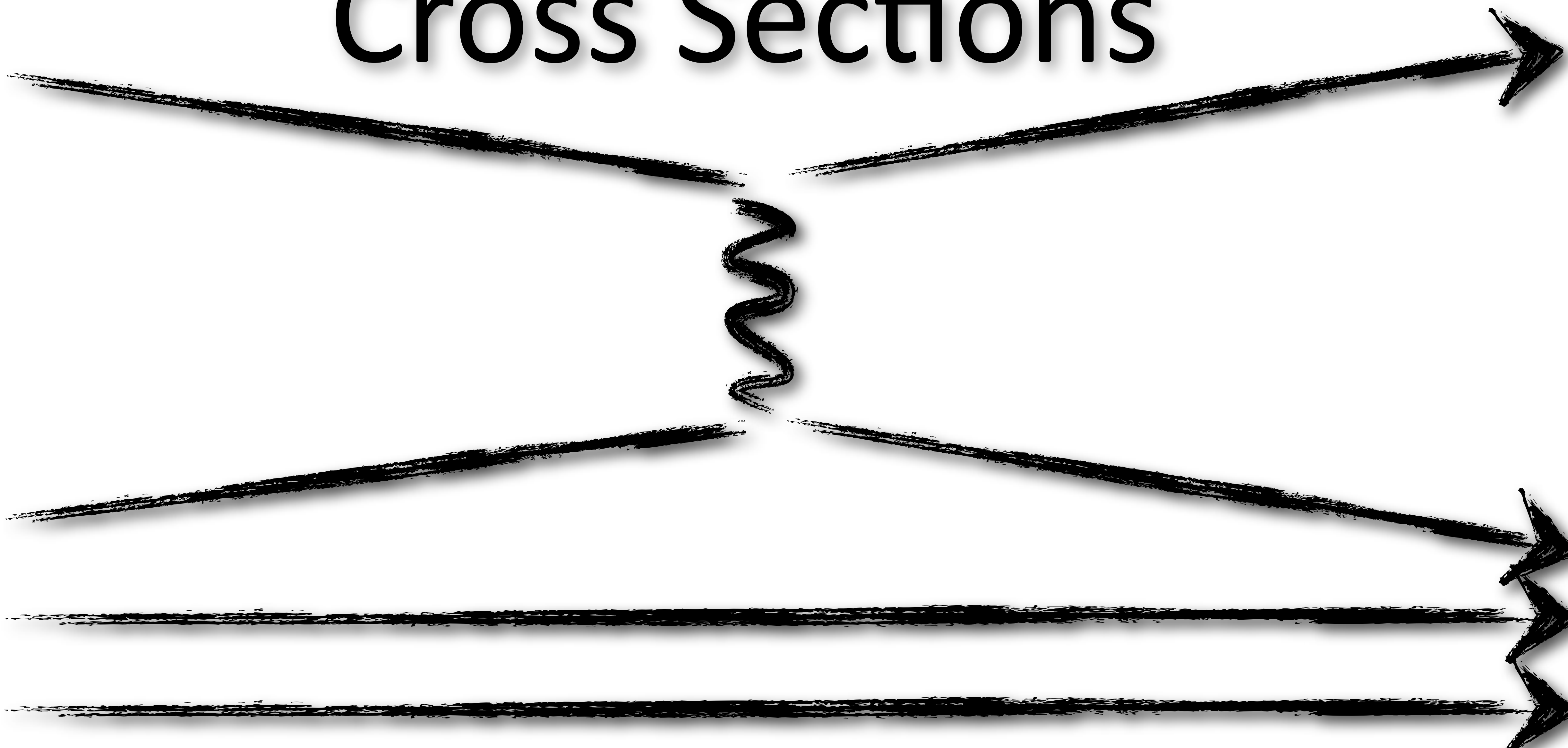
Introduction to

Neutrino Interaction Cross Sections



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International Neutrino Summer School 2011

2011 07 23

Outline

- Introduction to cross section calculations
- Focus on charged current quasi-elastic scattering

  ] Lecture 1

- Introduction to cross section experiments
  - Connections to neutrino oscillations
- Focus on CCQE

  ] Lecture 2

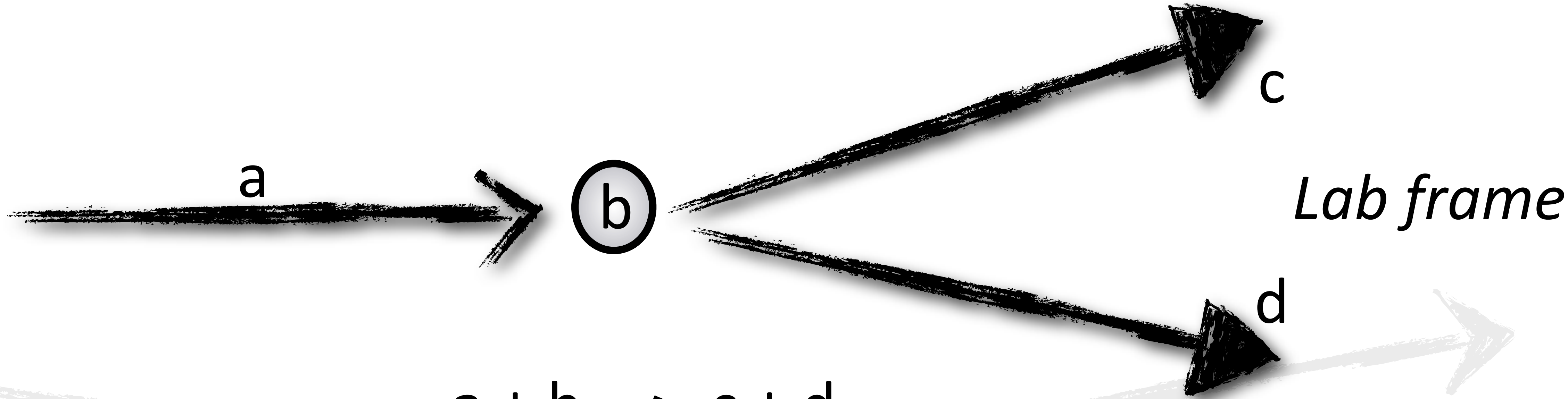
I know I'm supposed to know this already, but

What's a cross section?

