The Alaric parton shower

based on
Herren, Höche, Krauss, DR, Schönherr JHEP 10 (2023) 091 [arXiv:2208.06057]
Höche, Krauss, DR [arXiv:2404.14360]

Daniel Reichelt, 20 January 2024

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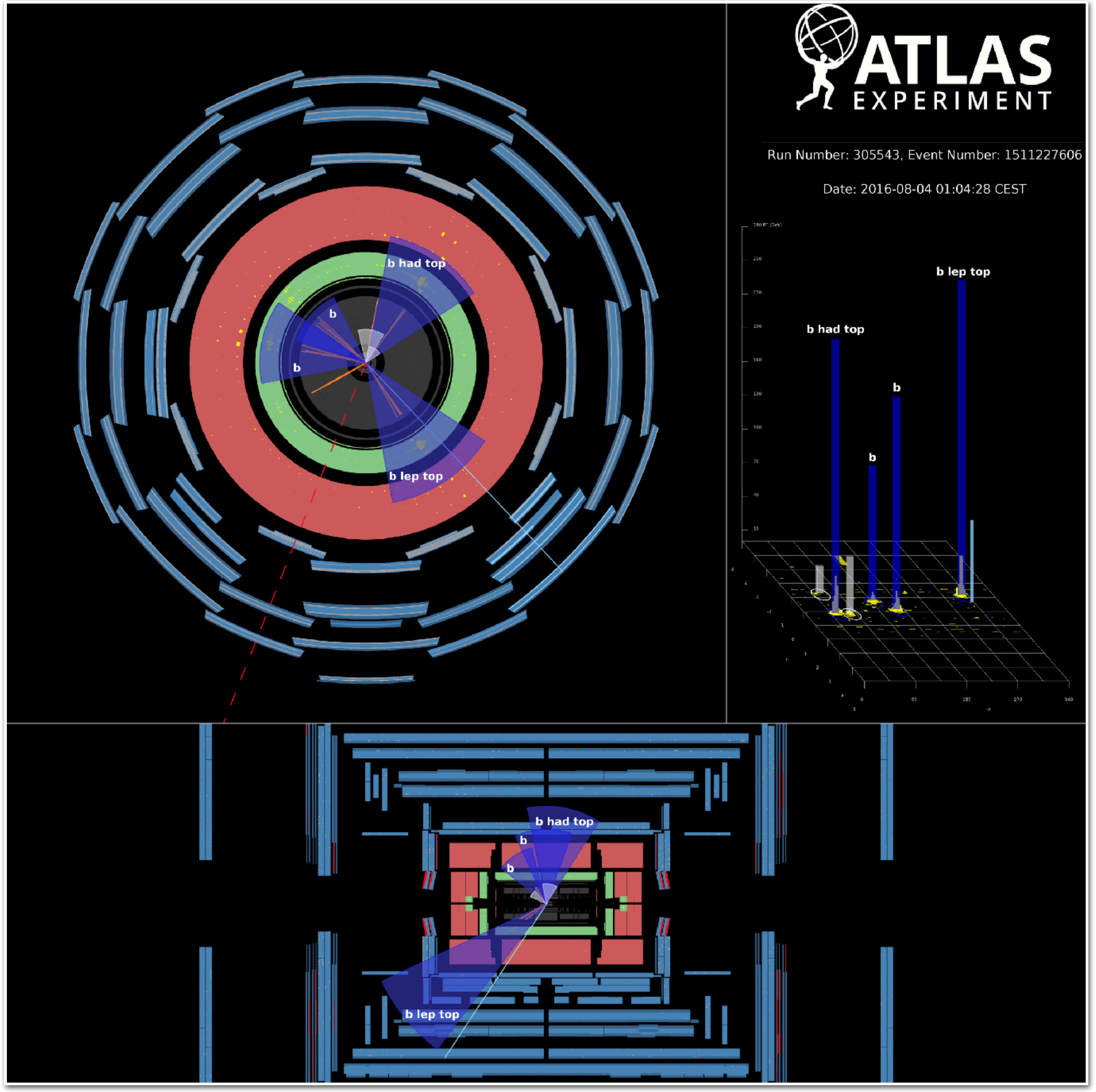
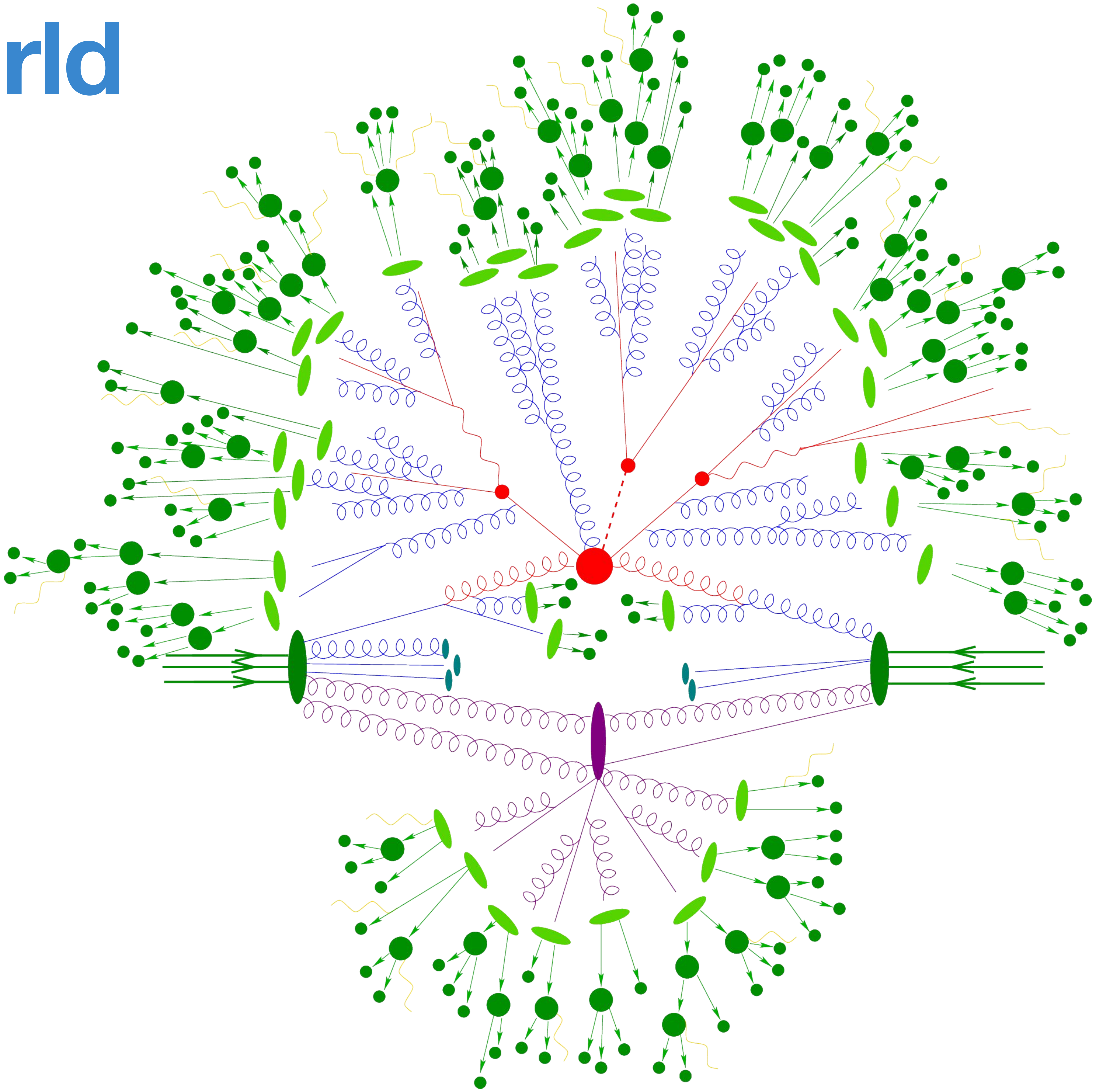
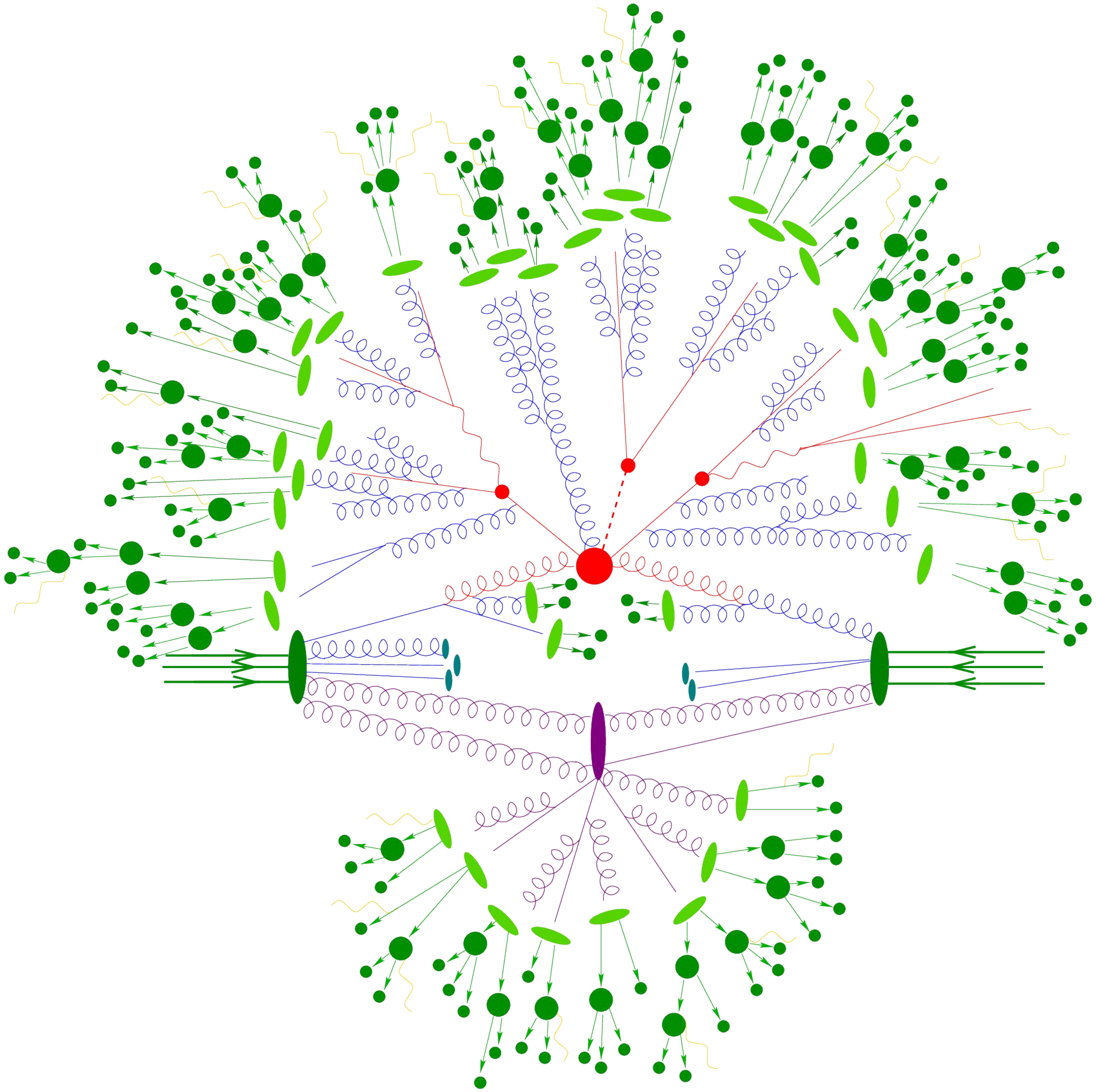
Colliders in the real world





Colliders for theorists

- Event simulation factorised into
  - Hard Process
  - Parton Shower
  - PDF/Underlying event
  - Hadronisation
  - QED radiation
  - Hadron Decays



