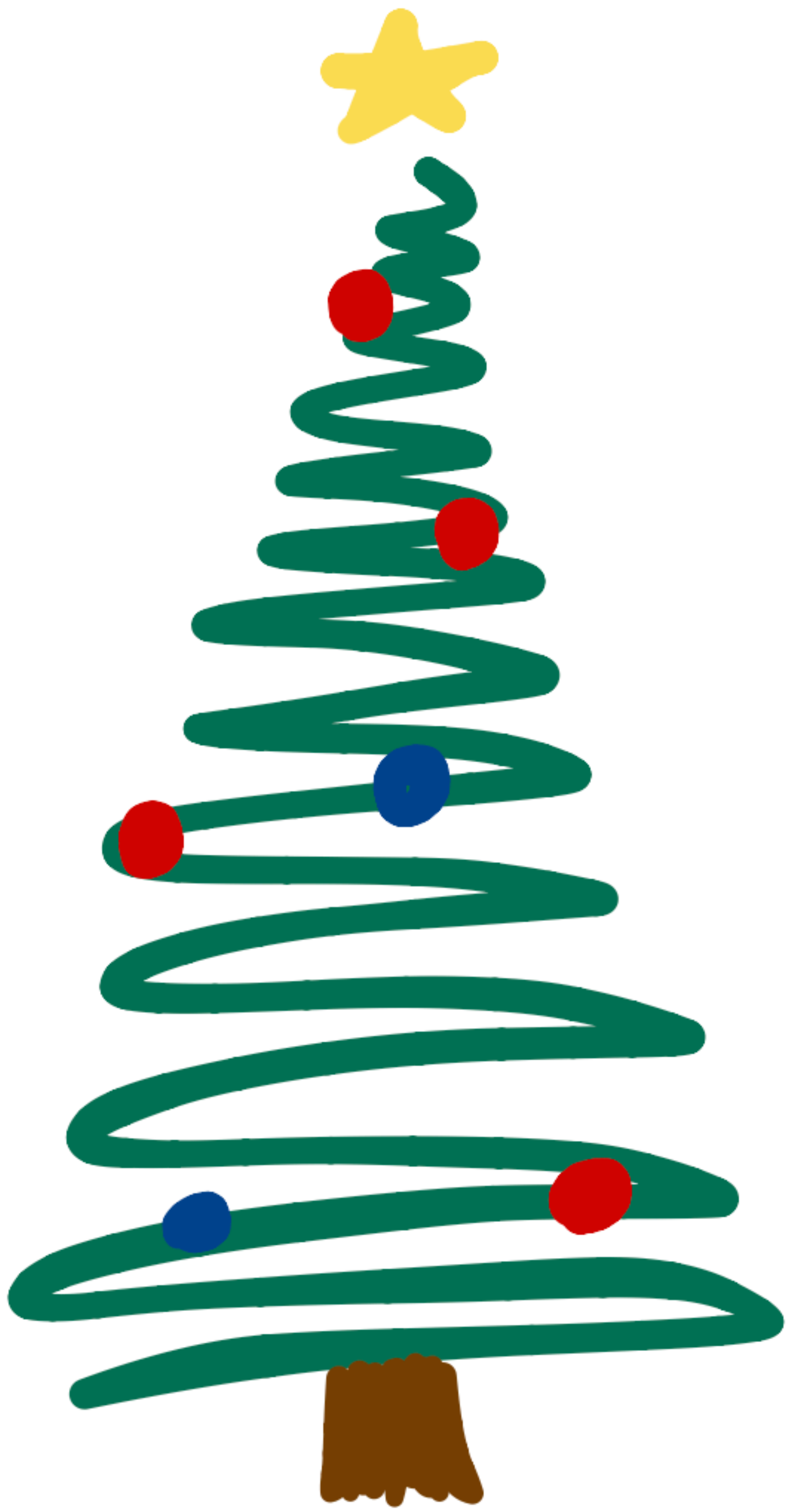




Summary & Outlook

- We presented the calculation of the pure **virtual contribution to the mixed QCD-EW corrections to Charged-current Drell-Yan**;
- The results have been obtained thanks to an **high level of automation** of every step of the calculation. In particular, concerning the evaluation of the Master Integrals. The latter has been carried out within the semi-analytical framework offered by **SeaSyde**;
- We showed how the semi-analytical framework could be exploited to provide numerical grids retaining the **exact dependence on the W mass**;