- A cross section is a number, with units of area, that gives the probability for any reaction to occur.
- Every particle physics experiment is centred around measuring rates and kinematics of particular reactions.
    - It is the most basic information we can calculate or measure about Nature.
- Thus a cross section sits at the core of everything you will ever do in particle physics.
    - Ever.

Disclaimer

- Since cross sections are so fundamental, lots of work has been done in this area (theory & experiment) by many people over many years.
- It is simply not possible to cover it all!
  - For those who do not work on cross sections, I will try to give you a flavour for why it is an important topic, why it is a difficult topic, and why it is interesting, by focusing on one process.
  - For those who do work on cross sections, I will include my own opinions with the aim of giving you some hope and advice.



$$a + b \rightarrow c + d$$

initial — final

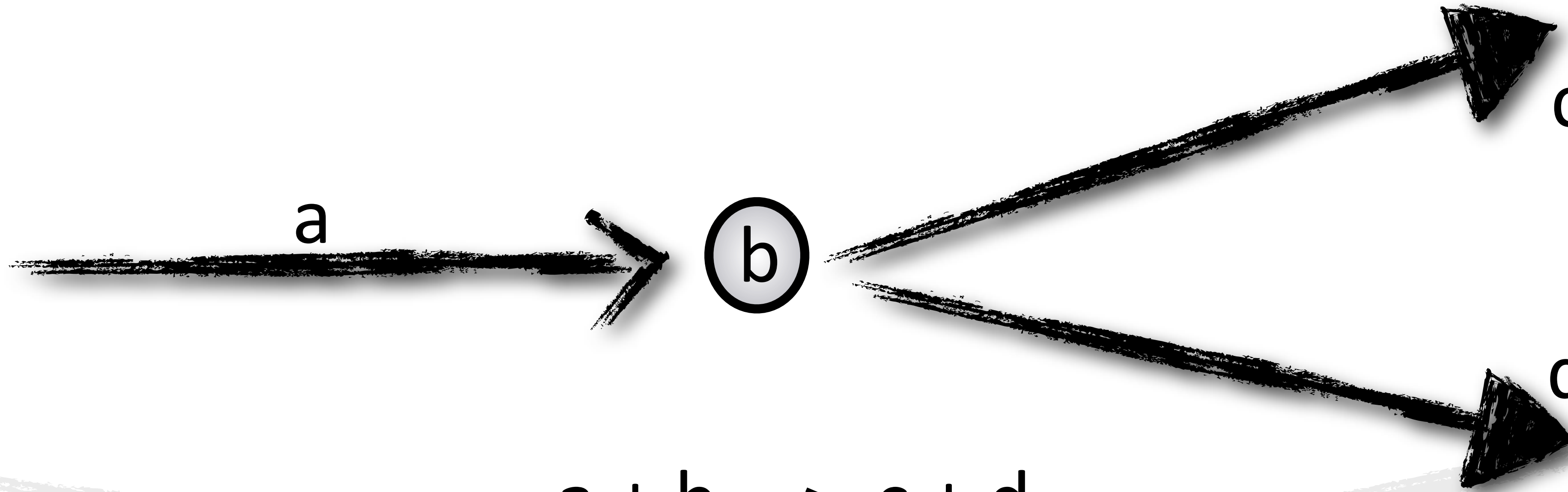
The cross section for this reaction is defined as the transition rate *W* per unit incident flux per target particle.

$\Phi \equiv$ flux of particles per unit time per unit area

$n_a \equiv$ density of particles in the incident beam

$v_i \equiv$ relative velocity of a and b

$$\Phi = n_a v_i$$



$$a + b \rightarrow c + d$$

initial   final

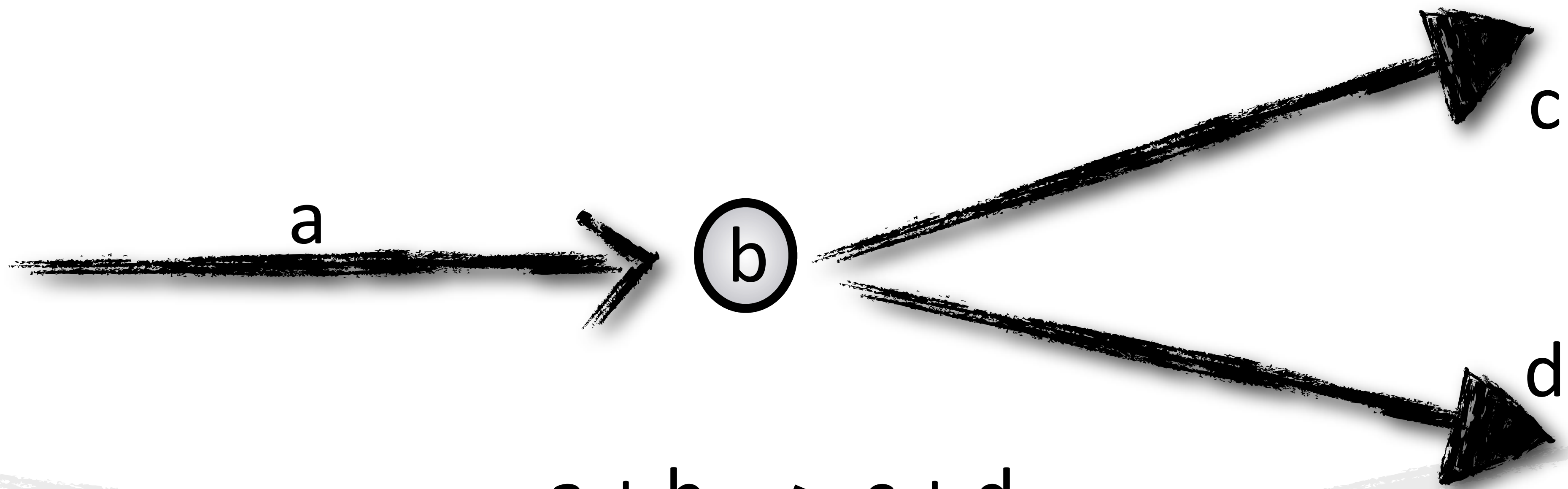
$n_b \equiv$ number of particles in target per unit area

$\sigma \equiv$ cross sectional area of each target particle

then, probability to hit a target is σn_b and the number of interactions per unit time per unit area is $n_a n_b \sigma v_i$

