Unintegrated Gluon Distributions with HELL and Applications with JETHAD

Francesco Giovanni Celiberto

Christmas Meeting

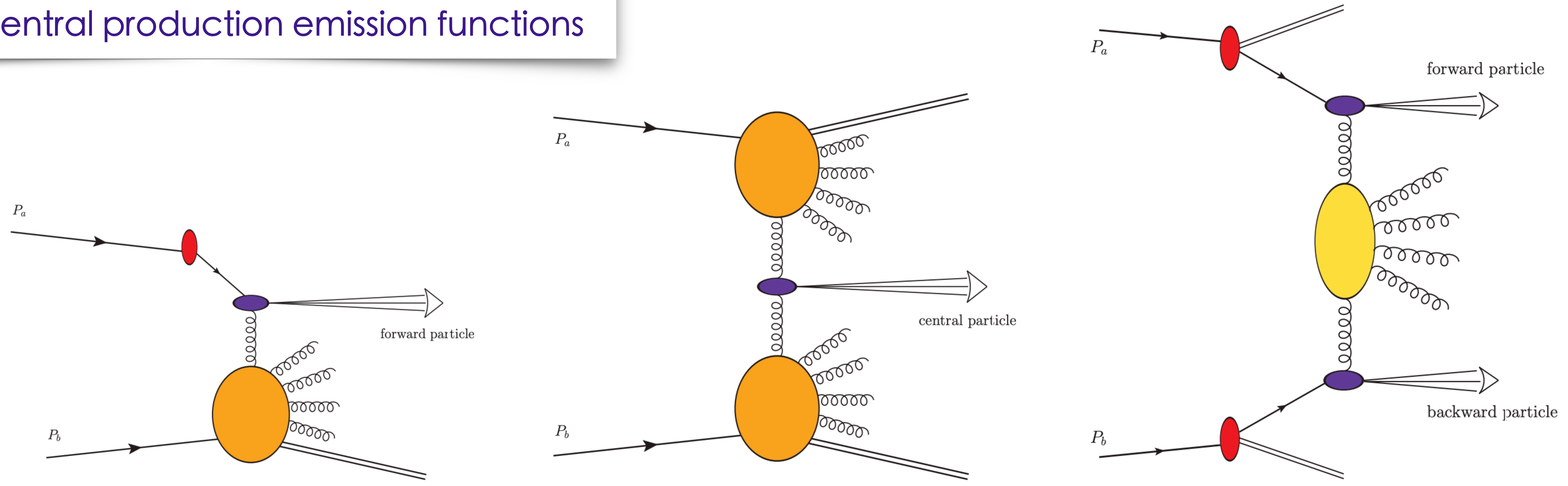
Milan

2024, December 19th

About the last Christmas Meeting...

High-energy factorization at a glance

Singly/double off-shell coefficient functions
Forward/central production emission functions



(a) Single forward (b) Single central (c) Forward-backward

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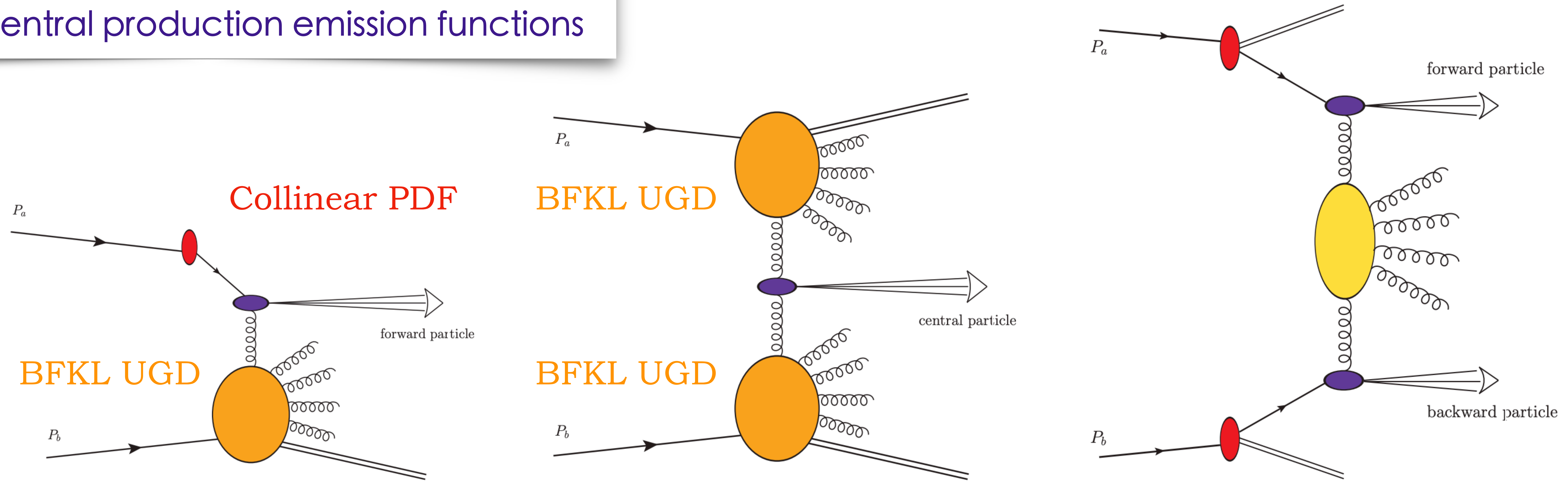

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Fast q/g + small-x g	gg induced
Hybrid factorization	High-energy factorization
BFKL + Threshold	BFKL or small-x improved PDFs

🔗 [M. Bonvini, S. Marzani (2018)]

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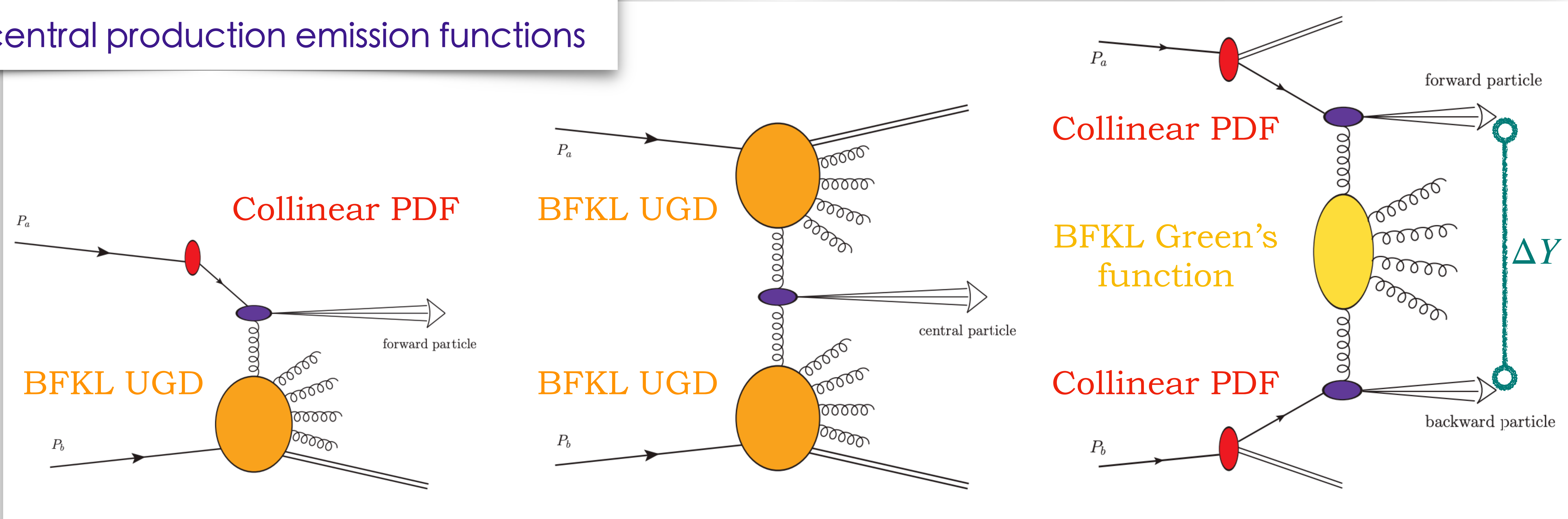
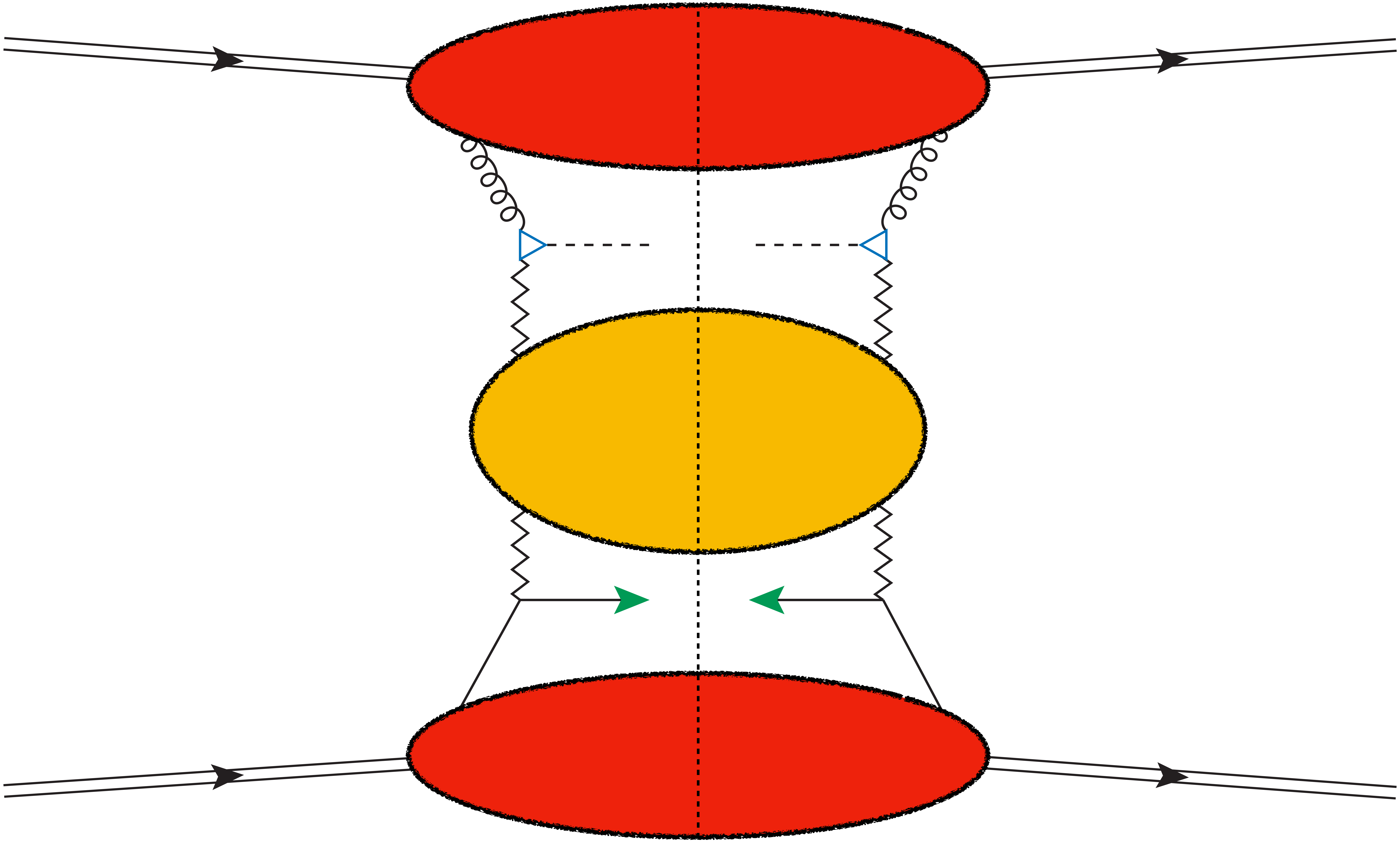


(a) Single forward | (b) Single central | (c) Forward-backward

(a) Single forward	(b) Single central	(c) Forward-backward
Fast q/g + small-x g	gg induced	Large rapidity distances, $\Delta Y \gg 1$
Hybrid factorization	High-energy factorization	High energies, moderate x
BFKL + Threshold	BFKL or small-x improved PDFs	PDFs + t-channel BFKL (NLL/NLO HyF)
	[M. Bonvini, S. Marzani (2018)]	

Anatomy of Higgs + jet in hybrid factorization (HyF)



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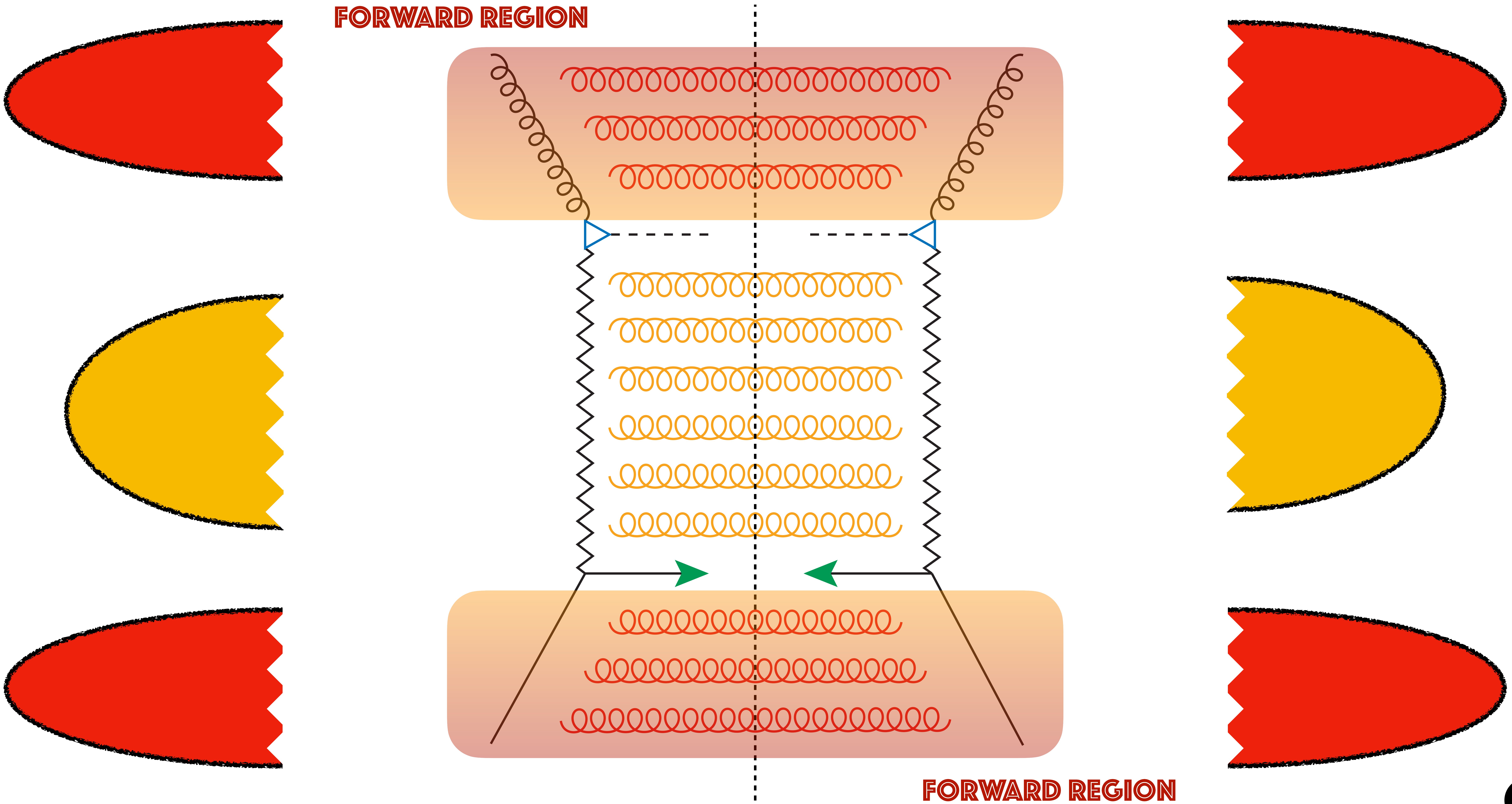

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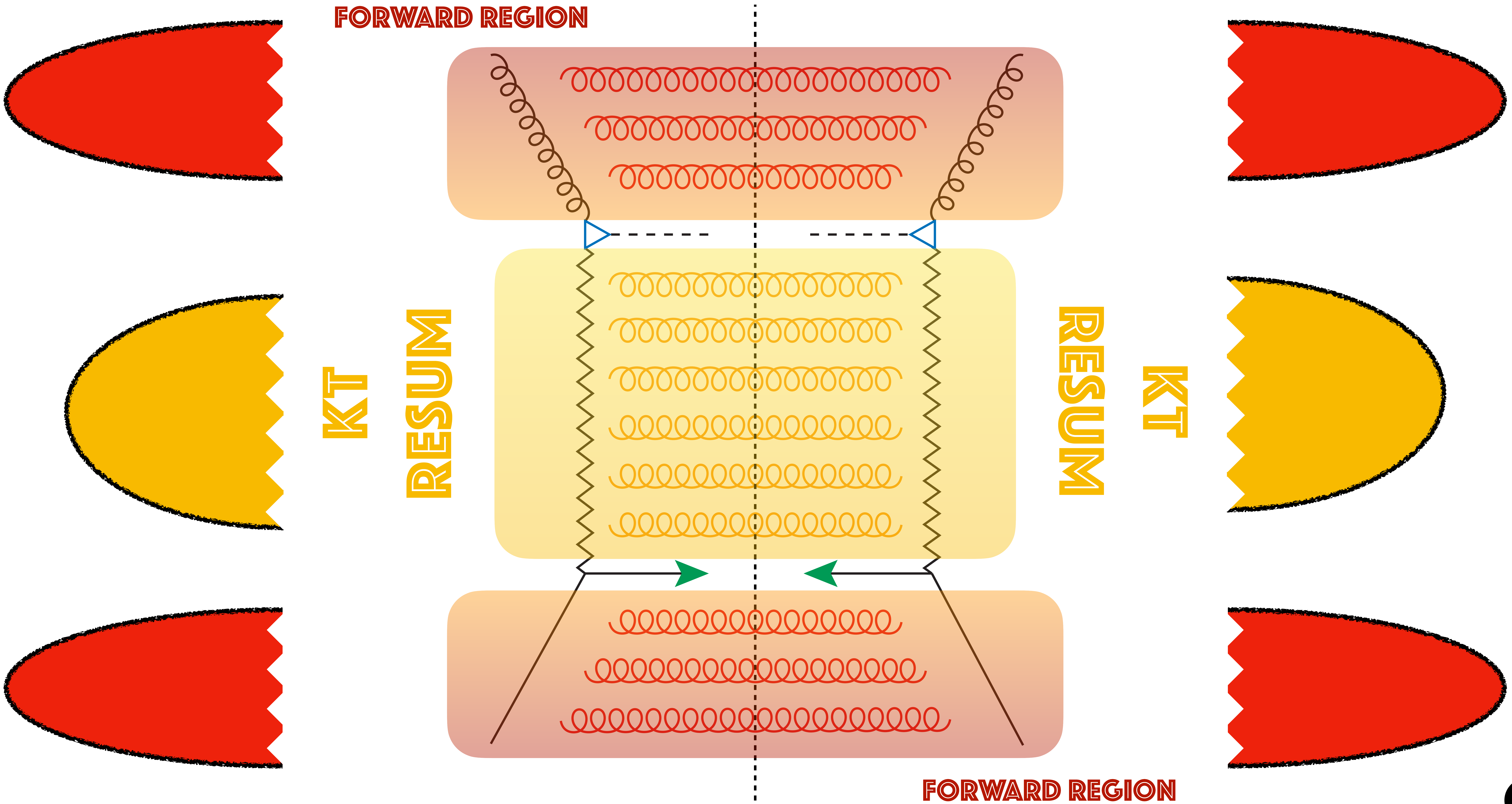


1

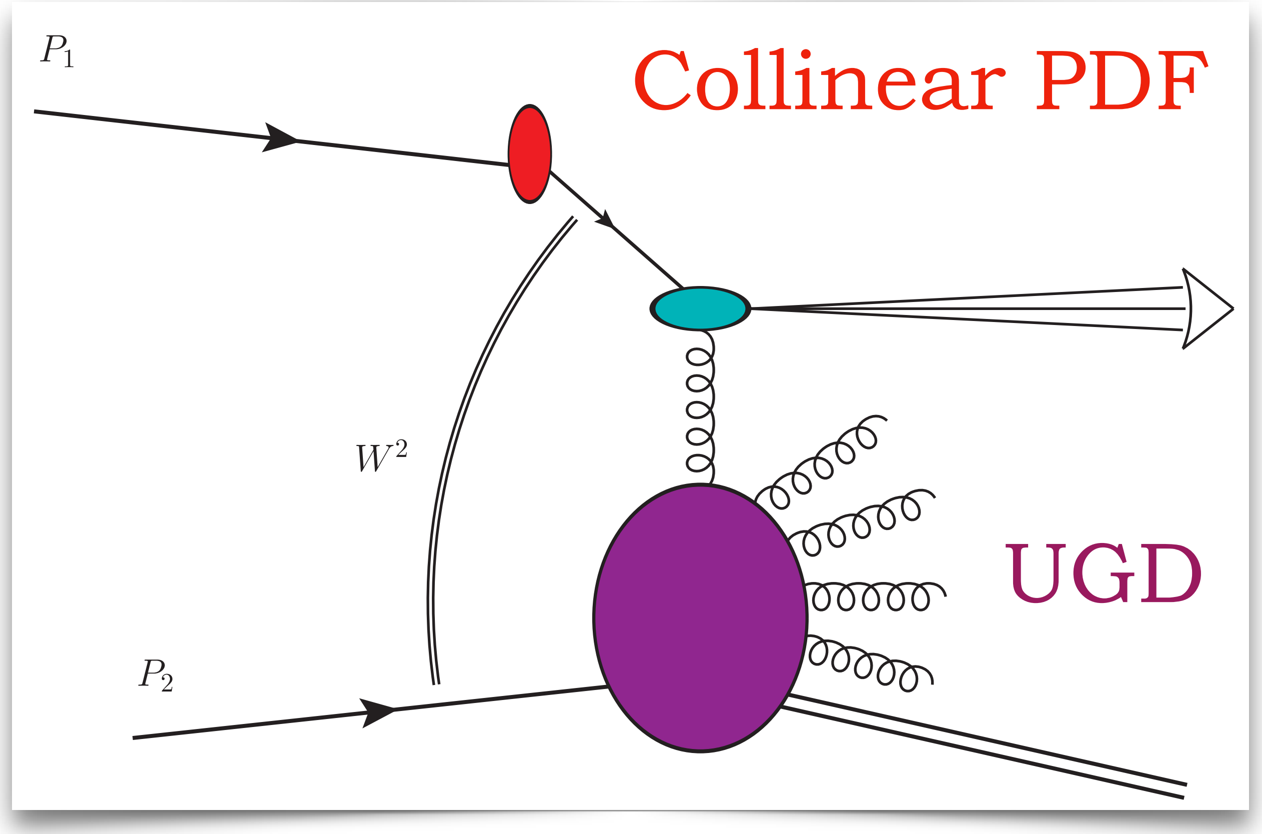
High-energy factorization and the UGD

Hybrid or pure factorization?

Forward emissions

* *Asymmetric* config. $\leftrightarrow$ fast parton + small-x gluon

* Hybrid **high-energy**/**collinear** factorization



* *Distinctive signals* of small-x dynamics **expected**

* Phenomenology:
  forward jet, Drell-Yan, Higgs or vector meson

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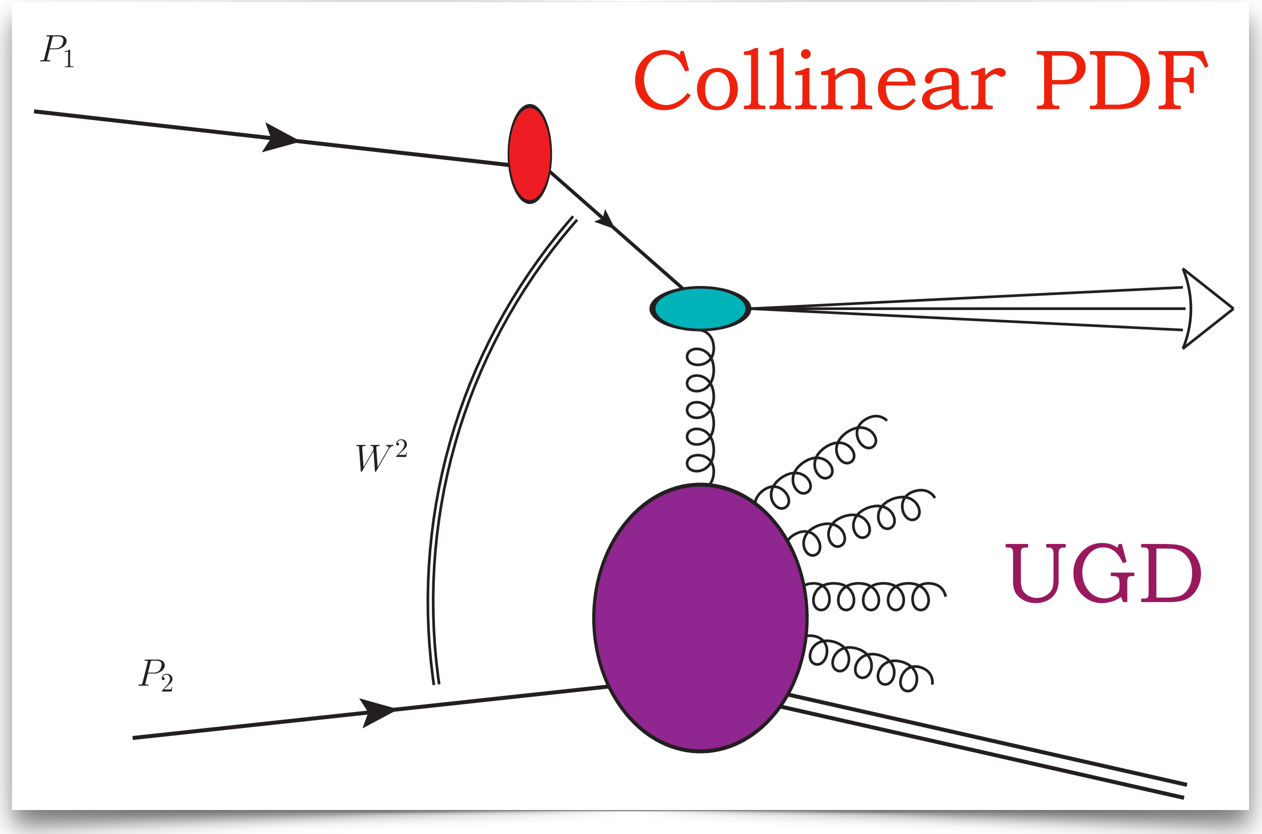

* *Distinctive signals* of small-x dynamics **expected**
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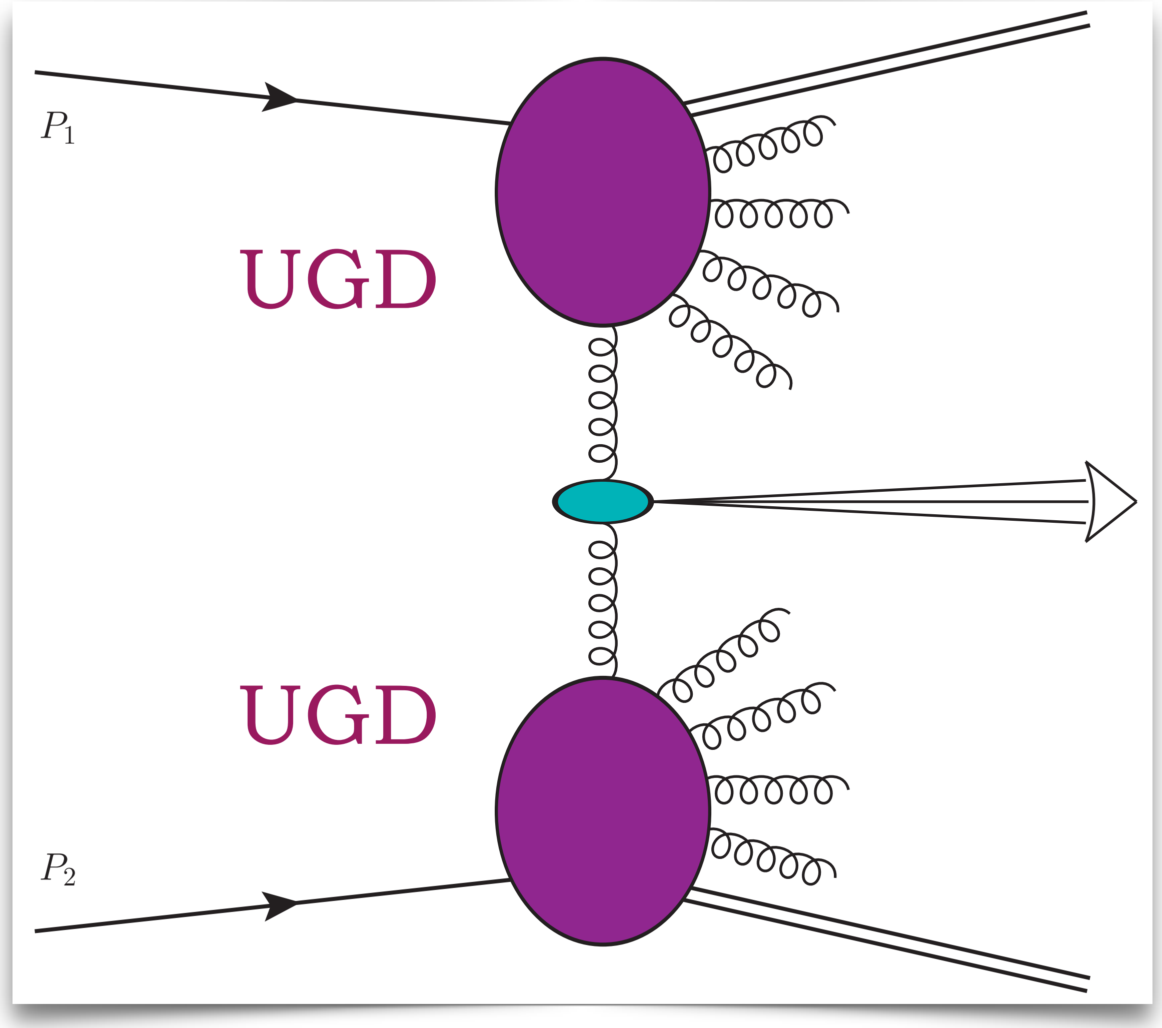
Central emissions

* *Gluon induced* ↔ small-x gluons
* Pure **high-energy** factorization



* Small-x dynamics to **enhance** f.o. description
* Phenomenology:
  central jet, Higgs or vector meson

Exclusive counterparts and ***lepto-hadronic*** channels

Omnes viae small-x ducunt

Incomplete list of small-x formalisms → *linear* (BFKL) or *saturation* (BK/JIMWLK) effects embodied

Unintegrated parton densities

A (hybrid) high-energy factorization established

- **BFKL UGD**: pure small-x evolution, Reggeons
- CCH HEF, CCFM, PRA **uPDFs**; 🔗 small-x **PB TMDs**

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Small-x improved collinear PDFs

DGLAP description improved via BFKL

- **ABF approach**: PDFs + small-x resummed splitting

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Small-x improved gluon TMDs

Nonperturbative content via an enhanced spectator model

* **Pavia model**: initial-scale f_1^g and g_{1L}^g matched to PDFs

🔗 [A. Bacchetta, F.G.C., M. Radici, P. Taels (2020)]

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Helicity and OAM at small-x

Need for sub-eikonal corrections, neglected by BFKL

- **BER**: DLA, flavor singlet and nonsinglet
- **KPS**: evolution via Wilson lines, saturation

CGC/JIMWLK gluon TMDs

Gluon recombination, kinematic + genuine higher twist

- **WW** vs **DP** gluon TMDs, **GTMDs**
- Small-x **iTMD**: interpolating TMD and BFKL regimes

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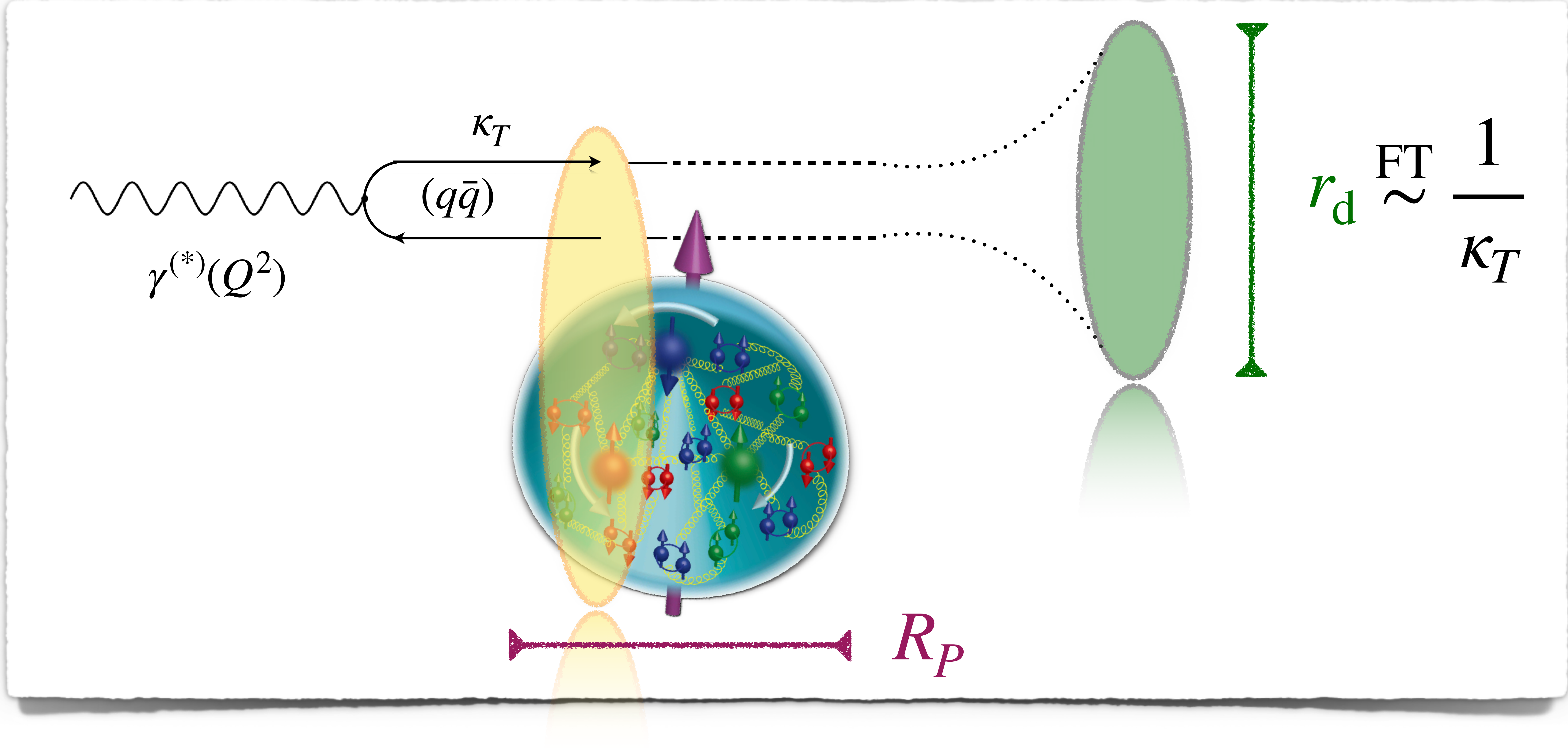
Forward scatterings & color dipoles



$$W_{\mu\nu} \propto \mathrm{Im}\left\{ i \int \mathrm{d}^4x\, e^{i\,q\cdot x} \langle P\,|\, \mathrm{T}\,[J_\mu(x)\, J_\nu(0)]\,|\,P\rangle \right\}$$

- Small-x $\Rightarrow$ ***Ioffe time*** $\gg$ R_P
- At least one J_μ outside proton…
- **…color dipole picture!**

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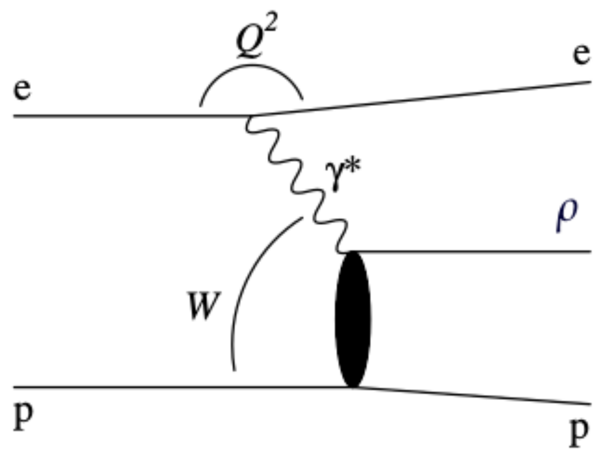
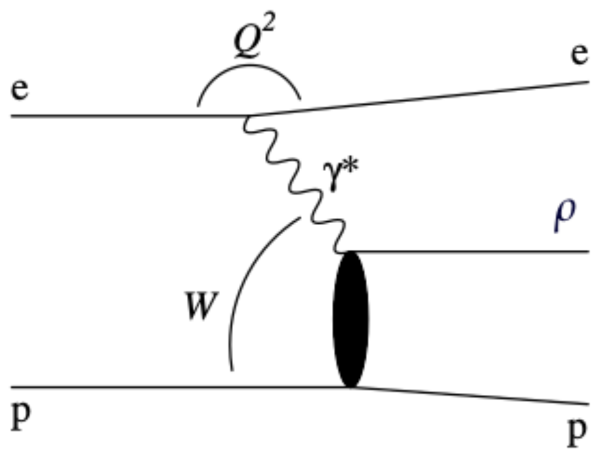
Exclusive ρ meson leptoproduction

Exclusive forward ρ-meson leptoproduction @HERA



- High-energy regime:
  $s \equiv W^2 \gg Q^2 \gg \Lambda^2_{\rm QCD} \implies$ small $x = \frac{Q^2}{W^2}$
- photon virtuality Q is the **hard scale** of the process

▶ **Process solved in helicity** $\implies$ so far **unexplored testfield** for UGD



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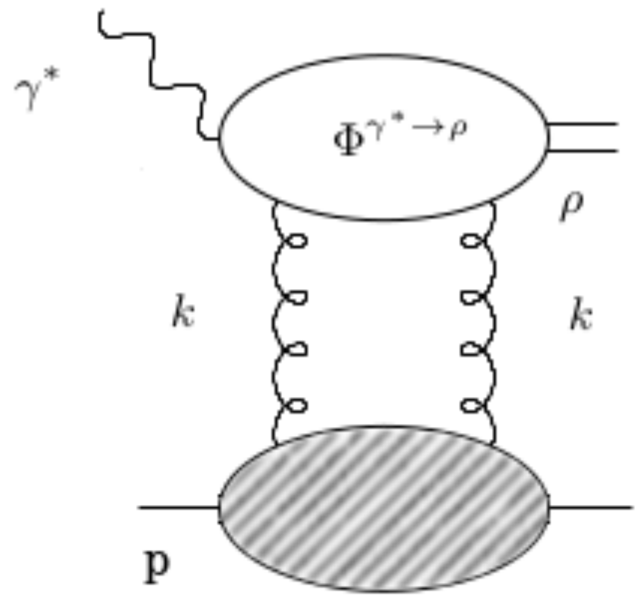
Leading **helicity amplitudes** are known

Assumption:

- $\mathcal{I}m_s \{\mathcal{A}(\gamma^* p \to \rho\, p)\}$
- same W- and t-dependence for T_{11} and T_{00} $\implies$ high-energy factorization
  - $\to$ same physical mechanism, scattering of small transverse size of dipole on the proton target, at work $\Rightarrow$ high-energy factorization

$$T_{\lambda_\rho \lambda_\gamma}(s; Q^2) = is \int \frac{d^2\kappa}{(\kappa^2)^2} \Phi^{\gamma^*(\lambda_\gamma) \to \rho(\lambda_\rho)}(\kappa^2, Q^2)\, \mathcal{F}(x, \kappa^2)\,, \quad x = \frac{Q^2}{s}$$

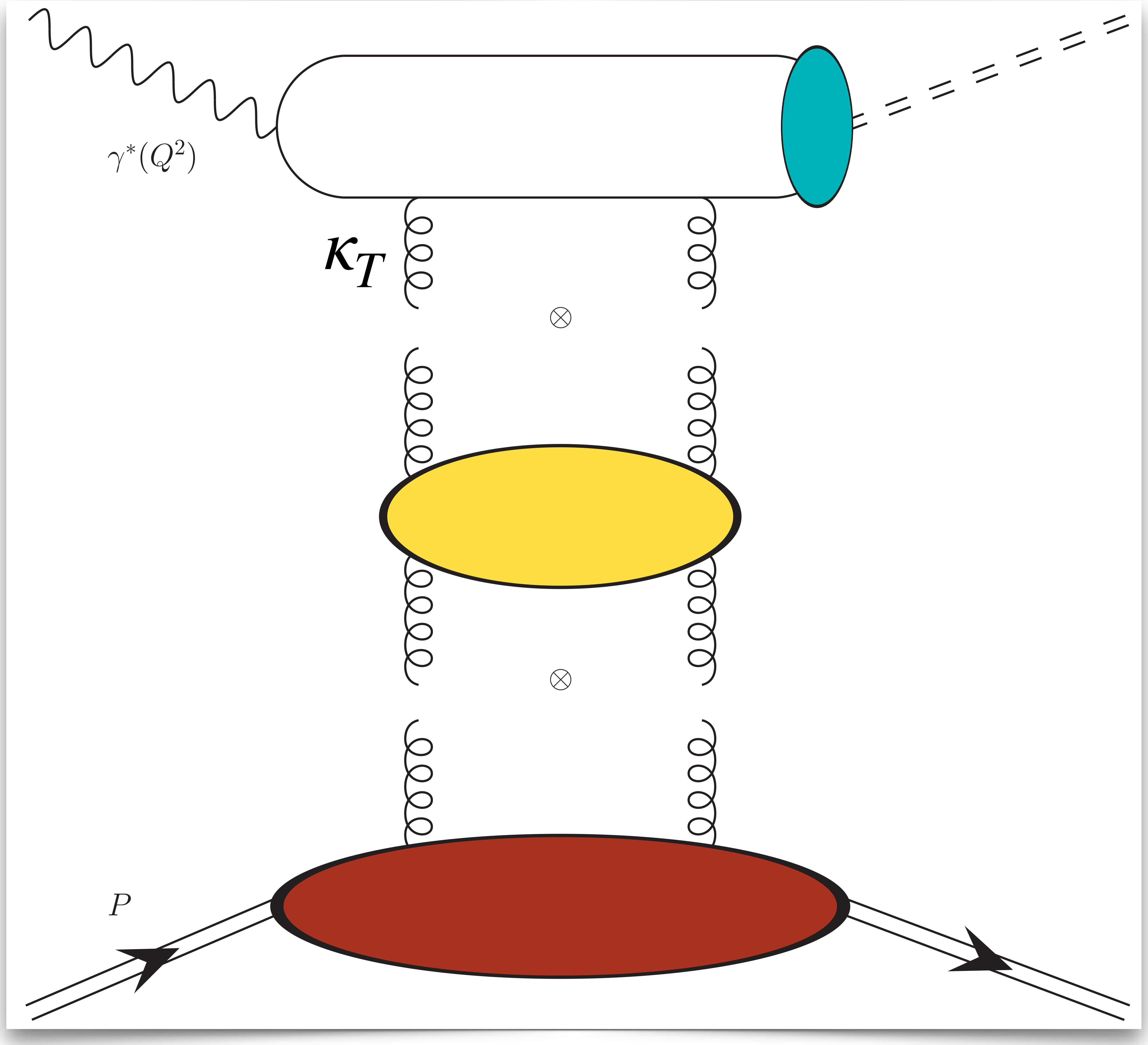


Interesting transitions:

- $\gamma^*_L \to \rho_L \xrightarrow{\text{encoded by}} \Phi^{\gamma^*_L \to \rho_L}$
- $\gamma^*_T \to \rho_T \xrightarrow{\text{encoded by}} \Phi^{\gamma^*_T \to \rho_T}$

$\implies$ **DAs** enter in $\Phi^{\gamma^* \to \rho}$

A factorization…of factorizations



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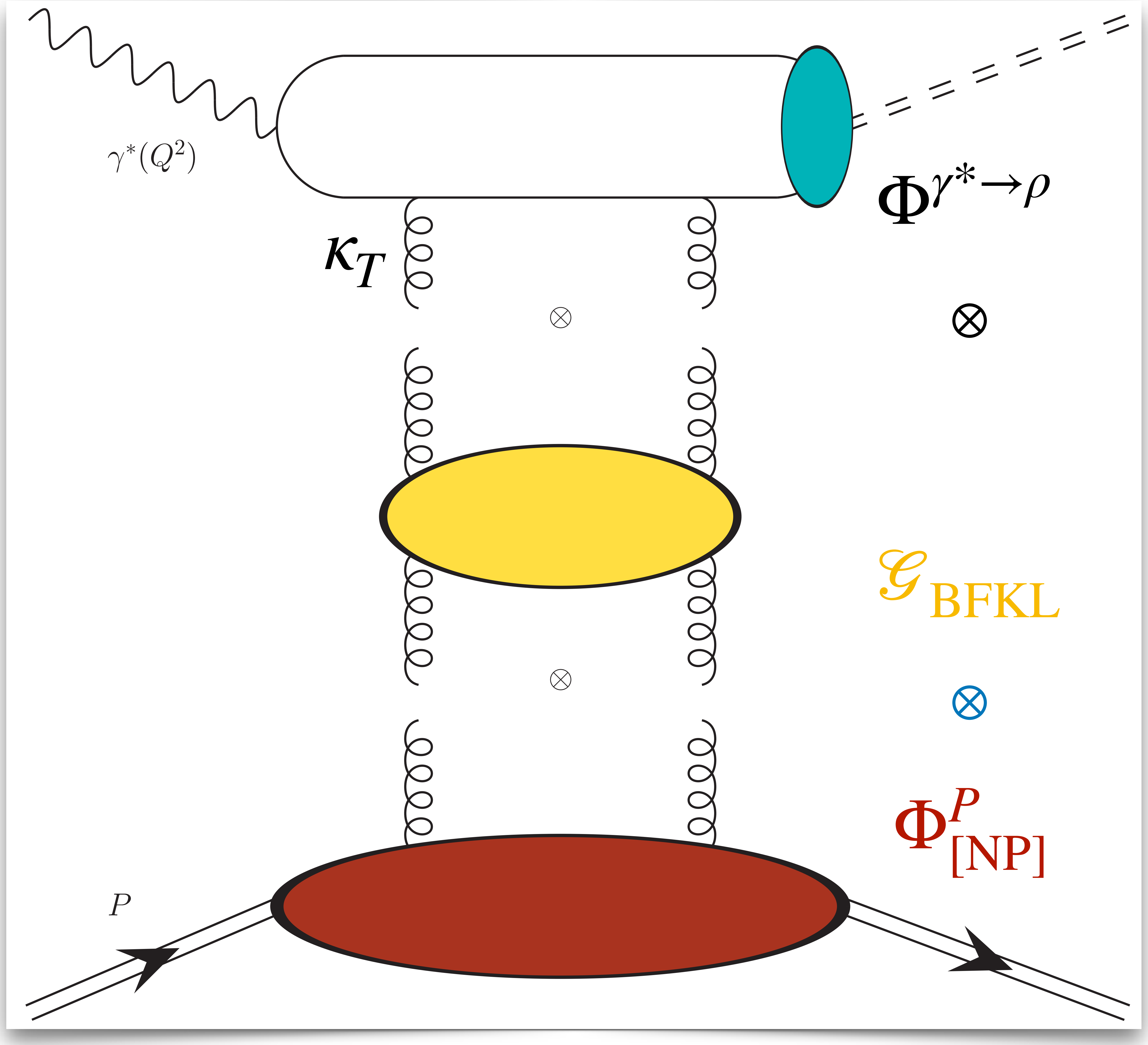

BFKL factorization

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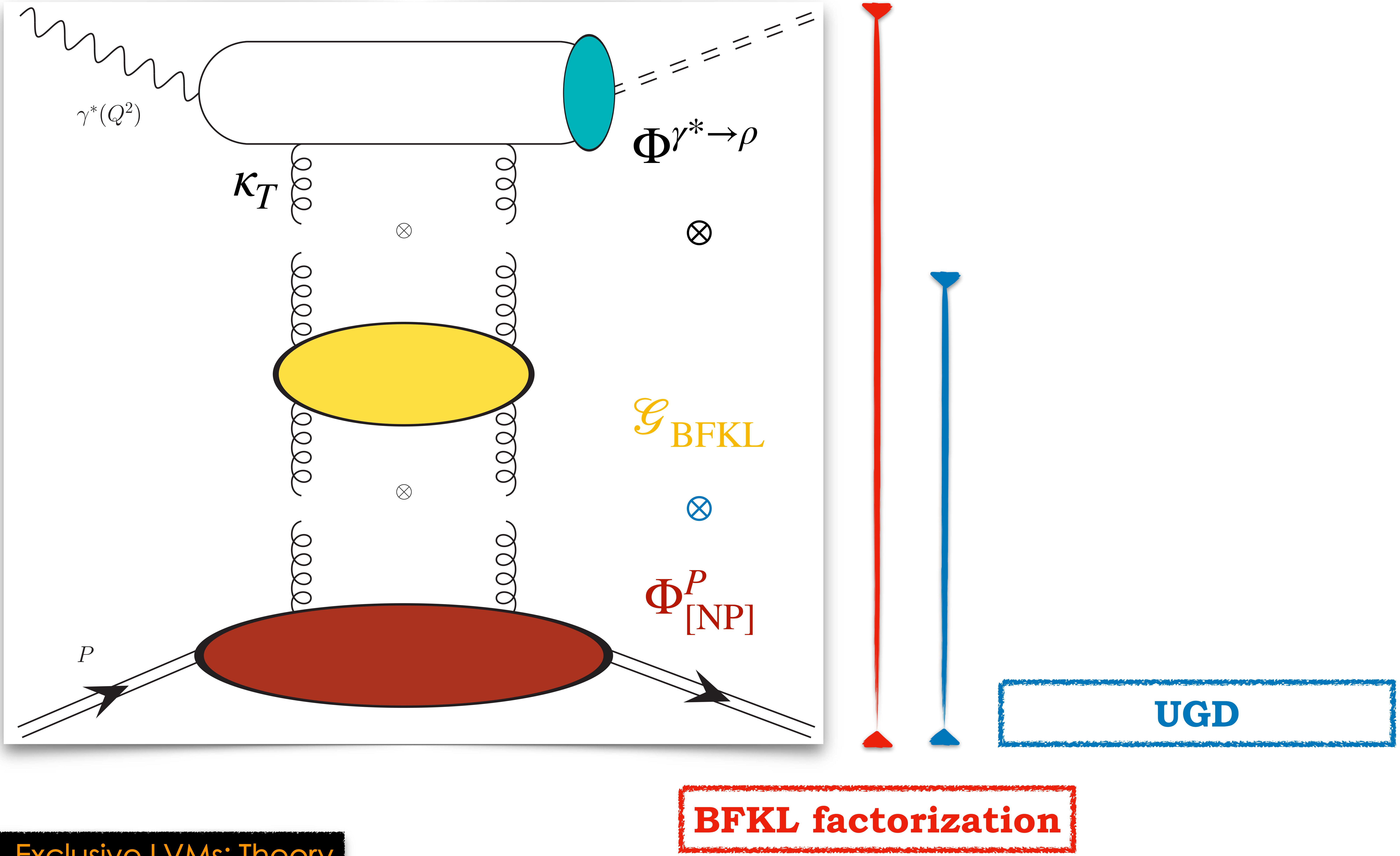

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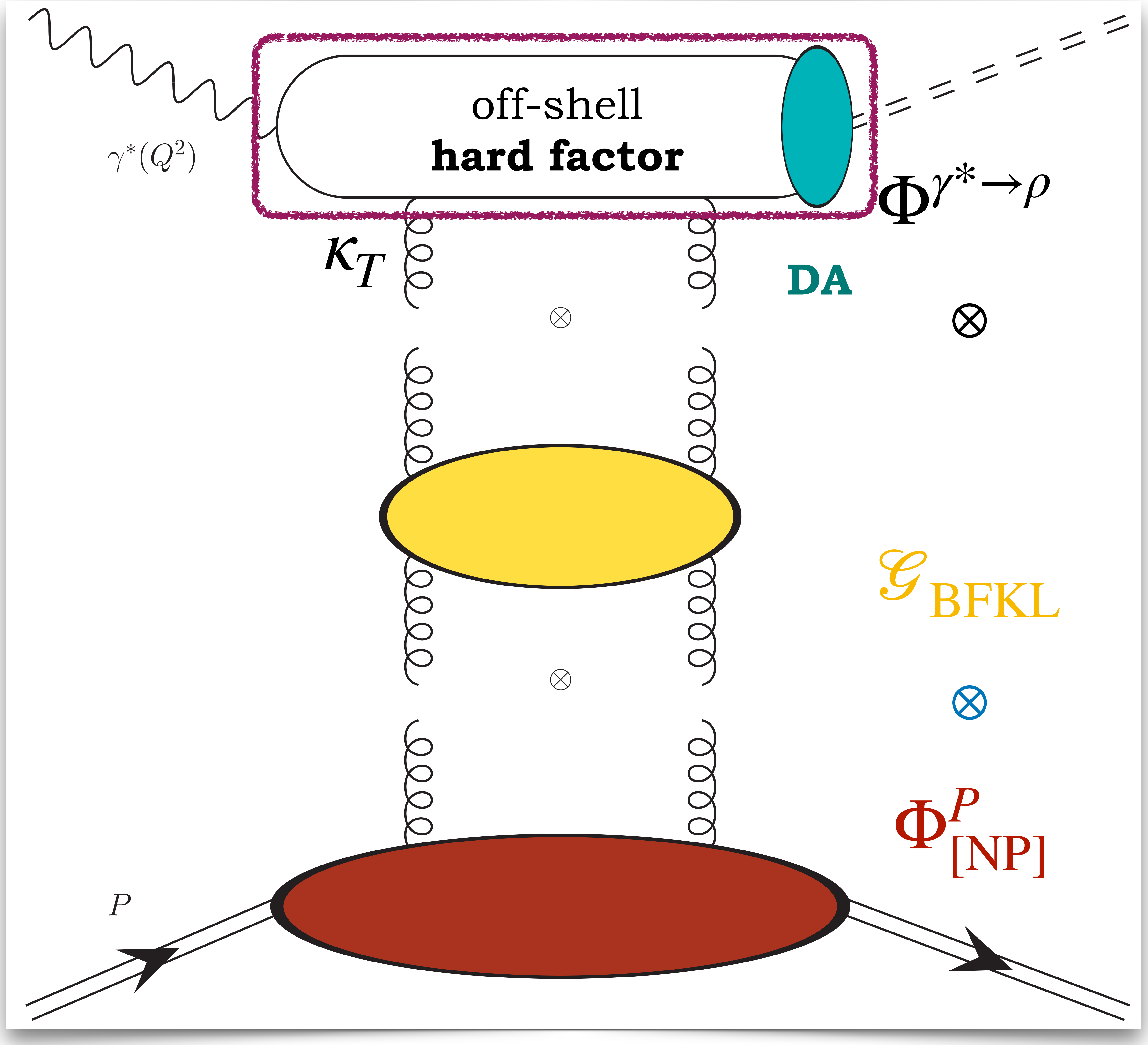

$\gamma \to \rho$ impact factor

$$\Phi^{\gamma^* \to \rho} \propto \int_0^1 \mathrm{d}z \, T_H^{\gamma^* \to \rho}(z, \kappa_T, Q, \mu_R, \mu_F) \, \phi^{\lambda_\rho}(z, \mu_F)$$

Collinear convolution $\Leftrightarrow$ large κ_T (**!**)

UGD

BFKL factorization

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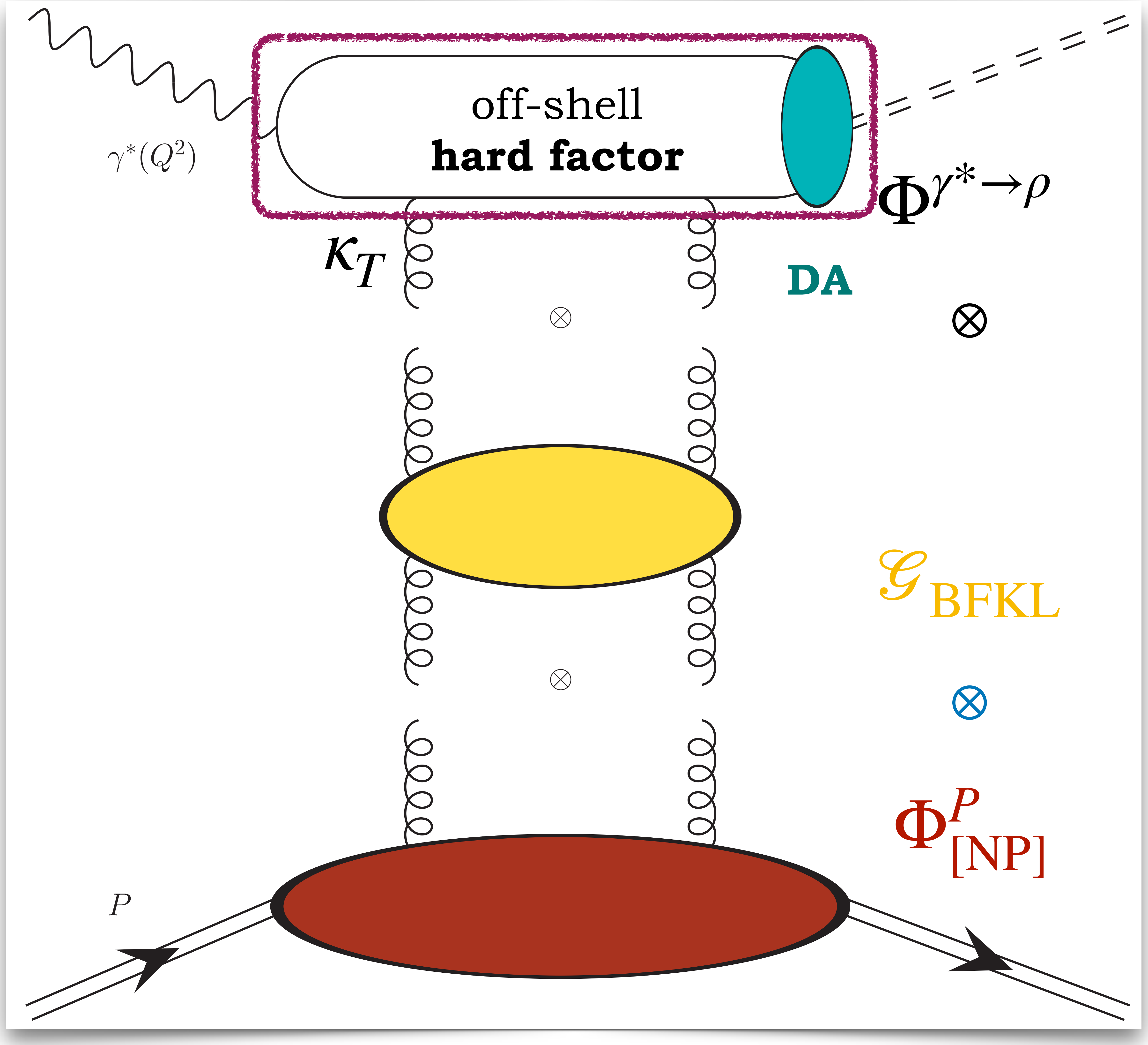

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Collinear convolution $\Leftrightarrow$ large κ_T (!)

