Updates on the Reverse Phase Operation

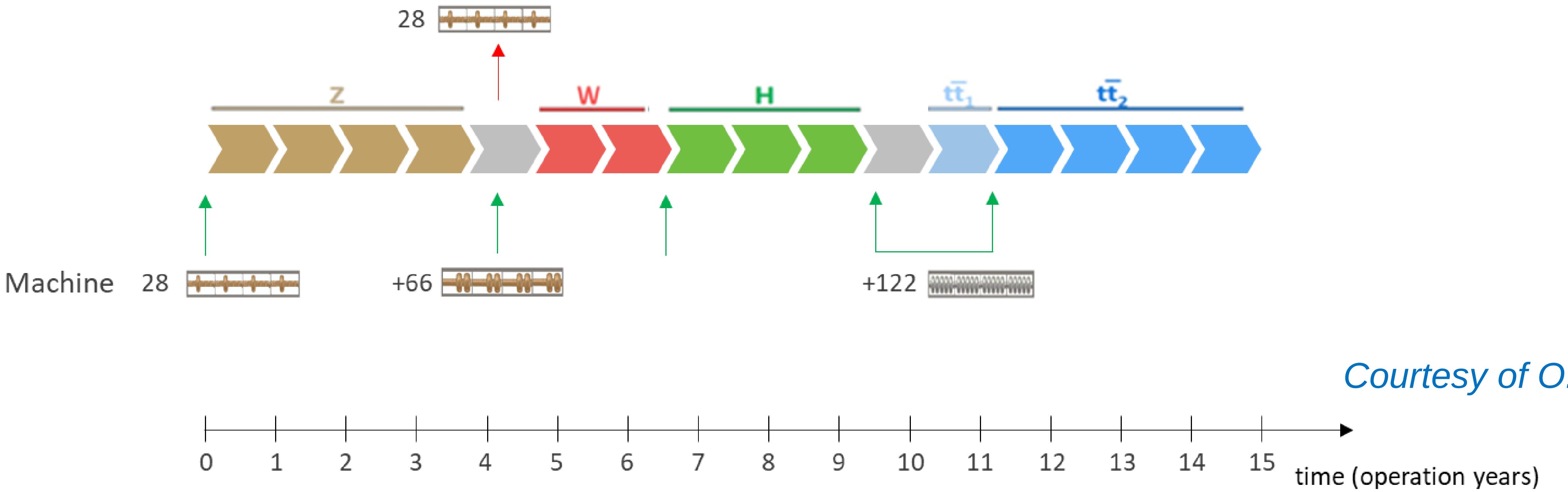
Ivan Karpov and Franck Peauger for FCC SRF WP1
Acknowledgments:
Xavier Buffat, Yann Dutheil, Giorgia Favia, Jiquan Guo (JLAB), Mauro Migliorati, Katsunobu Oide, Jorg Wenninger, Frank Zimmermann, Mikhail Zobov

196th Accelerator Design Meeting & 67th FCCIS WP2.2 Meeting, 06.11.2024

Baseline RF system configuration

	Energy (GeV)	Current (mA)	RF voltage (GV)
Z	45.6	1283	0.079
W	80	135	1.05
H	120	26.7	2.1
$t\bar{t}$	182.5	5	11.67

K. Oide, 29.05.2024



Courtesy of O. Brunner

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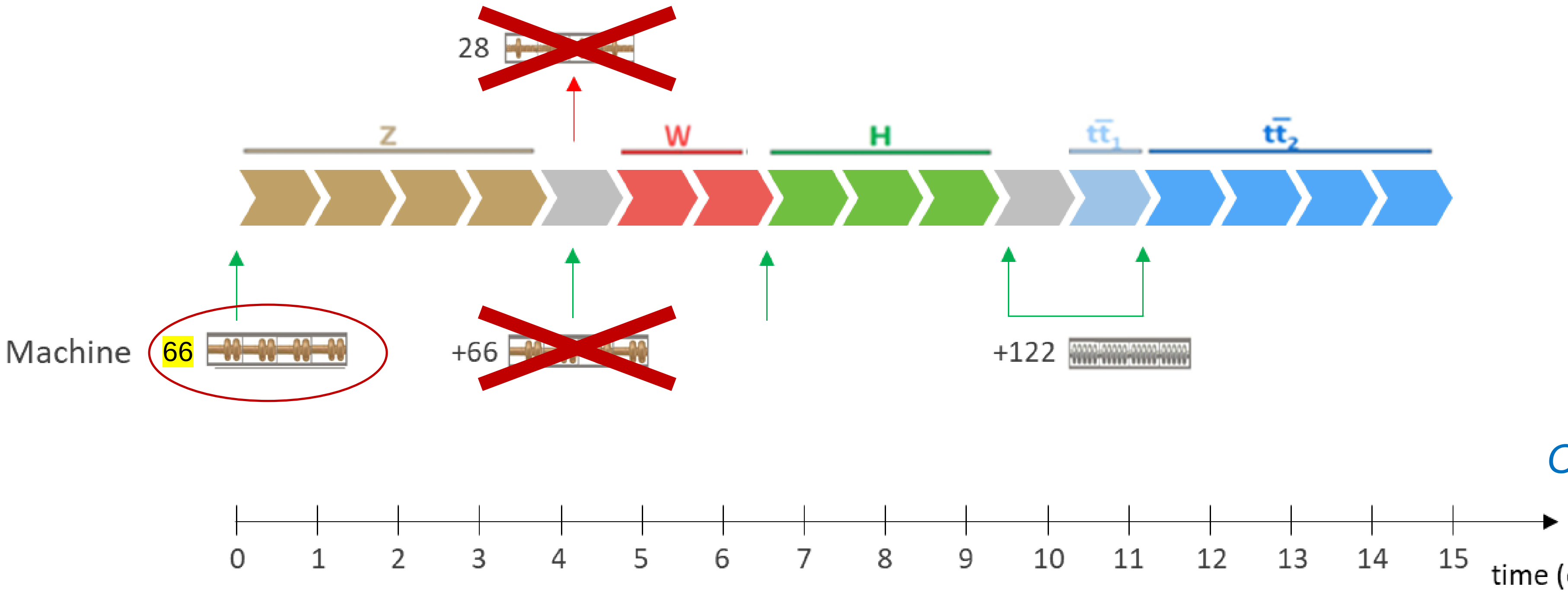


Courtesy of O. Brunner

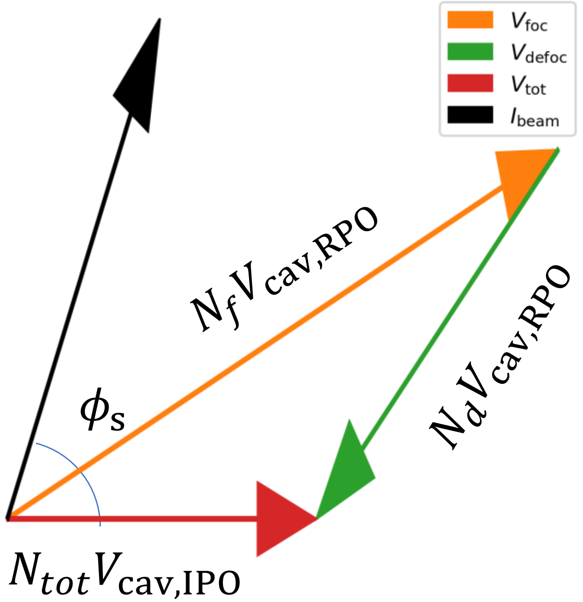
Reverse phase operation

Reverse phase operation (RPO) mode allows for increasing RF cavity voltage having optimal static beam loading compensation *(Y. Morita et al., SRF, 2009)*

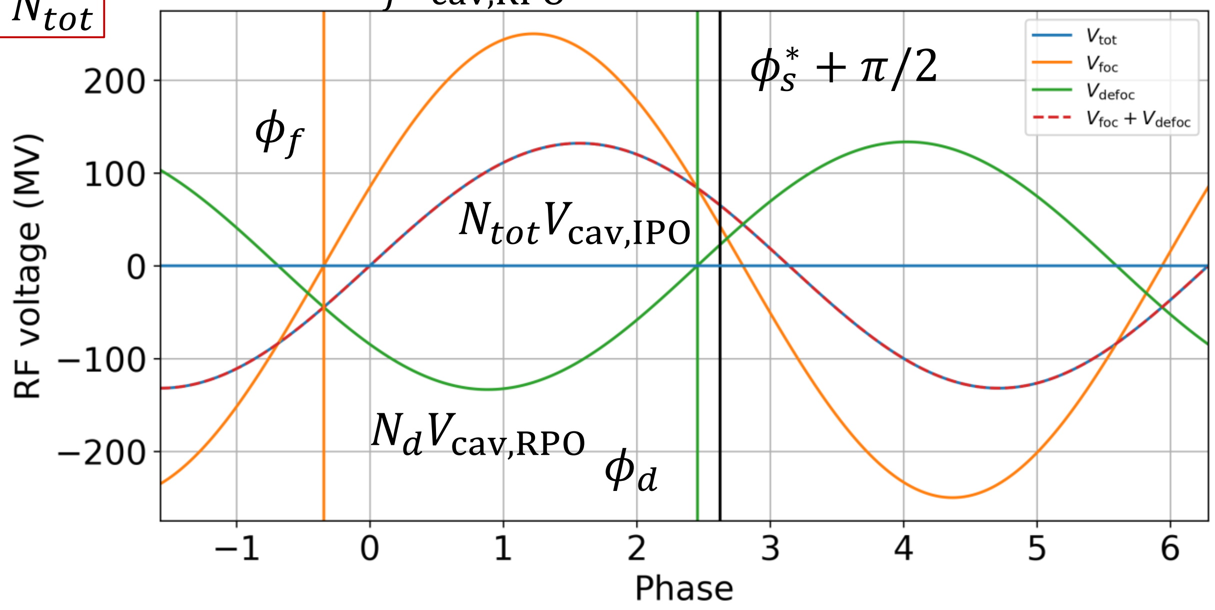
- Experimentally verified with high beam loading in KEKB *(Y. Morita et al., IPAC, 2010)*
- Baseline solution for EIC ESR *(e.g., J. Guo et al., IPAC, 2022)*

Phasors

$N_f + N_d = N_{tot}$



RF waves

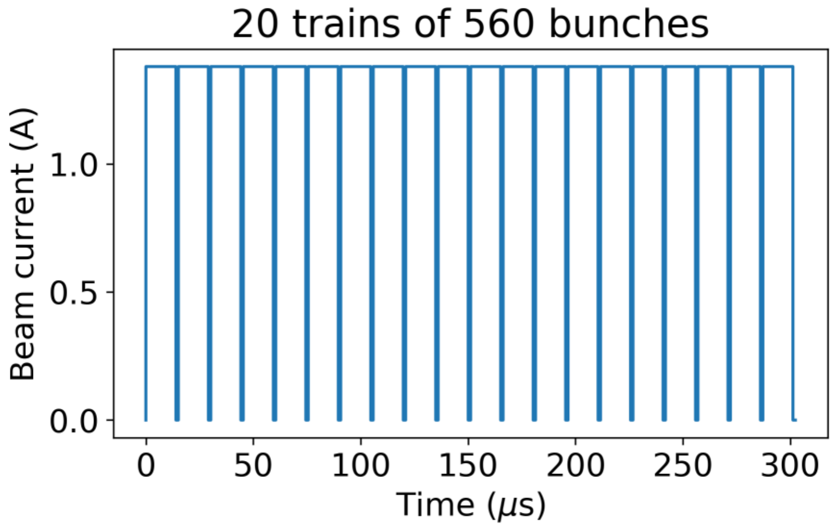


*Electron convention for synchronous phase: $\phi_{s,proton} = \frac{\pi}{2} + \phi_{s,electron}$

V_{tot} Z (MV)	V_{tot} W (MV)	V_{tot} ZH (MV)	V_{cav} (MV)	Q_L
88	1049	2099	7.95	9.21e5

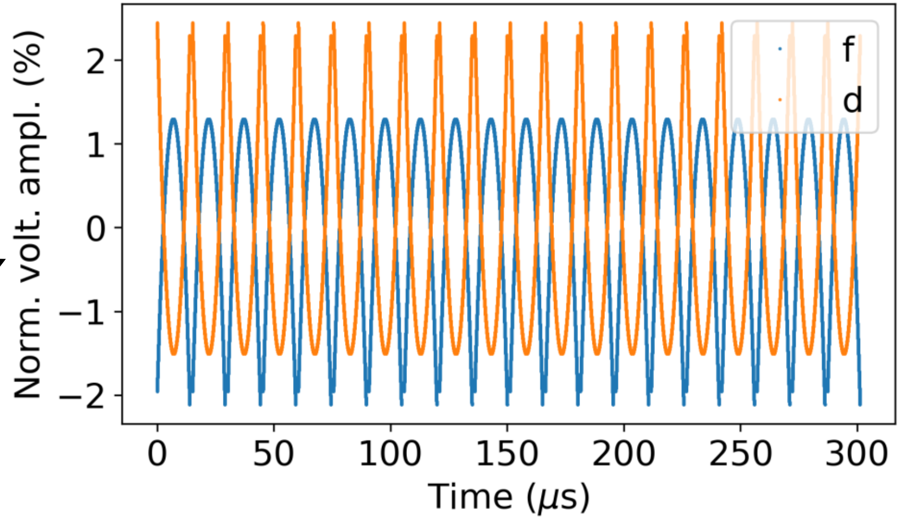
Transient beam loading

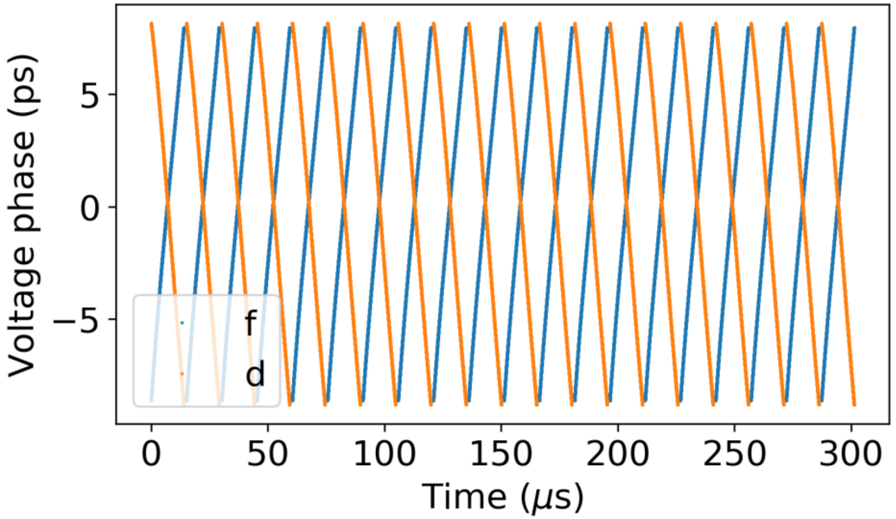
	N_f	N_d	V_{tot} **Z** (MV)	V_{cav} (MV)	Q_L
Current	71	61	88	7.95	9.21e5



$$\frac{a_{Vf,d}}{a_b}$$

$$\frac{\phi_{f,d}}{a_b}$$

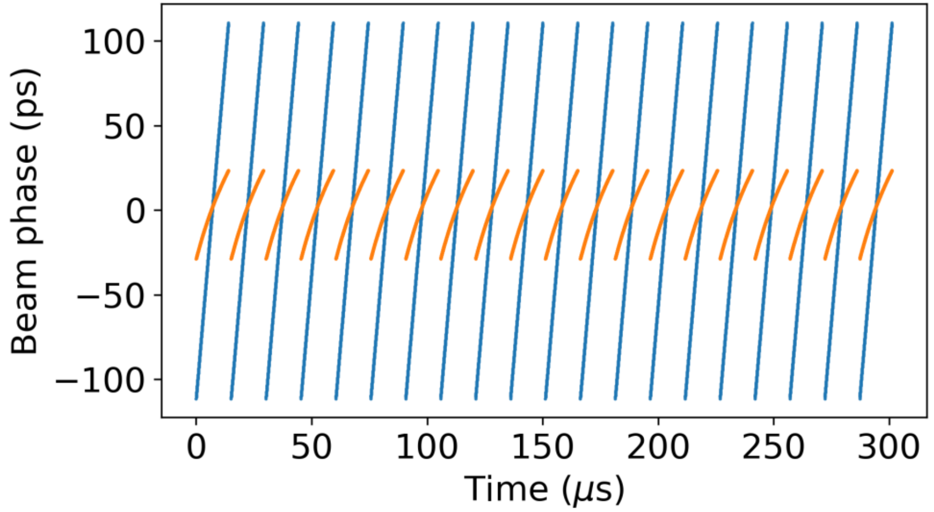




Gaps in machine filling will result in modulation RF parameters (voltage amplitude and phase) and therefore beam parameters (synchronous phase, synchrotron tune, and bunch length)

