Extraction of tPDFs from lattice QCD: The η_c-meson

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GPDs and their forward limit: tPDFs

Generalized parton distributions: Non-local light-like separated quark or gluon operators, evaluated between hadron states in non-forward kinematics and projected onto the light-front. [Fortsch.Phys.:42(1994)101, Phys.Lett.B:380(1996)417, Phys.Rev.D:55(1997)7114]

Example: Twist-two chiral-even quark GPD of a spinless hadron.

$$H_{q/h}(x,\xi,t,\mu^2) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ix\frac{p+p'}{2}\cdot z} \left\langle h\left(p'\right)\right| \overline{\psi}_q\left(-z/2\right)\gamma^+ \widehat{\mathcal{W}}\left[-z/2, z/2\right]\psi_q\left(z/2\right)\left|h\left(p\right)\right\rangle\Big|_{\substack{z^+=0\\ \boldsymbol{z}_\perp=\boldsymbol{0}_\perp}}$$

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t-dependent Parton Distribution Functions

$$q_h(x,t,\mu^2) = H_{q/h}(x,\xi=0,t,\mu^2) \quad [p\cdot z = p'\cdot z \equiv p^+]$$

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Properties:

- Universality *i.e.* hadron-specific objects.
- Contain parton distribution functions and electromagnetic form factors.
- Non-perturbative description of hadron structure: (3D) Tomography.

Pseudo-distributions in a nutshell (I)

Definition: Ioffe-time tPDF ($\nu \equiv -p \cdot z$)
[Nucl.Phys.B:311(1989)541, Phys.Rev.D:51(1995)6036, Phys.Rev.D:100(2019)116011]

$$q_h(\nu, t, \mu^2) \equiv \int_{-1}^{1} dx e^{i\nu x} q_h(x, t, \mu^2) = \frac{1}{2p^+} \left\langle h\left(p'\right) \left|\overline{\psi}_q\left(-z/2\right) \gamma^+ \widehat{\mathcal{W}}\left[-z/2; z/2\right] \psi_q\left(z/2\right)\right| h\left(p\right)\right\rangle\Big|_{\substack{z^+=0 \\ \boldsymbol{z}_\perp=\boldsymbol{0}_\perp}}$$

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1. Consider a matrix element with spacelike z
[Phys.Rev.D:96(2017)034025, Phys.Rev.D:96(2017)094503, Phys.Rev.D:100(2019)116011]

$$M^{\mu}_{q/h}\left(p, p', z\right) = (p + p')^{\mu} \mathcal{F}(\nu, t, z^2) - (p' - p)^{\mu} \mathcal{G}(\nu, t, z^2) + z^{\mu} \mathcal{Z}(\nu, t, z^2).$$

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3. Connection with Ioffe-time distributions

$$q_h(\nu, t, \mu^2) = \lim_{z^2 \to 0} \mathcal{F}(\nu, t, z^2)$$

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[Phys.Lett.B:767(2017)314]

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Light-cone matching: $$\widetilde{q}_h(\nu, t, z^2) = \int_0^1 dw\, \mathcal{C}^{\overline{\mathrm{MS}}}(w, z^2\mu^2, \alpha_s) q_h(w\nu, t, \mu^2)$$

Roadmap to tPDF from lattice QCD

Parton distribution (Light-cone)

$q = \langle h' | \mathcal{O}^\mu(z) | h \rangle \big|_{z^2=0}$

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Light-cone matching

$\widetilde{q} = \mathcal{C} \otimes q$

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Renormalization: Pseudo distribution

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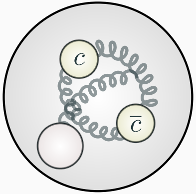
Lattice QCD Matrix elements

$\langle h(p') | \mathcal{O}^\mu(z) | h(p) \rangle$

Hands on

Case study: η_c meson

η_c meson	
Composition:	$c\bar{c}$
J^{PC}:	0^{-+}
Mass:	2983.9 ± 0.4 MeV
Width:	32.0 ± 0.7 MeV



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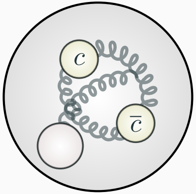

Goal: Lattice QCD extraction of the c-quark unpolarized tPDF of a η_c meson

$$q_{\eta_c}(\nu, t, \mu^2) = \frac{1}{2p^+} \langle \eta_c(p') | \overline{\psi}_c(-z/2) \gamma^+ \widehat{\mathcal{W}}[-z/2, z/2] \psi_c(z/2) | \eta_c(p) \rangle \Big|_{\substack{z^+=0 \\ \boldsymbol{z}_\perp = \boldsymbol{0}_\perp}}, \quad [p \cdot z = p' \cdot z]$$

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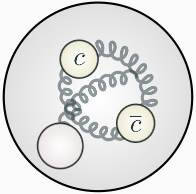

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- How does it emerge from the bounding of a pair $c\bar{c}$?
- Comparison with lighter 0^- mesons: Assess quark-mass effect on hadron structure.