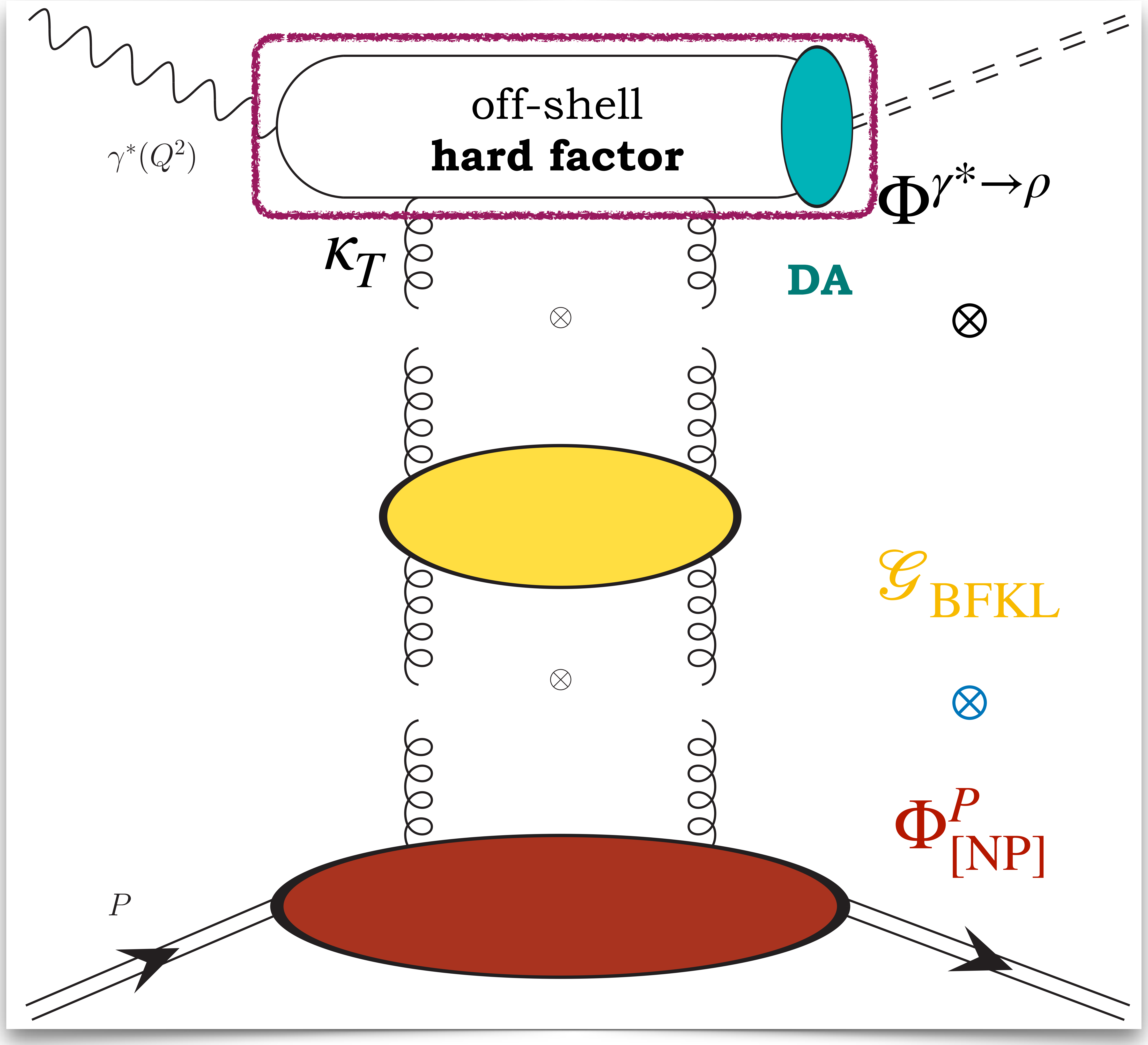
* $\gamma_L^* \to \rho_L$ transition [twist-2]

$$\frac{1}{\kappa_T^2}\Phi^{\gamma_L^*\to\rho_L} \underset{\kappa_T\to 0^+}{\sim} \text{constant}$$

UGD

BFKL factorization

A factorization...of factorizations



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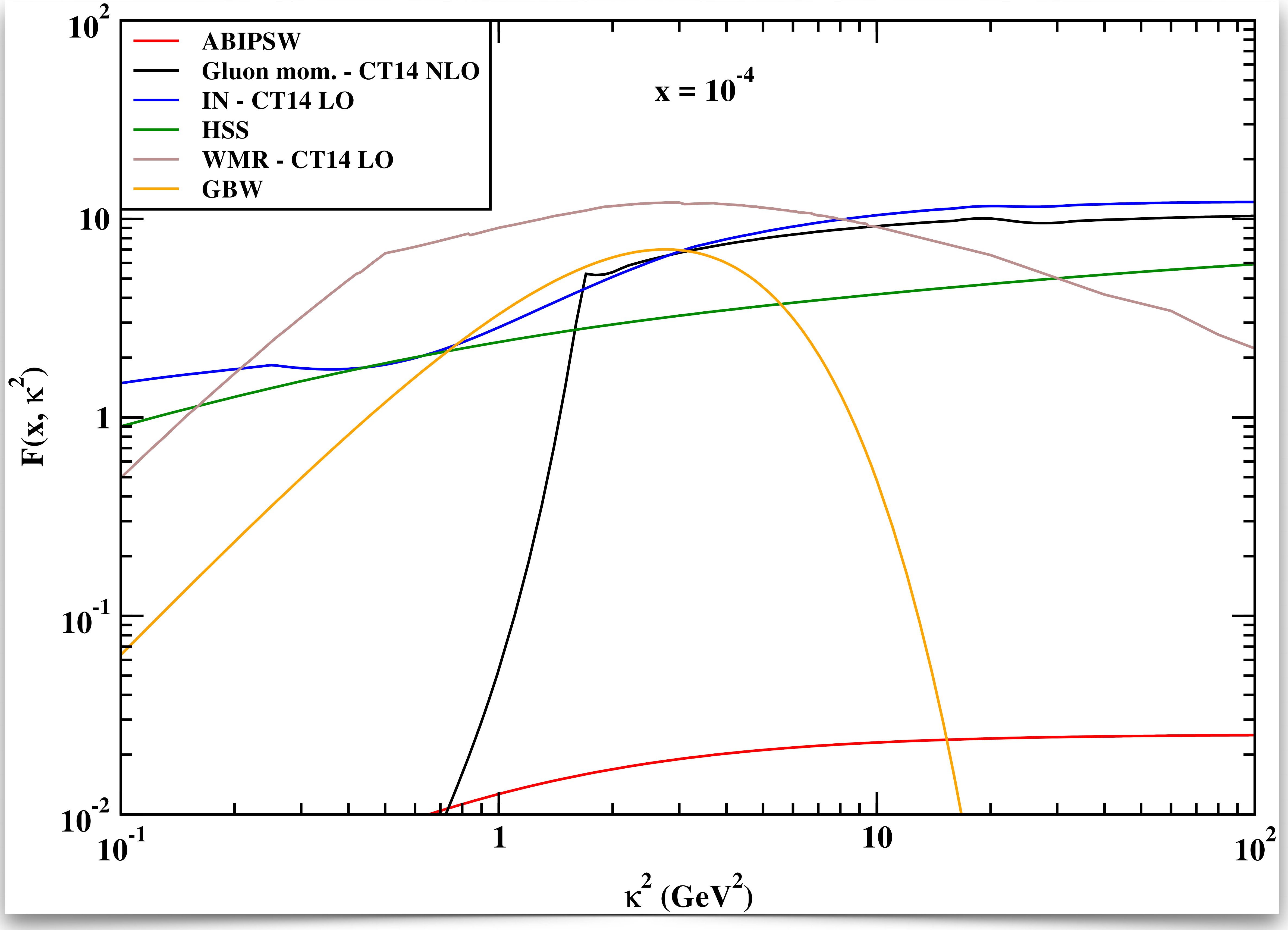
* $\gamma_T^* \to \rho_T$ transition [twist-3]

$$\frac{1}{\kappa_T^2} \Phi^{\gamma_T^* \to \rho_T} \underset{\kappa_T \to 0^+}{\sim} \ln \frac{\kappa_T^2}{Q^2}$$

UGD

BFKL factorization

UGD models



- **ABIPSW**: x-independent model

  [I. V. Anikin et al., *Phys. Rev. D* **84** (2011)]

- **Gluon mom. derivative**:

$$\mathcal{F}(x, \kappa^2) = \frac{d\, xg(x, \kappa^2)}{d \ln \kappa^2}$$

- **IN**: soft + hard component $\xrightarrow{\text{to probe}}$ different regions of κ

  [I. P. Ivanov and N. N. Nikolaev, *Phys. Rev. D* **65** (2002)]

- **HSS**: $\mathcal{G}_{\rm BFKL} \otimes$ [proton IF]

  [I. Bautista, A. Fernandez Tellez, M. Hentschinski, *Phys. Rev. D* **94** (2016) no.5, 054002]
  [G. Chachamis, M. Deák, M. Hentschinski, G. Rodrigo and A. Sabio Vera, *JHEP* **1509** (2015) 123]

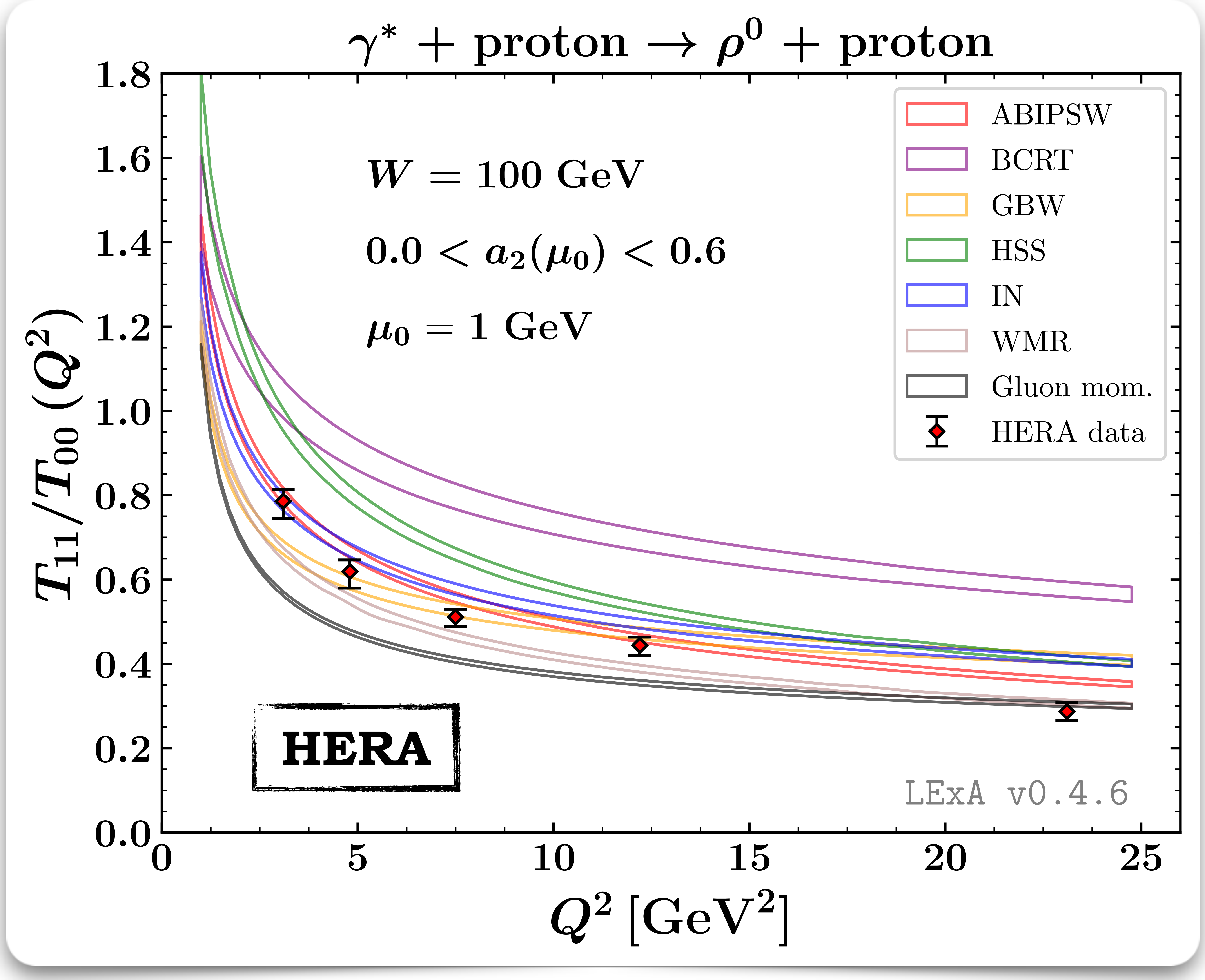
- **WMR**: angular ordering of gluon emissions

  [G. Watt, A.D. Martin, M.G. Ryskin, *Eur. Phys. J. C* **31** (2003) 73]

- **GBW**: FT of dipole cross section

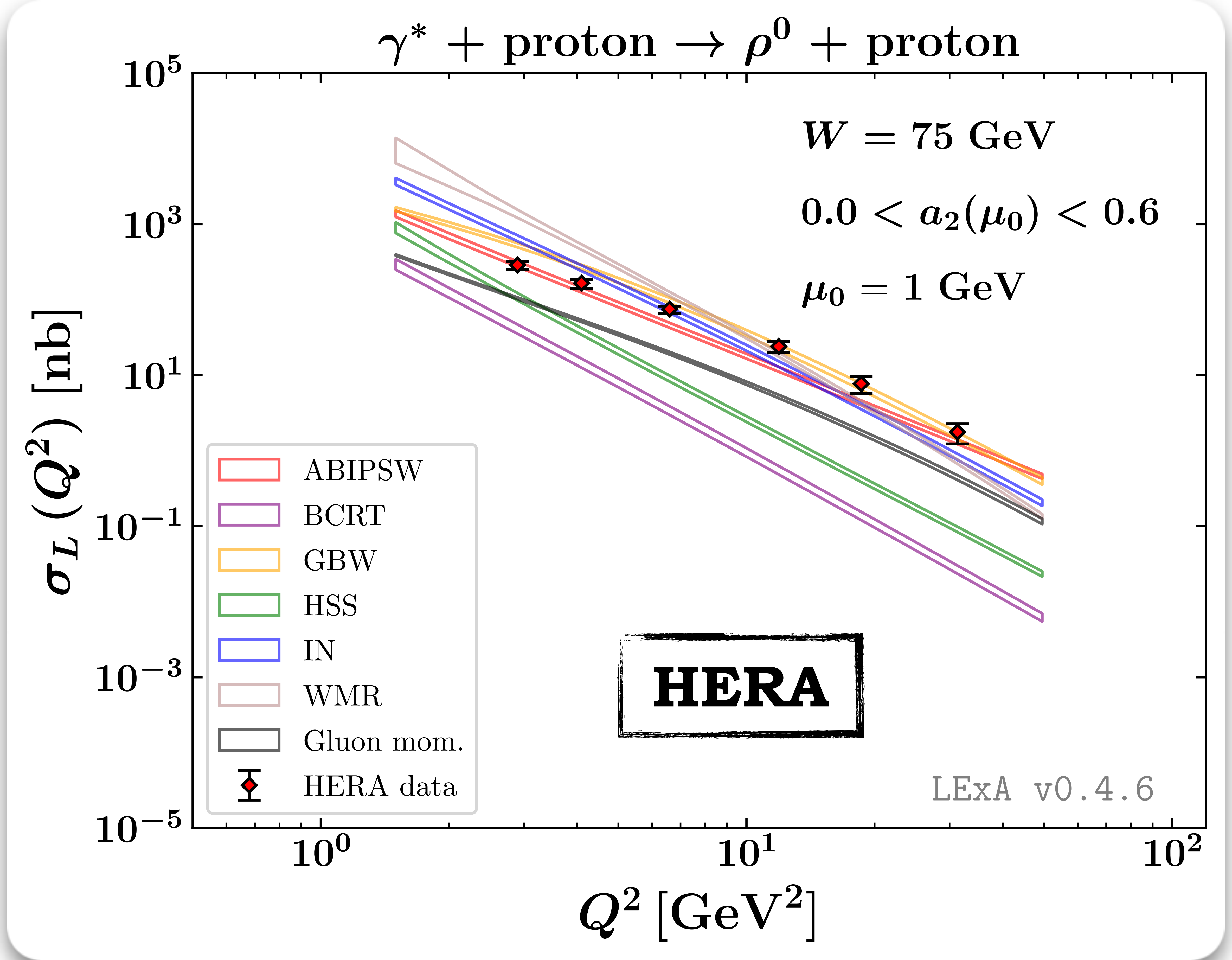
  [K.J. Golec-Biernat, M. Wüsthoff, *Phys. Rev. D* **59** (1998) 014017]

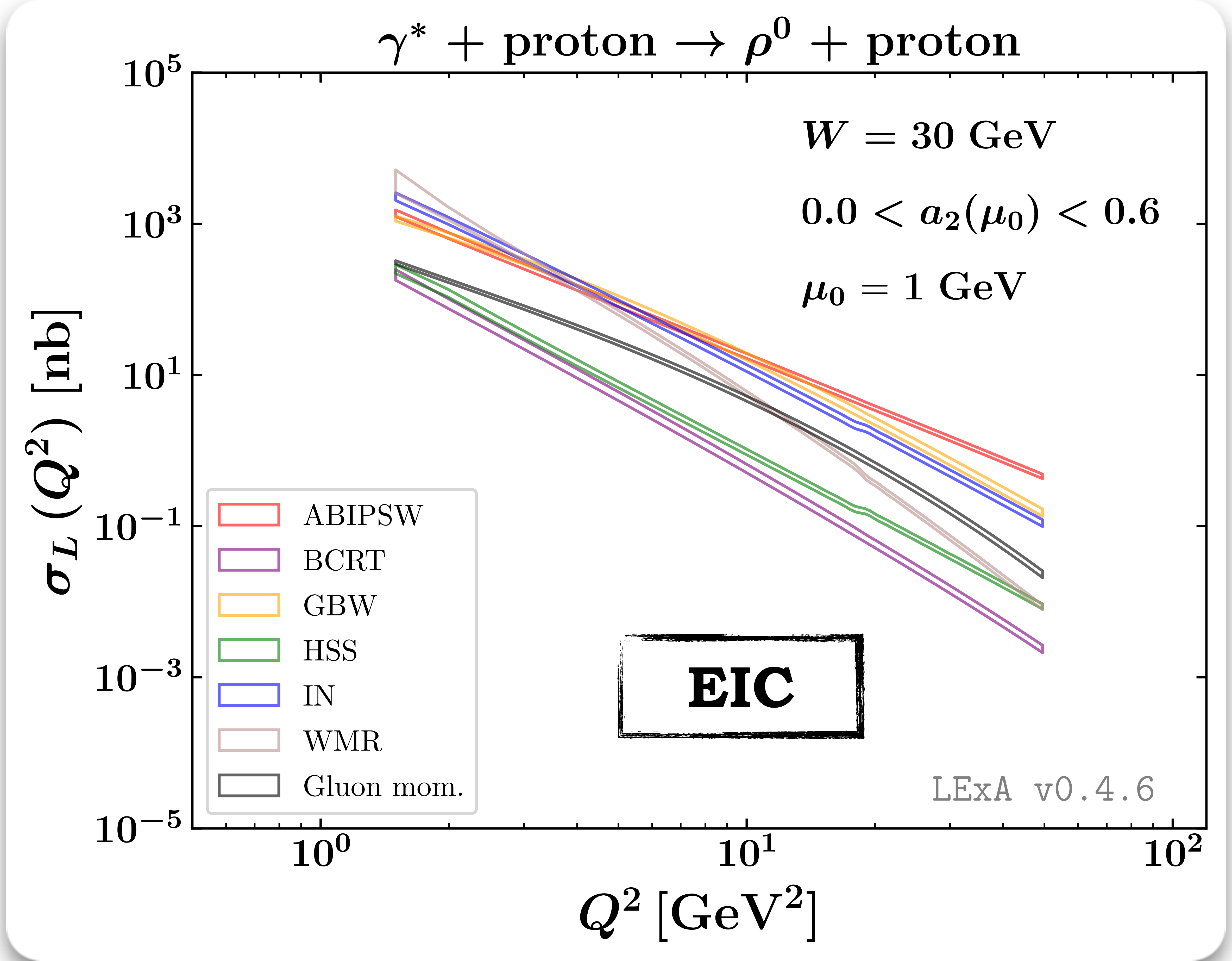
Exclusive forward ρ-meson: Helicity amplitudes



[A.D. Bolognino, F.G.C., D.Yu. Ivanov, A. Papa (2018)]
[A.D. Bolognino, PhD Thesis (2021)]

Exclusive forward ρ-meson: $\sigma_L(Q^2)$





(saturation effects in ρ-meson polarization @HERA) 🔗 [A. Besse, L. Szymanowski, S. Wallon (2013)]

(ρ-meson helicity amplitudes @HERA) 🔗 [A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa (2018)]

(extension to ϕ-meson emissions) 🔗 [A.D. Bolognino, A. Szczurek, W. Schäfer (2020)]

(in this slide) 🔗 [A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa, W. Schäfer, A. Szczurek (2021)]

3

UGDs from HELL

The HELL methodology

HELL1.0

ABF for DGLAP resum

Coefficient function resum

M. Bonvini, S. Marzani, T. Peraro

[Eur. Phys. J. C 76 (2016) 11, 597]

$$\alpha_s \ln(1/x) \lesssim 1$$

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HELL

High Energy Large Logarithms

A small-x resummation code

M. Bonvini, S. Marzani, C. Muselli

🔗 [JHEP 12 (2017) 117]

Improved β_0 resum

Full DIS(NC, CC, masses)

...road to HELL2.0

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...road to HELL3.0

DGLAP-evo predicted

at small-x at N[3]LO

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M. Bovini

🔗 [Eur. Phys. J. C 78 (2018) 10, 834]

F. Silvetti, M. Bovini

🔗 [Eur. Phys. J. C 83 (2023) 4, 267]

Inclusive Higgs @LHC

Differential $Q\bar{Q}$ @LHC

HELL3.0

Small-x resummation from ABF-HELL

Example: DIS cross section within QCD collinear factorization

Collinear factorization

DGLAP ***evolution***

$$\sigma(x, Q^2) = \int_x^1 \frac{dz}{z} C(z, \alpha_s(Q^2)) \, f\left(\frac{x}{z}, Q^2\right)$$

$$\mu^2 \frac{d}{d\mu^2} f(x, \mu^2) = \int_x^1 \frac{dz}{z} P(z, \alpha_s(\mu^2)) \, f\left(\frac{x}{z}, \mu^2\right)$$

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High-energy (small-*x*) *log enhancement* in ***coefficient functions*** & ***splitting functions***

Small-*x* logs are *single* logs

$$\alpha_s^n \frac{1}{x} \log^k \frac{1}{x} \quad (0 \leq k \leq n-1)$$

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¿ How to encode the small-x resummation ?

Small-x resummation from ABF-HELL

¡ Let us make us of k_t factorization !

Collinear factorization

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$$\sigma(x,Q^2) = \int_x^1 \frac{dz}{z} \int_0^\infty dk_t^2\, \mathcal{C}\left(z,k_t^2,\alpha_s\right) \mathcal{F}\left(\frac{x}{z},k_t^2\right)$$

$$-x\frac{d}{dx}\mathcal{F}(x,k_t^2) = \int_0^\infty \frac{dq_t^2}{k_t^2}\, \mathcal{K}\left(\frac{k_t^2}{q_t^2},\alpha_s(\cdot)\right) \mathcal{F}(x,q_t^2)$$

k_t factorization

BFKL evolution

Small-x resummation from ABF-HELL

¡ Let us make us of k_t factorization !

Collinear factorization

DGLAP evolution

$$\sigma(x,Q^2) = \int_x^1 \frac{dz}{z}\, C\big(z,\alpha_s(Q^2)\big)\, f\Big(\frac{x}{z},Q^2\Big)$$

$$\mu^2\frac{d}{d\mu^2} f(x,\mu^2) = \int_x^1 \frac{dz}{z}\, P\big(z,\alpha_s(\mu^2)\big)\, f\Big(\frac{x}{z},\mu^2\Big)$$

$$\sigma(x,Q^2) = \int_x^1 \frac{dz}{z} \int_0^\infty dk_t^2\, \mathcal{C}\big(z,k_t^2,\alpha_s\big)\, \mathcal{F}\Big(\frac{x}{z},k_t^2\Big)$$

$$-x\frac{d}{dx}\mathcal{F}(x,k_t^2) = \int_0^\infty \frac{dq_t^2}{k_t^2}\, \mathcal{K}\Big(\frac{k_t^2}{q_t^2},\alpha_s(\cdot)\Big)\, \mathcal{F}(x,q_t^2)$$

k_t factorization

BFKL evolution

UGD

$$\mathcal{F}(x,k_t^2) = \frac{d}{dk_t^2} f(x,k_t^2)$$

Small-x resummation from ABF-HELL

Problem: NLL/LL corrections to the BFKL kernel are large and negative ⇒ ***strong instabilities***

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Small-x resummation from ABF-HELL

Small-x & large-Q^2 regime:

Collinear factorization

k_t factorization

$$\sigma(x, Q^2) = \int_x^1 \frac{dz}{z} C(z, \alpha_s(Q^2)) \, f\left(\frac{x}{z}, Q^2\right)$$

$$= \int_x^1 \frac{dz}{z} \int_0^\infty dk_t^2 \, \mathcal{C}(z, k_t^2, \alpha_s) \, \mathcal{F}\left(\frac{x}{z}, k_t^2\right)$$

Resummed collinear coeff. function

Off-shell coeff. function

$$C(N, \alpha_s) = \int_0^\infty dk_t^2 \, \mathcal{C}\left(N, \frac{k_t^2}{Q^2}, \alpha_s\right) \mathcal{U}\left(N, \frac{k_t^2}{Q^2}\right)$$

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$$\mathcal{F}(N,k_t^2) = \mathcal{U}\left(N,\frac{k_t^2}{Q^2}\right) f(N,Q^2)$$

UGD **Evolutor** **PDF**

Resummed collinear coeff. function **Off-shell coeff. function**

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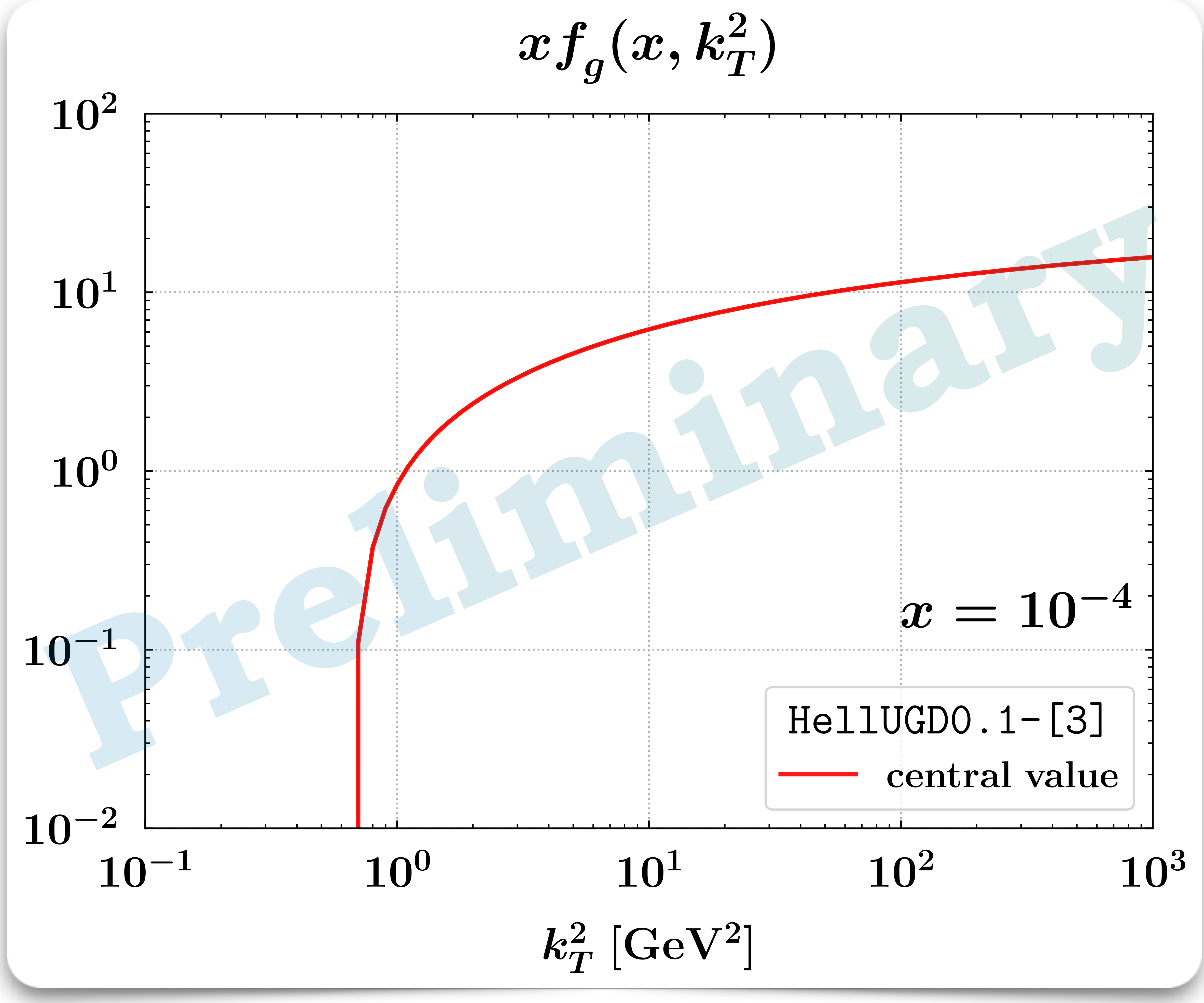
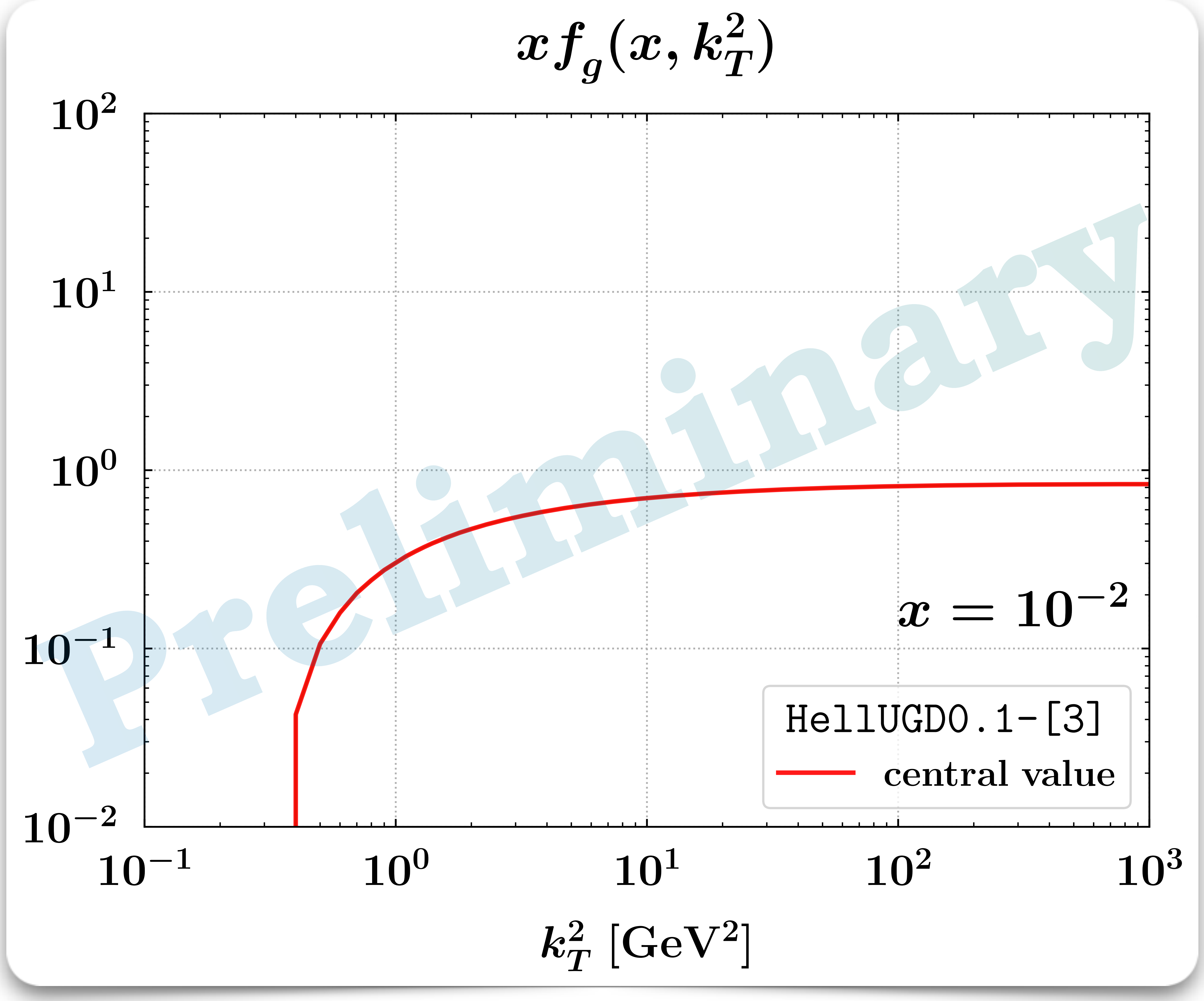
$$\mathcal{U}\left(N,\frac{k_t^2}{Q^2}\right) = \frac{d}{dk_t^2} \exp\int_{Q^2}^{k_t^2} \frac{d\nu^2}{\nu^2} \gamma(N,\alpha_s(\nu^2))$$

Resummed anomalous dimension

The HELL UGDs





4

Applications with JETHAD

The JETHAD technology

High-energy resummation

Hunting BFKL

in semi-hard reactions

Mueller-Navelet, light hadrons

ERIS super-module

$$\alpha_s \ln(s) \lesssim 1$$

JETHAD

v0.5.5

python + F

[Eur. Phys. J. C 81 (2021) 8, 691]

[Phys. Rev. D 105 (2022) 11, 114008]

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Forward Drell-Yan, onium, Higgs

𝔄ναμις, LExA, HATHOR super-modules

Proton structure

Small-x UGD

Gluon TMD PDFs

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from HF-NRevo

NRFF1.0 onium FFs

Vectors & pseudoscalars

JETHAD + DGLAP evolution operators

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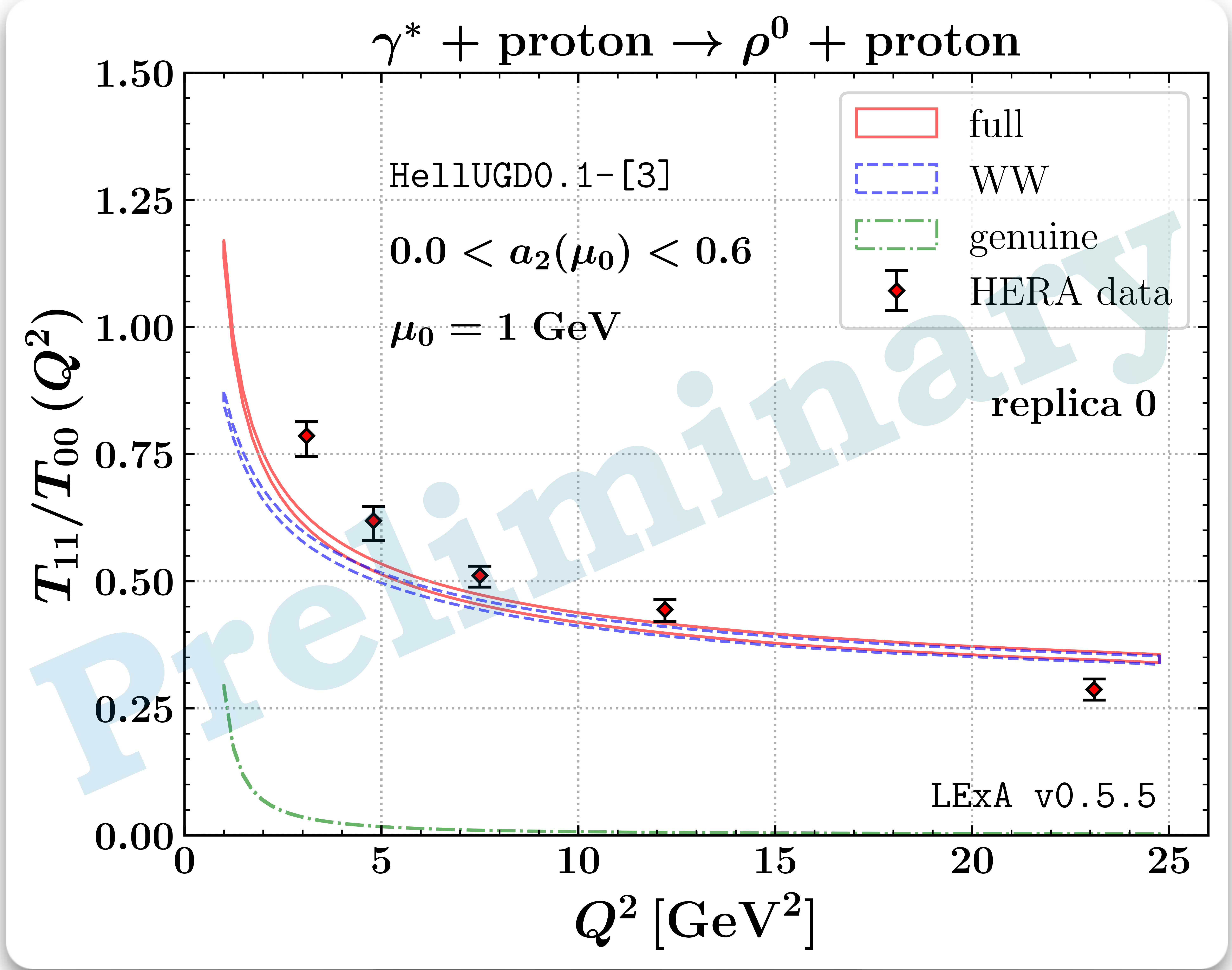
Helicity amplitudes and the HELL UGD



Forward Drell-Yan production

- LHC, **forward region** $\rightarrow$ $(l^+ l^-)$ produced in the fragmentation region of h_2
  - ◇ Asymmetric configuration: $x_1 \gg x_2$, down to $x_2 \simeq 10^{-6}$

$\Longrightarrow$ **possible small-x resummation effects expected!**

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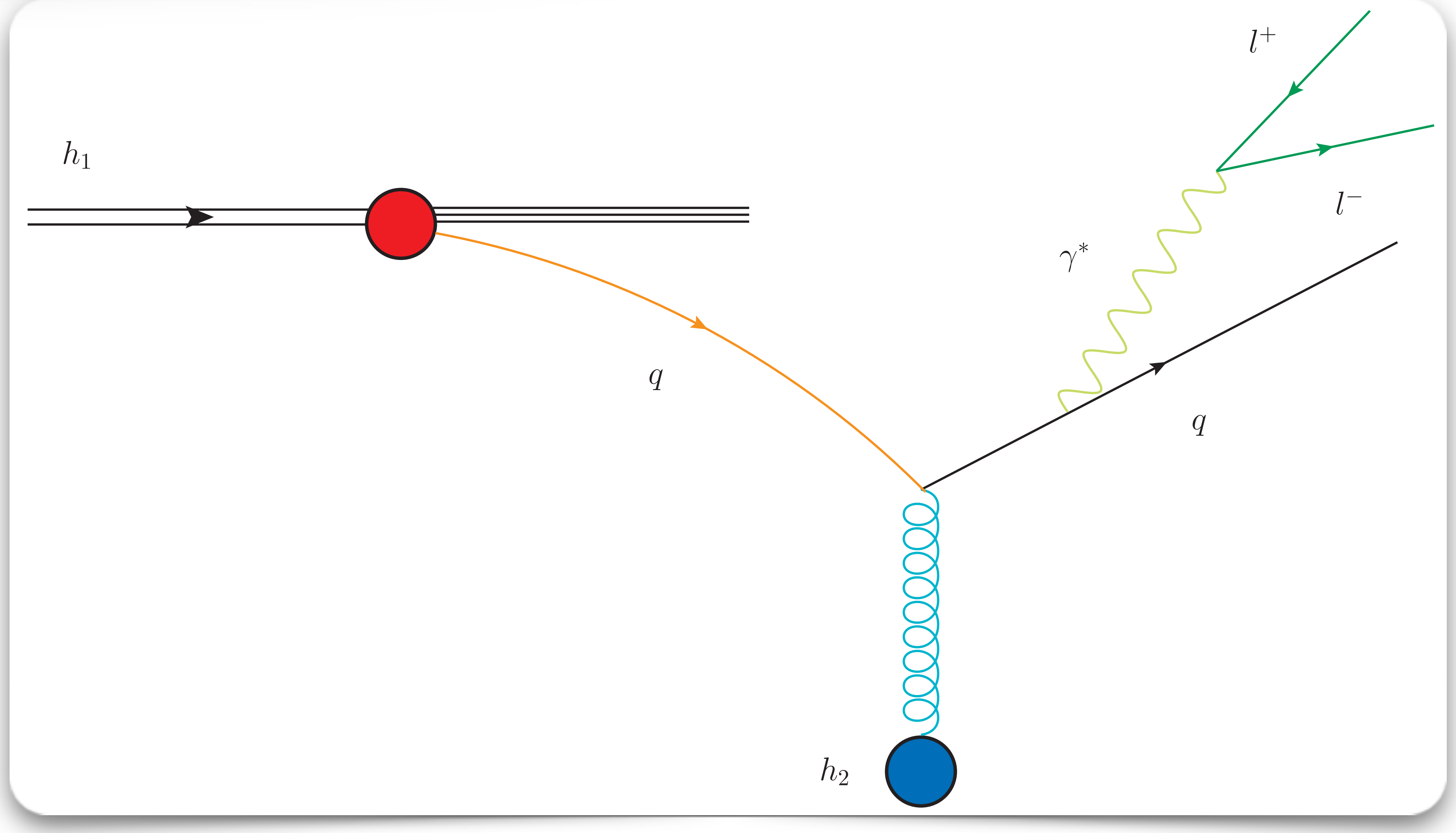
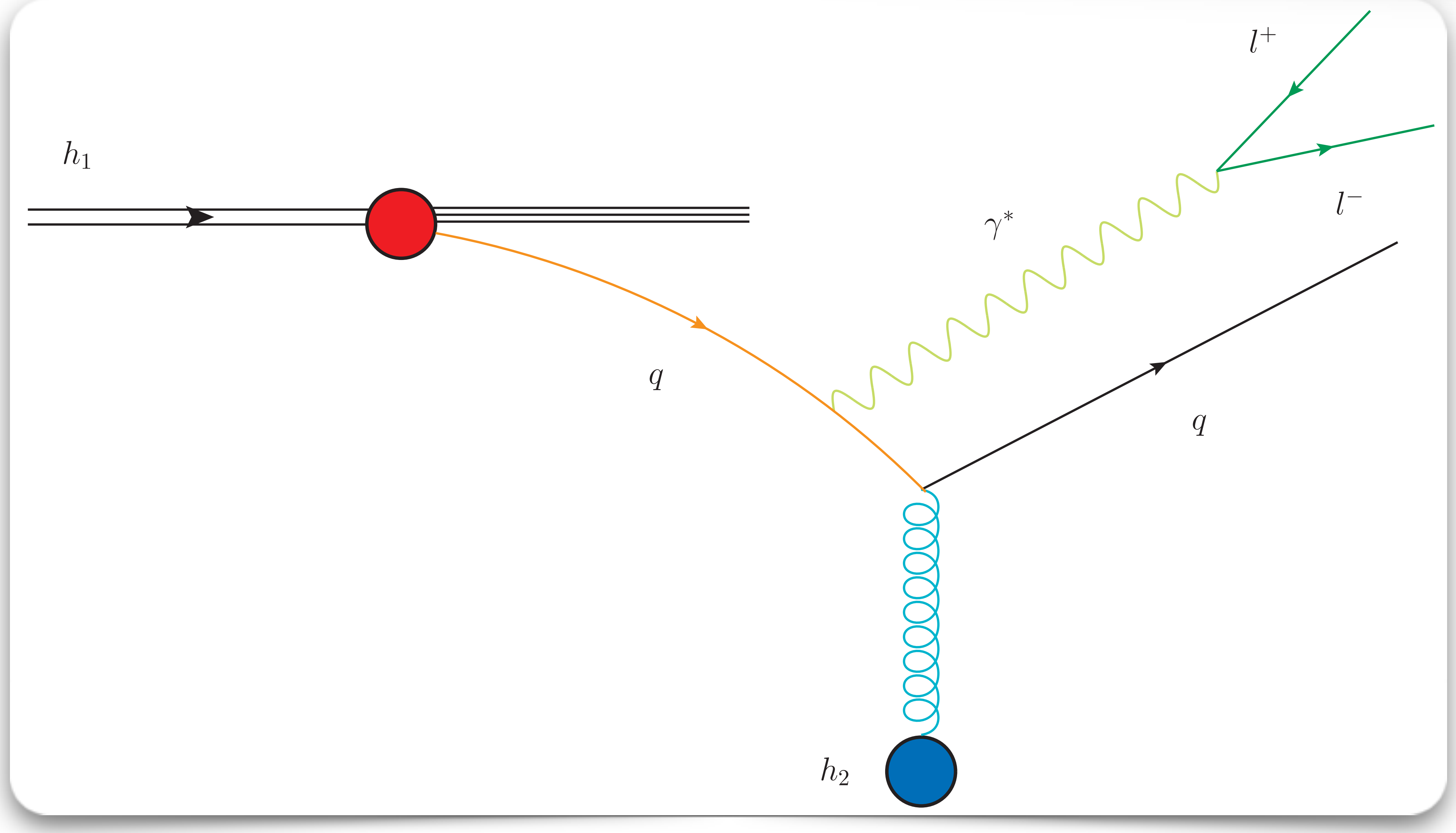


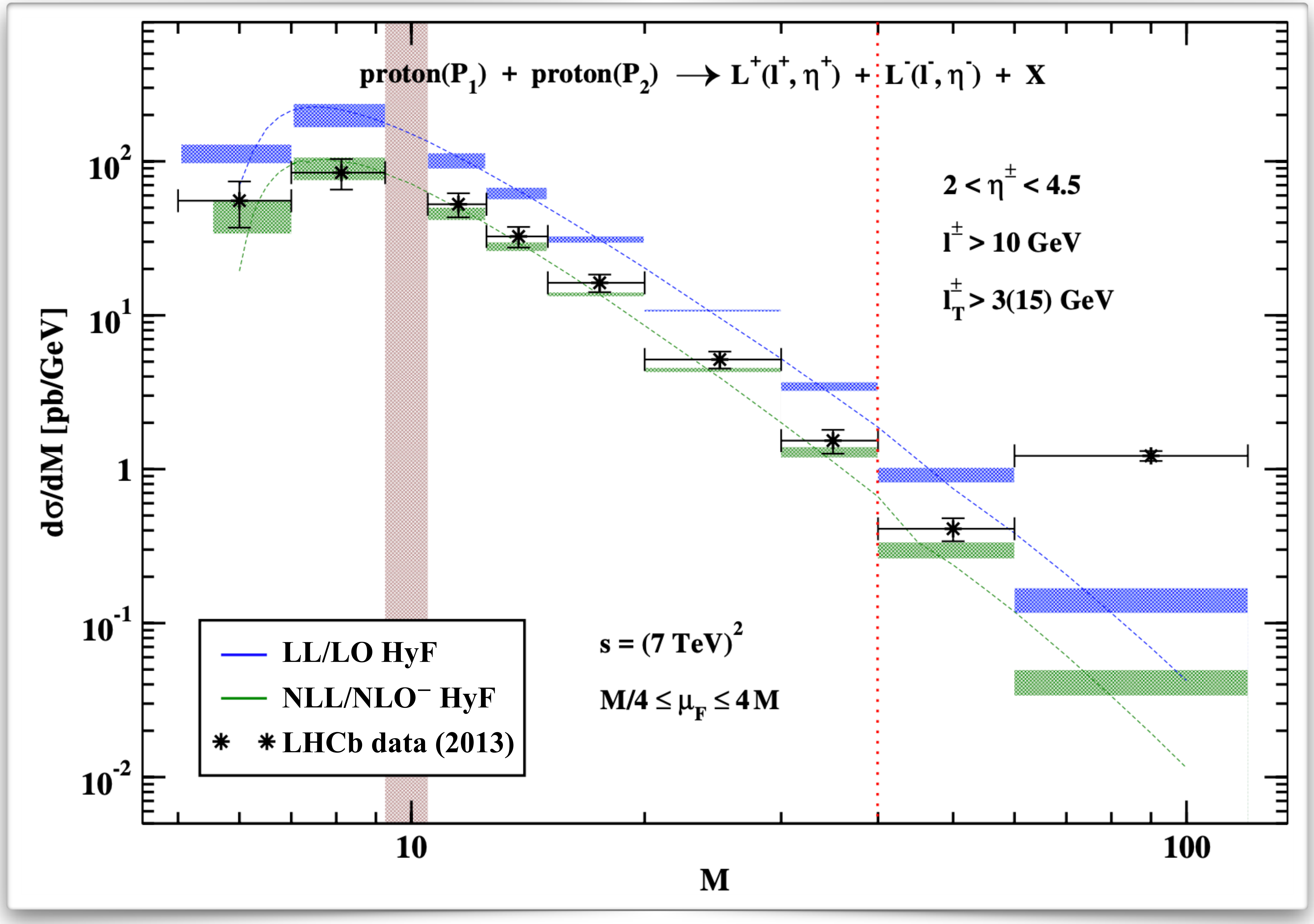


(LO DY emission functions + Mellin) 🔗 [G. Golec-Biernat, E. Lewandowska, A. M. Stasto, Phys. Rev. D 82 (2010) 094010]

(LO DY emission functions + Mellin) 🔗 [L. Motyka, M. Sadzikowski, T. Stebel, JHEP 05 (2015) 087]

(Twist decomposition + pheno) 🔗 [D. Brzemiński, L. Motyka, M. Sadzikowski, T. Stebel, JHEP 01 (2017) 005]

Forward Drell-Yan production @LHC (2018)



NLL/NLO^{-} studies @7TeV LHCb

[F. G. C., D Gordo Gómez, A. Sabio Vera, Phys. Lett. B 786 (2018) 201-206]

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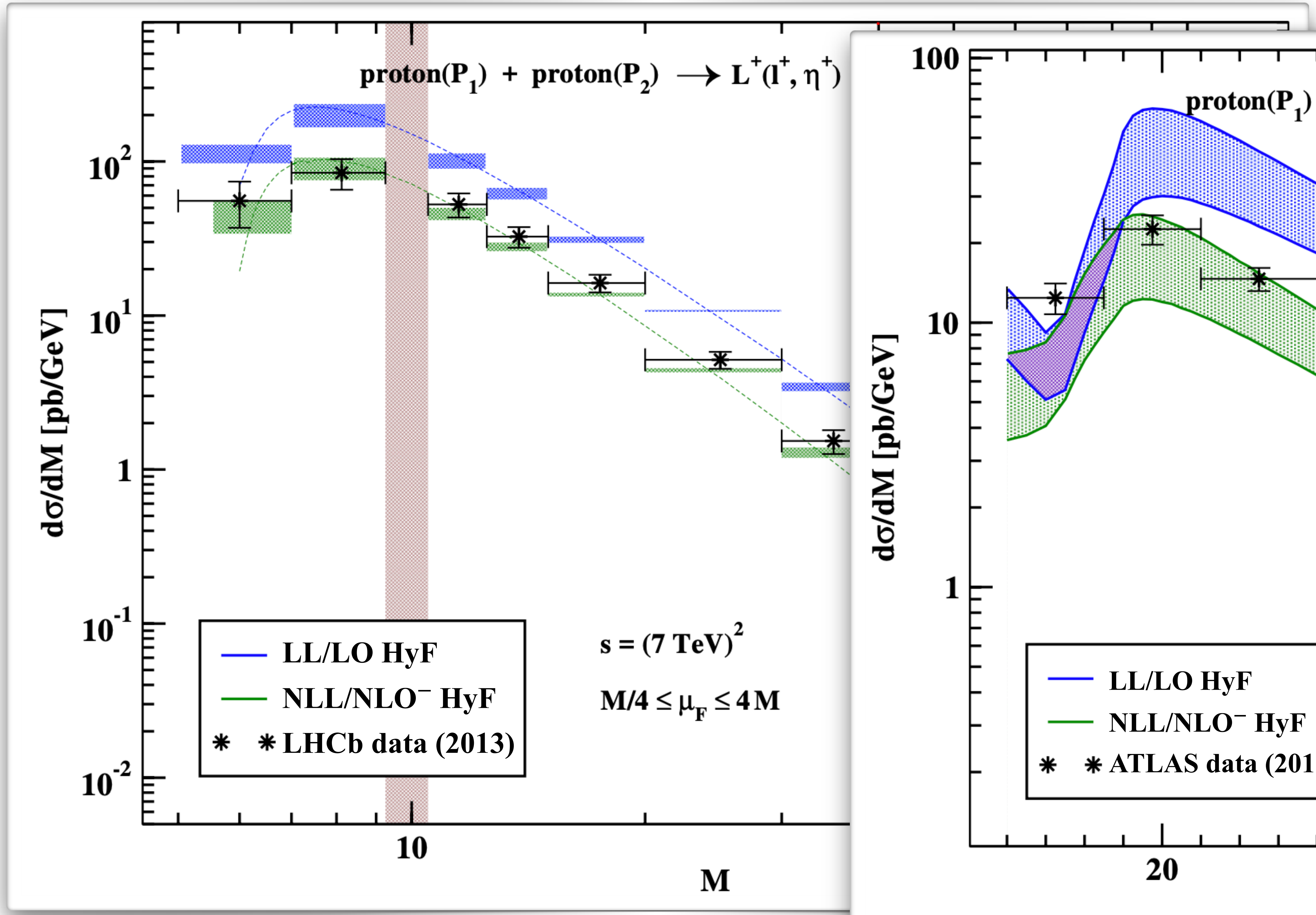

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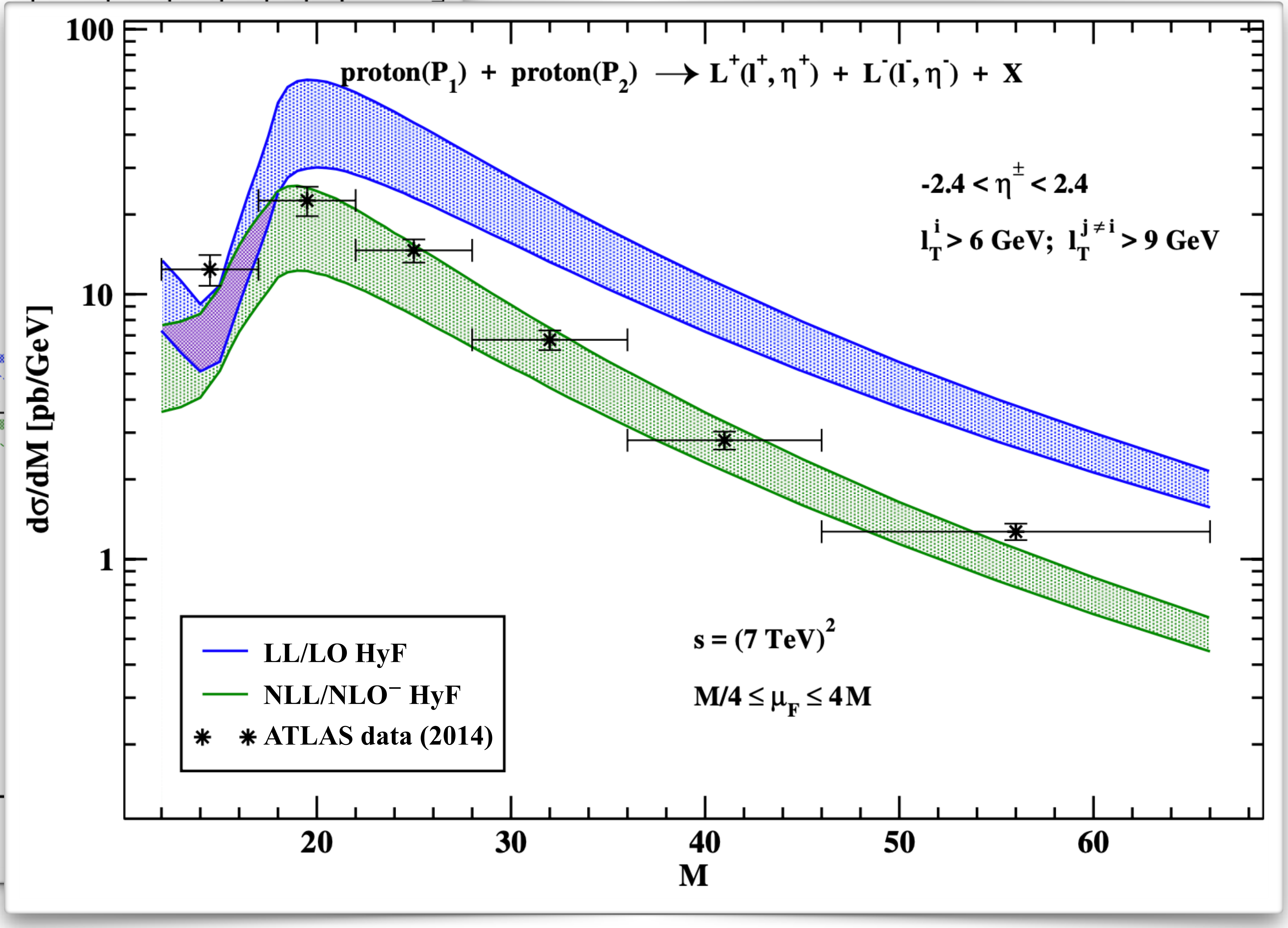

NLL/NLO$^-$ studies @7TeV ATLAS

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Road to the HELL UGD

BFKL + DGLAP resummations ⇒ Proton structure at small-x

Road to the HELL UGD

- BFKL + DGLAP resummations $\Rightarrow$ Proton structure at small-x

- ABF-HELL UGD $\Rightarrow$ solid methodology to hunt for precision

Road to the HELL UGD

BFKL + DGLAP resummations ⇒ Proton structure at small-x

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Formal development & pheno applications

Road to the HELL UGD

- BFKL + DGLAP resummations ⇒ Proton structure at small-x
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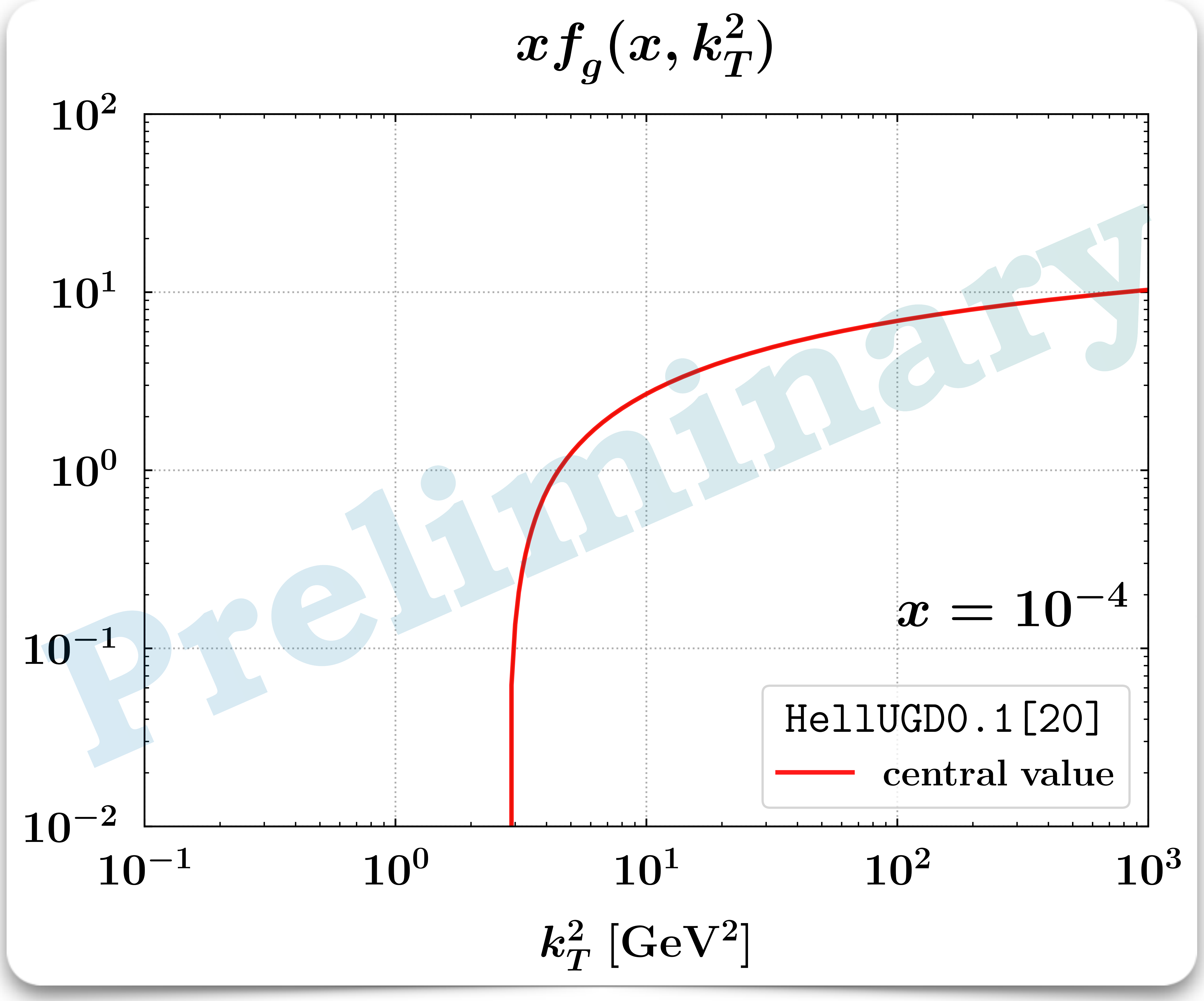
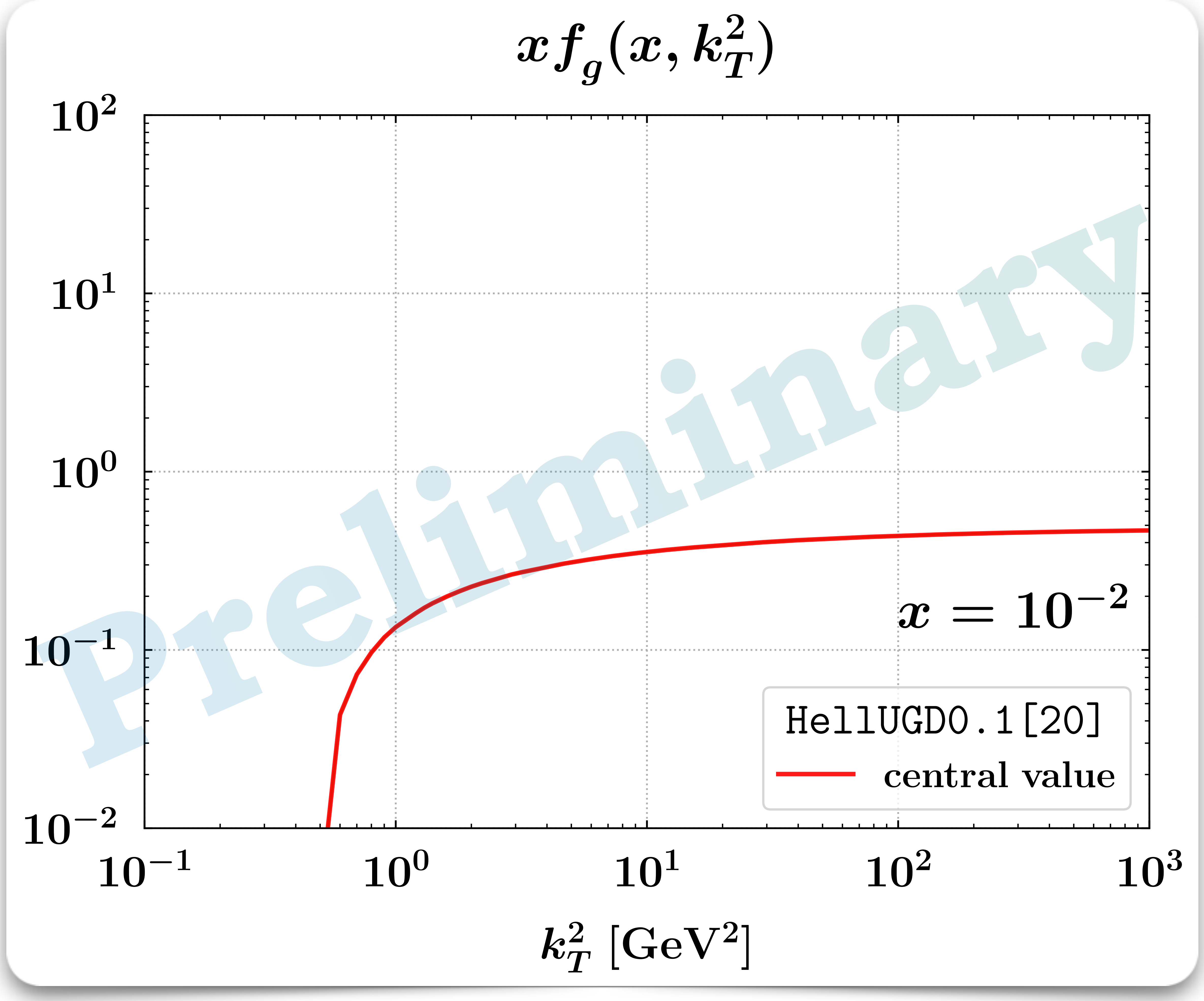
- ***¿ Possible extension to 3D tomography (TMD) ?***

EXTRAS

The HELL UGDs (II)





Higgs production from LHC to FCC

PHYSICAL REVIEW LETTERS **120,** 202003 (2018)

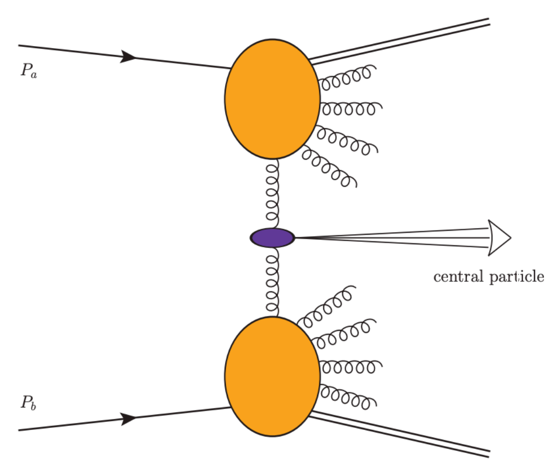
Double Resummation for Higgs Production

Marco Bonvini[1,*] and Simone Marzani[2,†]
[1]*INFN, Sezione di Roma 1, Piazzale Aldo Moro 5, 00185 Roma, Italy*
[2]*Dipartimento di Fisica, Università di Genova and INFN, Sezione di Genova, Via Dodecaneso 33, I-16146 Genova, Italy*

(Received 26 February 2018; published 16 May 2018)

We present the first double-resummed prediction of the inclusive cross section for the main Higgs production channel in proton-proton collisions, namely, gluon fusion. Our calculation incorporates to all orders in perturbation theory two distinct towers of logarithmic corrections which are enhanced, respectively, at threshold, i.e., large x, and in the high-energy limit, i.e., small x. Large-x logarithms are resummed to next-to-next-to-next-to-leading logarithmic accuracy, while small-x ones to leading logarithmic accuracy. The double-resummed cross section is furthermore matched to the state-of-the-art fixed-order prediction at next-to-next-to-next-to-leading accuracy. We find that double resummation corrects the Higgs production rate by 2% at the currently explored center-of-mass energy of 13 TeV and its impact reaches 10% at future circular colliders at 100 TeV.