So, transition rate per target particle is

$$W = \sigma\Phi = \sigma n_a v_i$$



$$a + b \rightarrow c + d$$

initial — final

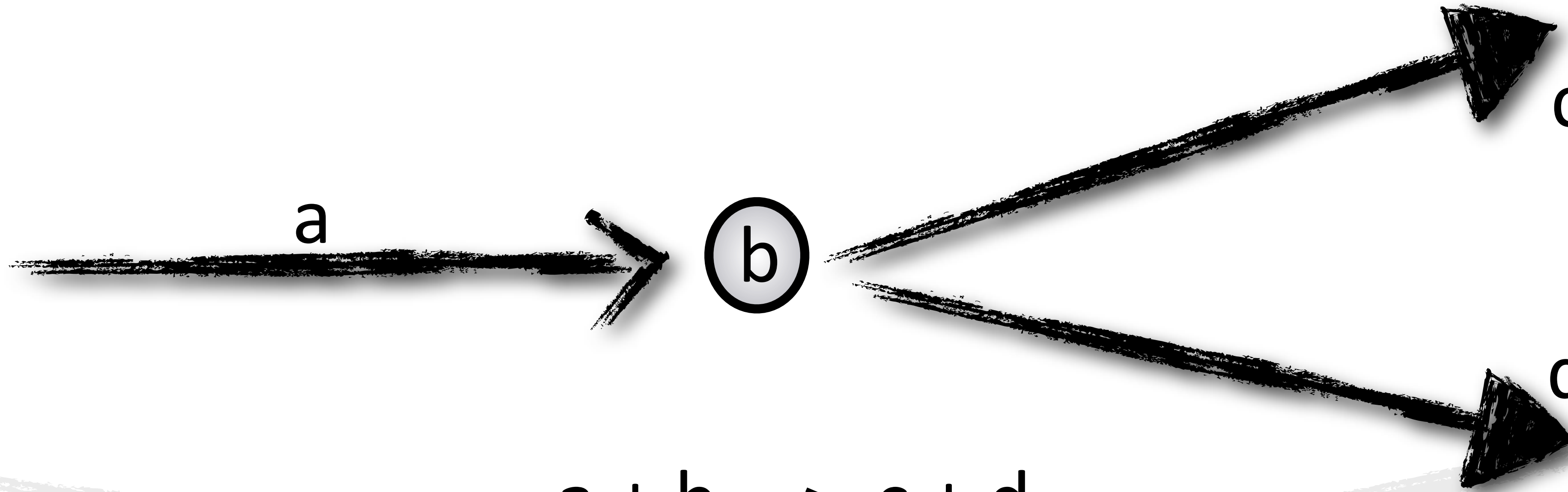
Fermi's "second" Golden Rule:

$$W = 1/h|M_{if}|^2\rho_f$$

M_{if} ≡ matrix element = *"the physics"*

(from perturbation theory: $M_{if} = \int\psi^*_f H'\psi_i d\tau$)

ρ_f ≡ density of states = *"phase space factor"*



$$a + b \rightarrow c + d$$

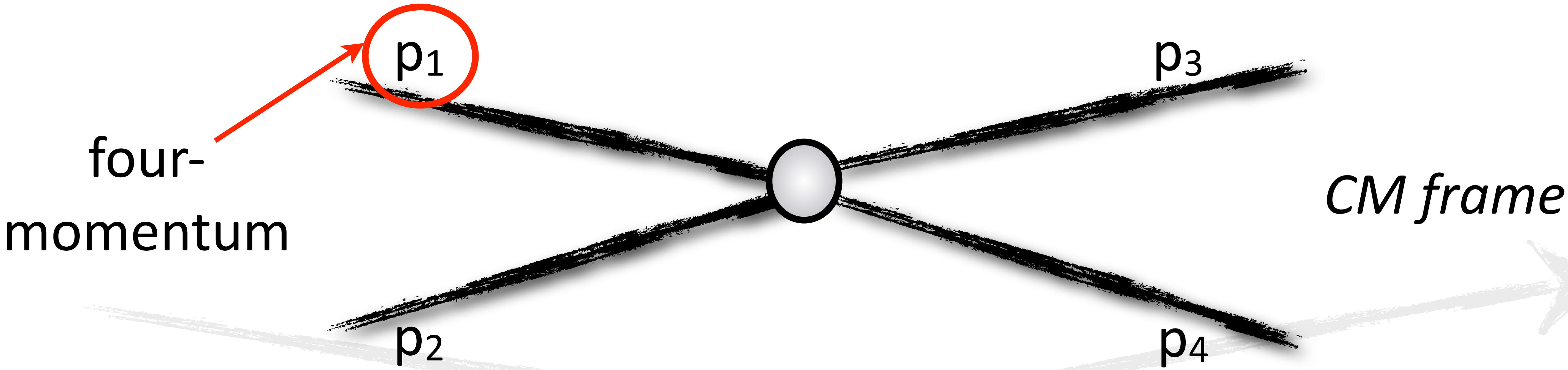
initial final

$$\sigma = W/\Phi \propto |M_{if}|^2 \rho_f$$

$$\rho_f \equiv = dn/dE_{CM} \propto p^2 d\Omega dp/dE_{CM}$$

(dropped final state spin factors)