Lattice QCD setup (I)

- $N_f = 2$ ensembles (CLS) [Nucl.Phys.B:865(2012)397,PoSLATTICE2013:(2014)475]
  - Wilson gauge action.
  - $\mathcal{O}(a)$-improved Wilson fermions.
  - $\kappa_u = \kappa_d \equiv \kappa_k$.
  - No Symanzik improvement for M^μ.

id	β	a [fm]	L/a	am_π	m_π [MeV]	$m_\pi L$	κ_c	κ_l
A5	5.2	0.0755(9)(7)	32	0.1265(8)	331	4.0	0.12531	0.13594
E5	5.3	0.0658(7)(7)	32	0.1458(3)	437	4.7	0.12724	0.13625
F7			48	0.0885(3)	265	4.3	0.12713	0.13638
N6	5.5	0.0486(4)(5)	48	0.0838(2)	340	4.0	0.13026	0.13667

- One hadron-interpolator and four smearings (source and sink).

$$\eta_c^s(x) = \overline{\psi}_c^s(x)\,\gamma_5\psi_c^s(x) \quad , \quad J^{PC} = 0^{-+}$$
$$\psi_q^s(x) = (1 + 0.125\Delta_{\text{APE}})^{N_s}\,\psi_q(x) \quad , \quad N_s \in \{0, 30, 50, 80\}$$

- Twisted boundary conditions and a symmetric frame.

Compute

$$M^0(p,p',z) = \langle \eta_c(p')|\overline{\psi}_c(-z/2)\gamma^0\widehat{\mathcal{W}}[-z/2,z/2]\psi_c(z/2)|\eta_c(p)\rangle = 2E\mathcal{F}(\nu,t,z^2)$$

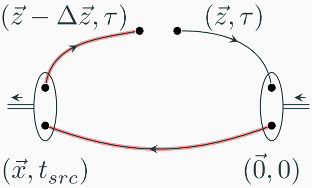
with,

- $p^\mu = (E, \boldsymbol{p}_\perp, p_3)$ and $p'^\mu = (E, -\boldsymbol{p}_\perp, p_3)$
- $z^\mu = (0,0,0,z_3)$

Matrix elements from LQCD

Computation of hadron three-point functions

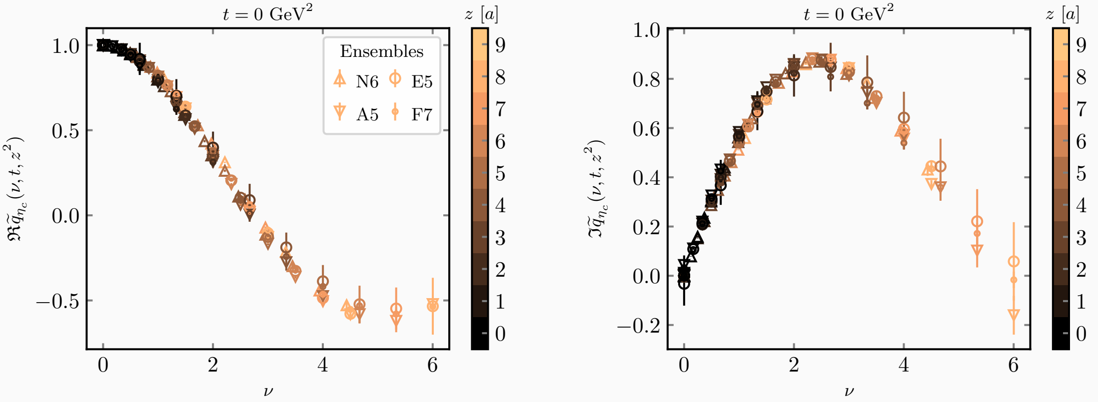
$$C_3^{(ss')}(\vec{p}, t_{src}) = \sum_{\vec{x},\vec{z}} e^{-i\vec{p}\cdot\vec{x}-i\vec{q}\cdot\vec{z}} \langle \eta_c^s(\vec{x}, t_{src}) \overline{\psi}_c(\vec{z}-\Delta\vec{z}, \tau)\, \gamma^0 \widehat{\mathcal{W}}[\vec{z}-\Delta\vec{z}, \tau; \vec{z}, \tau] \psi_c(\vec{z}, \tau)\, \overline{\eta}_c^s(\vec{0}, 0)\rangle$$