QCD in the soft limit

- factorisation in the soft limit ('Eikonal')

$$d\sigma_{n+1} = d\sigma_n \otimes d\Phi_{+1} \frac{\alpha_s}{2\pi} \sum_{k,i} \mathbb{T}_k \mathbb{T}_i \frac{p_k p_i}{(p_k q)(p_i q)}$$

$$\frac{p_k p_i}{(p_k q)(p_i q)} \sim 1/k_t^2$$

single emission phase space

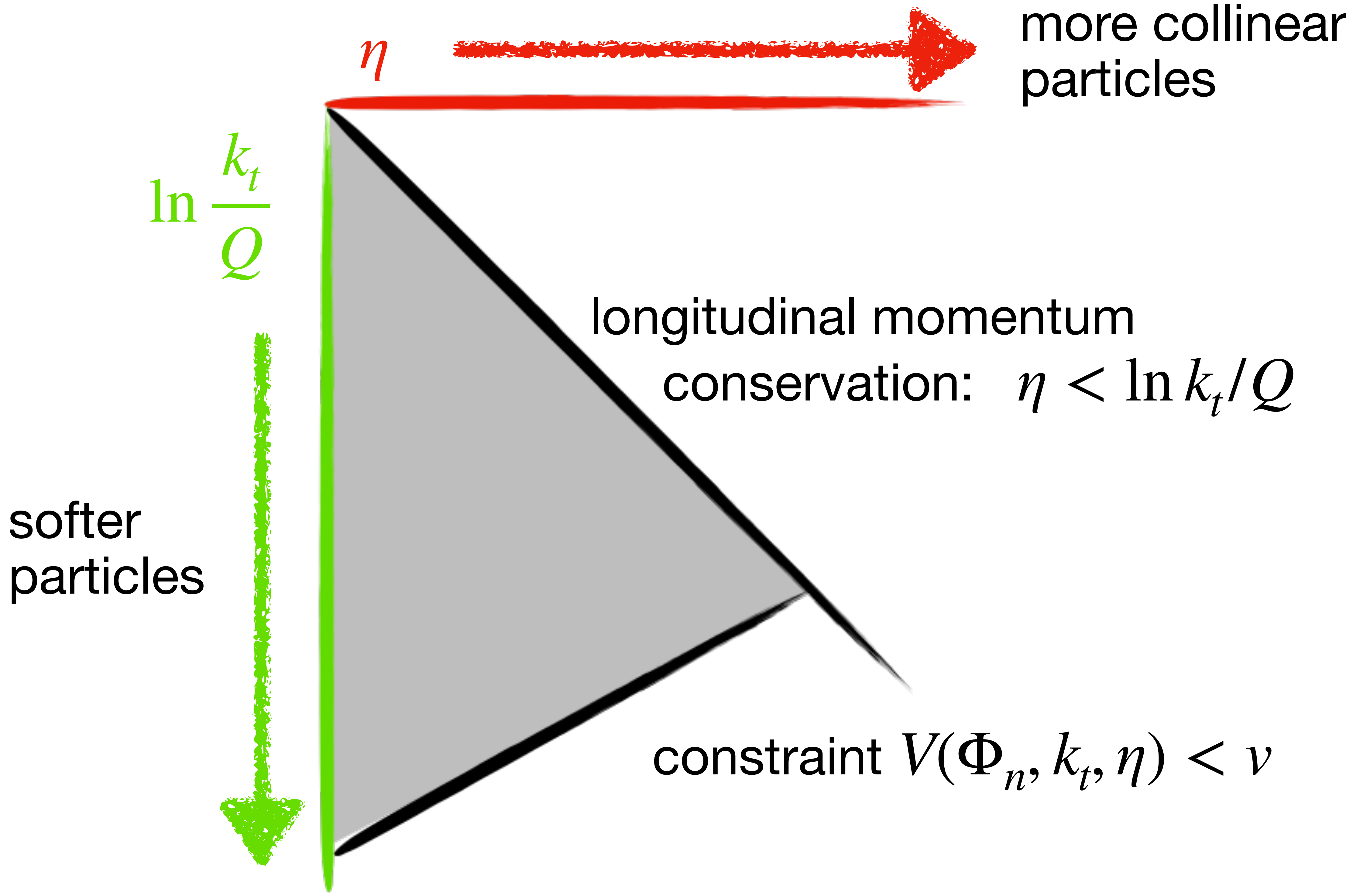
transverse momentum

rapidity

azimuthal angle

$$d\Phi_{+1} \sim dk_t^2 \, d\eta \, d\phi$$

- integrate over triangle in "Lund plane"



e.g. take $V(k_t, \eta) = k_t/Q$

$$\rightarrow \frac{\alpha_s}{2\pi} \int_{vQ}^{Q} \frac{dk_t}{k_t} \int_0^{\ln k_t/Q} d\eta \sim \frac{\alpha_s}{2\pi} \ln^2 1/v$$

Relation to structure of (NLL) resummation

- Additional components: collinear terms form DGLAP splitting kernels, running coupling and CMW scheme for α_s evolution → relevant for single emission only

- Simple event shape: $$\Sigma(v) = \int d\Phi_n e^{-R(v)} \mathcal{F}(v) \sim \exp(L g_1(\alpha_s L) + g_1(\alpha_s L))$$

$$R(v) = \int d\Phi_{+1} \frac{d\sigma_{+1}}{d\Phi_{+1}} \Theta(V(\Phi_{+1}) - v)$$

$$L \equiv \ln 1/v$$

$$g_i \equiv \sum_k \alpha_s^k L^k$$

- Multiple emissions expressed as

$$\mathcal{F}(v) = \lim_{\epsilon \to 0} \epsilon^{R'} \sum_m \int d\Phi_{+1} \frac{d\sigma_{+1}}{d\Phi_{+1}} \Theta\left(1 - \lim_{\rho \to 0} \frac{V(k_i^\rho)}{\rho v}\right)$$

For example thrust:

$$F(\tau) = \frac{\exp(-\gamma_E R')}{\Gamma(1 + R')}$$

Parton showers - Cliff notes version

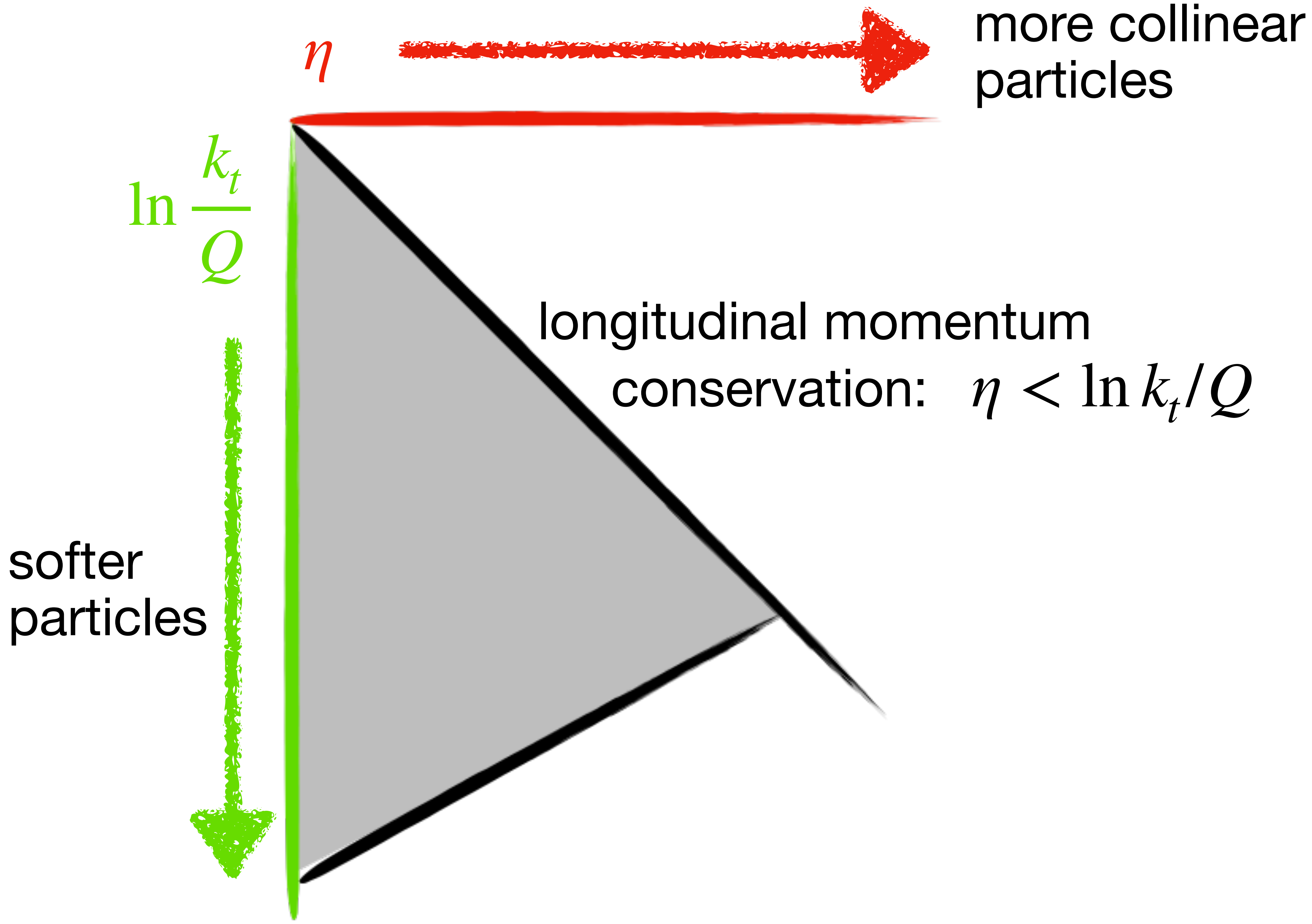
- no-emission probability (sudakov factor) $\sim \exp\left[-\int_{t_0}^{t_1} \frac{dk_t}{k_t} dz \frac{\alpha_S}{2\pi} P(z)\right]$

- Main ingredients to a shower:

1. splitting kernels $P(z)$ captures soft and collinear limits of matrix elements

2. fill phase space ordered in evolution variable (k_t, θ, q^2, …), definition of t, z away from exact limit (here all showers k_t ordered)

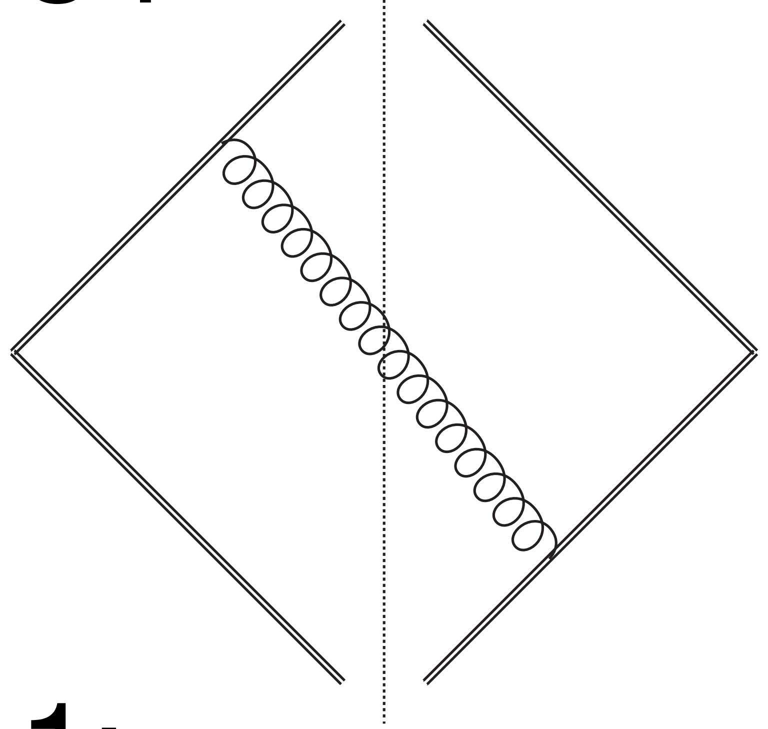
3. generate new final state after emission according to recoil scheme



Alaric Splitting Functions - Eikonal

Starting point: eikonal

$$\frac{p_i p_k}{(p_i p_j)(p_j p_k)} = \frac{1}{E_j^2} \frac{1 - \cos\theta_{ik}}{(1 - \cos\theta_{ij})(1 - \cos\theta_{jk})} \equiv \frac{W_{ik,j}}{E_j^2}$$



naive implementation leads to soft double counting need to split into ij and kj collinear terms [Marchesini, Webber '88]

Option 1:

$$W_{ik,j} = \tilde{W}^i_{ik,j} + \tilde{W}^k_{ki,j} \,, \qquad \text{where} \qquad \tilde{W}^i_{ik,j} = \frac{1}{2}\left(\frac{1 - \cos\theta_{ik}}{(1 - \cos\theta_{ij})(1 - \cos\theta_{jk})} + \frac{1}{1 - \cos\theta_{ij}} - \frac{1}{1 - \cos\theta_{jk}}\right)$$

- e.g. Angular ordered shower, downside: problems with NGLs

Option 2: follow [Catani, Seymour '97]

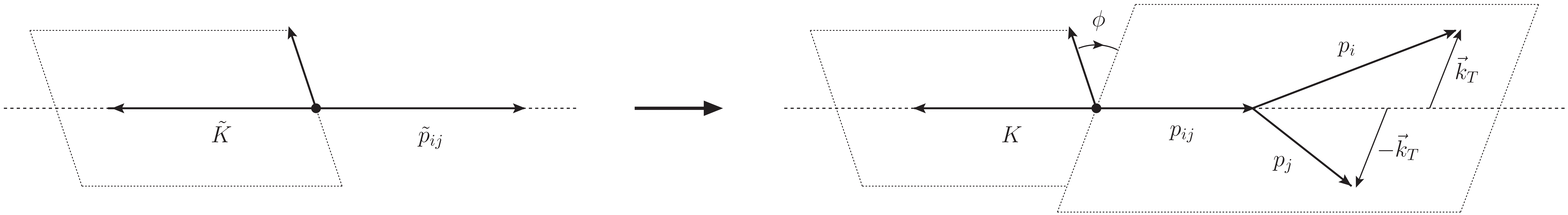
$$W_{ik,j} = \bar{W}^i_{ik,j} + \bar{W}^k_{ki,j} \,, \qquad \text{where} \qquad \bar{W}^i_{ik,j} = \frac{1 - \cos\theta_{ik}}{(1 - \cos\theta_{ij})(2 - \cos\theta_{ij} - \cos\theta_{jk})}$$

- full phase space coverage, splitting functions remain positive definite

Note related ideas in [Forshaw, Holguin, Plätzer '20]

Alaric Kinematics - splitting vs. radiation kinematics

- Before splitting:
- After splitting:



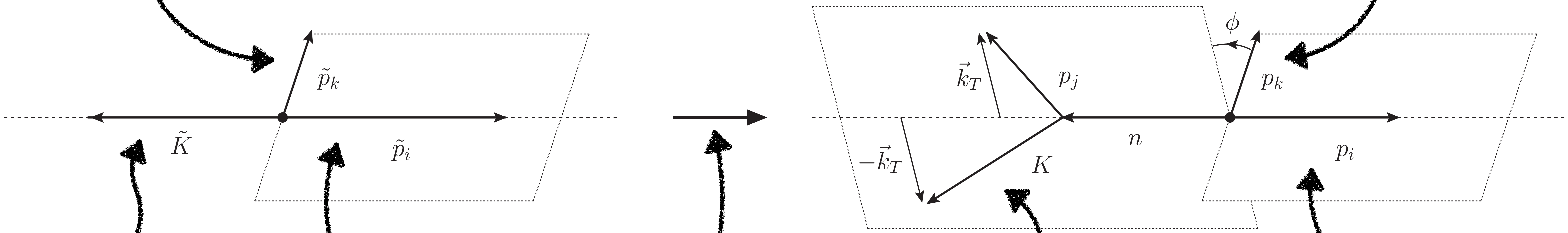
- traditional dipole scheme: share transverse momentum recoil between splitter and spectator
- advantage: treat both particles symmetric, seems like a natural choice for e.g. $g \to q\bar{q}$ splitting (at least naively)
- disadvantage: significant impact on emitter kinematics possible, only applicable to purely collinear splitting functions (see later)

