Particle tracking and identification with single-layer Timepix-series detectors

Benedikt Bergmann

Institute of Experimental and Applied Physics, Czech Technical University in Prague

Benedikt.bergmann@utef.cvut.cz

Preface

Outline the capabilities of Timepix-type detectors enabled by **high spatial granularity, nanosecond-scale** timestamping and **continuous measurement**.

Show examples of data analysis in different mixed radiation fields profiting from the single-layer **temporal and spatial coincidence analysis**, **particle identification** & precise **trajectory reconstruction**

Focus on possible use cases in **fundamental science.**

Radiation imaging detector

8 November 2024 Detector Seminar - CERN 30 November 2024 Detector Seminar - CERN 30 November 2024 Detector Seminar - CERN 30

0

Pattern recognition

Pattern recognition together with dE/dX information allows determination of incident particle type - and energy?

Working principle:

Detector response to highly ionizing radiation

$$
\left| \left\langle -\frac{dE}{dx} \right\rangle = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \beta^2 \gamma^2 W_{\text{max}}}{I^2} - \beta^2 - \frac{\delta(\beta \gamma)}{2} \right] \right|
$$

Analysis of δ-ray signal or Braggbehavior further enhances particle separation capability

Increased energy deposit in the medium at the end of its range (**Bragg-Peak**) Example: **350 MeV/A He track.**

8 November 2024 Detector Seminar - CERN 5

Timepix and Timepix3

- 256 x 256 pixels with 55 μ m pitch (1.98 cm²)
- Sensor layer (Silicon, GaAs, CdTe, ...) flip-chip bump bonded to the ASIC

Timepix

- Frame based readout (92 fps) dead time > 11 ms
- Measurement of **energy or time** (Δt = 20.8 ns)
- Threshold for noise free measurement: 3-5 keV

Timepix3

- Data-driven readout (Max. count rate 40 Mpix/s)
- Per-pixel dead time of 475 ns
- Measurement of **energy and time** (Δt = 1.56 ns)
- Threshold for noise free measurement: 3-5 keV

Solid-state Time-Projection Chamber:

3D reconstruction of particle tracks

z

e⁻ and h⁺ drift described by $v_e = -\mu_e \times E(z)$ $v_h = \mu_h \times E(z)$ Charge carrier drift motion:

 $\mu_{e/h}$: Mobility of e⁻/h⁺

Electric field parametrization:

$$
\text{Si: } \vec{E}(z) = \frac{U_B}{d} \vec{e}_z + \frac{2U_{dep}}{d^2} \left(\frac{d}{2} - z\right) \vec{e}_z ;
$$
\n
$$
\text{CdTe: } \vec{E}(z) = \frac{U_B}{d} \vec{e}_z
$$

 U_B : Bias voltage; U_{dep} : Depletion voltage; d: Sensor thickness

\rightarrow Look-up table: $z(t_{\text{meas}},E_{\text{meas}})$

Bergmann et al. Eur. Phys. J. C (2017) 77: 421. <https://doi.org/10.1140/epjc/s10052-017-4993-4> Bergmann et al., Eur. Phys. J. C (2019) 79: 165. <https://doi.org/10.1140/epjc/s10052-019-6673-z>

Test beam measurement: 3D track reconstruction – 500 µm thick silicon

Test measurement: 3D track reconstruction – 2 mm CdTe

Chapter 1 Measurements with table-top experiments

Spatial and temporal coincidencing schemes

3D reconstruction: Application as a single-layer Compton Camera

- Temporal coincidence detection of compton electron and scattered photon
- Energy information for apex calculation through kinematics reconstruction
- Vector between interactions defines axis

3D reconstruction: Single-layer Compton Camera -

5 mm CZT Timepix3 (110 µm pitch) irradiated with a **²²Na** at different lateral displacement.

P Smolyanskiy *et al* 2024 *Phys. Scr.* **99** 015301

Source position 2

Timepix3 ASIC

Frequency

 $10⁵$

 10^4

Single-cluster group Double-cluster group Multi-cluster group

 $\mu = 1278 \text{ keV}$
 $\sigma = 48 \text{ keV}$

 $\mu = 1082 \text{ keV}$ σ = 42 keV

T. Michel, J. Durst, J. Jakubek, "X-ray polarimetry by means of Compton scattering in the sensor of a hybrid photon counting pixel detector," NIMA 603, Issue 3, pp 384-392 (2009) <https://doi.org/10.1016/j.nima.2009.02.032>