DOI: 10.1103/PhysRevLett.120.202003



(*b*) Single central

High-energy resummation (BFKL) ⇒ PDFs at small-x

Altarelli-Ball-Forte to stabilize the NLL_{sx} BFKL kernel

$N^3LL_{lx}/LL_{sx}/N^3LO$ rapidity-inclusive coefficient functions

$$C_{ij}(x,\alpha_s) = C^{\rm fo}_{ij}(x,\alpha_s) + \Delta C^{\rm lx}_{ij}(x,\alpha_s) + \Delta C^{\rm sx}_{ij}(x,\alpha_s)$$

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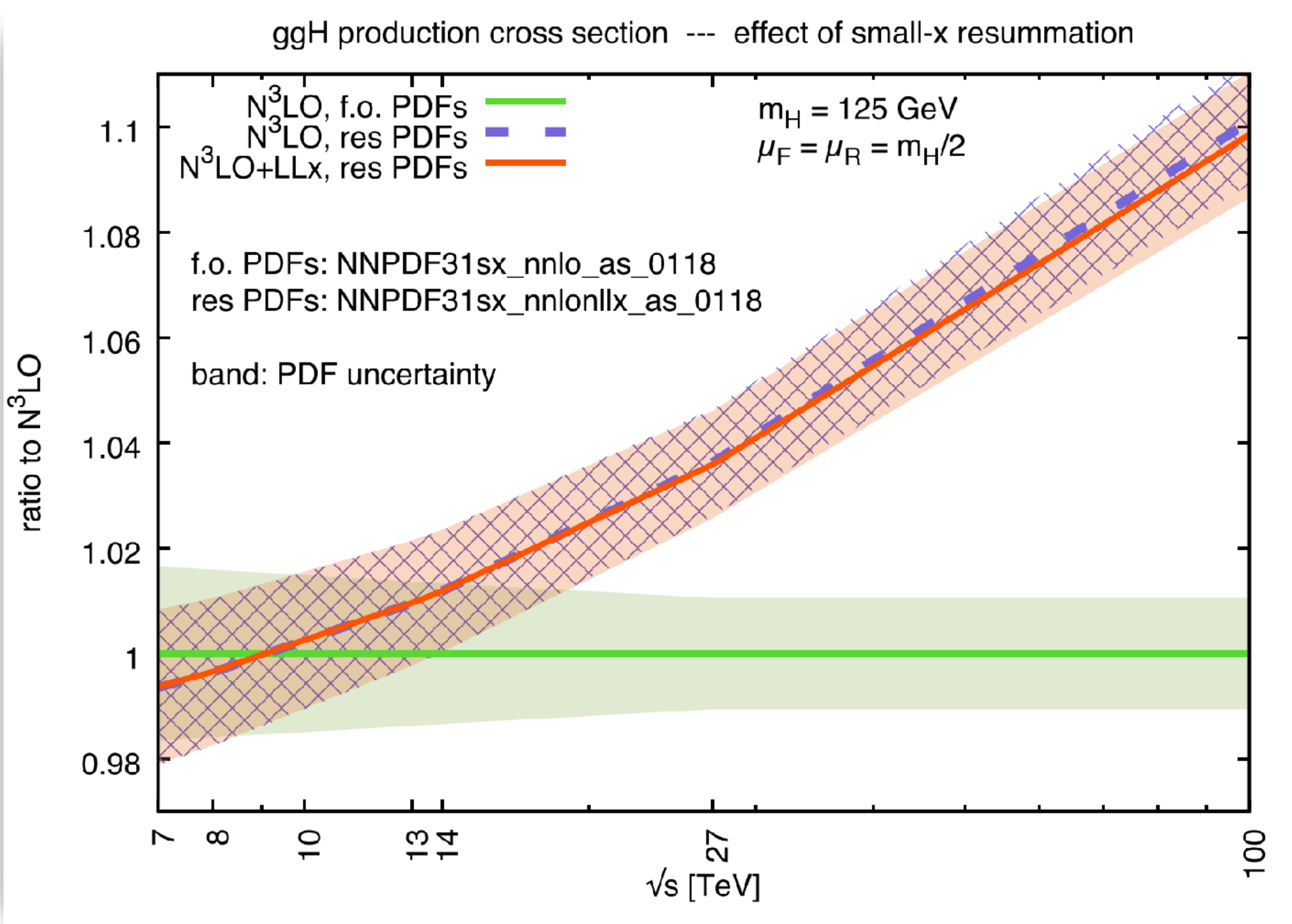

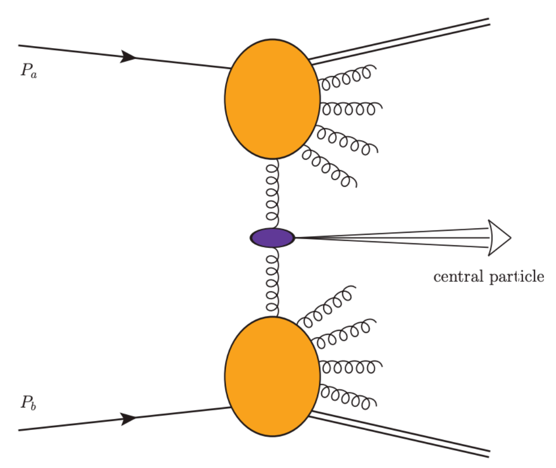

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(¡!) 100 TeV EW physics ⇔ small-x physics!

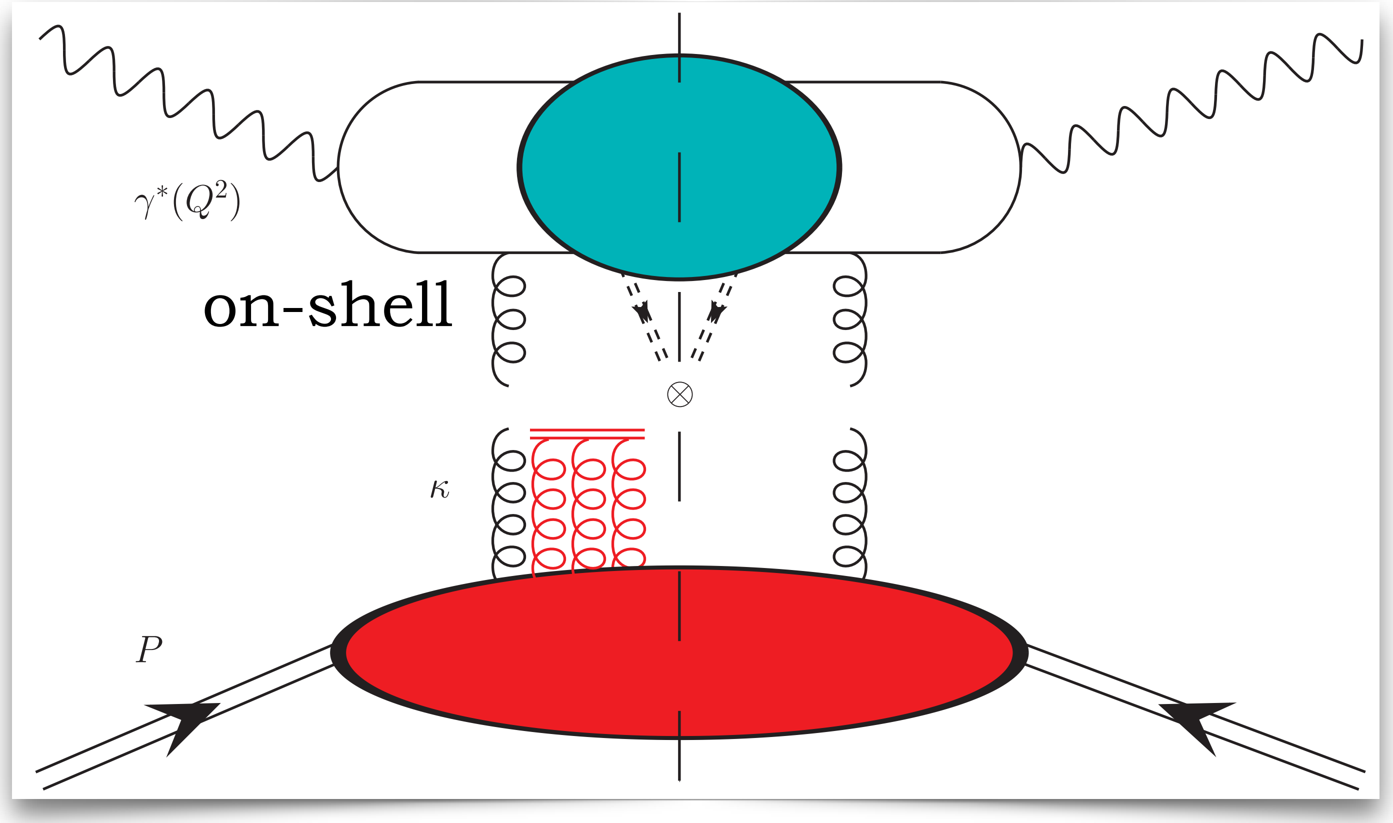
(¿?) Can LHC physics be resummation physics?

HIGH-ENERGY FACTORIZATION AND THE UGD

TMD versus HEF

TMD

* Semi-inclusive processes
* $\kappa_T \ll$ hardest scale
* Language of **parton correlators**
* Diagram: **SIDIS onium**



TMD
PDF

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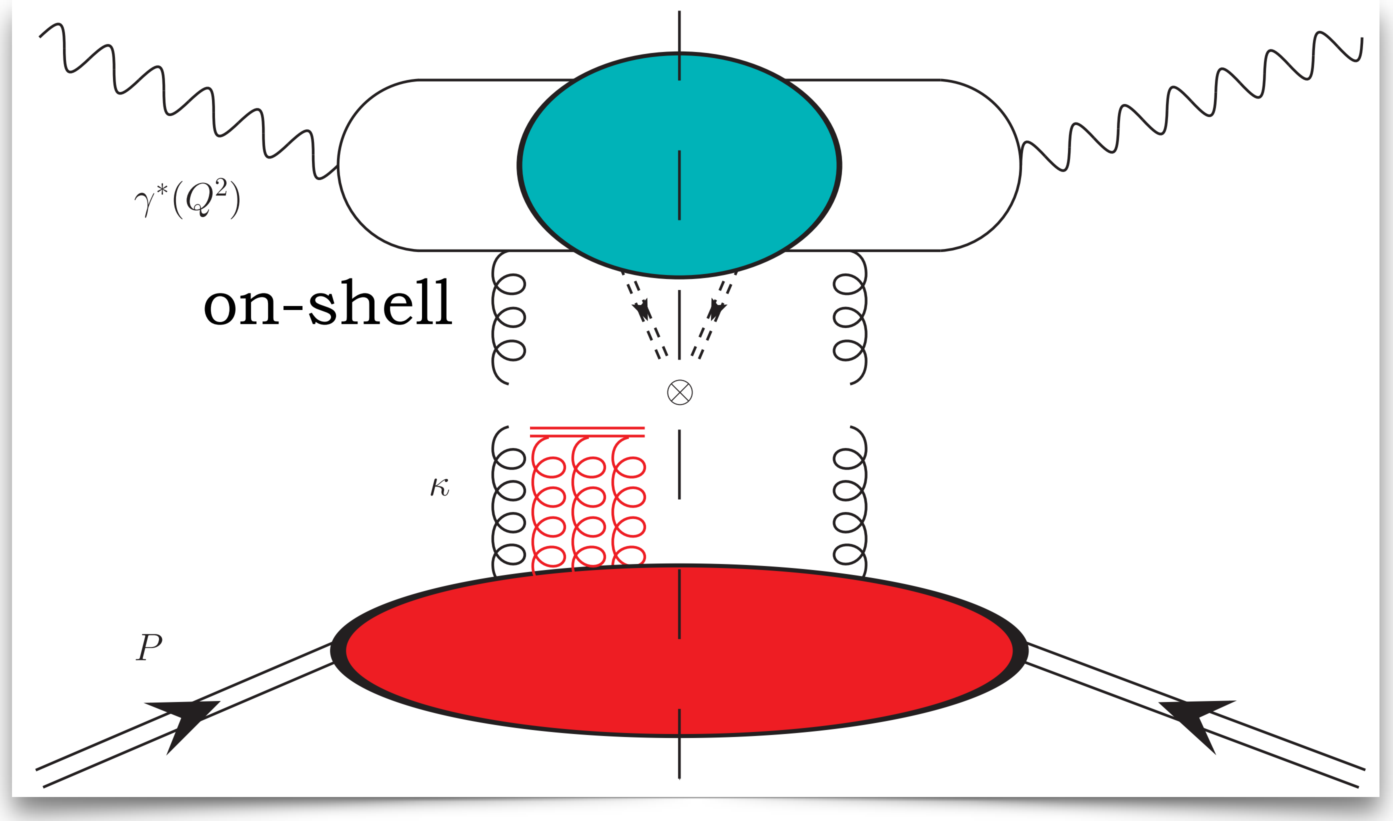

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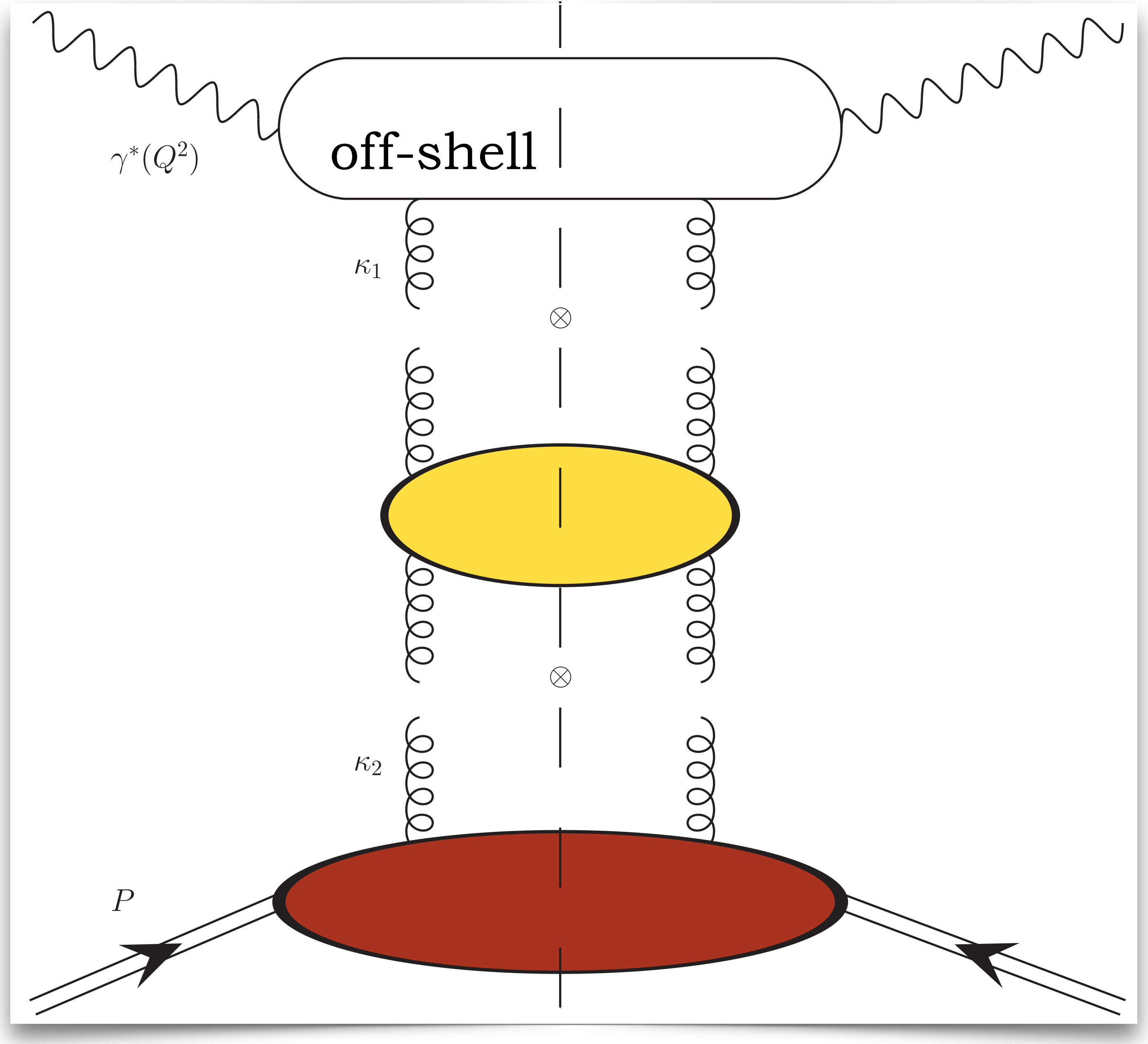

$$\Phi^{\gamma^* \to \gamma^*} \otimes \mathcal{G}_{\rm BFKL} \otimes \Phi^{P}_{[\rm NP]}$$

TMD versus HEF

IR-safe colorless $\{\Phi^{i\to o}\}$
(Fadin-Martin theorem)
🔗 [V.S. Fadin, A.D. Martin (1999)]

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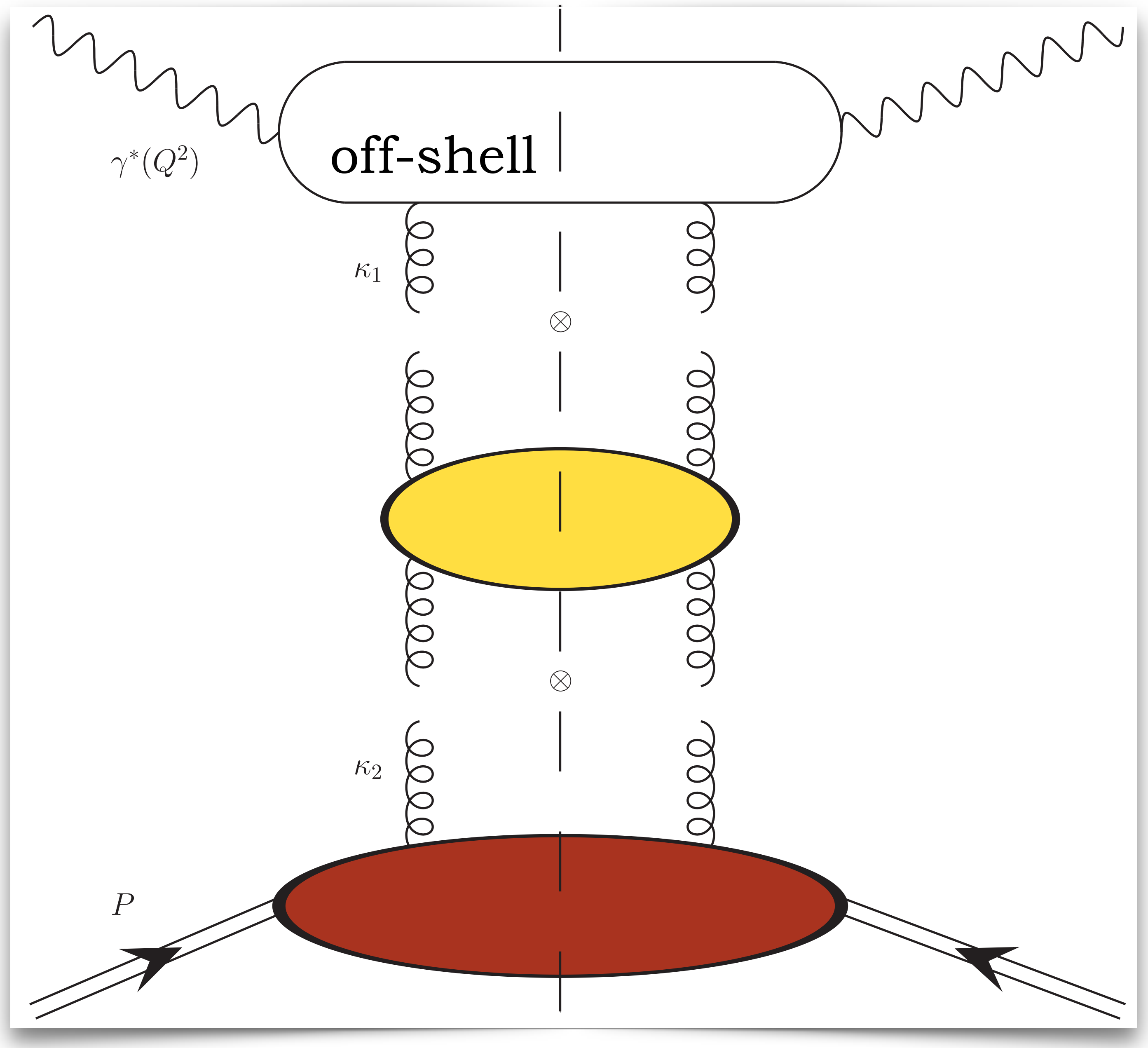

$\Phi^{\gamma^*\to\gamma^*}$

$\otimes$

$\mathcal{G}_{\rm BFKL}$

$\otimes$

$\Phi^{P}_{\rm [NP]}$

TMD versus HEF

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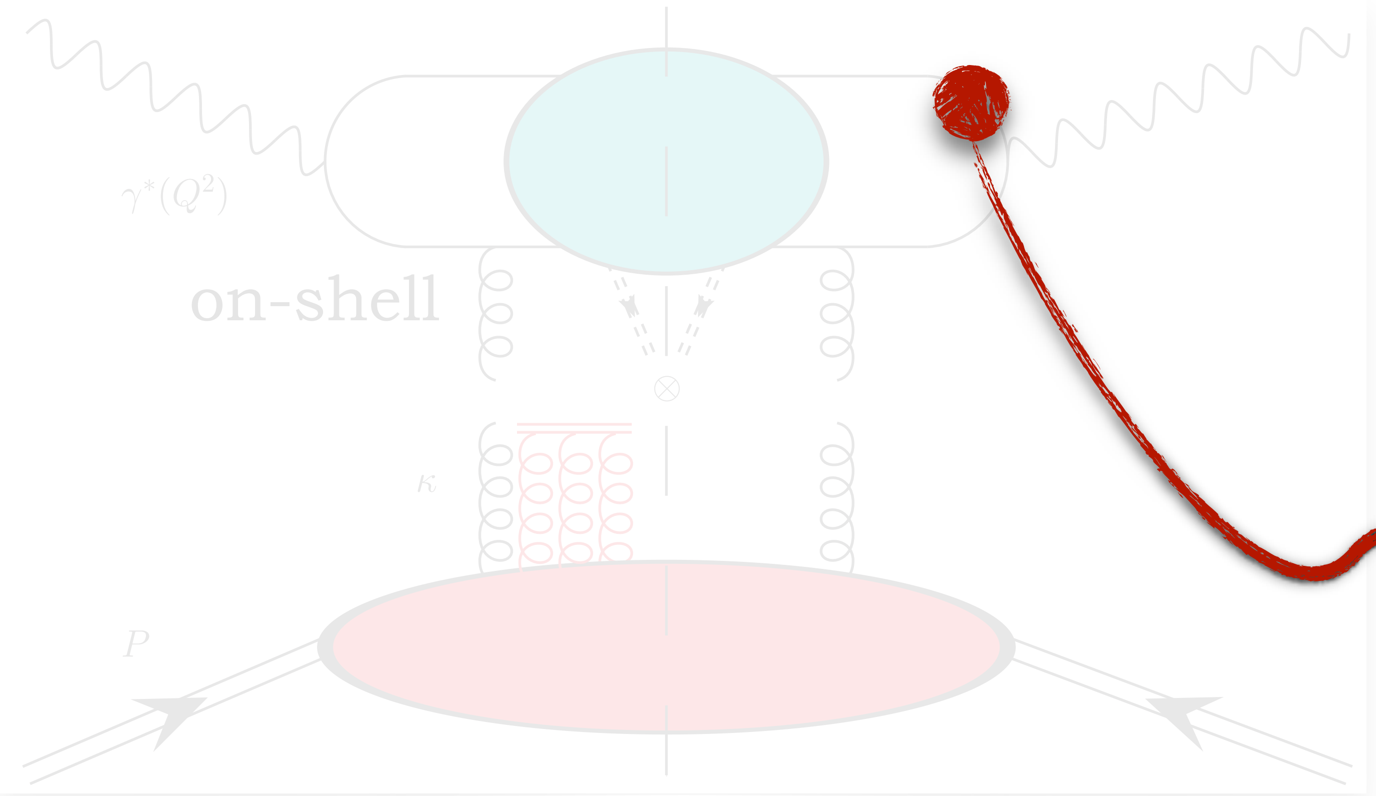
(Fadin-Martin theorem)

[V.S. Fadin, A.D. Martin (1999)]

IR diffusion pattern

(Bartels' cigar)

[J. Bartels, H. Lotter (1993)]



TMD PDF

Q^2

Q_0^2

HEF

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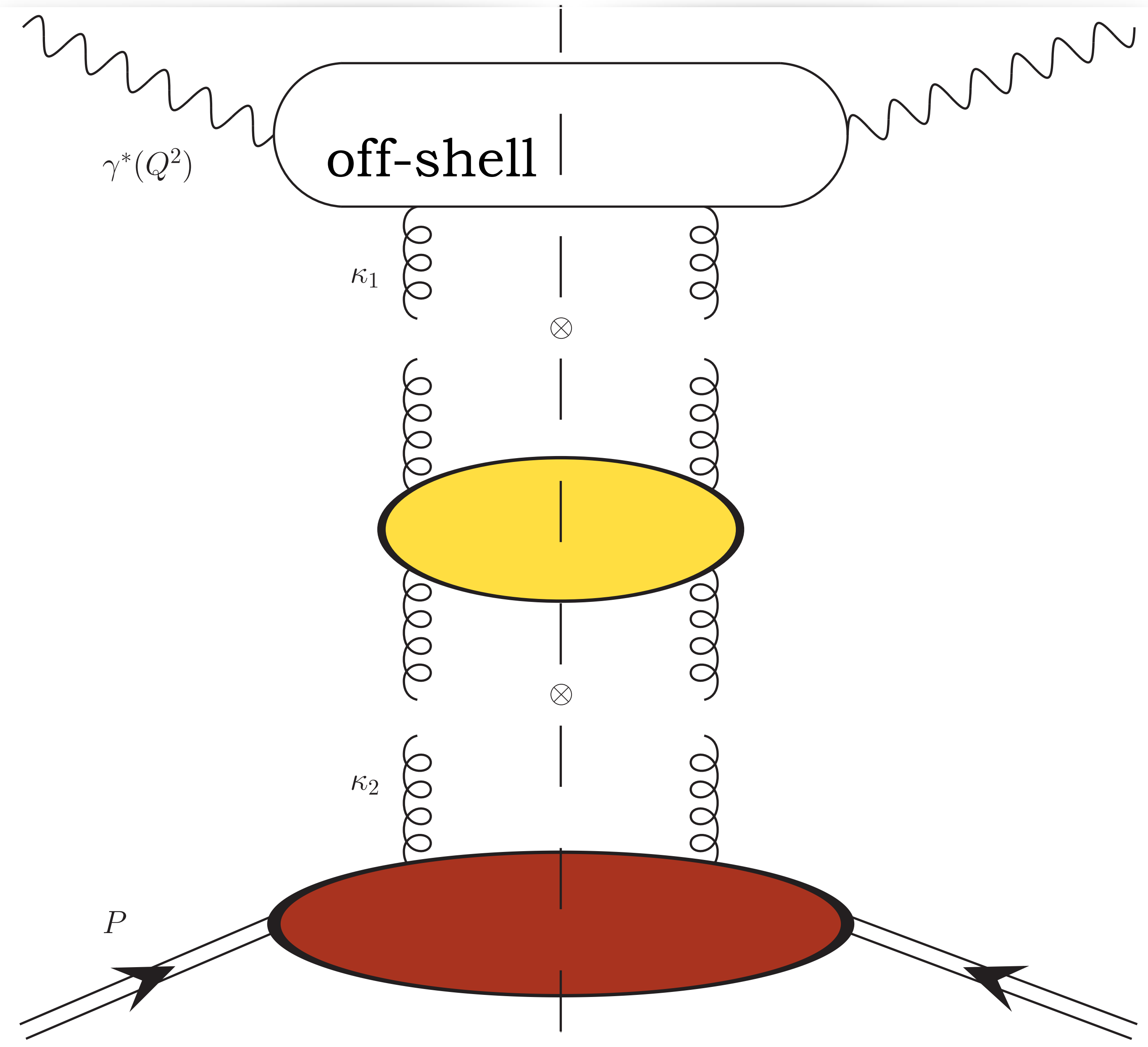


$\Phi^{\gamma^* \to \gamma^*}$

$\otimes$

$\mathcal{G}_{\text{BFKL}}$

$\otimes$

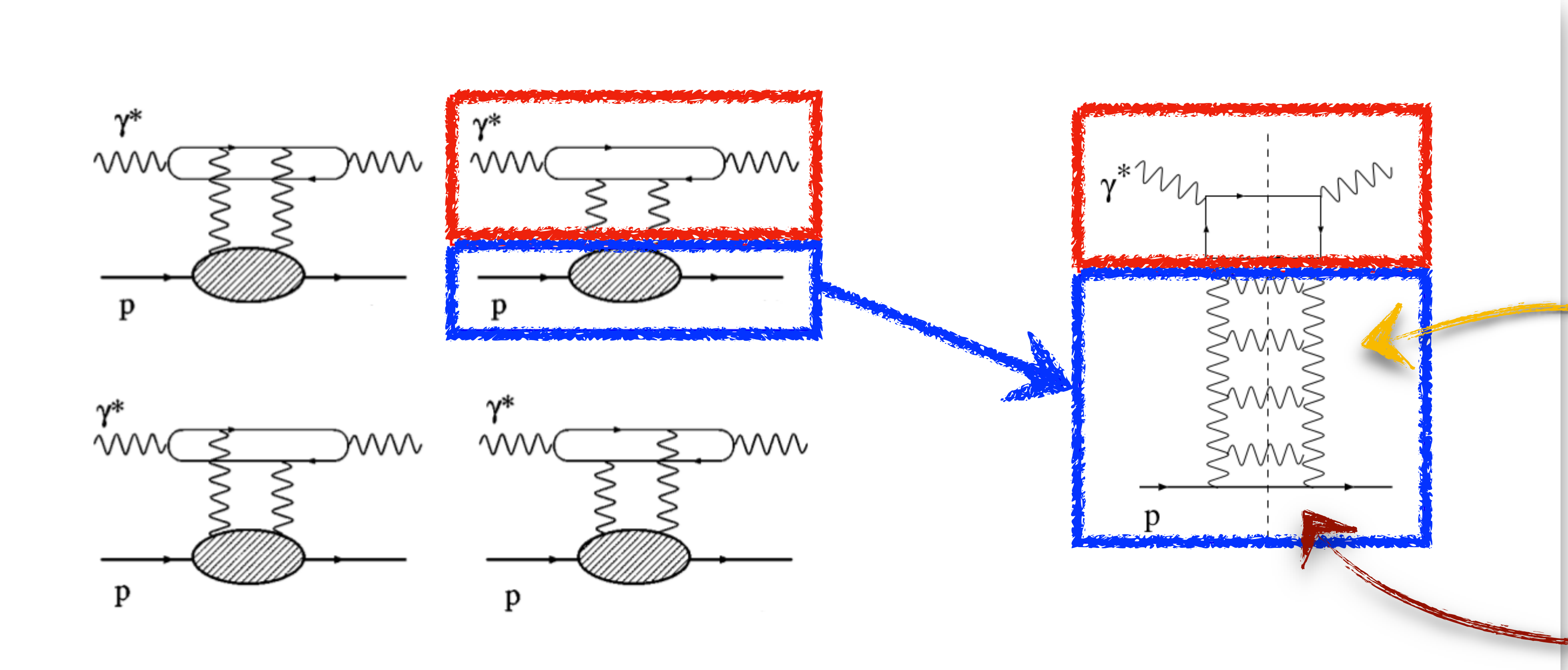
$\Phi^{P}_{[\text{NP}]}$

High-energy factorization and the UGD

- example: **virtual photoabsorption** in **high-energy factorization**

$$\sigma_{\text{tot}}(\gamma^* p \to X) \;\propto\; \mathcal{I}m_s\{\mathcal{A}(\gamma^* p \to \gamma^* p)\} \;\equiv\; \Phi_{\gamma^* \to \gamma^*} \circledast \mathcal{F}(x, \kappa^2)$$

◇ $\mathcal{F}(x, \kappa^2)$ is the **unintegrated gluon distribution** (**UGD**) in the proton



▶ Small-x limit: **UGD** $=$ [**BFKL gluon ladder**] $\circledast$ [**proton impact factor**]

◇ Takes into account the resummation of high-energy logs

◇ Describes the coupling of the gluon Green's function to the proton

▶ Proton impact factor is non-perturbative $\Longrightarrow$ UGD needs to be modeled!

EXCLUSIVE VECTOR MESONS

Diffractive slope

Empirical parametrization → introduces *smaller* uncertainties than UGD ones

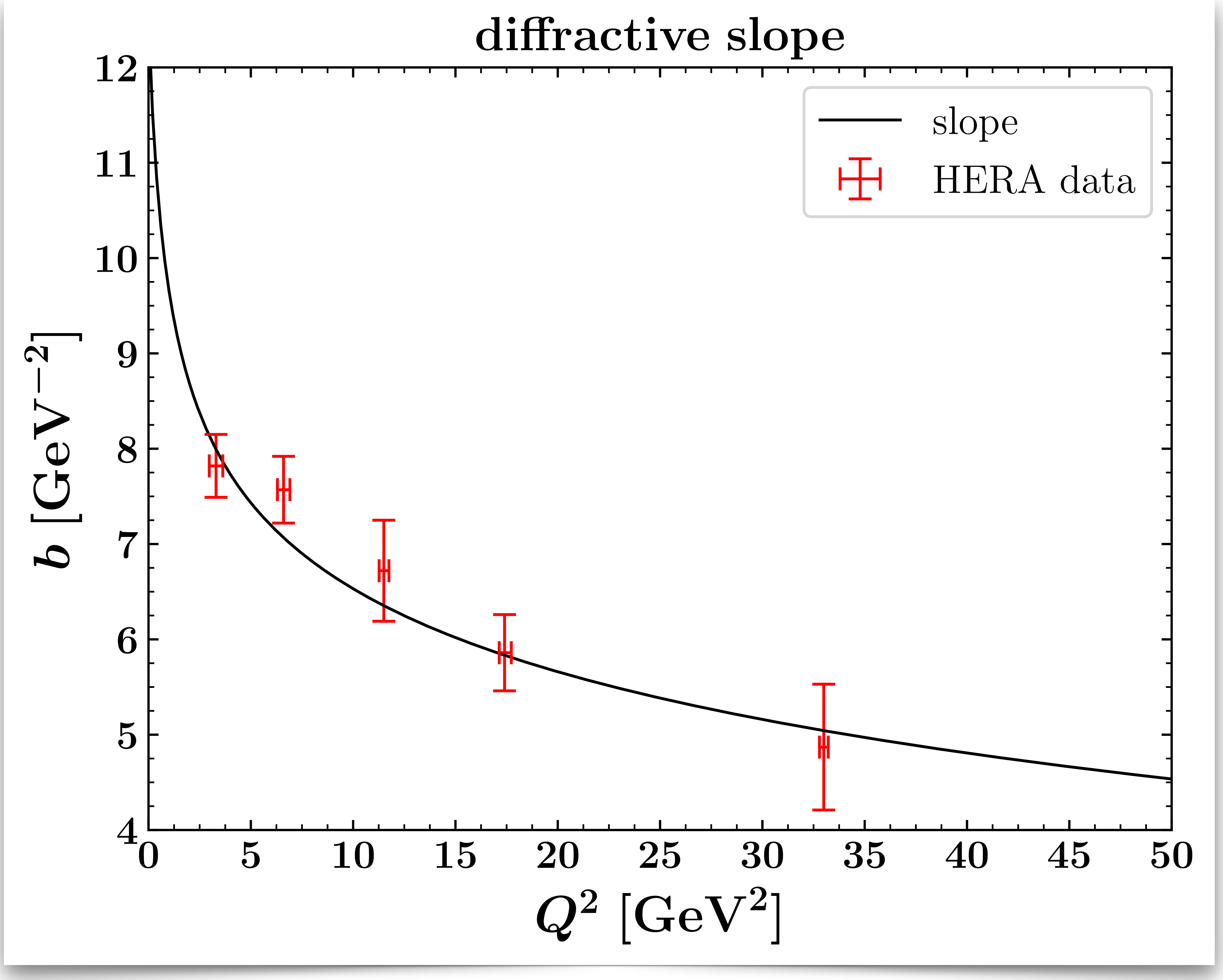
[J. Nemchik, N.N. Nikolaev, E. Predazzi, B.G. Zakharov, V. R. Zoller (1998)]

$$b(Q^2) = \beta_0 - \beta_1 \log\left[\frac{Q^2 + m_\rho^2}{m_{J/\psi}^2}\right] + \frac{\beta_2}{Q^2 + m_\rho^2}$$

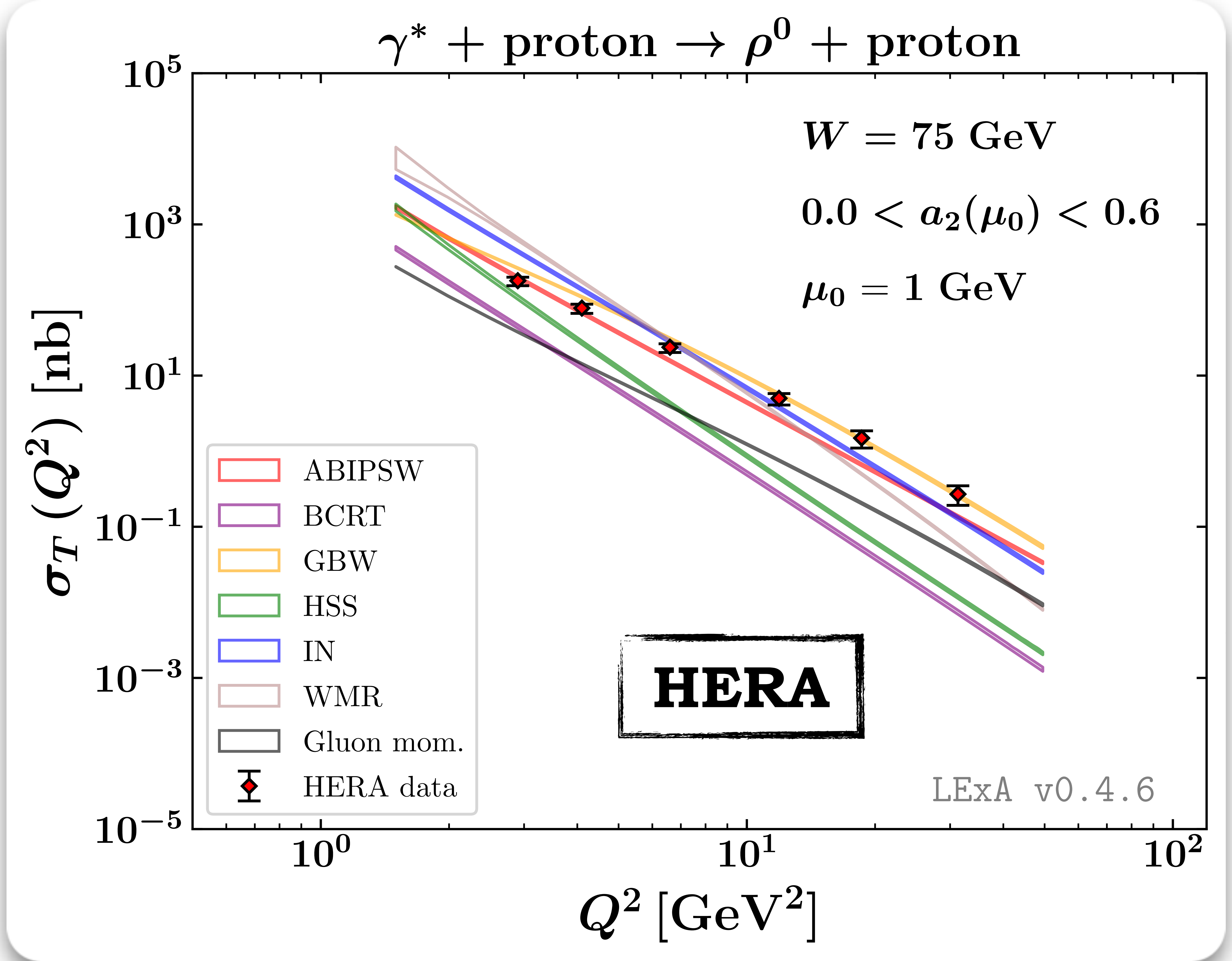
$$\sigma_L(\gamma^* p \to \rho p) = \frac{1}{16\pi b(Q^2)}\left|\frac{T_{00}(s, t=0)}{W^2}\right|^2$$

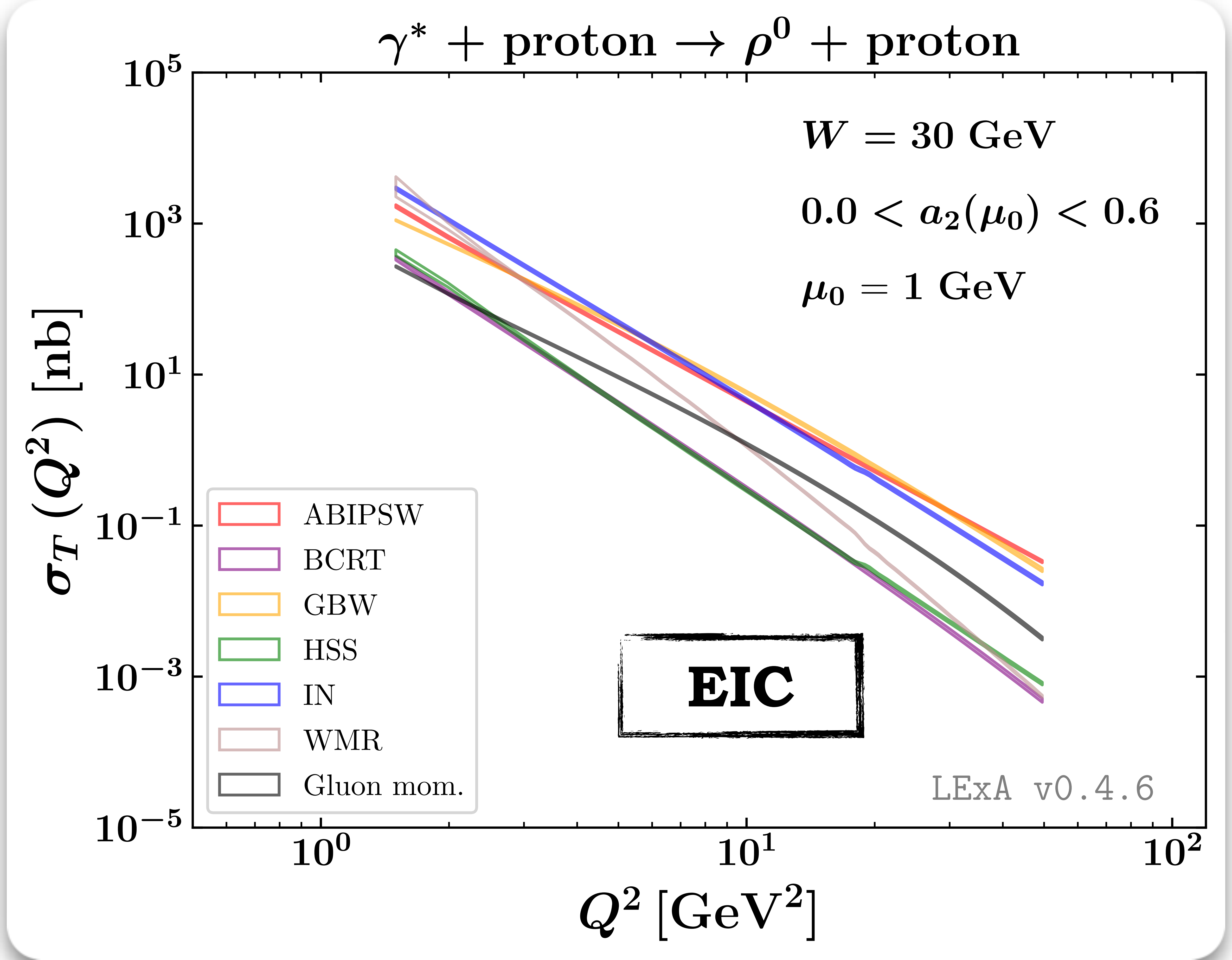
$$\sigma_T(\gamma^* p \to \rho p) = \frac{1}{16\pi b(Q^2)}\left|\frac{T_{11}(s, t=0)}{W^2}\right|^2$$

$$\beta_0 = 6.5\ \mathrm{GeV}^{-2},\ \beta_1 = 1.2\ \mathrm{GeV}^{-2} \text{ and } \beta_2 = 1.6$$



Exclusive forward ρ-meson: $\sigma_T(Q^2)$





(saturation effects in ρ-meson polarization @HERA) [A. Besse, L. Szymanowski, S. Wallon (2013)]

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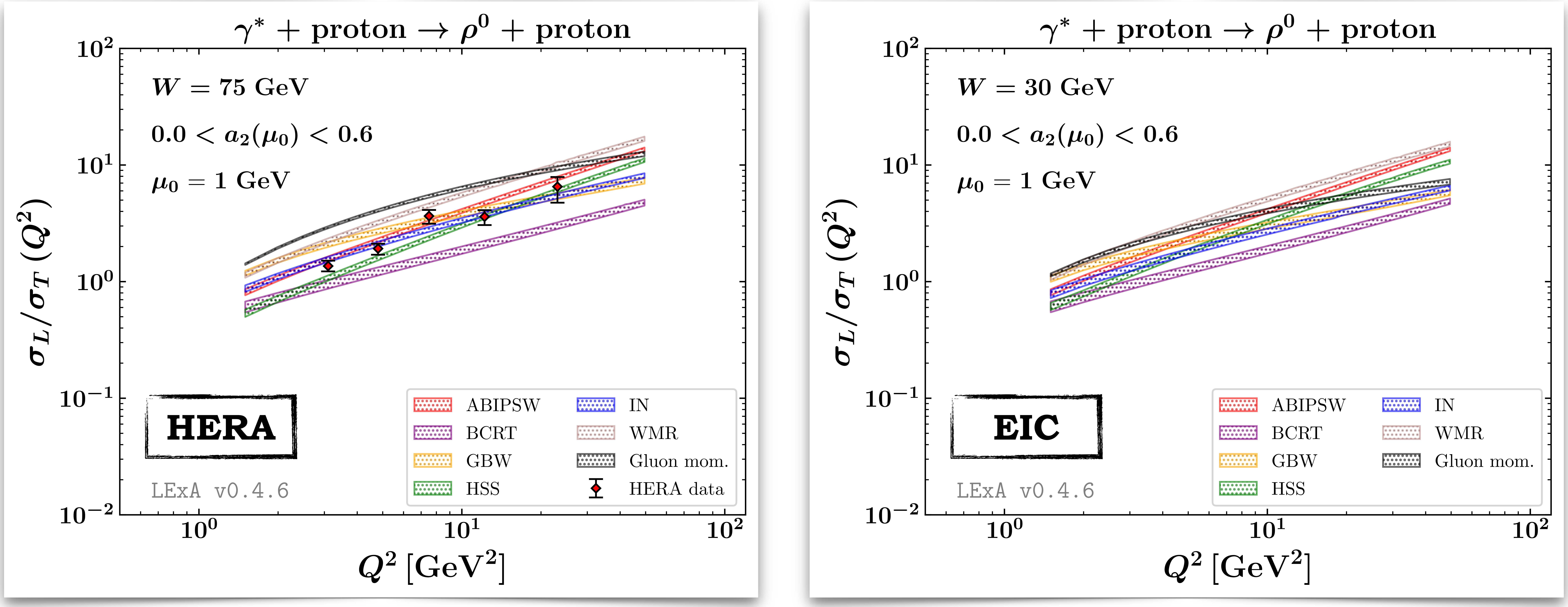

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Single forward emissions

Exclusive light VM: ρ^0, ω, ϕ

* *Small-size* dipoles $\Rightarrow$ large κ_T
* Collinear description: twist-2/-3 LVM NP **DAs**

$$\Phi^{\gamma^* \to \rho} \propto \int_0^1 \mathrm{d}z \, T_H^{\gamma^* \to \rho}(z, \kappa_T, Q, \mu_R, \mu_F) \, \phi^{\lambda_\rho}(z, \mu_F)$$

* Significance of small κ_T under investigation…
* HERA indication: no large-r_d dynamics
* LVMs as tools: discrimination among UGD models
* LVMs as tools: UGD <u>*extraction*</u> $\Leftarrow$ HERA + **EIC** <u>*fits*</u>

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Quarkonia

- Size of dipoles $\Rightarrow$ wide range of κ_T
- Description: **NRQCD** (combined with LFWFs)

$$\left[\,\text{LFWF} \otimes \mathscr{A}_{\text{dip.}}\right] \overset{\text{dilute}}{\longleftrightarrow} \left[\,\Phi^{\gamma^* \to J/\Psi} \otimes \text{UGD}\right]$$

- Validity of *small-size* dipoles questionable…
- NRQCD: large-$r_{\rm d}$ dynamics for $\Psi(2s)$ ($\Upsilon(2s)$?)