Alaric Kinematics - global recoil scheme

- Before splitting:
- After splitting:

colour spectator

$p_k = \tilde{p}_k$



$\sum$ other momenta k_i

splitter

$p_i = z\tilde{p}_i$

$K^2 = \tilde{K}^2$

$\tilde{K} + \tilde{p}_i = K + p_i + p_j$

$k_i^\mu \to \Lambda^\mu{}_\nu k_i^\nu$

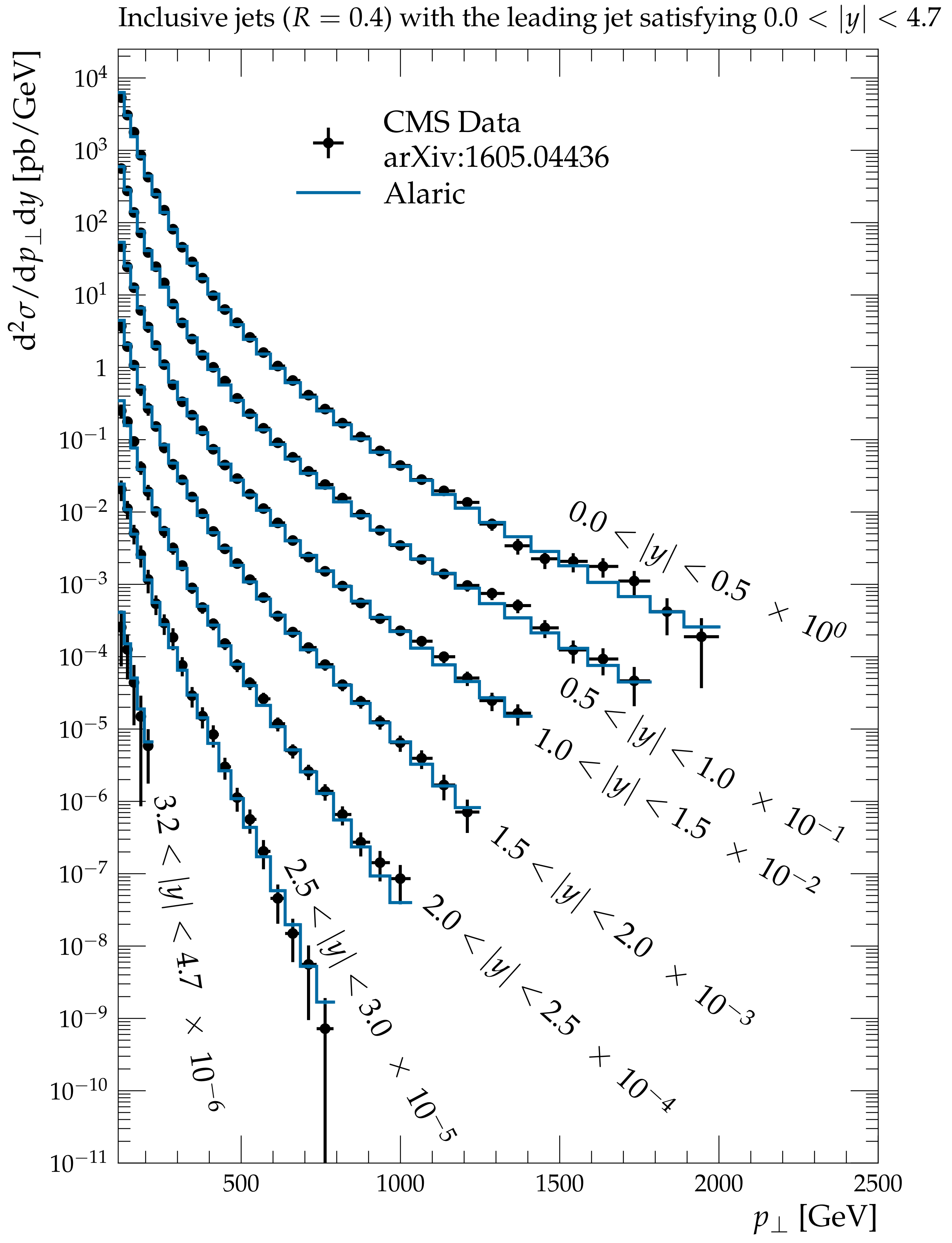
[Catani, Seymour ’97]

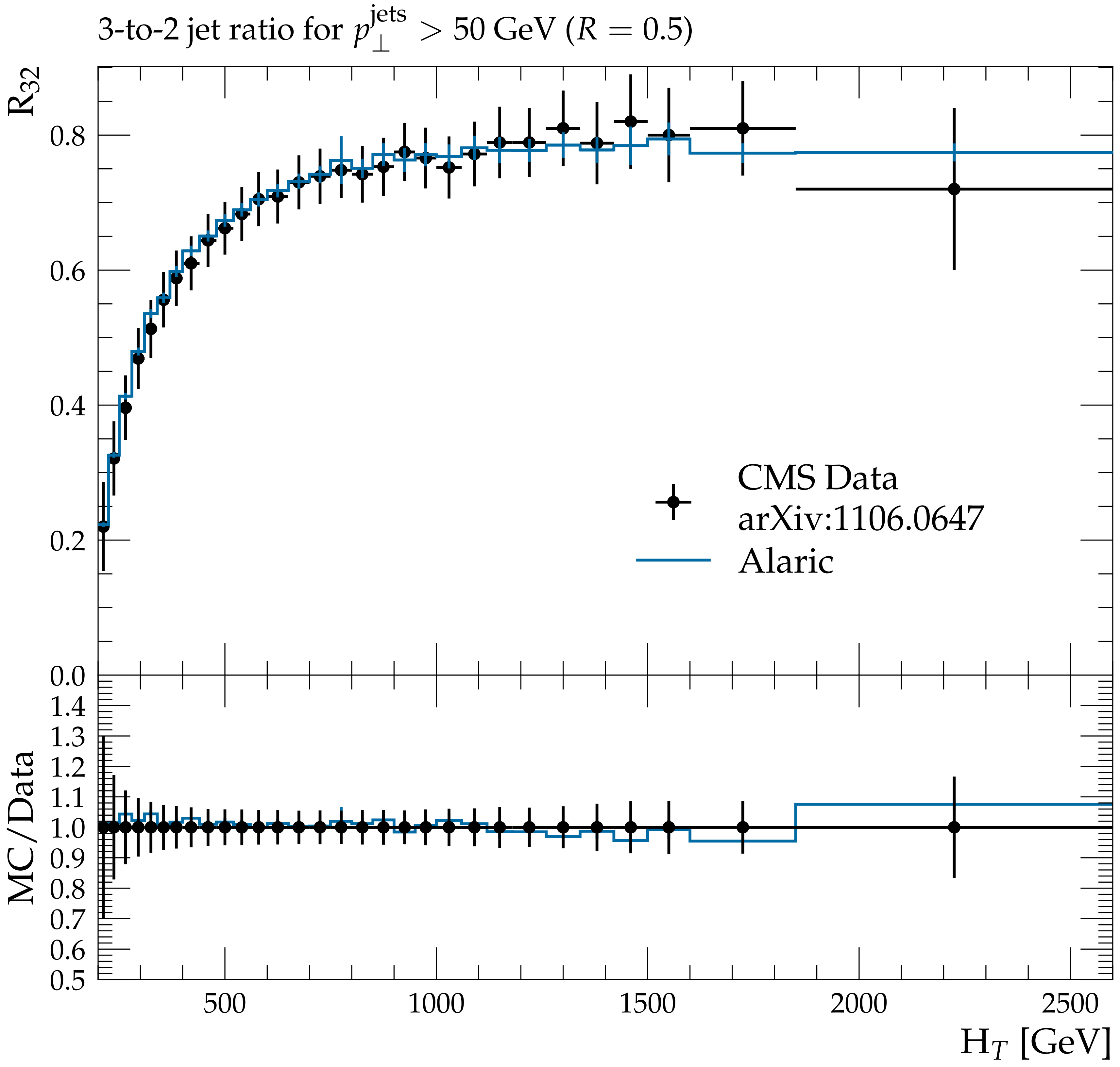
$$\Lambda^\mu{}_\nu = g^\mu{}_\nu - \frac{(K+\tilde{K})^\mu (K+\tilde{K})_\nu}{K\cdot\tilde{K} + \tilde{K}^2} + 2\frac{K^\mu \tilde{K}_\nu}{\tilde{K}^2} \to \Lambda^\mu{}_\nu \tilde{K}^\nu = K^\mu$$

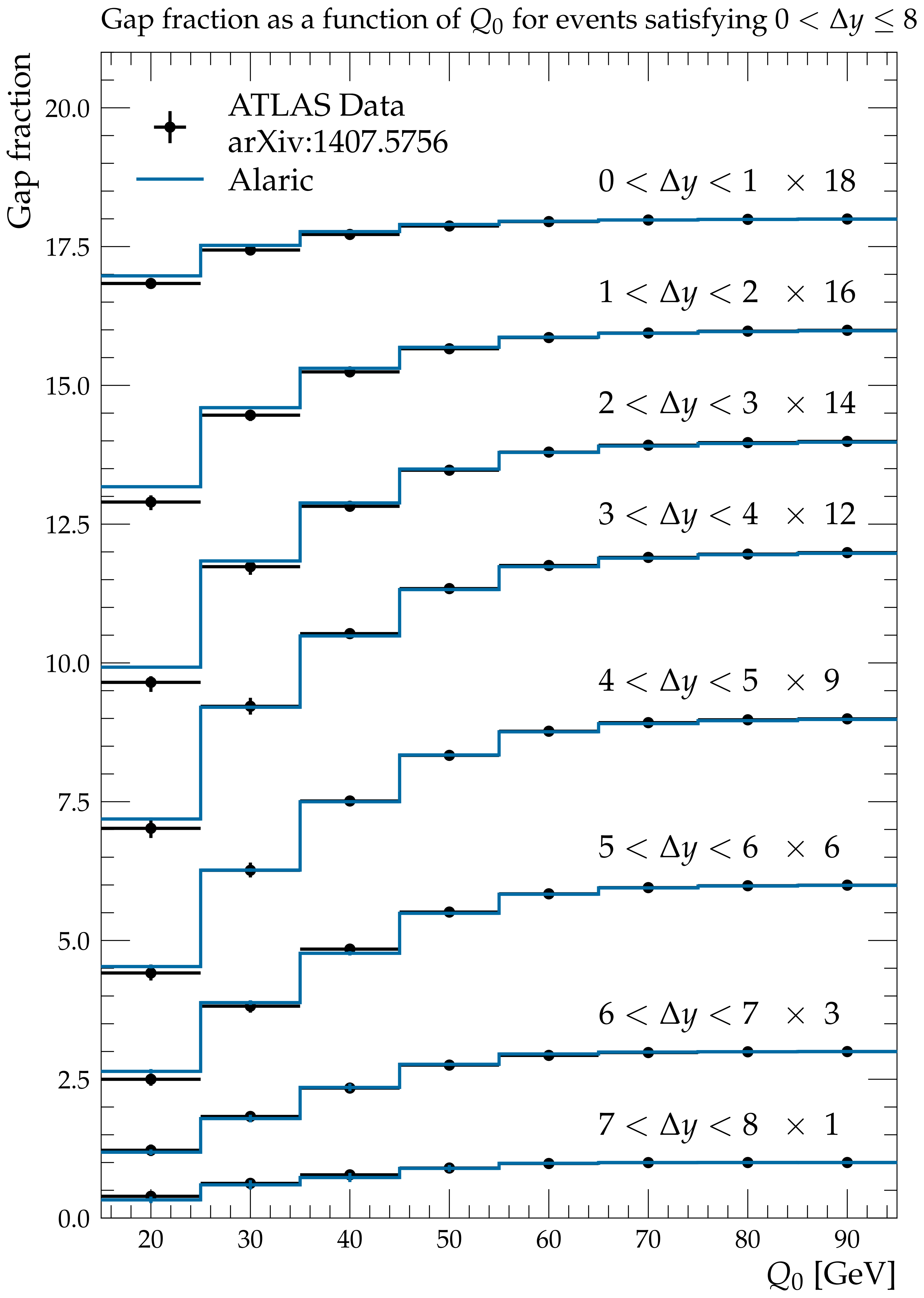
Alaric at the LHC — jets

[Höche, Krauss, DR ’24]

- satisfactory description of inclusive and dijet events
- transverse momentum spectrum of leading jet and ratio 3-to-2 jet rate