Timepix3 Compton polarimeter - principle

Differential cross section for Compton scattering off an electron is described by the Klein-Nishina formula:

$$
\frac{d\sigma}{d\Omega} = \frac{1}{2}r_0^2 \frac{E^2}{E_0^2} \left(\frac{E}{E_0} + \frac{E_0}{E} - 2 \cdot \sin^2\theta \left(\cos^2\phi\right)\right)
$$

Ф is the angle between the electric field vector of the incoming photon and the scattering plane

$$
M(\beta) = A \cos^2(\beta - \phi_0) + B
$$

\n
$$
\rightarrow \mu = \frac{A}{A+2B}
$$
 (modulation)
\nDegree of linear polarization $P = \frac{\mu}{\mu_{100}}$

Detected Compton-scatter pair

Timepix3 Compton polarimeter - principle

Precision measurement of the ²¹²Po and ²¹⁴Po half-life times with Timepix3

B. Bergmann, and J. Jelinek, "Measurement of the ²¹²Po, ²¹⁴Po and ²¹²Pb half-life times with Timepix3", *Eur. Phys. J. A* **58**, 106 (2022). <https://doi.org/10.1140/epja/s10050-022-00757-z>

The polonium decay signature

Measurements at the National Radiation Protection Institute (SURO)

B. Bergmann, and J. Jelinek, "Measurement of the ²¹²Po, ²¹⁴Po and ²¹²Pb half-life times with Timepix3", *Eur. Phys. J. A* **58**, 106 (2022). <https://doi.org/10.1140/epja/s10050-022-00757-z>

Delayed coincidence spectra - ²¹²Po and ²¹⁴Po

Measurement of the life time of excited states of ⁵⁷Fe

Energy spectrum and signals for half life time measurement Electron capture decay leaves the daughter atom in an

Chapter 2 Radiation field decomposition in low earth orbit

Electron & proton discrimination

Proton spectrum measurement

Radiation environment in LEO

Space Application of Timepix Radiation Monitor (SATRAM)

- **First Timepix in open space**
- Power consumption of **2.5 W**
- Total mass **380 g** (107 x 70 x 55 mm)
- Platform technology demonstrator

Proba-V

- Minisatellite (158 kg)
- Altitude \sim 820 km (LEO)
- 101.21 minutes orbit duration
- Inclination 98.6°
- Sun-synchronous
- Launched 7th March 2013

10 x times lower mass budget than space environment monitors with similar capabilities

<https://satram.utef.cvut.cz/>

SATRAM - Average dose rate 2015-2018 (mGy/h) - Orbit: 820 km

St. Gohl et al., "Study of the radiation fields in LEO with the Space Application of Timepix Radiation Monitor (SATRAM)", Advances in Space Research 63, Issue 5, pp. 1646-1660, (2019).

Electron and proton flux maps

e - fluxes 3 orders of magnitude larger than p⁺ fluxes

 \rightarrow Even small e⁻ misclassification distort p + flux measurement

Global electron flux distribution in 2015

 10^2 $\frac{\pi}{8}$ *St. Gohl et al., "Study of the radiation fields in LEO with the Space Application of Timepix Radiation Monitor (SATRAM)", Advances in Space Research 63, Issue 5, pp. 1646-1660 (2019)*.

 10

Proton energy spectrum reconstruction using dE/dX unfolding

 $R^{T}n(E) = \Phi(C)$ Solved by

Typical unfolding codes:

- Matrix inversion
- Richardson Lucy
- Gold deconvolution
- **Bayesian unfolding***

*G. D'Agostini, "A multidimensional unfolding method based on Bayes' theorem" Nucl. Inst. Meth. A **362**, 2-3, pp. 487-498 (1995) [https://doi.org/10.1016/0168-9002\(95\)00274-X](https://doi.org/10.1016/0168-9002(95)00274-X)

dE/dX unfolding: Response matrix

- Simulated in omnidirectional particle field for $e^-(E_e < 6 \text{ MeV})$ and **p**⁺ (E_p < 400 MeV)
- Methodology verification in monoenergetic proton beams

Resolution averaged over polar angles of 0, 45, 70 and 85 degrees

dE/dX unfolding: Application to measured SATRAM data

22,784 frames of 2 ms (**tmeas = ~46 s**) were found in the selected geographic region in the years 2014-2018.

The electron background was estimated by scaling the unfolded dE/dX spectrum from simulation to the amount of detected esignatures.