🔗 [K. Suzuki *et al.* (2000)]; 🔗 [J. Cepila *et al.* (2019)]; 🔗 [M. Hentschinski *et al.* (2020)]

- Onia as tools: <u>*scan*</u> of TMD/HEF <u>*intersection*</u> range

FORWARD DRELL-YAN

Inclusive forward Drell-Yan dilepton production

- LHC, **forward region** $\rightarrow$ $(l^+ l^-)$ produced in the fragmentation region of h_2
  - ◇ Asymmetric configuration: $x_1 \gg x_2$, down to $x_2 \simeq 10^{-6}$
  - $\Longrightarrow$ **possible small-x resummation effects expected!**
- **small-x** $\rightarrow$ evolution of sea $q\,(\bar{q})$ inside h_2 driven by gluon evolution
  - ◇ Dominance of sea $q\,(\bar{q})$ emerging in the last splitting (suppression of quark propagator at large rapidity)
- **high-energy factorization** $\rightarrow$ gluon exchange in the t-channel
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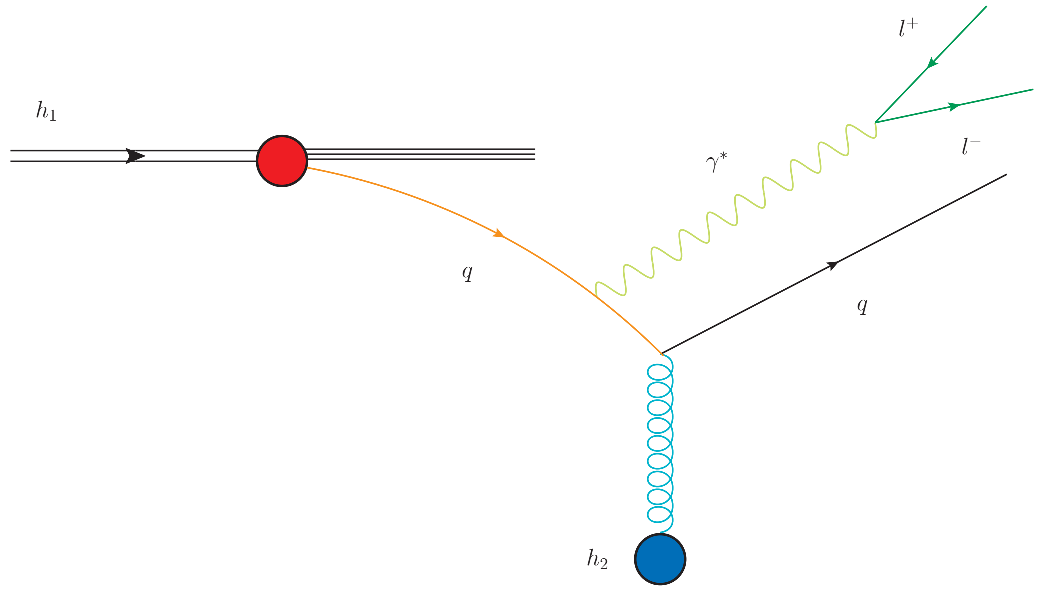

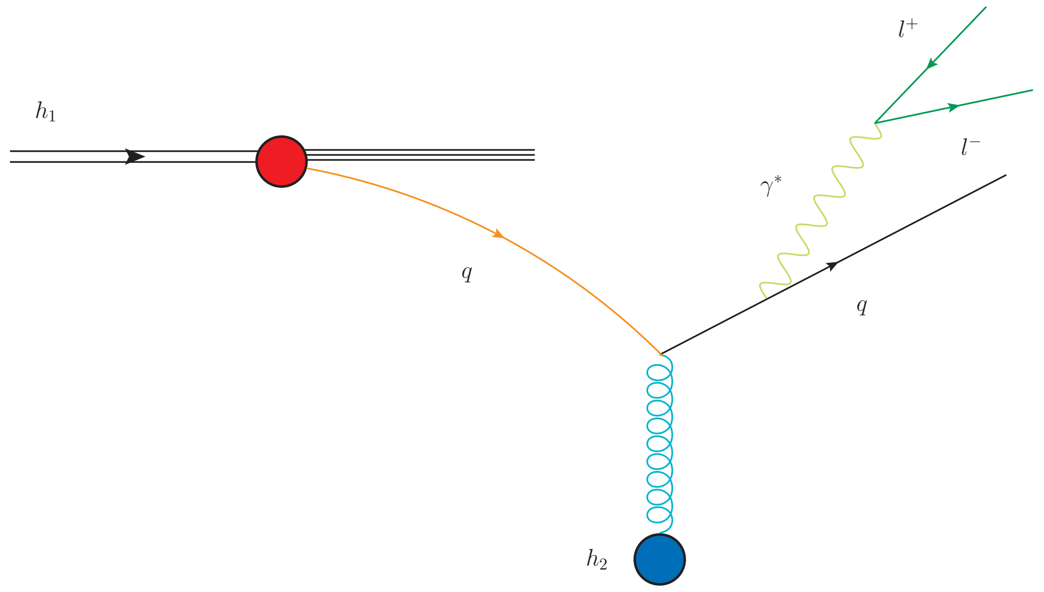

- Helicity structure functions in high-energy factorization:

$$\mathcal{W}_{[\lambda]} = \frac{2\pi M^2}{3} \int_{x_F}^{1} \frac{dz}{z^2} \sum_{r=q,\bar{q}} f_r\left(\frac{x_F}{z}, \mu_F\right) \int \frac{d\kappa_T\, d\phi_{\kappa_T}}{(\kappa_T^2)^2}\, \alpha_s(\mu_R)\, \mathcal{F}(x_g, \kappa_T^2)\, \Phi_{[\lambda]}(q_T, \vec{\kappa}_T, z)$$

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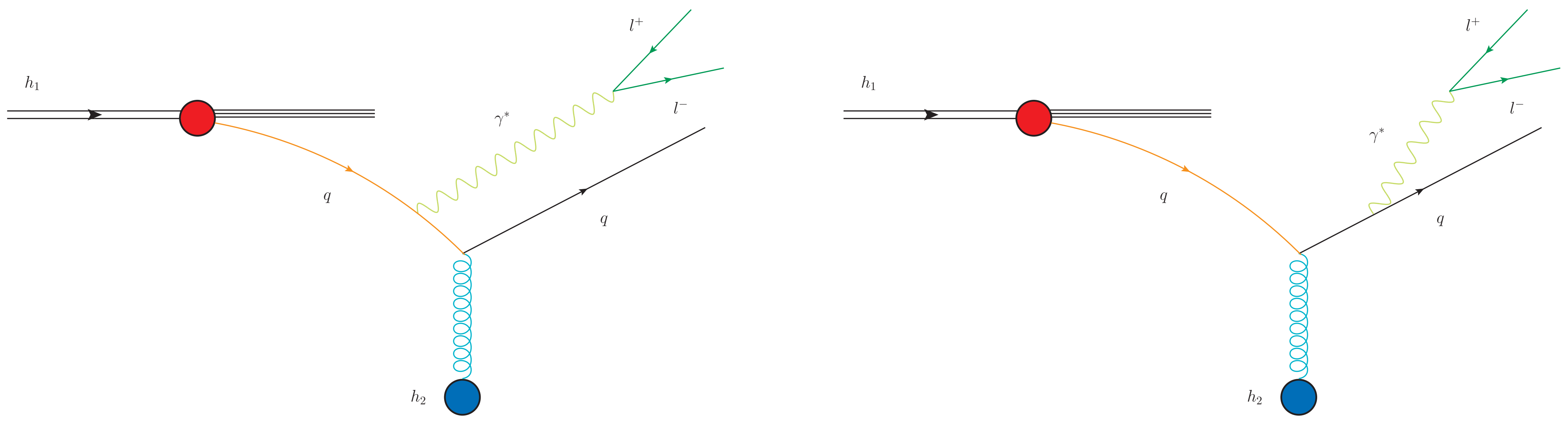

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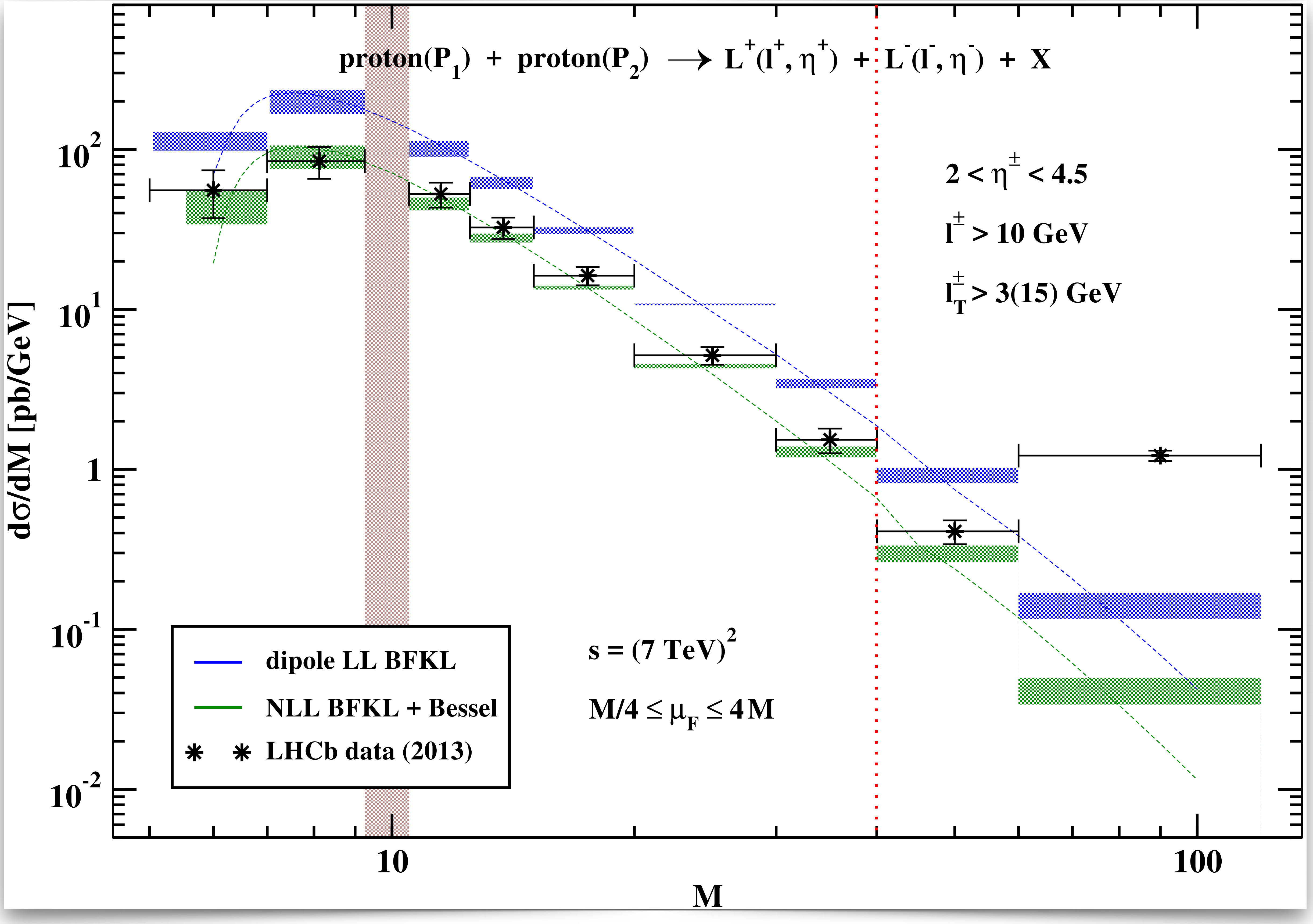

🔗 [D. Brzeminski, L. Motyka, M. Sadzikowski, T. Stebel (2017)]

🔗 [F.G.C., D. Gordo Gómez, A. Sabio Vera (2018)]

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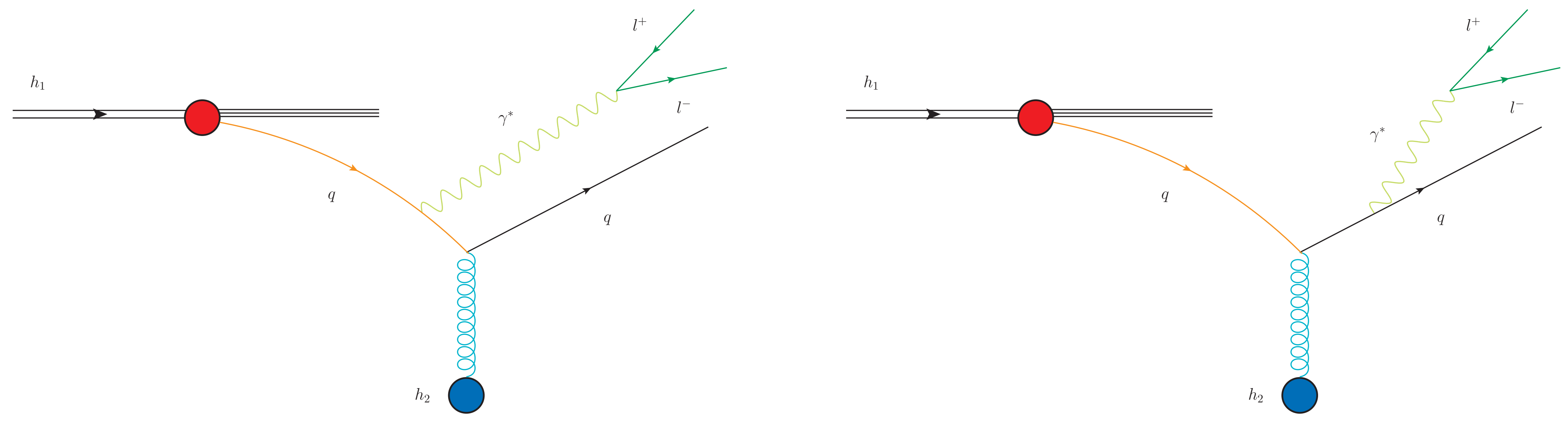

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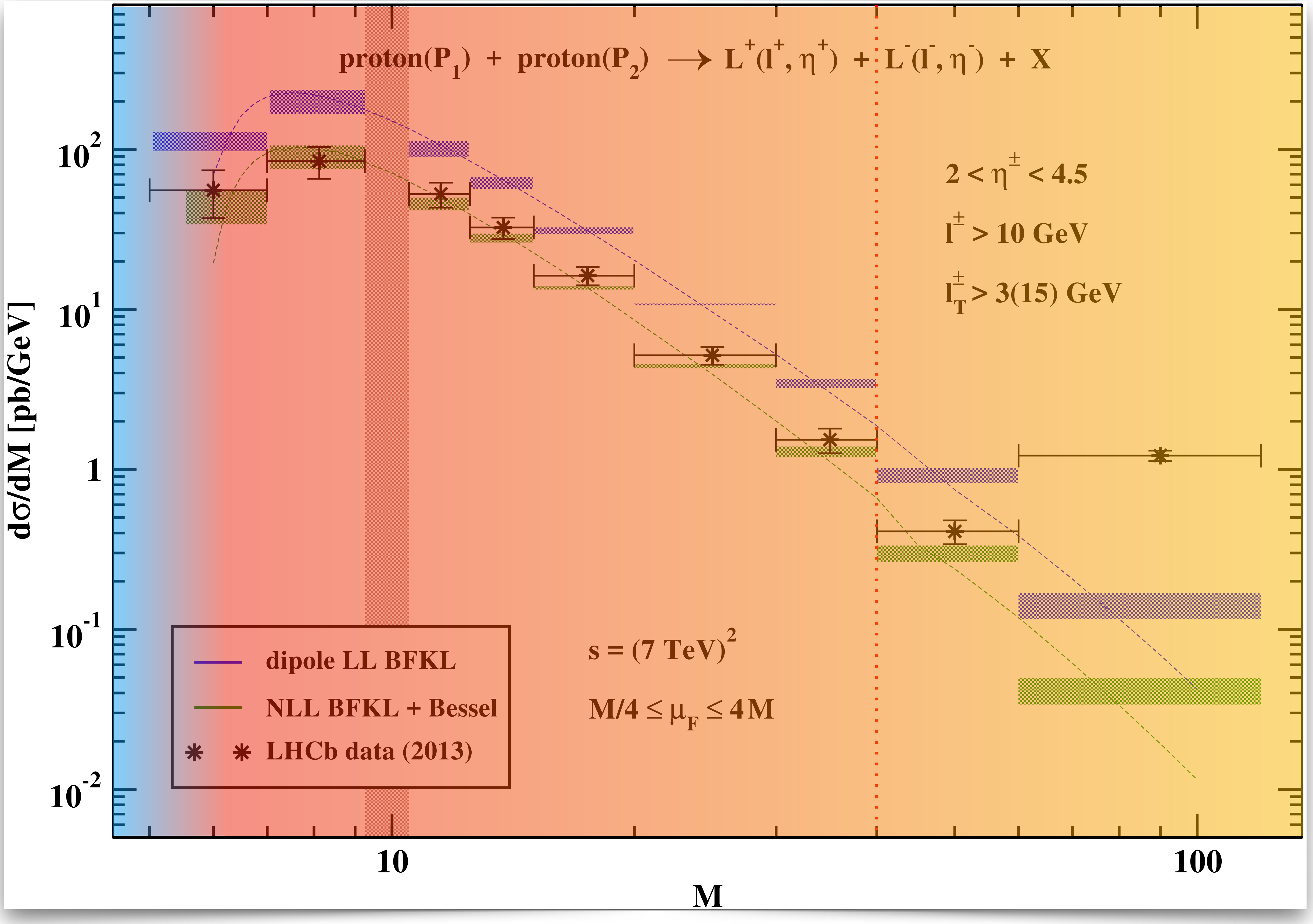
approaching limits of semi-hard regime



Z^0 contribution becoming relevant

Summary...

Exclusive polarized ρ-leptoproduction

[A.D. Bolognino, F.G. C., D.Yu. Ivanov, A. Papa, arXiv:1808.02395, to appear in *Eur. Phys. J. C*]

- T_{11}/T_{00} helicity-amplitude ratio to constrain the **UGD** in the HERA range
- High sensitivity to **distinct UGD models**
- Low sensitivity to **region of small-κ_T values**

  $\xrightarrow{\text{effect of}}$ dominance of **small-size dipoles**

Forward Drell–Yan dilepton production

[F.G. C., D. Gordo Gómez, A. Sabio Vera, *Phys. Lett. B* **786** (2018) 201]

- Good description of $\mathrm{d}\sigma/\mathrm{d}M$ in the **BFKL approach** at the LHC
- Same observable well described also by **fixed-order** calculations

  [LHCb Coll.: LHCb-CONF-2012-013, CERN-LHCb-CONF-2012-013]; [ATLAS Coll.: G. Aad et al., *JHEP* **1406** (2014) 112]

- Future data for Drell–Yan production in **forward directions**

  $\xrightarrow{\text{peerless to}}$ gauge need for **high-energy resummation**

SMALL-X GLUON TMDS

Gluon TMD PDFs at leading twist

gluon pol.

nucleon pol.	U	circ.	lin.
U	f_1^g		$h_1^{\perp g}$
L		g_1^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_1^g,\ h_{1T}^{\perp g}$

T-even

T-odd

Gluon TMD PDFs at leading twist

Unpolarized TMD PDF

Boer-Mulders

gluon pol.

nucleon pol.	U	circ.	lin.
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T-even

T-odd

Helicity TMD PDF

Worm-gear

Gluon TMD PDFs at leading twist

gluon pol. / nucleon pol.

nucleon pol. \ gluon pol.	U	circ.	lin.
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T	$f_{1T}^{\perp g}$	g_{1T}^g	h_1^g, $h_{1T}^{\perp g}$

Unpolarized TMD PDF

Propeller

Boer-Mulders

Sivers

Helicity TMD PDF

Linearity

Worm-gear

Butterfly

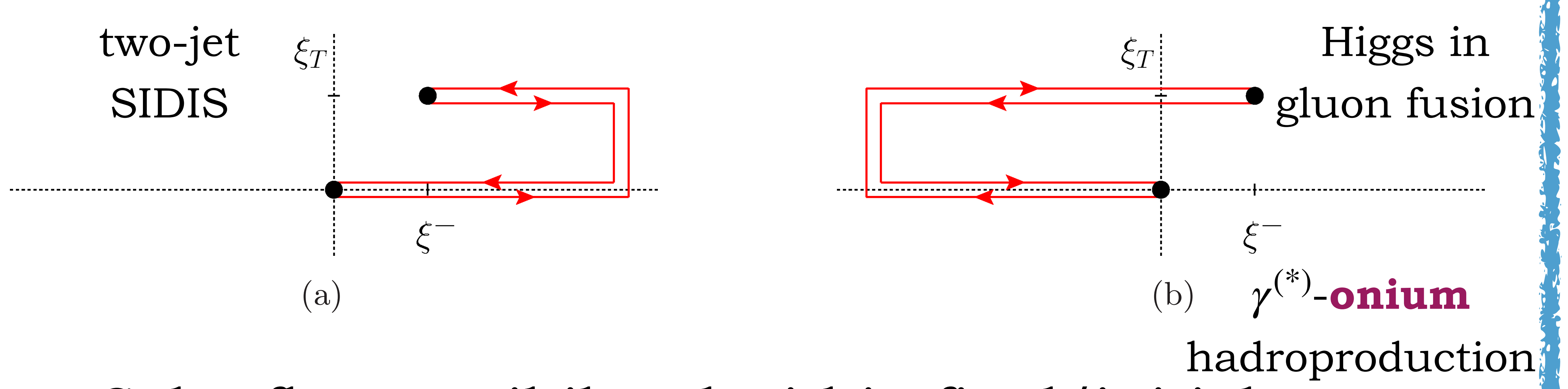
T-even

T-odd

Accessing f-type and d-type gluon TMD PDFs

f-type (WW)

(a) $[+,+]$ or (b) $[-,-]$



- Color flow annihilated within final/initial state
- *f*-type gluon TMDs $\rightarrow$ f^{abc} color structure
- Modified universality:

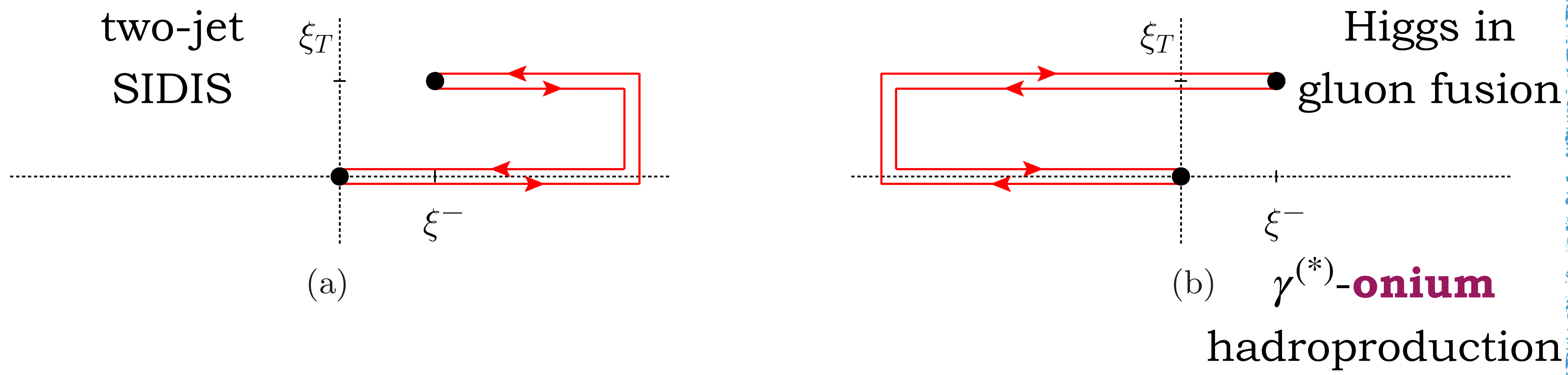
$$f_1^{[+,+]} = f_1^{[-,-]},$$
$$f_{1T}^{\perp[+,+]} = -f_{1T}^{\perp[-,-]}$$

- Phenomenology: Higgs, **quarkonia** or $\gamma\gamma$ in *pp*, two-jet SIDIS, heavy-quark pair SIDIS

Accessing f-type and d-type gluon TMD PDFs

f-type (WW)

(a) $[+,+]$ or (b) $[-,-]$



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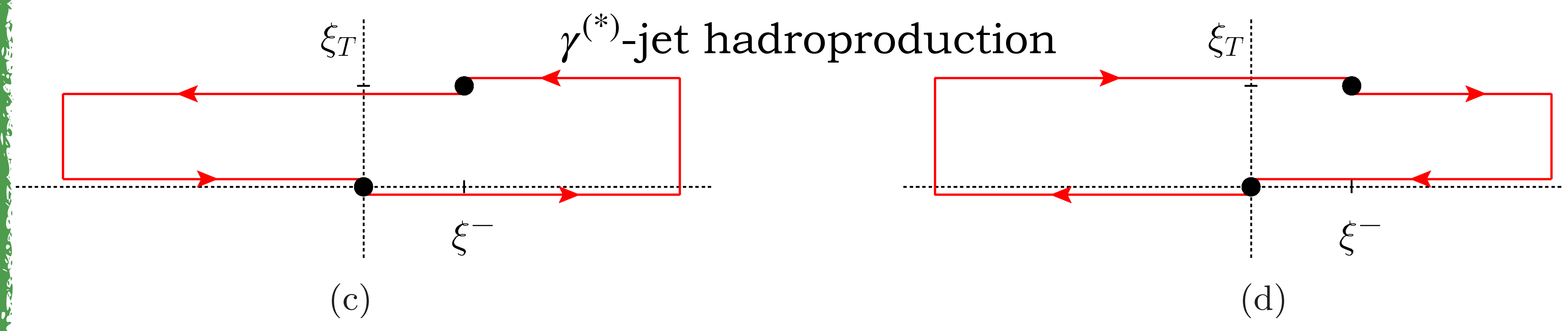
d-type (DP)

(c) $[+,-]$ or (d) $[-,+]$



- Color flow involving both initial and final states
- d-type gluon TMDs $\rightarrow$ d^{abc} color structure
- Modified universality:

$$f_1^{[+,-]} = f_1^{[-,+]}, \quad f_{1T}^{\perp[+,-]} = -f_{1T}^{\perp[-,+]}$$

- Phenomenology: single hadron or $\gamma^{(*)}$-jet hadroproduction, SIDIS or Drell-Yan (subleading)

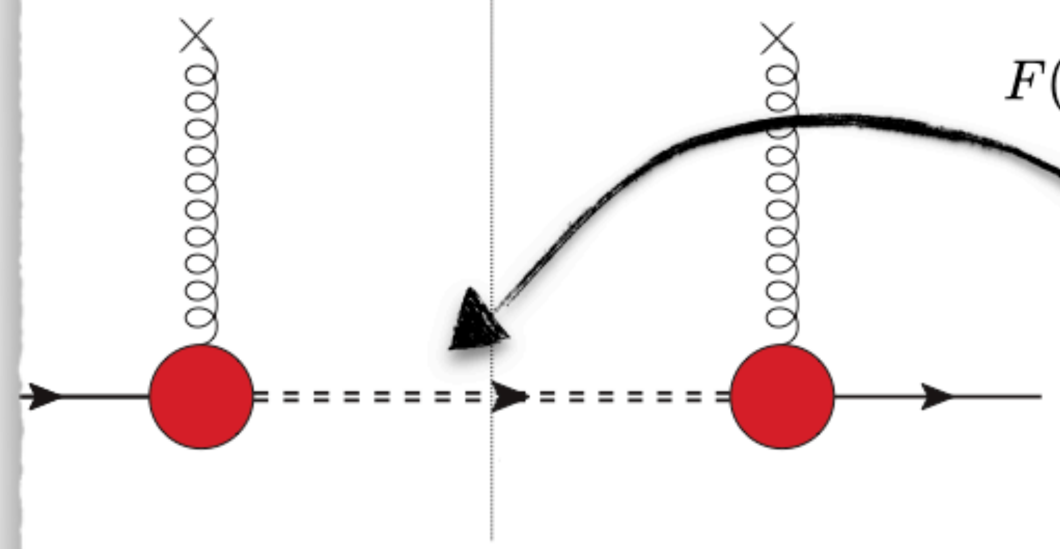
¡ Gauge link $\rightarrow$ two main **independent** sets of TMD PDFs, **not related** to each other !

Spectator-model gluon TMD PDFs

Our model at a glance

Spectator-system spectral-mass function

$$F(x, \boldsymbol{p}_T^2) = \int_M^\infty \mathrm{d}M_X \, \rho_X(M_X) \, \hat{F}(x, \boldsymbol{p}_T^2; M_X)$$

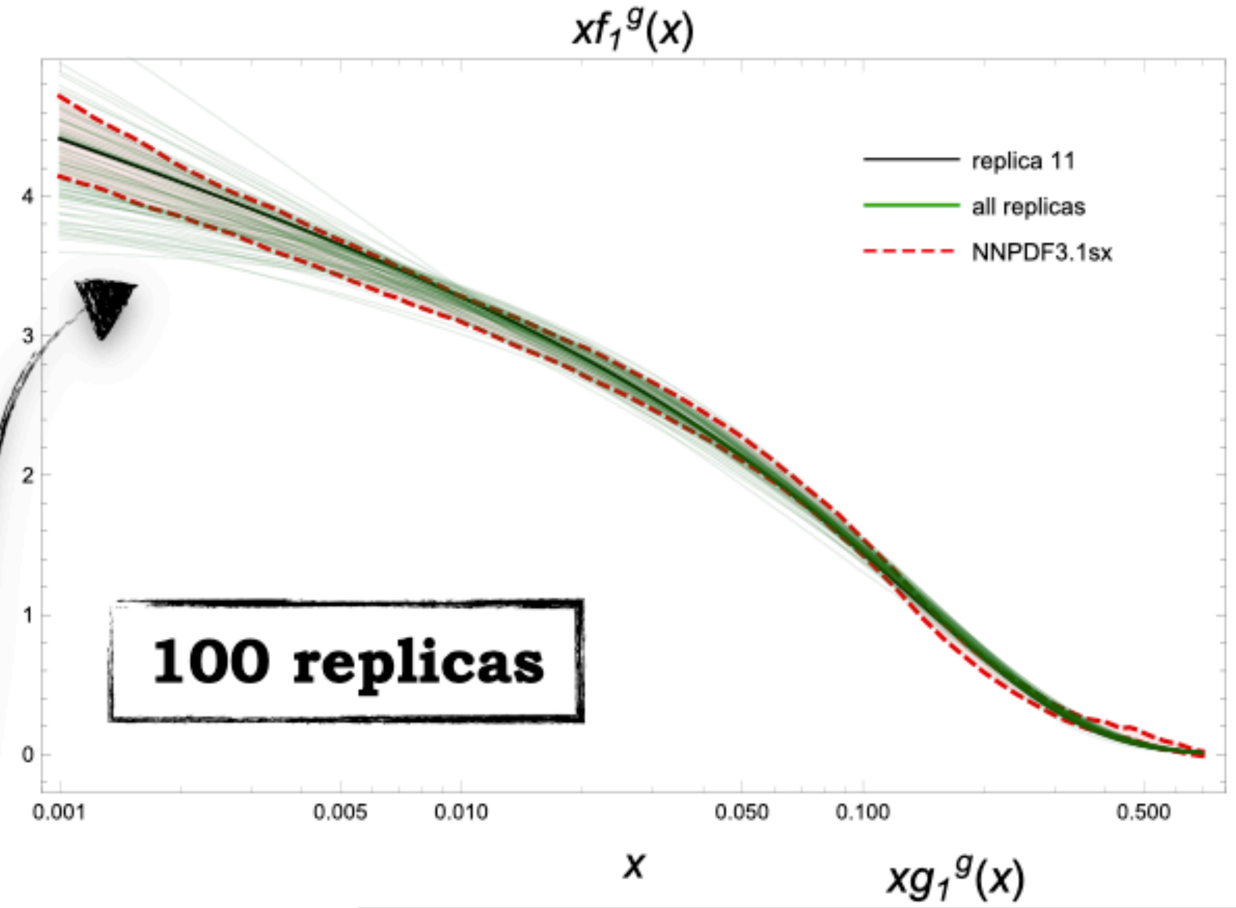


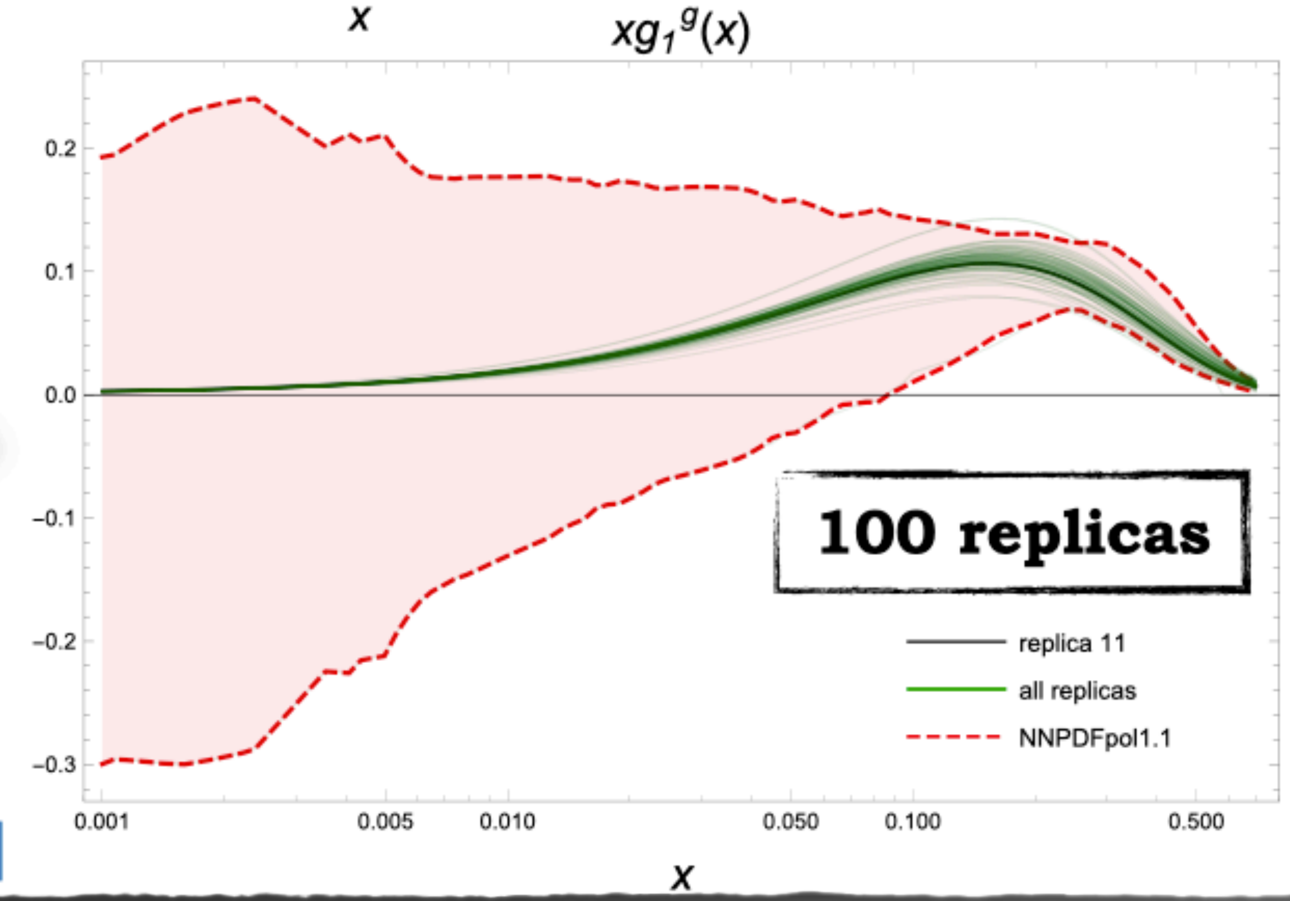
Instead of a single on-shell spectator, a continuum of spectators

Spectral function **learns** small- and moderate-x info encoded in **NNPDF** collinear parametrizations (`NNPDF3.1sx + NNPDFpol1.1`)

Link with collinear factorization

p_T-integrated TMDs **have to** reproduce PDFs at the lowest scale (Q_0) *before* evolution



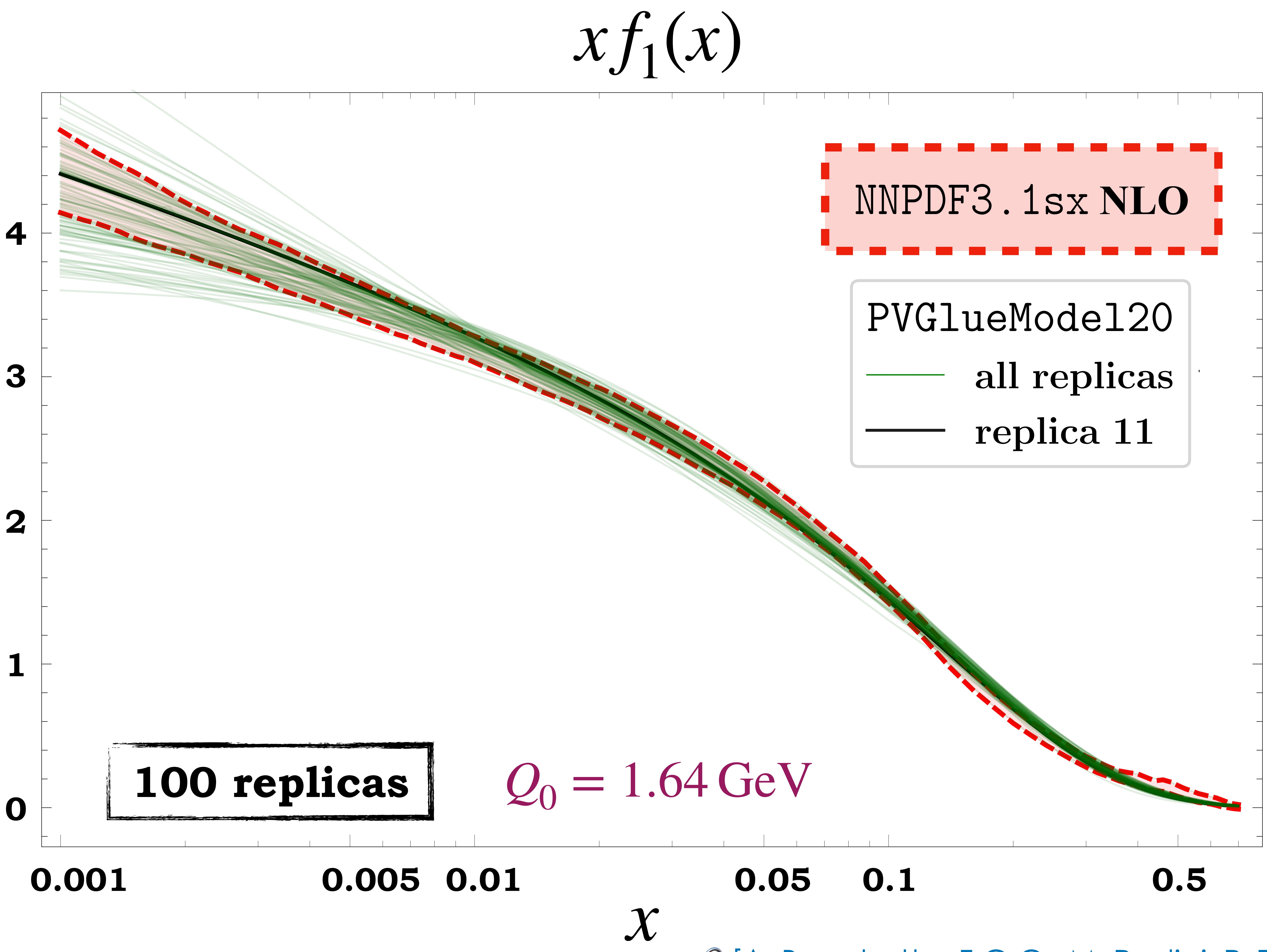


- ☑ **Simultaneous fit** of f_1 and g_1 PDFs
- ☑ Inclusion of small-x resummation effects (**BFKL**)
- ☑ Calculation of all leading-twist T-even gluon TMDs

2.3 Modeling gluon TMDs 🔗 [A. Bacchetta, F.G. C., M. Radici, P. Taels (2020)] 14

🔗 [my talk at the EIC User Group Early Career Workshop 2022]

Unpolarized gluon collinear PDF



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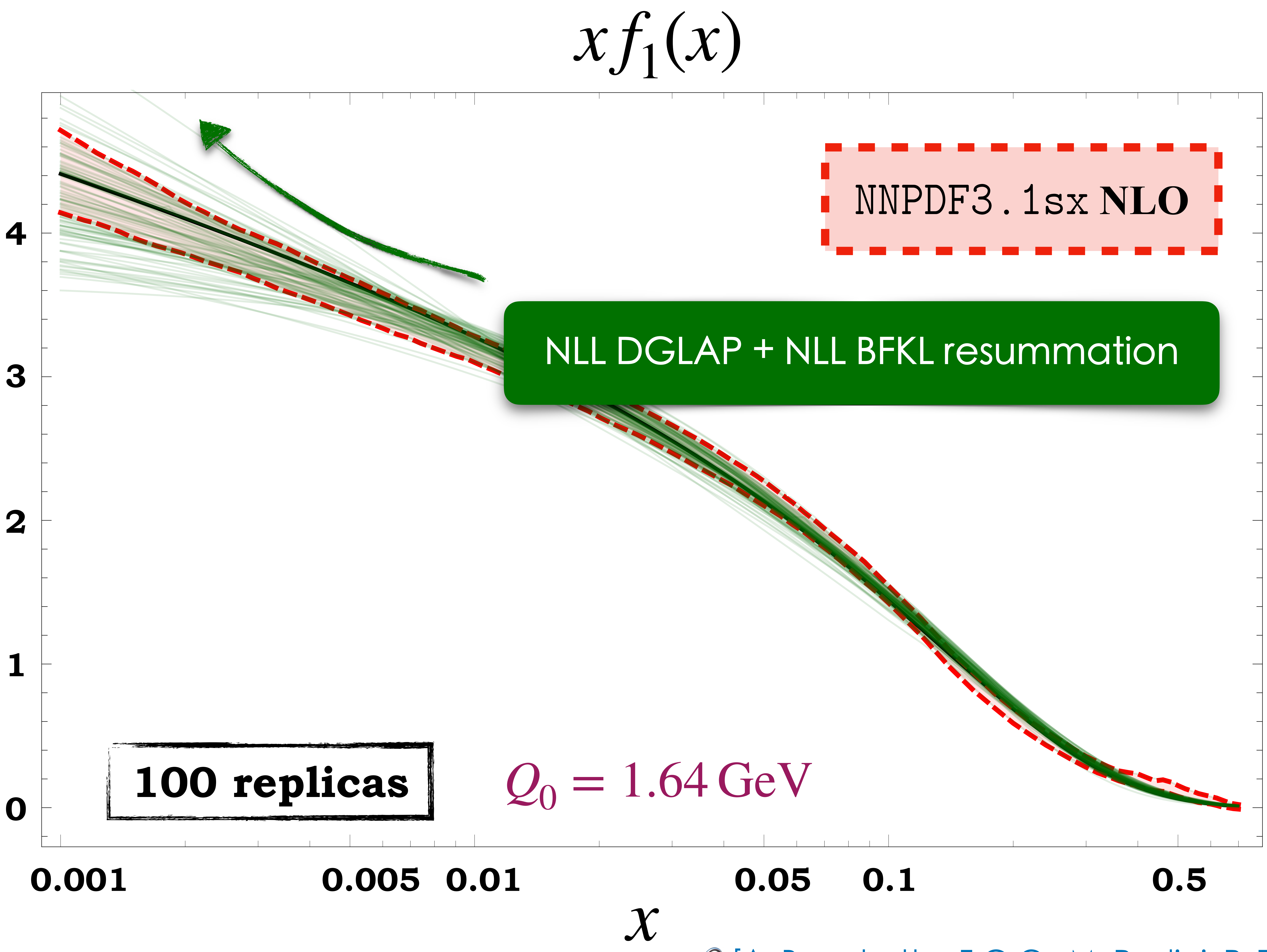

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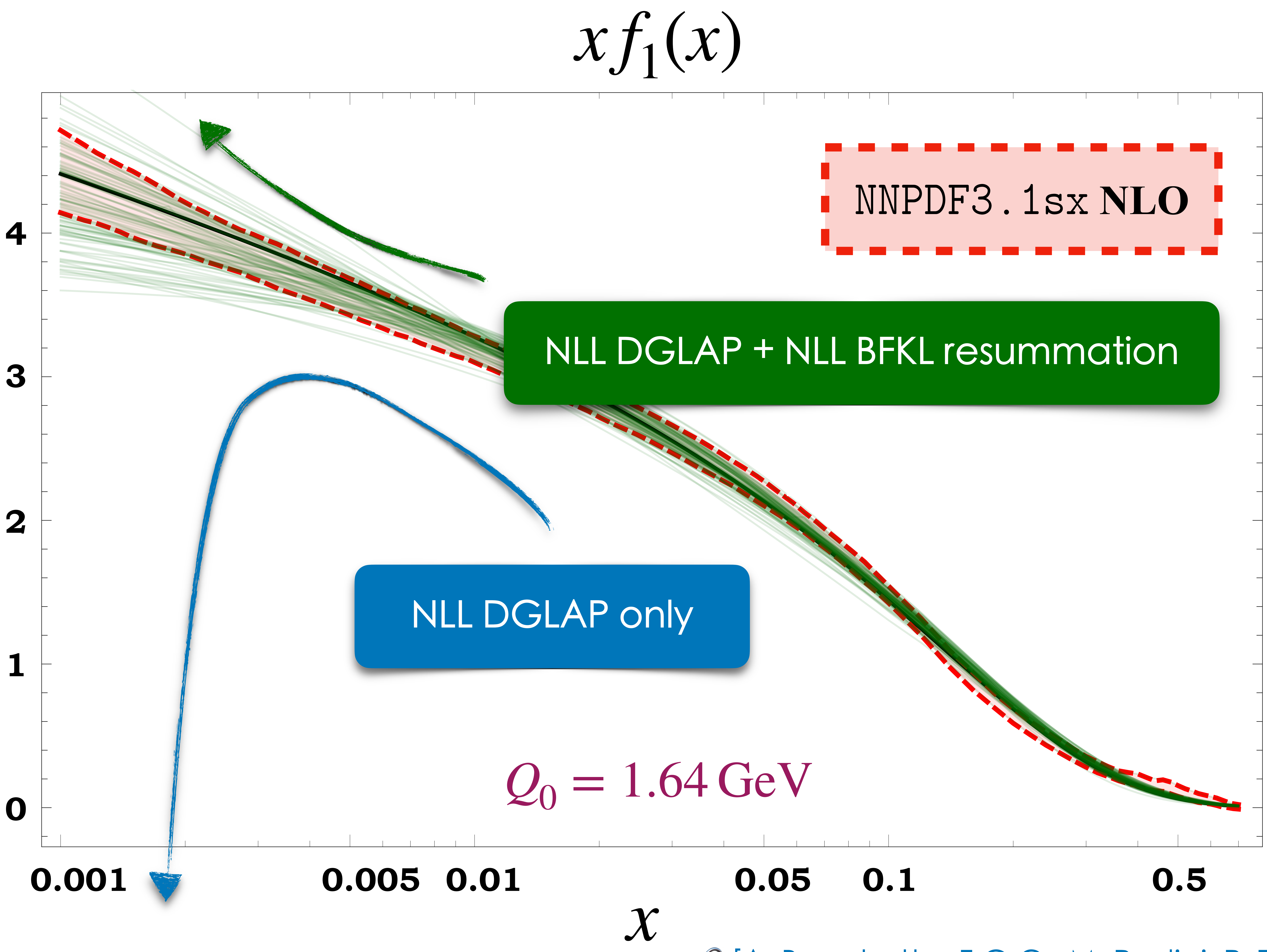

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Unpolarized gluon TMD PDF

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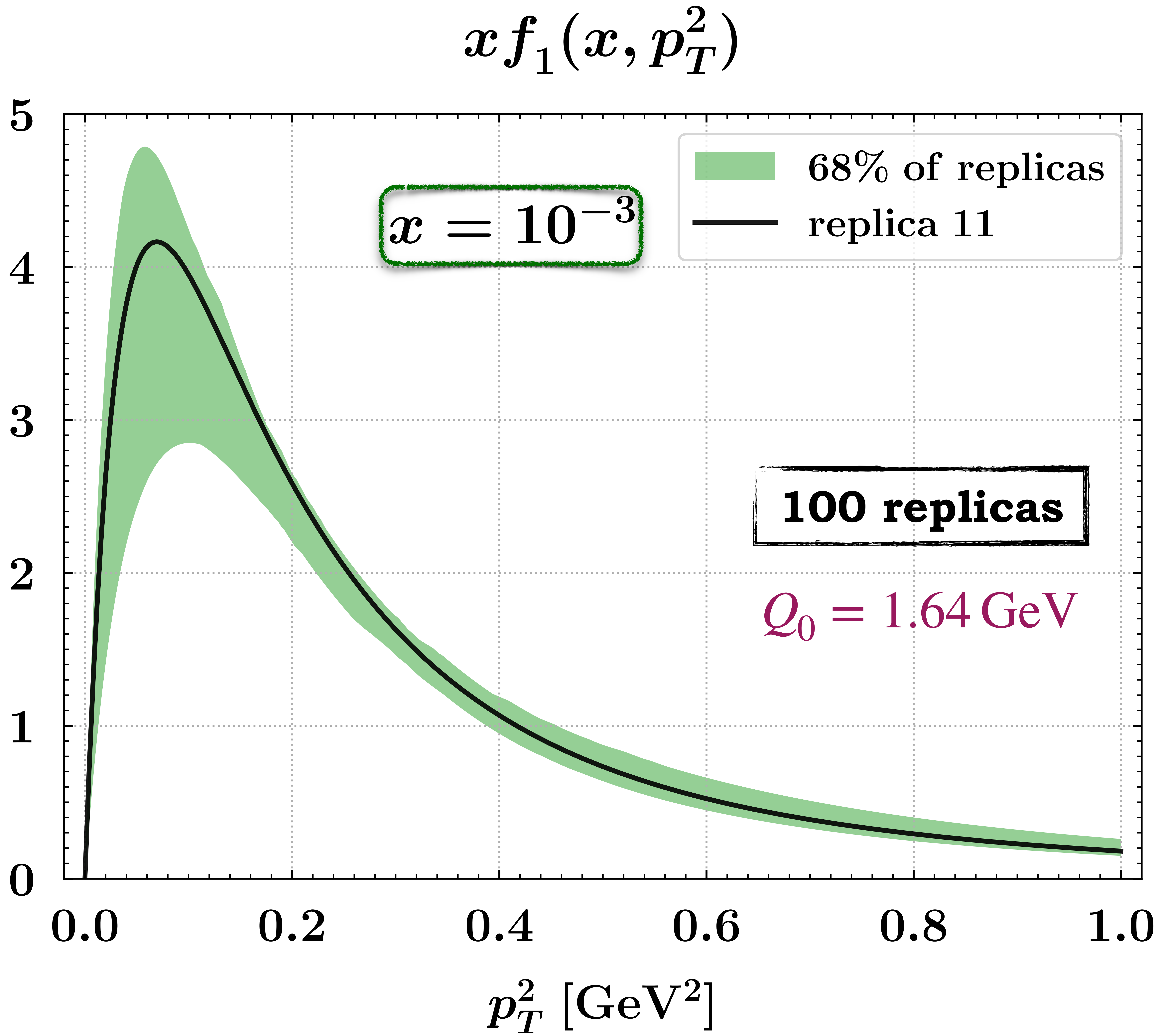
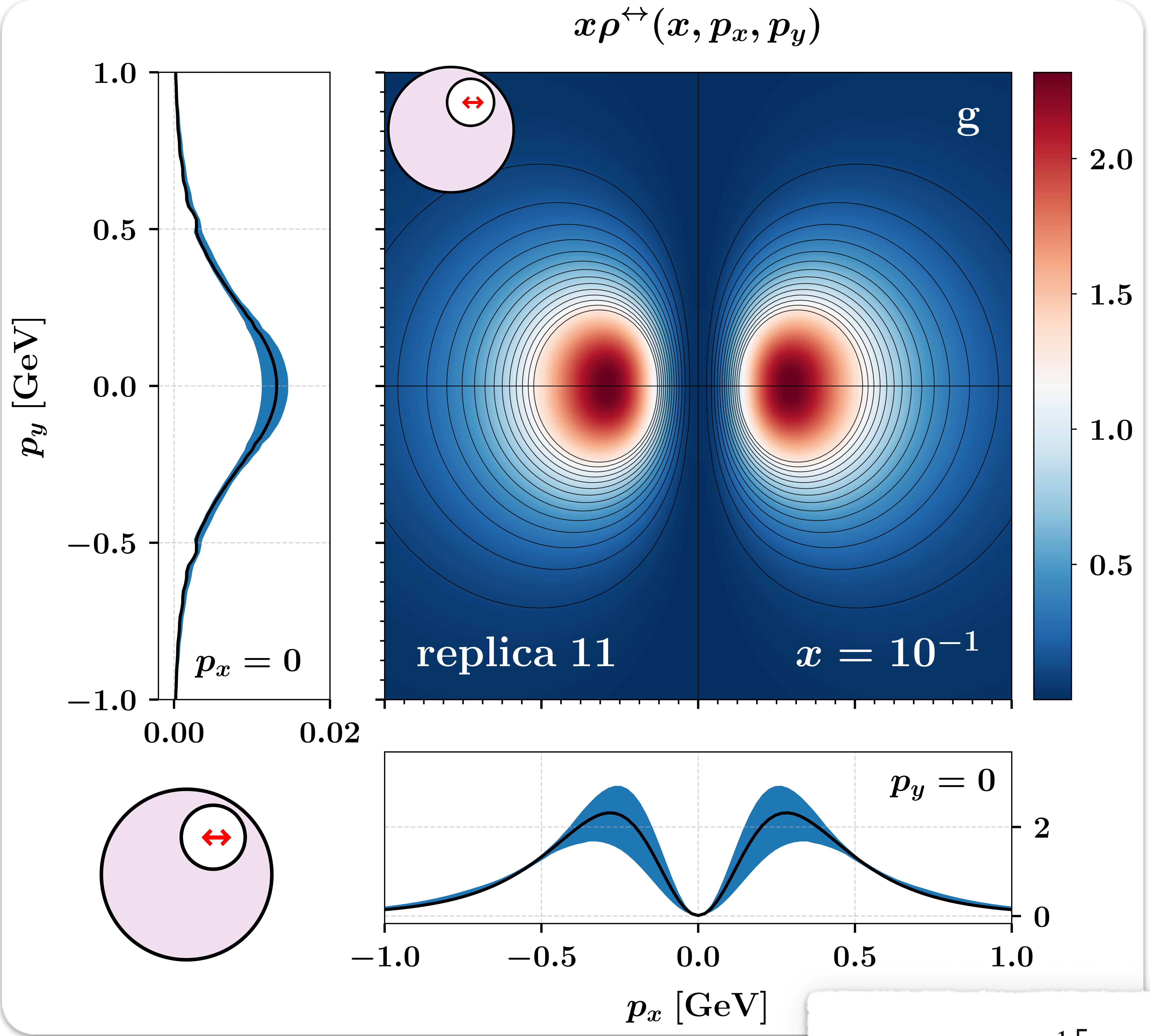


Gluon Boer-Mulders effect

🔗 [A. Bacchetta, F.G.C., M. Radici, P. Taels (EPJC 2020)]



$$x\rho^{\leftrightarrow}(x, p_x, p_y) = \frac{1}{2}\Big[xf_1^g(x, \boldsymbol{p}_T^2) + \frac{p_x^2 - p_y^2}{2M^2}\, xh_1^{\perp g}(x, \boldsymbol{p}_T^2)\Big]$$

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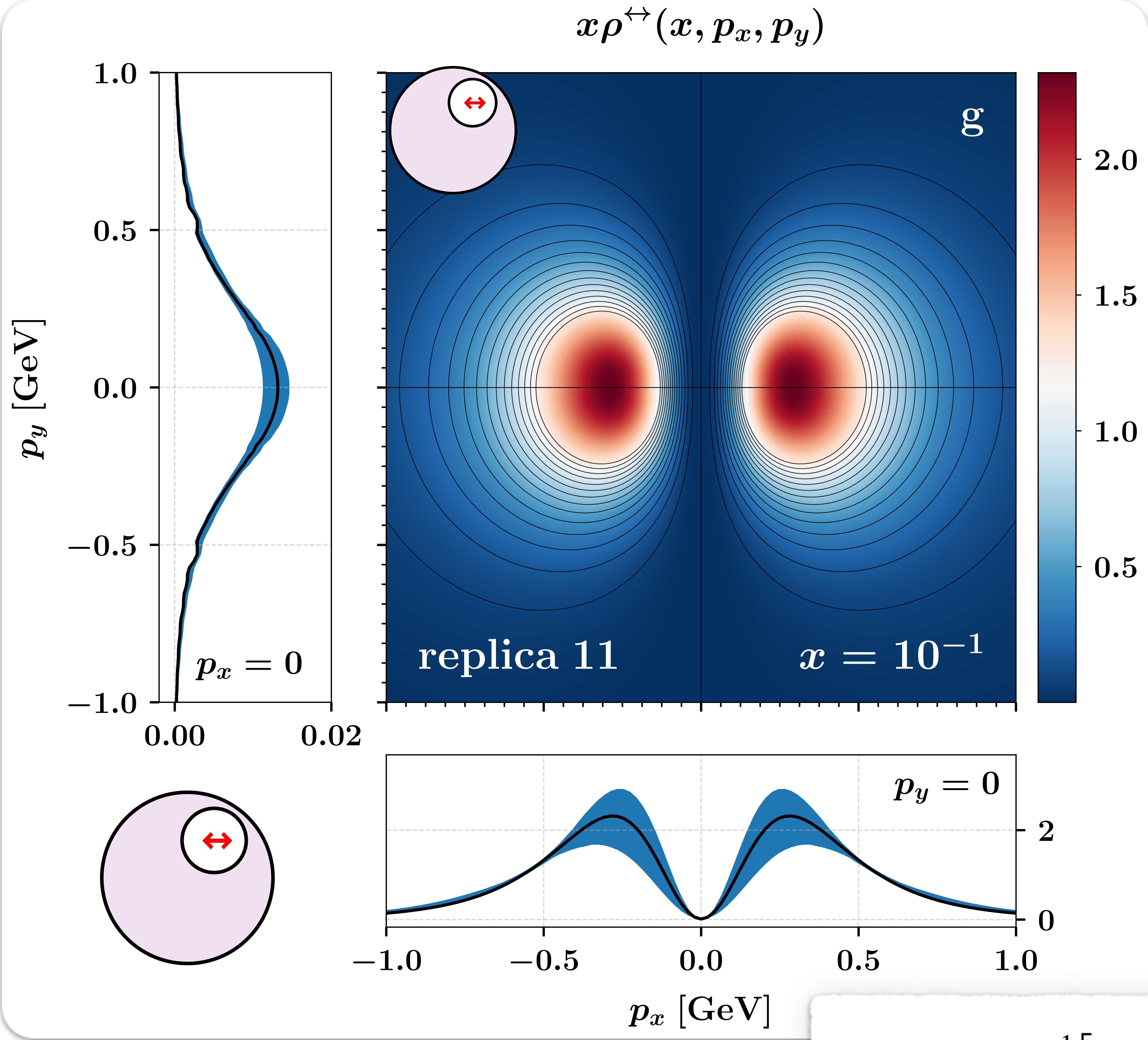

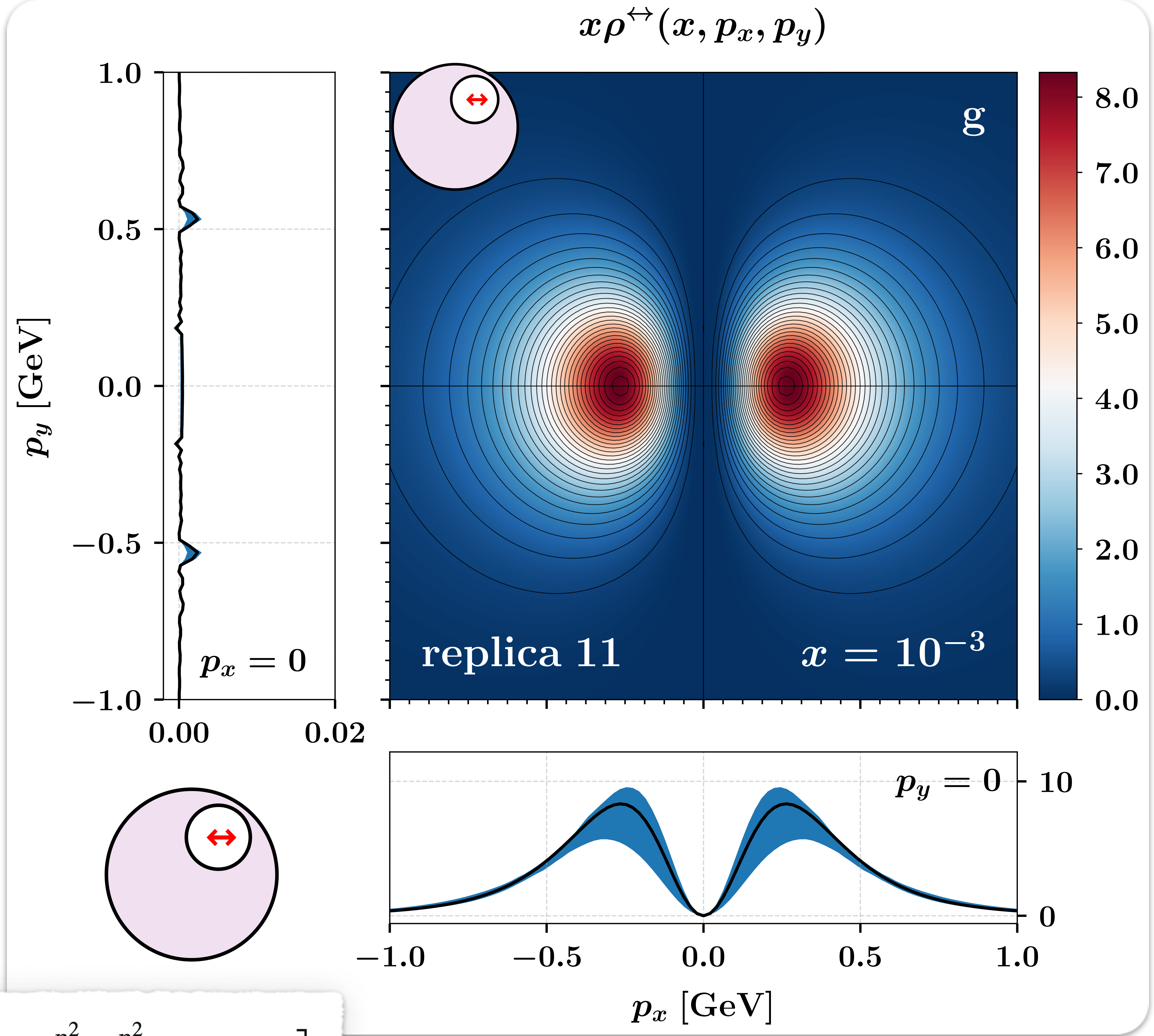

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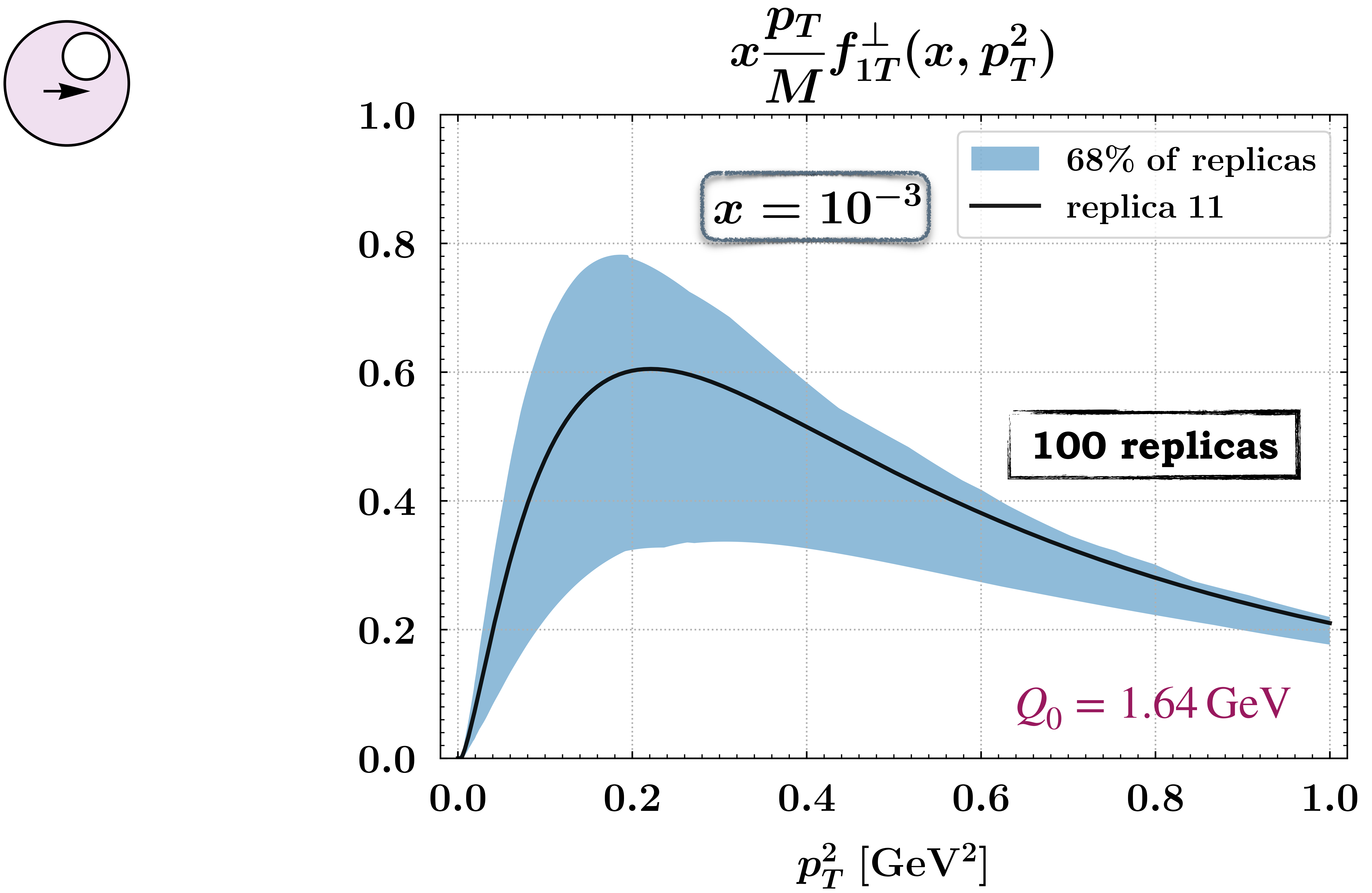
f-type Sivers gluon TMD

🔗 [A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]



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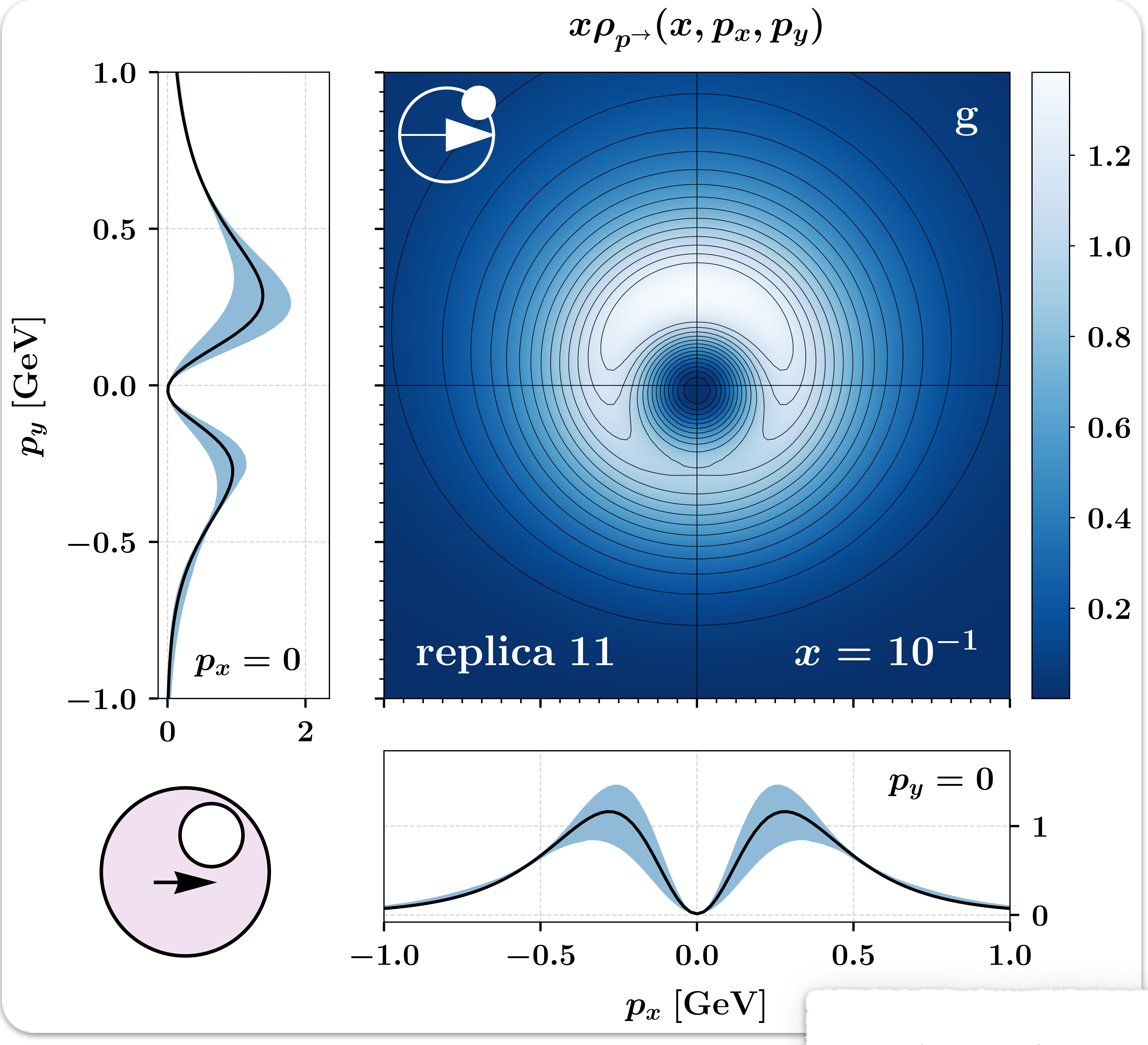