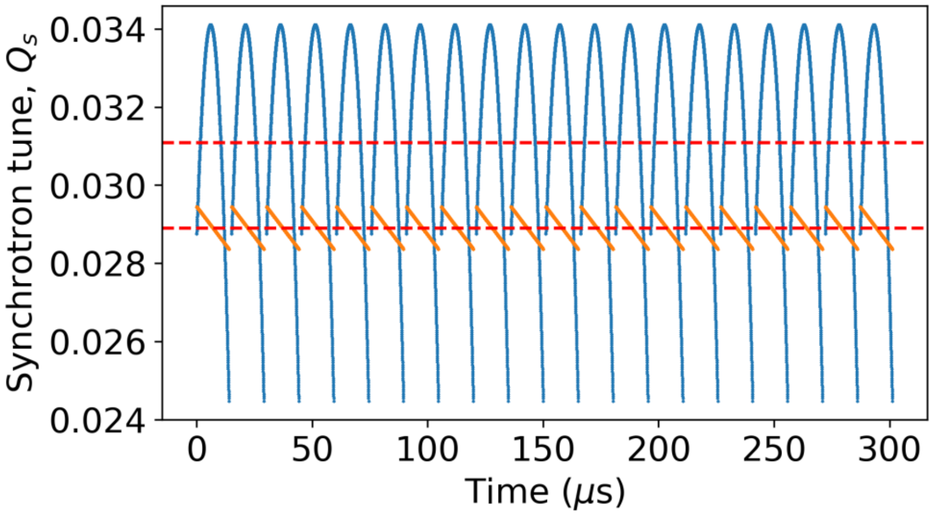
Bunch-by-bunch spread of beam parameters

$\frac{\phi_b}{a_b}$



σ_z ~ 50 ps

-10/+21 %

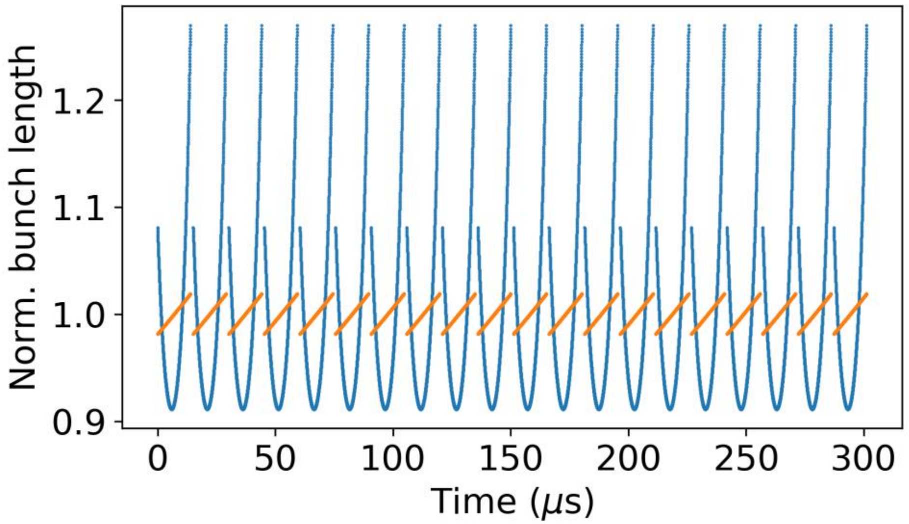


For identical rings, transients can be compensated by matching abort gaps (e.g., in PEPII, LHC,…)

Imbalance of charge results in different detuning for electron and positron beams
→ Slightly different transients (most critical during filling)

Peak-to-peak spread of ~30% in synchrotron tune and bunch length can have a significant impact on beam stability
→ We lose a factor of 15 wrt to 1-cell RF system

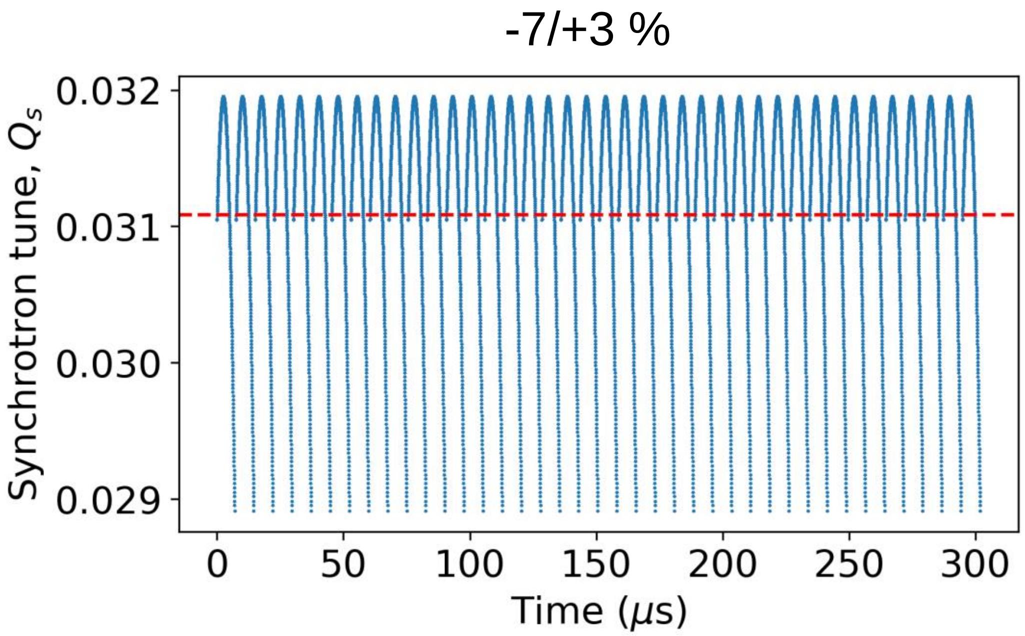
-9/+27 %



Possible scenarios

Peak-to-peak beam phase spread ∝ $\Delta\omega_{\mathrm{opt}}\tau_{\mathrm{gap}}N_{\mathrm{tot}}/(N_f - N_d)$

1. New filling scheme (e.g., 40 trains of 280 bunches)
   → Spread is reduced by a factor of ~3
   → Gaps become twice shorted (~600 ns) – potential significant impact on injection and extraction systems



2. Higher total RF voltage for Z?

Optimal quality factor $$Q_{L,\mathrm{opt}} = \frac{V_{\mathrm{cav}}^2 N_{\mathrm{tot}}}{2P_{\mathrm{SR}}(R/Q)}$$

Since $Q_{L,\mathrm{opt}}$ should be the same for Z, W, and ZH, V_{cav} cannot be changed

→ Optimal detuning is also unchanged $$\Delta\omega_{\mathrm{opt}} = -\frac{\omega_{\mathrm{rf}}(R/Q)|F_b|I_{b,dc}}{2V_{\mathrm{cav}}}\sqrt{1-\frac{U_0^2}{e^2V_{\mathrm{cav}}^2N_{\mathrm{tot}}^2}}$$

The only knob is to change $N_f - N_d$ by changing V_{tot}: $$V_{\mathrm{cav}} = \frac{V_{\mathrm{tot}}}{N_{\mathrm{tot}}}\sqrt{\frac{U_0^2}{e^2V_{\mathrm{tot}}^2} + \left(1-\frac{U_0^2}{e^2V_{\mathrm{tot}}^2}\right)\frac{N_{\mathrm{tot}}^2}{(N_f - N_d)^2}}$$

Significantly higher RF voltage

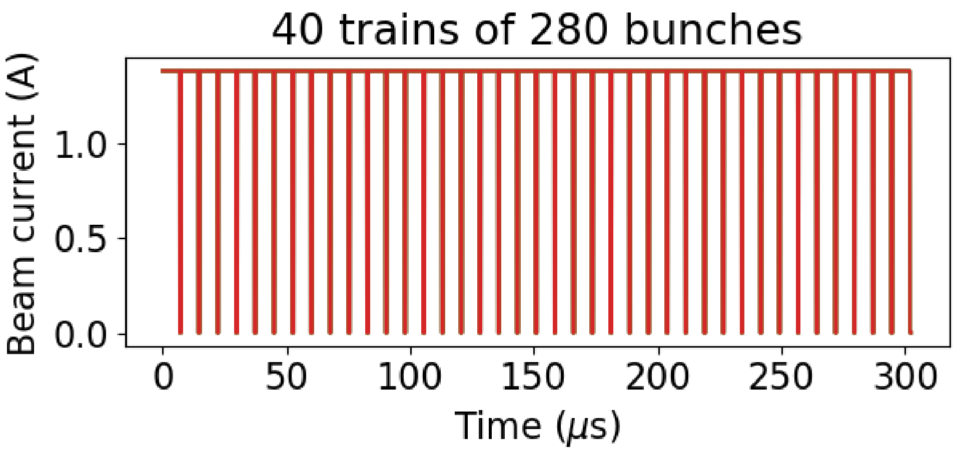
	N_f	N_d	V_{tot} **Z** (MV)	V_{cav} (MV)	Q_L
Current	71	61	88	7.95	9.21e5
Option 2	78	54	195	7.95	9.21e5

Higher RF voltage reduces parameter spread to ~5%
Peak-to-peak Qs spread 2.5e-3
→ Did not work because of significant reduction of beam lifetime *(K. Oide, 09.10.2024)*

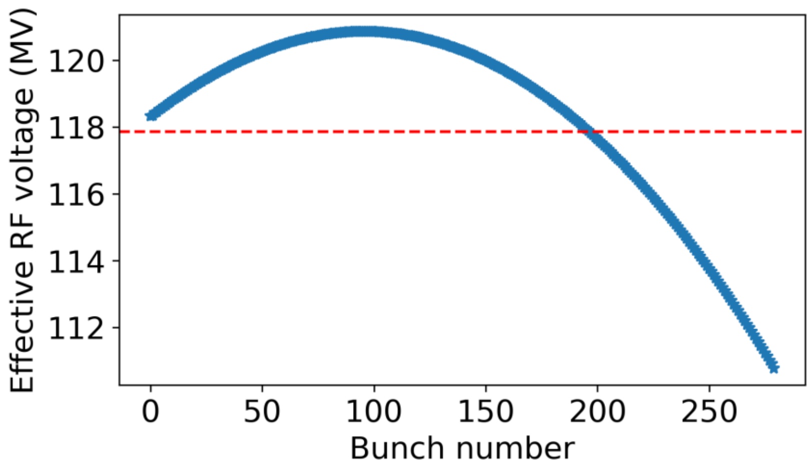
Options with a smaller abort gap

Preliminary studies showed that 600 ns abort gap duration is feasible *(G. Favia et al.)*

A new filling scheme is 40 trains of 280 bunches spaced by 25 ns



Option #	V_{nom} (MV)	V_{min} (MV)	V_{max} (MV)	$Q_{s,nom}$	$Q_{s,min}$	$Q_{s,max}$	$\Delta Q_s / Q_s$
Baseline	88.48	78.86	92.47	0.0311	0.0289	0.0319	10%
1	103.00	94.83	106.43	0.0341	0.0324	0.0347	7%
2	117.86	110.77	120.86	0.0368	0.0355	0.0373	5%
3	132.96	126.71	135.61	0.0394	0.0383	0.0398	4%



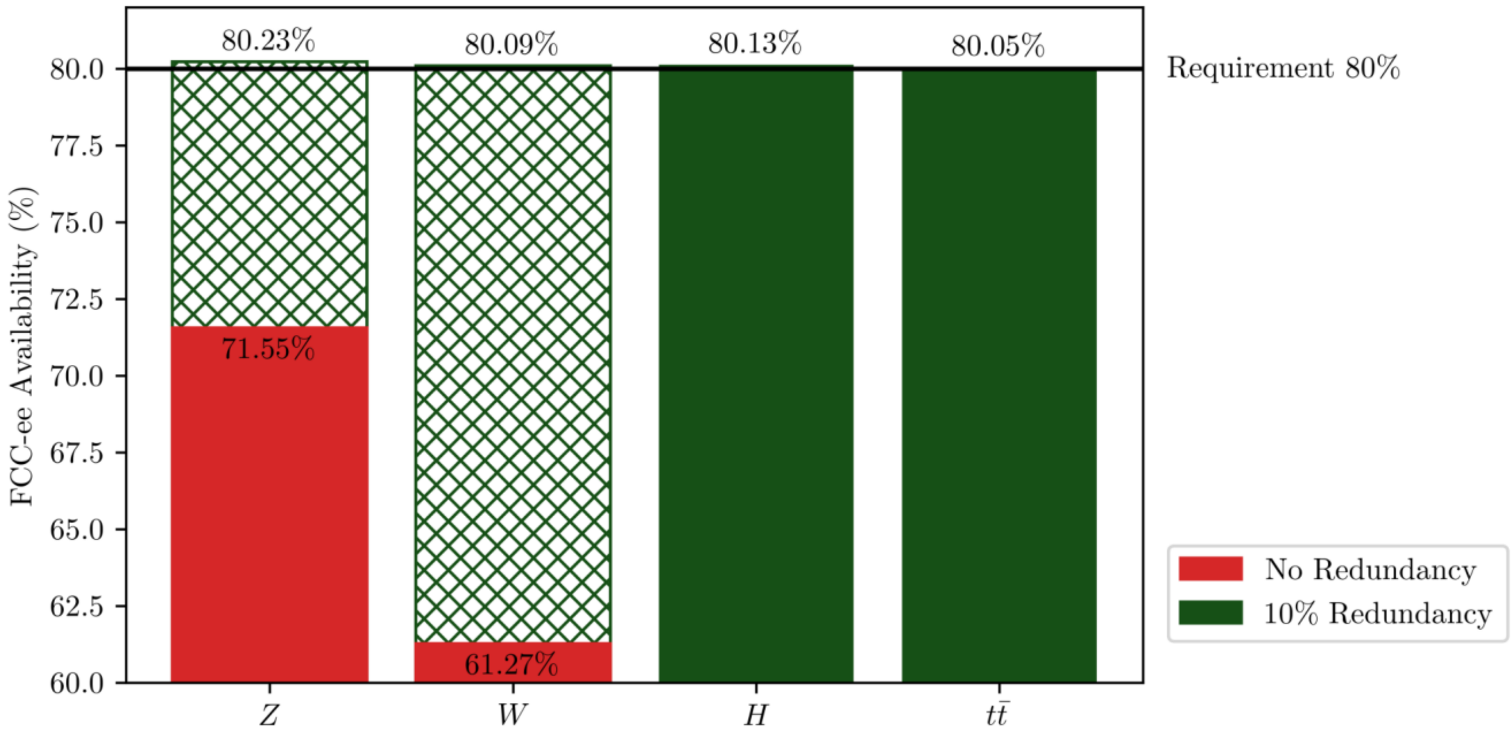
Option 2 looks promising for lifetime *(see slides of K. Oide)*, while beam stability aspects are not fully conclusive yet *(see slides of X. Buffat)*