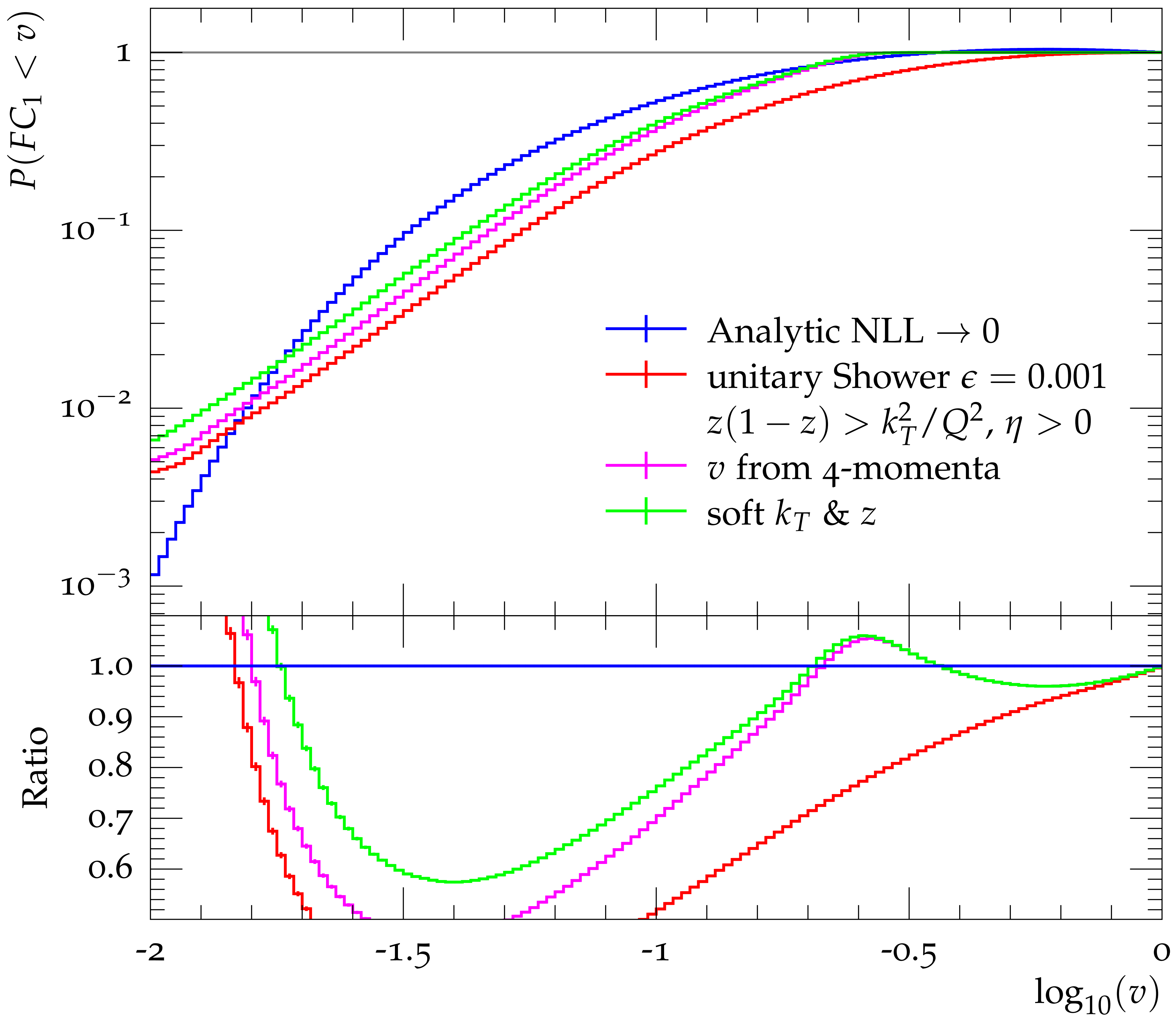






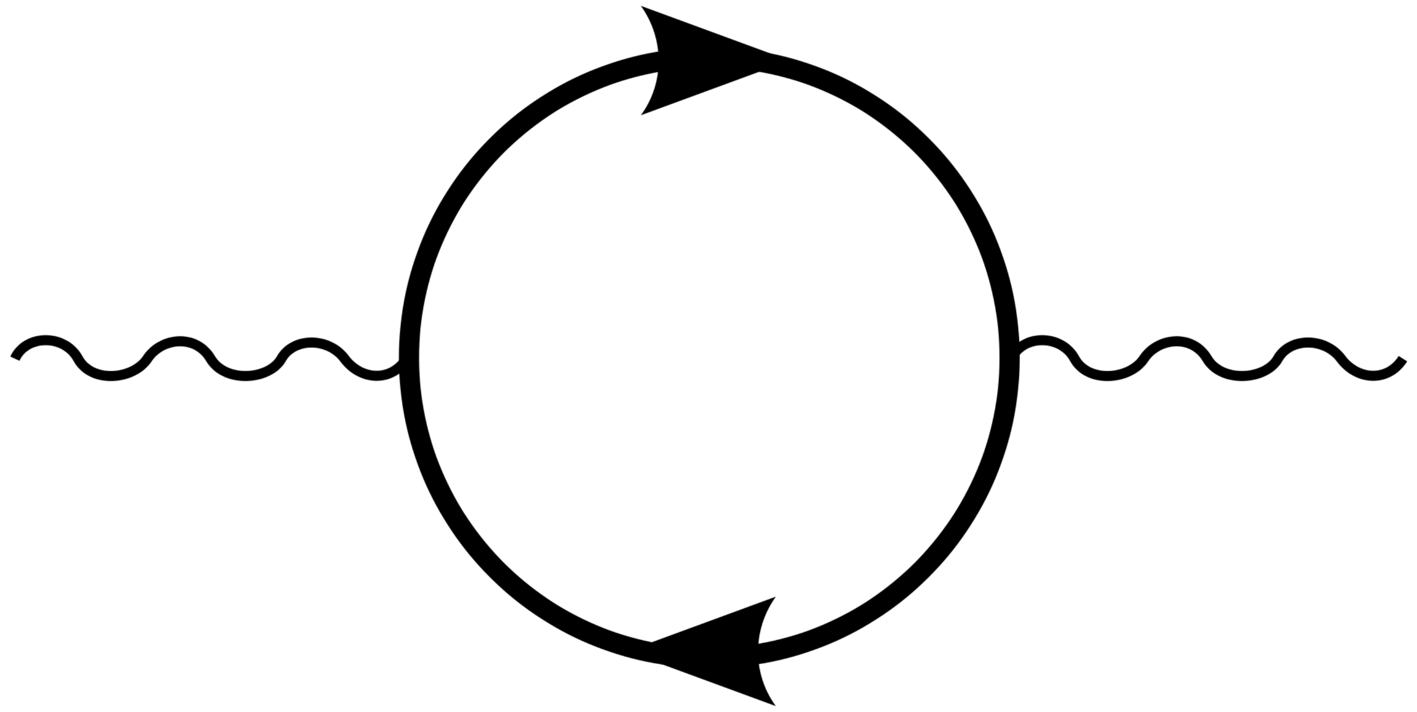
Beyond (leading) logarithmic accuracy

- So far, LL picture with significant use of NLL ingredients
  - collinear splitting functions, CMW scheme,…
  - ⇒ formalisation towards NLL/NNLL accuracy
- and beyond
  - recoil scheme and implied integration boundaries
  - ⇒ subleading effect play a significant role in phenomenological successful parton showers, more systematic understanding desirable, see also [Höche, Siegert, DR ’17]



Collinear Splitting Functions

- Calculate for example $g \to q\bar{q}$



assume Sudakov decomposition like

$$p_i^\mu = z_i \hat{p}_{ij}^\mu + \frac{-k_t^2}{z_i\, 2p_{ij}\bar{n}}\, \bar{n}^\mu + k_t^\mu \;,$$

$$p_j^\mu = z_j \hat{p}_{ij}^\mu + \frac{-k_t^2}{z_j\, 2p_{ij}\bar{n}}\, \bar{n}^\mu - k_t^\mu$$

polarisation tensor:

$$d^{\mu\nu}(p,n) = -g^{\mu\nu} + \frac{p^\mu n^\nu + p^\nu n^\mu}{pn}$$

$$P_{g\to q}^{\mu\nu}(p_i, p_j) = \frac{T_R}{2p_{ij}^2}\, d^\mu{}_\rho(p_{ij},\bar{n}) \mathrm{Tr}[\not{p}_i \gamma^\rho \not{p}_j \gamma^\sigma\,] d^\nu{}_\sigma(p_{ij},\bar{n})$$

- evaluate in collinear limit: $\to T_R \left[-\, g^{\mu\nu} + 4 z_i z_j \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \right]$

Collinear Splitting Functions

assume Sudakov decomposition like

$$p_i^\mu = z_i \hat{p}_{ij}^\mu + \frac{-k_t^2}{z_i\, 2p_{ij}\bar{n}}\, \bar{n}^\mu + k_t^\mu \ ,$$

$$p_j^\mu = z_j \hat{p}_{ij}^\mu + \frac{-k_t^2}{z_j\, 2p_{ij}\bar{n}}\, \bar{n}^\mu - k_t^\mu$$

derivation of splitting functions leads to:

$$P^{(\mathrm{F})}_{qq\,\|}(p_i, p_j, \bar{n}) = C_F\,(1-\varepsilon)(1-z_i)$$

$$P^{(\mathrm{F})}_{gg\,\|}(p_i, p_j, \bar{n}) = 2C_A\, z_i z_j \ ,$$

$$P^{(\mathrm{F})}_{gq\,\|}(p_i, p_j, \bar{n}) = T_R \left[1 - \frac{2\, z_i z_j}{1-\varepsilon} \right] .$$

actual shower kinematics:

$$p_i = z\, \tilde{p}_i \ ,$$

$$p_j = (1-z)\, \tilde{p}_i + v\big(\tilde{K} - (1-z+2\kappa)\, \tilde{p}_i\big) - k_\perp \ ,$$

$$K = \tilde{K} - v\big(\tilde{K} - (1-z+2\kappa)\, \tilde{p}_i\big) + k_\perp \ ,$$

$$p_i = \frac{z}{1-v(1-z+\kappa)}\, \hat{p}_{ij} + \frac{z}{1-v(1-z+\kappa)}\, k_\perp + \mathcal{O}\left(\frac{k_\perp^2}{2\tilde{p}_i \tilde{K}}\right) \ ,$$

$$p_j = \frac{(1-z)(1-v) - v\kappa}{1-v(1-z+\kappa)}\, \hat{p}_{ij} - \frac{z}{1-v(1-z+\kappa)}\, k_\perp + \mathcal{O}\left(\frac{k_\perp^2}{2\tilde{p}_i \tilde{K}}\right)$$

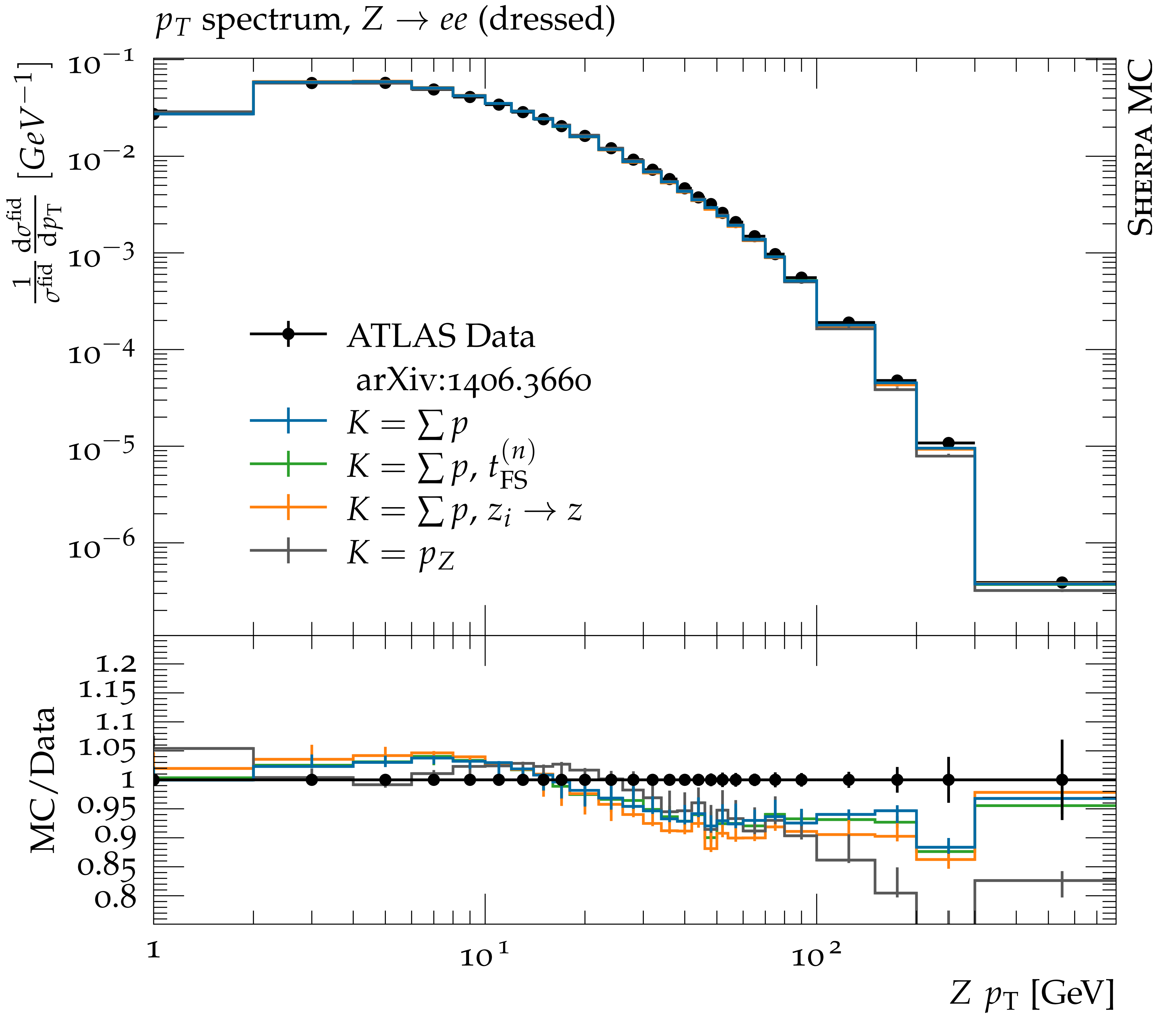
ultimately, “proper” splitting variables:

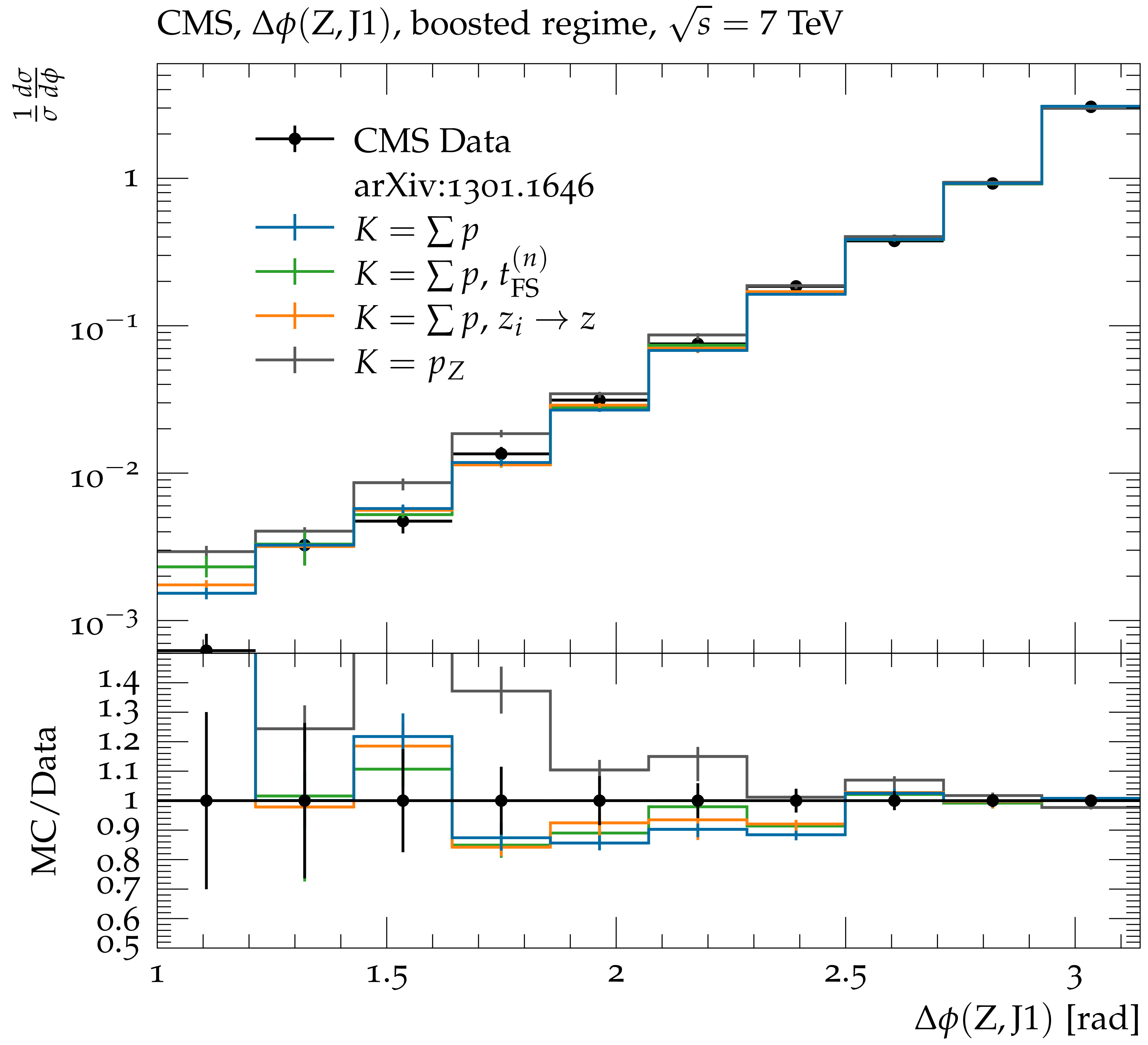
$$z_i = \frac{z}{1-v(1-z+\kappa)} \ ,$$

$$z_j = 1 - \frac{z}{1-v(1-z+\kappa)}$$

Alaric — subleading effects in Z+jets

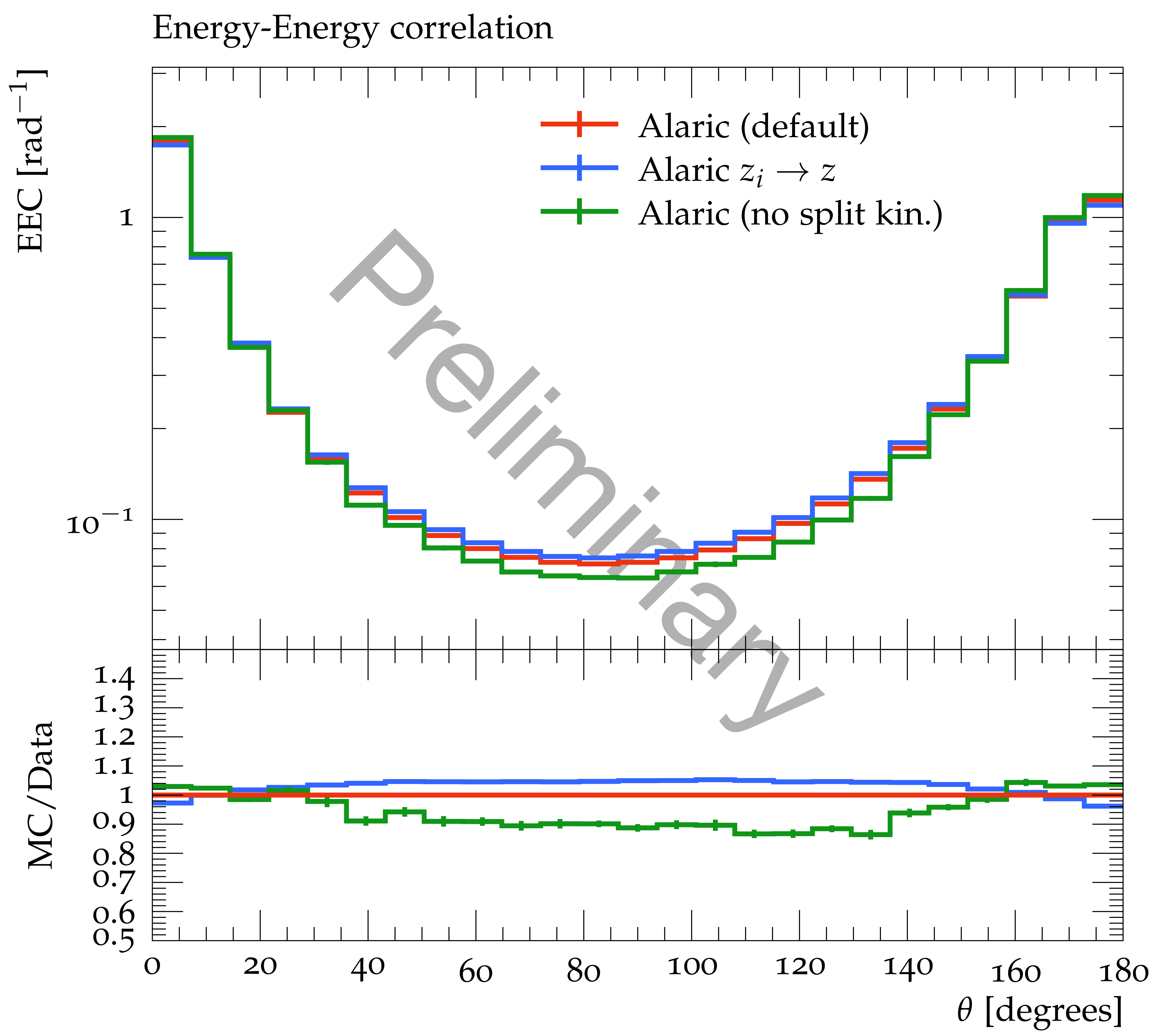
- effects/choices beyond NLL accuracy:
  - choice of evolution variable (up to factors of $z \sim 1$)
  - identify PS parameter z with z_i, z_j
  - choice of recoil momentum K (NLL accuracy needs "hard" K)





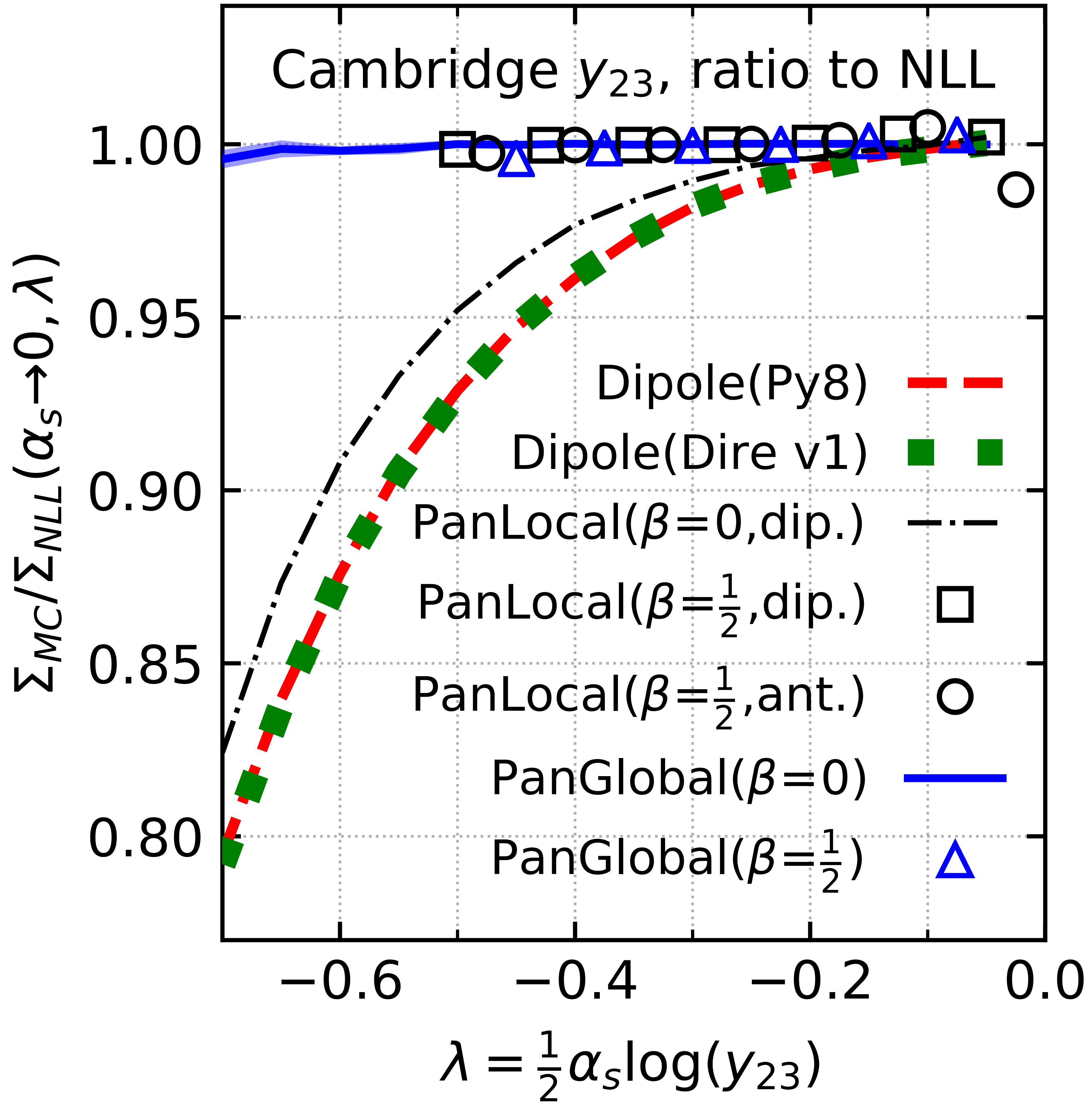
Towards subleading effects for lepton colliders

- same variations available for lepton colliders (as far as they are applicable)
- example here: EEC from $q\bar{q}$ final state at the Z pole
- systematic variations not captured by e.g. scale variations
  - additional uncertainty
  - kinematics enter splitting functions, hope for systematic reduction at higher orders



New Parton Showers - NLL accuracy

- typical claim based on accuracy of splitting functions etc.
  - parton showers ~ NLL accurate if CMW scheme for strong coupling is used
- observation in [Dasgupta, Dreyer, Hamilton, Monni, Salam '18] (PanScales collaboration):
  - subtleties arise in distribution of recoil for subsequent emissions ⇒ phase space where accuracy is spoiled if soft gluon absorbs recoil
  - + in colour assignment
  - also: set of tests for shower accuracy [Dasgupta, Dreyer, Hamilton, Monni, Salam '20]



New Parton Showers - NLL accuracy

- Several solutions/re-evaluations of parton shower concepts:
- [Dasgupta, Dreyer, Hamilton,Monni, Salam, Soyez ’20], [vanBeekveld, Ferrario Ravasio, Hamilton, Salam, Soto-Ontoso,Soyez ’22]
  - partitioning of splitting functions and appropriate choice of evolution variable can lead to NLL accurate shower for local and global recoil strategies
- [Forshaw, Holguin, Plätzer ’20]
  - Connections between angular ordered and dipole showers
- [Nagy, Soper ’11]
  - local transverse, global longitudinal recoil
- [Herren, Krauss, DR, Schönherr, Höche ’22]
  - global recoil, enables analytic comparison to resummation and proof of NLL accuracy
- [Preuss ’24]
  - global recoil in antenna shower Vinca