- Consider connected diagrams only: Sequential propagator technique.
- Project ground state ($\eta_c(1s)$) according to GEVP.
- Compute ratios to isolate matrix elements: [PoSLATTICE2005:(2006)360]

$$R(\tau) = \frac{C_3^{(\mathrm{P})}(\vec{p}, \vec{p}', t_{src}, \tau)}{\sqrt{C_2^{(\mathrm{P})}(\vec{p}', t_{src})\, C_2^{(\mathrm{P})}(\vec{p}, t_{src})}} \sqrt{\frac{C_2^{(\mathrm{P})}(\vec{p}, t_{src}-\tau)\, C_2^{(\mathrm{P})}(\vec{p}', \tau)}{C_2^{(\mathrm{P})}(\vec{p}', t_{src}-\tau)\, C_2^{(\mathrm{P})}(\vec{p}, \tau)}} \to \frac{M^0(p, p', z)}{4\sqrt{E(\vec{p})\, E(\vec{p}')}}$$

Pseudo-tPDF data



- Pseudo-distribution data approaching a universal line for all ensembles.
- Signal-to-noise ratio significantly good over the entire kinematic range.

Light-cone matching

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Light-cone matching

1. Light-cone operator product expansion: $z^2 \to 0$
[Nucl.Phys.B:27(1971)541]

$$\begin{aligned}\widetilde{q}_h(\nu, t, z^2) &\sim \sum_i \sum_{n=0}^{\infty} C_n^{(i),\overline{\text{MS}}}(z^2\mu^2)\langle h(p')|\widetilde{\mathcal{O}}_{(i)}^{\{\mu\mu_1\cdots\mu_n\}}|h(p)\rangle^{\overline{\text{MS}}} z_\mu \prod_{k=1}^{n} z_{\mu_k} + \text{h.t.} \\ &= \sum_i \sum_{n=0}^{\infty} C_n^{(i),\overline{\text{MS}}}(z^2\mu^2)(-2\nu)^n(1 + z^2 \widetilde{f}_n^{(i)}(\nu, t, z^2, m^2))a_n^{(i)}(t, \mu^2) + \text{h.t.}\end{aligned}$$

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2. Recast as convolution: Light-cone matching to $\overline{\mathrm{MS}}$ distribution
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Challenges of the light-cone matching

- Integral relation
- Acts on light-cone distributions

From pseudo to light-cone distributions (I)

Idea: Ioffe-time distributions, onto and outside the light-front, are analytic functions in the Ioffe-time variable: [see e.g. ISBN:0-8493-8273-4]

$$q_h^{(i)}(\nu, t, \mu^2) = \sum_{m=0}^{\infty} A_m^{(i)}(t, \mu^2)\nu^m, \qquad \widetilde{q}_h(\nu, t, z^2) = \sum_{m=0}^{\infty} \widetilde{A}_m(t, z^2)\nu^m.$$

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- Connection with Mellin moments

$$\mathfrak{m}_m^{(i)}(t, \mu^2) = i^m m! A_m^{(i)}(t, \mu^2), \qquad \mathfrak{m}_m^{(i)}(t, \mu^2) = \int_{-1}^1 dx x^m q_h^{(i)}(x, t, \mu^2).$$

From pseudo to light-cone distributions (II)

Strategy: Combined fit to lattice data on pseudo distribution

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1. Fit pseudo-distribution data to

$$\widetilde{q}_h(\nu, t, z^2; e_i) = \sum_{m=0}^{N} \widetilde{\mathcal{A}}_m(t, z^2; e_i)\nu^m = \sum_{m=0}^{N} 2^m \sum_i C_m^{(i)}(z^2\mu^2)\mathcal{A}_m^{(i)}(t, \mu^2; e_i)\nu^m$$

From pseudo to light-cone distributions (II)