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Gluon Sivers effect

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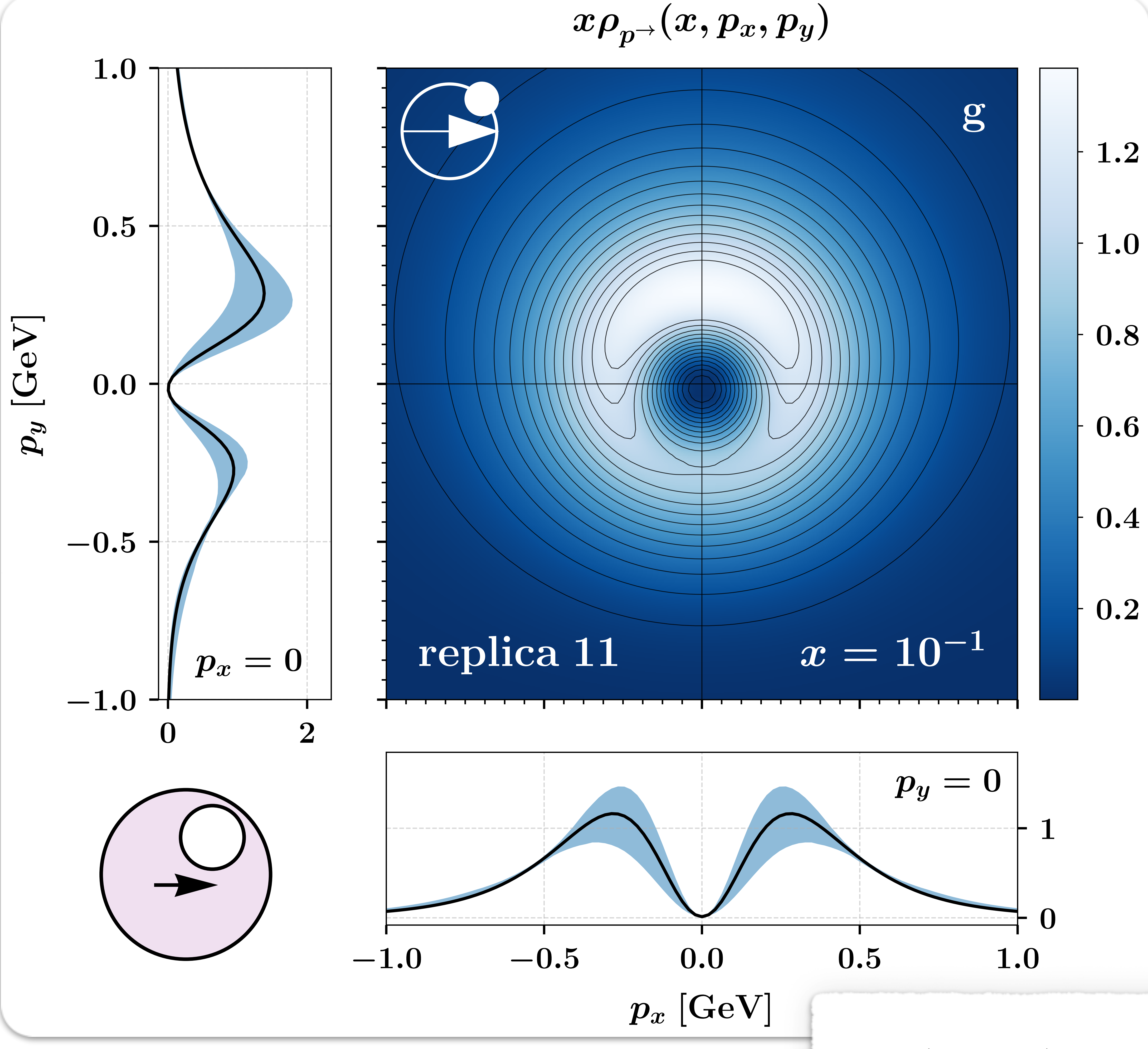

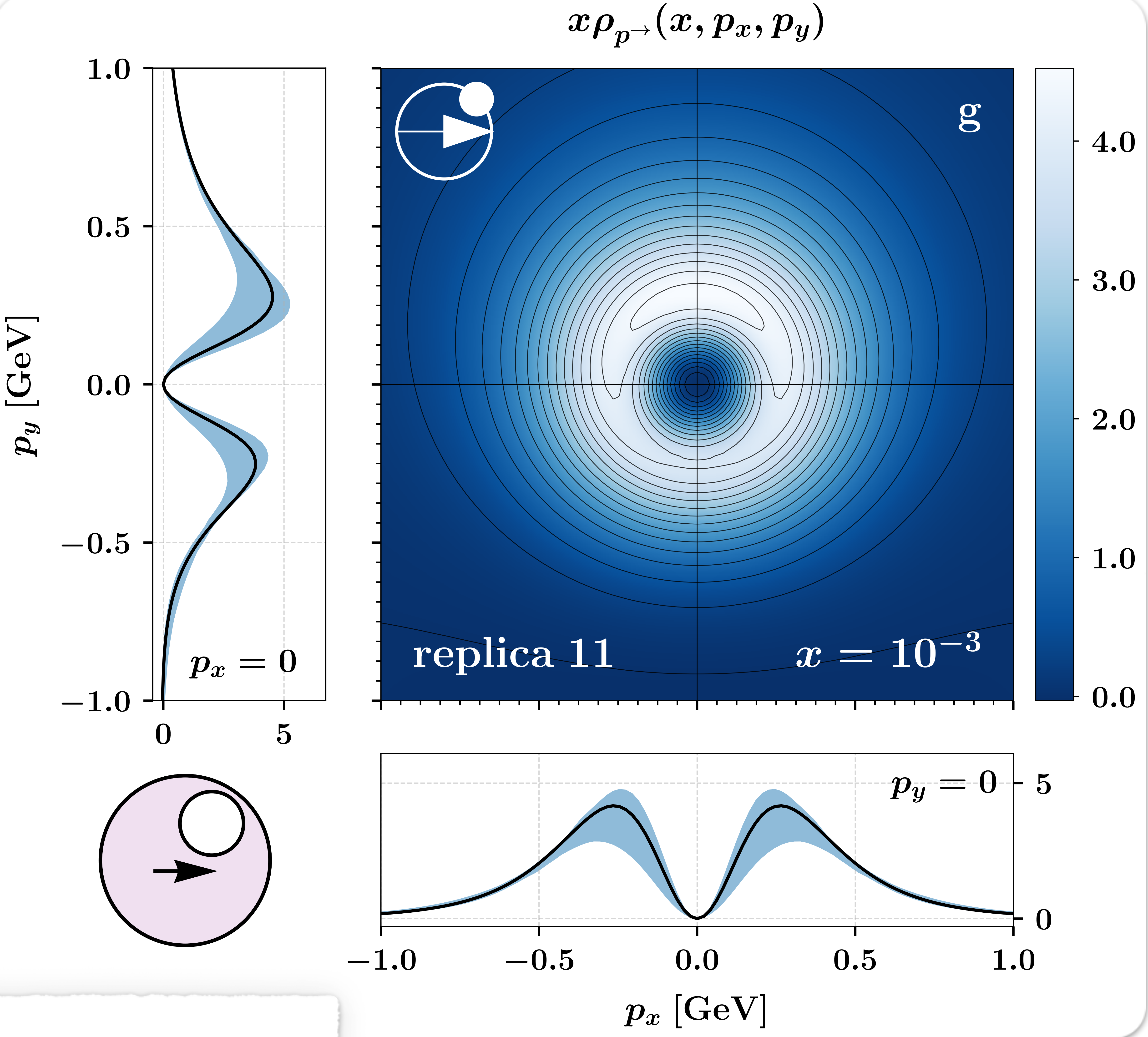

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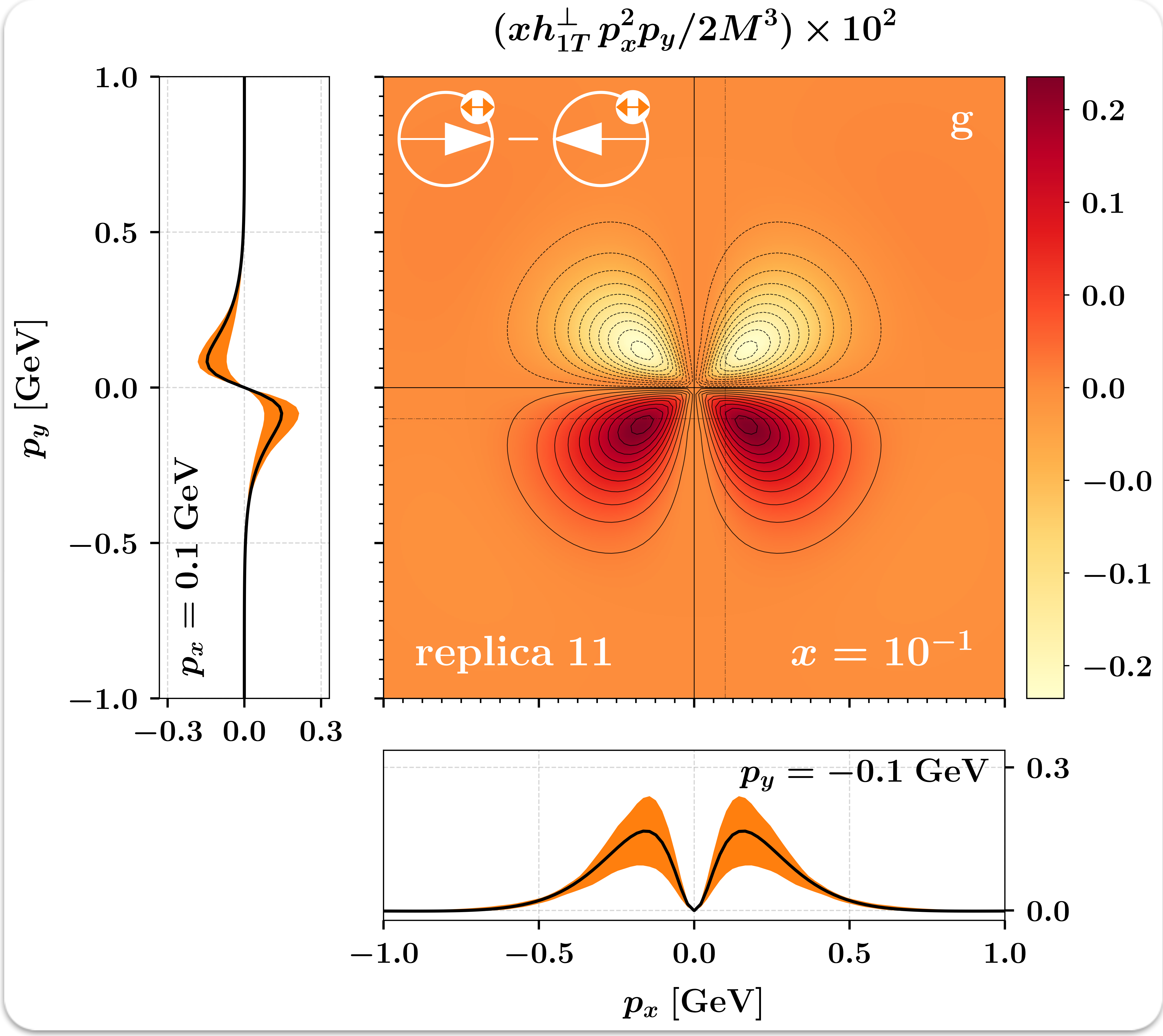
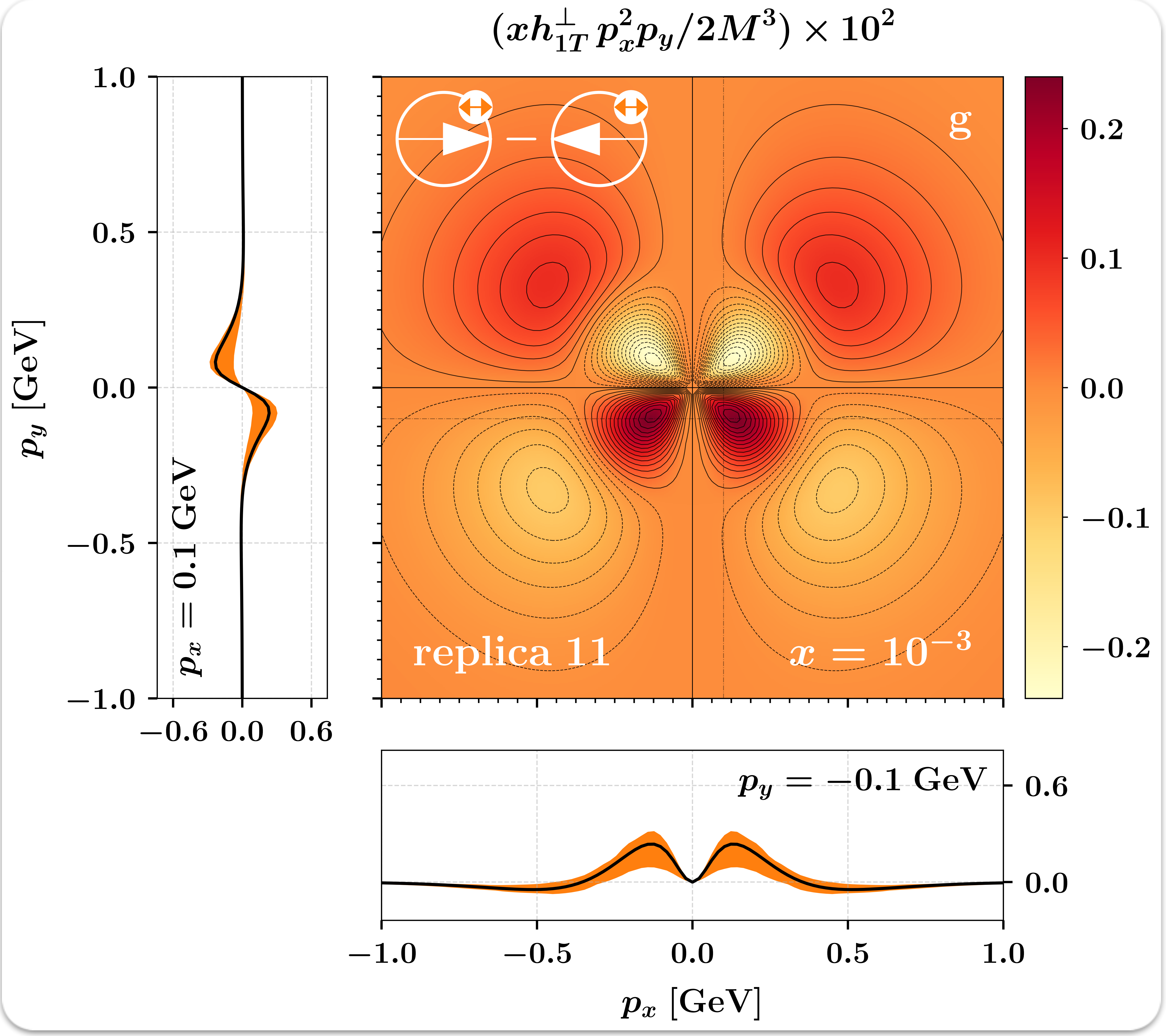
Gluon propeller effect

[A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]

Gluon butterfly effect

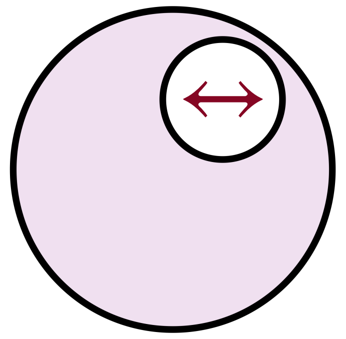
[A. Bacchetta, F.G.C., M. Radici (EPJC 2024)]





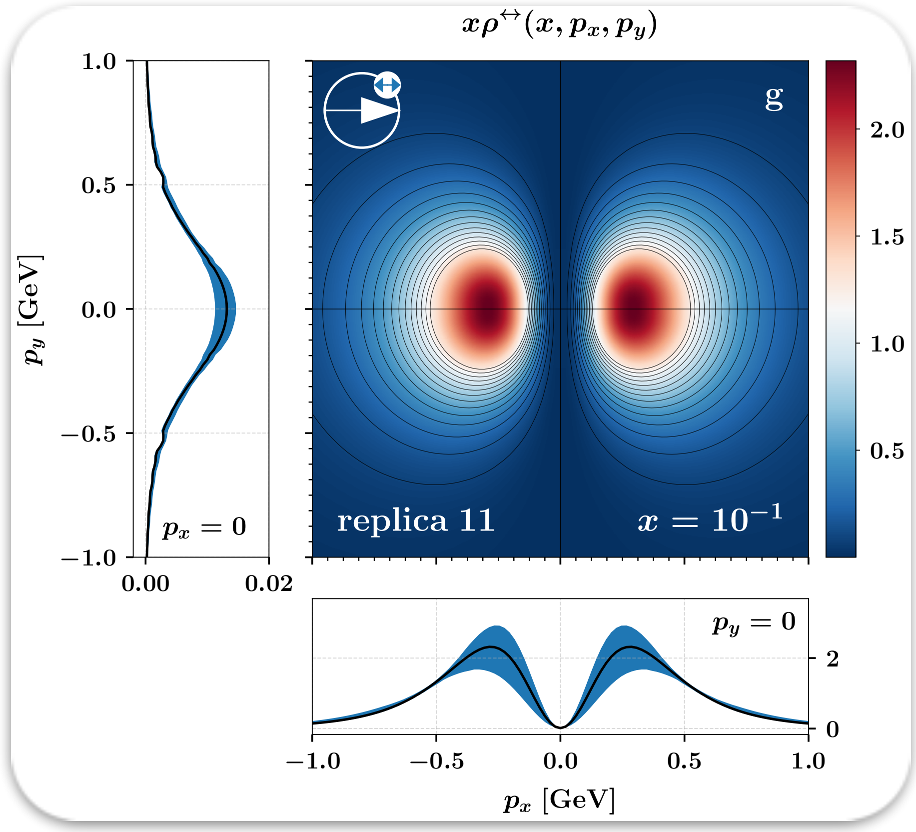
Quarkonia: Precision & Exploration

TMD exploration: Assets



EIC, LHCb, LHCspin

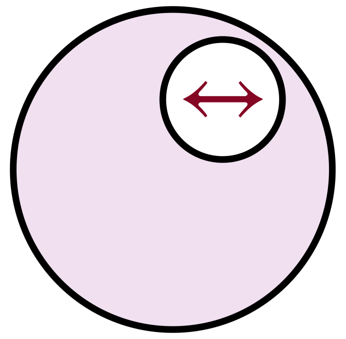
Boer-Mulders

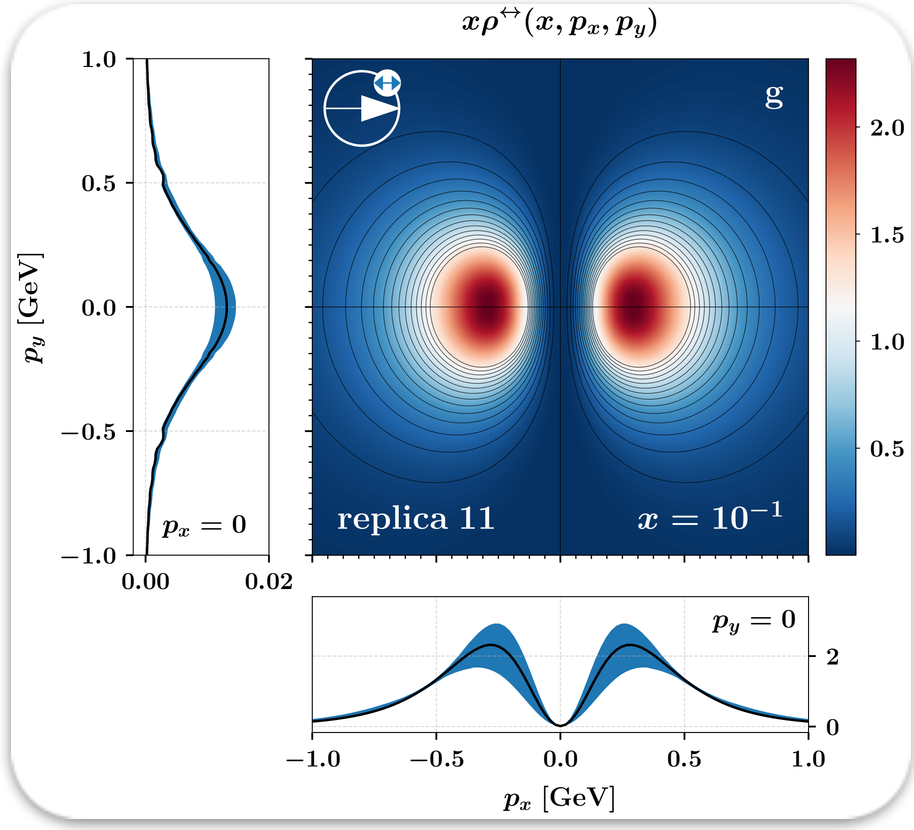


🔗 [A. Bacchetta, F.G. C., M. Radici, P. Taels (EPJC 2020)]

Quarkonia: Precision & Exploration

TMD exploration: Assets



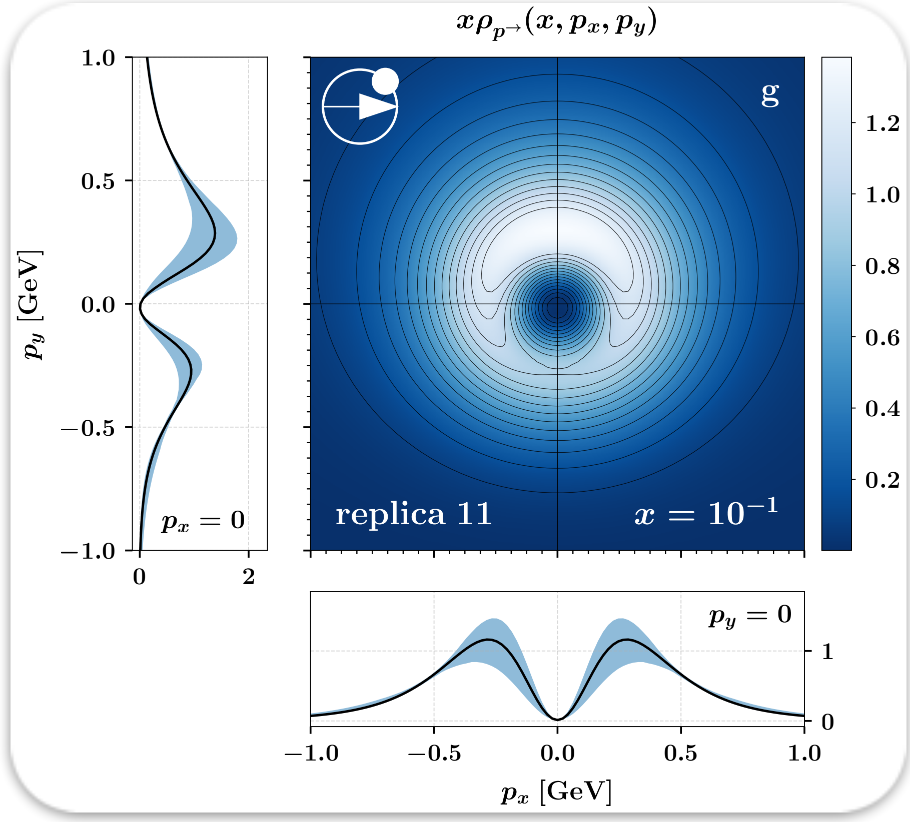


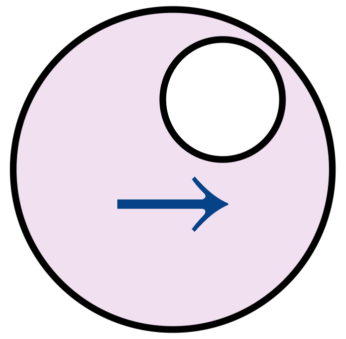
EIC, LHCb, LHCspin

Boer-Mulders

🔗 [A. Bacchetta, F.G. C., M. Radici, P. Taels (EPJC 2020)]

🔗 [A. Bacchetta, F.G. C., M. Radici (EPJC 2024)]





EIC, LHCspin

Sivers

Quarkonia: Precision & Exploration

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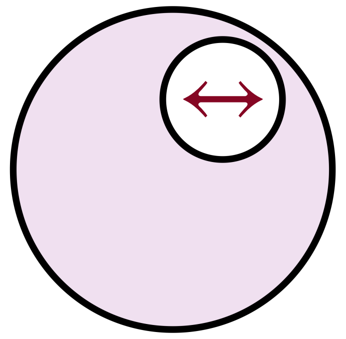

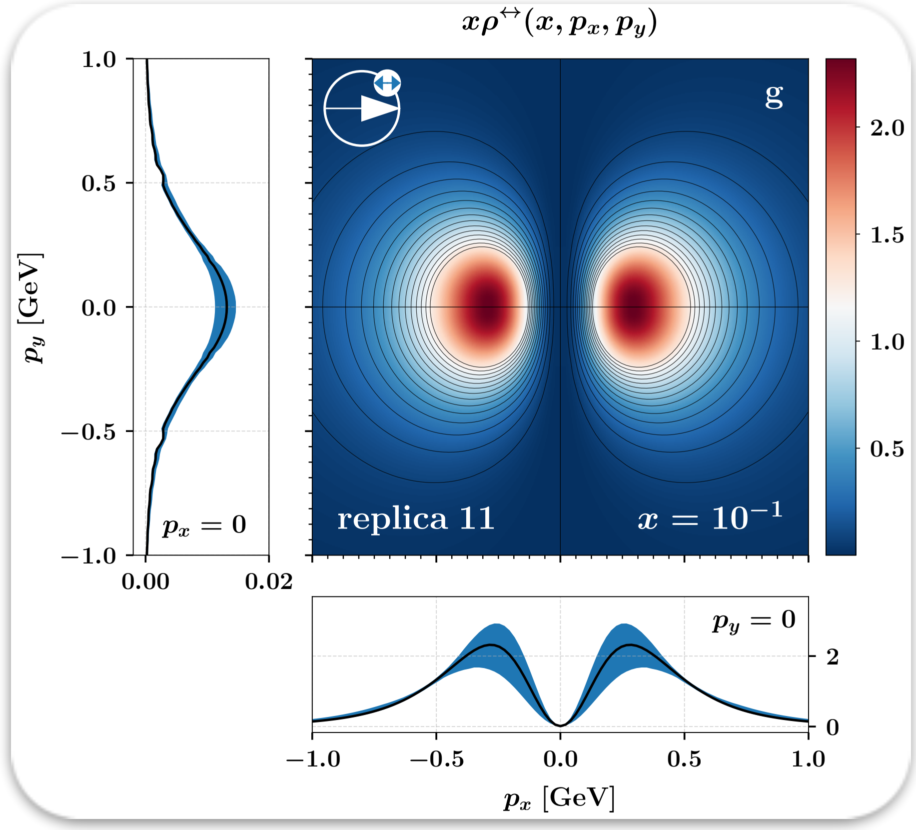

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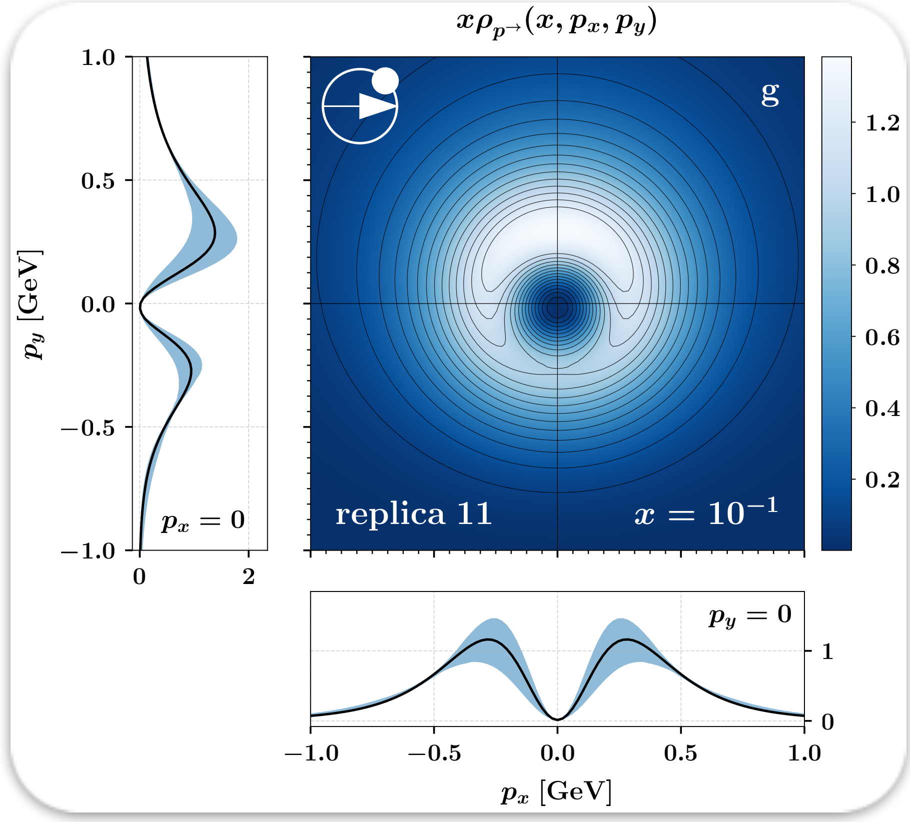
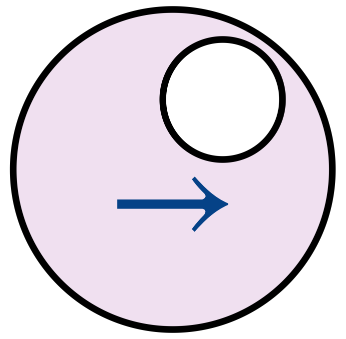


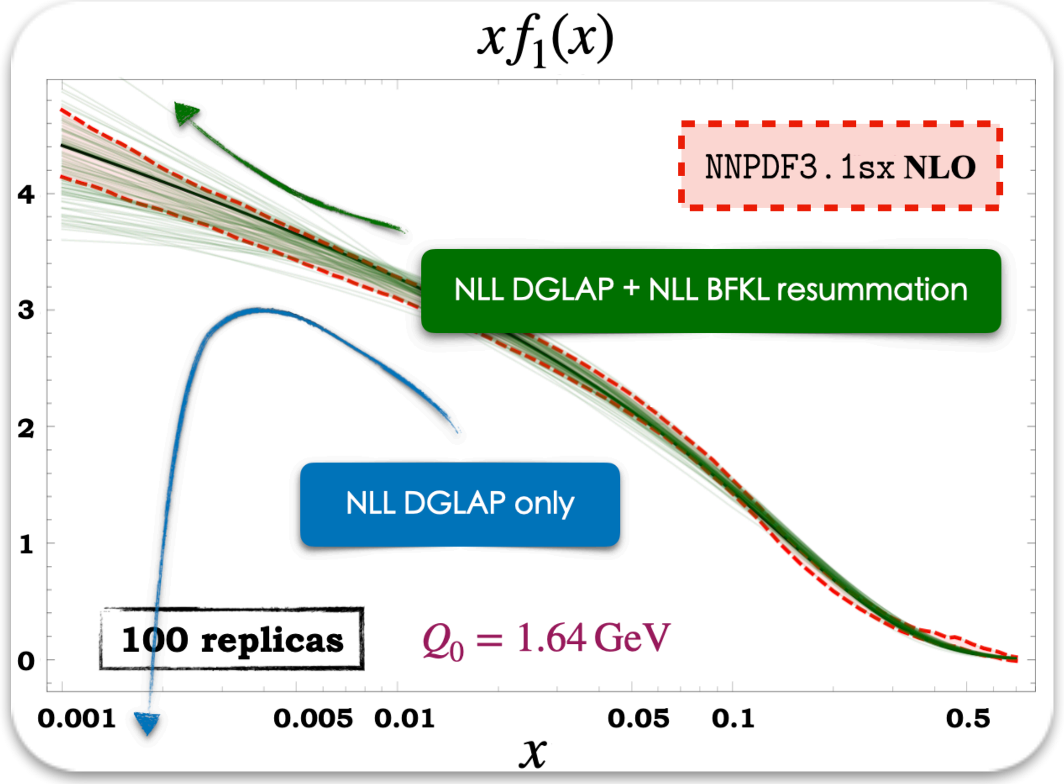


EIC, LHCspin

Sivers

Precision QCD: Challenges

Forward Onia ⇒ Gluon PDF & TMD Positivity

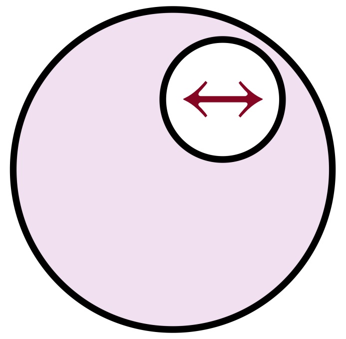


Low x and low Q²

[G. Altarelli et al. (1998)]
[A. Candido et al. (2020)]
[J. Collins et al. (2022)]
[A. Candido et al. (2023)]

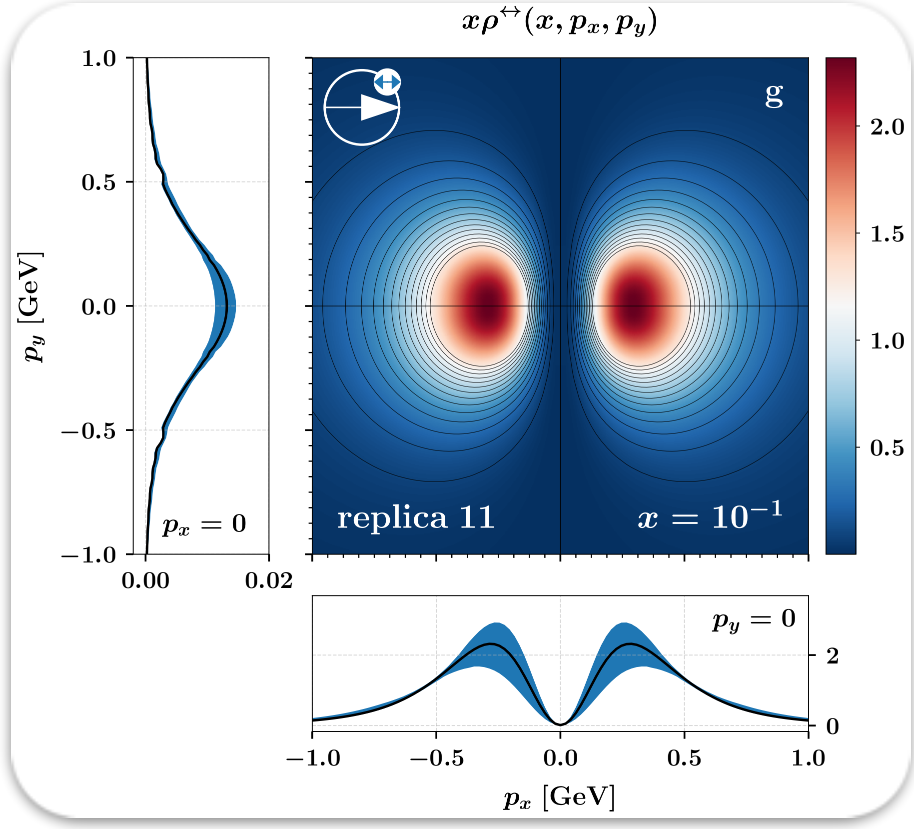
Quarkonia: Precision & Exploration

TMD exploration: Assets



EIC, LHCb, LHCspin

Boer-Mulders



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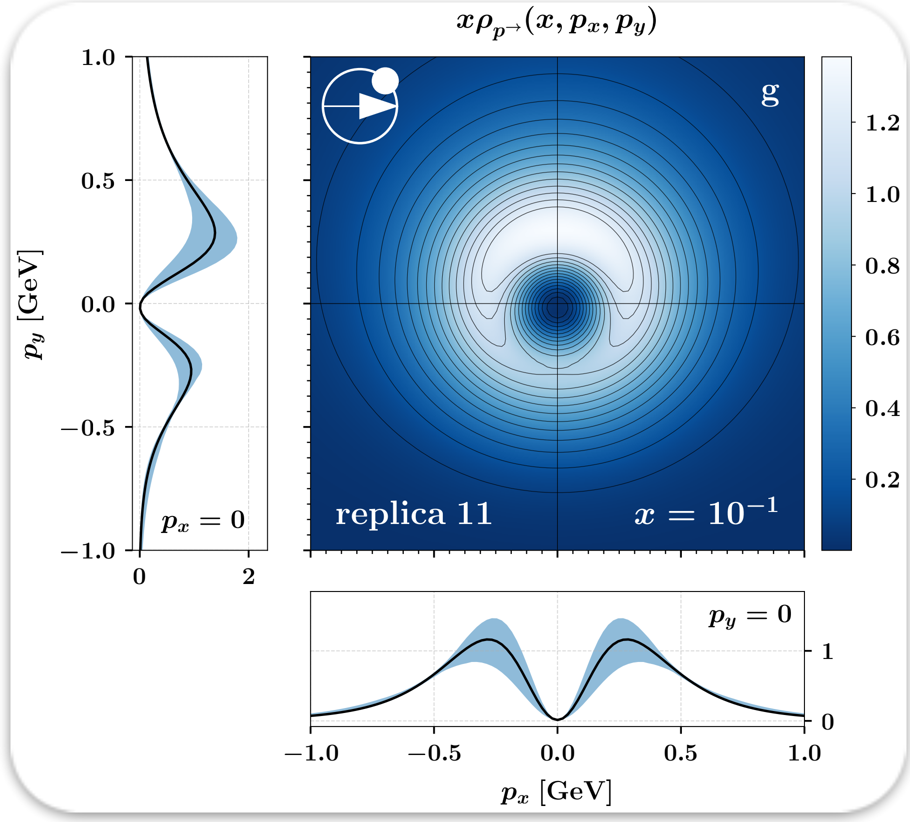

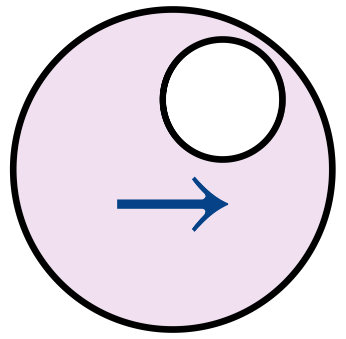

EIC, LHCspin

Sivers

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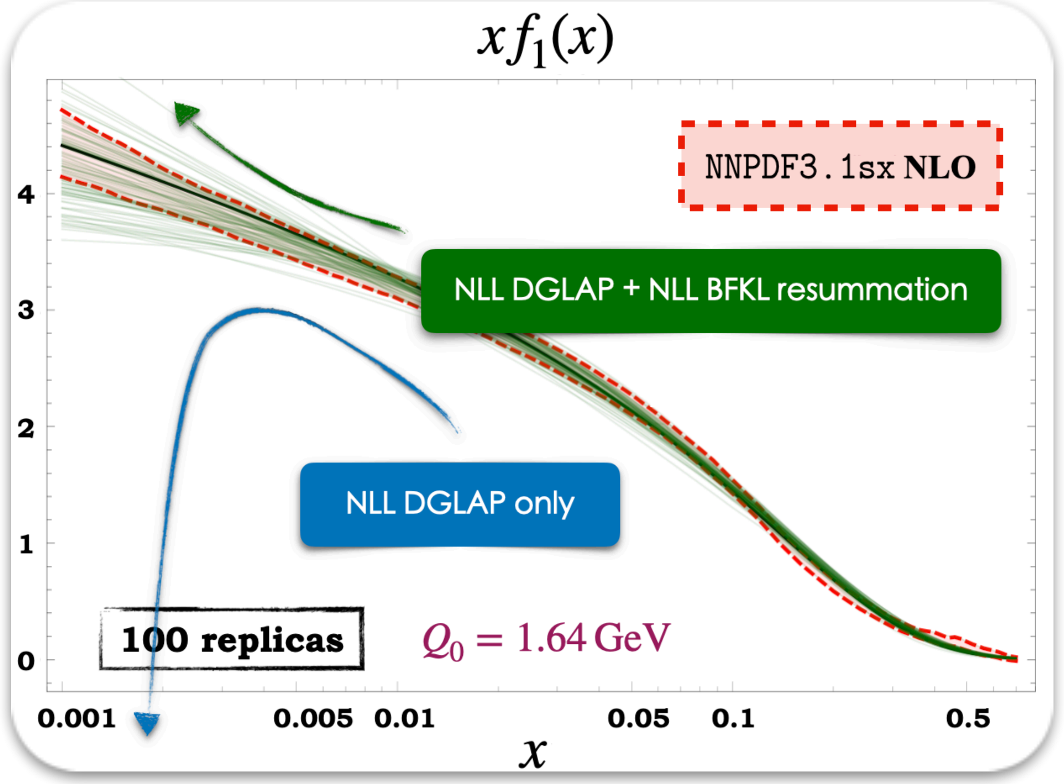

Low x and low Q^2

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[A. Candido et al. (2020)]
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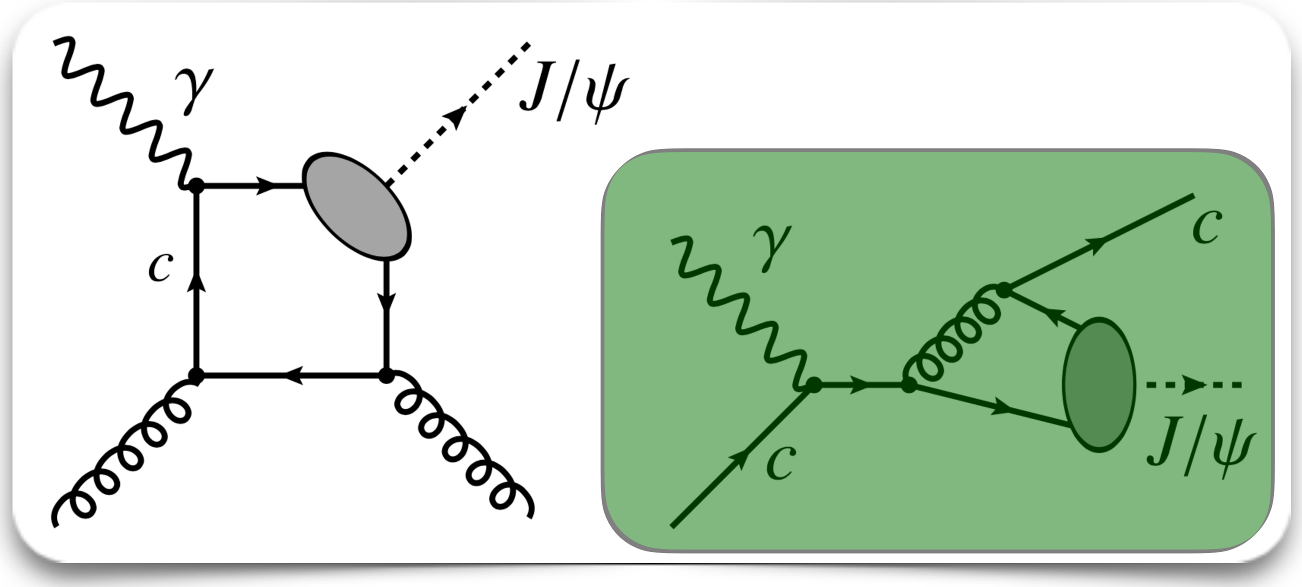
[A. Candido et al. (2023)]

$\gamma + g \rightarrow J/\psi + c \Rightarrow$ Valence Intrinsic Charm @EIC

Large x
Moderate to large p_T



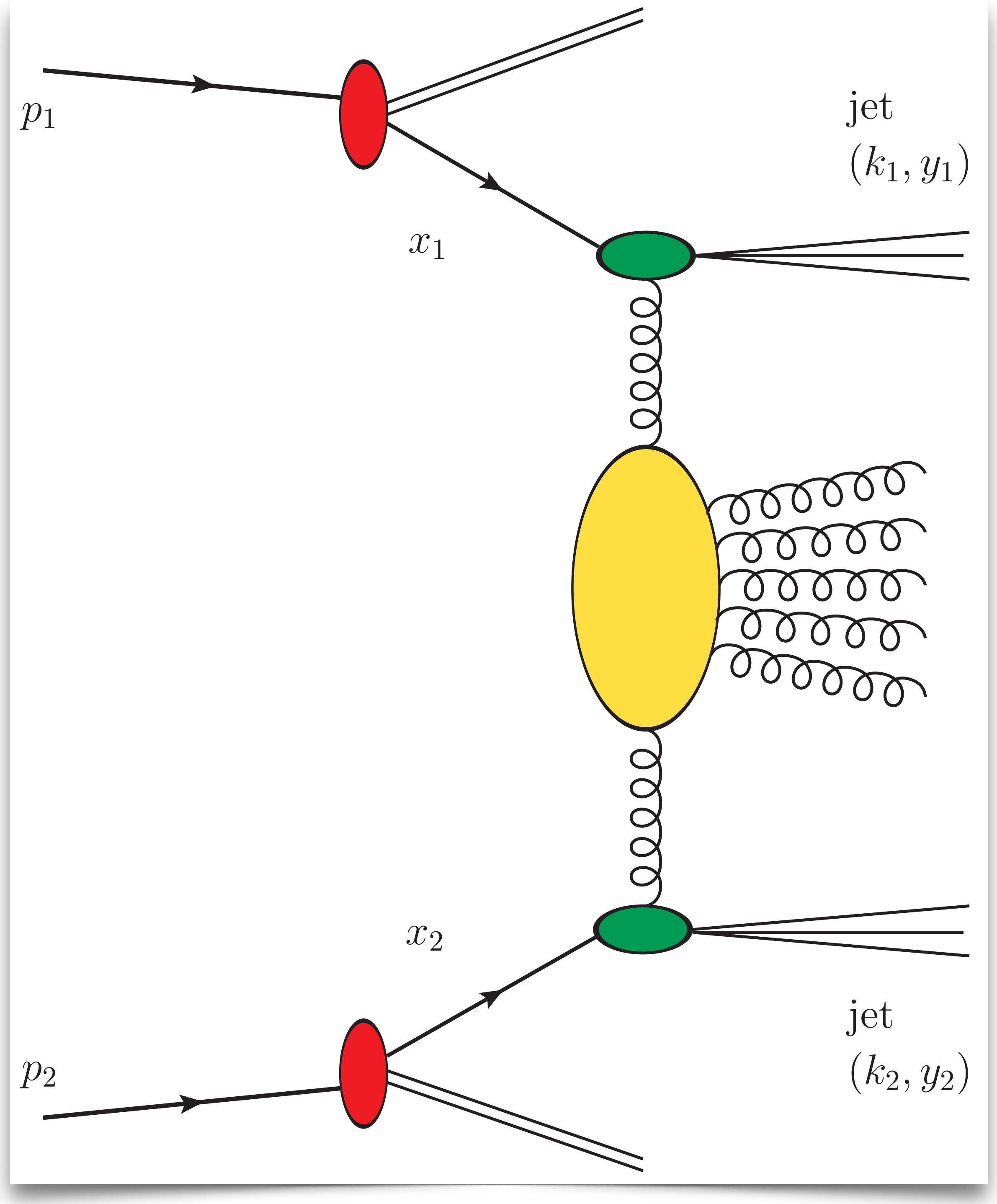
(J/ψ @EIC) [C. Flore, J.-P. Lansberg, H.-S. Shao, Y. Yedelkina (2020)]
(Intrinsic Charm + valence) studies [NNPDF Collaboration (2022, 2023)]

MUELLER-NAVELET JETS

Mueller-Navelet jets @LHC & resummation instabilities

- Inclusive hadroproduction of two jets with high p_T and large rapidity separation, ΔY
- Moderate x (collinear PDFs), but t-channel p_T (BFKL resummation) $\Rightarrow$ hybrid factorization (HyF)

$$\frac{d\sigma}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2} = \sum_{r,s=q,g} \int_0^1 dx_1 \int_0^1 dx_2\, f_r(x_1, \mu_F)\, f_s(x_2, \mu_F)\, \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu_F)}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2}$$



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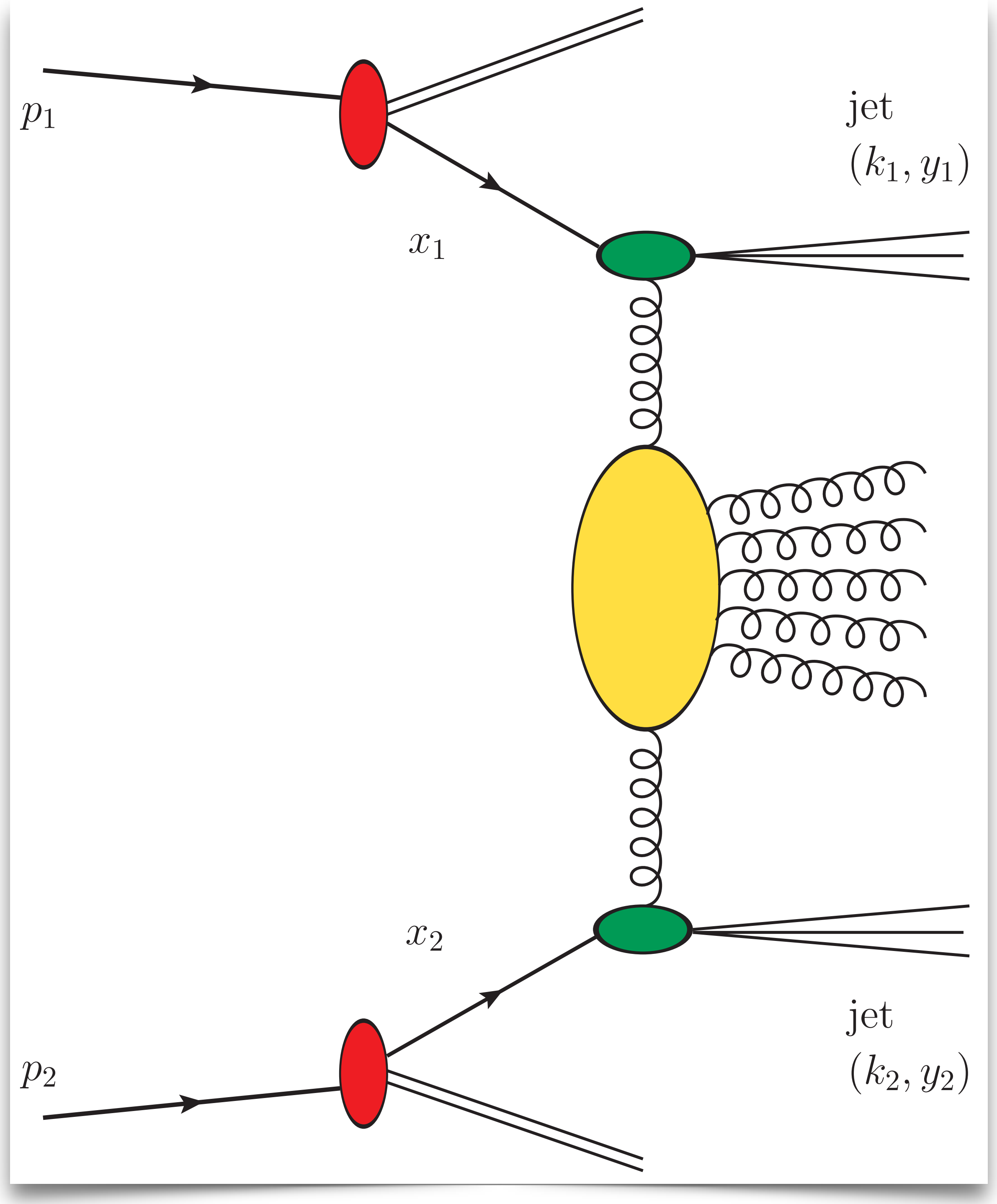

NLO(+)

NLL

NLO(+)

$$\frac{d\hat{\sigma}_{r,s}(x_1 x_2 s,\mu)}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2} = \frac{1}{(2\pi)^2}$$
$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\, \mathcal{V}_J^{(r)}(\vec{q}_1, s_0, x_1, \vec{k}_1)$$
$$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2)$$
$$\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}}\, \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{k}_2)$$

jet vertices (off-shell coefficient functions)

BFKL Green's function

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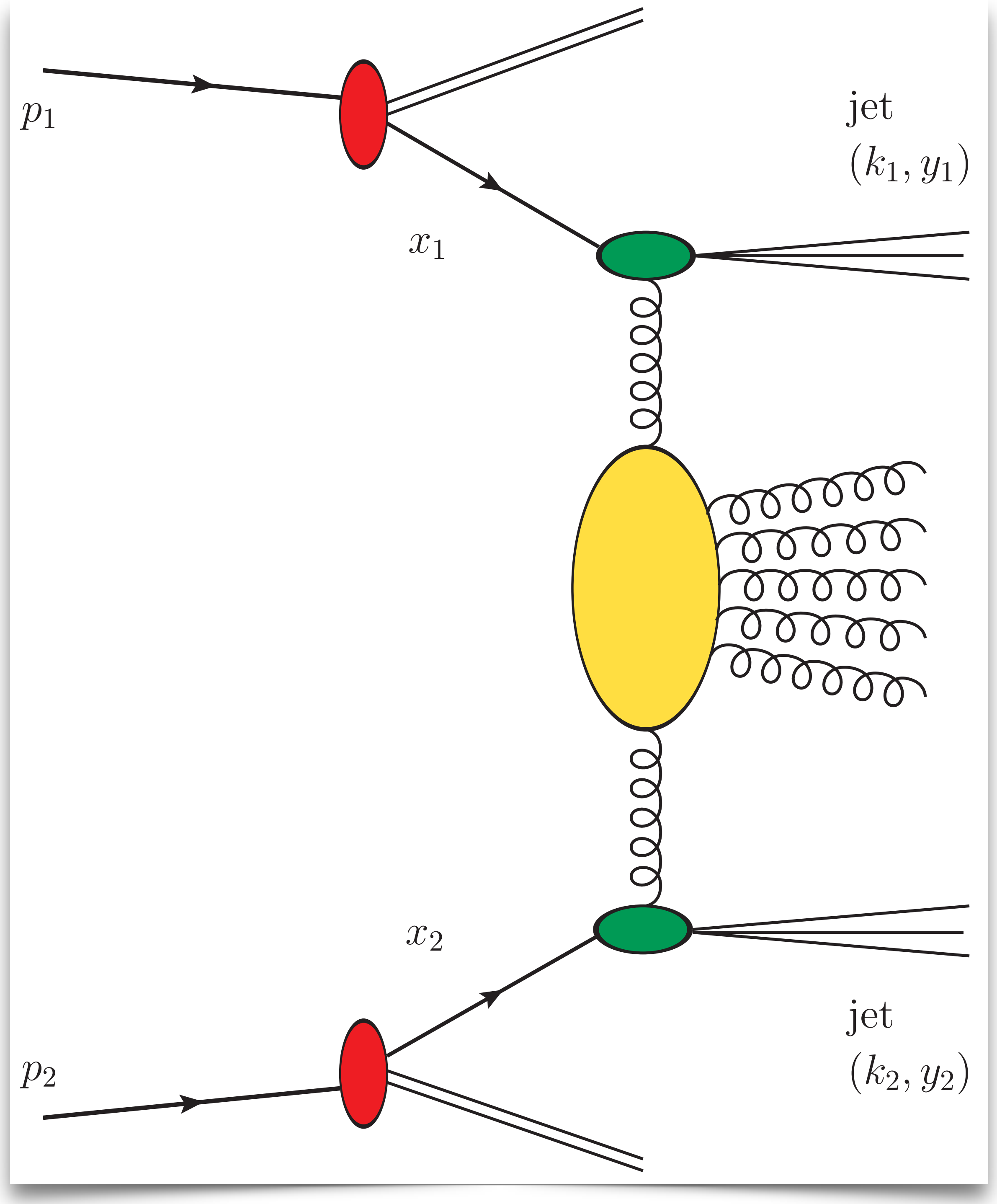

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NLO$^{(+)}$ $$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\, \mathcal{V}_J^{(r)}(\vec{q}_1, s_0, x_1, \vec{k}_1)$$

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BFKL Green's function

- NLL/LL instabilities + NLO missing threshold $\Rightarrow$ precision studies hampered

Mueller-Navelet jets @LHC & resummation instabilities

- Strong manifestation of higher-order instabilities via scale variation (¡!)
- ¡ At natural scales: NLL/LL ratio large, no agreement with data, unphysical values !
- BLM scales, theory vs experiment: CMS @7TeV with symmetric p_T-ranges, only