Compare: resummation e.g. in CAESAR

- factorisation of matrix elements in soft collinear limit well known
- how to extract NLL observable independent (i.e. without additional information)?
- method from [Banfi, Salam, Zanderighi '05]: need explicit implementation of soft-collinear limit*:

$$k_t^\rho = k_t \rho$$

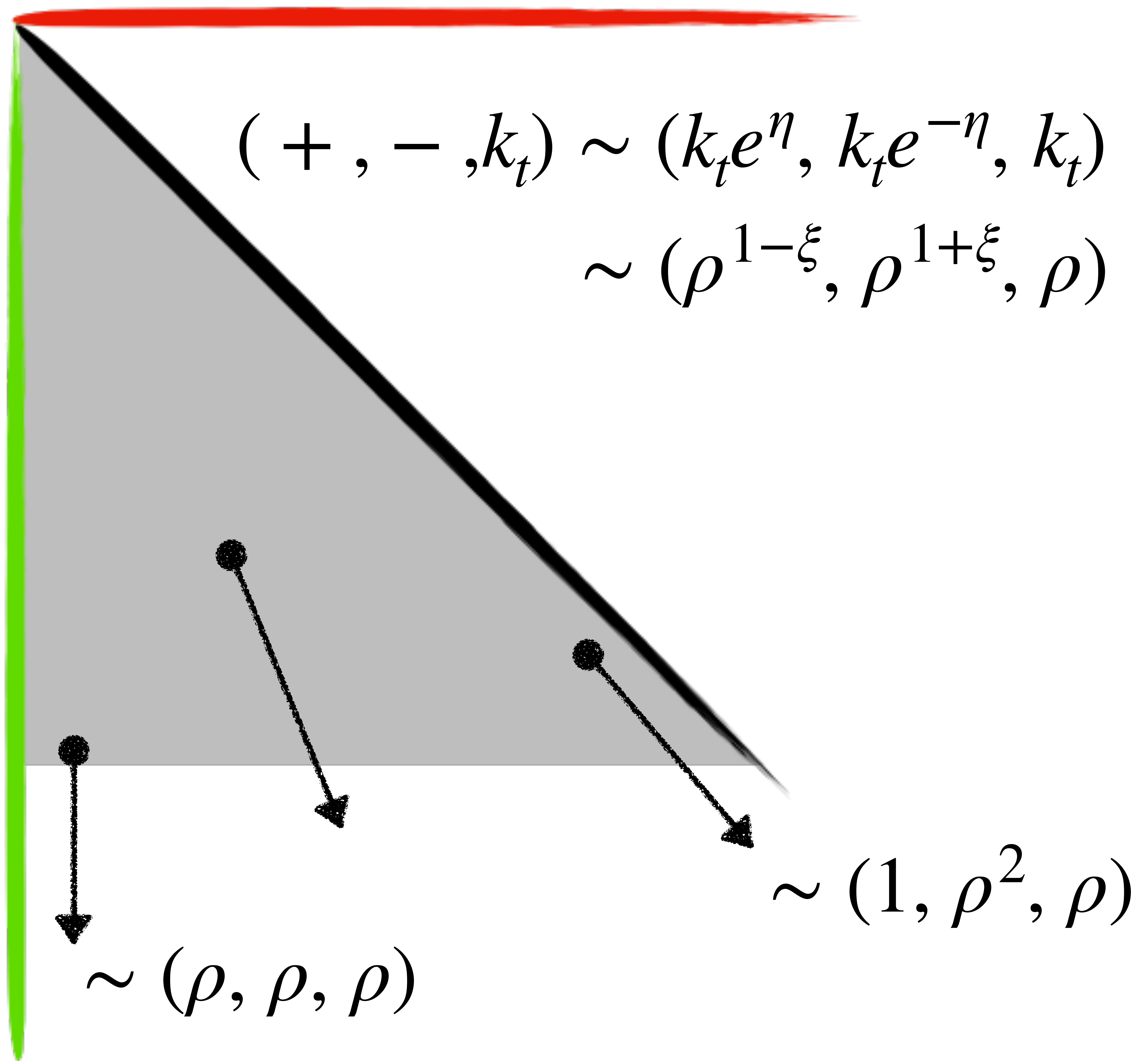
$$\xi = \frac{\eta}{\eta_{\mathrm{max}}}$$

$$\eta^\rho = \eta - \xi \ln \rho$$

and assume

$$V(k_i^\rho) = \rho V(k_i)$$

$\rightarrow$ numerically evaluate phase space integrals in this limit



* example assuming $V(k_t, \eta) \sim k_t/Q$ for brevity

Effect of recoil on accuracy

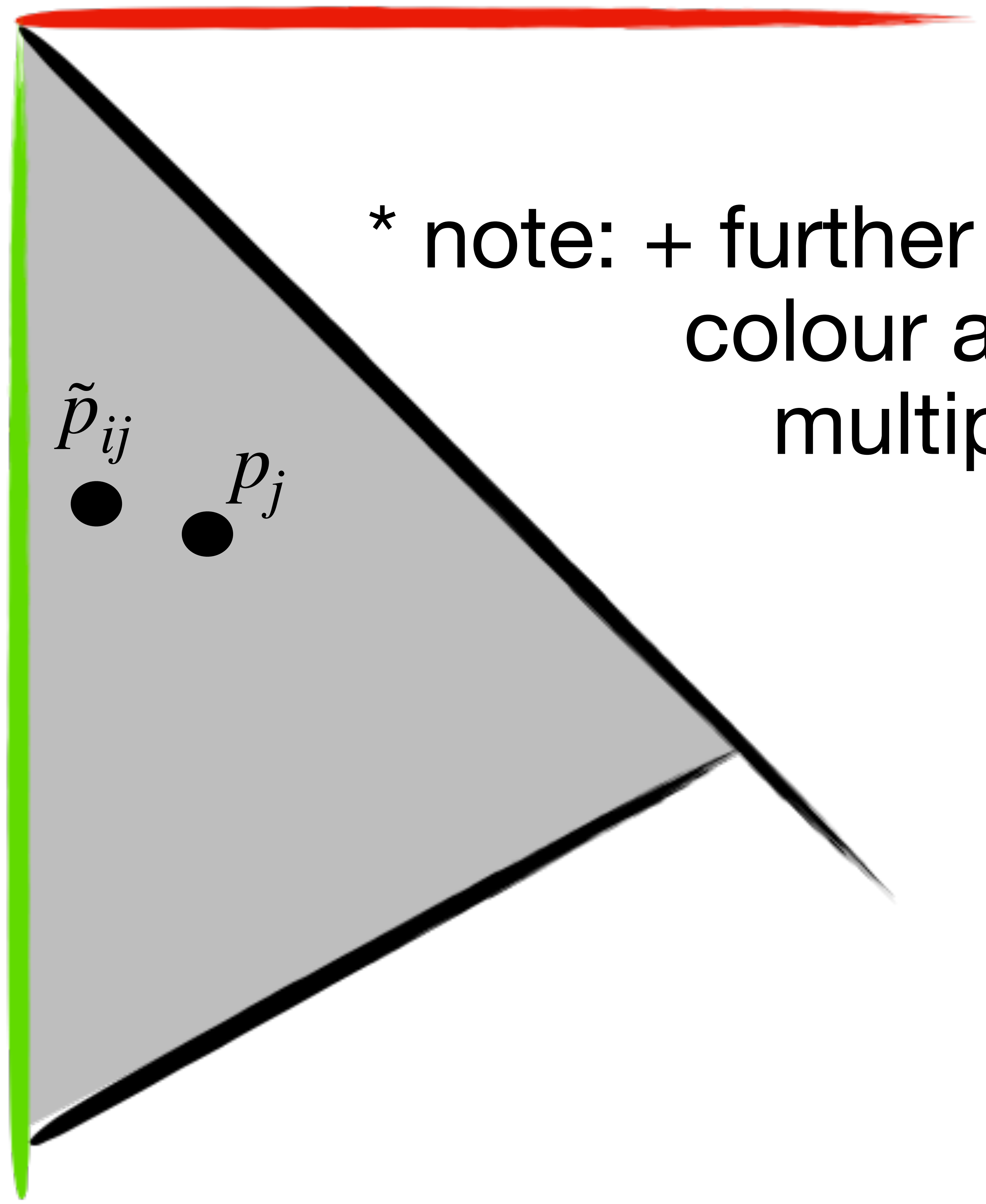
- question: do recoil effects indeed vanish in soft limit (i.e. $\rho \to 0$)?*
  [Dasgupta,Dreyer,Hamilton,Monni,Salam '18]

- consider situation where we first emit $\tilde{p}_{ij}$ from p_a, p_b, then emit p_j, $\tilde{p}_{ij} \to p_i, p_j$

- transverse momentum of p_i will be $k_t^i \sim k_t^{ij} + k_t^j \to k_t^{ij}$ as $\frac{k_t^j}{k_t^i} \to 0$

- but, relevant limit is simultaneous rescaling $\frac{\Delta k_t^i}{k_t^i} \to \frac{\rho k_t^j}{\rho k_t^i} = \mathcal{O}(1)$

$$p_i = z\tilde{p}_{ij} + (1-z)y\tilde{p}_k + k_\perp$$
$$p_j = (1-z)\tilde{p}_{ij} + zy\tilde{p}_k - k_\perp$$
$$p_k = (1-y)\tilde{p}_k \ .$$



* note: + further problems for colour assignment in multiple emissions

Alaric - Analytic tracing of recoil effects

$$\Lambda^{\mu}{}_{\nu}(K,\tilde{K}) = g^{\mu}_{\nu} + \tilde{K}^{\mu} A_{\nu} + X^{\mu} B_{\nu}$$

vanishes in soft limit

work out $\rho \to 0$ limit: $A^{\nu} \xrightarrow{\rho\to 0} 2\,\frac{\tilde{K}X}{\tilde{K}^2}\,\frac{\tilde{K}^{\nu}}{\tilde{K}^2} - \frac{X^{\nu}}{\tilde{K}^2}\,,$ and $B^{\nu} \xrightarrow{\rho\to 0} \frac{\tilde{K}^{\nu}}{\tilde{K}^2}$

apply to soft momentum p_l:

$$\frac{\Delta p_l^{0,3}}{p_l^{0,3}} \sim \rho^{1-\max(\xi_i,\xi_j)}$$

$$\frac{\Delta p_l^{1,2}}{p_l^{1,2}} \sim \rho^{(1-\xi_l)(\max(\xi_i,\xi_j)-\xi_l)}$$

compare to $\frac{\Delta k_t}{k_t} \sim \mathcal{O}(1)$ from local dipole scheme

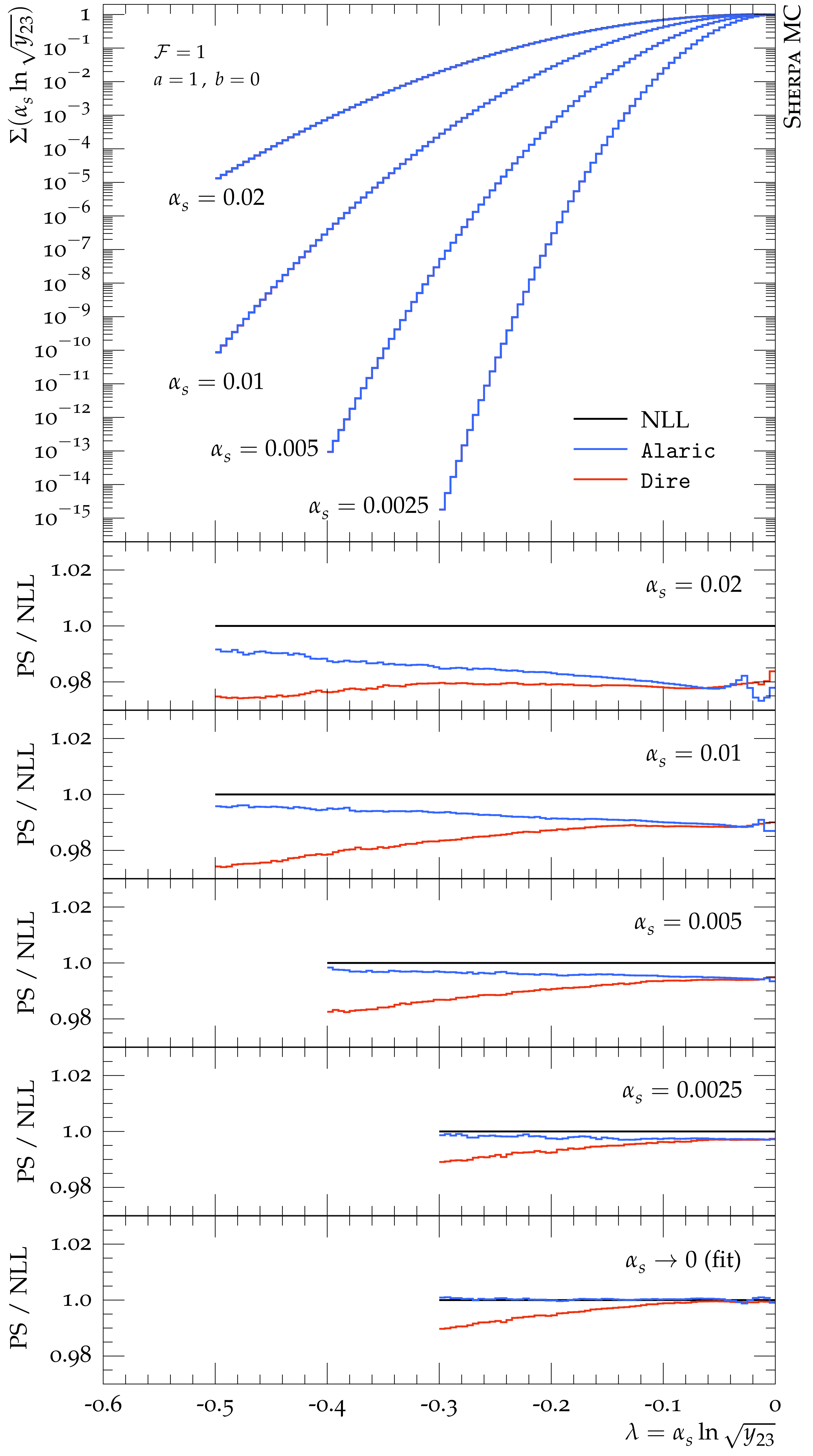
Alaric - Numerical validation I

- Limit $\alpha_s \to 0$ with $\lambda = \alpha_s L =$ const. of

$$\frac{\Sigma^{\text{Shower}}}{\Sigma^{\text{NLL}}} \sim \exp\left(f^{LL}_{\text{Shower}} - Lg_1(\alpha_s^n L^n)\right)$$
$$\times \exp\left(f^{NLL}_{\text{Shower}} - g_2(\alpha_s^n L^n)\right)$$
$$\times \exp\left(\mathcal{O}(\alpha_s^{n+1} L^n)\right)$$