Bergmann et al. *Instruments* 2024, **8**(1), 17; <https://www.mdpi.com/2410-390X/8/1/17>

8 November 2024 Detector Seminar - CERN 31

Chapter 3 Radiation field decomposition for luminosity measurement in ATLAS

The value of pattern recognition for neutron-gamma discrimination and bunchsensitive luminosity measurement

Neutron detection with Timepix: ATLAS-Timepix3 device design

- ⁶LiF (89% Li enrichment): ⁶Li + n → α + ³H + 4.78 MeV
- **PE** (\sim 1 mm): recoil protons from elastic scattering
- **PE + Al** (80 µm): fast neutrons above 3.5 MeV
- **Free:** Background subtraction (non-neutron field and neutron interactions in silicon)

 $(\mu, \text{minimum ionizing light ions}, \ldots)$

Pattern recognition

Proper selection of neutron converters and application of pattern recognition allows for **reliable neutron-γ separation**

Neutron detection with Timepix:

Converter effect and γ-discrimination

8 November 2024 Detector Seminar - CERN 34

Neutron converter efficiencies

Converter efficiency:

$$
\varepsilon_i = \frac{N_i - \frac{A_i}{A_{Si}} N_{Si}}{\Phi_{source} t}
$$

i: converter region LiF, PE, PE+Al

Converter efficiency calibration was done in time-of-flight experiments at

- The **Los Alamos neutron Science Center** (LANSCE) neutrons 1-600 MeV
- **n_TOF at CERN:** neutrons meV 400 MeV

Neutron spectrum hardness assessment through comparison of signal in the PE regions

Timepix(3) in ATLAS

Timepix and Timepix3 **detector networks** were installed in the ATLAS experiment

- Study the radiation fields during and after collision periods
- Measurement of the luminosity

ATLAS environment Continuous measurement of the radiation level

 $8N$ s november 2022 σ Start time (UTC)

Activation analysis:

Equation to describe the growth and decay of induced radioactivity

$$
M_{act}^{i} = \sum_{k=1}^{n} M_{act}^{i-1,k} \times e^{-\lambda_k t} + \theta (M_{tot}^{i} - M_{act}^{i-1}) \times \sum_{k=1}^{n} (M_{tot}^{i} - M_{act}^{i-1,k}) \times Y_k \times (1 - e^{-\lambda_k t})
$$

Decay of atoms
activated before i-th
time bin
bin
(valid
only during collisions)

- λ decay constant, $\lambda = \ln(2)/t_{1/2}$; $t_{1/2}$ is the half-life time
- *Y^k* **activation yield; how many clusters do we have to measure to create on instable isotope** *k*
- *i* index of the time bin
- M_{tot} total count rate measured in the given Timepix3 time bin (normalized to unit time)
 M_{net} count rate caused by all activation products
- count rate caused by all activation products
- *t* time period between the end of (*i-1*)-th bin and the end of *i*-th bin
- $\theta(x)$ Heavyside-function

Activation analysis: Determination of the input half-life times

Activation analysis: Application of the iterative formula to the measured data

The count rate from activation at a specific point in time depends on:

- Previous collision periods
- Time from start/end of the collision period

Short half-lifes lead to systematic drift within a single run **Long half-lifes** will be seen in long-term studies \rightarrow baseline shift

The presented activation modelling is possible due to triggerless continuous measurement!

Thermal neutron signals in ATLAS

TPX07: Integral HETE frame (**8.33min**)

TPX04: Integral HETE frame (**1.67min**) measured on May 31, 2016

TPX09: Integral HETE frame (**16.67min**) measured on May 31, 2016

Thermal neutron fluxes in ATLAS

Luminosity measurement during 2018 *pp* collisions at √s=13 TeV using Timepix3 in ATLAS

(Relative) luminosity measurement with Timepix3 in ATLAS

Luminosity measurement through the counting of particle traces (clusters) left in Timepix3.

$$
\mathcal{L}_{\text{Timepix3}} = C \frac{N_{\text{clusters/TN}}}{t}
$$

Scaling factor *C* determined by comparison of Timepix3 and LUCID $_{c12}$ run integrated luminosity during a anchor run.

Luminosity measurement Evaluation of the short-term precision

Relative devitation of Timepix3 compared with LUCID $_{c12}$ within a Fill (lumi block by lumi block)

- Cluster counting with good agreement after activation subtraction
- Neutron counting not affected by activation but lower statistical precision

https://<mark>/doi.org/10024140/epjc/s10052-023-11747-w</mark> (c) (a) Reference data are from **ATLAS**, see also: Aad, G., Abbott, B., Abeling, K. *et al.* Luminosity determination in *pp* collisions at √s=13 TeV using the ATLAS detector at the LHC. *Eur. Phys. J. C* **83**, 982 (2023).