- When included in the **MATRIX framework**, for the evaluation of the fiducial cross sections, these results will allow a consistent simultaneous analysis of both NC and CC DY processes at NNLO QCD-EW level;
- Finally, the techniques employed in this calculation are completely general, and can be applied to **other relevant 2->2 process** at **NNLO QCD-EW** level or even **NNLO EW**.

THANK YOU

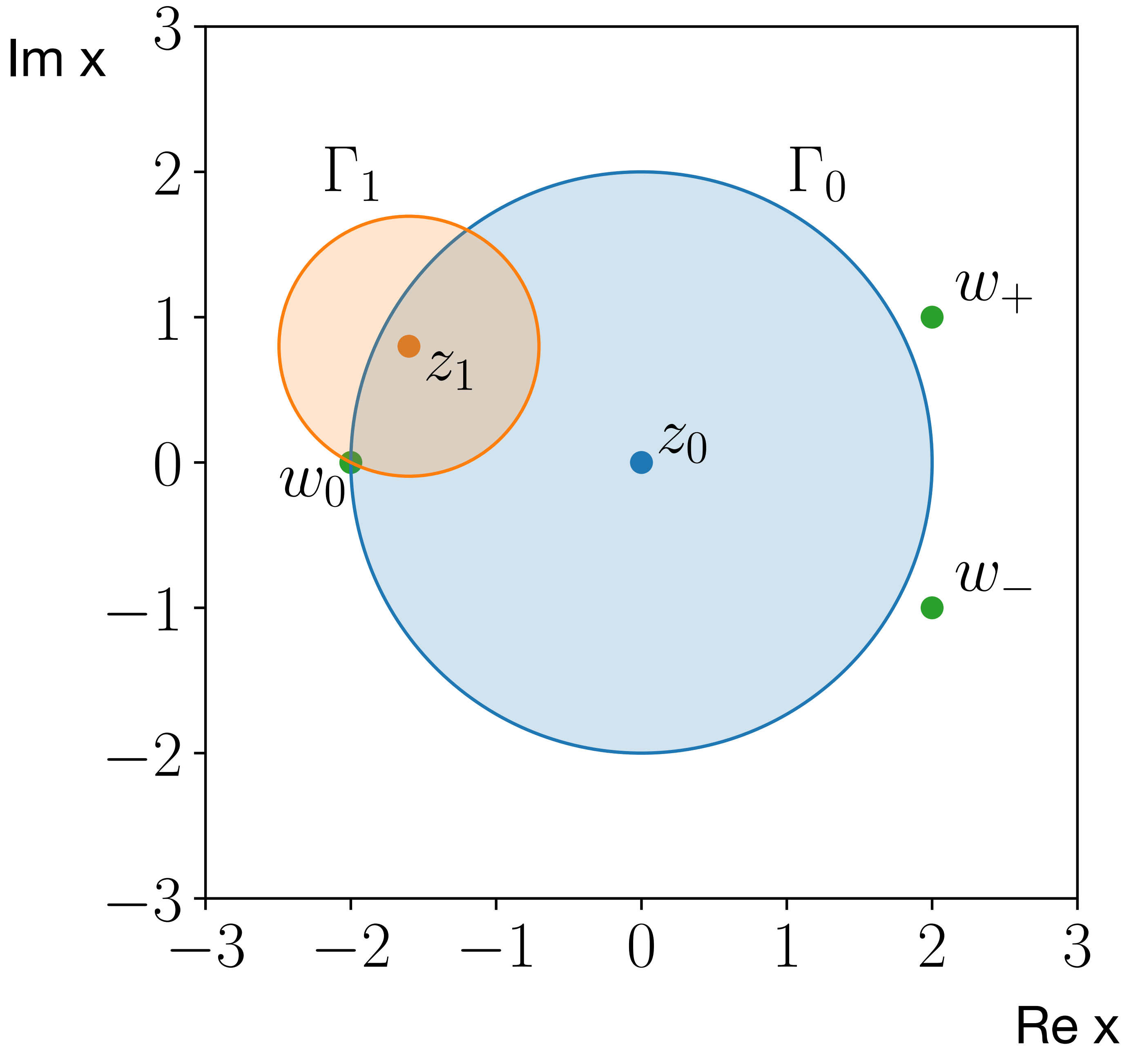


SeaSyde

[**TA**, R. Bonciani, S. Devoto, N.Rana, A.Vicini, arXiv:2205.03345]