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Caveat: $C_m^{(i)}(z^2\mu^2)$, as they are, do not handle $\mathcal{O}(z^2)$

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From pseudo to light-cone distributions (II)

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2. Continuum limit

$$\widetilde{q}_h(\nu, t, z^2; e_i) = \sum_{m=0}^{N} 2^m \sum_i C_m^{(i)}(z^2\mu^2)\mathcal{A}_m^{(i)}(t, \mu^2)\nu^m + \sum_{n=0}^{N} z^2\mathcal{B}_m\nu^m + a^2\sum_{n=0}^{N} \mathcal{D}_m\nu^m$$

From pseudo to light-cone distributions (III)

Reconstruct light-cone Ioffe-time $\overline{\text{MS}}$:

$$q_h^{(i)}(\nu, t, \mu^2) = \sum_{m=0}^{N} \mathcal{A}_m^{(i)}(t, \mu^2)\nu^m$$

Note: Treat real- and imaginary part of pseudo distribution data separately

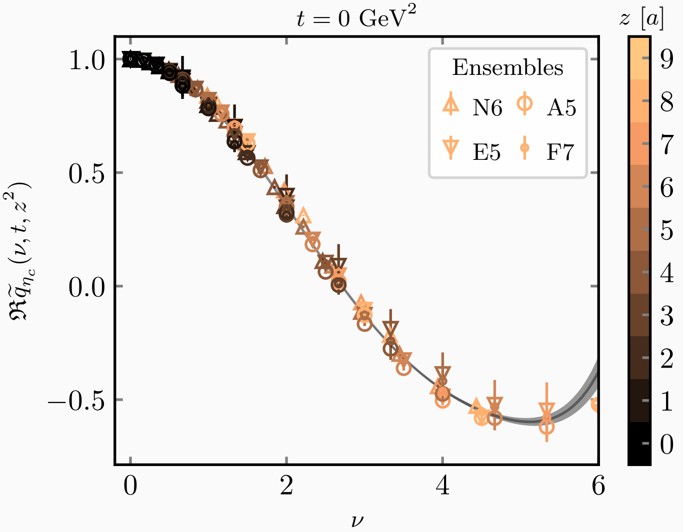
- Real part: $\Re\widetilde{q}_h$

  $q_{ns}(\nu, t, z^2) = \Re\widetilde{q}_h(\nu, t, z^2) = \int_{-1}^{1} dx \cos(\nu x)\widetilde{q}_h(x, t, z^2)$ Even distribution in Ioffe-time

- Imaginary part: $\Im\widetilde{q}_h$

  $q_s(\nu, t, z^2) = \Im\widetilde{q}_h(\nu, t, z^2) = \int_{-1}^{1} dx \sin(\nu x)\widetilde{q}_h(x, t, z^2)$ Odd distribution in Ioffe-time

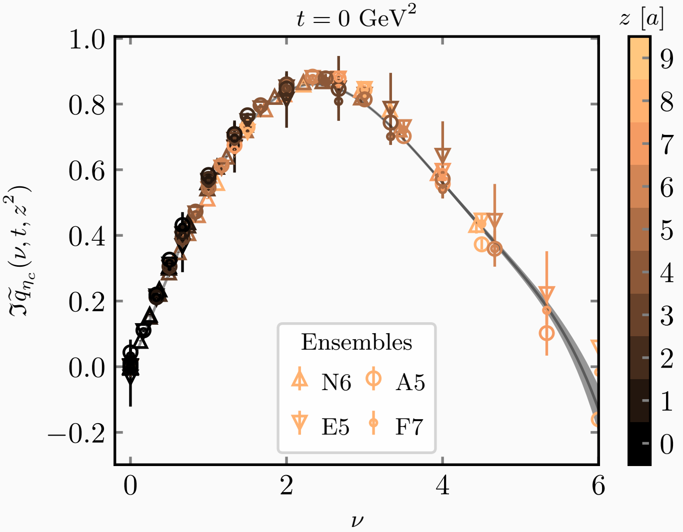
Ioffe-time tPDFs: Non-singlet distribution



$\mathcal{A}_2(t, \mu^2)$	$-0.0634(4)$
$\mathcal{A}_4(t, \mu^2)$	$0.0009(3)$
$\mathcal{A}_6(t, \mu^2)$	$-0.000004(3)$