Activation subtracted cluster counting

Timepix3 luminosity measurement: Long-term stability

Reference data are from **ATLAS**, see also: Aad, G., Abbott, B., Abeling, K. *et al.* Luminosity determination in *pp* collisions at √s=13 TeV using the ATLAS detector at the LHC. *Eur. Phys. J. C* **83**, 982 (2023). <https://doi.org/10.1140/epjc/s10052-023-11747-w>

Timepix3 fill-by-fill luminosity measurement is consistent with other luminometers.

Luminosity fraction

Resolving the bunch structure for luminosity measurement with BCID

Resolving bunches with ATLAS-Timepix3

Timepix3 detectors synchronized with LHC orbit clock allows to resolve bunch slots separated by 25 ns.

Time structure consists of **trains** of **filled** slots interruptd by empty slots

Decomposiong the temporal response Isolated bunch

Decomposiong the temporal response Isolated bunch - Quantitative

Delayed-particle signal present a challenge for proper assignment of particle counts to BCID

 \rightarrow Find the particle signature with best peakto-tail ratio

Bunch-by-bunch luminosity:

Time spectrum decomposition for luminosity determination

Bunch -by -bunch luminosity: Comparison with other lumino meters

Reference data are from **ATLAS**, see also: Aad, G., Abbott, B., Abeling, K. *et al.* "Luminosity determination in *pp* collisions at √s=13 TeV using the ATLAS detector at the LHC." *Eur. Phys. J. C* **83**, 982 (2023).

[https://doi.org/10.1140/epjc/s10052](https://doi.org/10.1140/epjc/s10052-023-11747-w) -023 - 11747 - w

Overall good agreement both with LUCID and Track Counting \rightarrow Precision is statistics limited

Chapter 4 Minimum ionizing particle tracking for interaction point length reconstruction

Timepix3 within MoEDAL

Installation of 2 Timepix3 detectors in MoEDAL in **September 2018**. Timepix3 are placed at 1.1 m distance to IP8 with a relatively unobstructed view

Continuous quasi dead-time free measurement (in real time) keeping a permanent record of **all particle traces**

- Tracking and identification of **all** particles
- Online outlier detection to search for exotics (highly ionizing events)

Timepix3 in MoEDAL:

Time-resolved measurement of the directionality map *Pb-Pb* collision period

Timepix3 at MoEDAL:

The origin of the secondary peaks

- Two Fills were found where the secondary peaks were not present. These coincide with Fills where the Velo detectors were retracted from the beam pipe
- The time after beam alignment that the peaks appear corresponds approximately to the difference between beam duration and the time the Velo detectors are inserted.
- \rightarrow Peaks are due to scattering in the Velo Detectors

Even with small area detector, changes of the radiation field characteristics, e.g., induced by changed material composition along the beam line are obervable.

59 https://lhcb-outreach.web.cern.ch/detector/vertex-locator-velo/

Timepix3 at MoEDAL: Beam spot reconstruction

Determine the angular spread of the particles along the major axis:

- Fit slices along *ϴ* with double gaussians to get $\phi(\theta)$
- Evaluate the projection of the integral of the central gaussian along the spot axis

Timepix3 at MoEDAL: Beam spot reconstruction

 $\sigma_{\text{max}}^2 = \sigma_{\text{det}}^2 + \sigma_{\text{scat}}^2 + \sigma_{\text{IP}}^2$ $\rightarrow \sigma_{IP} = \sqrt{\sigma_{meas.}^2 - \sigma_{det.}^2}$ $\frac{1}{2}$ $\sigma_{det.} = (1.5 \pm 0.1)$ deg. Extracted from simulation and test beam $\sigma_{scat.} = \frac{0.013 \text{ GeV}}{8n}$ $\ln(\frac{x}{y})$ X_0) $\left[1 + 0.038 \ln \left(\frac{x}{y}\right)\right]$ $\left[\frac{x}{X_0}\right]$ << σ_{det} . x z ydet. L $\rightarrow \sigma_{Z} =$ $(x_{det.}^2 + y_{det.}^2 + z^2) \times \sqrt{x_{det.}^2 + z^2}$ y_{det.} z σ_{IP}