https: //github.com/TommasoArmadillo/SeaSyde

- Power series have a limited **radius of convergence** which is determined by the position of the **nearest singularity**.
- We need to be able to extend the solution beyond the radius of convergence, to the entire **complex plane**.



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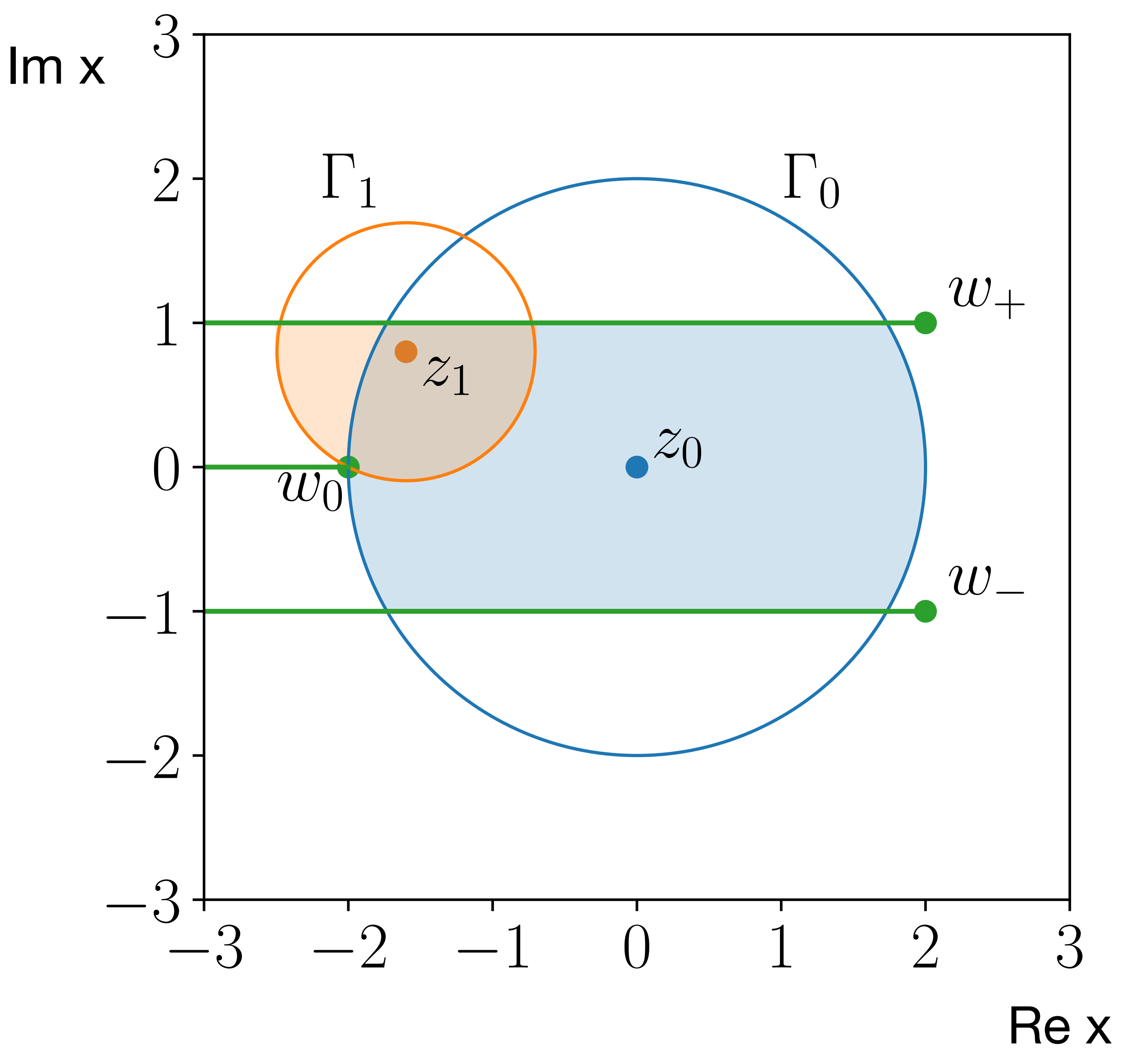