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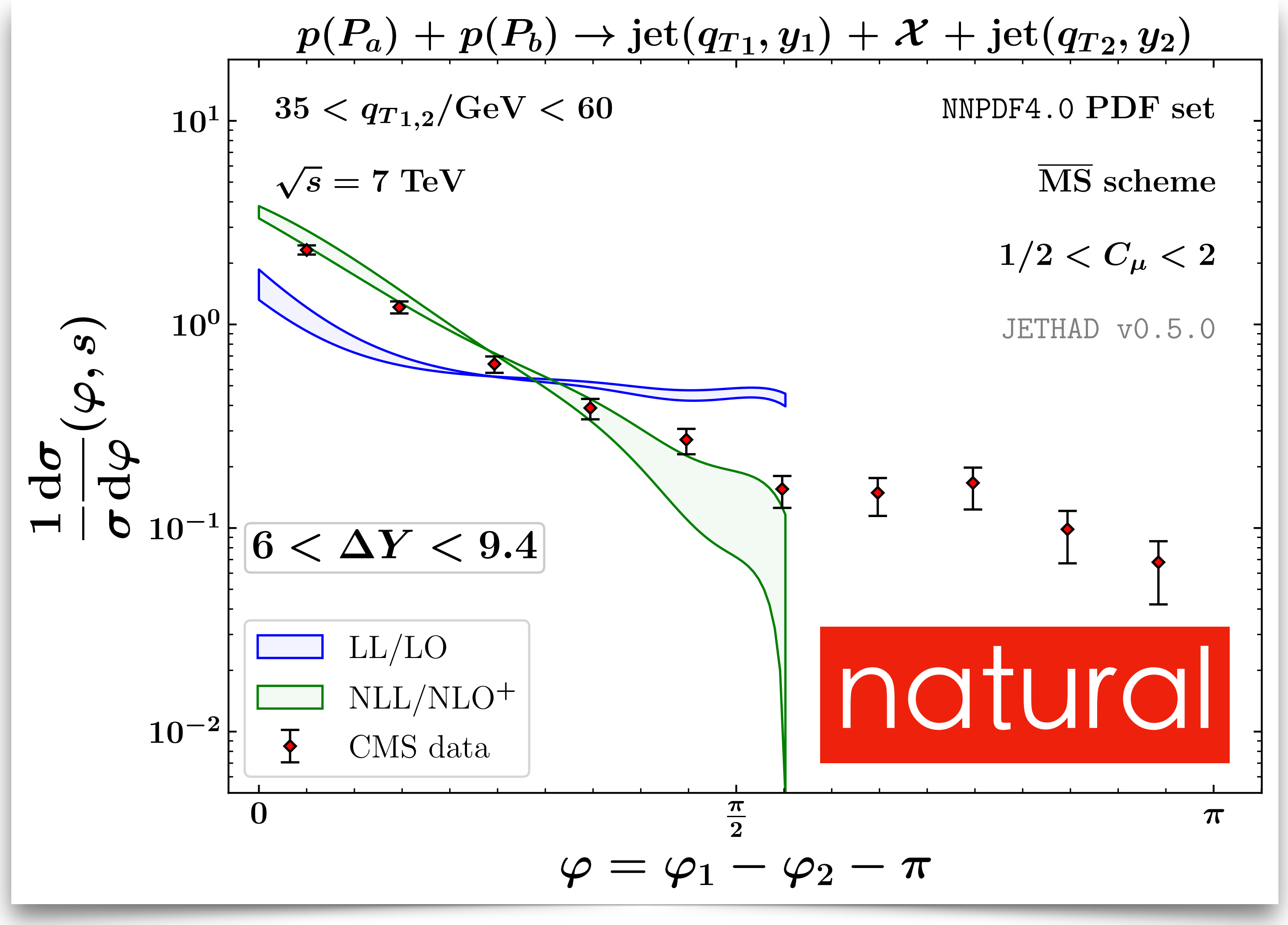
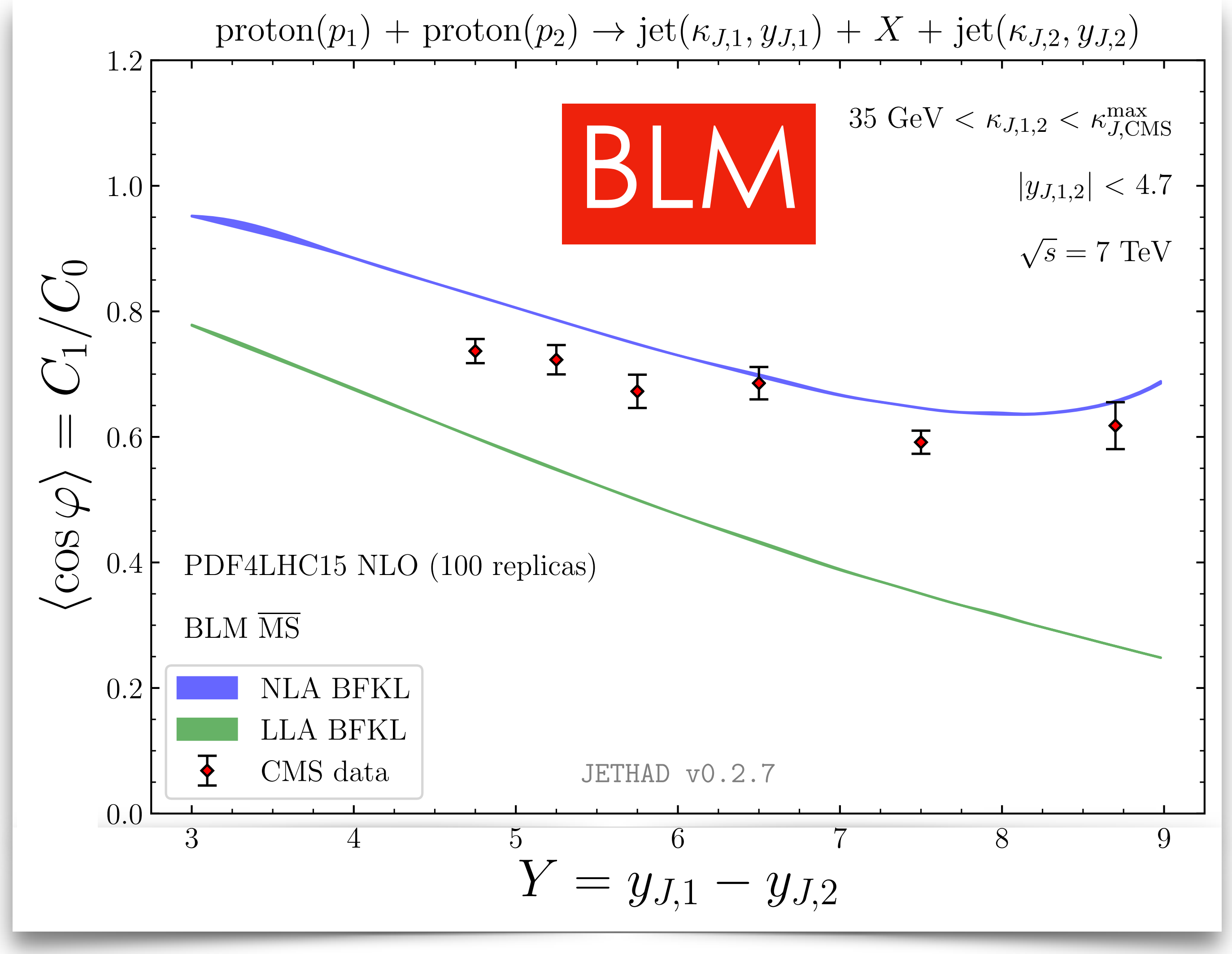
[CMS Collaboration, JHEP 08 (2016) 139]
[B. Ducloué et al., Phys. Rev. Lett. 112 (2014) 082003]
[F. Caporale et al., Eur. Phys. J. C 74 (2014) 10, 3084]





(left figure) [F. G. C., A. Papa, Phys. Rev. D 106 (2022) 11, 114004]
(right figure) [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]

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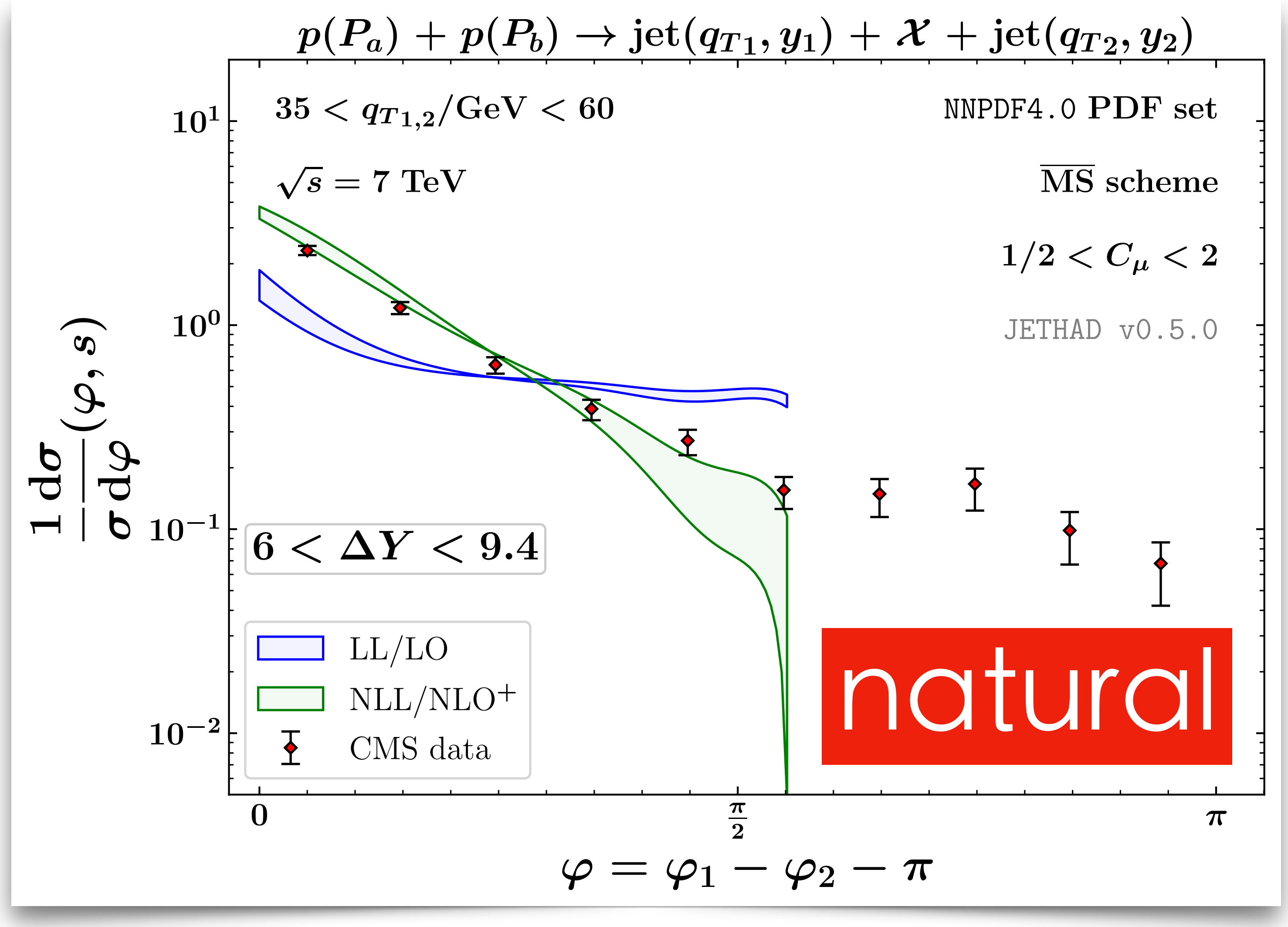

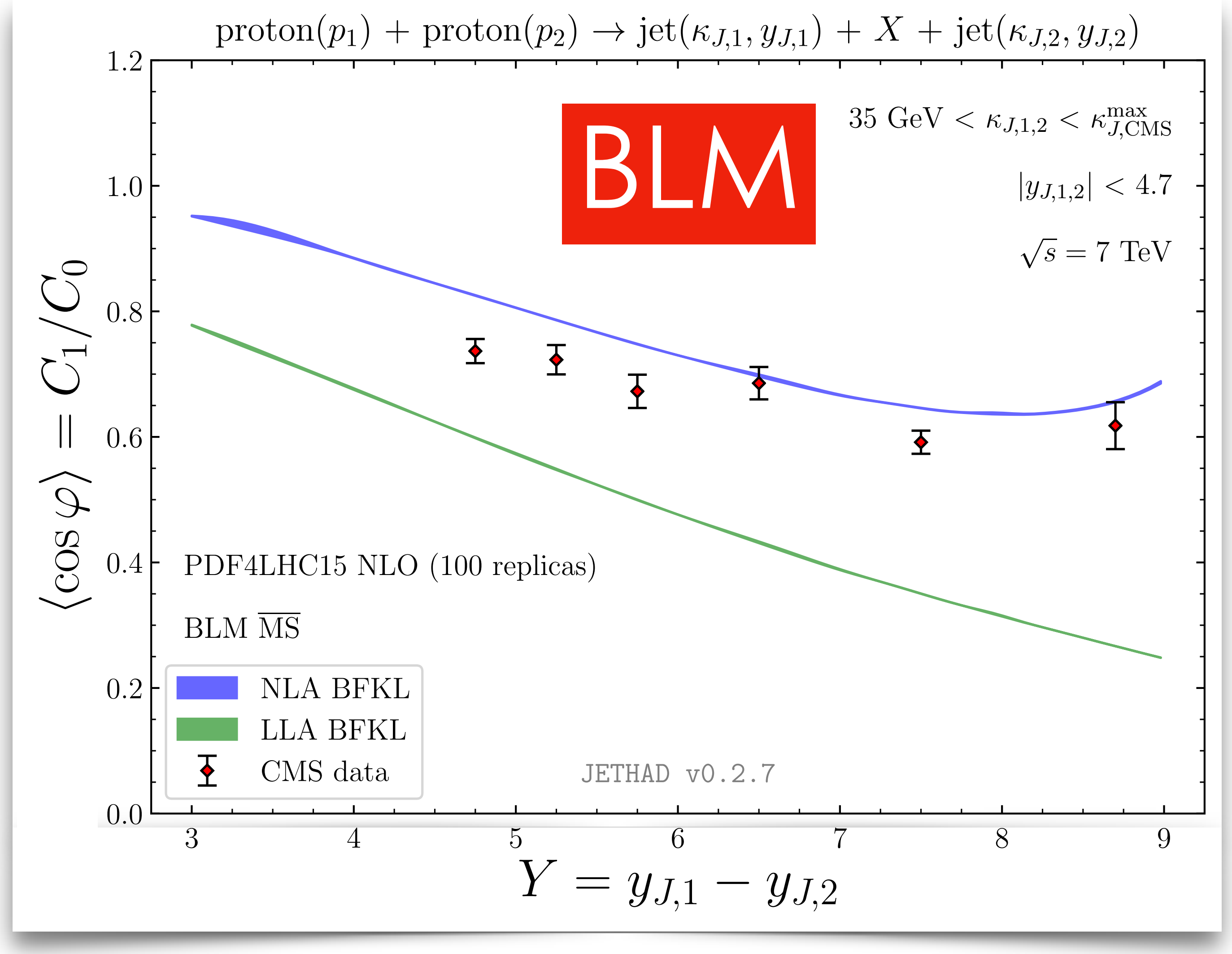

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- $\mu_R^{\text{BLM}} \gg \mu_R^{\text{nat.}} \;\Rightarrow\; d\sigma^{\text{BLM}}/d\sigma^{\text{nat.}} \sim 10^{-(1\div 2)}$ $\Rightarrow$ precision studies hampered
- Unsuccessful scale optimization → processes featuring natural stability (¿?)

Azimuthal-angle multiplicity

$$\frac{1}{\sigma}\frac{d\sigma(\Delta Y, s)}{d\varphi} = \frac{1}{2\pi}\left\{1 + 2\sum_{n=1}^{\infty}\cos(n\varphi)\langle\cos(n\varphi)\rangle\right\}$$

MUELLER-NAVELET JETS

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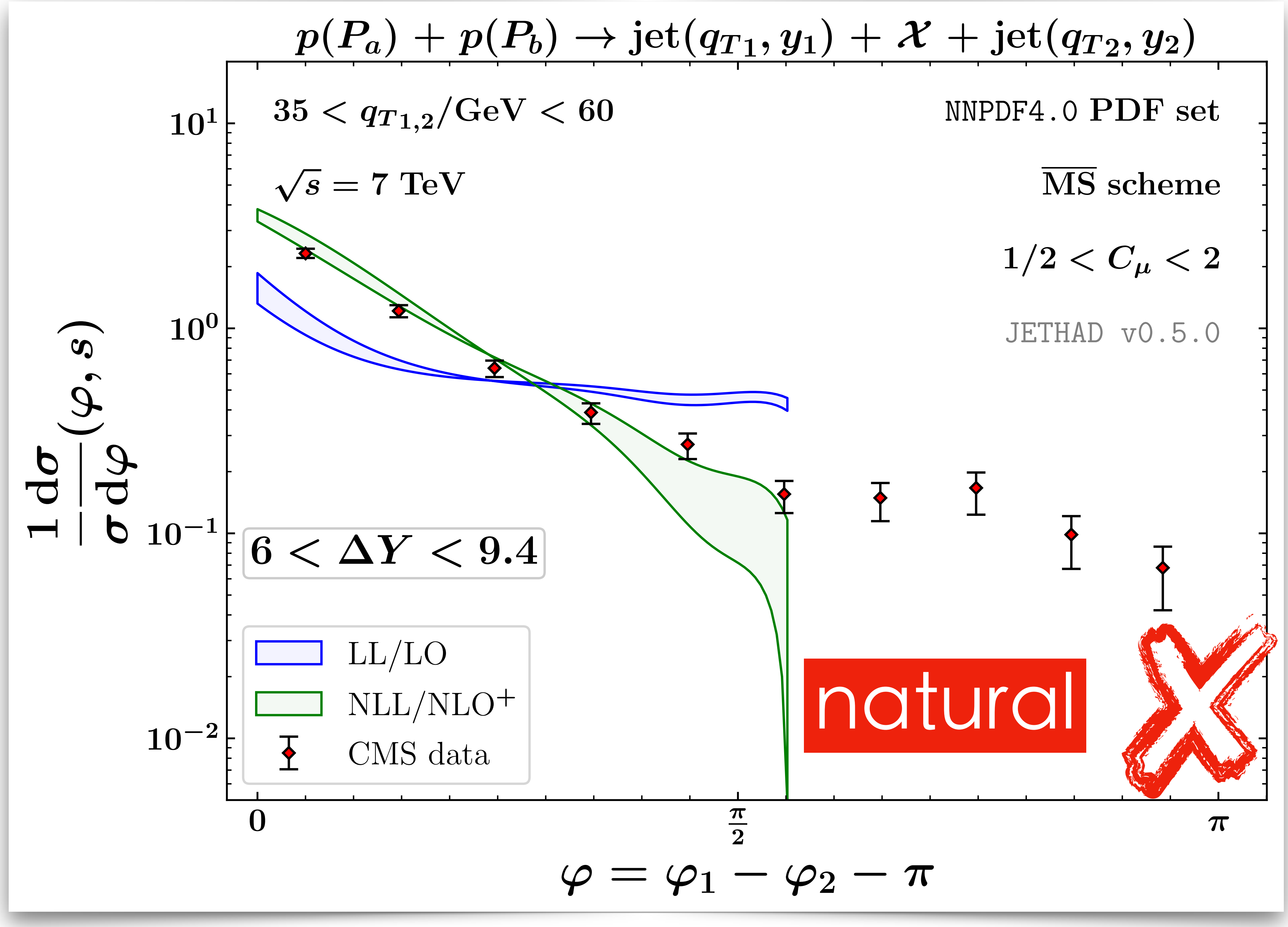

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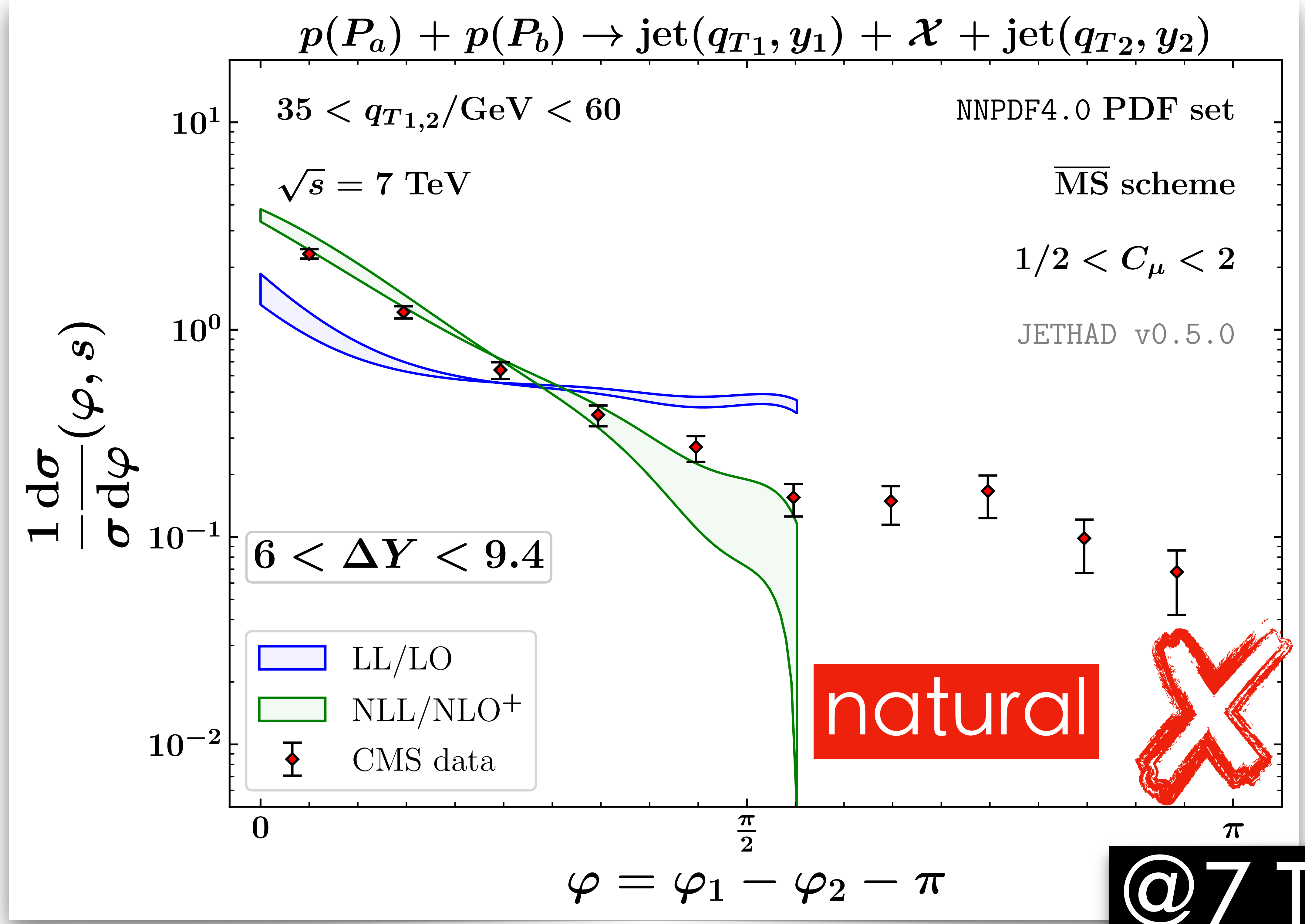
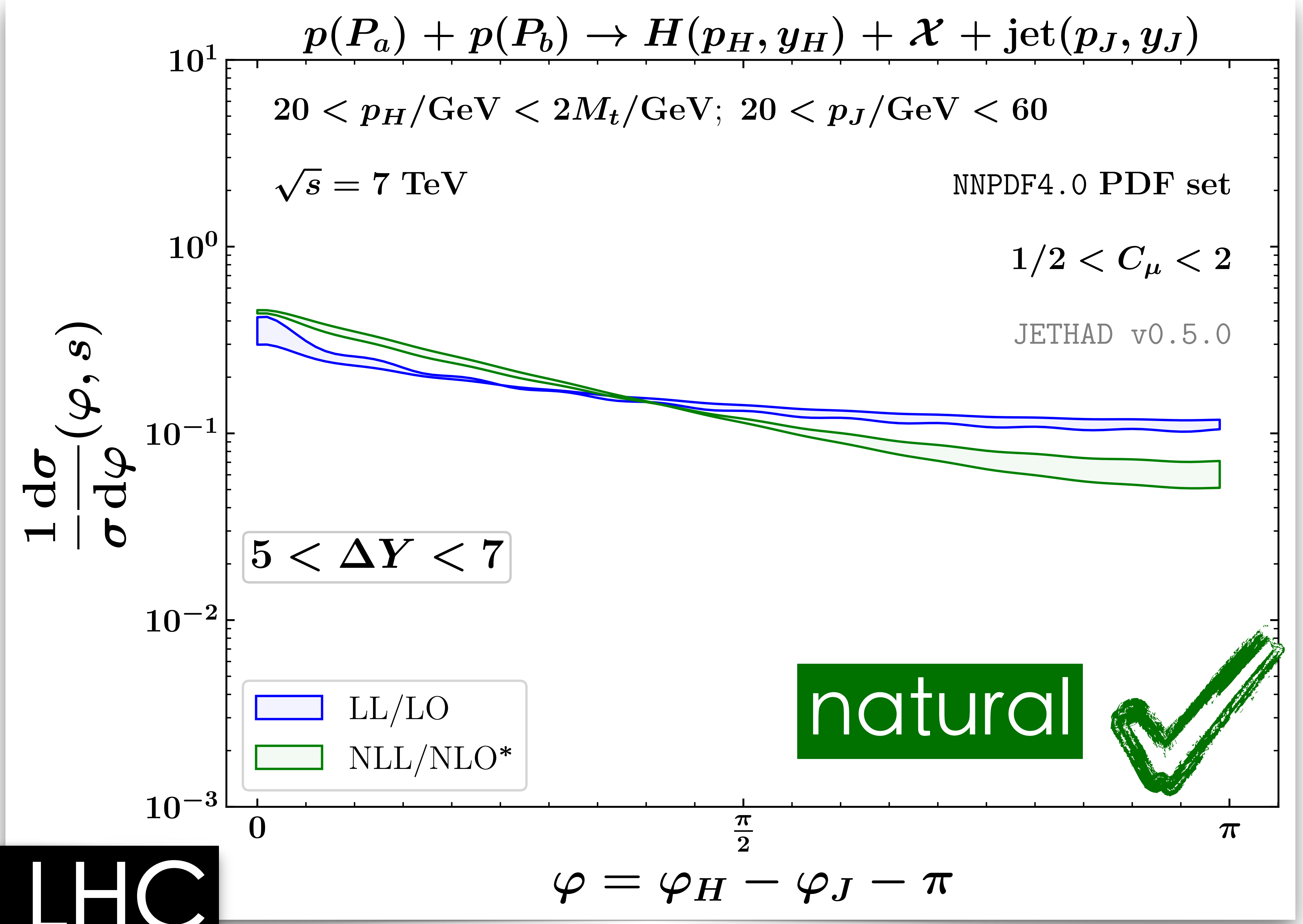
HIGGS + JET

(figure below) [F. G. C. et al., Eur. Phys. J. C 81 (2021) 4, 293]

(NLO Higgs coefficient function) [F. G. C. et al., JHEP 08 (2022) 092]





@7 TeV LHC

Hybrid factorization via the JETHAD code python + F

[F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]. [F. G. C. , Phys. Rev. D 105 (2022) 11, 114008]

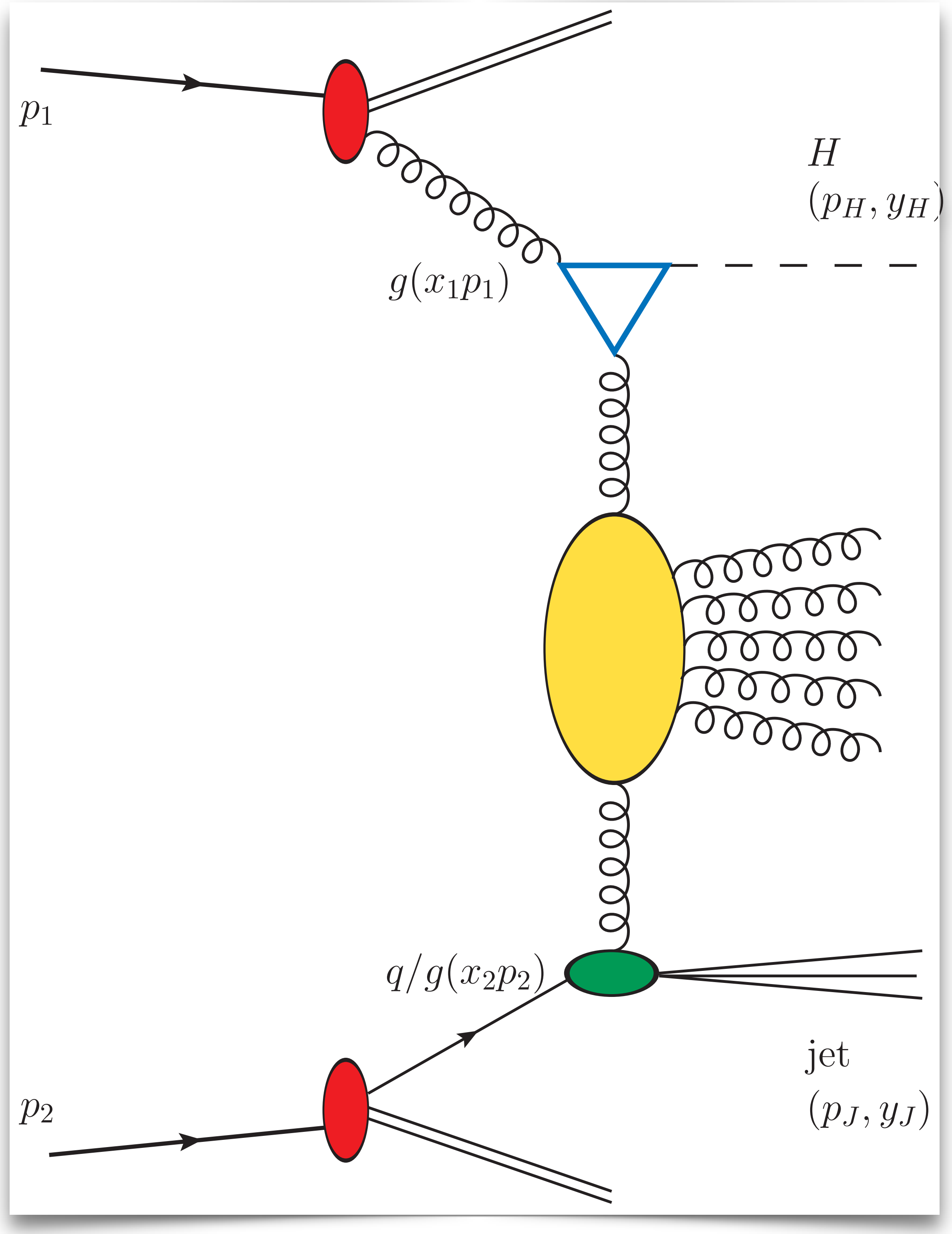
HIGGS + JET DISTRIBUTIONS

From Mueller-Navelet to Higgs and heavy flavor

Pheno path: hunt for channels leading to a NLL stabilization pattern at natural scales (¡!)

HIGGS BOSON

Stabilizers ⇔ large Higgs transverse masses



(Higgs + jet, NLL/NLO*) 🔗 [F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]

(NLO Higgs emission function) 🔗 [F. G. C., M. Fucilla et al., JHEP 08 (2022) 092]

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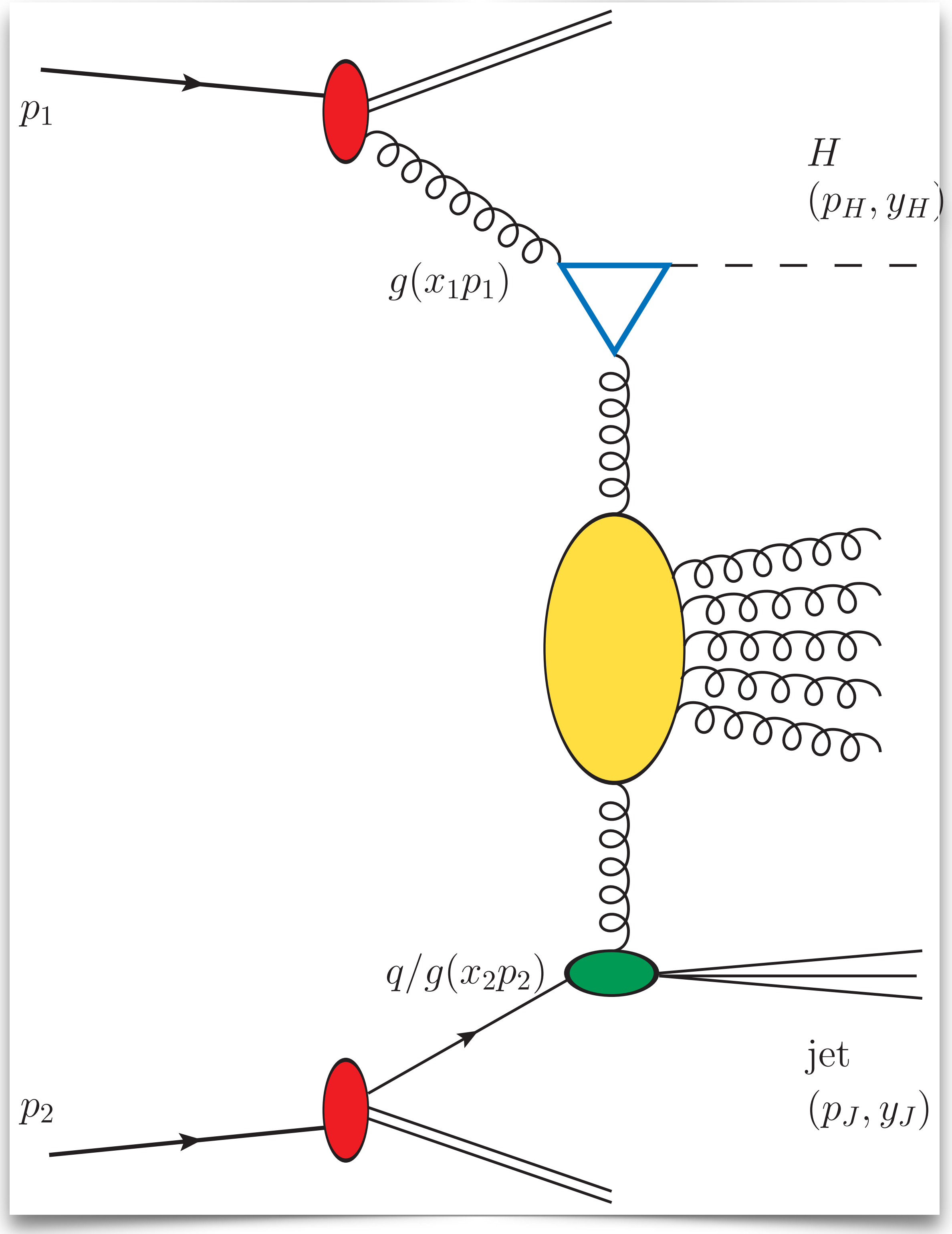

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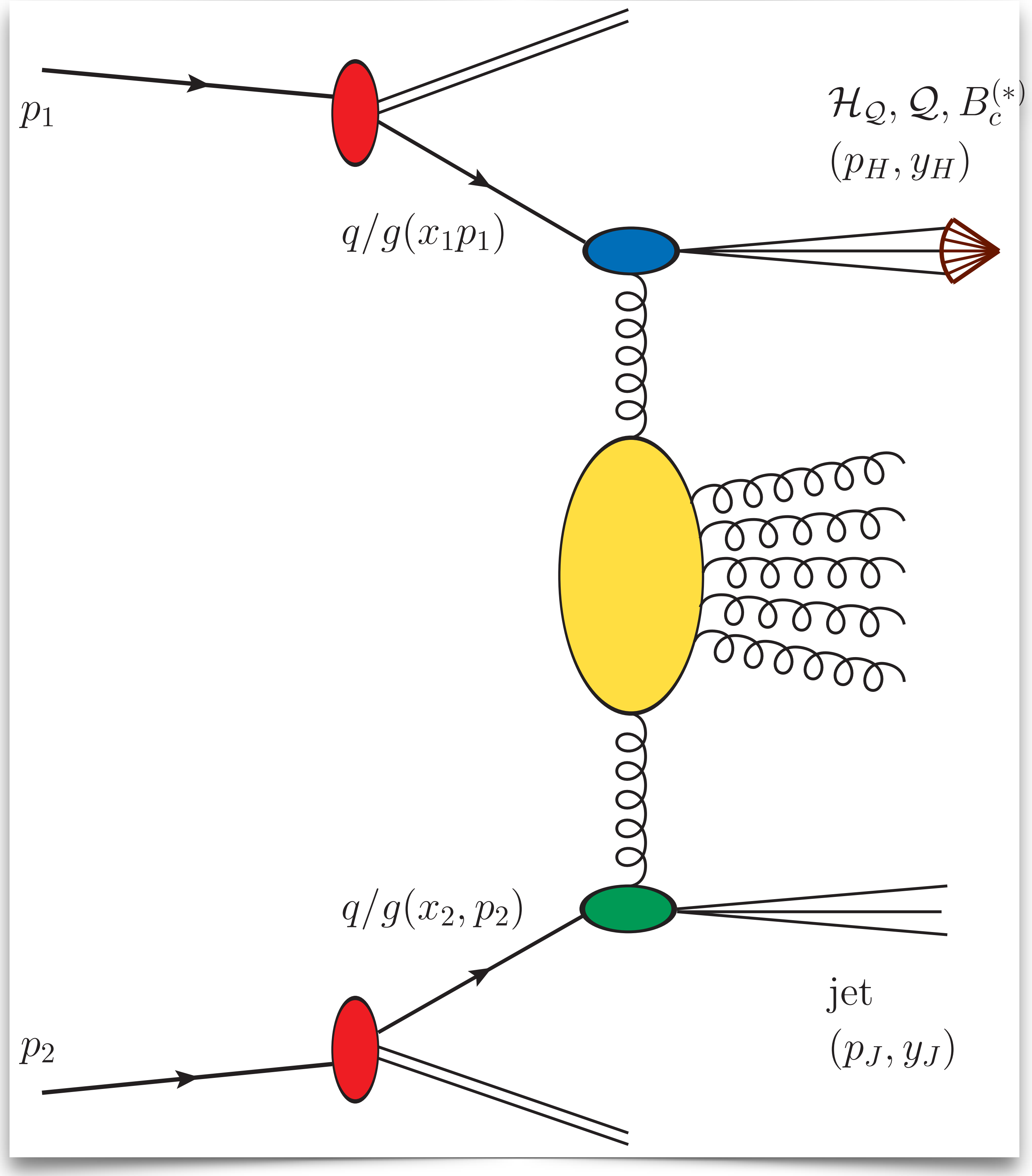
HEAVY FLAVOR AT LARGE p_T

Stabilizers ⇔ gluon fragmentation channels



($\Lambda_c^\pm$ baryons, NLL/NLO) 🔗 [F. G. C. et al., Phys. Rev. D 104 (2021) 11, 114007]

(J/ψ or Υ, NLL/NLO) 🔗 [F. G. C. et al., Eur. Phys. J. C 82 (2022) 10, 929]

($B_c^\pm(^1S_0)$ or $B_c^{*\pm}(^3S_1)$, NLL/NLO) 🔗 [F. G. C., Phys. Lett. B 835 (2022) 137554]

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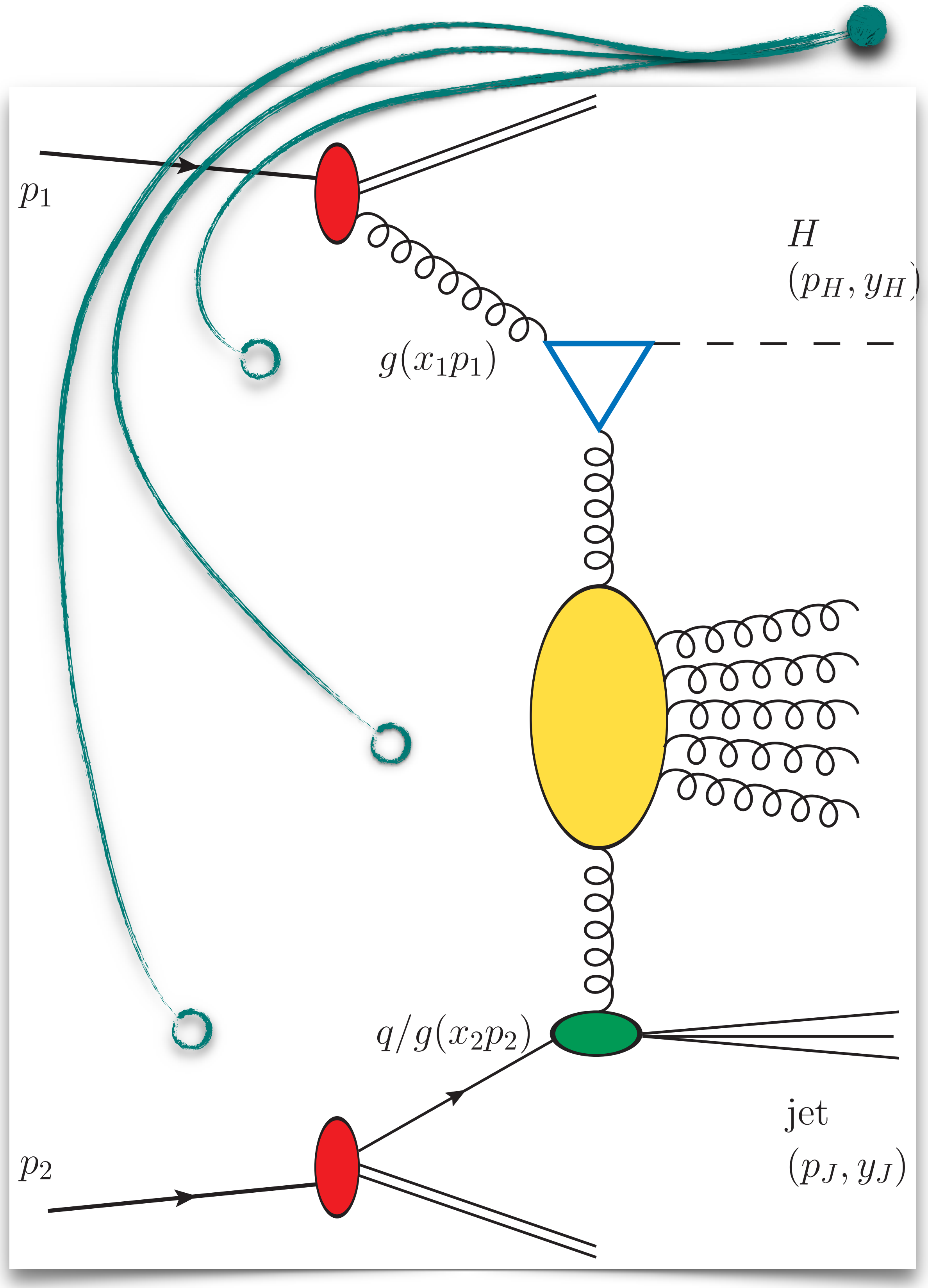
Higgs boson

Stabilizers ⇔ large Higgs transverse masses

$\mu_{F,R} \sim M_{H,\perp}$



NLO*

= LO + NLO^RGE

NLL

NLO

Heavy flavor at large p_T

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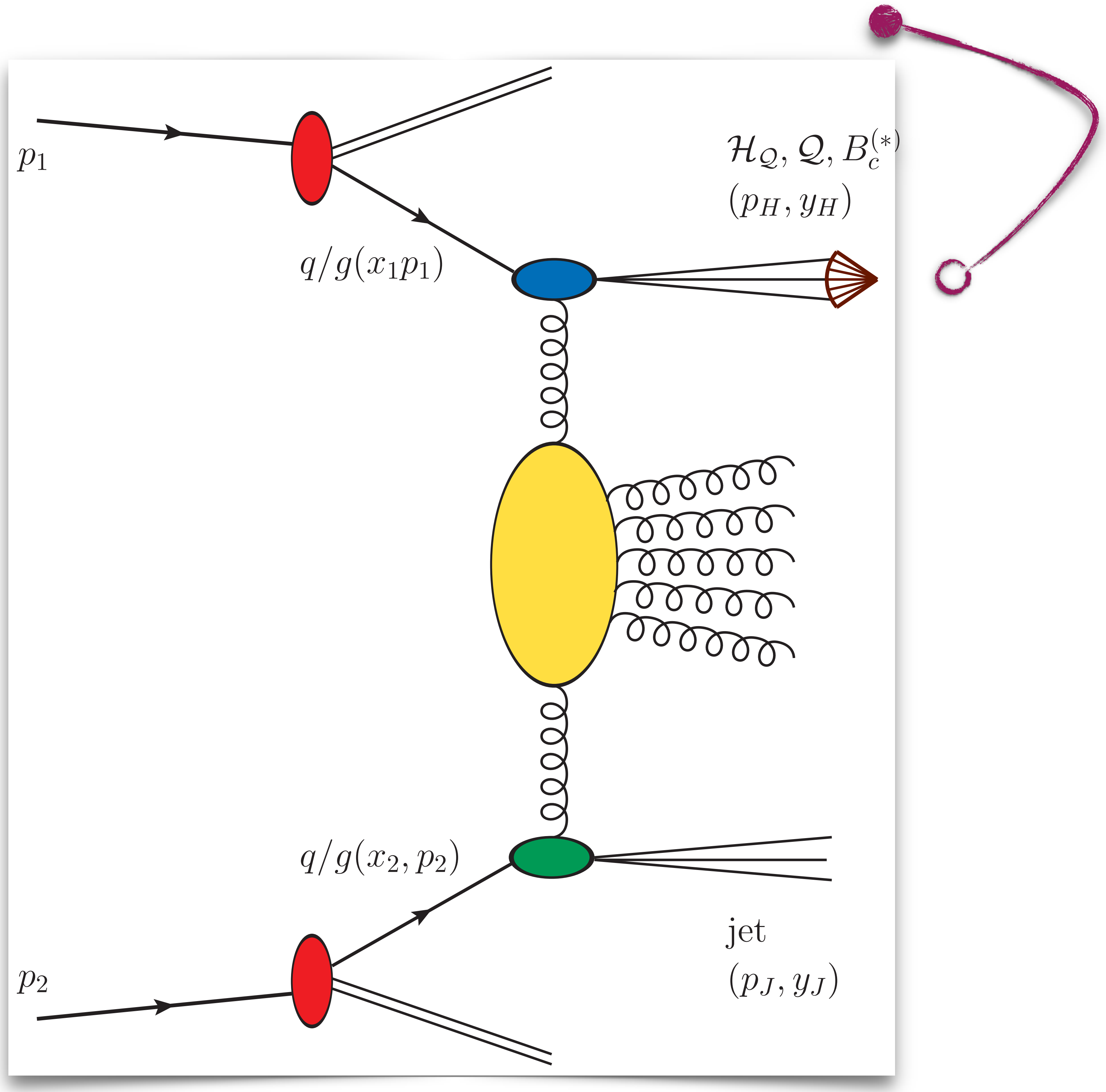

NLO^(+)

NLL

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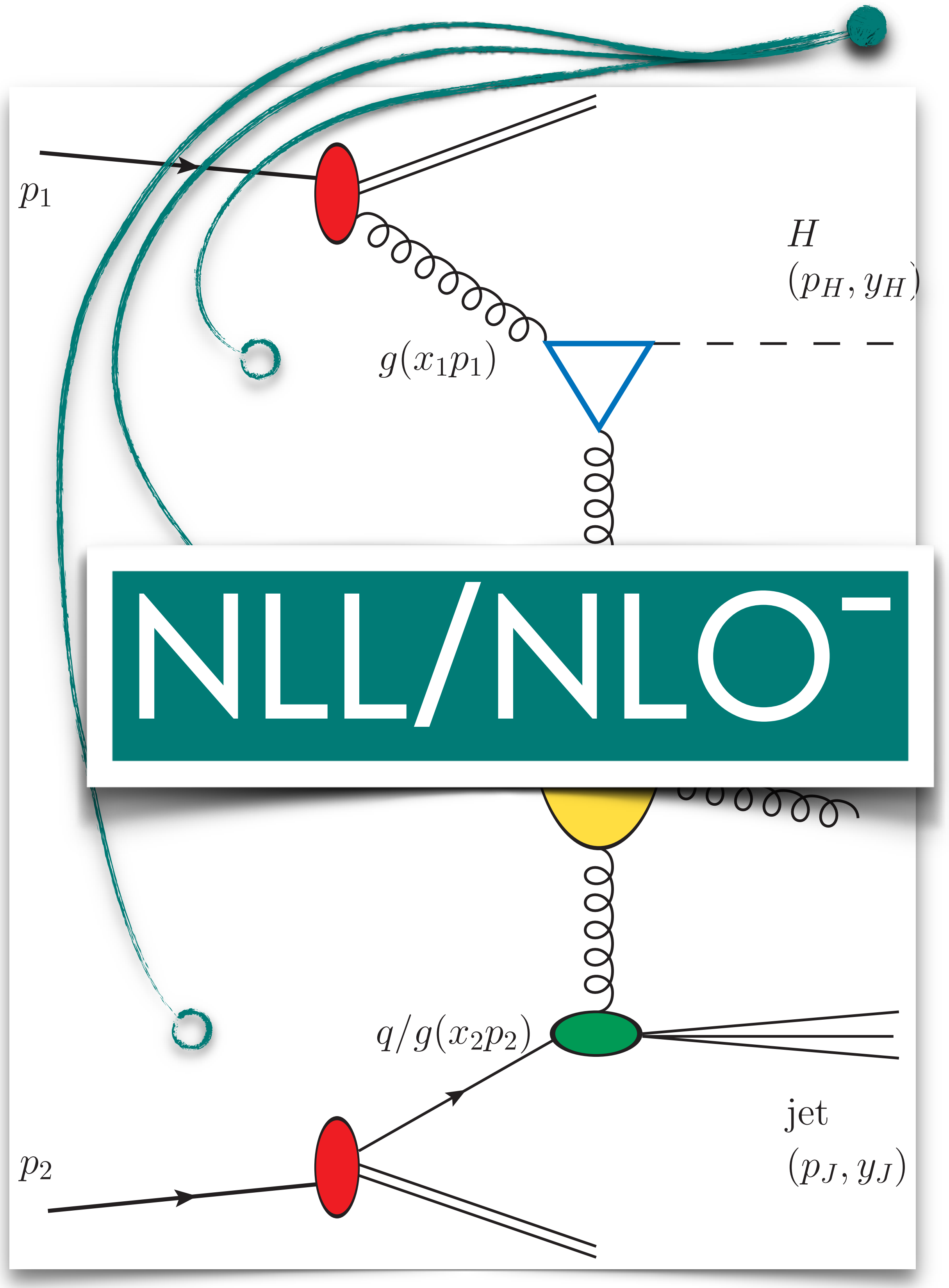

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NLL

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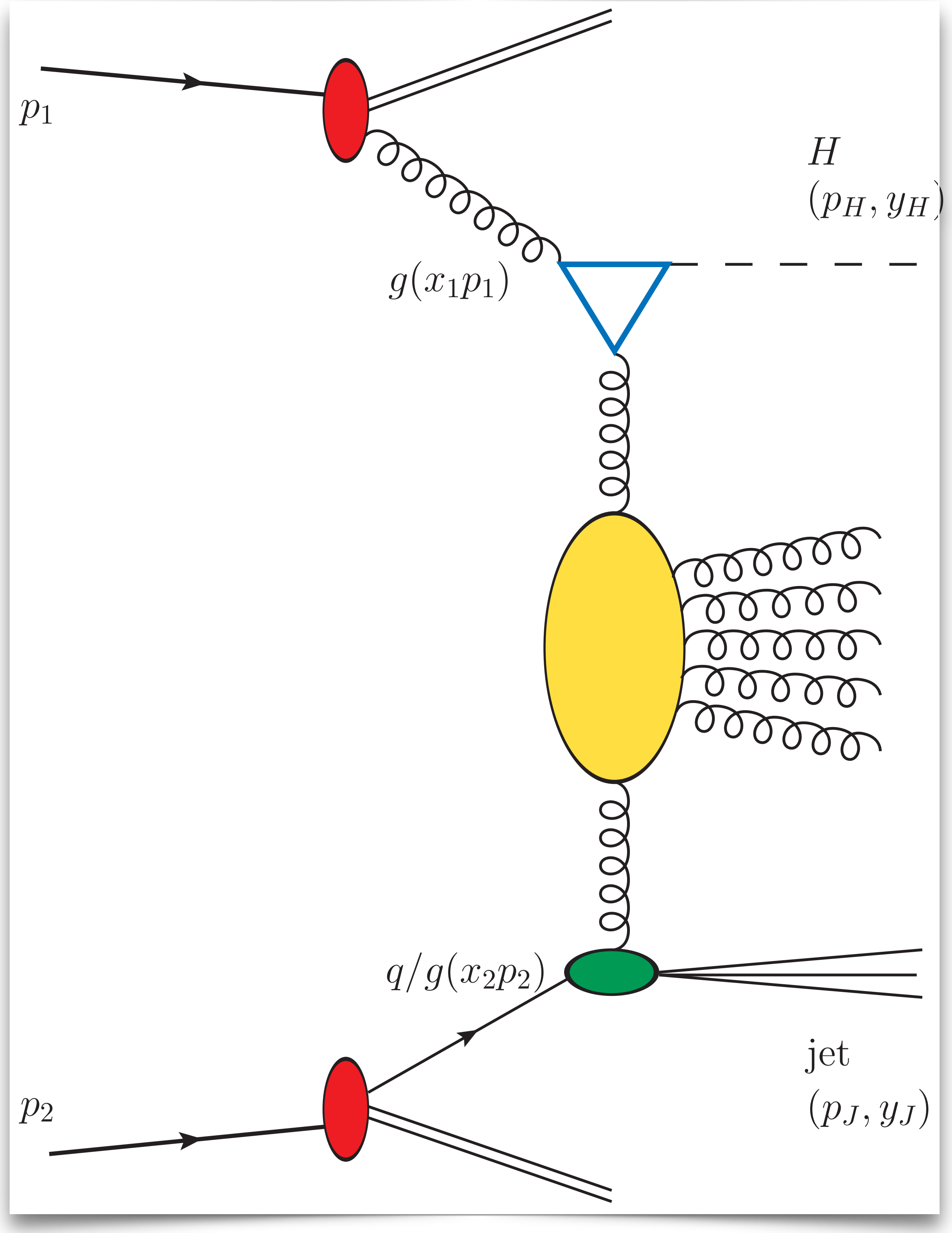
Inclusive Higgs + jet at the LHC

- Inclusive h.p. of a Higgs + jet system with high p_T and large rapidity separation, ΔY
- Large energy scales expected to stabilize the high-energy resummed series

$$\frac{d\sigma}{dx_1 dx_2\, d|\vec{p}_H|\, d|\vec{p}_J| d\varphi_H d\varphi_J} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\varphi)\, \mathcal{C}_n\right]$$



$$\varphi = \varphi_H - \varphi_J - \pi$$

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Higgs vertex (off-shell coefficient function)

jet vertex (off-shell coefficient function)

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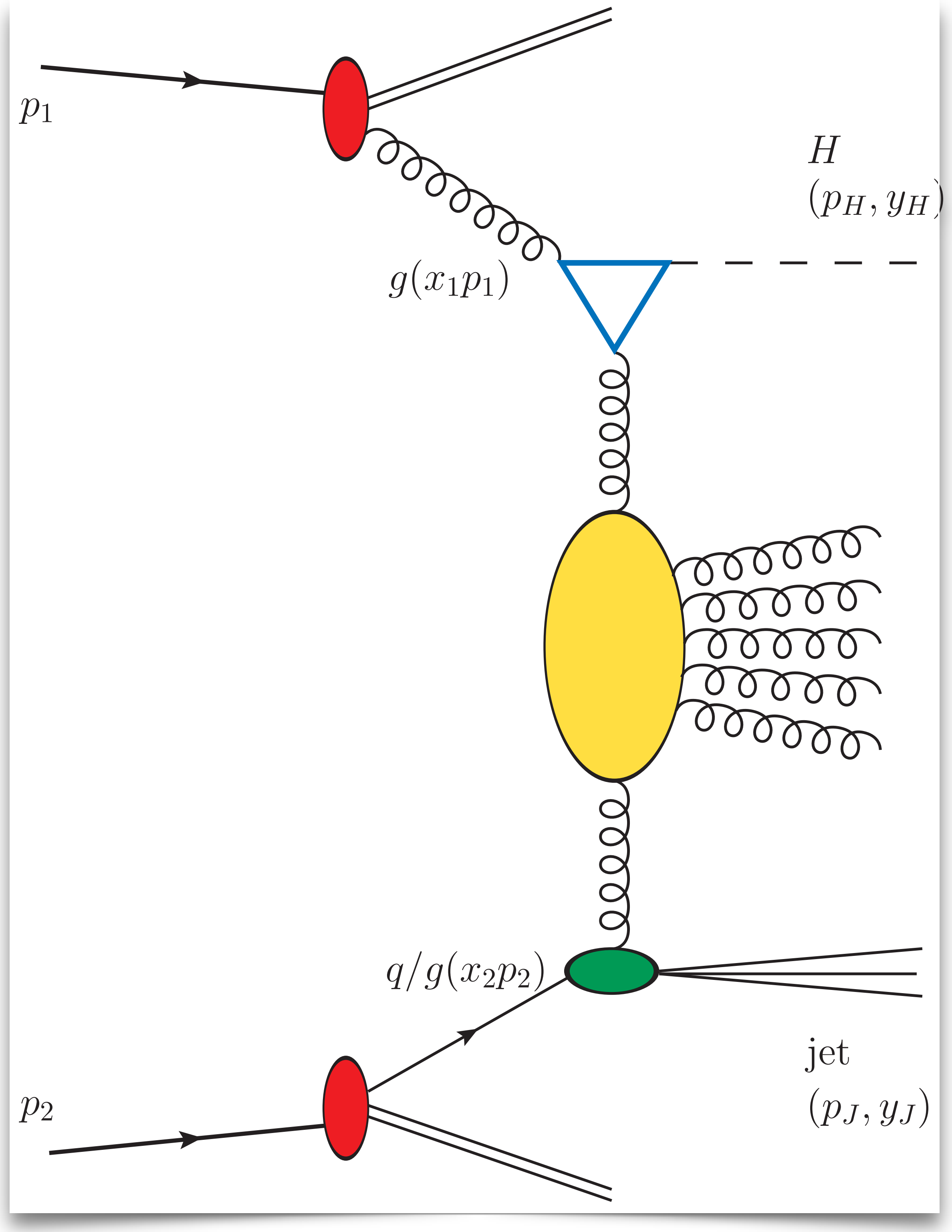

$$\frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu)}{dy_H\, dy_J\, d^2\vec{p}_H\, d^2\vec{p}_J} = \frac{1}{(2\pi)^2}$$

NLO* $\quad \times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\, \mathcal{V}_H^{(r)}(\vec{q}_1, s_0, x_1, \vec{p}_H)$

NLL $\quad \times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2)$

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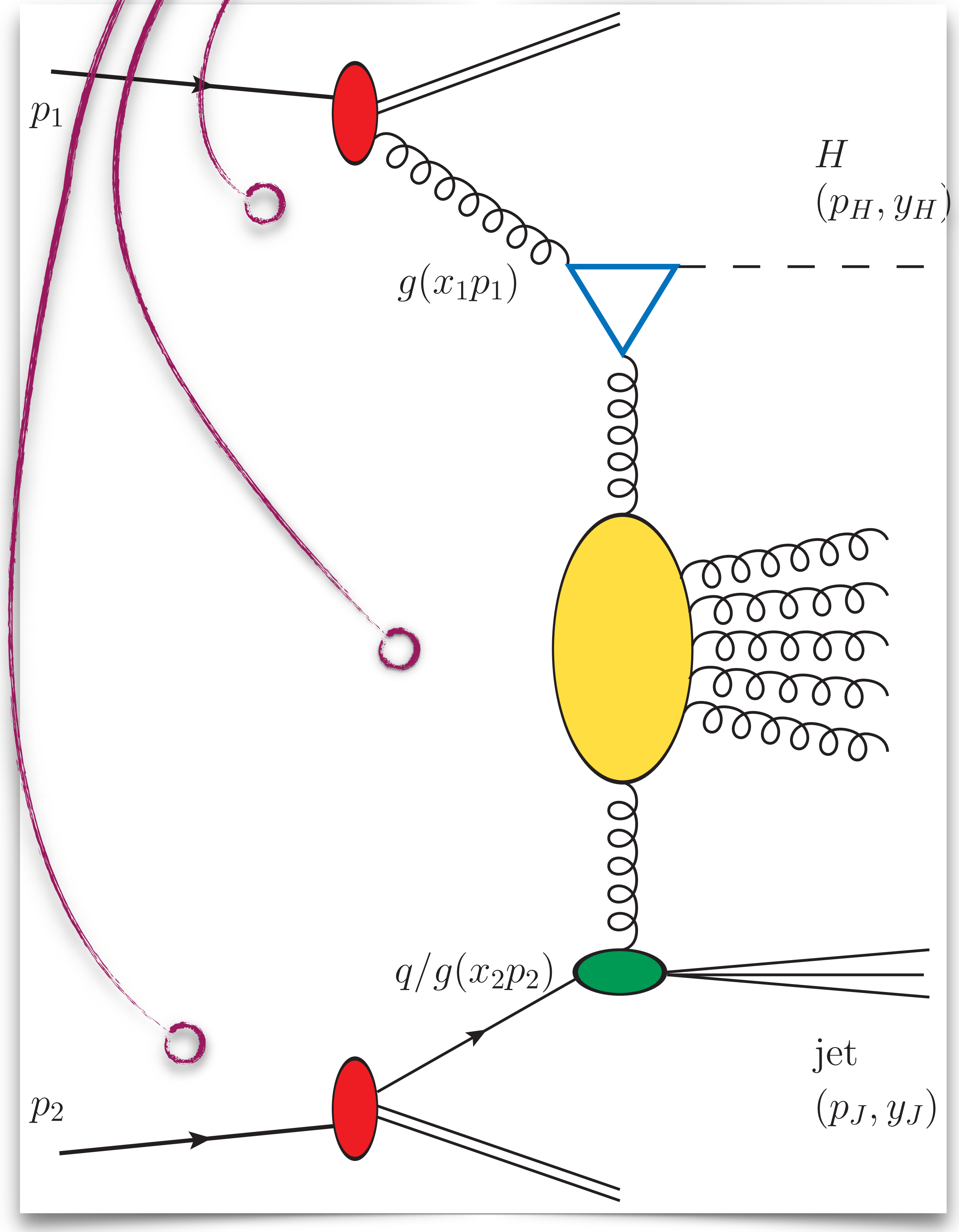
BFKL Green's function