Timepix3 at MoEDAL: Fill-by-fill variation of the interaction point size

29 fills during ion physics in 2018

- The measured beam spot size per fill shows two distinct sizes
- Change of size coincides with reducing the bunch spacing

Australia, May 2019, pp. 2258-2261. doi:10.18429/JACoW-IPAC2019-WEYYPLM2

MAPP

MoEDAL

medipix Acknowledgements

S. Pospisil, S. Gohl, J. Jelinek, D. Garvey, P. Smolyanskiy, P. Burian, L. Javora, P. Manek, M. Campbell, E. Heijne, C. Granja, A. Owens, E. Bosne, J. Pinfold, R. Soluk, M. Suk, M. Raymond, M. Ciapetti, ...

Presented results would not be possible without the Medipix collaborations.

The work has profited from funding by the Czech Science Foundation Junior Star Grant with No. GM23-04869M.

Conclusion

- In laboratory table-top experiments, half-life time measurement of Poisotopes and the excited levels of Fe-57 were done demonstrative reliable measurement of **decay times down to 8 ns**
- **Electron and proton** separation and **proton spectrum measurement** were shown for the LEO space radiation environment.
- The power of pattern recognition was outline by the example of **gammaneutron discrimination** and decomposition of the **single-bunch response** in the ATLAS radiation environment.
- Precise **particle tracking** was used for determination of the interaction point length within the MoEDAL experiment at IP8 during PbPb physics in 2018. Changes in the charged particle component of the radiation field were observed during insertion of Velo.

Outlook

Timepix4 @ CERN SPS heavy ion test beam

Heavy ion tracks measured with Timepix4 Courtesy of Petr Burian

Thank vou very mucl Therm you set y the for your attention!

Timepix4 is available:

- 200 ps time binning
- 350 Mhits/s (8 x improvement)
- 7 cm² area (3.5 x improvement)

Working principle: Modes of operation - Timepix

Timepix pixel processing:

t

Working principle:

Timepix3

• **Simultaneous** measurement of ToT and ToA • Local oscillator creates **640 MHz** clock for precise time stampingTHL (threshold level) $t (ns)$ f ToA \parallel 640 MHz 40 MHz ToT 8 November 2024 TOA CERN Detector EP Seminar 69

• **Data-driven** readout with continuously running

40 MHz base clock

Working principle: e - , photons, low energy α-particles

X- and $γ$ -ray photons are detected through conversion to electrons \rightarrow Difficult to separate

Highly localized charge deposition

- \rightarrow Spread due to repulsion and diffusion during drift motion
- → **Subpixel spatial resolution** (dx \sim 400 nm)

B. Bergmann *et al* 2022 *JINST* **17** C01025

Ion resolving and particle separation capability Relativistic fragmented ion beam

Mixed field of relativistic ion fragments created by Pb beam on target.

Observed peaks relate to different ion charge:

$$
\frac{dE}{dX} = \frac{dE_{Z=1}}{dX}Z^2
$$

Resolving power up to Z=11

10

0

10

20

30

40

50

15

20

25

30

35

Energy (MeV)

40

 $10⁴$

Frequency

10

5

10

70

Energy (MeV)

60

Instruments for measurements in LEO

Next Generation Radiation Monitor (NGRM)

- Mass \sim 1 kg
- Consumption ~1-2 W

EPT (Energetic Particle Telescope)

- Mass: **4.6 kg**
- Consumption: 5.6 W

ICARE-NG:

- Mass: **2.4 kg**
- Consumption: 3 W

Timepix devices in LEO

- Single-layer particle discrimination
- Small dimensions and low mass
- Large field of view

Space flights

- REM on ISS (since 2012: different versions; MiniPIX TPX3 by Advacam deployed in 2021)
- SATRAM on Proba-V (launch in 2013, 820 km)
- LUCID-Timepix (2014-2017, 635 km)
- VZLUSAT-1 (launch in 2017, 510 km)
- RISESAT (launch in 2019, 500 km)
- VZLUSAT-2 (launch in 2022, 500 km, CdTe 2 mm)
- **HardPix** SWIMMR* project (launched in 2023)

*Space Weather Instrumentation, Measurement, Modelling and Risk

Space radiation in LEO measured by TPX3, integrated frame, 200 s, energy display

Comparison with other radiation detectors in LEO

SATRAM vs. EPT (Energetic Particle Telescope)*) : Electron fluxes

S. Gohl, B. Bergmann, M. Kaplan et al., "Measurement of electron fluxes in a Low Earth Orbit with SATRAM and comparison to EPT data", *Adv. Space Res.*, https://doi.org/10.1016/j.asr.2023.05.033

*) EPT and SATRAM are both on Proba-V.