$\mu = 3$ GeV

Ioffe-time tPDFs: Singlet distribution

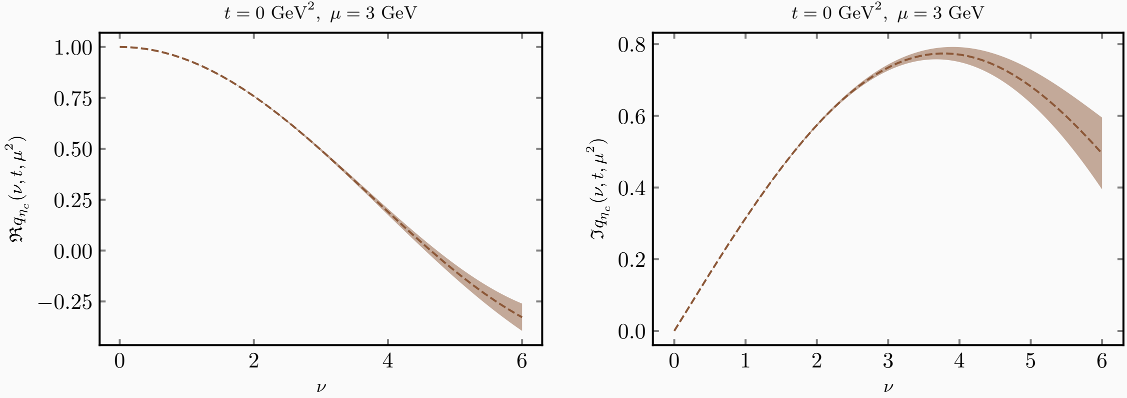


Preliminary!

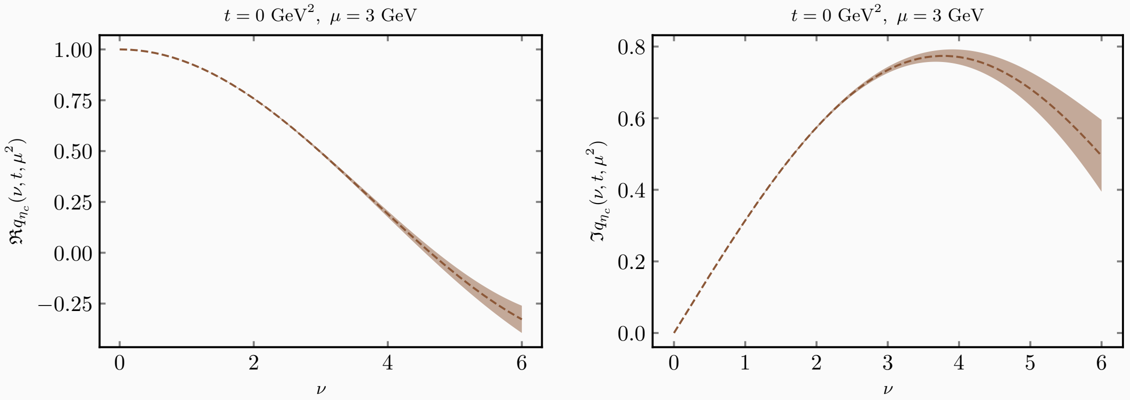
$\mathcal{A}_1(t, \mu^2)$	0.3239(9)
$\mathcal{A}_3(t, \mu^2)$	−0.0096(2)
$\mathcal{A}_5(t, \mu^2)$	0.000090(6)
$\mathcal{A}_7(t, \mu^2)$	−0.00000029(5)

$\mu = 3$ GeV

Ioffe-time distributions



Ioffe-time distributions



n	1	2	3	4	5	6	7
$\mathfrak{m}_n(t, \mu^2)$	0.3239(9)	0.1279(9)	0.057(1)	0.0216(7)	0.011(1)	0.0030(2)	0.0015(2)

Conclusions and future steps

Summary

- Ongoing study of η_c meson structure through tPDFs within lattice QCD.
- Exploitation Ioffe-time distrbutions' analiticity.
  - Model bias reduction to truncation error.
  - Transparent treatment of target mass corrections.
- Extraction of η_c meson (Ioffe-time) tPDF for $\nu \lesssim 4 - 6$.

Future steps

- Refine analysis in the singlet sector: Gluon coupling.
- Extend kinematics and statistics: t-values and new ensembles.
- Refine analysis of lattice artifacts: e.g. finite volume or quark masses.
- Reconstruction of x-space distributions.