$$\boxed{\sigma(a+b \rightarrow c+d) \propto |M_{if}|^2 p_f^2}$$



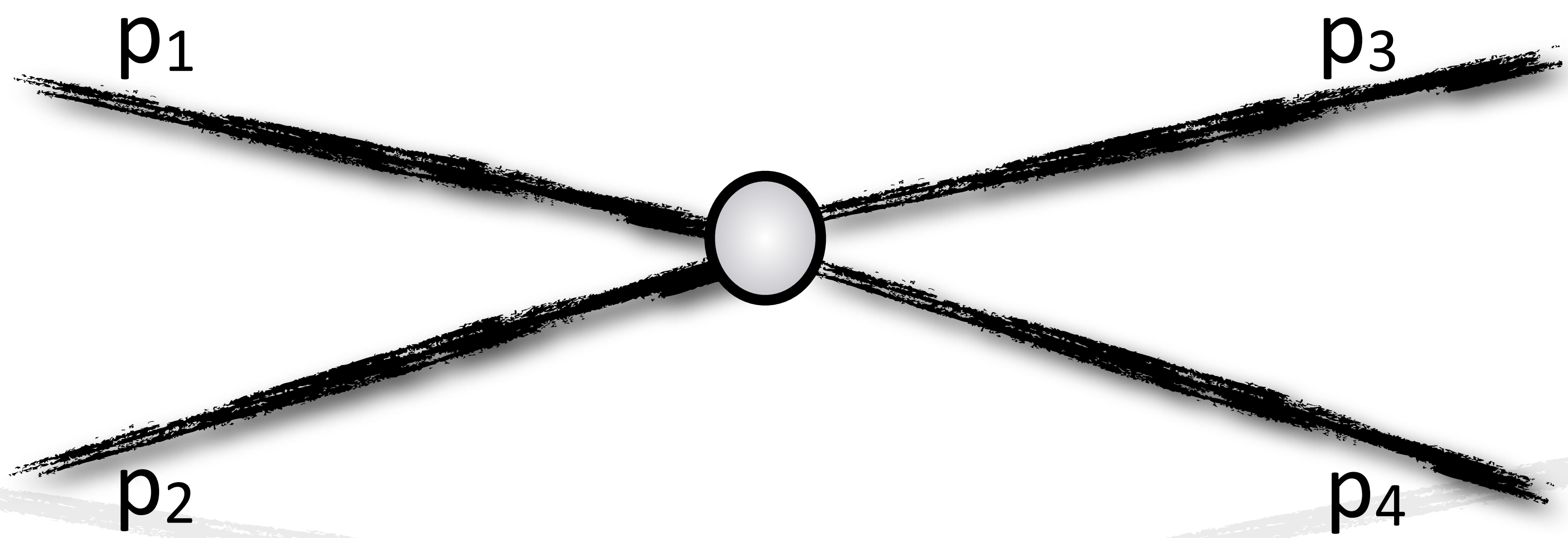
Mandelstam variables

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$
$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$$
$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2$$

invariant mass

momentum transfer

Lorentz invariant quantities!



$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$
$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$$
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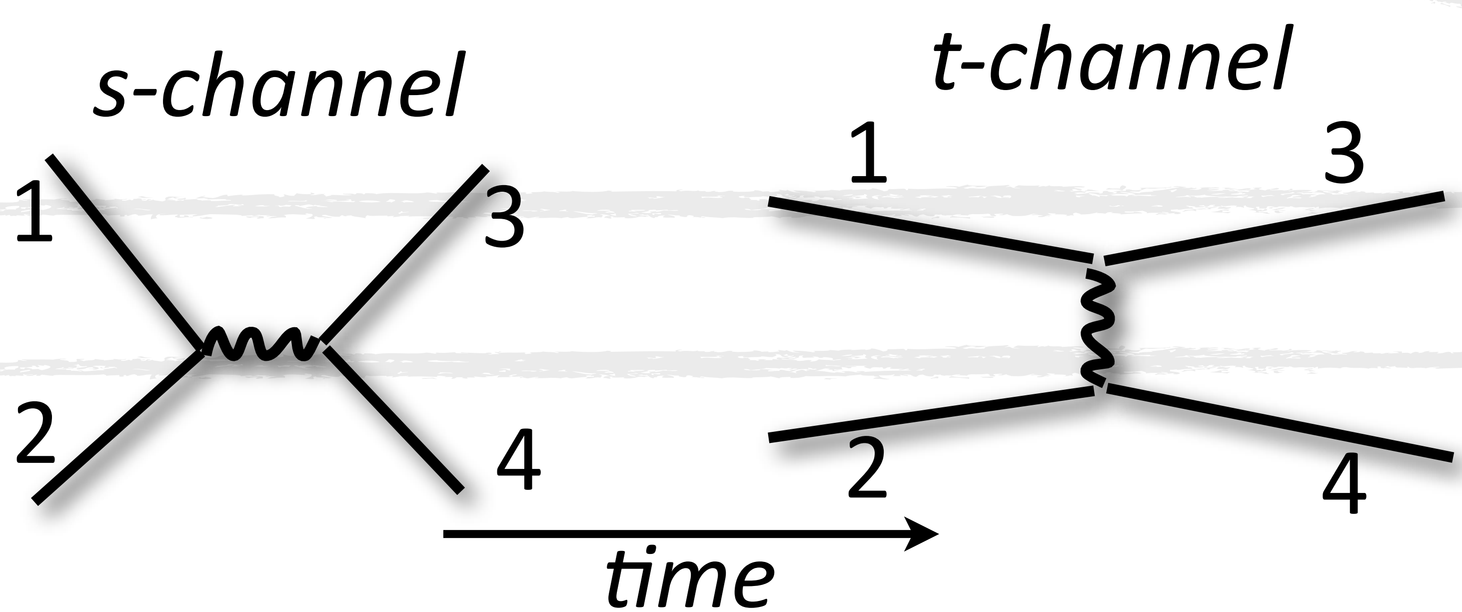
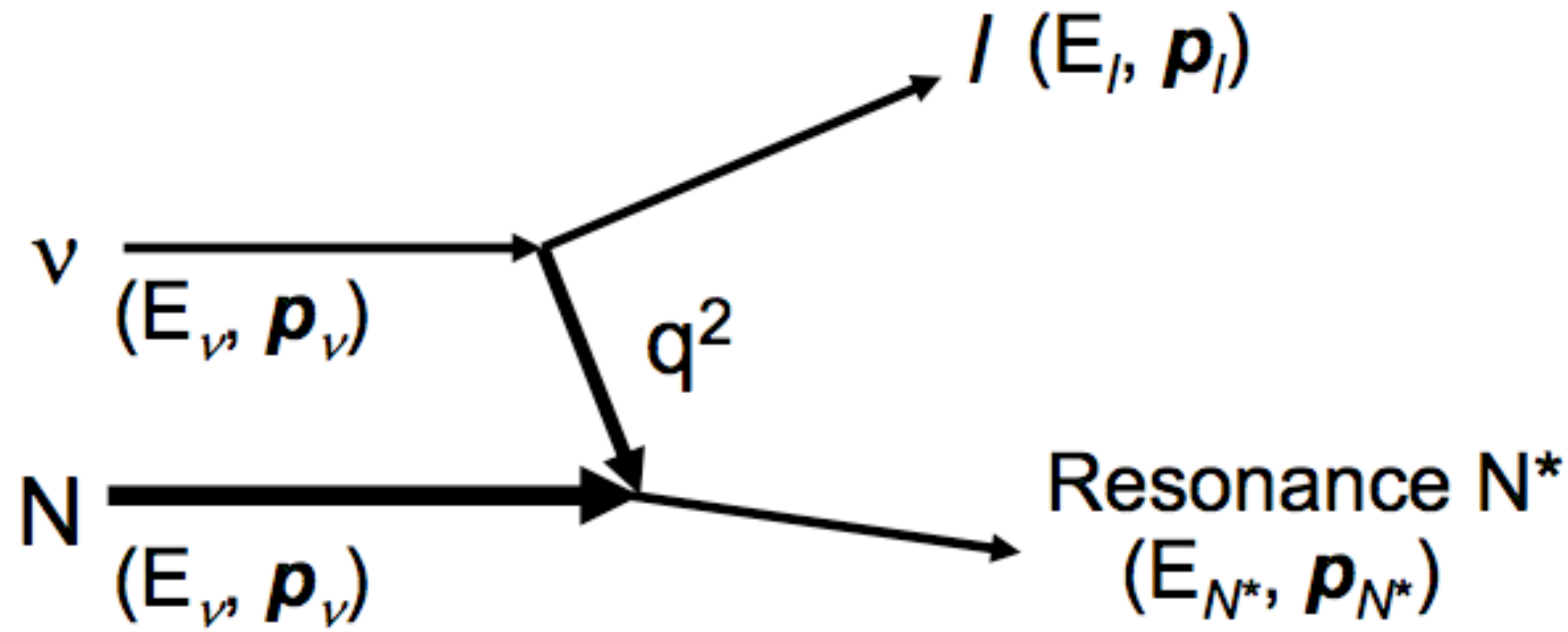