Items to be addressed

- Coupled-bunch instabilities
- Higher-order-mode power losses
- Transient beam loading
- Availability aspects:
  - Reverse phasing with tripped cavities
  - Beam-induced voltage
  - Coupled bunch instabilities due to fundamental mode without feedback
- Sensitivity of RPO on cavity parameters (e.g., spread of Q_L, input power, …)
- Impact on FCC-ee booster with all cavities needed for H being installed from the beginning
- Possibility of powering several cavities with a single RF source

Discussed earlier
Discuss today
Ongoing studies
Not started

Availability challenges



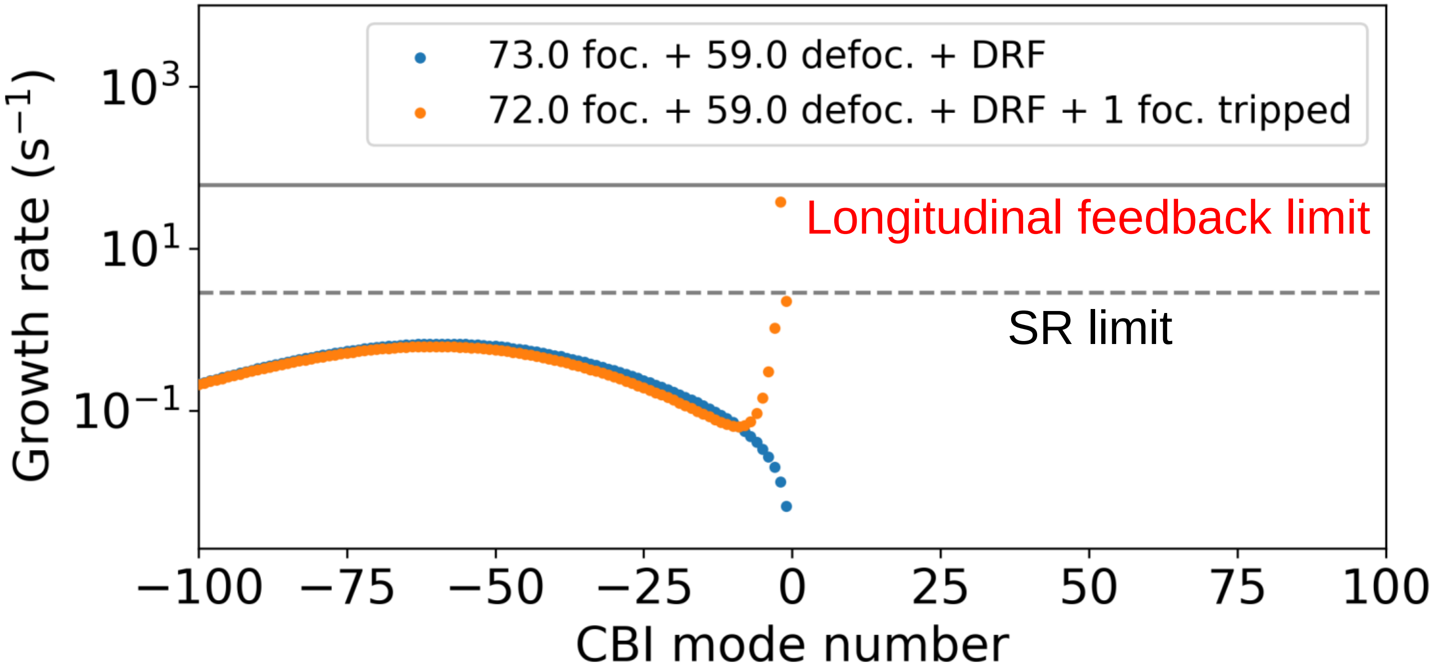
Availability goals require 10% (minimum 4%) redundancy of the RF system *(J. Heron, FCC Week 2024)*

Critical questions for Z mode with RPO:

- Coupled-bunch instability due to fundamental impedance
- Cavity damage due to strong beam-induced fields
- Missing RF voltage

Impact of fundamental impedance

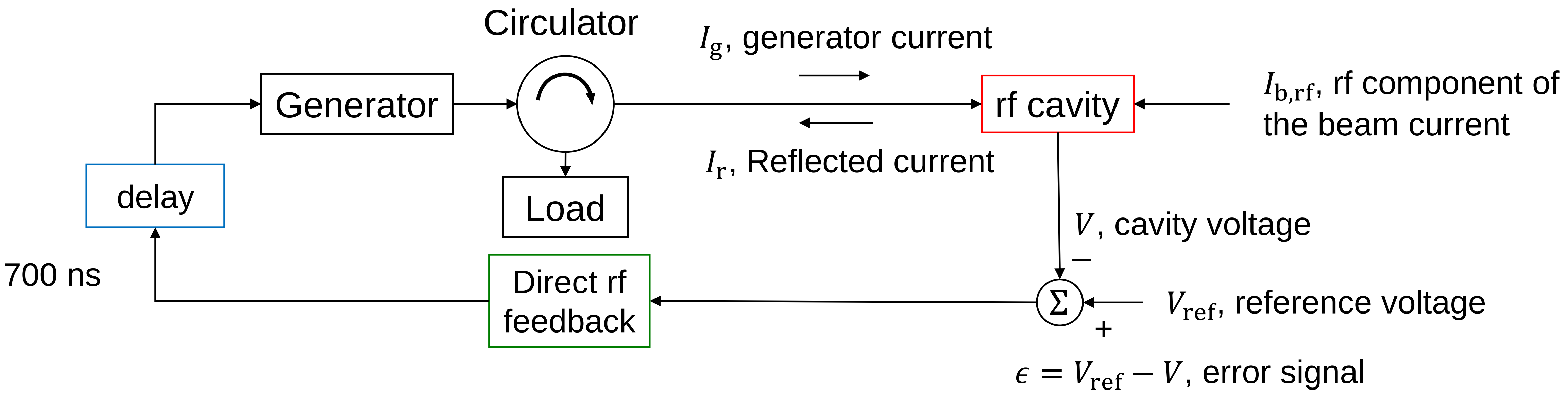
Instability growth rates with 1 tripped focusing cavity



Coupled-bunch instability due to fundamental mode could be suppressed by a longitudinal feedback system (main RF system as kicker) with damping time of $2T_s$ *(see, D. Teytelman, FCC week, 2019),* but RF power requirements need to be evaluated

→ We are at the limit with one missing cavity

Simplified beam-cavity interaction model

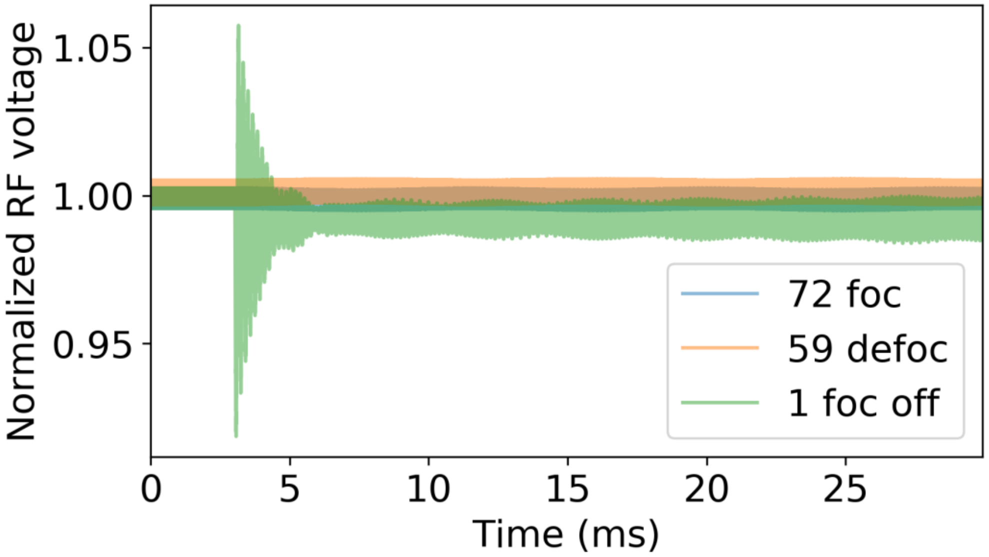


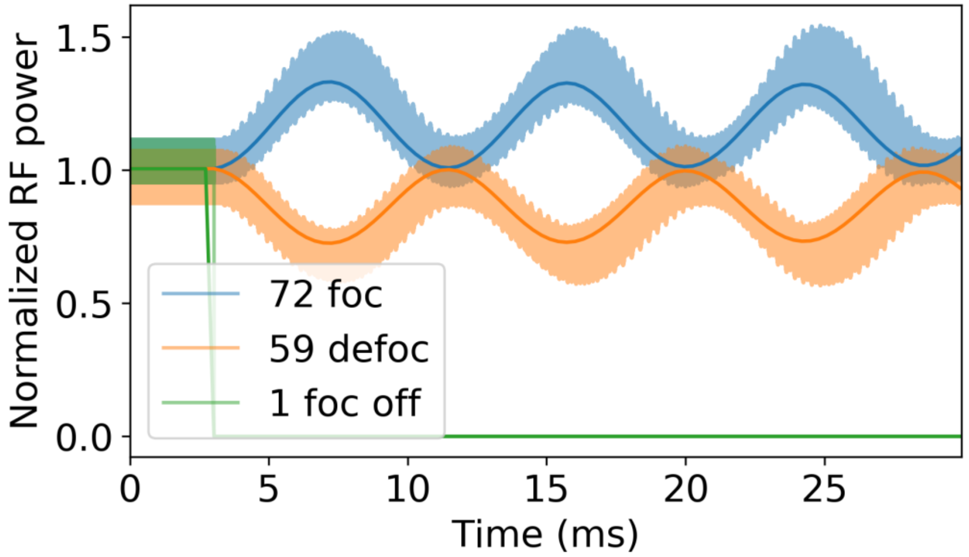
Coupled differential equations are solved for three groups:
- Focusing (N_f cavities)
- Defocusing (N_d cavities)
- Tripped (N_{off} cavities)

Combined with longitudinal equations of motion for one particle per bunch

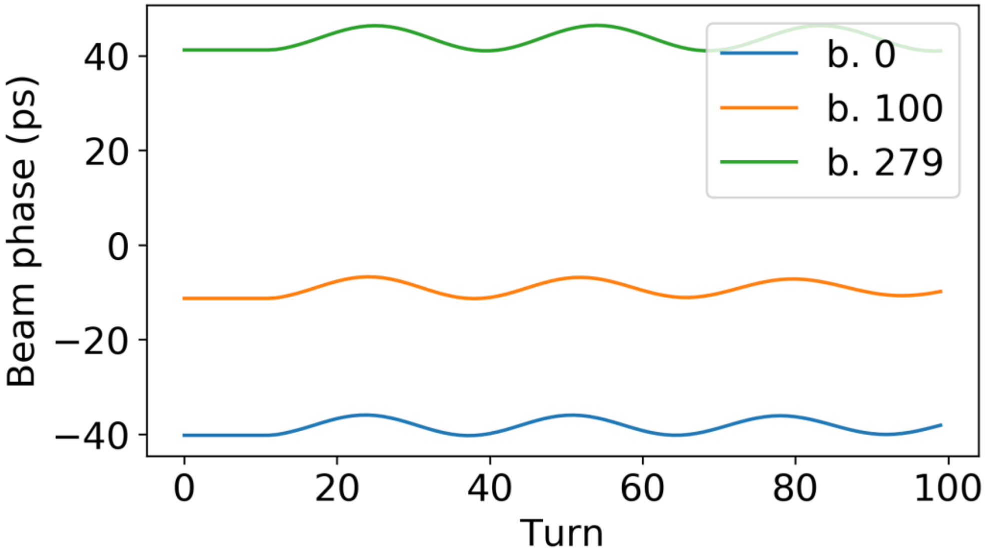
Longitudinal damper is not implemented yet

Trip of focusing cavity





- Short RF voltage transients ~6%
- Peak power of other cavities is modulated at synchrotron frequency (mean <33%, peak <53%)
- Initial bunch oscillation amplitude is ~10% of rms bunch length
- Beam is unstable without longitudinal damper due to uncompensated impedance



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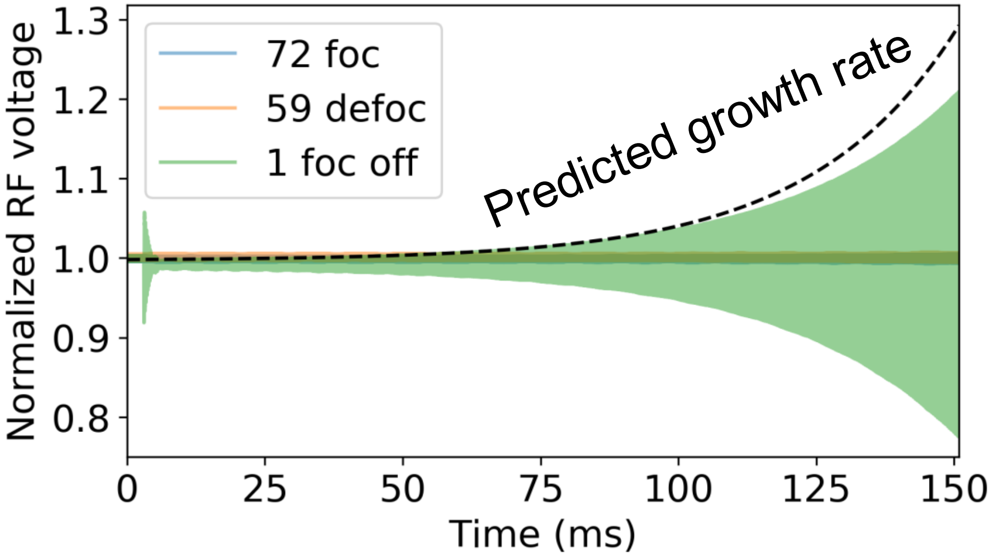