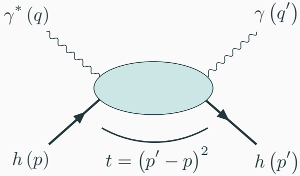
Thank you!

Back-up slides

t-dependent Parton Distribution Functions (tPDFs)

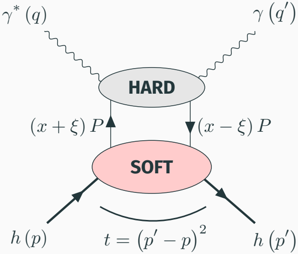
Hadron structure

How do quarks and gluons combine to make hadrons up?



Generalized Bjorken limit
$Q^2 \to \infty$ with $Q^2 >> t$
and ξ fixed.

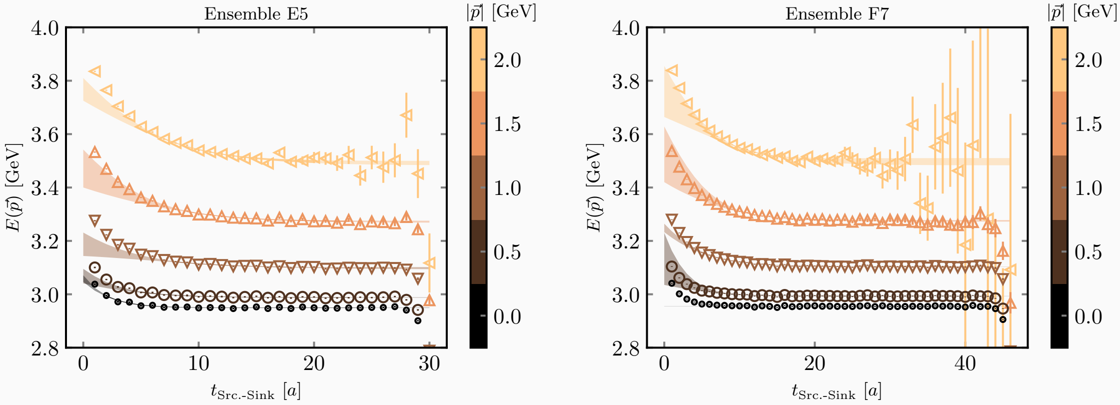
Factorization
[Phys.Rev.D59(1999)074009]



$$\mathcal{H}\left(\xi,t,Q^2\right)=\sum_{p=q,g}\int_{-1}^{1}\frac{dx}{\xi}\mathcal{K}^p\left(\frac{x}{\xi},\frac{Q^2}{\mu_F^2},\alpha_s\left(\mu_F^2\right)\right)\boldsymbol{H^p\left(x,\xi,t,\mu_F^2\right)}$$

Generalized Parton distributions: **Off-forward parton distribution functions**

Two-point functions: Effective masses

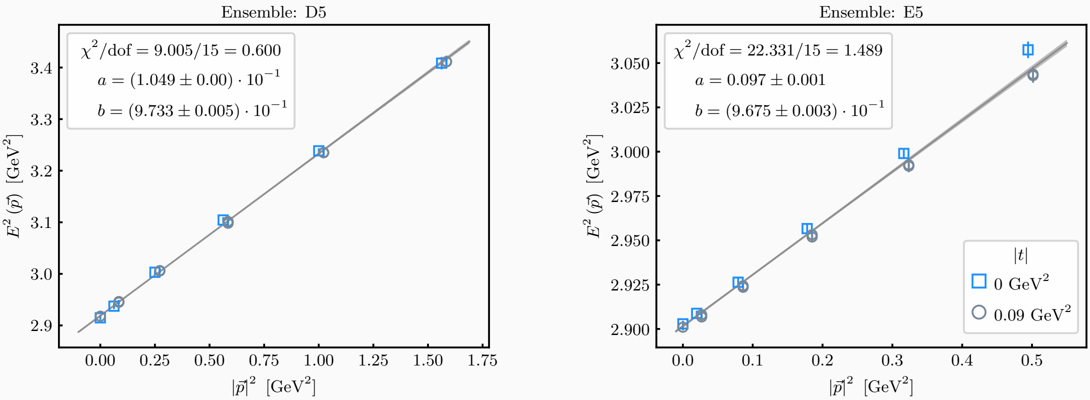


Energy spectrum compatible with expectation within finite-volume and cut-off effects.