More definitions



q^2 : 4 momentum transfer

$$q^2 \equiv (E_l - E_\nu)^2 - (\boldsymbol{p}_l - \boldsymbol{p}_\nu)^2 \quad (= -Q^2) = t$$

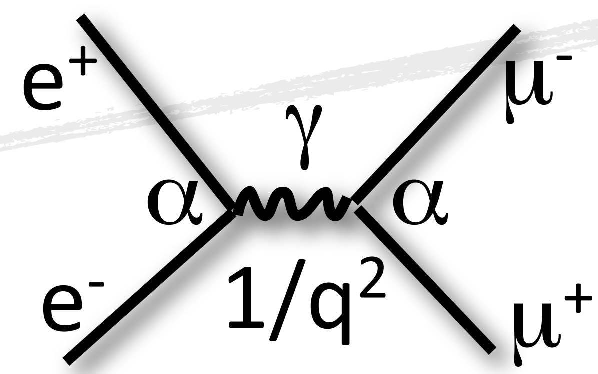
W : Invariant Mass of N*

$$W \equiv \sqrt{E_{N*}^2 - p_{N*}^2}$$

QED interactions

toy example

$\sigma(e^+e^-\rightarrow\mu^+\mu^-)$



$$M_{if} = \frac{4\pi\alpha}{q^2}$$

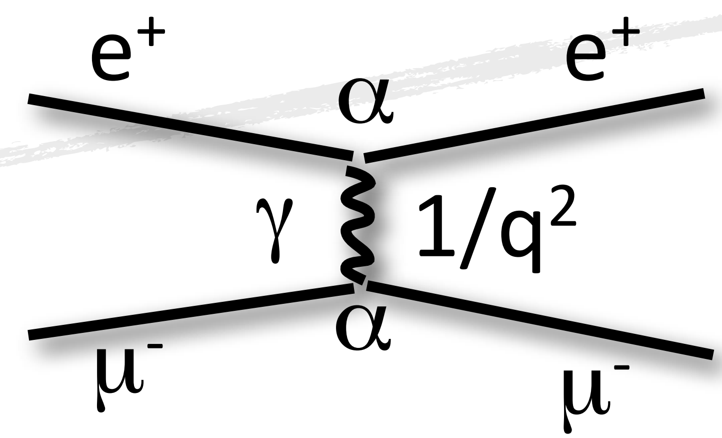
$$\frac{d\sigma}{d\Omega}(e^+e^- \rightarrow \mu^+\mu^-) = \frac{\alpha^2}{4s}(1+\cos^2\theta) = \frac{\alpha^2}{8p^2}\left(\frac{t^2+u^2}{s^2}\right)$$

$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s}$$

QED interactions

toy t-channel

$\sigma(e^+\mu^- \rightarrow e^+\mu^-)$



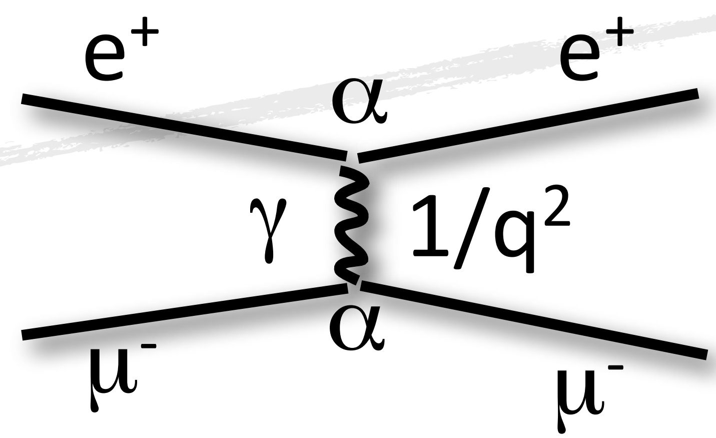
$$\frac{d\sigma}{d\Omega}(e^-\mu^+ \rightarrow e^-\mu^+) = \frac{\alpha^2}{8p^2}\left(\frac{s^2+u^2}{t^2}\right)$$

$$= \frac{\alpha^2}{8p^2\sin^4(\theta/2)}\left[1+\cos^4(\theta/2)\right]$$

QED interactions

toy t-channel

$\sigma(e^+\mu^- \to e^+\mu^-)$



In the muon rest frame

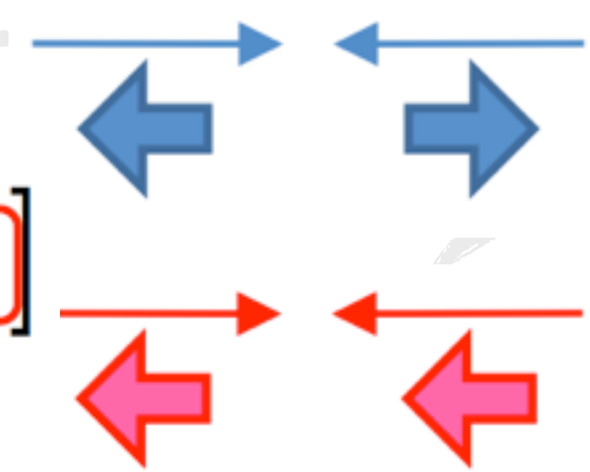
$$E_\mu^{'} = \gamma p(1-\cos\theta), E_e = 2\gamma p, y = \frac{E_\mu^{'}}{E_e} = \frac{1-\cos\theta}{2}$$