60 seconds integration time

8 November 2024

SATRAM vs. ICARE-NG: Proton fluxes

SATRAM vs. EPT: Proton spectrum measurement

EPT data digitized from:

G. López Rosson, V. Pierrard, *"Analysis of proton and electron spectra observed by EPT/PROBA-V in the South Atlantic Anomaly",* Adv. Space Res. **60**, Issue 4, pp. 796-805 (2017). <https://doi.org/10.1016/j.asr.2017.03.022>

SATRAM data agree within EPT data points on a one sigma level

Near-future missions (of our Timepix radiation detectors)

- **SWIMMR2** D-Orbit satellite orbit >1000 km launch in October 2024
- 2 modules outside of the Lunar Gateway as a part of the ESA **ERSA** (European Radiation Sensors Array) – **2024**
- **HEKI** study radiation field influence on a superconducting magnet by Robinson-Paihau research institute in New Zealand using 2x HardPix detectors. Launch to ISS/Nanoracks in **2024**.
- **Cassini** European Commission In-orbit demonstration mission. Managed by ESA and provided by ISISPACE 6U Cubesat - Launch **2025**
- Equipped with neutron converters selected in the MoonPool ESA call for ideas → **neutron detection for water mapping**

HardPix was developed with ESA projects

B. Bergmann, S. Pospisil, I. Caicedo, J. Kierstead, H. Takai and E. Frojdh, "Ionizing Energy Depositions After Fast Neutron Interactions in Silicon," in *IEEE Transactions on Nuclear Science*, vol. 63, no. 4, pp. 2372-2378, Aug. 2016, doi: 10.1109/TNS.2016.2574961

Interactions of fast neutrons in silicon

Time-of-Flight technique: 1-600 MeV neutrons interacting in silicon

Neutron interactions in silicon show a large variety of signatures resembling the different ways neutrons interact in the sensor

- Small clusters similar to photon interactions
- Large clusters with high energy depositions like stopped charged particles
- Tracks similar to penetrating particles
- →**Can we decompose this signatures?**

Low energy X - and γ -rays, low energy electrons

 (1) Dot

Neutron energy deposition spectra

Interpretation: Edges from backscattering

Maximal energy transfer to the recoil silicon:

n, T_n
\nSi
\n
$$
T_{Si,max} = \frac{4M_{Si}m_n}{(M_{Si} + m_n)^2}T_n
$$
\n
$$
T_{Si,max} = 0.133 \times T_n
$$

Calculation: $E_n = 3.8$ MeV $\rightarrow T_{S i, max} = 505$ keV

<u>Measured:</u> $T_{Si,max} = 379 keV$

$$
f_{meas,IEL} = \frac{E_{edge}}{T_{Si, max}} = \frac{E_{edge}}{0.133 \cdot T_n}
$$

B. Bergmann, S. Pospisil, I. Caicedo, J. Kierstead, H. Takai and E. Frojdh, "Ionizing Energy Depositions After Fast Neutron Interactions in Silicon," in *IEEE Transactions on Nuclear Science*, vol. 63, no. 4, pp. 2372-2378, Aug. 2016, doi: 10.1109/TNS.2016.2574961

Partition function of IEL and NIEL

$$
f_{meas,IEL} = \frac{E_{edge}}{T_{Si, max}} = \frac{E_{edge}}{0.133 \cdot T_n}
$$

Measurement: A. R. Sattler. *Phys. Rev.*, 138:A1815- 1821, Jun 1965.

Theoretical predictions:

$$
f_{IEL} = \frac{k \times g(\varepsilon)}{1 + k \times g(\varepsilon)}
$$

M. T. Robinson and I. M. Torrens. *Phys. Rev. B*, 9: 5008-5024, Jun 1974. $k = 0.1462$, $\varepsilon = 1.014 \times 10^{-2} \times Z^{-7/3} \times E$, $g(\varepsilon) = 3.4008 \times \varepsilon^{1/6} + 0.40244 \times \varepsilon^{3/4} + \varepsilon$

A. Akkerman and J. Barak. *IEEE Transactions on Nuclear Science*, 53(6): 3667–3674, Dec 2006. $g(\varepsilon) = 0.90656 \times \varepsilon^{1/6} + 1.6812 \times \varepsilon^{3/4} + 0.7442 \varepsilon$