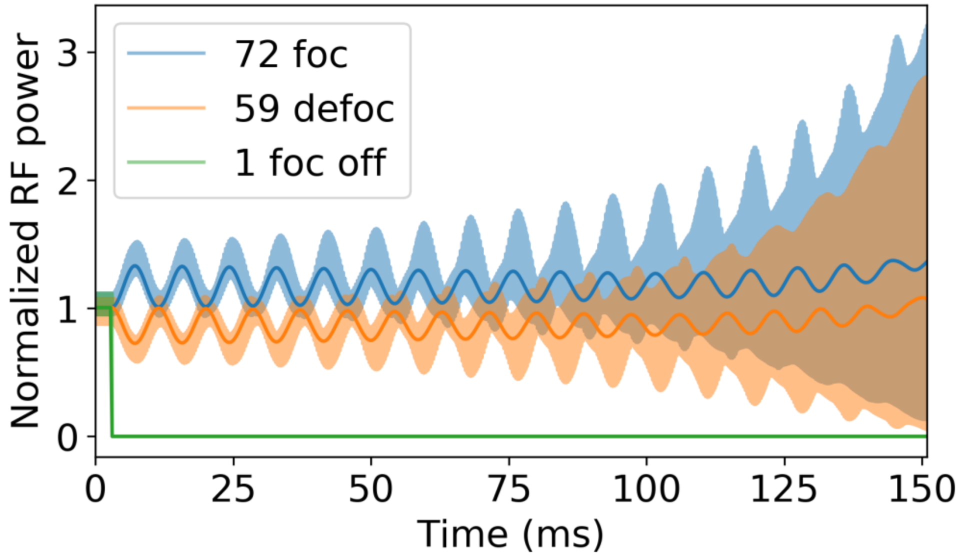

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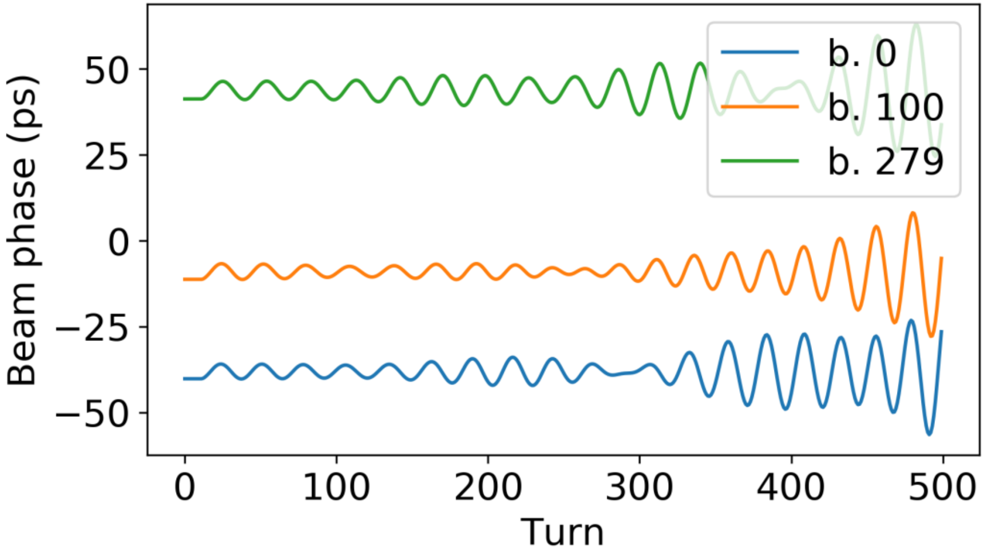
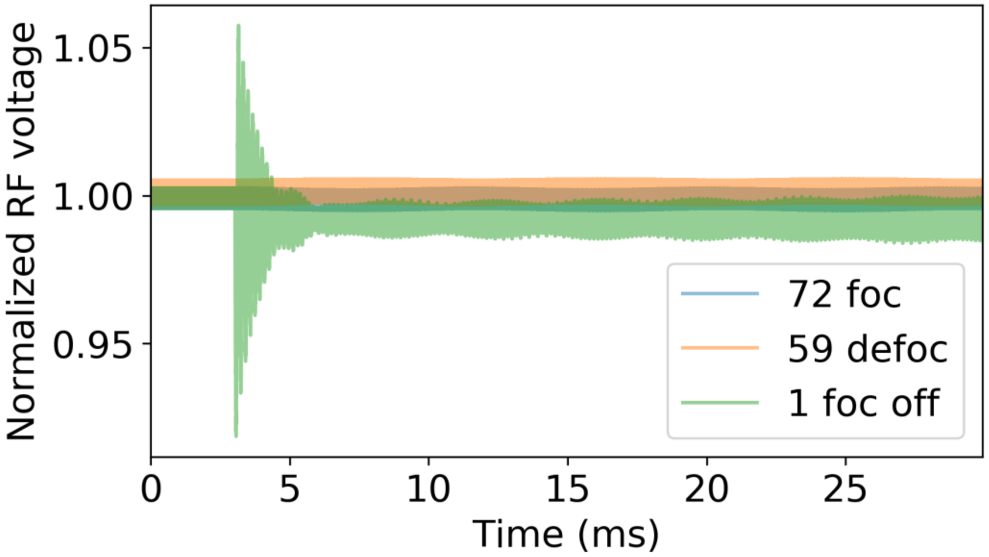
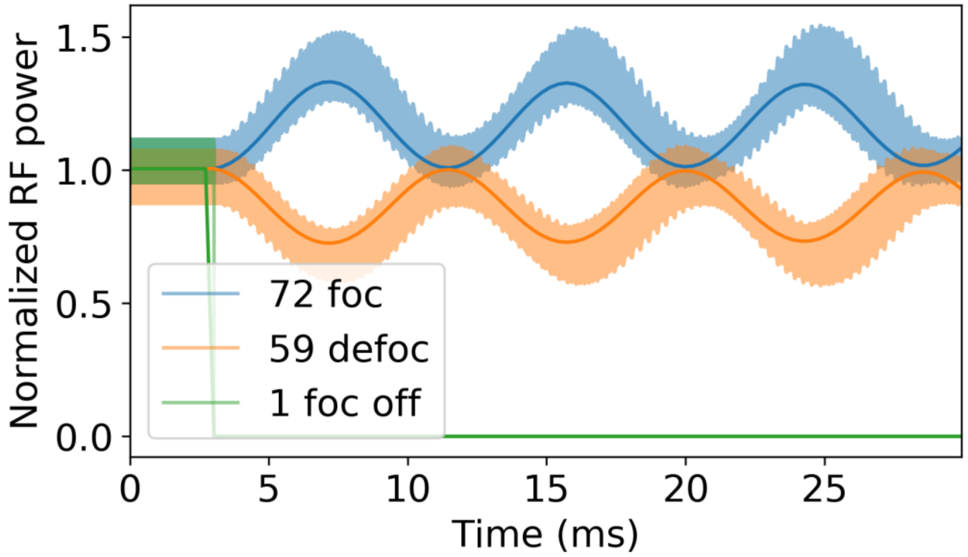
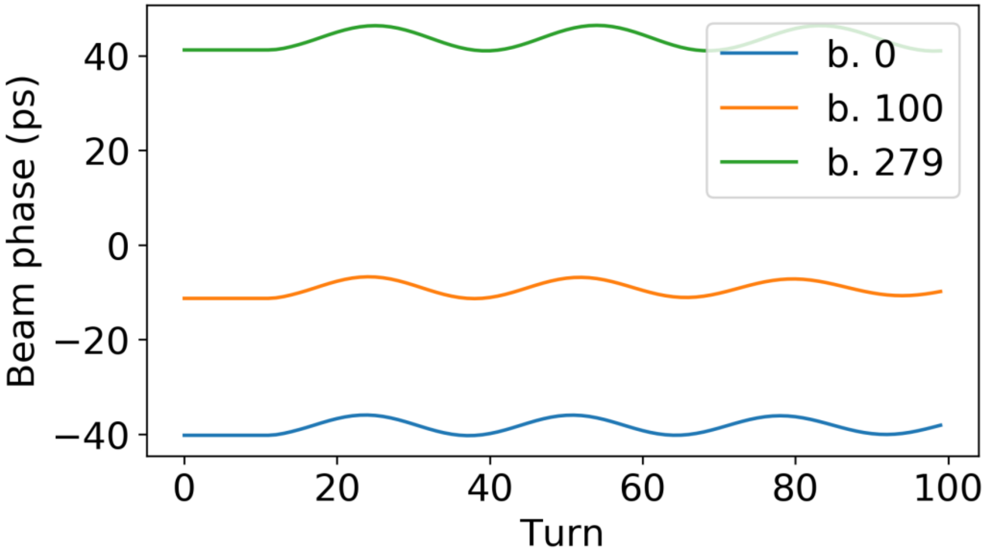