"inelasticity"

$$\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2} = 1-y$$

$$d\Omega = 2\pi d(\cos\theta) = 4\pi dy$$

$$\frac{d\sigma}{dy}(e^-\mu^+ \to e^-\mu^+) = \frac{2\pi\alpha^2 s}{q^4}\left[1+(1-y)^2\right]$$



QED → Weak

- Weak scattering has the same general form.
- Some details are very different!

- (We've also only considered interactions between point particles. Scattering from nucleons adds richness…)

QED → Weak

$$M_{if} = \frac{\left(g/\sqrt{2}\right)^2}{q^2 + M_W^2} \approx \frac{g^2}{2M_W^2}$$

massive propagator!

$$\frac{d\sigma}{dq^2}(\nu e^- \to \nu e^-) = \frac{2\pi}{v}\left|M_{if}\right|^2 p^2 \frac{dp}{dE_f}\frac{d\Omega}{dq^2}\frac{2}{(2\pi)^3}$$

$$= \frac{2}{\pi}\left(\frac{g^2}{8M_W^2}\right)^2 = \frac{G^2}{\pi}, \quad G \equiv \frac{\sqrt{2}g^2}{8M_W^2}$$

$G=1.16639(1)\times 10^{-5}\,\mathrm{GeV}^{-2}$

$$\frac{d\sigma}{dy}\sigma(\nu e^- \to \nu e^-) = \frac{G^2}{\pi}s, \quad \sigma(\nu e^- \to \nu e^-) = \frac{G^2}{\pi}s$$

QED → Weak

$$\frac{d\sigma}{dy}\sigma(\nu e^- \to \nu e^-) = \frac{G^2}{\pi}s, \quad \sigma(\nu e^- \to \nu e^-) = \frac{G^2}{\pi}s$$

$y = q^2/4p^2 = q^2/s$

$sdy = dq^2$

$$s = (p_\nu + p_e)^2 = 2m_e E_\nu + m_e^2 \approx 2m_e E_\nu$$

The cross section is proportional to the neutrino energy!

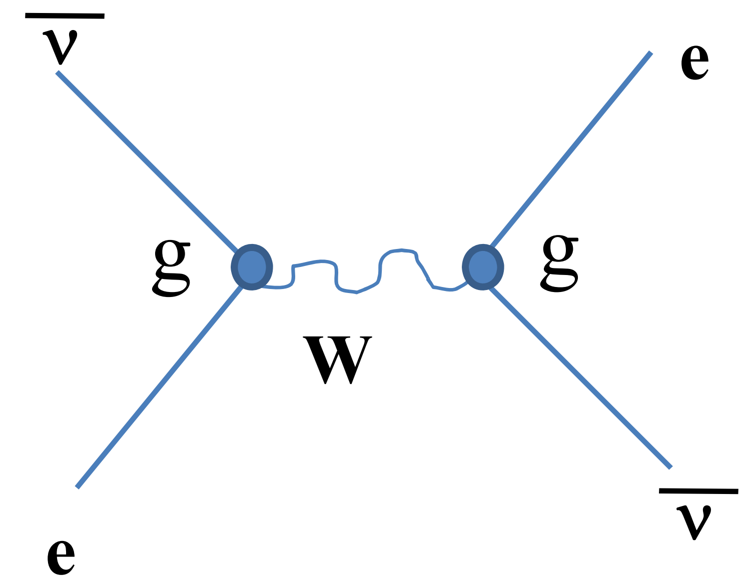
Spin effects

Antineutrino scattering

– s→-t

$$\frac{d\sigma}{dy}\sigma(\bar{\nu}e^- \to \bar{\nu}e^-) = \frac{G^2 s}{\pi}(1-y)^2$$

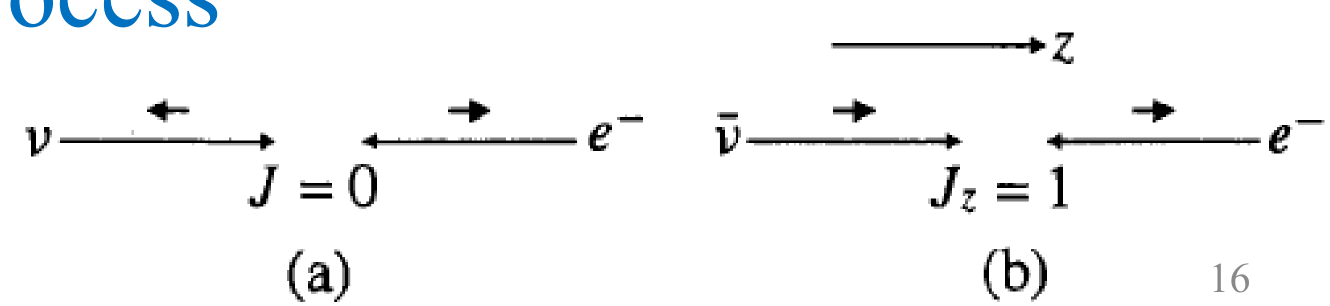
$$\sigma(\bar{\nu}e^- \to \bar{\nu}e^-) = \frac{G^2}{3\pi}s = \frac{1}{3}\sigma(\nu e^- \to \nu e^-)$$



– At $y=(E_e/E_\nu)=1$, the cross section is zero.

– The average energy is $E_\nu/4$. (Homework?)