Systematics: - Fit range: Model averaging (AIC) [Phys.Rev.D:103(2021)114502]

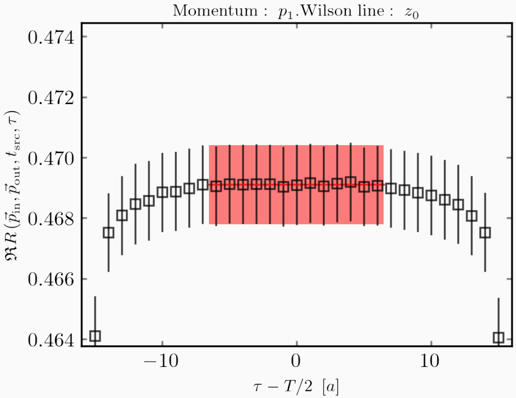
- Excited states: GEVP [Nucl.Phys.B:259(1985)58, JHEP:04(2009)094]

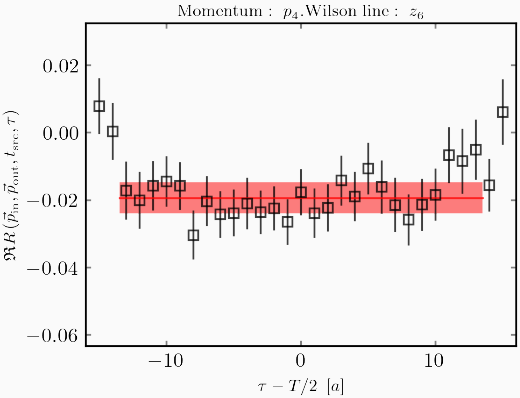
Two-point functions: Dispersion relations



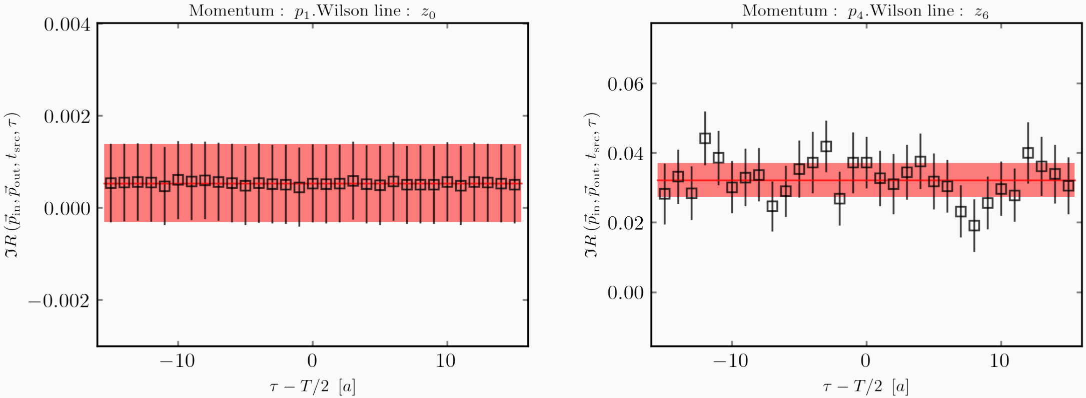
Consistency check: Expected energy-momentum dispersion relations fulfilled.

Three-point functions: Ratio fits





Three-point functions



Light-cone matching (I)

$$\langle h(p')|\mathcal{O}^{\mu}(z)|h(p)\rangle z_{\mu}\big|_{\mathrm{R}} \sim \sum_{n=0}^{\infty} C_n^{\overline{\mathrm{MS}}}(z^2\mu^2)\langle h(p')|\widetilde{\mathcal{O}}^{\{\mu\mu_1\cdots\mu_n\}}|h(p)\rangle z_{\mu}\prod_{k=1}^{n} z_{\mu_k} + \mathrm{h.t}$$

$$= C_0^{\overline{\mathrm{MS}}}(z^2\mu^2)\langle h(p')|\overline{\psi}_c(0)\gamma^{\mu}\psi_c(0)|h(p)\rangle z_{\mu}$$

$$+ C_1^{\overline{\mathrm{MS}}}(z^2\mu^2)\langle h(p')|\overline{\psi}_c(0)\gamma^{\{\mu}iD^{\mu_1\}}\psi_c(0)|h(p)\rangle z_{\mu}z_{\mu_1} + \mathrm{h.t.}$$