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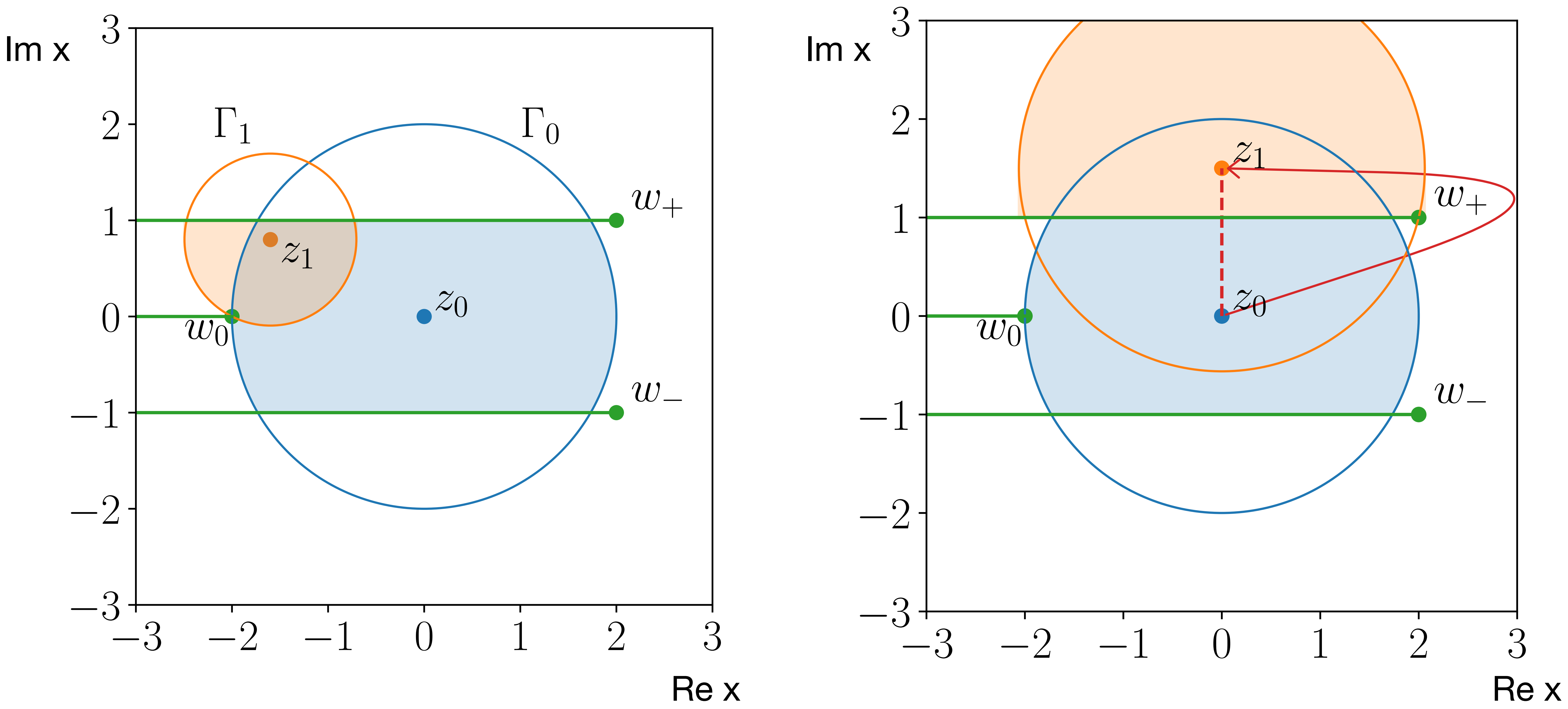
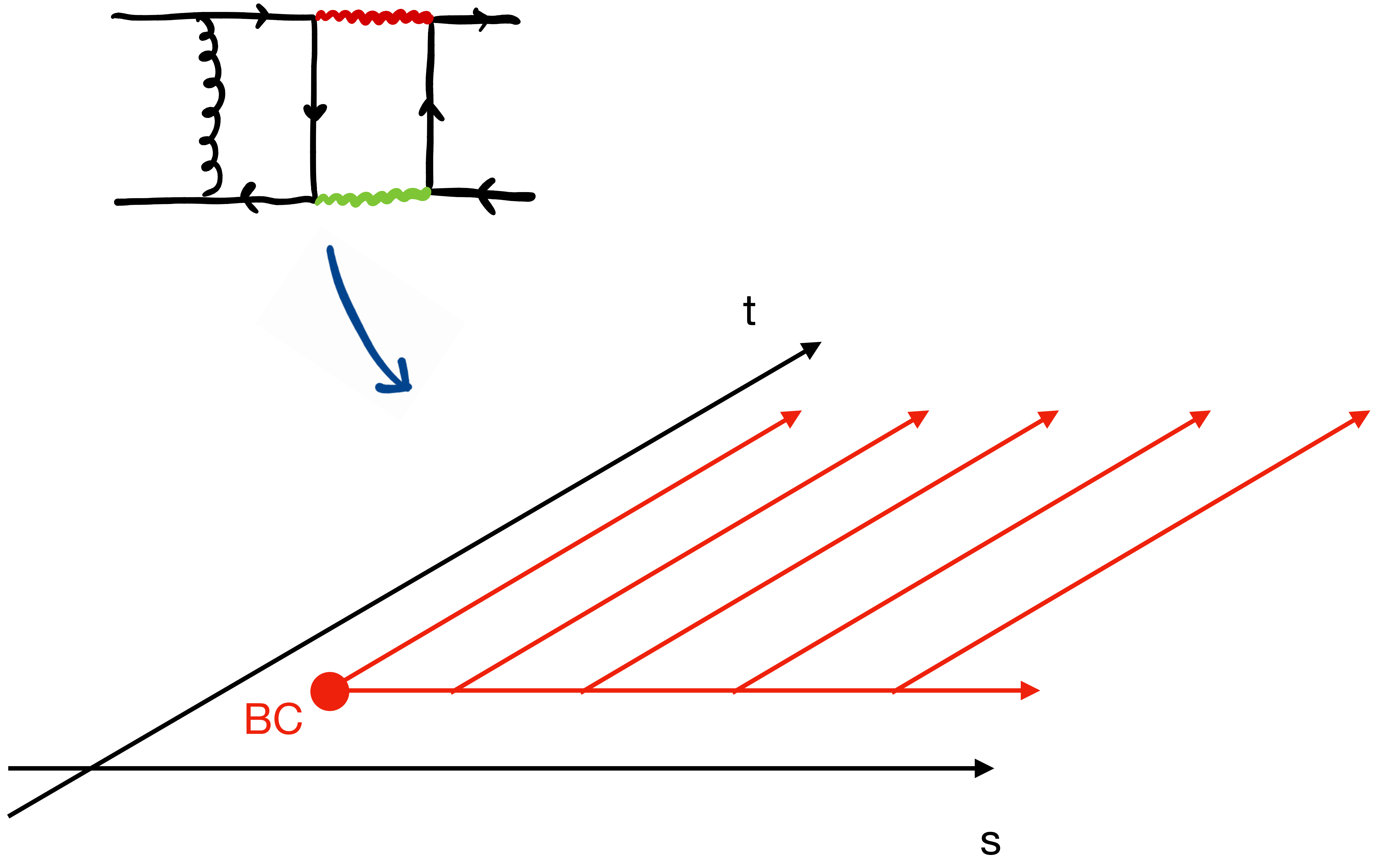