Trip of defocusing cavity



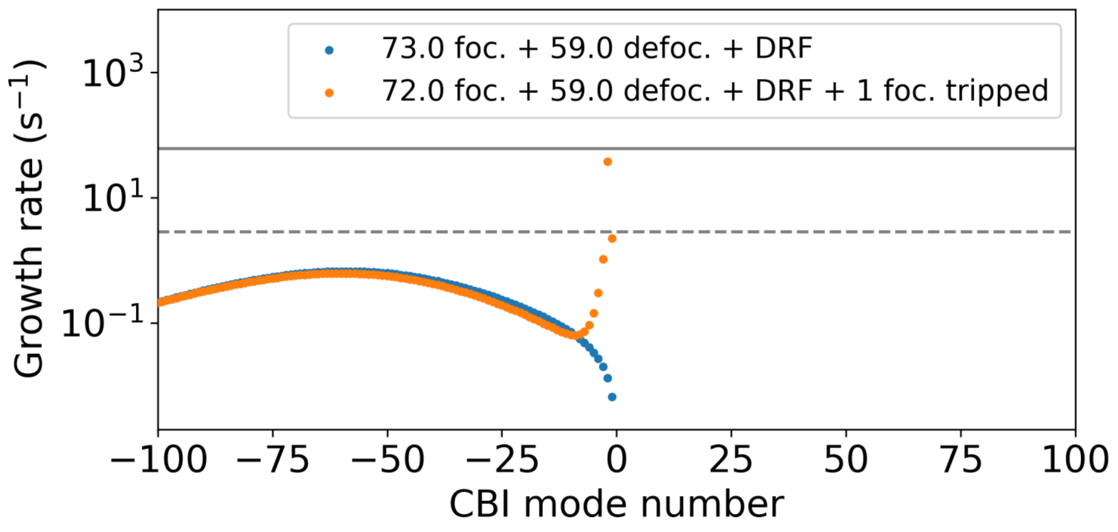


Similar behavior to focusing cavities

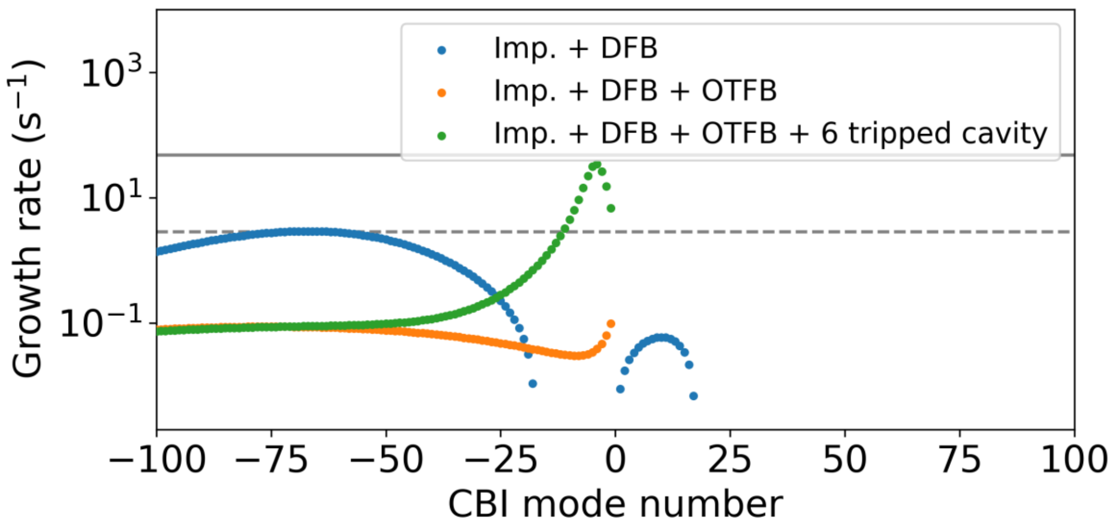


Comparison with single-cell RF system

2-cell RPO

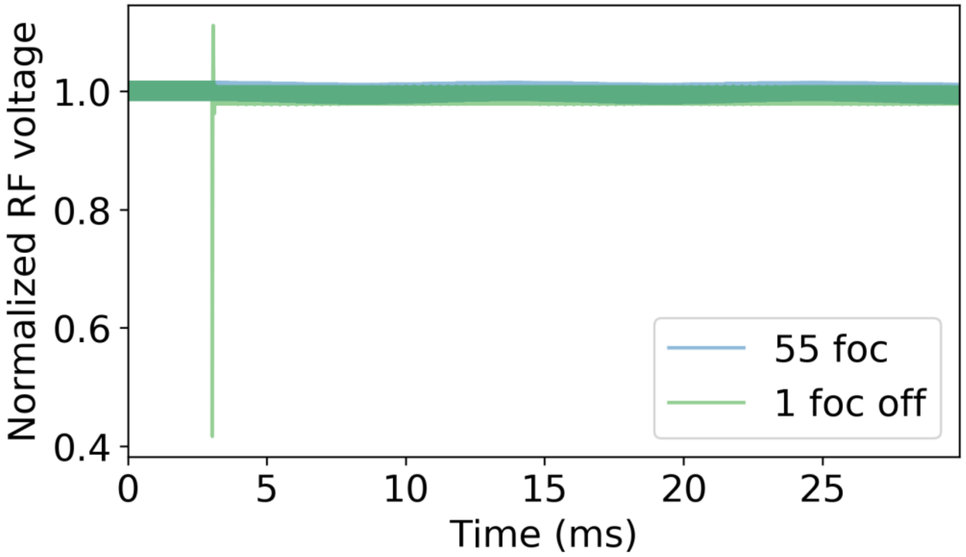


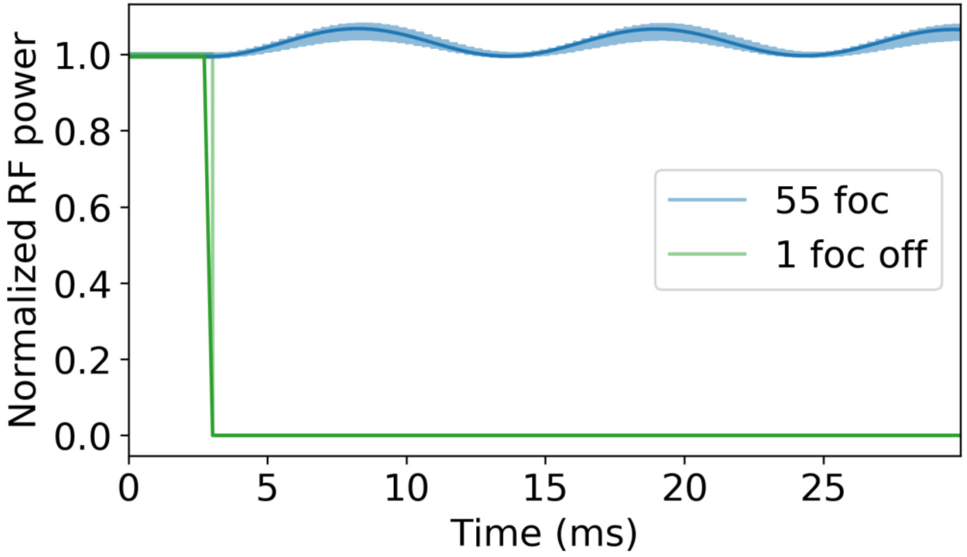
1-cell Normal



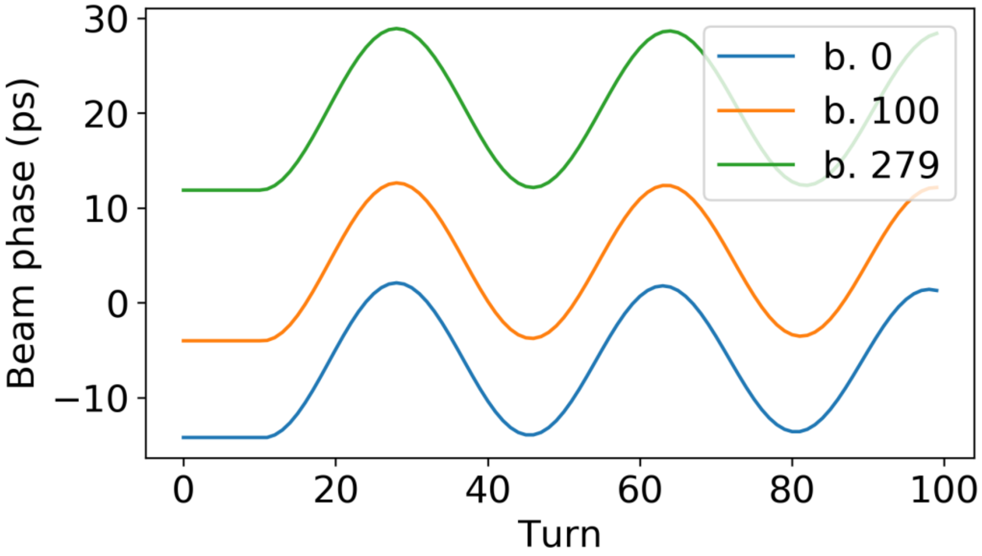
→ Margin for 6 simultaneously tripped cavities (10%) for 1-cell RF system

Single tripped cavity

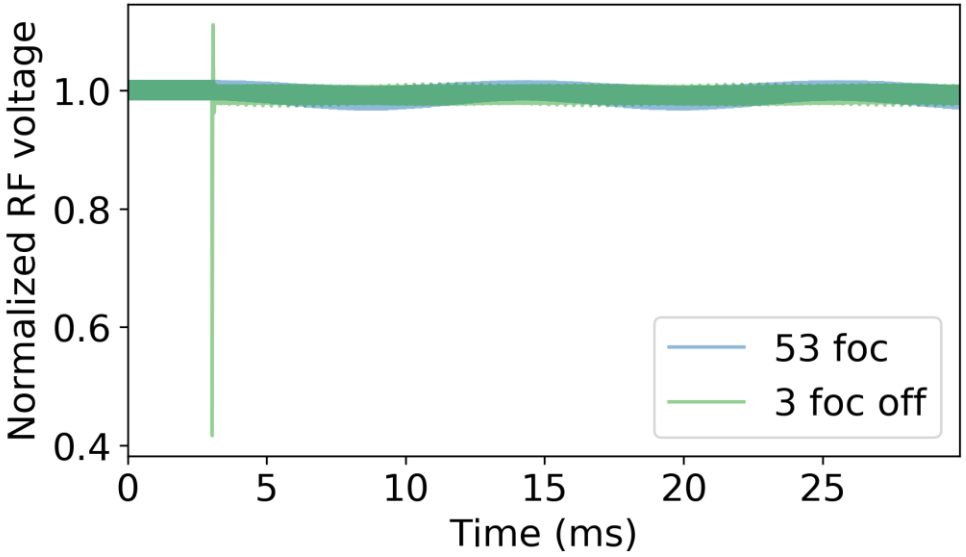


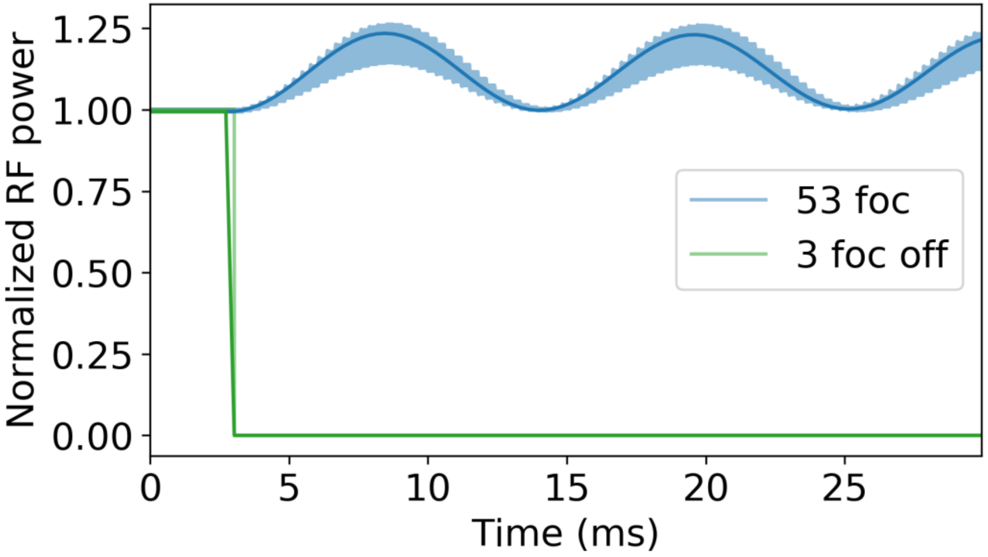


- Similar short RF voltage transients ~11%
- Peak power of other cavities is modulated at synchrotron frequency (mean <7%, peak <8%)
- Initial bunch oscillation amplitude is ~35% of rms bunch length

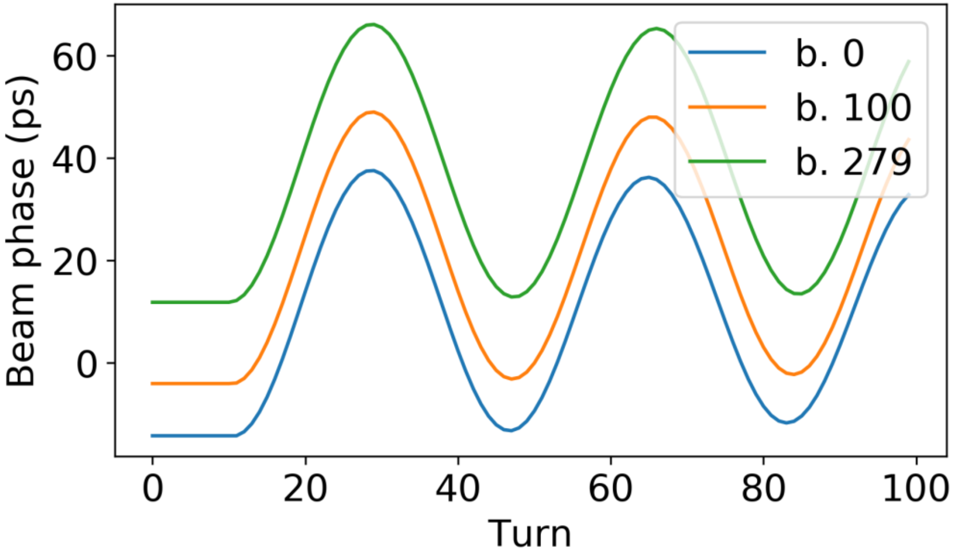


Three simultaneously tripped cavity

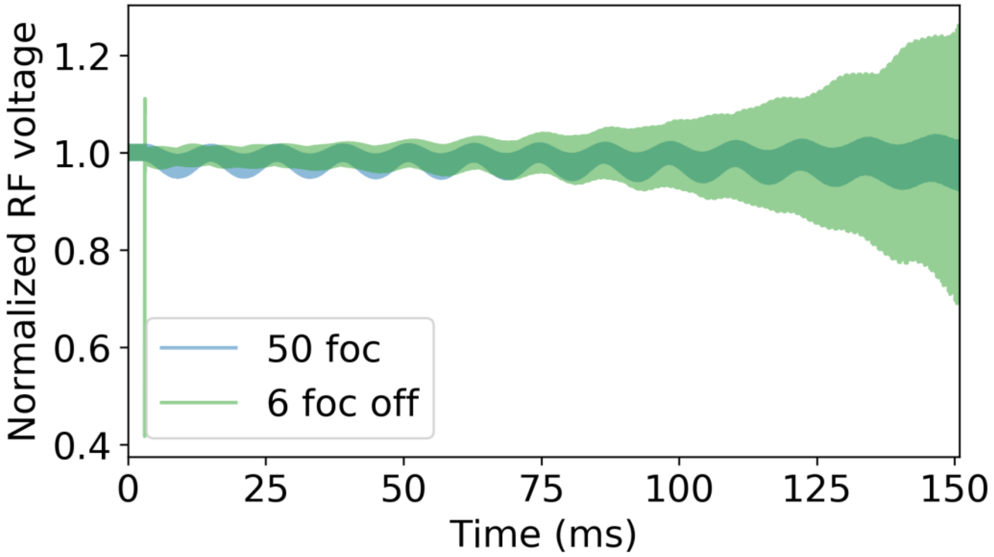


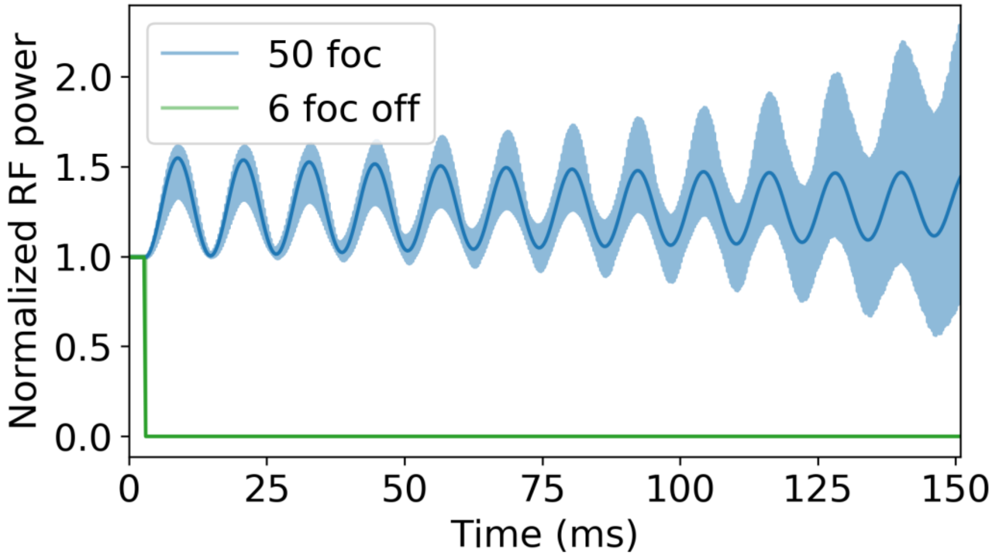


- Similar short RF voltage transients ~11%
- Peak power of other cavities is modulated at synchrotron frequency (mean <23%, peak <25%)
- Initial bunch oscillation amplitude is ~100% of rms bunch length