– Review the QED process



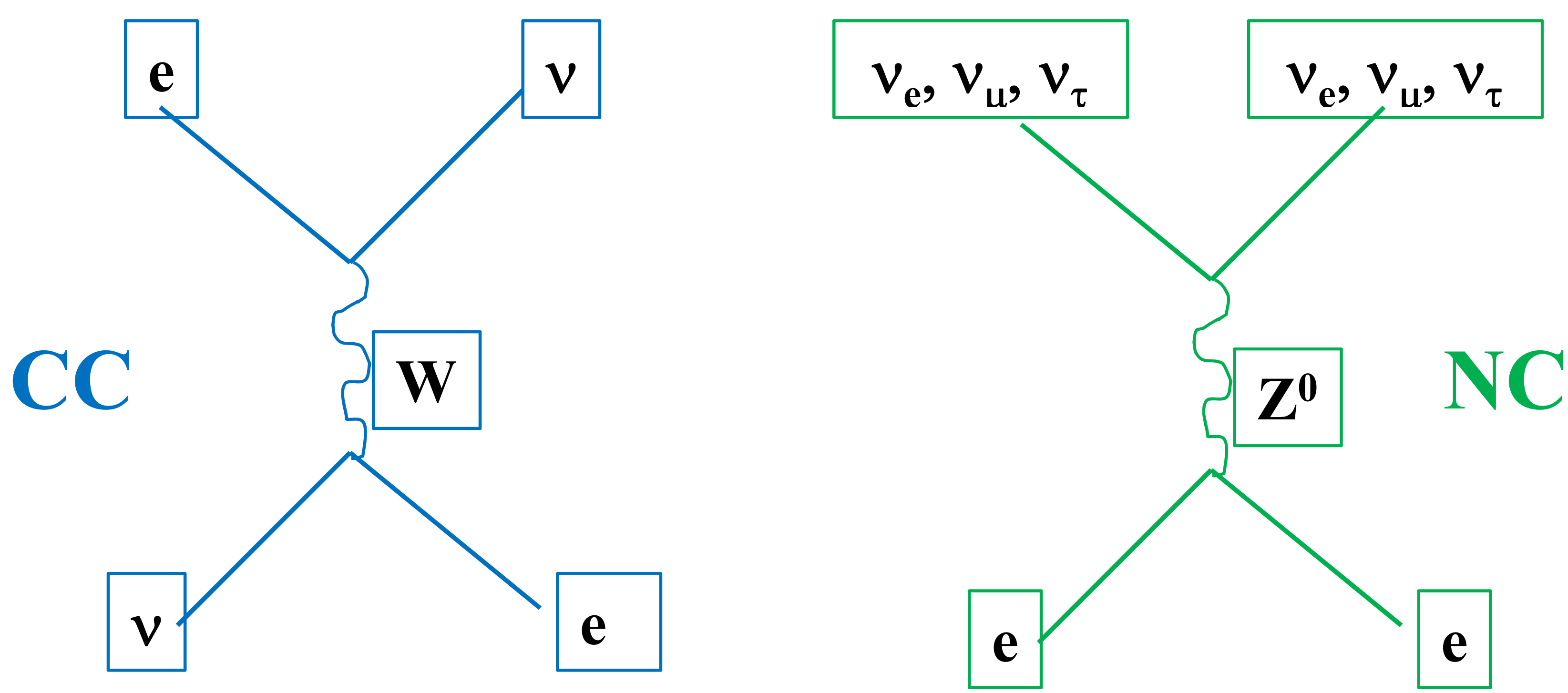
Charged+Neutral Current

$$H_W^{eff} = \frac{G_F}{\sqrt{2}}\left\{\left[\bar{e}\gamma_\mu(1-\gamma_5)\nu_e\right]\left[\bar{\nu}_e\gamma^\mu(1-\gamma^5)e\right]+2\rho\left[\bar{\nu}_l\gamma_\mu\frac{1}{2}(1-\gamma_5)\nu_l\right]\left[\bar{e}\gamma^\mu(g_V-g_A\gamma_5)e\right]\right\}$$

$$-i\frac{g}{\cos\theta_W}(J_\mu^{\;3}-\sin^2\theta_W J_\mu^{\;EM})Z^\mu$$

$$g_V = g_L + g_R = T_3 - 2\sin^2\theta_W\cdot Q = -\frac{1}{2}+2\sin^2\theta_W$$

$$g_A = g_L - g_R = T_3 = -\frac{1}{2} \qquad \sin^2\theta_W = 0.223$$



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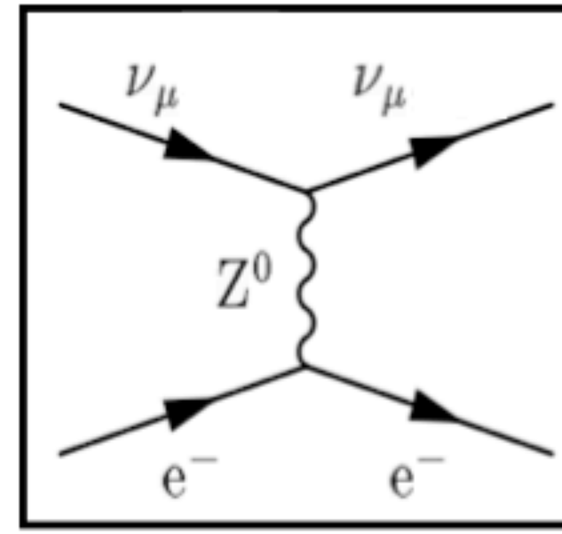
NC couplings

$$\sin^2\theta_W = 0.223$$

	g_L	g_R
e, μ, τ	$-\frac{1}{2}+\sin^2\theta_W$	$\sin^2\theta_W$
ν	$\frac{1}{2}$	0
u,c,t	$\frac{1}{2}-2/3*\sin^2\theta_W$	$-2/3*\sin^2\theta_W$
d,s,b	$-\frac{1}{2}+1/3*\sin^2\theta_W$	$1/3*\sin^2\theta_W$

$$-i\frac{g}{\cos\theta_W}(J_\mu{}^3 - \sin^2\theta_W J_\mu{}^{EM})Z^\mu$$

- process in which we 1st discovered NC's!
- purely-leptonic process, so σ calculation is very straightforward



$$\sigma = \frac{2G_F^2 m_e}{\pi}\left[\left(g_L^2 + \frac{g_R^2}{3}\right)E_\nu - g_L g_R \frac{m_e}{2}\right]$$

$$g_L = \sin^2\theta_W \underset{\mu,\tau}{\overset{e}{\pm}} \frac{1}{2}$$

$$g_R = \sin^2\theta_W$$

some facts

- σ is ~ linear with E_ν (generic feature of point-like scattering)
- $\sigma(\nu_e\, e^-) > \sigma(\nu_{\mu,\tau}\, e^-)$ (ν_e can scatter both by NC & CC)
- σ is small:

$$\sigma \sim s = (E_{CM})^2 = 2m_{target}E_\nu$$

4 orders of magnitude less likely than scattering off nucleons at 1 GeV!