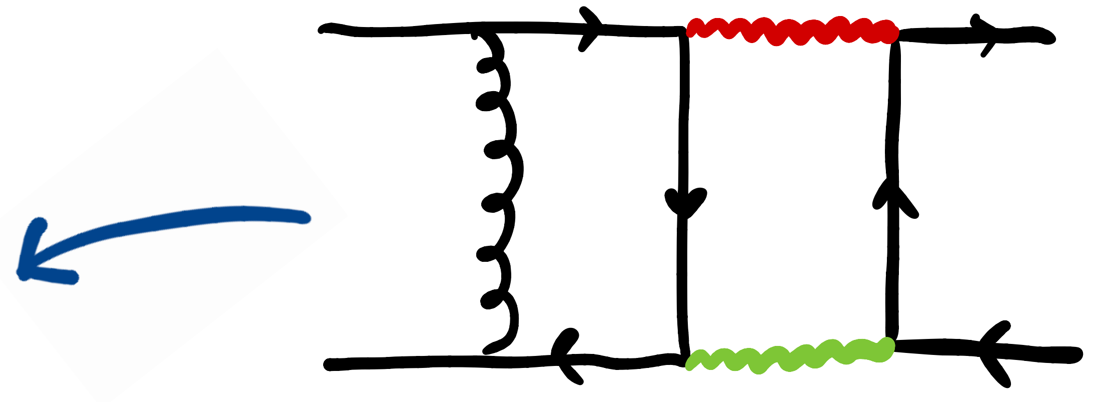
*For simplicity, we are not showing all the intermediate circles.