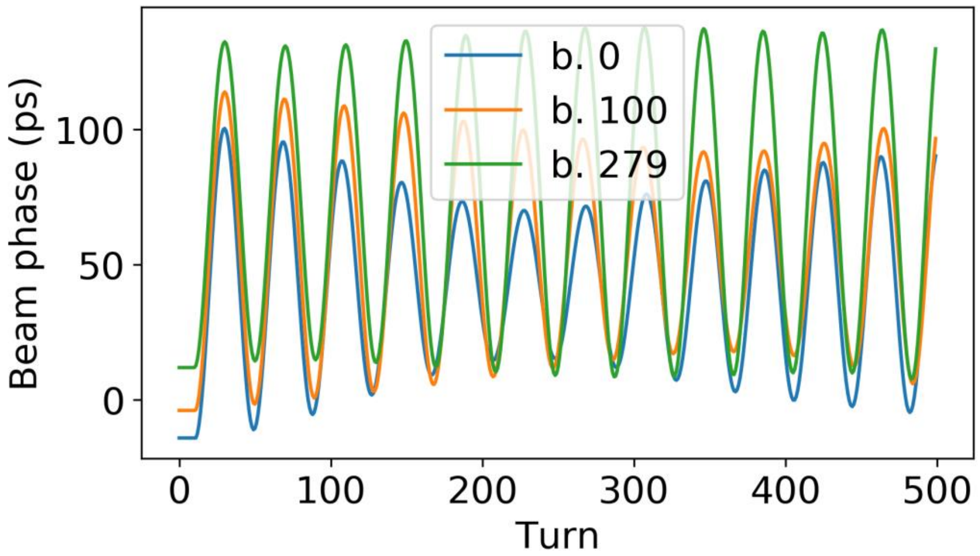


Six simultaneously tripped cavity

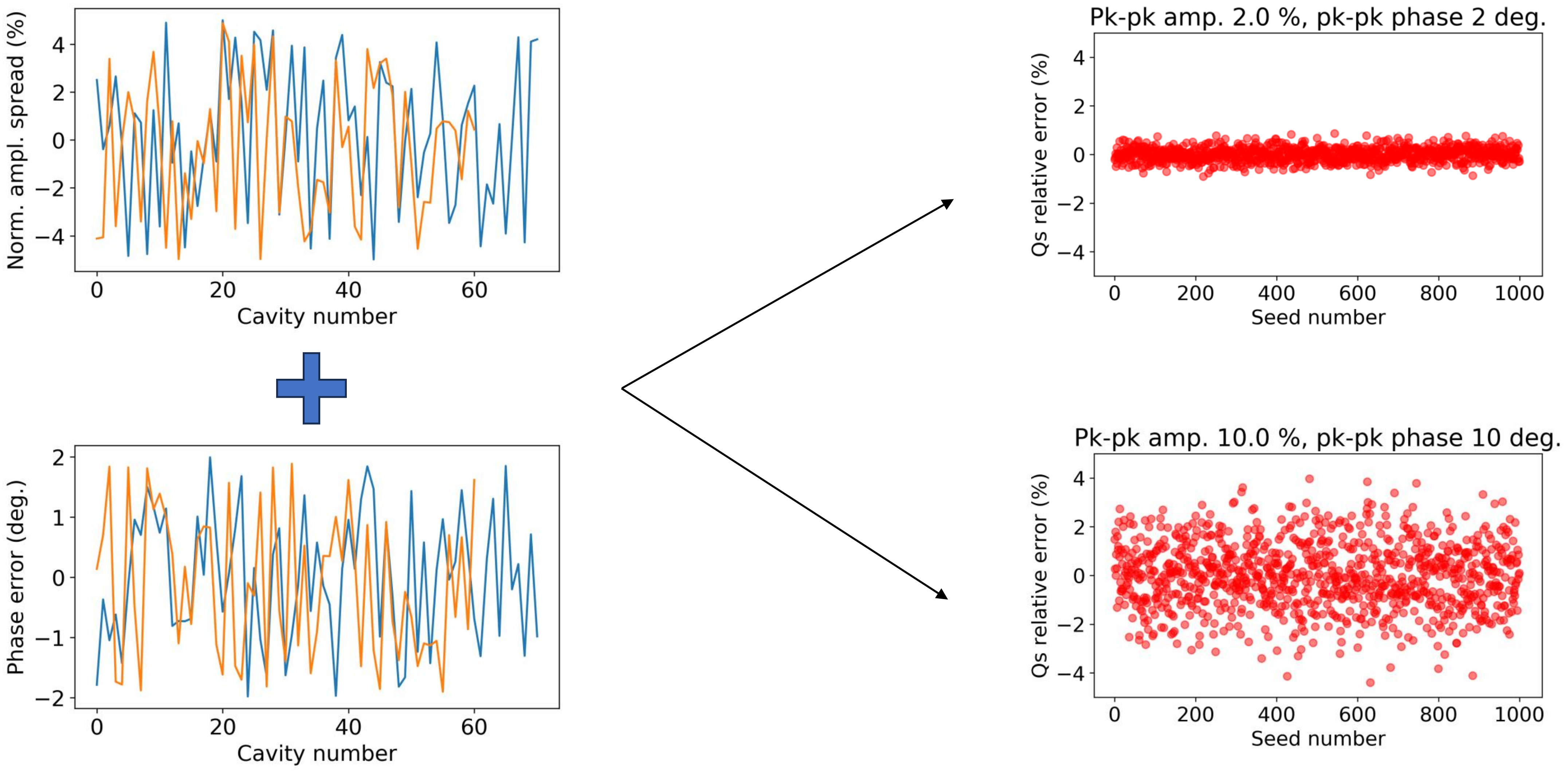




- Similar short RF voltage transients ~11%
- Peak power of other cavities is modulated at synchrotron frequency (mean <55%, peak <61%)
- Initial bunch oscillation amplitude is ~240% of rms bunch length



Parameter sensitivity of RPO



Small, but visible impact of parameter spread on global parameters (e.g., Q_s)

Summary

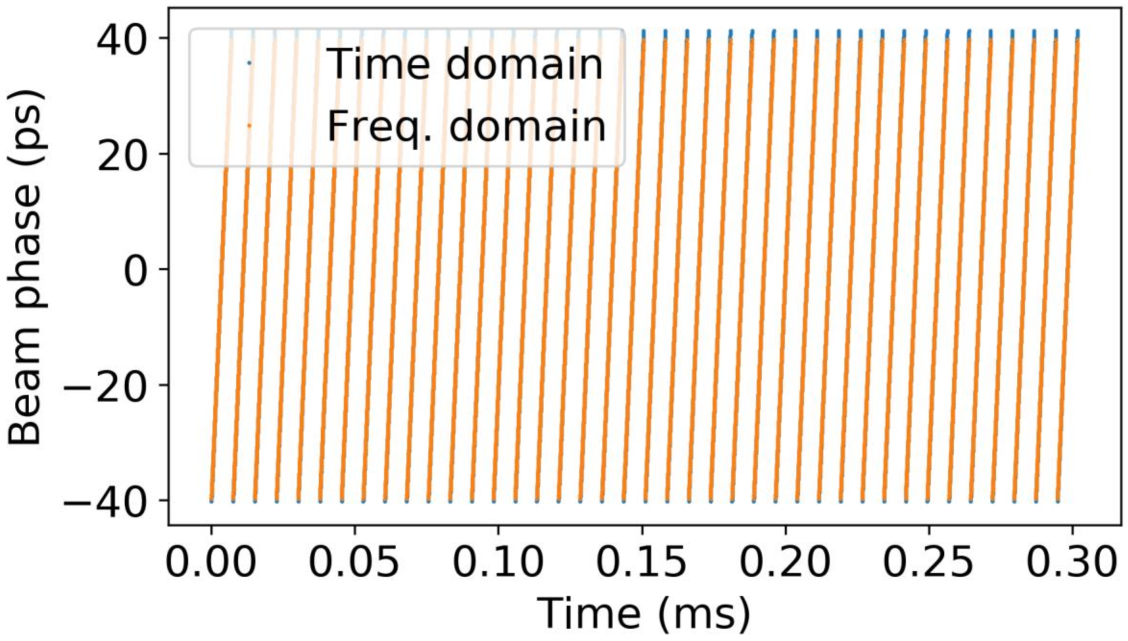
Reverse Phase Operation (RPO) mode aims to avoid hardware modification of RF system between Z, W, and ZH modes

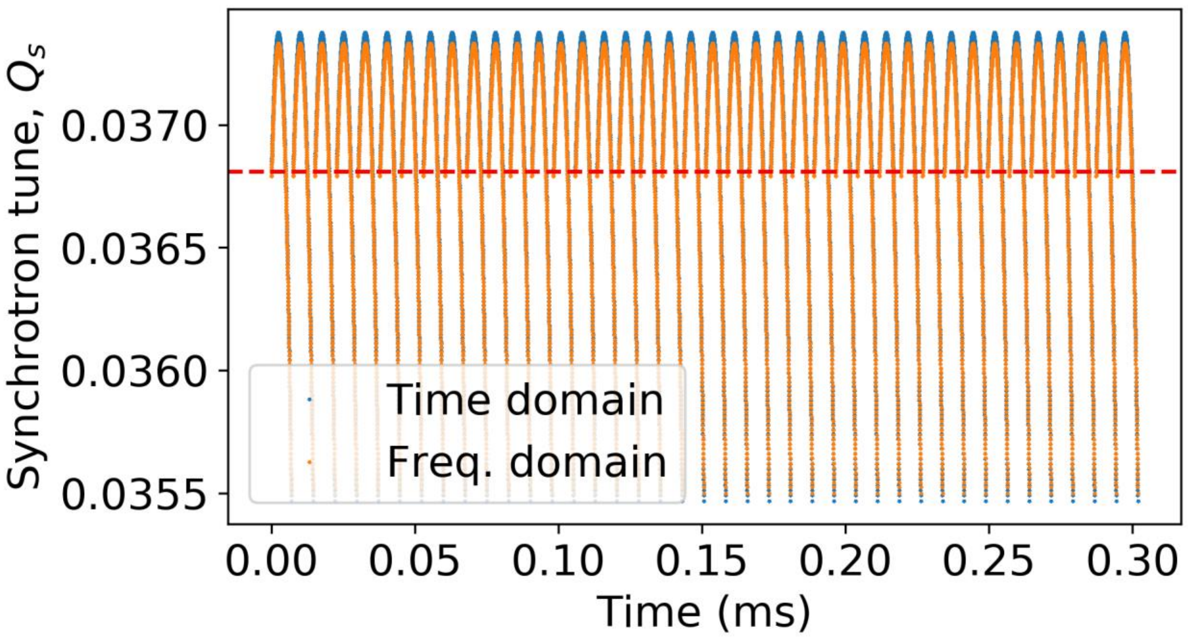
- Synchrotron frequency and bunch length spread due to transient beam loading could be a potential showstopper.
- Thanks to reduction of gap length and ~50% increase of total RF voltage a new parameter set was found although it requires further verifications
- Dynamic beam-cavity interaction model was developed to evaluate transient behavior during cavity trips
- First results show no risk of rapid increase of induced voltage in the tripped cavity, while RF power transients need to be further looked at

Thank you for your attention!

Backup slides

Time- vs frequency domain analysis





Preliminary parameter set

FCC-ee collider parameters for the GHC lattice at Z, Oct. 29, 2024.

Beam energy	[GeV]	45.6			
Layout		PA31-3.0			
# of IPs		4			
Circumference	[km]	90.658728			
Bend. radius of arc dipole	[km]	10.021			
Energy loss / turn	[GeV]	0.0390			
SR power / beam	[MW]	50			
Beam current	[mA]	1283			
Colliding bunches / beam		11200	11220		
Colliding bunch population	[10^{11}]	2.180	2.176		
Hor. emittance at collision ε_x	[nm]	0.70			
Ver. emittance at collision ε_y	[pm]	1.90	2.18	2.40	2.53
Lattice ver. emittance $\varepsilon_{y,\text{lattice}}$	[pm]	0.76	0.71	1.09	1.06
Arc cell		Long 90/90			
Momentum compaction α_p	[10^{-6}]	28.67			
Arc sext families		75			
$\beta^*_{x/y}$	[mm]	110 / 0.7		130 / 0.7	
Transverse tunes $Q_{x/y}$		218.158 / 222.200	218.187 / 222.220	218.167 / 222.220	218.175 / 222.220
Chromaticities $Q'_{x/y}$		+5 / +5			
Energy spread (SR/BS) σ_δ	[%]	0.039 / 0.110	0.039 / 0.121	0.039 / 0.123	0.039 / 0.127
Bunch length (SR/BS) σ_z	[mm]	5.53 / 15.7	4.70 / 14.6	4.31 / 13.7	4.11 / 13.4
RF voltage 400/800 MHz	[GV]	0.079 / 0	0.103 / 0	0.120 / 0	0.130 / 0
Harm. number for 400 MHz		121200			
RF frequency (400 MHz)	MHz	400.787129			
Synchrotron tune Q_s		0.0289	0.0340	0.0371	0.0388
Long. damping time	[turns]	1171			
RF acceptance	[%]	1.06	1.41	1.62	1.74
Energy acceptance (DA)	[%]	±1.0			
Beam crossing angle at IP θ_x	[mrad]	±15			
Crab waist ratio	[%]	50			
Beam-beam ξ_x/ξ_y[a]		0.0022 / 0.0985	0.0025 / 0.0981	0.0034 / 0.1008	0.0036 / 0.1006
X-Z threshold param. Q_s/ξ_x		13.1	13.6	10.9	10.8
Piwinski angle $(\theta_x \sigma_{z,\text{BS}})/\sigma^*_x$		26.9	25.0	21.4	21.0
Lifetime (q + BS + lattice)	[sec]	13000	2200	9600	2460
Lifetime (lum)[b]	[sec]	1320	1320	1320	1330
Luminosity / IP	[10^{34}/cm^{2}s]	145.2	145.0	145.1	144.8

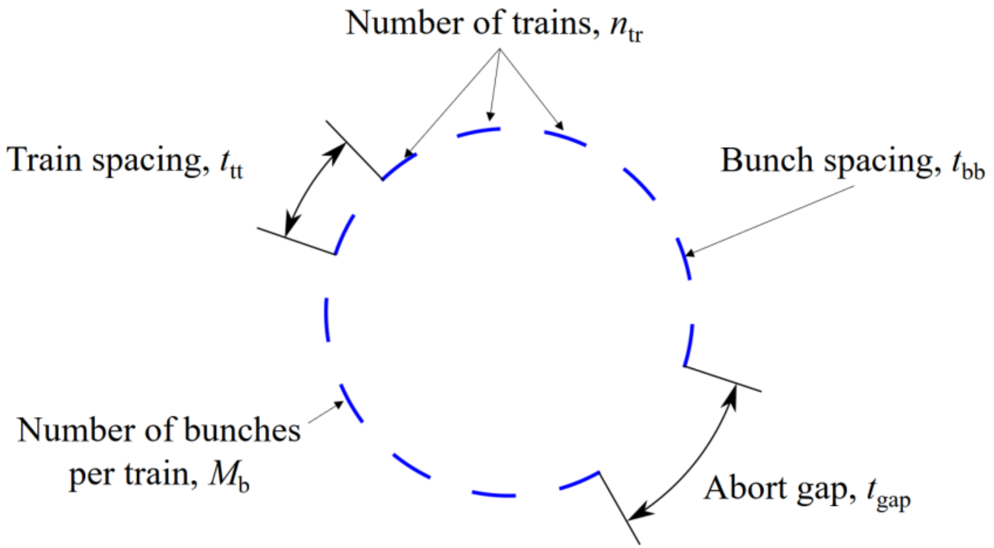
[a] incl. hourglass.
[b] only the energy acceptance is taken into account for the cross section, no beam-size effect.