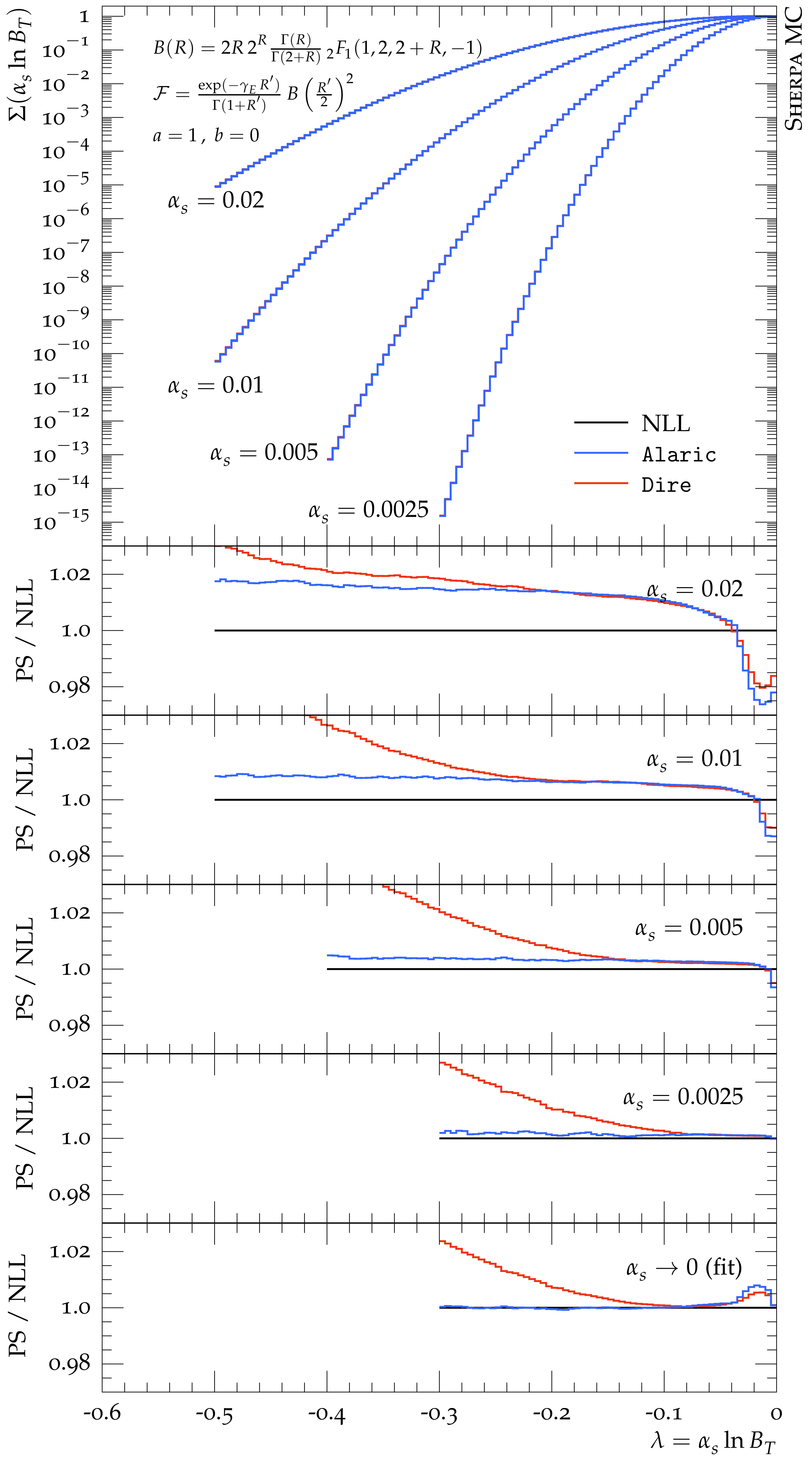
$\to 1$ if shower reproduces LL, NLL logs

- Observable: jet resolution y_{23} in Cambridge jet measure, $\mathcal{F} = 1 \to$ only largest emission matters, check that additional shower emissions vanish

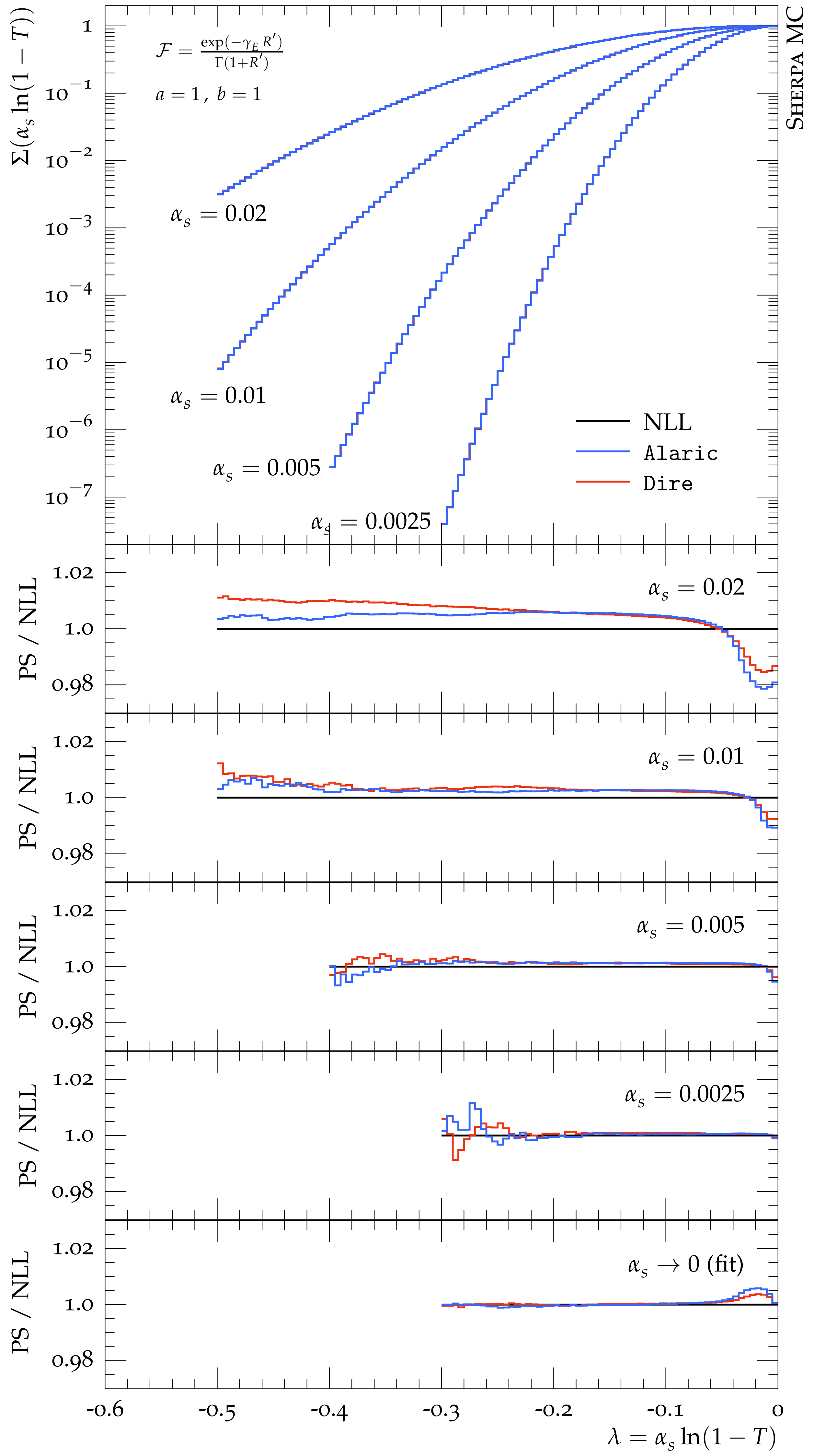


Alaric - Numerical validation II

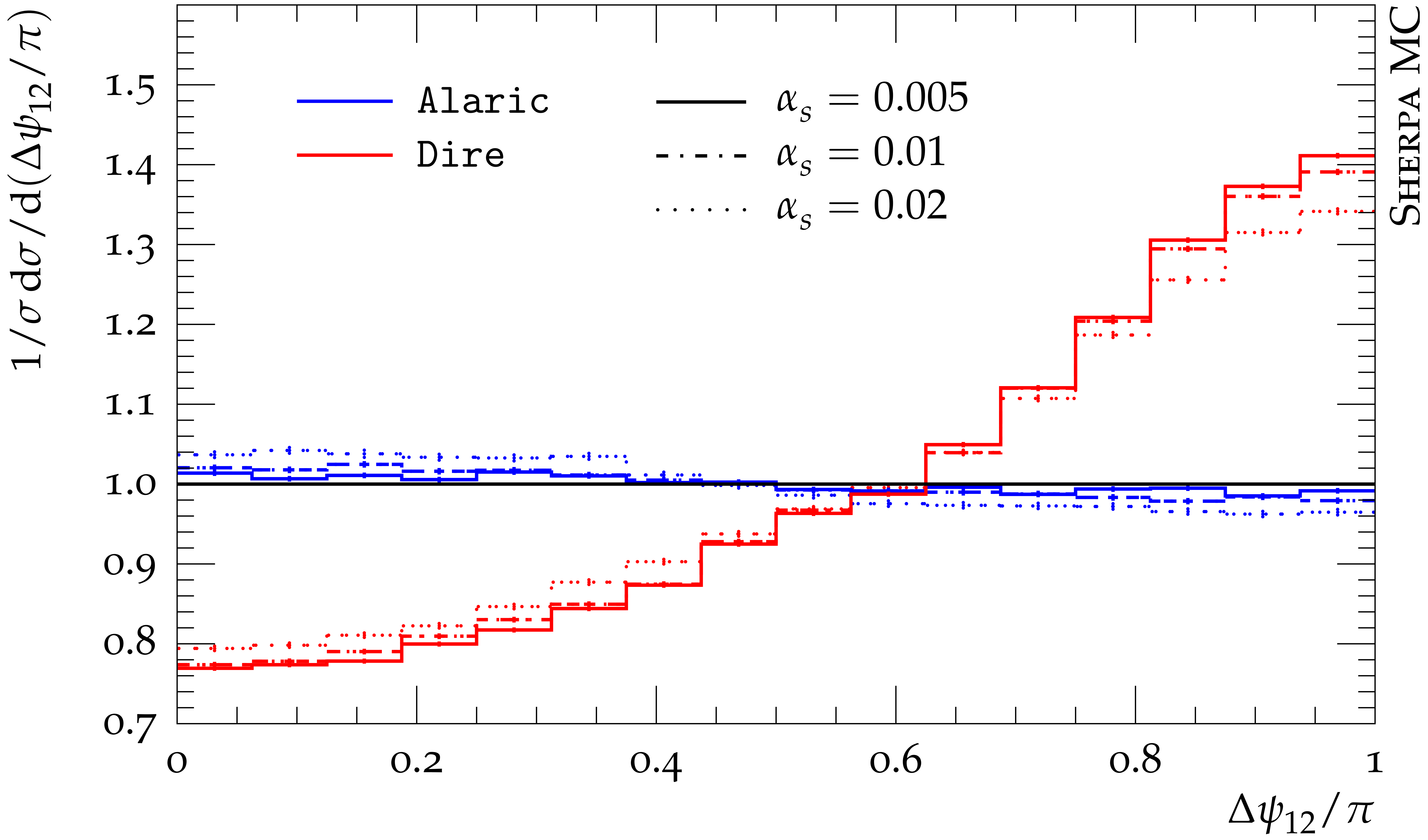
- total broadening $B_T = B_L + B_R$
- scaling k_t like, similar to y_{23}
- but non-trivial $\mathcal{F}$ function



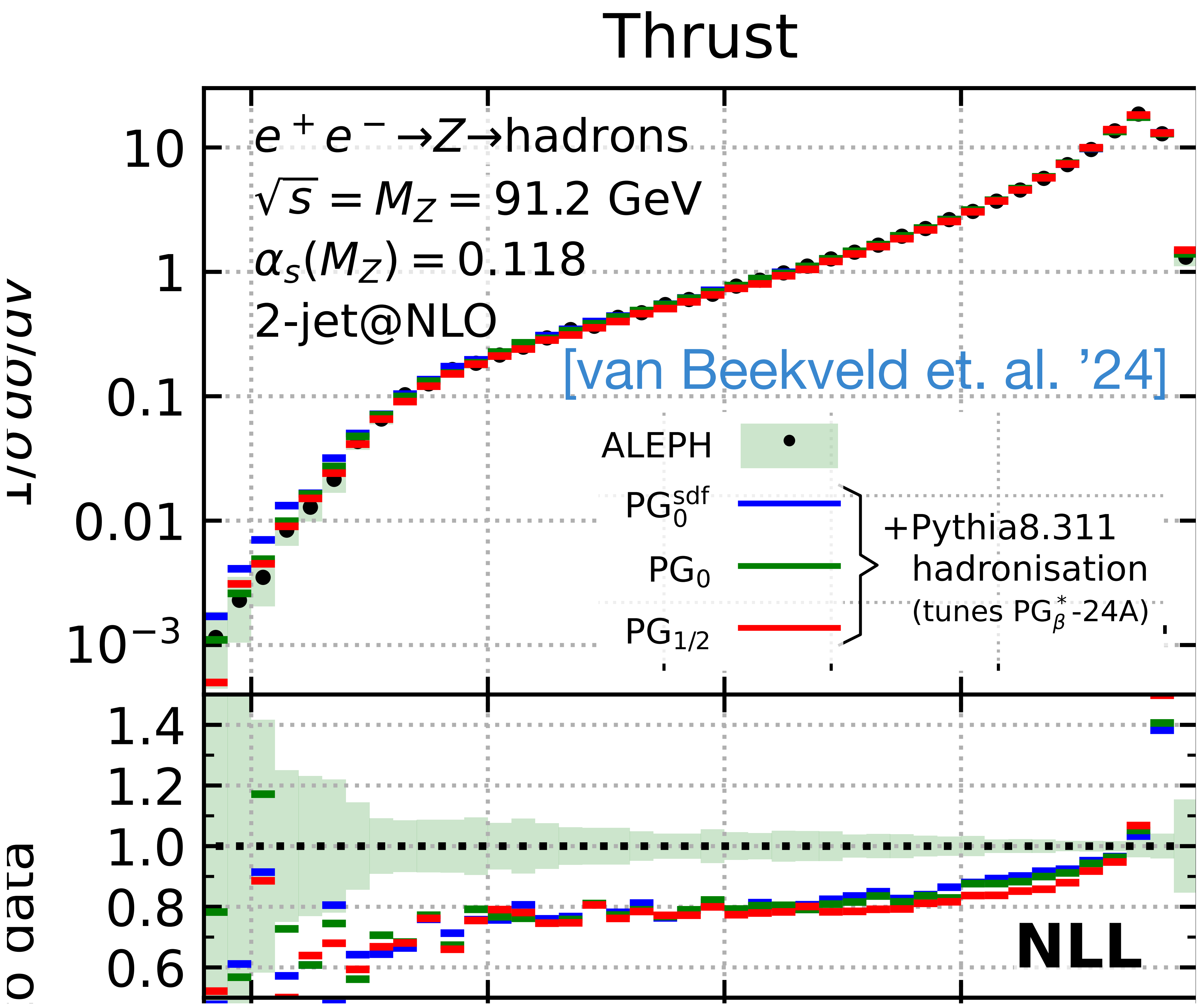
- thrust $\tau = 1 - t$
- scaling like virtuality $k_t e^{-\eta}$
- standard function $\mathcal{F} = \dfrac{\exp(-\gamma_E R)}{\Gamma(1 + R')}$
- no evidence for NLL violation even for standard showers