- $\boldsymbol{n=0}$

$$C_0^{\overline{\mathrm{MS}}}(z^2\mu^2)(2P^{\mu}a_0(\mu^2) - \Delta^{\mu}b_0)z_{\mu} = -2\nu C_0^{\overline{\mathrm{MS}}}(z^2\mu^2)a_0(\mu^2)$$

Light-cone matching (II)

- $\boldsymbol{n = 1}$

$$C_1^{\overline{\text{MS}}}(z^2\mu^2)\Big[(2P^\mu a_{1,0}(\mu^2) - \Delta^\mu b_{1,0}(\mu^2))P^{\mu_1} + (2P^{\mu_1} a_{1,0}(\mu^2) - \Delta^{\mu_1} b_{1,0})(\mu^2)P^\mu$$

$$-\frac{1}{2}g^{\mu\mu_1}(2P^2 a_{1,0}(\mu^2) - P\cdot\Delta b_{1,0}(\mu^2)) + \frac{1}{2}(2P^\mu a_{1,1}(\mu^2) - \Delta^\mu b_{1,1}(\mu^2))\Delta^{\mu_1}$$

$$+\frac{1}{2}(2P^{\mu_1} a_{1,1}(\mu^2) - \Delta^{\mu_1} b_{1,1}(\mu^2))\Delta^\mu - \frac{1}{4}g^{\mu\mu_1}(2(P\cdot\Delta)a_{1,1}(\mu^2) - \Delta^2 b_{1,1}(\mu^2))\Big] z_\mu z_{\mu_1}$$

$$= C_1^{\overline{\text{MS}}}(z^2\mu^2)\left[4\nu^2\left(1 - \frac{z^2P^2}{4\nu^2}\right)a_{1,0}(\mu^2) + \frac{z^2 t}{4}b_{1,1}(\mu^2)\right]$$

Light-cone matching (III)

$$\langle h(p')|\mathcal{O}^{\mu}(z)|h(p)\rangle z_{\mu}\big|_{\mathrm{R}} \sim -2\nu C_0^{\overline{\mathrm{MS}}}(z^2\mu^2)a_{0,0}(\mu^2) + 4\nu^2 C_1^{\overline{\mathrm{MS}}}(z^2\mu^2)\left(1-\frac{z^2P^2}{4\nu^2}\right)a_{1,0}(\mu^2)+\cdots$$

$$= -2\nu\left[\widetilde{a}_{0,0}(z^2) - 2\nu(1+z^2\widetilde{f}_1(\nu,z^2,t,m^2))\widetilde{a}_{1,0}(z^2)+\cdots\right]$$

with

$$\widetilde{a}_{n,0}(z^2) \equiv C_n^{\overline{\mathrm{MS}}}(z^2\mu^2)a_{n,0}(\mu^2)$$

Therefore

$$\widetilde{q}_h(\nu,t,z^2) \sim \widetilde{a}_{0,0}(z^2) - 2\nu(1+z^2\widetilde{f}_1(\nu,z^2,t,m^2)\widetilde{a}_{1,0}(z^2)+\cdots$$

One-loop light-cone matching

One loop matching kernel in the non-singlet sector [Phys. Rev. D 100, 1160011 (2019)]

$$\mathcal{C}(\alpha, z^2\mu^2, \alpha_s) = \delta(1-\alpha) - \frac{\alpha_s}{2\pi} C_F \left\{ \log\left(-z^2\mu^2 \frac{e^{2\gamma_E+1}}{4}\right) \left(\frac{1+\alpha^2}{1-\alpha}\right)_+ + 4\left(\frac{\log(1-\alpha)}{1-\alpha}\right)_+ - 2(1-\alpha)_+ \right\} + \mathcal{O}(\alpha_s^2)$$

Moments

$$C_n(z^2\mu^2, \alpha_s) = 1 - \frac{\alpha_s}{2\pi} C_F \left\{ \log\left(-z^2\mu^2 \frac{e^{2\gamma_E+1}}{4}\right) \left[2H(n) - \left(\frac{1}{n+1} + \frac{1}{n+2}\right) - \frac{3}{2}\right] + 4\sum_{j=1}^{n} \frac{H(j)}{j} - 2\left(\frac{1}{n+1} - \frac{1}{n+2}\right) \right\} + \mathcal{O}(\alpha_s^2)$$