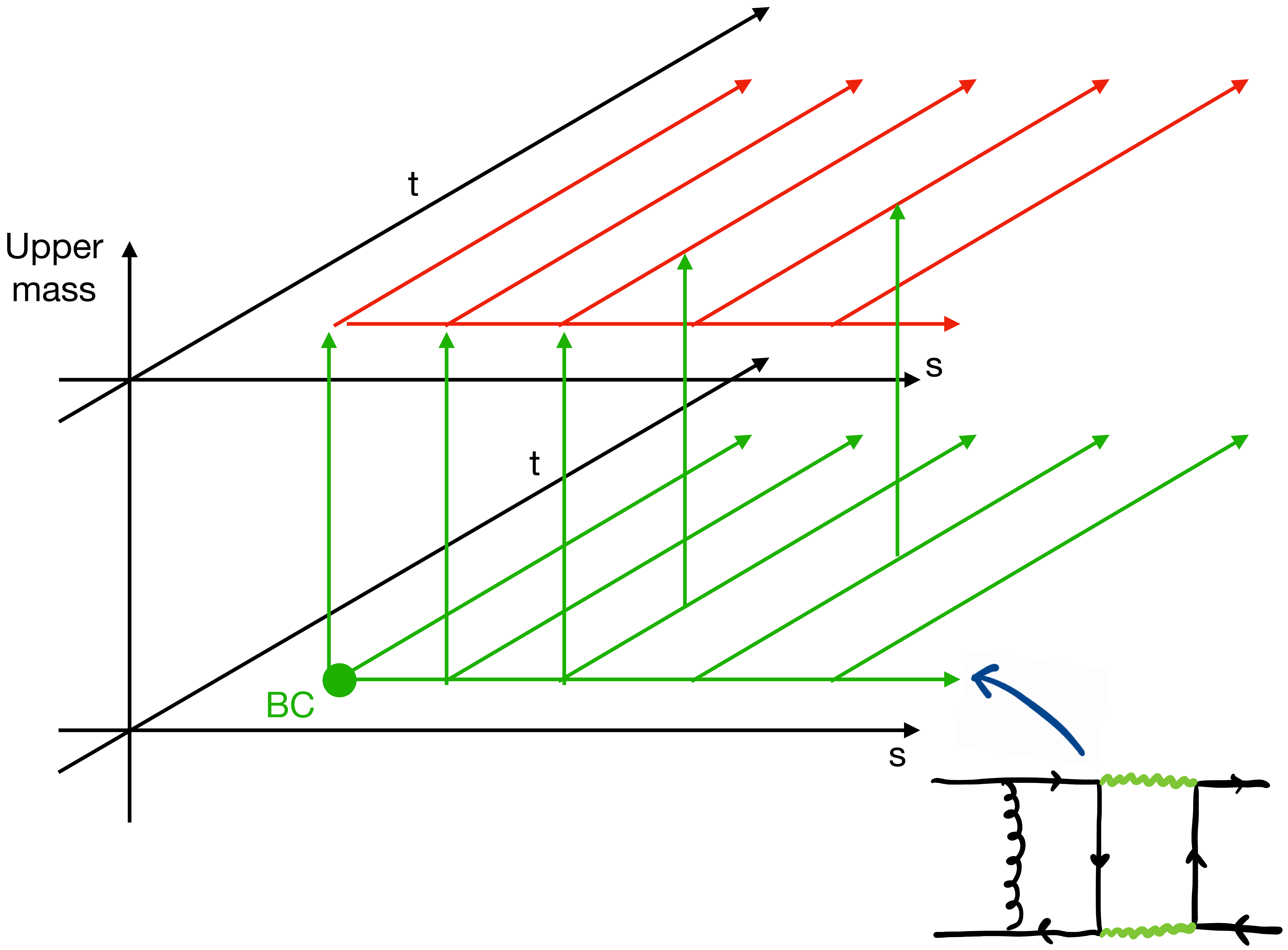
Creating a grid



- This approach is completely general and **easy to automate**;
- We have to solve a 56x56 system of differential equations w.r.t. to the Mandelstam variables s and t;
- Since we are not putting the system in canonical form, these are usually quite complicated and the solution might require some time;
- The computation of a grid with 3250 points required ∼3 weeks on 26 cores.

Mass evolution





- We can re-use the grid from the **Neutral-current Drell-Yan**;
- We have to solve a **36x36** system of differential equations w.r.t. to the Mandelstam variables s and t;
- Then, for every point, we have to solve a **56x56, but easier, system** w.r.t. one mass;
- We used this as a **cross-check**.

Background-Field Method (BGF)

- We chose to perform the calculation using the **background-field method**:

$$\mathscr{L}_{SM} = \mathscr{L}_C(\hat{V} + V) + \mathscr{L}_{GF}(V) + \mathscr{L}_{FP}$$

$$\mathscr{L}_C = \mathscr{L}_{YM} + \mathscr{L}_H + \mathscr{L}_F$$

- The fields are split into background fields $\hat{V}$ and quantum ones V;
- The quantum fields are the variables of integration in the functional integral, i.e. they appear only in loops.
- Even though we have more fields, the expressions are usually simpler

- $\mathscr{L}_{GF}(V)$ breaks gauge-invariance only of the quantum fields, for this reasons very simple and **QED-like Ward identities** are satisfied at any order in perturbation theory for the background ones.

SeaSyde

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A SIMPLE EXAMPLE

$$\begin{cases} f'(x) + \frac{1}{x^2 - 4x + 5} f(x) = \frac{1}{x+2} \\ f(0) = 1 \end{cases}$$

$$f_{hom}(x) = x^r \sum_{k=0}^{\infty} c_k x^k$$

$$f_{hom}(x) = 5 - x - \frac{3}{10}x^2 + \frac{11}{150}x^3 + \ldots$$

$$\begin{cases} rc_0 = 0 \\ \frac{1}{5}c_0 + c_1(r+1) = 0 \\ \frac{4}{25}c_0 + \frac{1}{5}c_1 + c_2(2+r) = 0 \\ \frac{11}{125}c_0 + \frac{4}{25}c_1 + \frac{1}{5}c_2 + c_3(3+r) = 0 \\ \ldots \end{cases}$$

SEASYDE

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SEASYDE

A SIMPLE EXAMPLE

$$\begin{cases} f'(x) + \dfrac{1}{x^2 - 4x + 5} f(x) = \dfrac{1}{x+2} \\ f(0) = 1 \end{cases}$$

$$f_{part}(x) = f_{hom}(x) \int_0^x dx' \, \frac{1}{(x'+2)} \, f_{hom}^{-1}(x')$$
$$= \frac{1}{2}x - \frac{7}{40}x^2 + \frac{2}{75}x^3 + \dots$$

$$f(x) = c\, f_{hom}(x) + f_{part}(x)$$
$$= 1 + \frac{3}{10}x - \frac{47}{200}x^2 + \frac{3}{250}x^3 + \dots$$