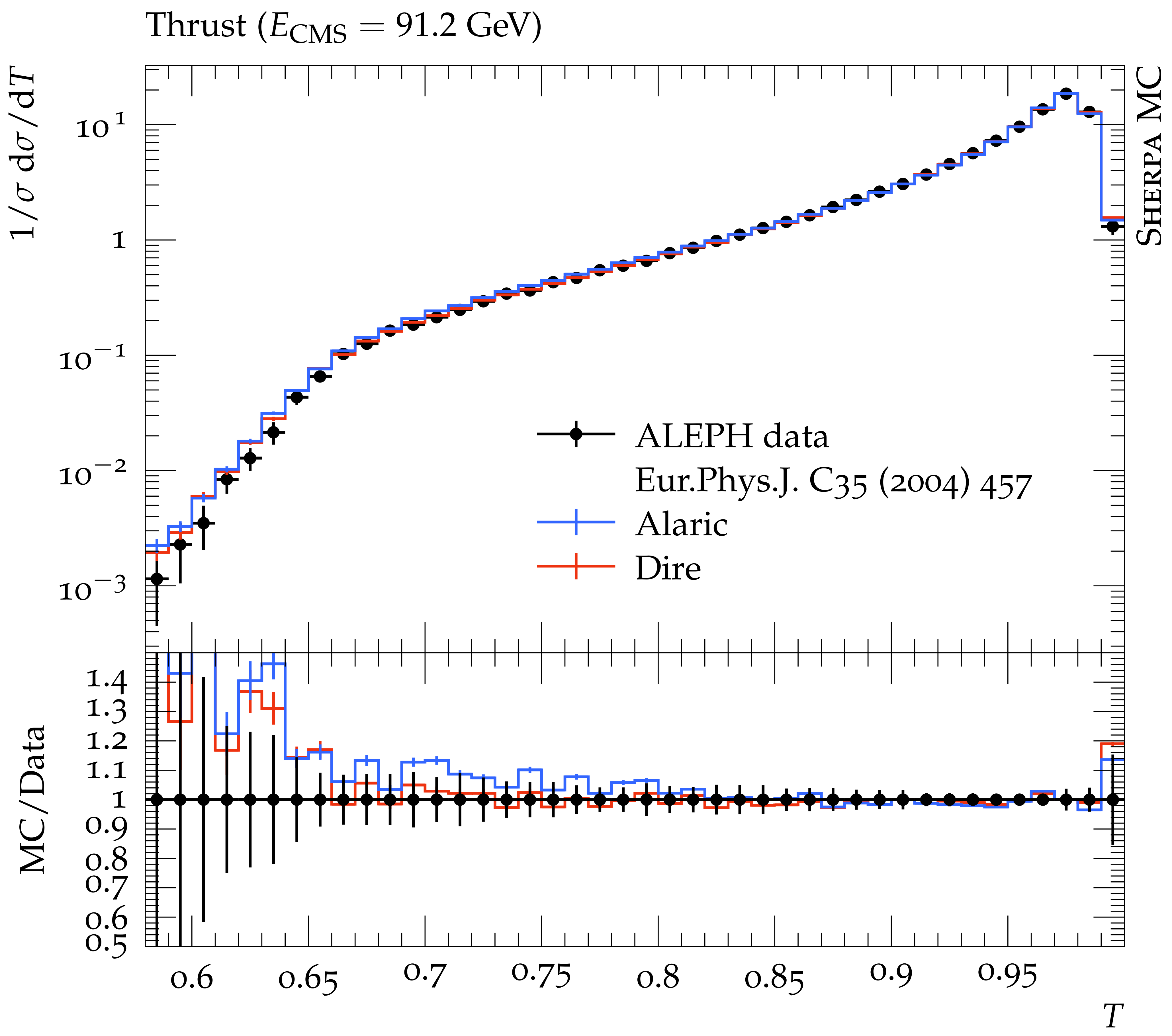


Alaric - Numerical validation III



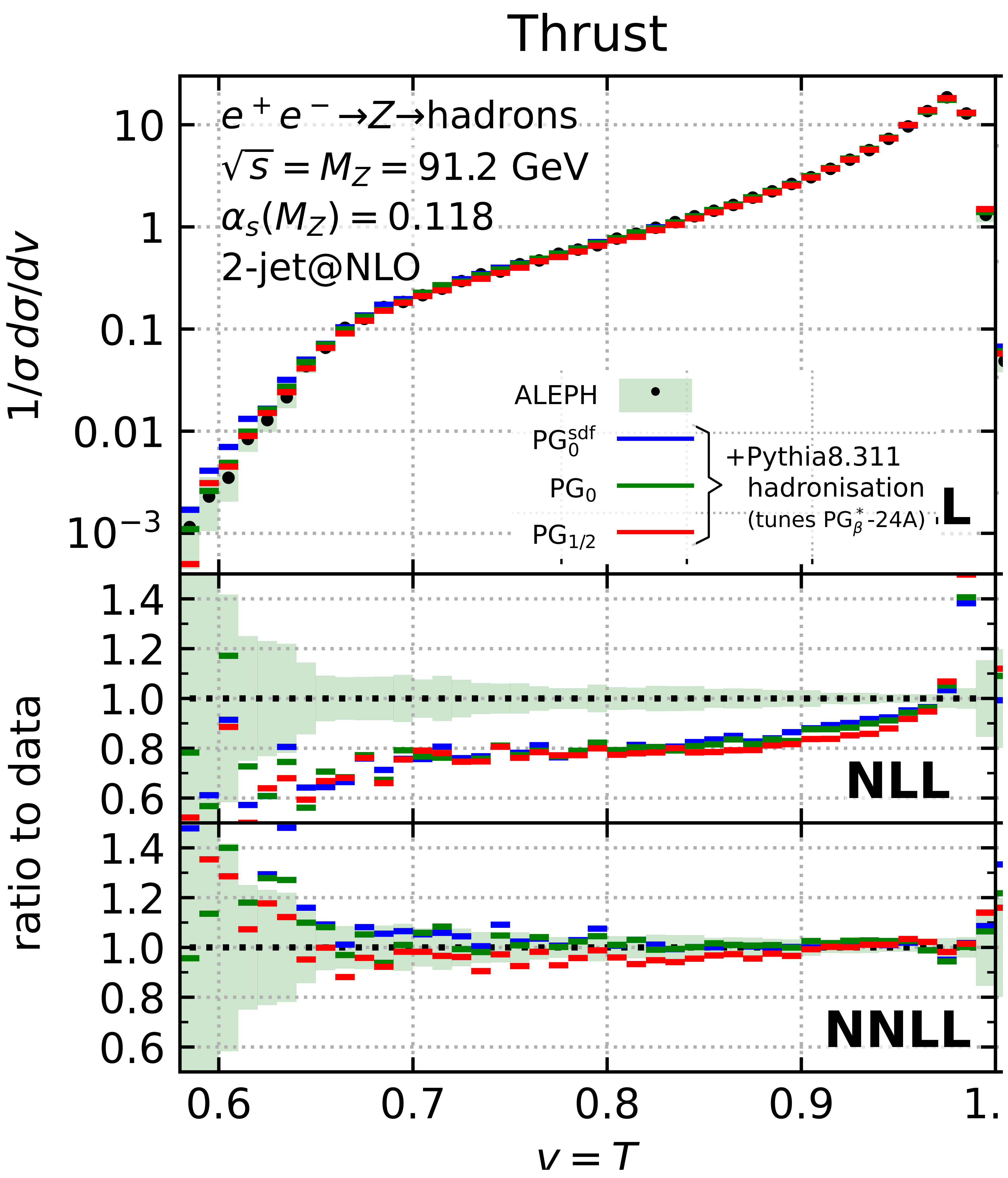
Pheno with NLL showers





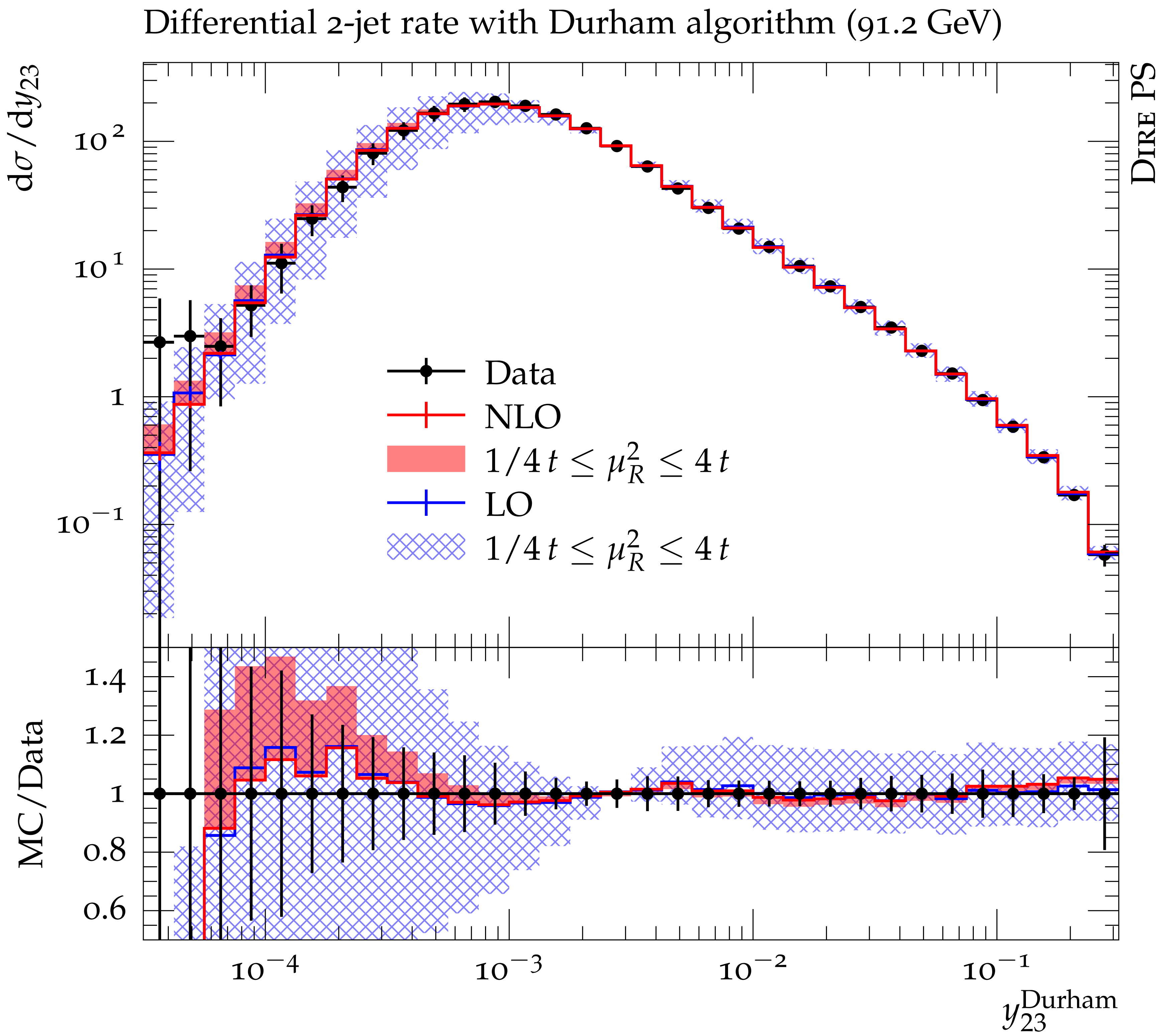
- PanScales shower and Alaric @ NLL accuracy
  - significantly different conclusion about ability of NLL shower to describe data (similar level of tuning, $\alpha_S = 0.118$ is fixed, string fragmentation parameters in Pythia 8 tuned to LEP data)

Towards NNLL



[van Beekveld et. al. '24]

- Conclusion from PanScales studies: NNLL needed to describe even simple observables
- Achieved by multiplicative matching of NLO splitting kernels + correction terms capturing effect of inclusive gluon emissions



[Höche, Prestel '17]

- Appears to be in contrast with small effects found so far in implementing higher order splitting functions (though not in complete NNLL framework yet) [Höche, Prestel '17], [Dulat, Höche, Prestel '18], [Gellersen, Höche, Prestel]

Summary

- Alaric parton shower
  - partial fractioning of eikonal leading to positive splitting functions filling full phase space
  - global kinematics for soft splitting functions, guarantees NLL and analytic tracking of accuracy
- new developments:
  - CKKW merging
  - systematic variations of NLL ambiguities
- future
  - MC@NLO matching on the way (enabling NLL' accuracy in the soft limit)
  - higher order splitting functions with Dire technique + new kinematics
  - spin correlations to complete radiation pattern