Inclusive Higgs + jet at the LHC

- Inclusive h.p. of a Higgs + jet system with high p_T and large rapidity separation, ΔY
- Large energy scales expected to stabilize the high-energy resummed series



$$\frac{d\sigma}{dx_1 dx_2\, d|\vec{p}_H|\, d|\vec{p}_J| d\varphi_H d\varphi_J} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\varphi)\, \mathcal{C}_n\right]$$

$$\varphi = \varphi_H - \varphi_J - \pi$$

$\mu_{F,R} \sim M_{H,\perp}$

$\mu_R \sim \sqrt{M_{H,\perp}\, p_J}$

$\mu_{F,R} \sim p_J$

NLO*

NLL

NLO*

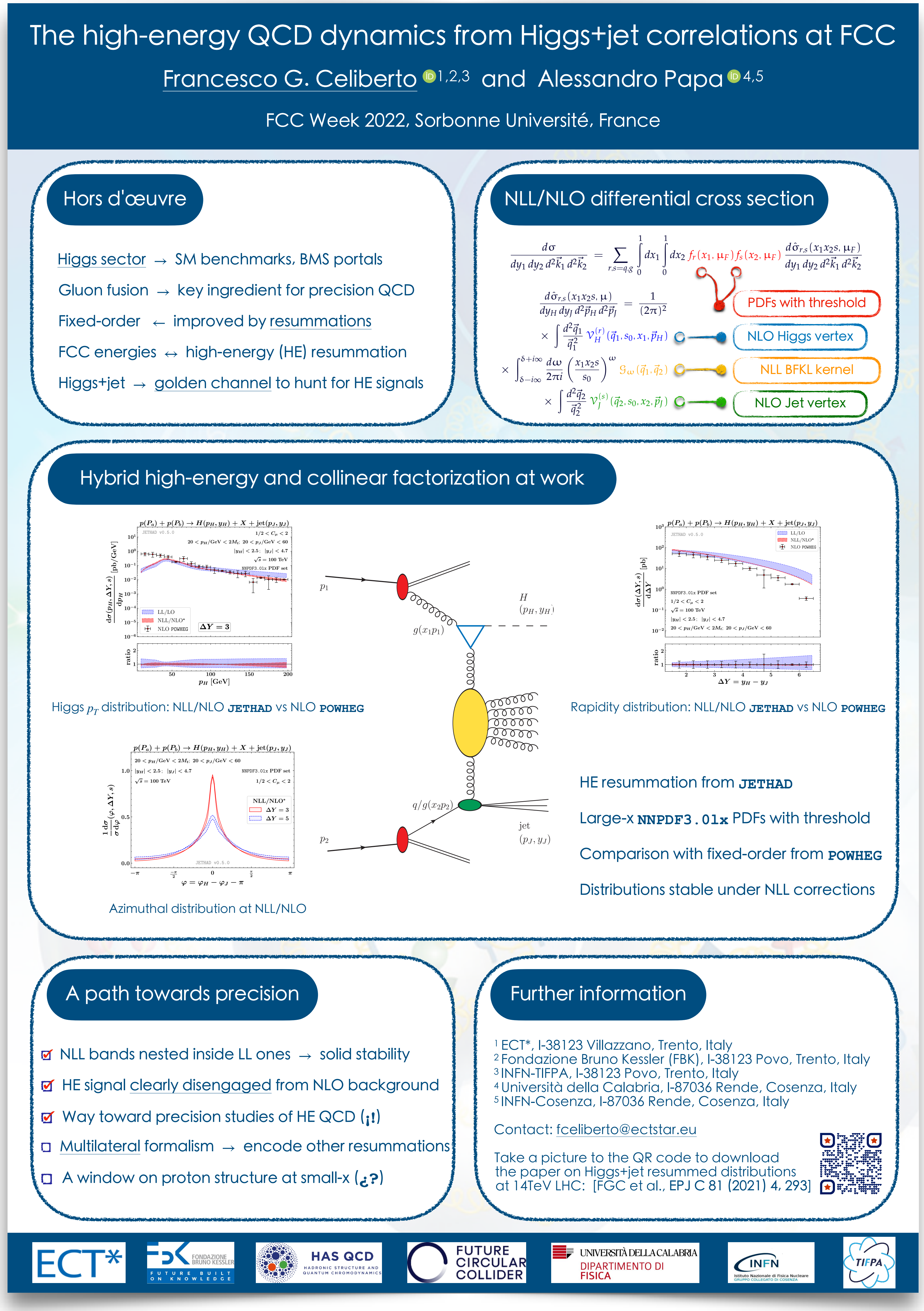
$$\frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu)}{dy_H\, dy_J\, d^2\vec{p}_H\, d^2\vec{p}_J} = \frac{1}{(2\pi)^2}$$
$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\, \mathcal{V}_H^{(r)}(\vec{q}_1, s_0, x_1, \vec{p}_H)$$
$$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2)$$
$$\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}}\, \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{p}_J)$$

Higgs vertex (off-shell coefficient function)

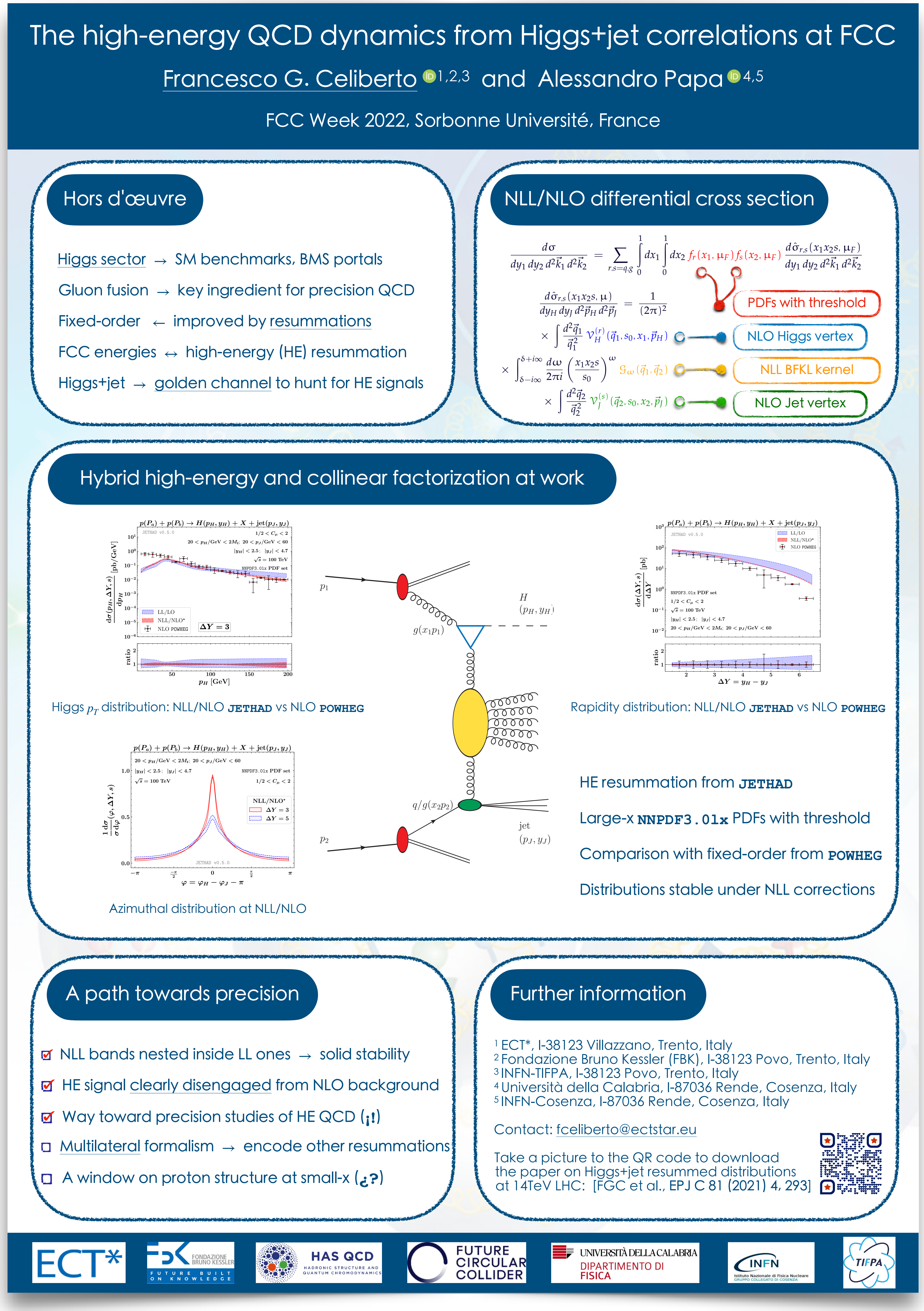
jet vertex (off-shell coefficient function)

BFKL Green's function

Higgs + jet highlights from the FCC Week 2022

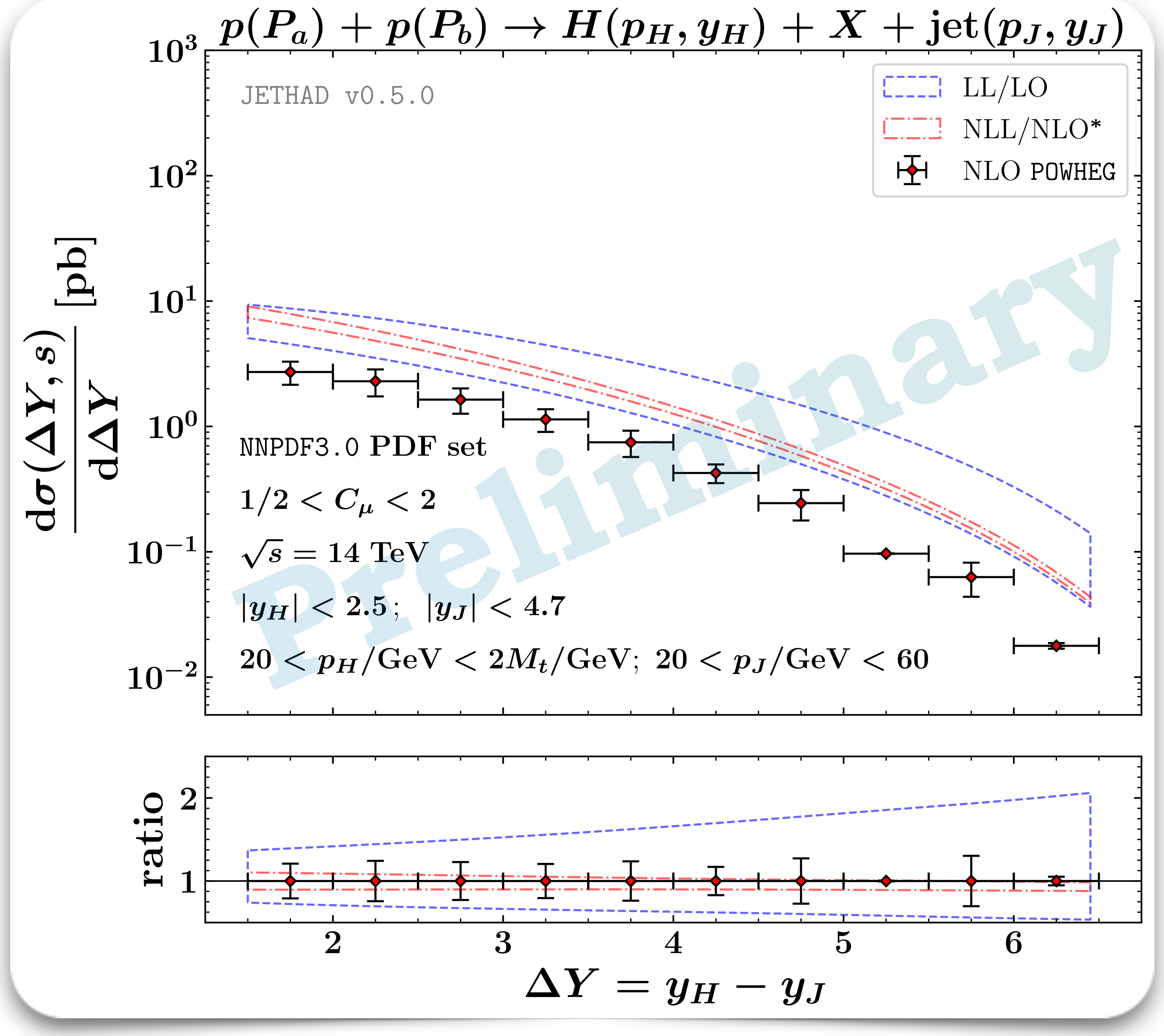


Higgs + jet highlights from the FCC Week 2022

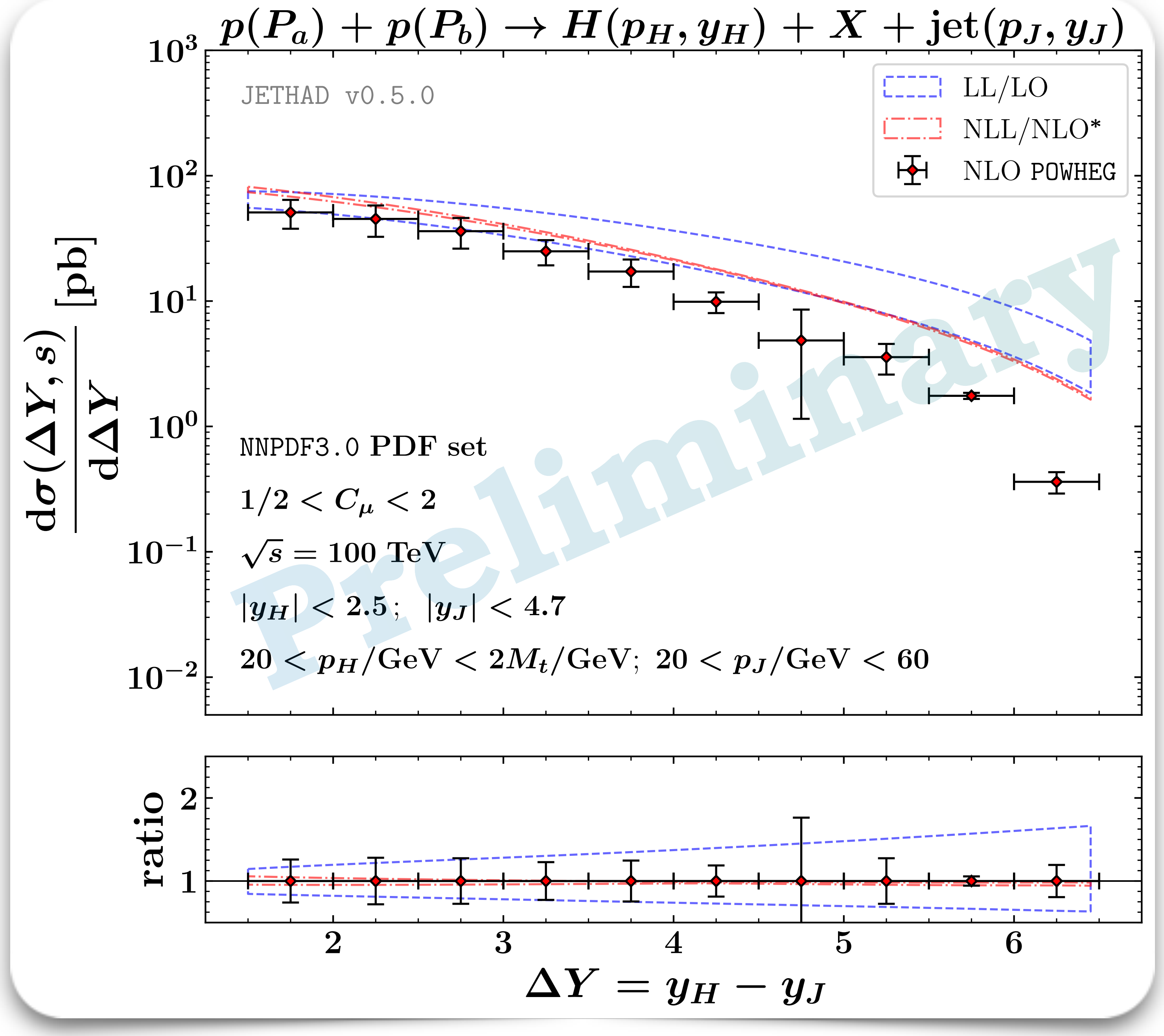


$$C_n(\Delta Y, s) = \int_{p_H^{\rm min}}^{p_H^{\rm max}} d|\vec{p}_H| \int_{p_J^{\rm min}}^{p_J^{\rm max}} d|\vec{p}_J| \int_{y_H^{\rm min}}^{y_H^{\rm max}} dy_H \int_{y_J^{\rm min}}^{y_J^{\rm max}} dy_J \, \delta\left(y_H - y_J - \Delta Y\right) \mathcal{C}_n$$

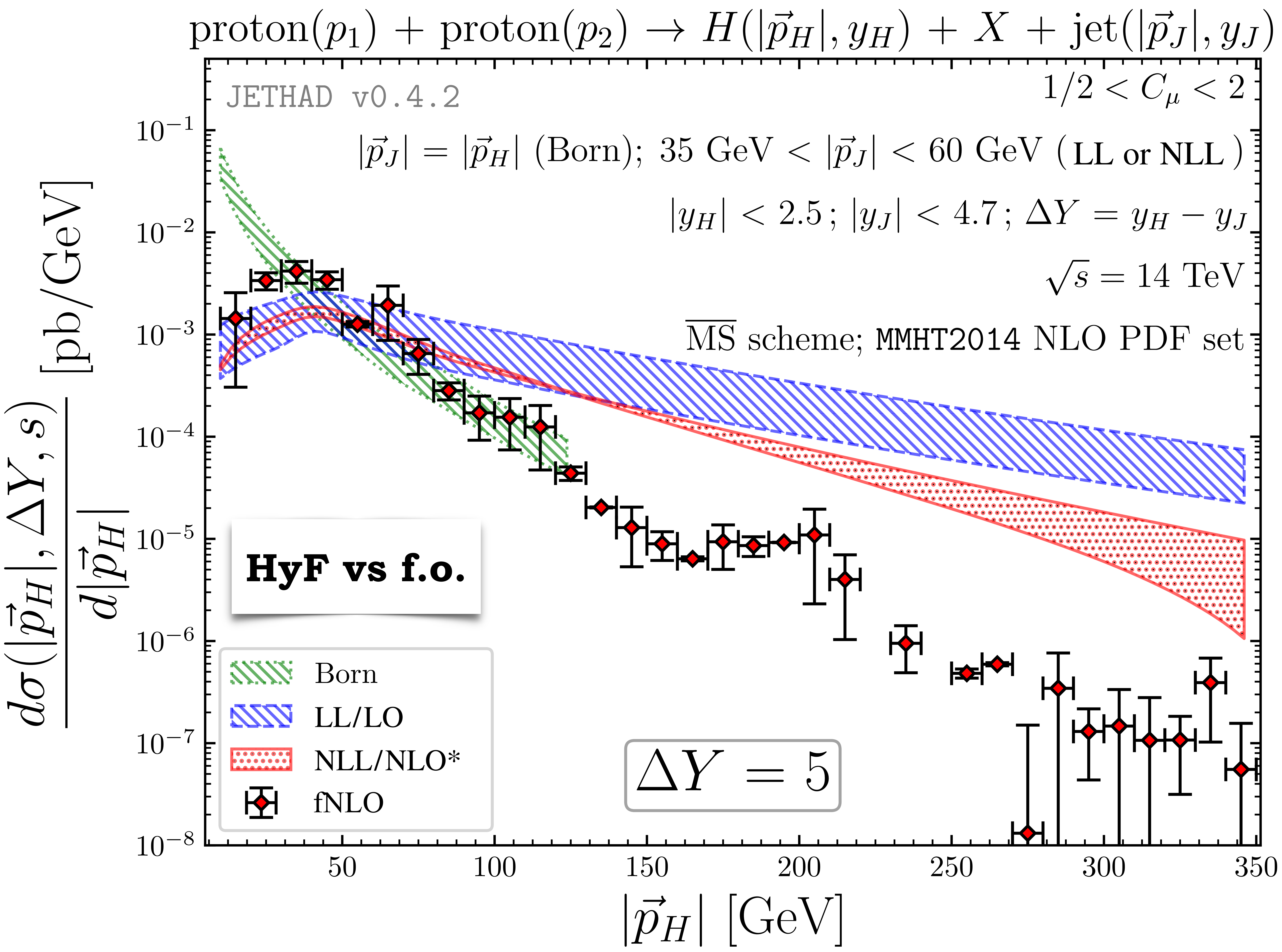
Rapidity distribution: NLL/NLO* JETHAD vs NLO POWHEG



14 TeV LHC

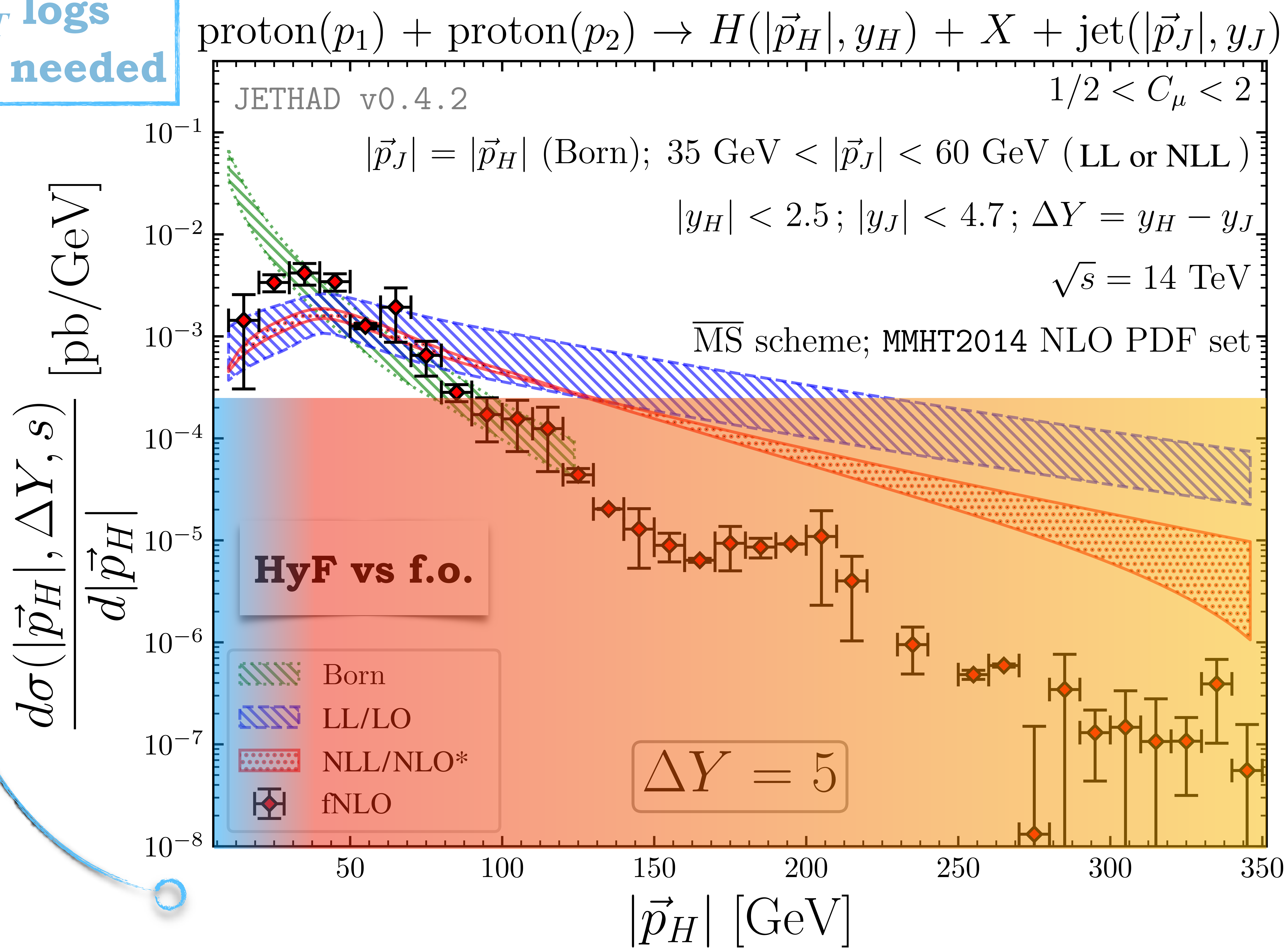


100 TeV FCC

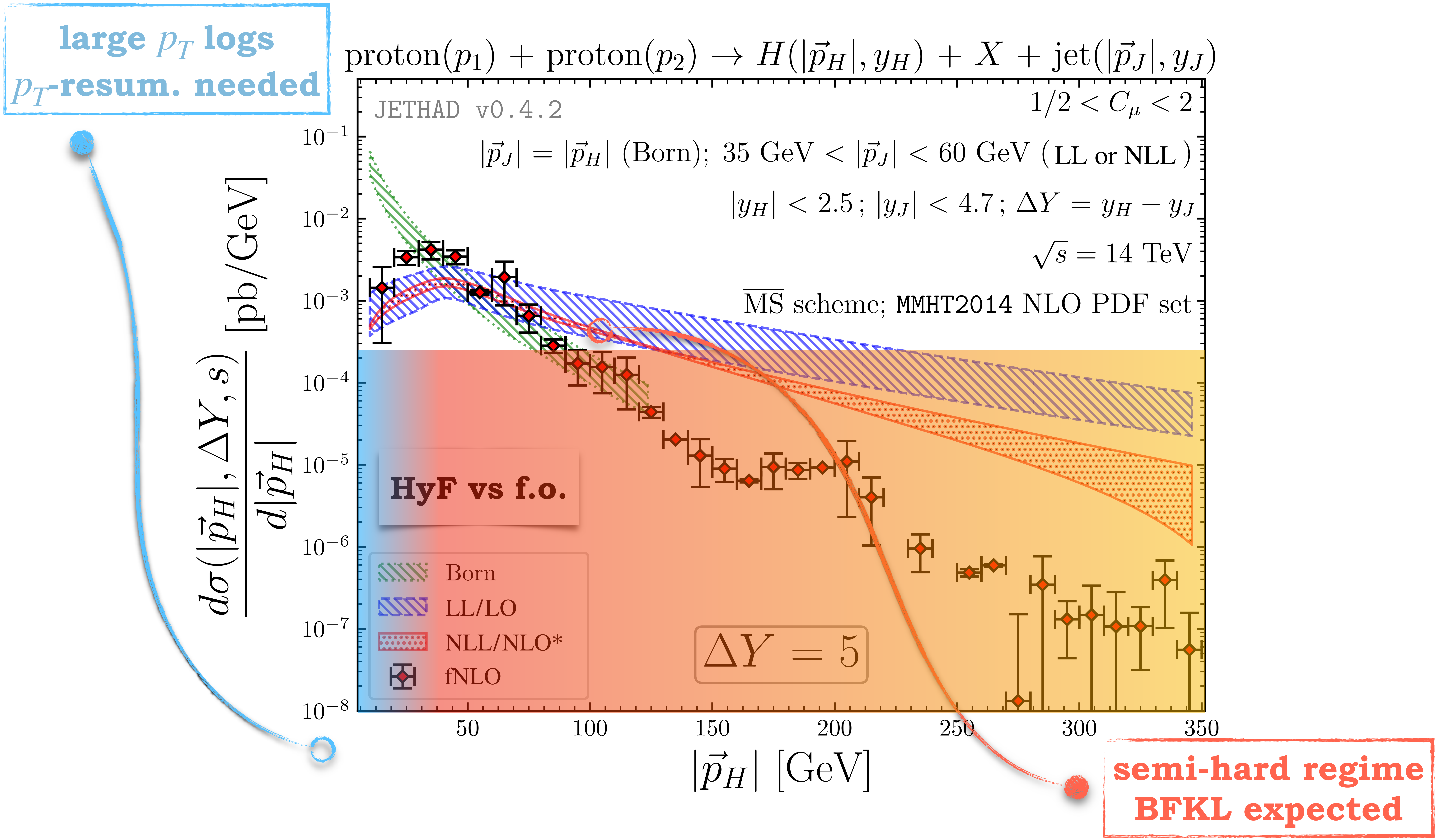


(Higgs + jet, NLL/NLO* HyF) 🔗 [F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]

large p_T logs
p_T-resum. needed



(Higgs + jet, NLL/NLO* HyF) 🔗 [F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]



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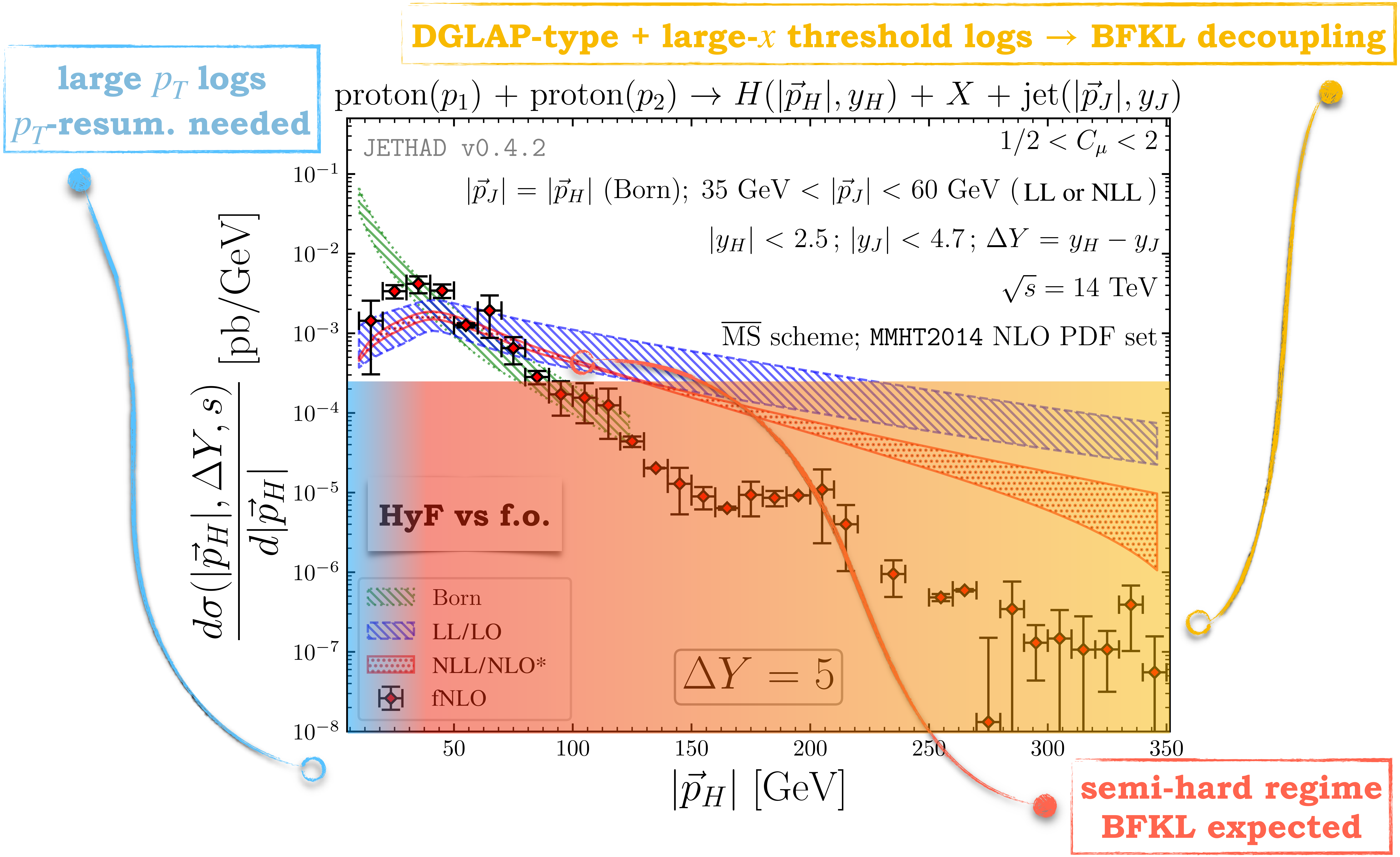

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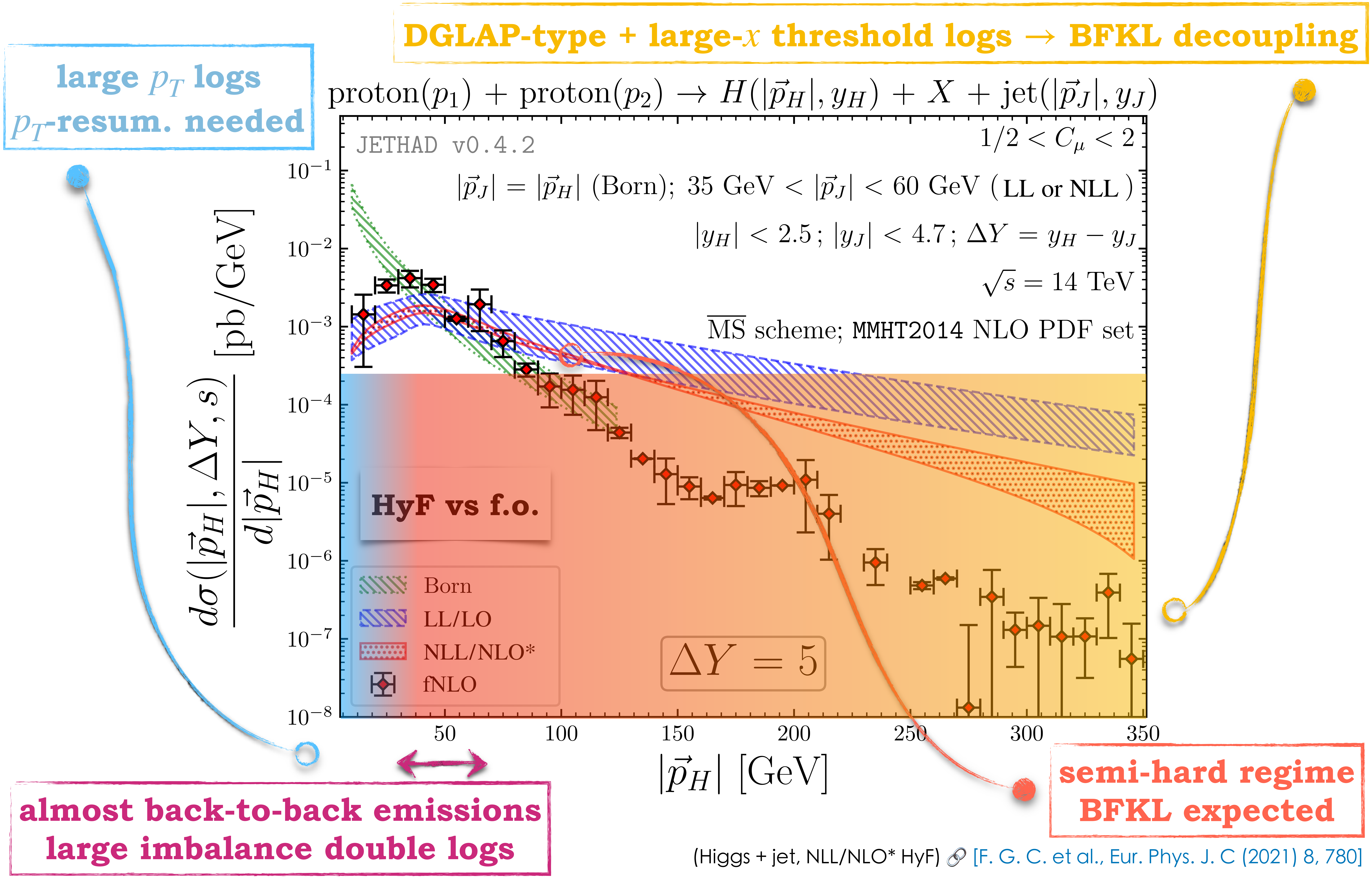
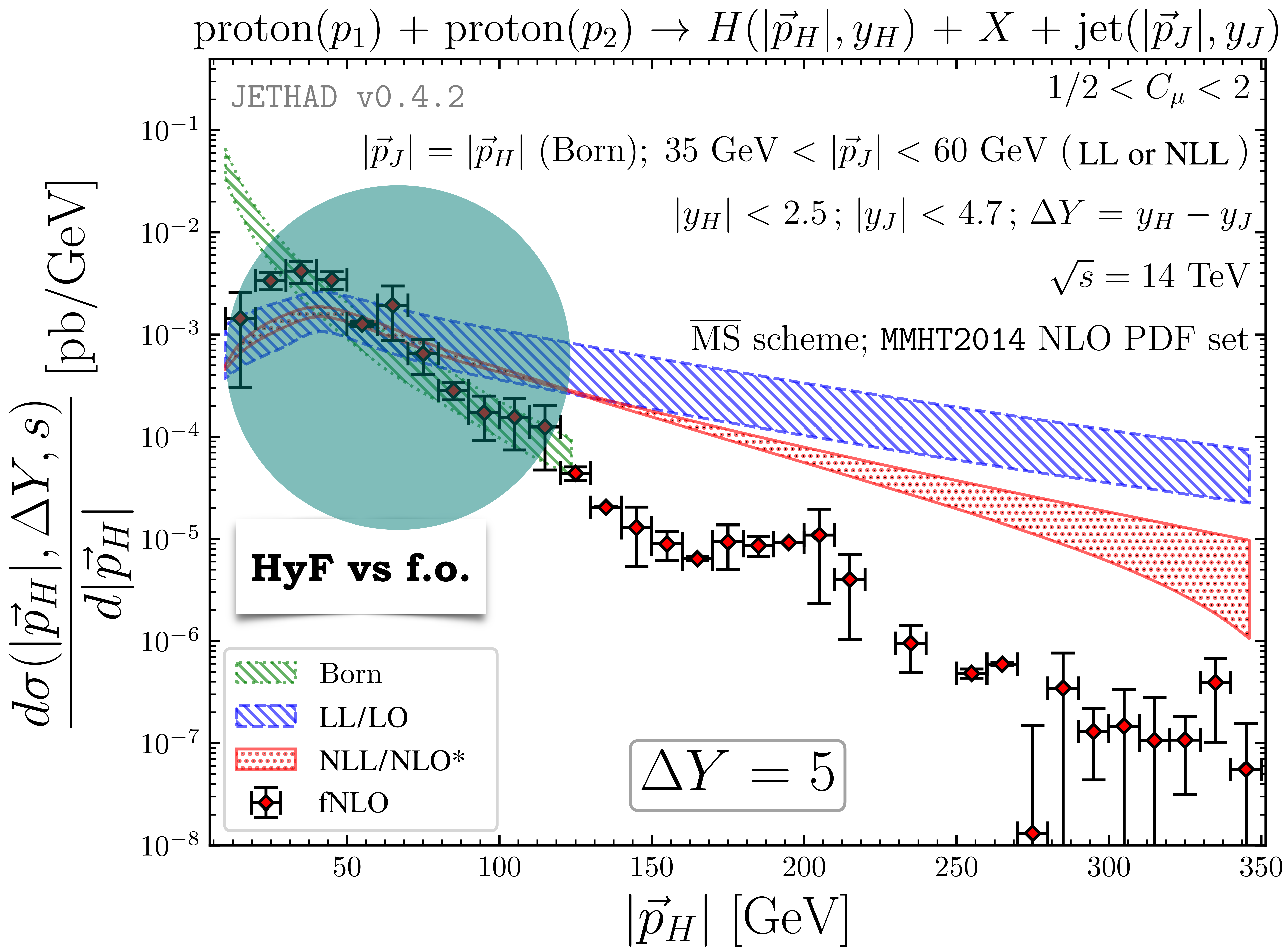


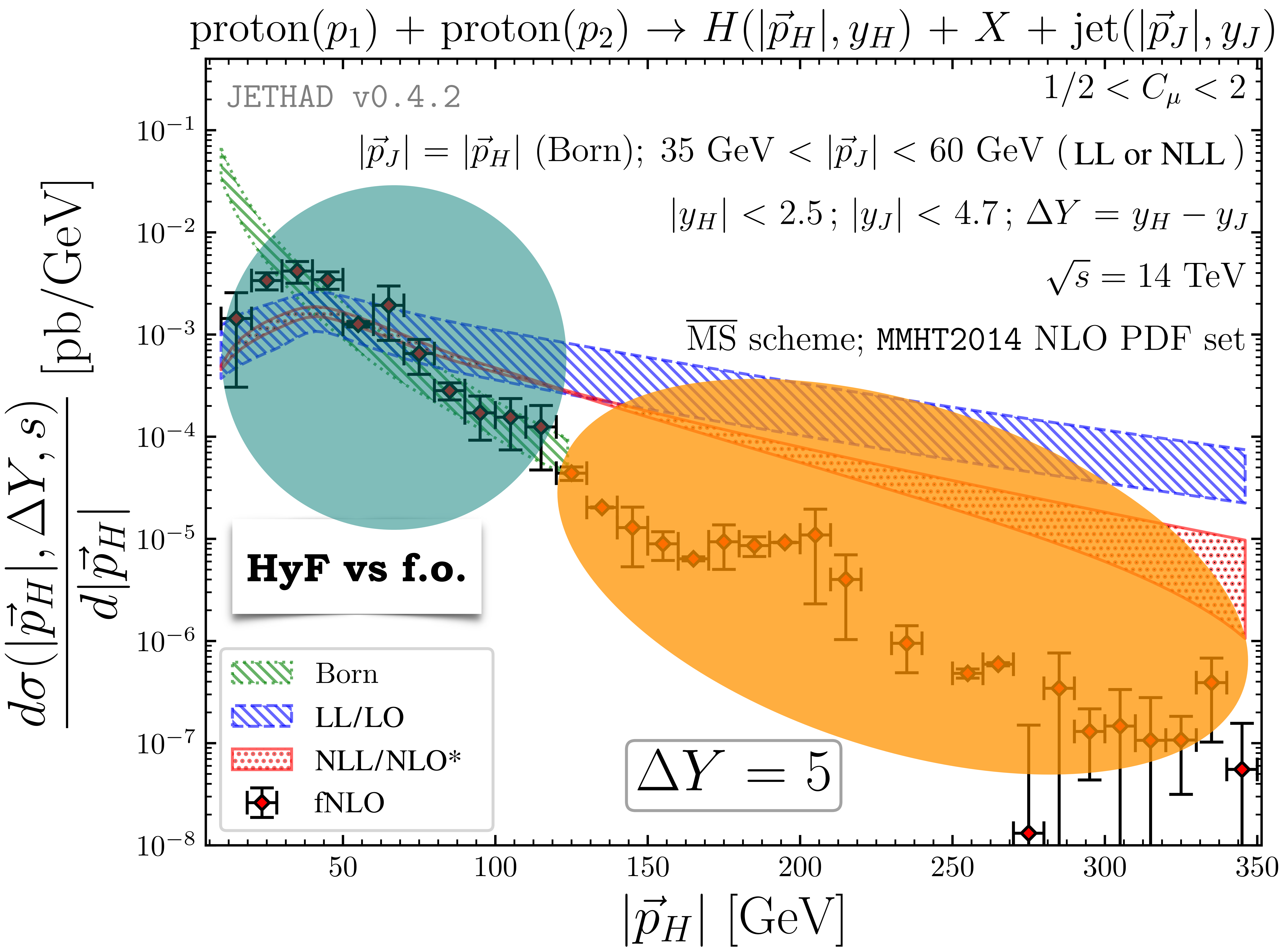
(Higgs + jet, NLL/NLO* HyF) 🔗 [F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]

(Higgs + jet, NLL/NLO* HyF) 🔗 [F. G. C., D Yu. Ivanov, M, M. A. Mohammed, A. Papa, Eur. Phys. J. C (2021) 8, 780]



¡ **Precision corrections** *expected*

(Higgs + jet, NLL/NLO* HyF) 🔗 [F. G. C., D Yu. Ivanov, M, M. A. Mohammed, A. Papa, Eur. Phys. J. C (2021) 8, 780]



¡ Precision corrections ***expected***, but **HyF** *predicts* **large deviations** from **f.o. !**

NLL accurate predictions matched to NLO via the JETHAD Method

Matching NLL to NLO with JETHAD

¡ **Precision corrections** *expected* ⇔ *need* for an accurate NLL-to-NLO **Matching procedure** !

JETHAD Method → NLL/NLO Additive Matching (analytic: BFKL kernel + coefficient functions)

$$\underbrace{d\sigma^{\mathrm{NLL/NLO^-}}(\Delta Y, \varphi, s)} = \underbrace{\underbrace{d\sigma^{\mathrm{NLO}}(\Delta Y, \varphi, s)}_{\text{NLO fixed order}}}_{\text{NLO POWHEG w/o PS}} + d\sigma^{\mathrm{NLL^-}}(\Delta Y, \varphi, s) - \Delta d\sigma^{\mathrm{NLL/NLO^-}}(\Delta Y, \varphi, s)$$

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$$
\underbrace{\mathrm{d}\sigma^{\mathrm{NLL/NLO}^{-}}(\Delta Y, \varphi, s)}
=
\underbrace{\underbrace{\mathrm{d}\sigma^{\mathrm{NLO}}(\Delta Y, \varphi, s)}_{\textbf{NLO}\ \text{fixed order}}}_{\textbf{NLO}\ \texttt{POWHEG}\ \text{w/o PS}}
+
\underbrace{\underbrace{\mathrm{d}\sigma^{\mathrm{NLL}^{-}}(\Delta Y, \varphi, s)}_{\textbf{NLL}^{-}\ \textbf{resum}\ \text{(HyF)}}
-
\underbrace{\Delta\mathrm{d}\sigma^{\mathrm{NLL/NLO}^{-}}(\Delta Y, \varphi, s)}_{\textbf{NLL}^{-}\ \textbf{expanded}\ \text{at NLO}}}_{\textbf{NLL}^{-}\ \texttt{JETHAD}\ \text{w/o NLO}^{-}\ \text{double counting}}
$$

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HELL + ggHiggs

$N^3LL_{lx}/LL_{sx}/N^3LO$

Inclusive Higgs

[M. Bonvini, S. Marzani (2018)]

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HELL + ggHiggs

$N^3LL_{lx}/LL_{sx}/N^3LO$

Inclusive Higgs

[M. Bonvini, S. Marzani (2018)]

HEJ framework

NLL_{sx}^-/NLO

Higgs + jet(s)

[J. R. Andersen et al. (2022)]

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[M. Bonvini, S. Marzani (2018)]

HEJ framework

$\mathrm{NLL_{sx}^{-}/NLO}$

Higgs + jet(s)

[J. R. Andersen et al. (2022)]

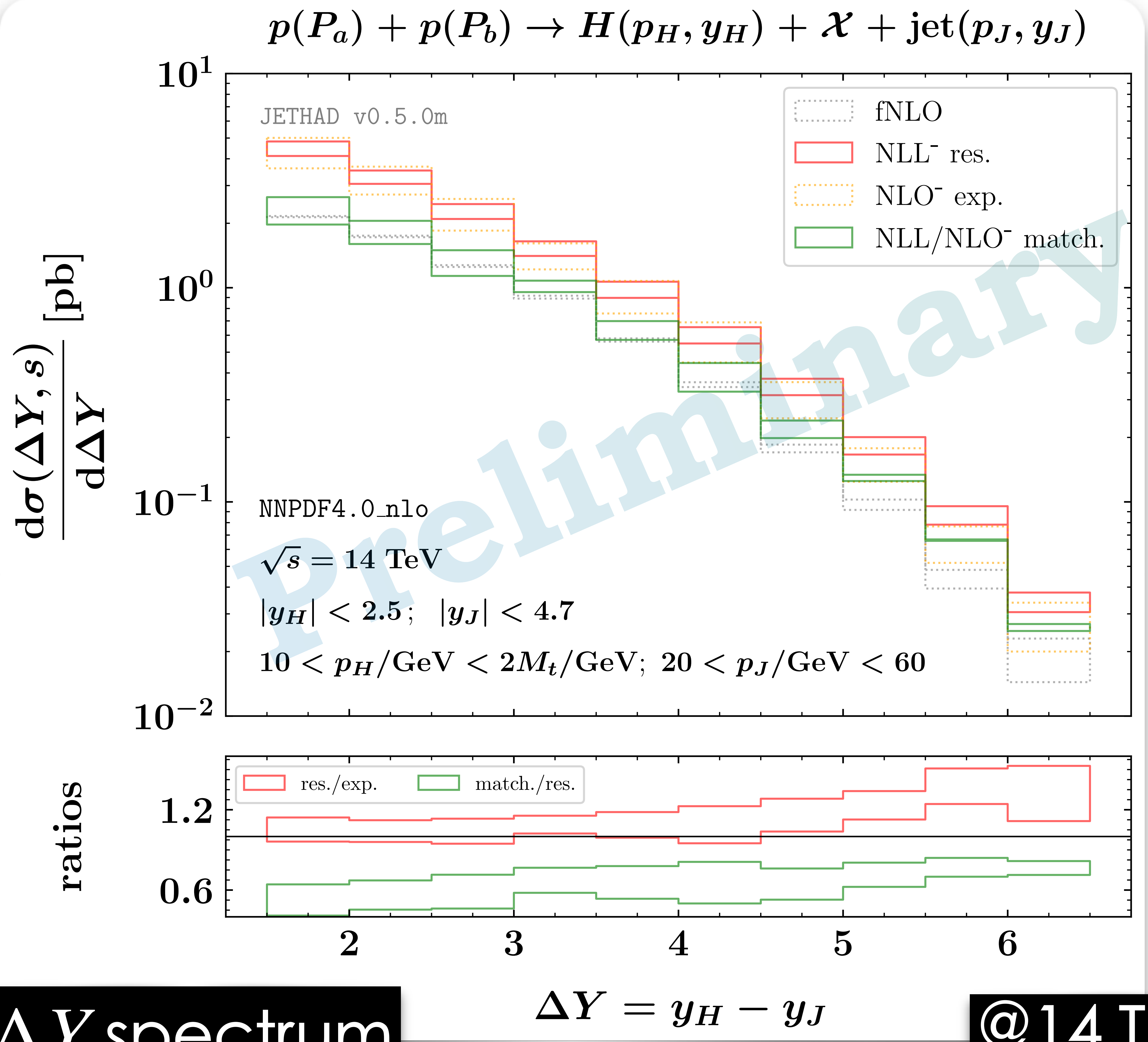
RadISH + MCFM-8.3

$\mathrm{NNLL_{TM}/NLO}$

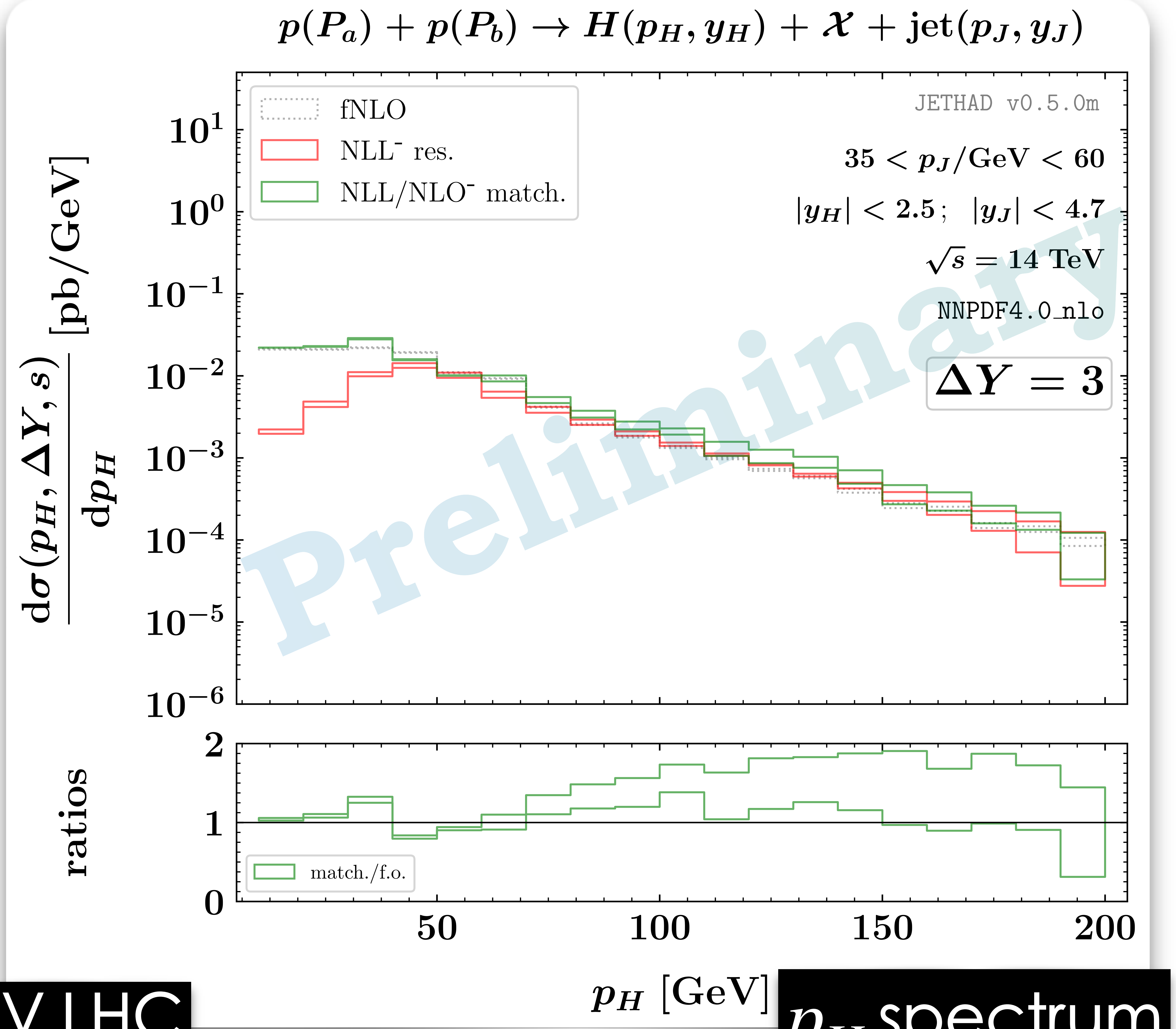
Higgs + jet

[P.F. Monni et al. (2020)]

The Higgs + jet spectrum from POWHEG + JETHAD



ΔY spectrum

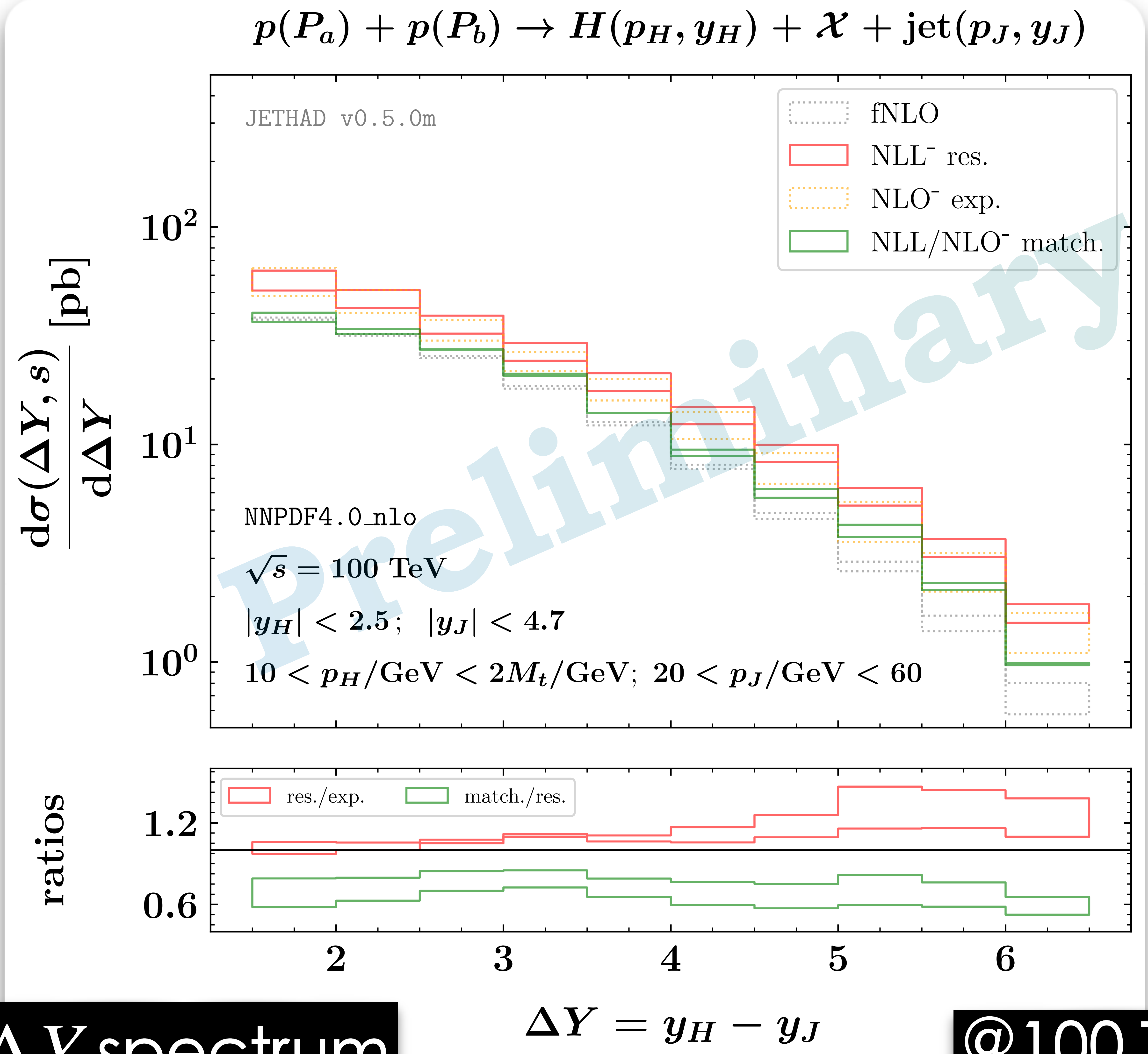


@14 TeV LHC

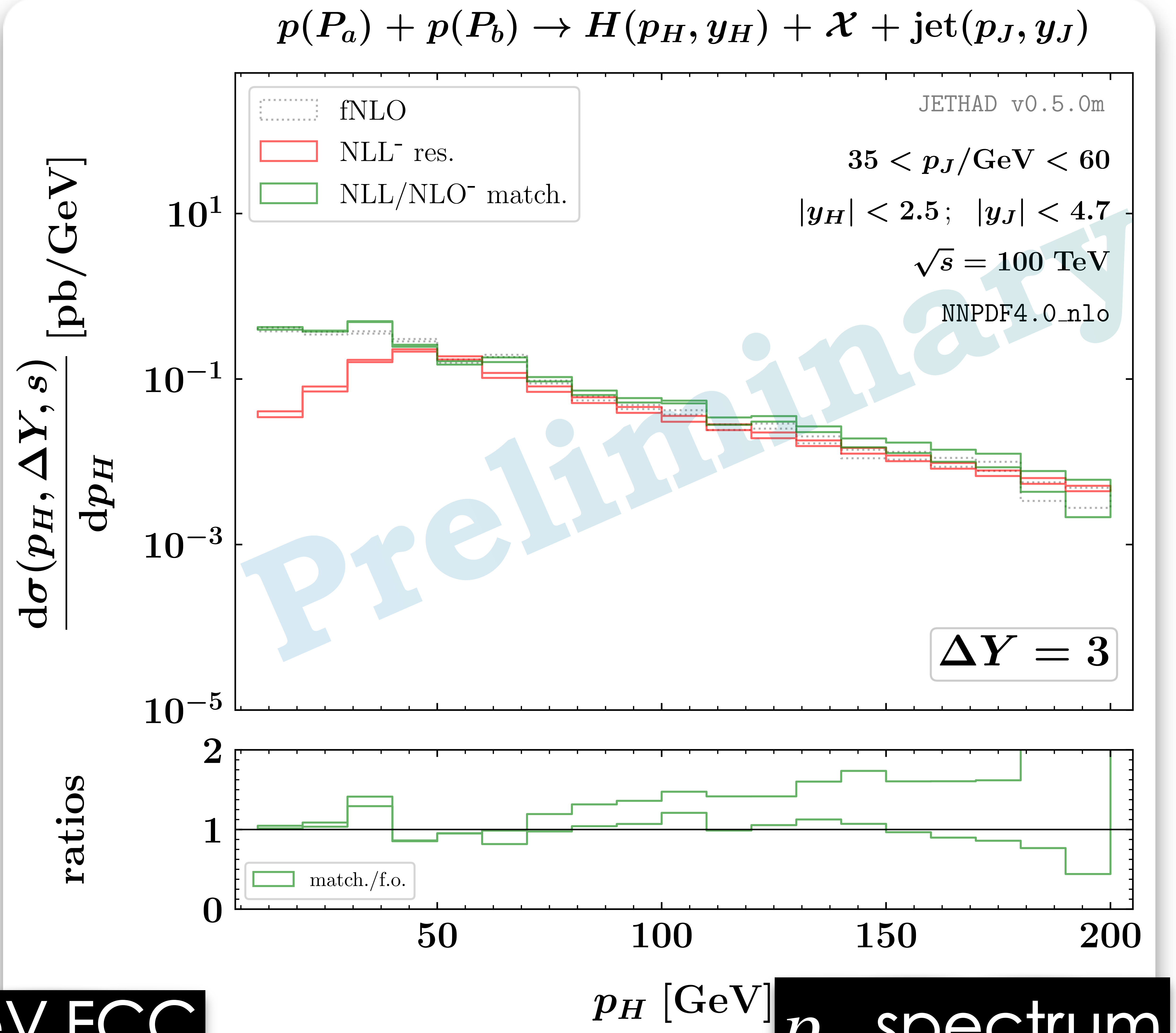
p_H spectrum

NLL matched to NLO fixed-order `POWHEG + JETHAD` (in progress)