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SEASYDE

A SIMPLE EXAMPLE

$$\begin{cases} f'(x) + \dfrac{1}{x^2 - 4x + 5} f(x) = \dfrac{1}{x+2} \\ f(0) = 1 \end{cases}$$

$$\begin{aligned} f(x) &= c\, f_{hom}(x) + f_{part}(x) \\ &= 1 + \frac{3}{10}x - \frac{47}{200}x^2 + \frac{3}{250}x^3 + \dots \end{aligned}$$

- This procedure can be generalised to **systems of differential equations**;
- The method has been firstly implemented in the Mathematica package **DiffExp** for a **real kinematic variable** [F.Moriello, arXiv:1907.13234], [M.Hidding, arXiv:2006.05510]
- The great advantage of this approach is that we can reach **arbitrary precision** just by adding more terms in the serie

Recent developments

Theoretical developments:

- **2-loop virtual Master Integrals with internal masses:** [U. Aglietti, R. Bonciani, arXiv:0304028, arXiv:0401193],[R. Bonciani, S. Di Vita, P. Mastrolia, U. Schubert, arXiv:1604.08581], [M.Heller, A.von Manteuffel, R.Schabinger arXiv:1907.00491], [M.Long,R,Zhang,W.Ma,Y,Jiang,L.Han,,Z.Li,S.Wang, arXiv:2111.14130],[X.Liu, Y.Ma, arXiv:2201.11669]
- **Altarelli-Parisi splitting functions including QCD-QED effects** [D. de Florian, G. Sborlini, G. Rodrigo, arXiv:1512.00612]
- **Renormalisation** [G.Degrassi, A.Vicini, hep-ph/0307122],[S.Dittmaier,T.Schmidt,J.Schwarz, arXiv:2009.02229], [S.Dittmaier, arXiv:2101.05154]

On-shell Z and W production:

- **pole approximation of the NNLO QCD-EW corrections** [S.Dittmaier, A.Huss, C.Schwinn, arXiv:1403.3216, 1511.08016]
- **analytical total Z production cross section including NNLO QCD-QED corrections** [D. de Florian, M.Der, I.Fabre, arXiv:1805.12214]
- **fully differential on-shell Z production including exact NNLO QCD-QED corrections** [M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:1909.08428] [S.Hasan, U.Schubert, arXiv:2004.14908]
- **analytical total Z production cross section including NNLO QCD-EW corrections** [R. Bonciani, F. Buccioni, R.Mondini, A.Vicini, arXiv:1611.00645], [R. Bonciani, F. Buccioni, N.Rana, I.Triscari, A.Vicini, arXiv:1911.06200], [R. Bonciani, F. Buccioni, N.Rana, A.Vicini, arXiv:2007.06518, arXiv:2111.12694]
- **fully differential Z and W production including NNLO QCD-EW corrections** [F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2005.10221], [A. Behring, F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2009.10386, 2103.02671]

Complete Drell-Yan:

- **neutrino-pair production including NNLO QCD-QED corrections** [L. Cieri, D. de Florian, M.Der, J.Mazzitelli, arXiv:2005.01315]
- **2-loop amplitudes** [M.Heller, A.von Manteuffel, R.Schabinger, arXiv:2012.05918],[TA, R.Bonciani, S. Devoto, N.Rana, A.Vicini, arXiv:2201.01754]
- **NNLO QCD-EW corrections to neutral-current DY including leptonic decay** [R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, A.Vicini, arXiv:2106.11953],[F.Buccioni, F.Caola, H.Chawdhry, F.Devoto, M.Heller,A.von Manteuffel, K.Melnikov, R.Röntsch, C.Signorile-Signorile, arXiv:2203.11237]
- **NNLO QCD-EW corrections to charged-current DY including leptonic decay (2-loop contributions in pole approximation)**. [L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, F.Tramontano, arXiv:2102.12539]