K. Oide, 29.10.2024

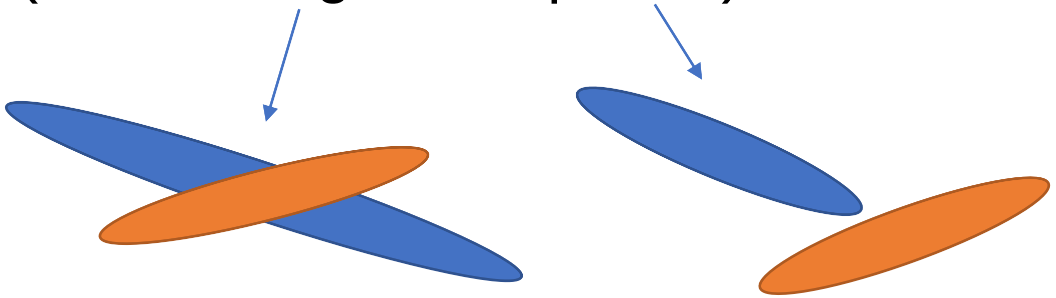
Despite ~30% reduction of lifetime, ~120 MV option looks promising

To be confirmed in full self-consistent simulations with impedance

Transient beam loading



Gaps in machine filling will result in modulation beam parameters (bunch length and phase)



→ Modulations might impact luminosity and/or beam stability

Conventional approaches:

- Small-signal model in frequency domain *(F. Pedersen, 1992)*
- Particle tracking simulations (difficult for 11200 bunches in FCC-ee Z)
- Steady-state time domain method *(J. Tückmantel, 2011)*

→ Small-signal model and time-domain methods were adapted for the RPO case of FCC

Reduced Pedersen model

General equations of beam-cavity interactions with reverse phase operation (RPO) mode (adaptation of formalism in *J. Tückmantel, 2011*):

$$I_{gf}(t) = \frac{V_f(t)}{2(R/Q)}\left(\frac{1}{Q_L} - 2i\frac{\Delta\omega_f}{\omega_{\mathrm{rf}}}\right) + \frac{I_{\mathrm{b,rf}}(t)}{2} + \frac{dV_f(t)}{dt}\frac{1}{\omega_{\mathrm{rf}}(R/Q)}$$

$$I_{gd}(t) = \frac{V_d(t)}{2(R/Q)}\left(\frac{1}{Q_L} - 2i\frac{\Delta\omega_d}{\omega_{\mathrm{rf}}}\right) + \frac{I_{\mathrm{b,rf}}(t)}{2} + \frac{dV_d(t)}{dt}\frac{1}{\omega_{\mathrm{rf}}(R/Q)}$$

Energy balance $$V_{\mathrm{tot}}\cos\phi_s = N_f A_f \cos(\phi_s - \phi_b + \phi_{cf} + \phi_f) + N_d A_d \cos(\phi_s - \phi_b + \phi_{cd} + \phi_d)$$

To calculate beam-induced modulation we assume:

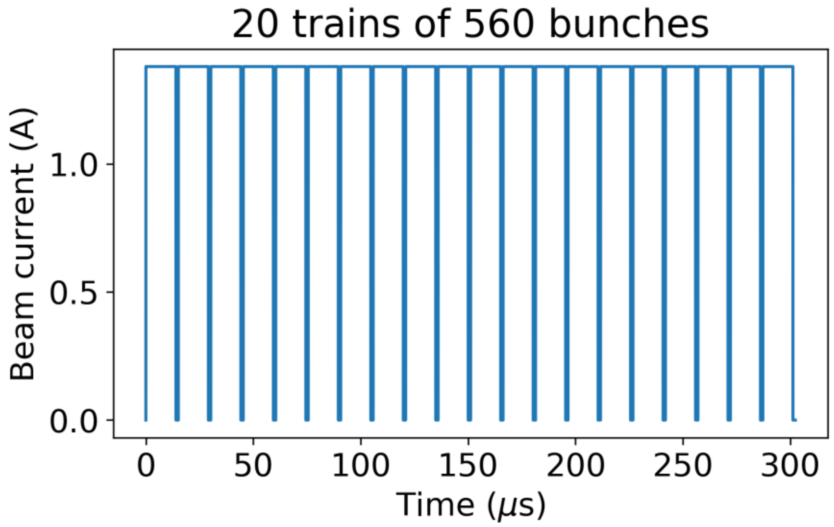
- $I_{gf,d}(t)$ = constant – no beam loading compensation
- $V_{f,d}(t) = A_{f,d}(t)e^{i\phi_f(t)+i\phi_{cf,d}}$, $I_{b,\mathrm{rf}}(t) = A_b(t)e^{-i\phi_s+i\phi_b(t)}$

Then, system of equations is linearized to obtain transfer functions: $\frac{a_{Vf,d}}{a_b}, \frac{\phi_{f,d}}{a_b}, \frac{\phi_b}{a_b}$

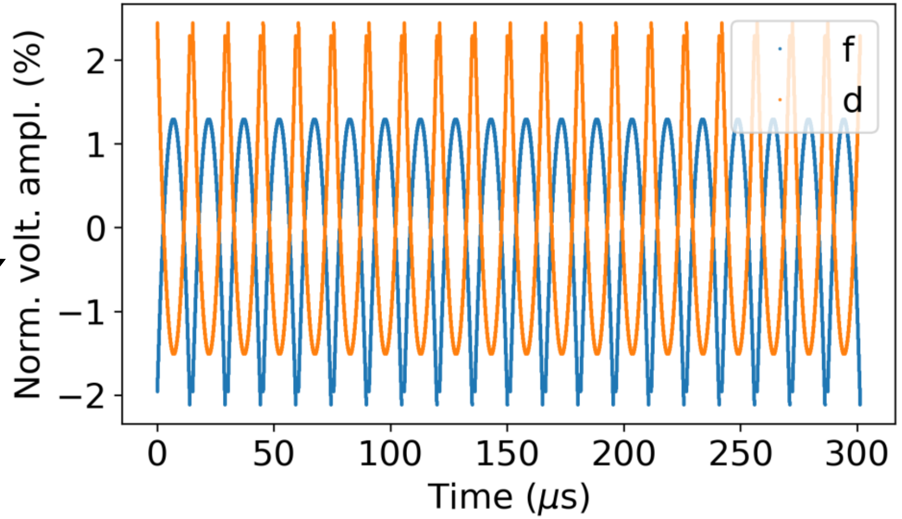
$A_{f,d} = V_{\mathrm{cav}}(1 + a_{Vf,d}), A_b(t) = |F_b|I_{b,\mathrm{dc}}\,(1 + a_b)$

Bunch-by-bunch spread of cavity parameters

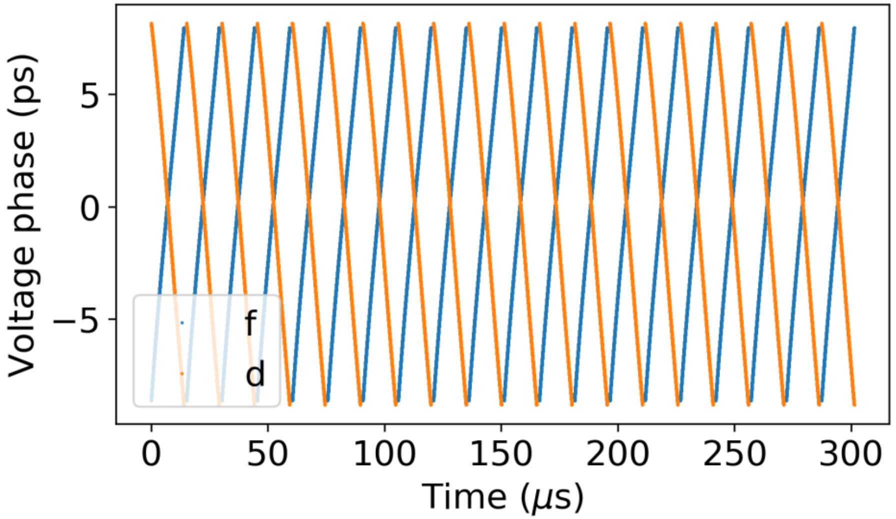
	N_f	N_d	V_{tot} Z (MV)	V_{cav} (MV)	Q_L
Current	71	61	88	7.95	9.21e5



$$\frac{a_{Vf,d}}{a_b}$$



$$\frac{\phi_{f,d}}{a_b}$$



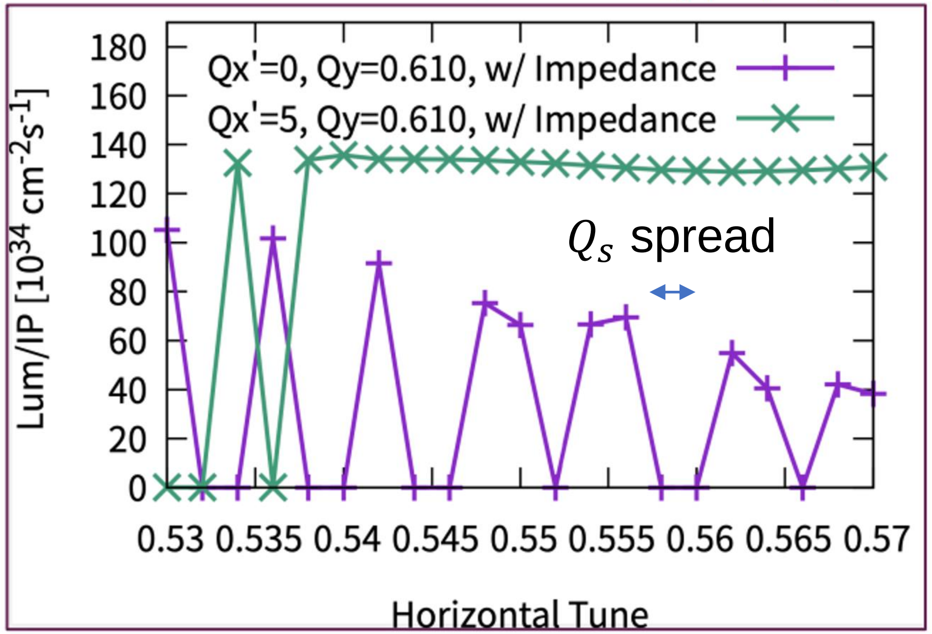
Note, the designed rms bunch length is 50 ps (with beamstrahlung)

Critical impact of spread

Interplay between beam-beam and coupling impedance

A positive chromaticity has a beneficial effect on the beam-beam. Self-consistent simulations show a luminosity per IP close to the nominal value of 141 x 10^{34} $cm^{-2}s^{-1}$ by properly choosing the collider working point.

Y. Zhang, IHEP, Beijing, China



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M. Migliorati, FCC Week 2024

→ No stable region for a horizontal tune can be found in presence of large Q_s spread. Possible mitigations need to be studied

Impact on parameters (oversimplified scaling)

FCC-ee collider parameters for the GHC lattice as of Aug. 2, 2024.

Beam energy	[GeV]	45.6	80	120	182.5
Layout		PA31-3.0			
# of IPs		4			
Circumference	[km]	90.658728			
Bend. radius of arc dipole	[km]	10.021			
Energy loss / turn	[GeV]	0.0390	0.369	1.86	9.94
SR power / beam	[MW]	50			
Beam current	[mA]	1283	135	26.8	5.0
Colliding bunches / beam		11200	1852	300	64
Colliding bunch population	$[10^{11}]$	2.16	1.38	1.69	1.48
Hor. emittance at collision ε_x	[nm]	0.70	2.16	0.66	1.51
Ver. emittance at collision ε_y	[pm]	1.9	2.0	1.0	1.36
Lattice ver. emittance $\varepsilon_{y,\mathrm{lattice}}$	[pm]	0.87	1.20	0.57	0.94
Arc cell		Long 90/90		90/90	
Momentum compaction α_p	$[10^{-6}]$	28.67		7.52	
Arc sext families		75		146	
$\beta^*_{x/y}$	[mm]	110 / 0.7	220 / 1	240 / 1	[illegible]
Transverse tunes $Q_{x/y}$		218.158 / 222.220	218.185 / 222.2[illegible]	[illegible]98.2[illegible]	[illegible]
Chromaticities $Q'_{x/y}$		0 / +5	0 / +5	[illegible]	[illegible]
Energy spread (SR/BS) σ_δ	[%]	0.039 / 0.110	0.069 / 0.105	0.102 / 0.176	[illegible]
Bunch length (SR/BS) σ_z	[mm]	5.57 / 15.6	3.46 / 5.28	[illegible]	1.91 / 2.32
RF voltage 400/800 MHz	[GV]	0.079 / 0	1.00 / 0	[illegible]	2.1 / 9.20
Harm. number for 400 MHz		[illegible]			
RF frequency (400 MHz)	MHz	40[illegible]			
Synchrotron tune Q_s		0.0289	0.0809	[illegible]	0.0881
Long. damping time	[turns]	1171	218	[illegible]	19.4
RF acceptance	[%]	1.06	3.32	[illegible]06	3.06
Energy acceptance (DA)	[%]	±1.0	±1.0	[illegible].9	-2.8/+2.5
Beam crossing angle at IP θ_x	[mrad]	±15			
Crab waist ratio	[%]	70	55	[illegible]	[illegible]
Beam-beam ξ_x/ξ_y[a]		0.0022 / 0.0977	0.013 / 0.12[illegible]	[illegible]130	[illegible]136
Piwinski angle $(\theta_x \sigma_{z,\mathrm{BS}})/\sigma^*_x$		26.6	3.6	[illegible]	[illegible]
Lifetime (q + BS + lattice)	[sec]	11800	4500	6000	[illegible]
Lifetime (lum)[b]	[sec]	1330	960	600	[illegible]
Luminosity / IP	$[10^{34}/\mathrm{cm}^2\mathrm{s}]$	143	20	7.5	[illegible]

[a] incl. hourglass.
[b] only the energy acceptance is taken into account for the cross section, no beam size effect.

3.3/9.4

$\sigma_z \propto \frac{1}{\sqrt{V_{tot}}}$

0.195/0

0.0483

2.35

xx/0.162

$\xi_y \propto \frac{N_p}{\sigma_z}$

?

$L \propto \xi_y$

K. Oide, 2024

X-Z instability
(K. Ohmi, 2016**)**

Higher Q_s
→ stronger low order resonance but more space available between them

Bunches are ~50% shorter (assuming the same σ_δ)
→ stronger beamstrahlung (ξ_y increase is smaller)
→ stronger impact of longitudinal impedance?

Resonant depolarization
Figure of merit SMI $=\nu_s \sigma_\delta / Q_s$ ~ 1.3-1.4 for baseline
→ is reduced to ~0.85 (SMI<1 is preferred)

Many more aspects to be re-analyzed…

Motivation

RF power for SRF cavities with circulators is minimized for optimal parameters:

Optimal detuning $\Delta\omega_{\text{opt}} = -\frac{\omega_{\text{rf}}(R/Q)|F_b|I_{b,\text{dc}}\sin\phi_s}{2V_{\text{cav}}}$

$$F_b = 2\frac{\mathcal{F}[\lambda(t)]_{\omega=\omega_{\text{rf}}}}{\mathcal{F}[\lambda(t)]_{\omega=0}}$$

Optimal quality factor $Q_{\text{ext,opt}} = \frac{V_{\text{cav}}}{|F_b|(R/Q)I_{b,\text{dc}}\cos\phi_s}$

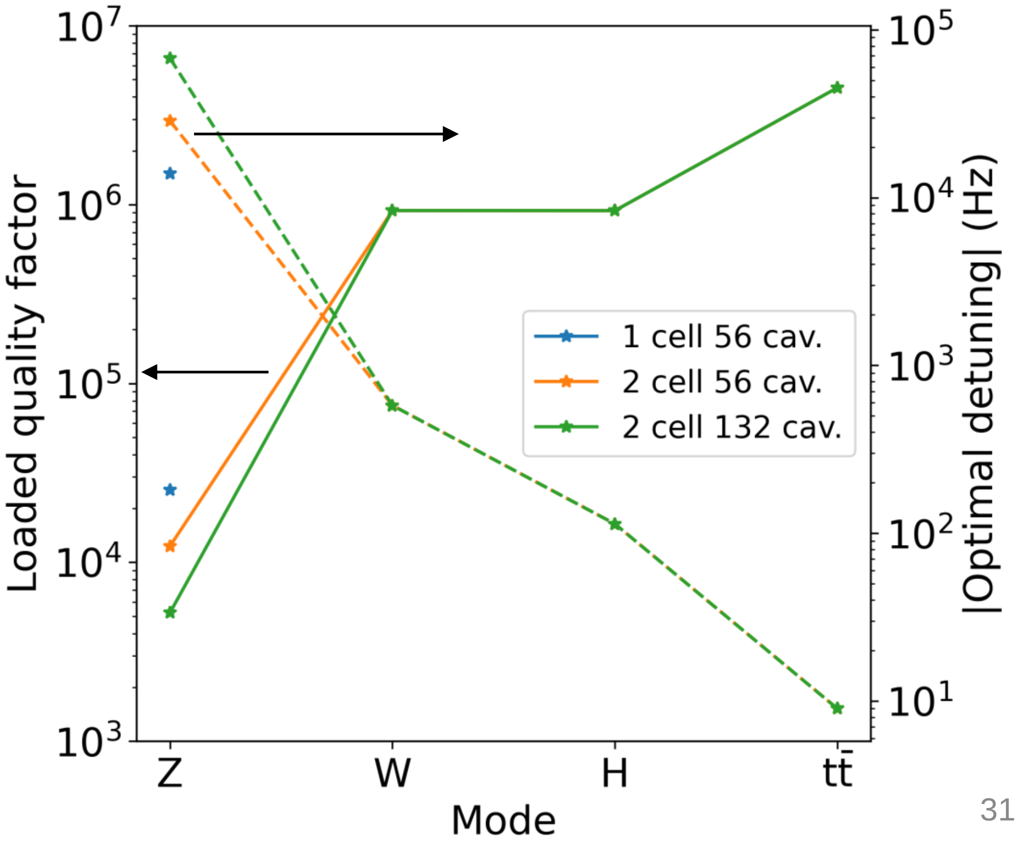
Keeping 2-cell cavities for Z, W, H, (and $t\bar{t}$):