The Higgs + jet spectrum from POWHEG + JETHAD



ΔY spectrum

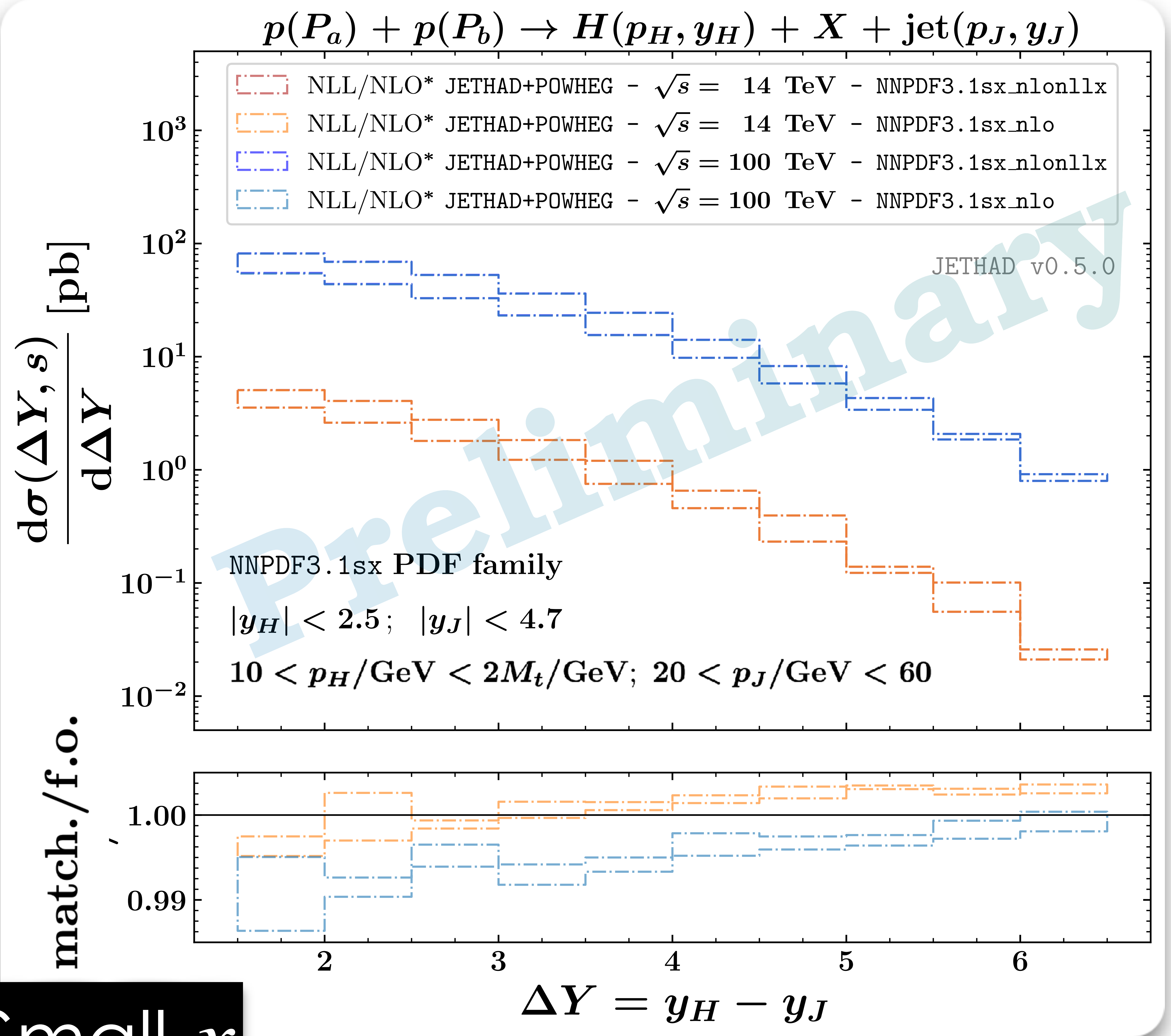


@100 TeV FCC

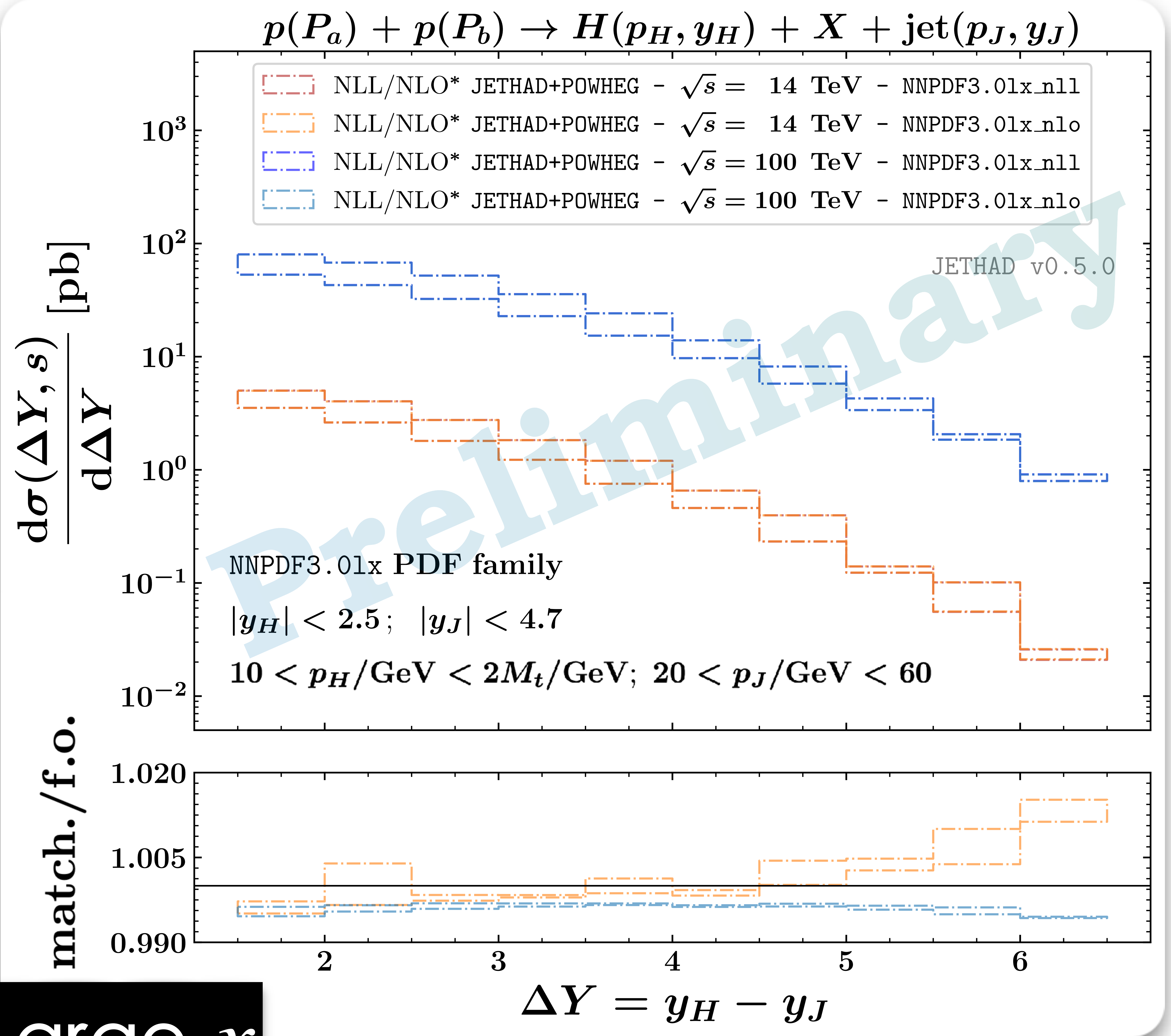
p_H spectrum

NLL matched to NLO fixed-order JETHAD + POWHEG (in progress)

Small-x and large-x enhancement from PDFs



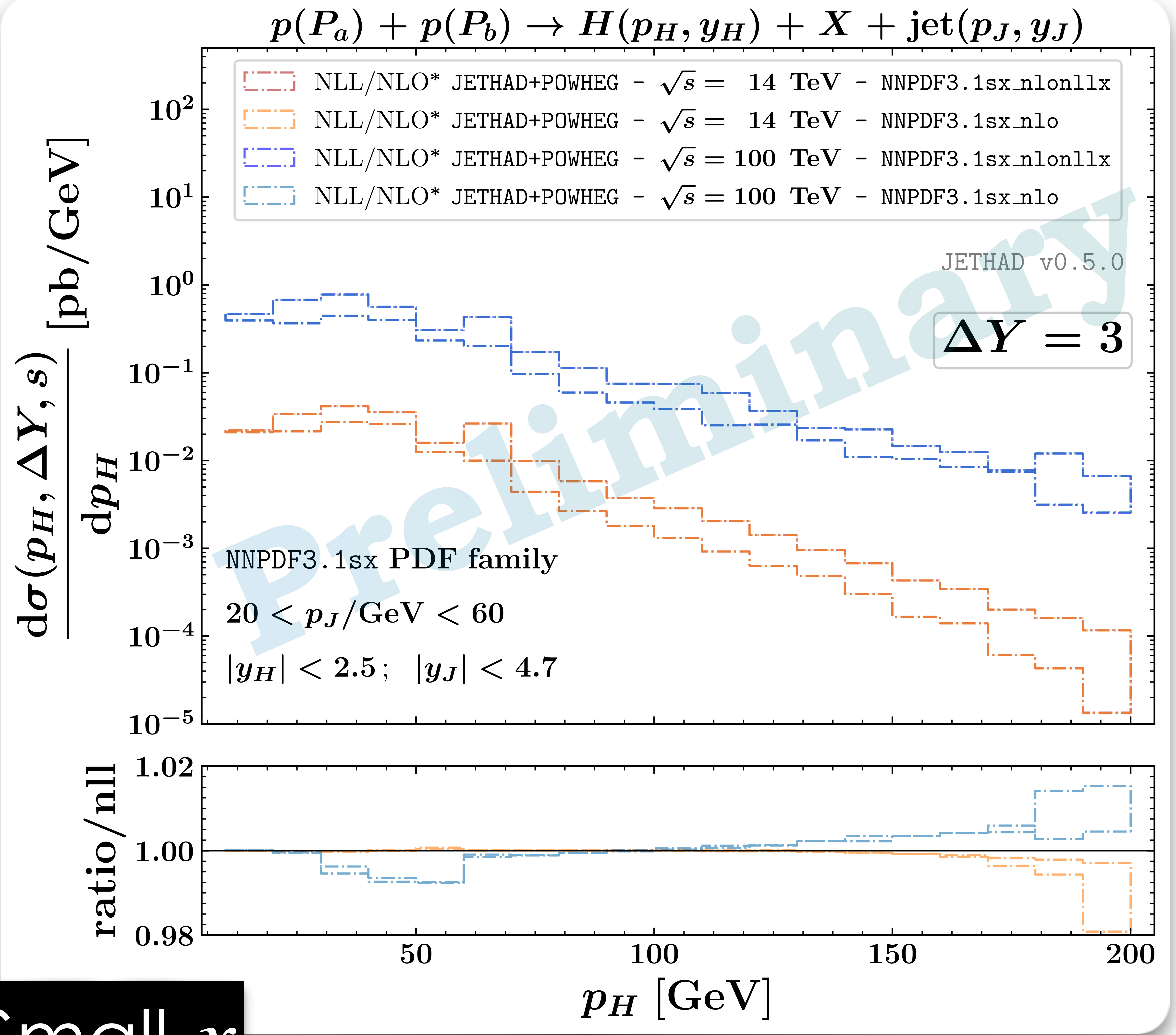
Small-x



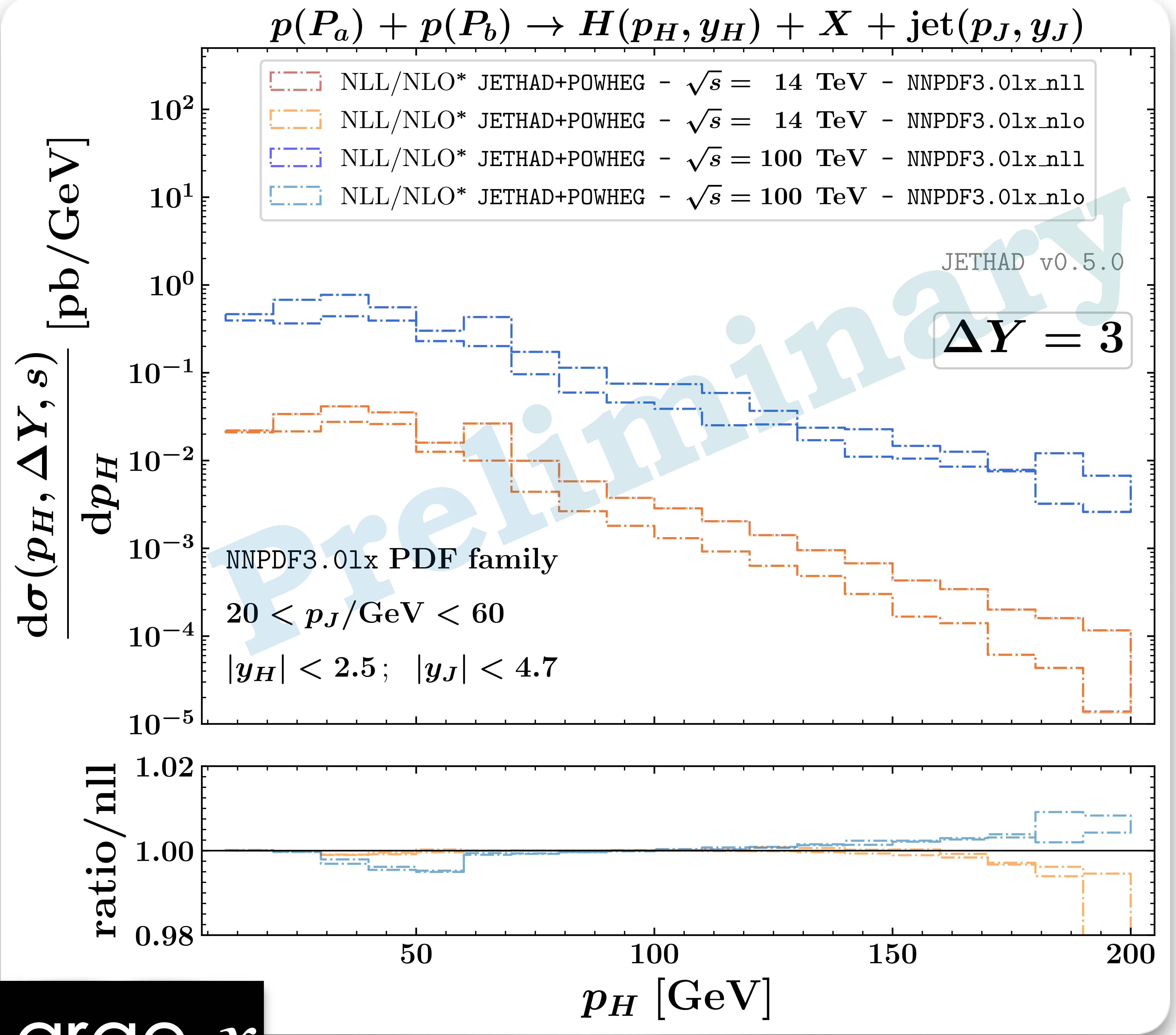
Large-x

Impact of large-x threshold logs on NLO emission functions to be gauged

Small-x and large-x enhancement from PDFs



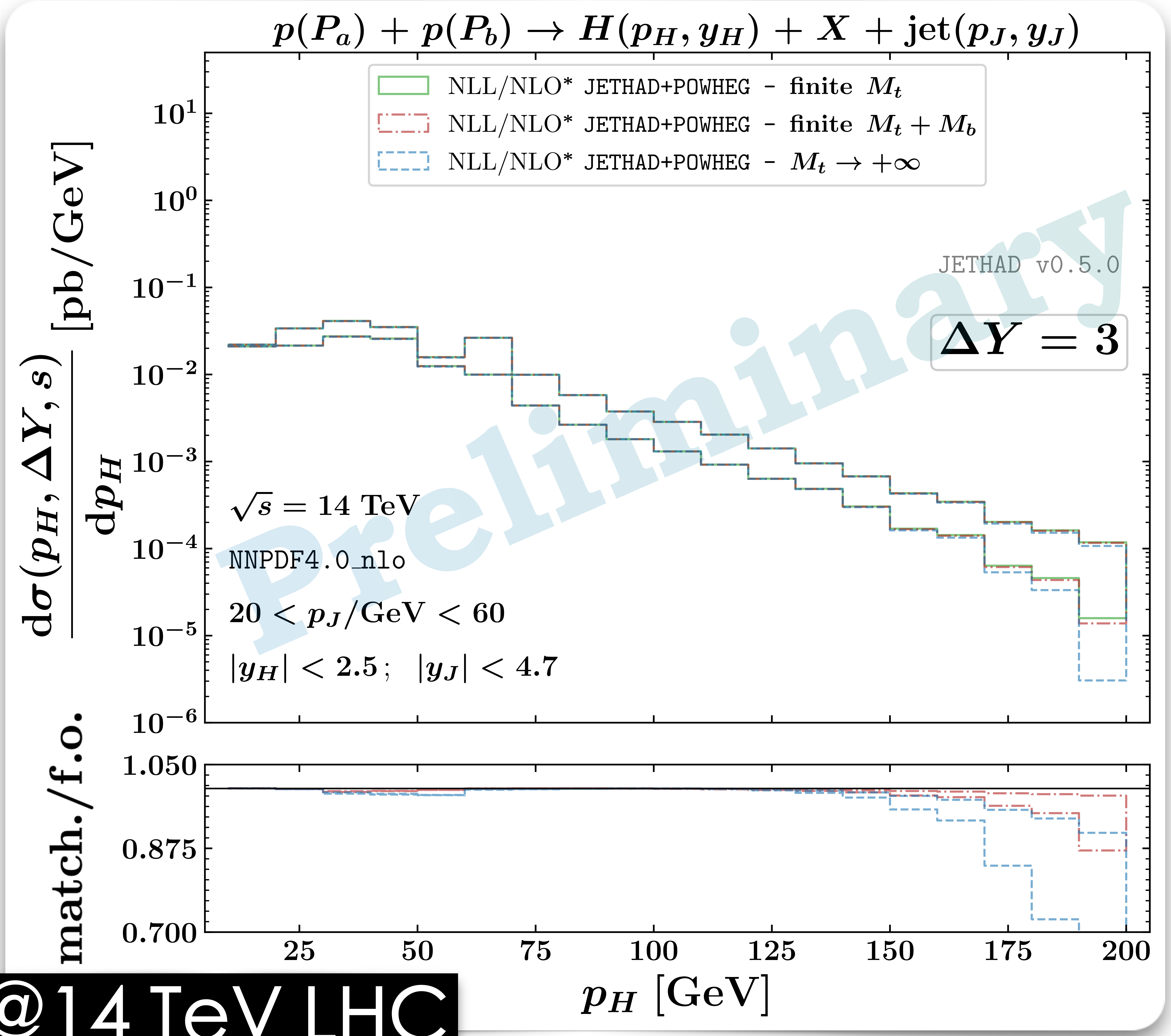
Small-x



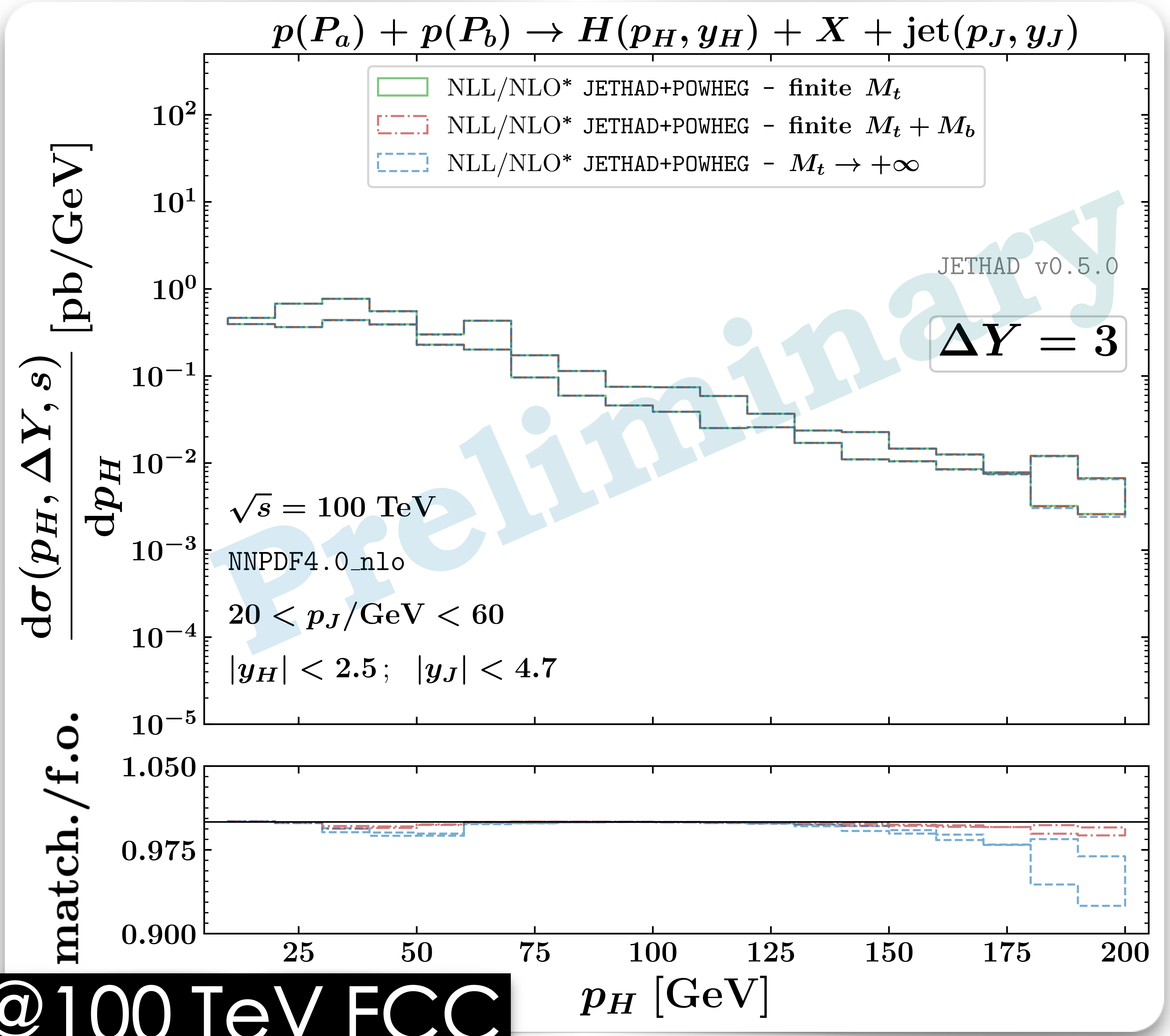
Large-x

Impact of large-x threshold logs on NLO emission functions to be gauged

Finite top- and bottom-mass corrections



@14 TeV LHC

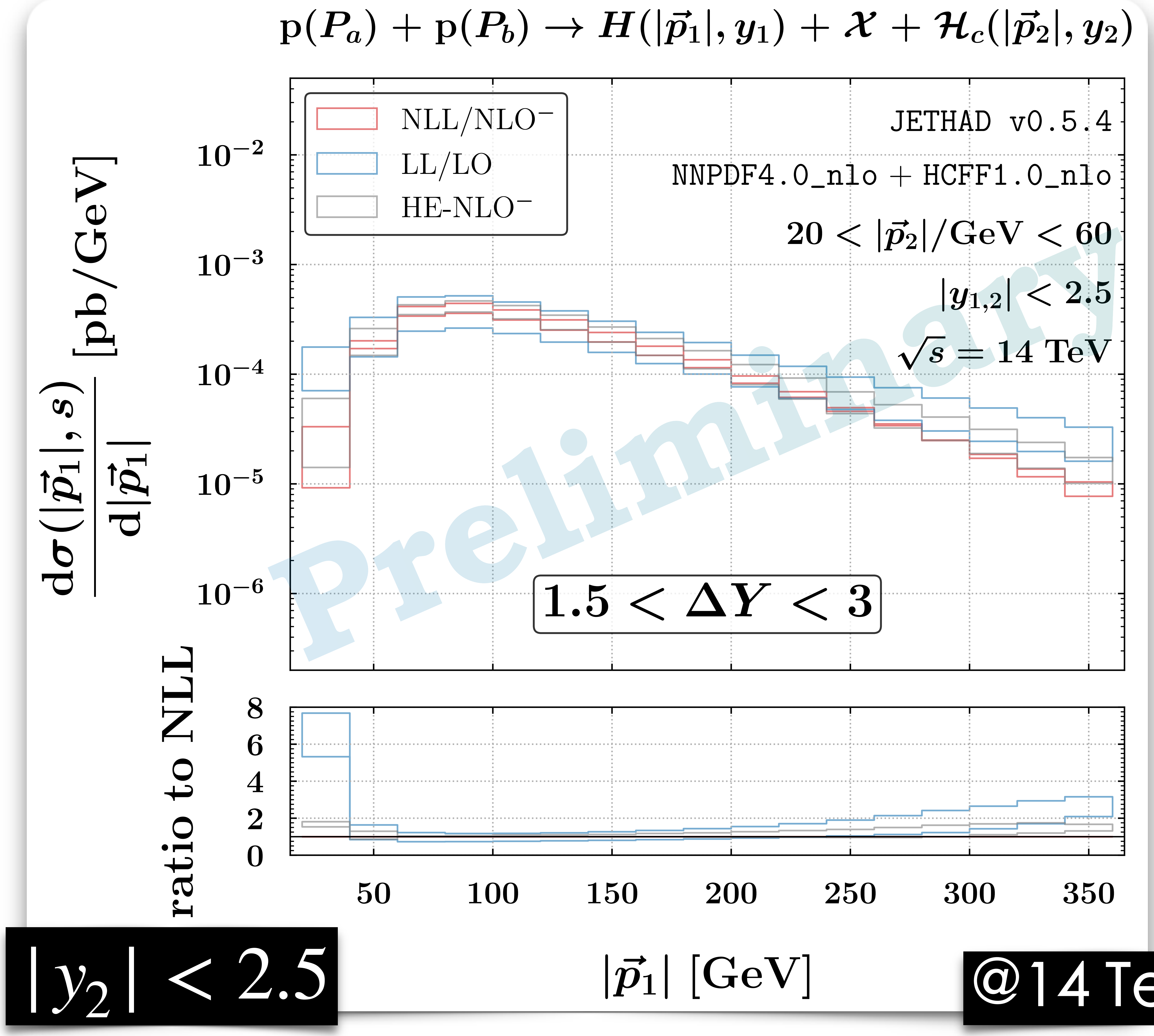


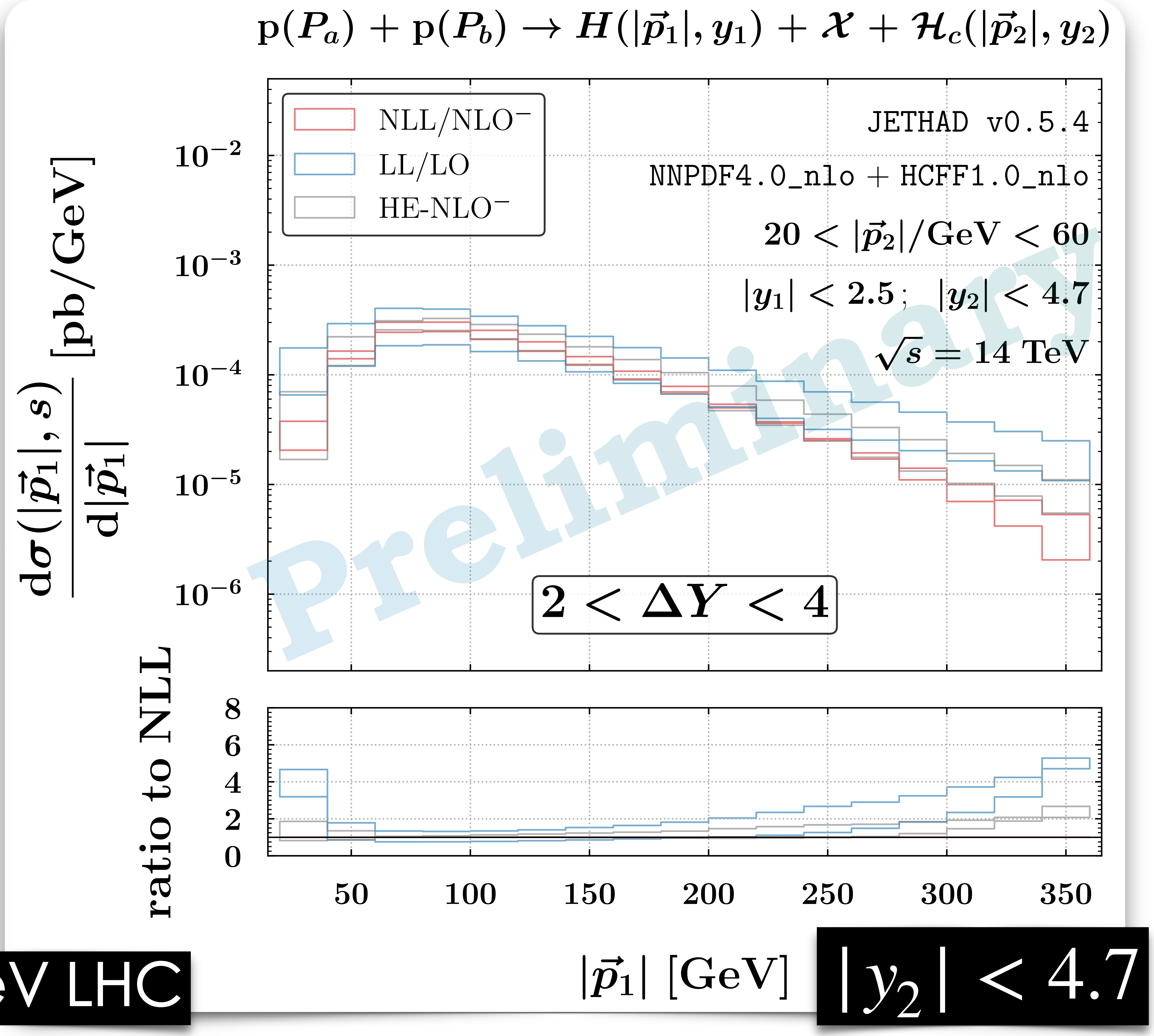
@100 TeV FCC

(Physical masses at NLO) [F. G. C., L. Delle Rose, M. Fucilla, G. Gatto, A. Papa (2024)]

The NLL/NLO^{-} spectrum of Higgs + charm

The Higgs + charm spectrum at NLL/NLO$^-$ HyF





@14 TeV LHC

(NLL/NLO$^+$ HyF) [F. G. C., L. Delle Rose, M. Fucilla, G. Gatto, A. Papa (in progress)]

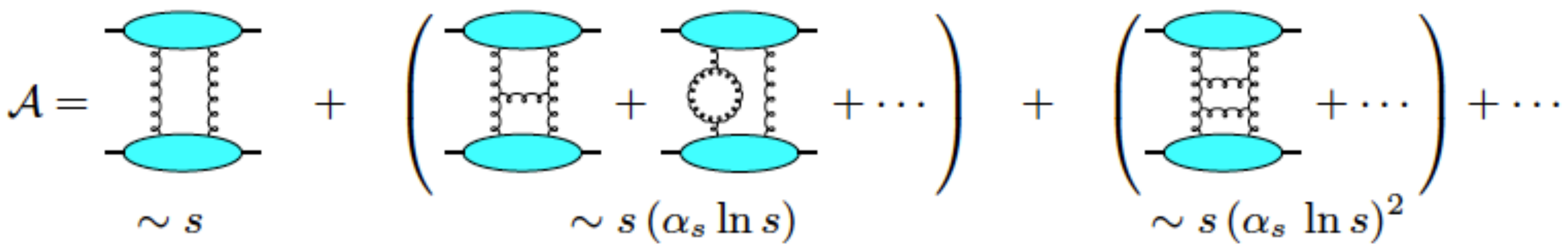
BASICS OF BFKL

The high-energy resummation

- **BFKL resummation**: [V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977); Y.Y. Balitskii, L.N. Lipatov (1978)]

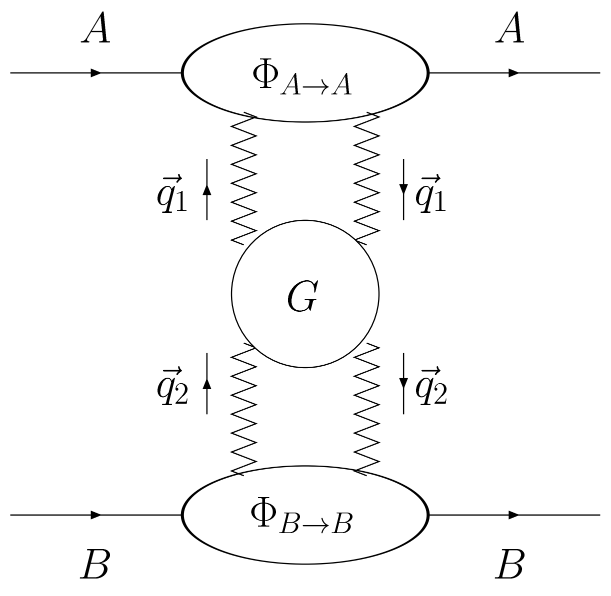
  $\xrightarrow{\text{based on}}$ **gluon Reggeization**

  leading logarithmic approximation (LL): $\alpha_s^n (\ln s)^n$



  next-to-leading logarithmic approximation (NLL): $\alpha_s^{n+1} (\ln s)^n$

Total cross section for $A + B \to X$: $\sigma_{AB}(s) = \frac{\mathcal{I}m_s\{\mathcal{A}_{AB}^{AB}\}}{s}$ $\Leftarrow$ *optical theorem*



▶ $\mathcal{I}m_s\left\{\mathcal{A}_{AB}^{AB}\right\}$ factorization:

convolution of the **Green's function** of two interacting Reggeized gluons with the **impact factors** of the colliding particles

Green's function is **process-independent**, describes energy dependence and obeys BFKL equation; impact factors are known in the **NLL just for few processes**

The high-energy resummation

$$\mathrm{Im}_s(\mathcal{A}) = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}} \Phi_A(\vec{q}_1, s_0) \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}} \Phi_B(-\vec{q}_2, s_0) \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2)$$

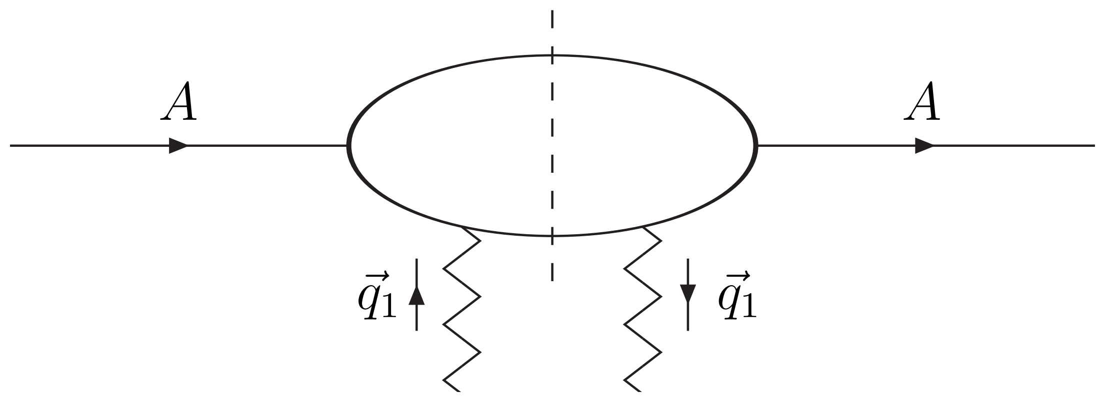
- **Green's function** is **process-independent** and takes care of the **energy dependence**

  ⟶ determined through the **BFKL equation**

  [Ya.Ya. Balitskii, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

- **Impact factors** are **process-dependent** and depend on the hard scale, but not on the energy

  ⟶ known in the NLA just for few processes



- Successful tests of NLA BFKL in the **Mueller–Navelet** channel with the advent of the LHC; nevertheless, *new BFKL-sensitive observables* as well as *more exclusive final-state reactions* are needed (**di-hadron**, **hadron-jet**, **heavy-quark pair**, **multi-jet**, production processes,...)

(**MN jets**) [B. Ducloué, L. Szymanowski, S. Wallon (2014); F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015, 2016)]
(**di-hadron**) [F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]
(**four-jet**) [F. Caporale, F.G.C., G. Chachamis, A. Sabio Vera (2016)]
(**multi-jet**) F. Caporale, F.G.C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016, 2017, 2017)]
(**heavy-quark pair**) [F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2018); A.D. Bolognino, F.G.C., D.Yu. Ivanov, M. Fucilla, A. Papa (2018)]
(**hadron-jet**) [M.M.A. Mohammed, MD thesis (2018); A.D. Bolognino, F.G.C., D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

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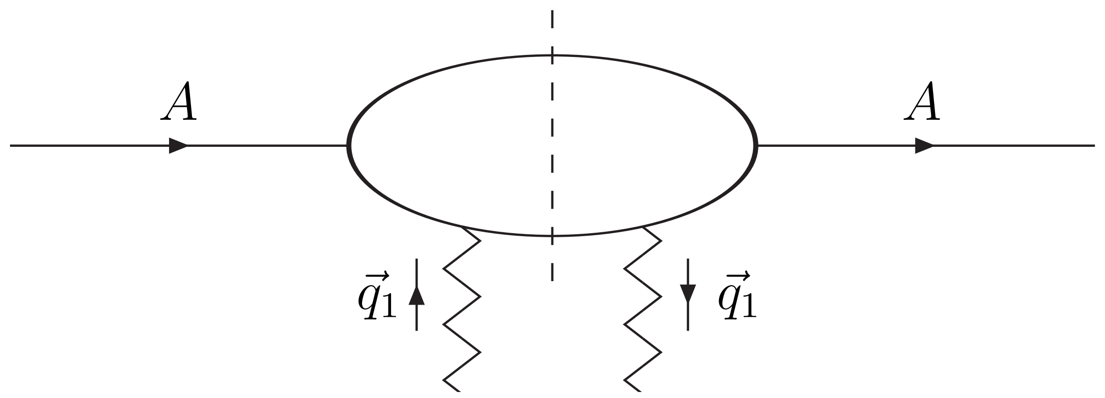

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(**heavy-quark pair**) [F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2018); A.D. Bolognino, F.G.C., D.Yu. Ivanov, M. Fucilla, A. Papa (2018)]
(**hadron-jet**) [M.M.A. Mohammed, MD thesis (2018); A.D. Bolognino, F.G.C., D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

¡ NEW ! NLO HIGGS

(κ_T space) [M. Hentschinski et al. (2021)]
(κ_T & Mellin) [F.G.C. et al. (2022)]

The high-energy resummation

Gluon Reggeization in perturbative QCD

◇ Gluon quantum numbers in the t-channel: 8^- representation

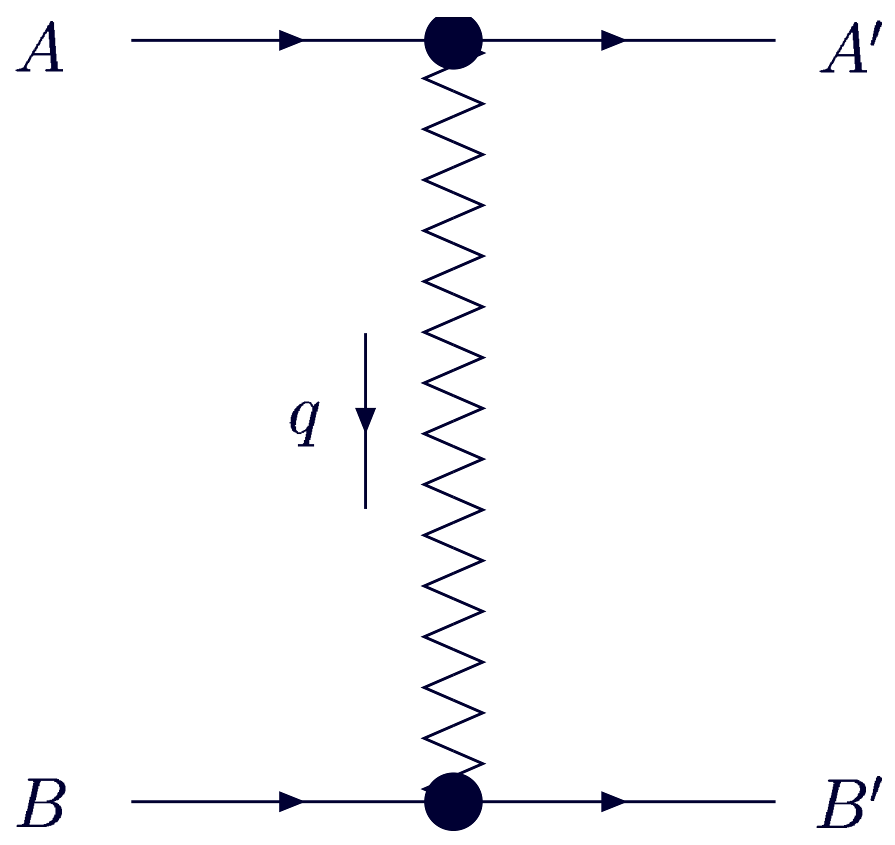
◇ Regge limit: $s \simeq -u \to \infty$, t not growing with s

$\to$ amplitudes governed by **gluon Reggeization** $\to D_{\mu\nu} = -i\frac{g_{\mu\nu}}{q^2}\left(\frac{s}{s_0}\right)^{\alpha_g(q^2)-1}$

$\xrightarrow{\text{feature}}$ all-order resummation: **LLA** $[\alpha_s^n(\ln s)^n]$ + **NLA** $[\alpha_s^{n+1}(\ln s)^n]$

$\xrightarrow{\text{consequence}}$ factorization of elastic and real part of inelastic amplitudes

$\xrightarrow{\text{example}}$ Elastic scattering process: $A + B \longrightarrow A' + B'$



$$(\mathcal{A}_8^-)_{AB}^{A'B'} = \Gamma^c_{A'A}\left[\left(\frac{-s}{-t}\right)^{j(t)} - \left(\frac{s}{-t}\right)^{j(t)}\right]\Gamma^c_{B'B}$$

$$j(t) = 1 + \omega(t)\,, \qquad j(0) = 1$$

$\omega(t) \to$ Reggeized gluon trajectory

$\Gamma^c_{A'A} = g\langle A'|T^c|A\rangle\Gamma_{A'A} \to$ PPR vertex

$T^c \to$ fundamental (q) or adjoint (g)

- QCD is the unique SM theory where all elementary particles reggeize
- Possible extensions: N=4 SYM, AdS/CFT,...

The high-energy resummation

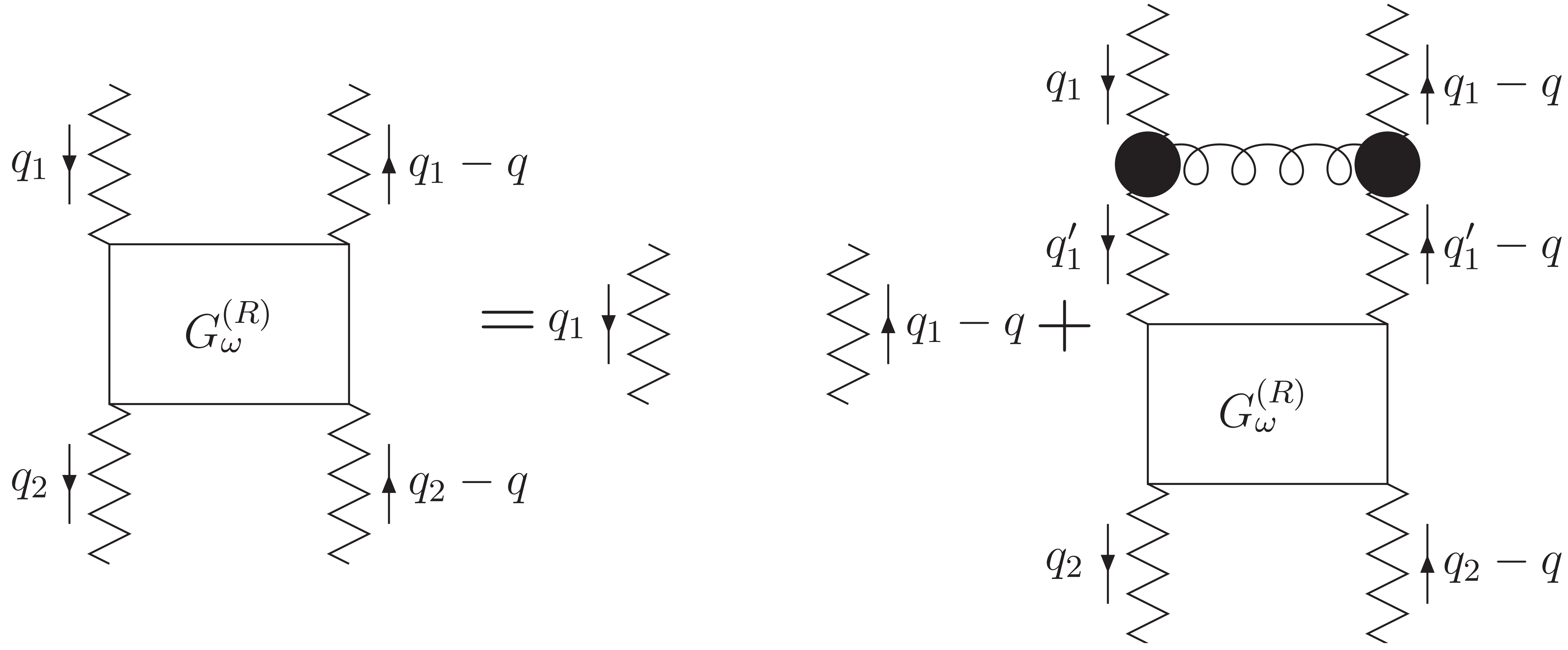
$$\mathcal{I}m_s\{\mathcal{A}\} = \frac{s}{(2\pi)^{D-2}}\int\frac{d^{D-2}q_1}{\vec{q}_1^{\,2}}\Phi_A(\vec{q}_1,s_0)\int\frac{d^{D-2}q_2}{\vec{q}_2^{\,2}}\Phi_B(-\vec{q}_2,s_0)\int_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\left(\frac{s}{s_0}\right)^{\omega}G_\omega(\vec{q}_1,\vec{q}_2)$$

- **Green's function** is **process-independent** and takes care of the **energy dependence**

$\longrightarrow$ determined through the **BFKL equation**

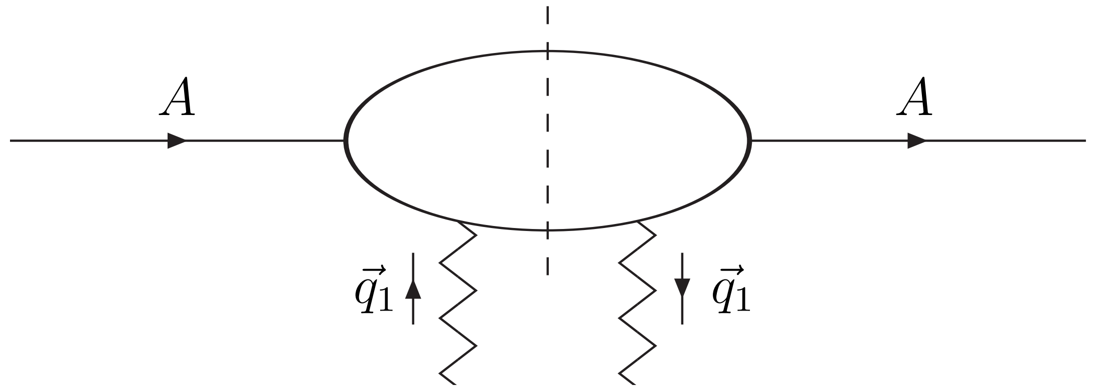
[Ya.Ya. Balitskii, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

$$\omega\, G_\omega(\vec{q}_1,\vec{q}_2) = \delta^{D-2}(\vec{q}_1-\vec{q}_2) + \int d^{D-2}q\, K(\vec{q}_1,\vec{q})\, G_\omega(\vec{q},\vec{q}_1)\,.$$



The high-energy resummation

- **Impact factors** are **process-dependent** and depend on the hard scale, but not on the energy $\longrightarrow$ known in the NLA just for few processes



◇ **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
[M. Ciafaloni, G. Rodrigo (2000)]

◇ $\gamma^* \longrightarrow V$, with $V = \rho^0,\ \omega,\ \phi$, forward case

[D.Yu. Ivanov, M.I. Kotsky, A. Papa (2004)]

◇ forward jet production

[J. Bartels, D. Colferai, G.P. Vacca (2003)]
(exact IF) [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, A. Perri (2012)]
(small-cone IF) [D.Yu. Ivanov, A. Papa (2012)]
(several jet algorithms discussed) [D. Colferai, A. Niccoli (2015)]

◇ forward identified hadron production

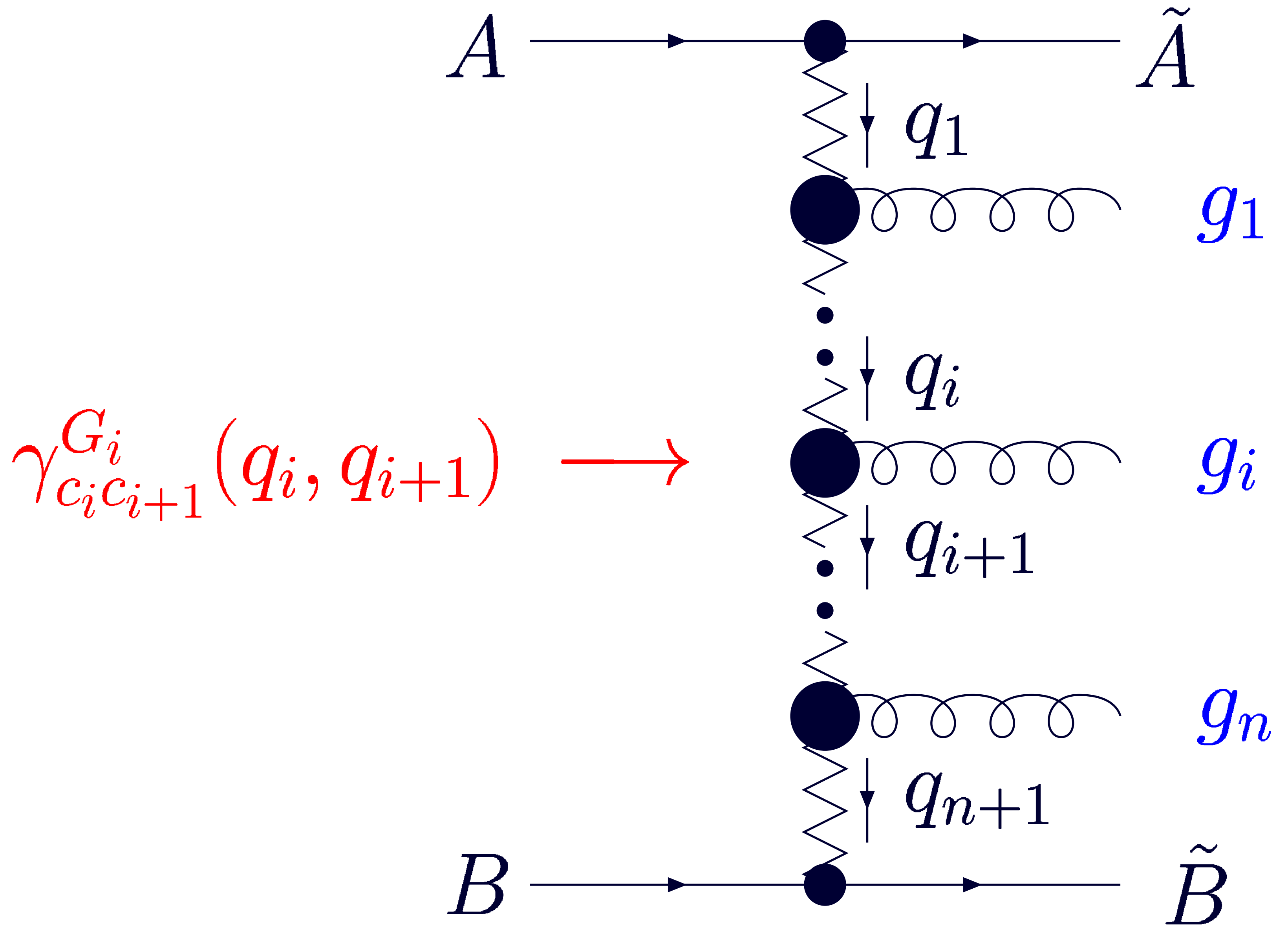
[D.Yu. Ivanov, A. Papa (2012)]

◇ $\gamma^* \longrightarrow \gamma^*$

[J. Bartels *et al.* (2001), I. Balitsky, G.A. Chirilli (2011, 2013)]

BFKL in the LLA (I)

Inelastic scattering process $A + B \;\rightarrow\; \tilde{A} + \tilde{B} + n$ in the LLA



multi-Regge kinematics

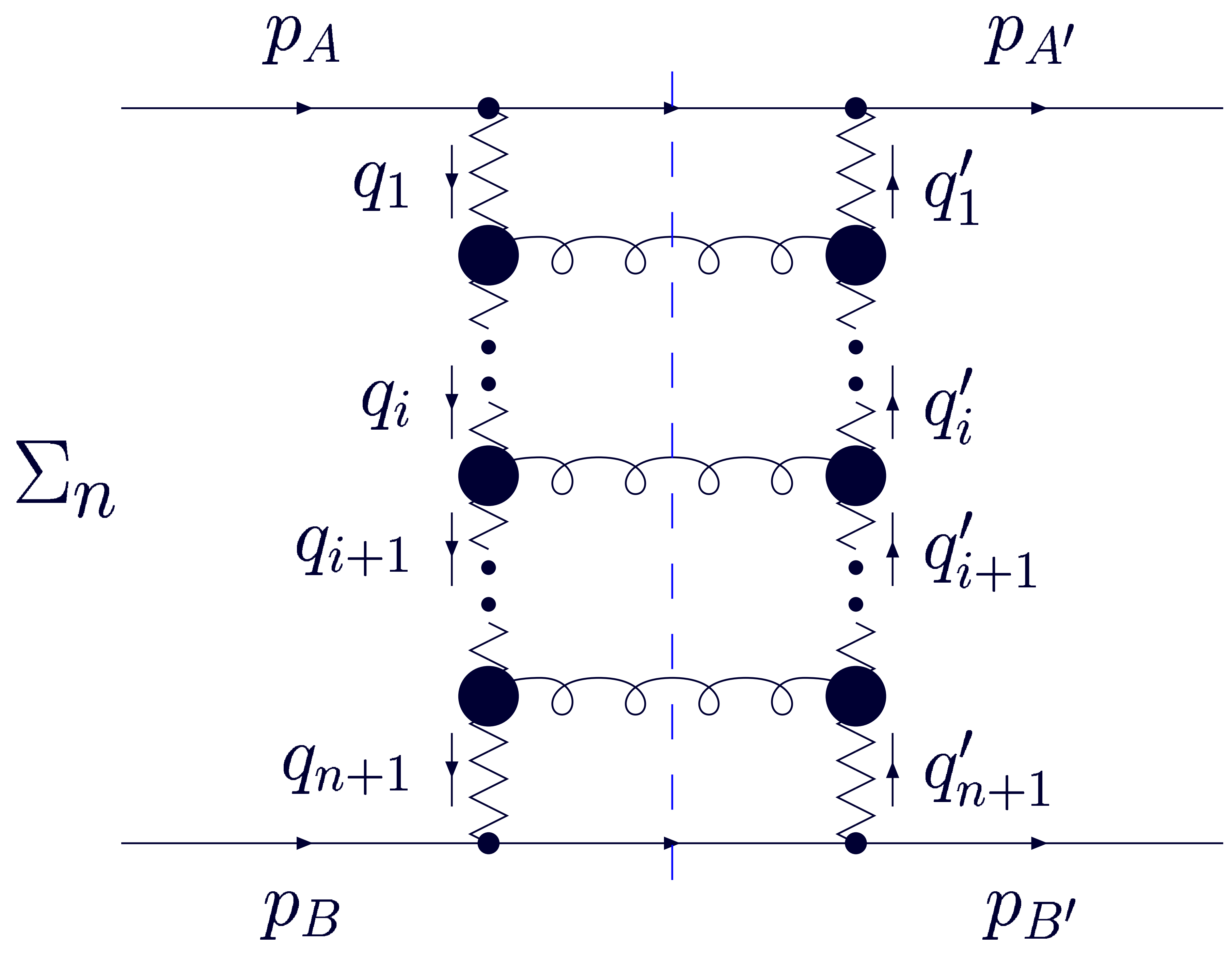
$$\mathrm{Re}\mathcal{A}^{\tilde{A}\tilde{B}+n}_{AB} = 2s\,\Gamma^{c_1}_{\tilde{A}A}\left(\prod_{i=1}^{n}\gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1})\left(\frac{s_i}{s_R}\right)^{\omega(t_i)}\frac{1}{t_i}\right)\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_R}\right)^{\omega(t_{n+1})}\Gamma^{c_{n+1}}_{\tilde{B}B}$$

$\gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1}) \;\rightarrow\;$ RRG vertex

$s_R \;\rightarrow\;$ energy scale, irrelevant in the LLA

BFKL in the LLA (II)

Elastic amplitude $A + B \longrightarrow A' + B'$ in the LLA via s-channel unitarity



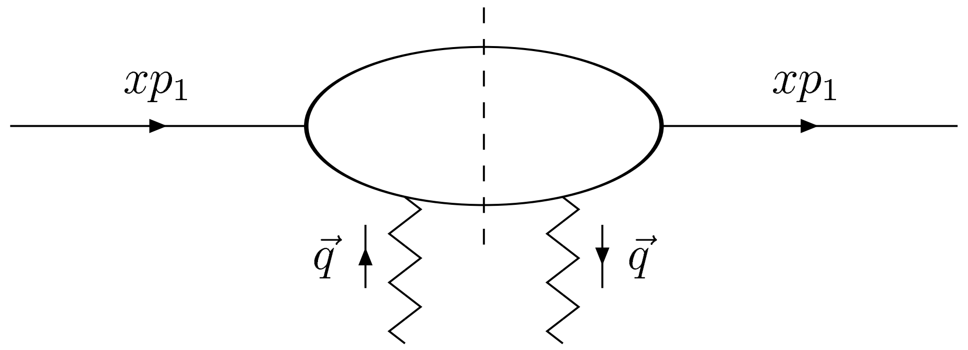
$$\mathcal{A}_{AB}^{A'B'} = \sum_{\mathcal{R}} (\mathcal{A}_{\mathcal{R}})_{AB}^{A'B'}, \qquad \mathcal{R} = 1 \text{ (singlet)}, 8^- \text{ (octet)}, \ldots$$

The 8^- color representation is important for the bootstrap, i.e. the consistency between the above amplitude and that with one Reggeized gluon exchange

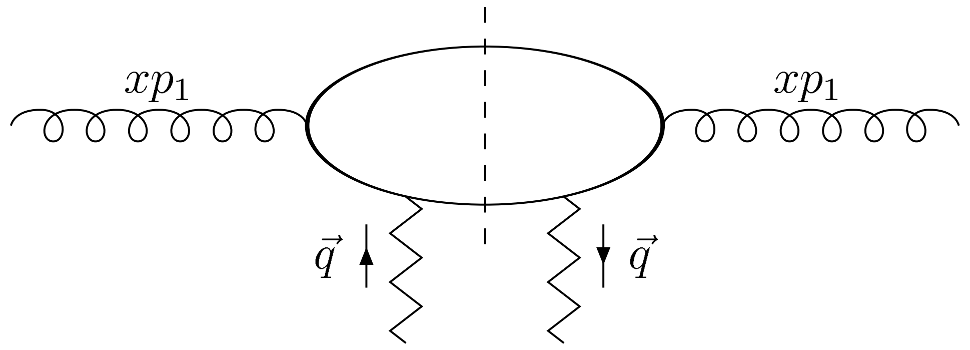
Hybrid factorization at work

Forward-jet impact factor

- take the impact factors for **colliding partons**
  [V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
  [M. Ciafaloni and G. Rodrigo (2000)]

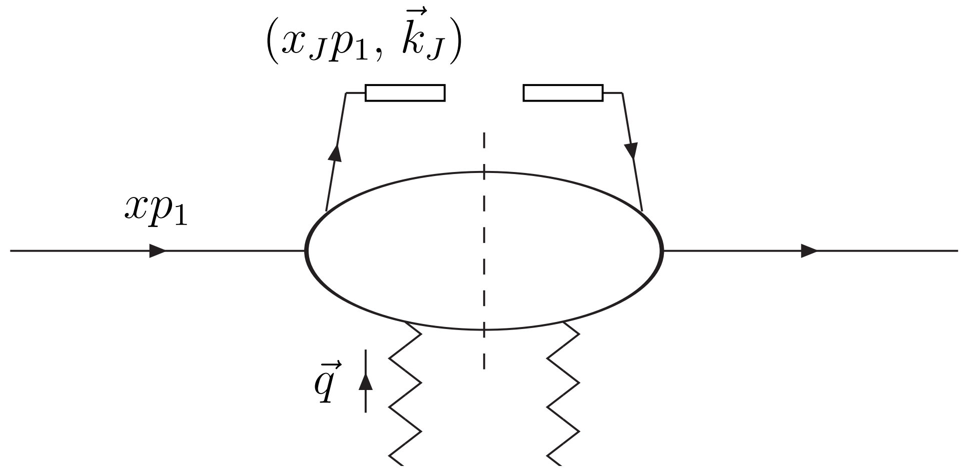


quark vertex

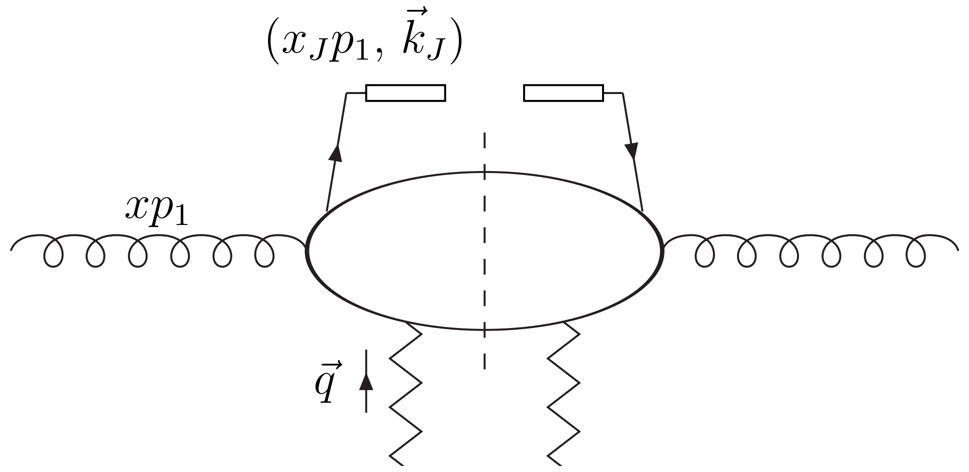


gluon vertex

- "open" one of the integrations over the phase space of the intermediate state to allow one parton to generate the jet



quark jet vertex



gluon jet vertex

- use QCD collinear factoriz.: $\sum_{s=q,\bar{q}} f_s \otimes$ [quark vertex] $+ f_g \otimes$ [gluon vertex]