→ Large range for $Q_{\text{ext,opt}}$ adjustment (a factor of ~75-600) starting from $\sim 5 \times 10^3$: possible FPC solutions was studied *(S. Gorgi Zadeh and E. Montesinos, CERN SRF, 2024; see also slides of F. Gerigk, FCC Week 2024)*

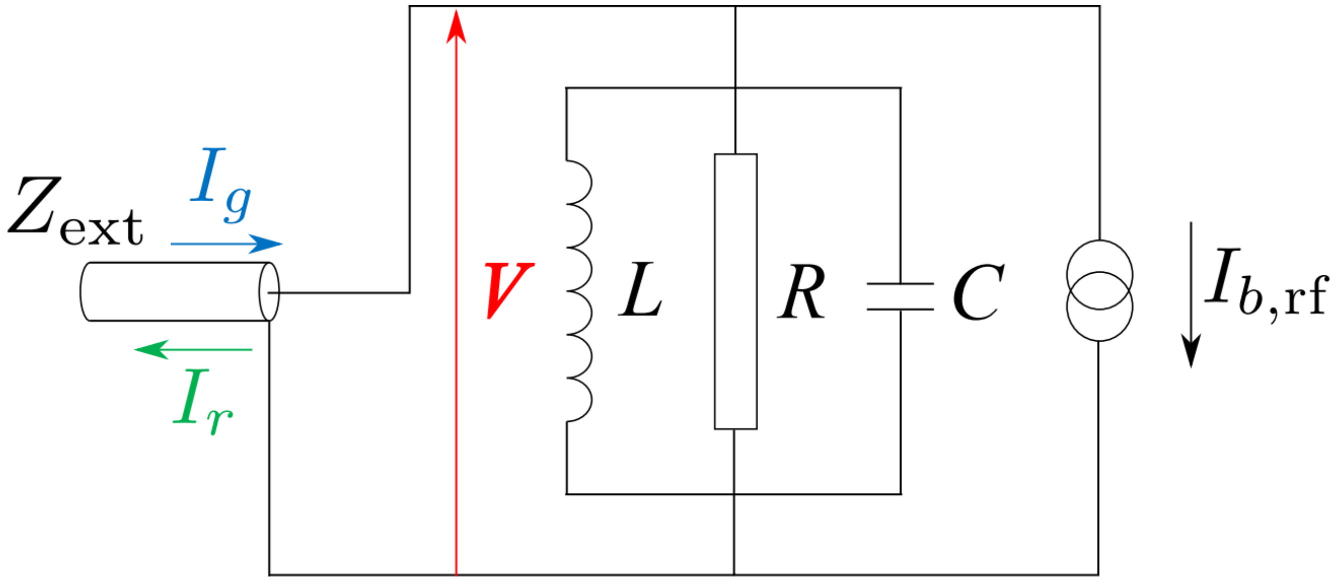
→ Incresed detuning enhances instability due to fundamental mode

Can the voltage per cavity be increased for Z mode?

Optimal parameters for different scenarios



Beam loading model: main equation



Generator current
$$I_g = \frac{V}{2(R/Q)}\left(\frac{1}{Q_0} + \frac{1}{Q_{\text{ext}}} - 2i\,\frac{\Delta\omega}{\omega_{\text{rf}}}\right) + \frac{I_{b,\text{rf}}}{2}$$

Generator power
$$P_g = \frac{1}{2} Z_{\text{ext}} |I_g|^2 = \frac{1}{2}(R/Q)Q_{\text{ext}}|I_g|^2$$

Fixed parameters are V, (R/Q), Q_0, ω_{rf}, $I_{b,\text{rf}}$, while V, $\Delta\omega$, and Q_{ext} can be adjusted

See, e.g., J. Tückmantel, CERN Report No. CERN-ATS-Note-2011- 002 TECH, 2011

RF power requirements

Constraints:
- The same $Q_{\text{ext,opt}}$ for all cavities to avoid a movable fundamental power coupler design
- The same $P_{g,\text{opt}}$ to have the identical power sources and uniform power distribution (role of variations is under study)

$$Q_{\text{ext,opt}} = \frac{|V_{\text{cav}}|}{|F_b|(R/Q) I_{b,\text{dc}} \cos(\phi_s + \phi_c)}$$

$$P_{g,\text{opt}} = \frac{|V_{\text{cav}}||F_b| I_{b,\text{dc}} \cos(\phi_s + \phi_c)}{2}$$

→ Cavity voltage must be the same for all cavities: $\cos(\phi_s + \phi_{\text{foc}}) = \cos(\phi_s + \phi_{\text{defoc}})$ → $\phi_{\text{foc}} = -2\phi_s - \phi_{\text{defoc}}$

Starting with energy gain per turn

$$N_{\text{foc}} |V_{\text{cav}}| \cos(\phi_s + \phi_{\text{foc}}) + N_{\text{defoc}} |V_{\text{cav}}| \cos(\phi_s + \phi_{\text{defoc}}) = V_{\text{tot}} \cos\phi_s \qquad \times \frac{|F_b| I_{b,\text{dc}}}{2}$$

$$N_{\text{foc}} \frac{|V_{\text{cav}}||F_b| I_{b,\text{dc}} \cos(\phi_s + \phi_{\text{foc}})}{2} + N_{\text{defoc}} \frac{|V_{\text{cav}}||F_b| I_{b,\text{dc}} \cos(\phi_s + \phi_{\text{defoc}})}{2} = \frac{|F_b| I_{b,\text{dc}}}{2} V_{\text{tot}} \cos\phi_s$$

$$\cos\phi_s = \frac{U_0}{V_{\text{tot}}} \qquad |F_b| \approx 2$$

$$P_{g,\text{foc}} = P_{g,\text{defoc}} = P_{g,\text{opt}}$$

$$N_{\text{foc}} + N_{\text{defoc}} = N_{\text{tot}}$$

$$N_{\text{foc}} P_{g,\text{foc}} + N_{\text{defoc}} P_{g,\text{defoc}} = I_{b,\text{dc}} U_0 = P_{\text{SR}}$$

$$P_{g,\text{opt}} = \frac{P_{SR}}{N_{\text{tot}}}$$

→ No RF power overshoot is needed for RPO if optimal detuning and optimal quality factor are used

Reverse phasing mode equations

Preservation of energy gain

$$N_{\text{foc}}|V_{\text{cav}}|\cos(\phi_s+\phi_{\text{foc}})+N_{\text{defoc}}|V_{\text{cav}}|\cos(\phi_s+\phi_{\text{defoc}})=V_{\text{tot}}\cos\phi_s$$

Preservation of synchrotron tune

$$N_{\text{foc}}|V_{\text{cav}}|\sin(\phi_s+\phi_{\text{foc}})+N_{\text{defoc}}|V_{\text{cav}}|\sin(\phi_s+\phi_{\text{defoc}})=V_{\text{tot}}\sin\phi_s$$

→ Cavity voltage

$$|V_{\text{cav}}|=\frac{V_{\text{tot}}}{N_{\text{tot}}}\sqrt{\frac{U_0^2}{V_{\text{tot}}^2}+\left(1-\frac{U_0^2}{V_{\text{tot}}^2}\right)\frac{N_{\text{tot}}^2}{(N_{\text{foc}}-N_{\text{defoc}})^2}}$$

Optimal detuning

$$\Delta\omega_{\text{opt}}=-\frac{\omega_{\text{rf}}(R/Q)|F_b|I_{b,\text{dc}}}{2V_{\text{cav}}}\sqrt{1-\frac{U_0^2}{V_{\text{cav}}^2N_{\text{tot}}^2}}$$

See, also A. Blednykh et al, EIC-ADD-TN-33, 2022

Phases

$$\phi_{\text{foc}}=-\phi_s+\arccos\left(\frac{V_{\text{tot}}\cos\phi_s}{N_{\text{tot}}V_{\text{cav}}}\right)\qquad\phi_{\text{defoc}}=-\phi_s-\arccos\left(\frac{V_{\text{tot}}\cos\phi_s}{N_{\text{tot}}V_{\text{cav}}}\right)$$

The aim is to keep V_{cav}, $P_{g,\text{opt}}$, and $Q_{\text{ext,opt}}$ for Z, W, and ZH modes
→ Cavity voltage can be change in discrete steps of $N_{\text{foc}}-N_{\text{defoc}}=2,4,\ldots$

Derivations for arbitrary cavity phase (1/2)

Generator current $$I_g = \frac{V}{2(R/Q)}\left(\frac{1}{Q_0} + \frac{1}{Q_{\text{ext}}} - 2i\frac{\Delta\omega}{\omega_{\text{rf}}}\right) + \frac{I_{b,\text{rf}}}{2}$$

($\frac{1}{Q_0} \to 0$)

$$F_b = 2\frac{\mathcal{F}[\lambda(t)]_{\omega=\omega_{\text{rf}}}}{\mathcal{F}[\lambda(t)]_{\omega=0}}$$

Complex quantities: I_g, V, and $I_{b,\text{rf}}$ → $I_g = |I_g|e^{i\phi_L}$, $V = |V_{\text{cav}}|e^{i\phi_c}$, $I_{b,\text{rf}} = |F_b|I_{b,\text{dc}}e^{-i\phi_s}$

$$|I_g|e^{i\phi_L} = \frac{|V_{\text{cav}}|e^{i\phi_c}}{2(R/Q)}\left(\frac{1}{Q_{\text{ext}}} - 2i\frac{\Delta\omega}{\omega_{\text{rf}}}\right) + \frac{|F_b|I_{b,\text{dc}}e^{-i\phi_s}}{2} \quad \Big| \times e^{-i\phi_c}$$

$$|I_g|e^{i\phi_L - i\phi_c} = \frac{|V_{\text{cav}}|}{2(R/Q)}\left(\frac{1}{Q_{\text{ext}}} - 2i\frac{\Delta\omega}{\omega_{\text{rf}}}\right) + \frac{|F_b|I_{b,\text{dc}}e^{-i\phi_s - i\phi_c}}{2}$$

Then splitting in real and imaginary parts:

Derivations for arbitrary cavity phase (2/2)

$$|I_g|e^{i\phi_L - i\phi_c} = \frac{|V_{\text{cav}}|}{2(R/Q)Q_{\text{ext}}} + \frac{|F_b|I_{b,\text{dc}}\cos(\phi_s+\phi_c)}{2} - i\left[\frac{|V_{\text{cav}}|}{(R/Q)}\frac{\Delta\omega}{\omega_{\text{rf}}} + \frac{|F_b|I_{b,\text{dc}}\sin(\phi_s+\phi_c)}{2}\right]$$

$$P_g = \frac{1}{2}(R/Q)Q_{\text{ext}}|I_g|^2$$

$$= \frac{1}{2}(R/Q)Q_{\text{ext}}\left[\frac{|V_{\text{cav}}|}{2(R/Q)Q_{\text{ext}}} + \frac{|F_b|I_{b,\text{dc}}\cos(\phi_s+\phi_c)}{2}\right]^2$$

→ Minimized for $Q_{\text{ext,opt}} = \frac{|V_{\text{cav}}|}{|F_b|(R/Q)I_{b,\text{dc}}\cos(\phi_s+\phi_c)}$

$$+\frac{1}{2}(R/Q)Q_{\text{ext}}\left[\frac{|V_{\text{cav}}|}{(R/Q)}\frac{\Delta\omega}{\omega_{\text{rf}}} + \frac{|F_b|I_{b,\text{dc}}\sin(\phi_s+\phi_c)}{2}\right]^2$$

→ = 0 for $\Delta\omega_{\text{opt}} = -\frac{\omega_{\text{rf}}(R/Q)|F_b|I_{b,\text{dc}}\sin(\phi_s+\phi_c)}{2|V_{\text{cav}}|}$

Setting $\phi_c = 0$ recovers classical equations for optimal parameters
Adjusting ϕ_c, $Q_{\text{ext,opt}}$ can be modified to meet certain constraints

The minimum power

$$P_{g,\text{opt}} = \frac{|V_{\text{cav}}||F_b|I_{b,\text{dc}}\cos(\phi_s+\phi_c)}{2}$$

Reverse phasing mode equations

Constraints: $|V_{\text{cav}}|$ and $P_{g,\text{opt}}$ are the same for focusing and defocusing cavities

$$P_{g,\text{opt}} = \frac{|V_{\text{cav}}||F_b|I_{b,\text{dc}}\cos(\phi_s+\phi_c)}{2}$$

$\rightarrow \cos(\phi_s + \phi_{\text{foc}}) = \cos(\phi_s + \phi_{\text{defoc}}) \rightarrow \phi_{\text{foc}} = -2\phi_s - \phi_{\text{defoc}}$

Preservation of energy gain

$$N_{\text{foc}}|V_{\text{cav}}|\cos(\phi_s+\phi_{\text{foc}}) + N_{\text{defoc}}|V_{\text{cav}}|\cos(\phi_s+\phi_{\text{defoc}}) = V_{\text{tot}}\cos\phi_s$$

Preservation of synchrotron tune

$$N_{\text{foc}}|V_{\text{cav}}|\sin(\phi_s+\phi_{\text{foc}}) + N_{\text{defoc}}|V_{\text{cav}}|\sin(\phi_s+\phi_{\text{defoc}}) = V_{\text{tot}}\sin\phi_s$$

	RPO	Classical
Optimal quality factor	$Q_{\text{ext,opt}} = \frac{V_{\text{cav}}^2 N_{\text{tot}}}{V_{\text{tot}}(R/Q)\|F_b\|I_{b,\text{dc}}\cos\phi_s}$	$Q_{\text{ext,opt}} = \frac{V_{\text{cav}}}{\|F_b\|(R/Q)I_{b,\text{dc}}\cos\phi_s}$
Optimal detuning	$\Delta\omega_{\text{opt}} = -\frac{\omega_{\text{rf}}(R/Q)\|F_b\|I_{b,\text{dc}}}{2V_{\text{cav}}}\sqrt{1-\frac{\cos^2\phi_s V_{\text{tot}}^2}{V_{\text{cav}}^2 N_{\text{tot}}^2}}$	$\Delta\omega_{\text{opt}} = -\frac{\omega_{\text{rf}}(R/Q)\|F_b\|I_{b,\text{dc}}\sin\phi_s}{2V_{\text{cav}}}$