$$\sigma = \frac{2G^2 m_e E_\nu}{\pi}\left[c_L^2 + \frac{1}{3}c_R^2 - \frac{1}{2}c_L c_R \frac{m_e}{E_\nu}\right]$$

$$\sigma(\nu_e e^- \to \nu_e e^-) = 0.952 \times 10^{-43}(E_\nu/10\,\mathrm{MeV})\ \mathrm{cm}^2$$
$$\sigma(\bar{\nu}_e e^- \to \bar{\nu}_e e^-) = 0.399 \times 10^{-43}(E_\nu/10\,\mathrm{MeV})\ \mathrm{cm}^2$$
$$\sigma(\nu_\mu e^- \to \nu_\mu e^-) = 0.155 \times 10^{-43}(E_\nu/10\,\mathrm{MeV})\ \mathrm{cm}^2$$
$$\sigma(\bar{\nu}_\mu e^- \to \bar{\nu}_\mu e^-) = 0.134 \times 10^{-43}(E_\nu/10\,\mathrm{MeV})\ \mathrm{cm}^2$$

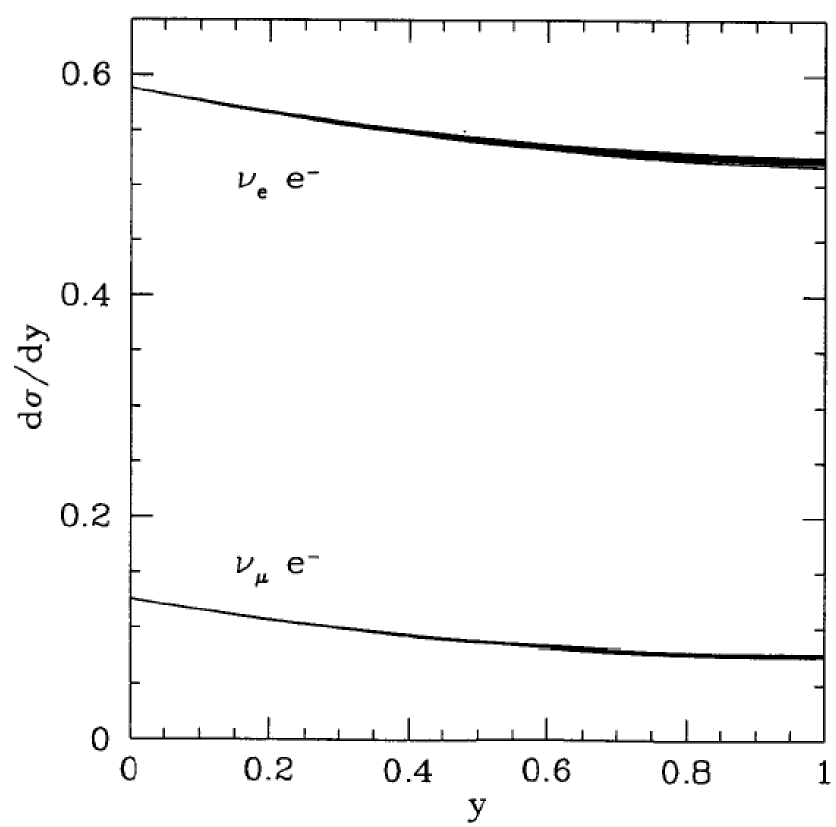


Fig. 3.5. Differential cross section of $\nu_e e^-$ and $\nu_\mu e^-$ scattering for $E = 5$–$10\,\mathrm{MeV}$. The curve is moving upwards (downwards) slightly with the energy of neutrinos for $\nu_e e$ ($\nu_\mu e$) scattering.

- Detection of anti-neutrinos.
  - Is the cross section smaller than ν+e→ν+e scattering?
    - $\sigma(\nu_e+e\rightarrow \nu_e+e) \sim 3\times \sigma(\overline{\nu}_e+e\rightarrow \overline{\nu}_e+e)$
  - We can use a proton in H_2O or CH(KamLAND, ..)!
    - Inverse beta decay: $\overline{\nu}p\rightarrow e^+n$.
    - Native expectation (assuming a proton as a Dirac particle):

$$\sigma(\overline{\nu}e^- \rightarrow \overline{\nu}e^-) = \frac{G^2}{3\pi}s \Rightarrow \sigma(\overline{\nu}p \rightarrow e^+n) \sim \frac{G^2}{3\pi}s$$

$$s = (p_\nu + p_{e(p)})^2 = 2m_{e(p)}E_\nu + m_{e(p)}^2 \approx 2m_{e(p)}E_\nu$$

    - In reality, the proton is NOT a Dirac particle.

$$H_W^{eff} = \frac{G_F \cos\theta_C}{\sqrt{2}}\left[\overline{e}\gamma_\mu(1-\gamma_5)\nu_e\right]\left[\overline{u}(p')\gamma^\mu(1-g_A\gamma^5)u(p)\right]$$

Parameterising the non-point-like nucleons

- **Form Factor of the proton:**
  - Fourier transformation of the charge distribution.
  - In the case of exponential distribution:

    ρ (r)=ρ(0)exp(-mr)

$$F\left(|q|^2\right) = N\int e^{-mr} e^{i\vec{q}\cdot\vec{x}} d^3x = N\int r^2 dr e^{-mr}(2\pi)\int_{-1}^{1} d(\cos\theta) e^{i|q|r\cos\theta}$$

$$= \frac{2\pi N}{i|q|}\int_0^\infty r dr\left[e^{-(m-i|q|)r} - e^{-(m+i|q|)r}\right] = \frac{8\pi N}{m^3\left(1+|q|^2/m^2\right)^2}$$

Normalization: $N\int e^{-mr} d^3x = 1 \Rightarrow N = m^3/8\pi$

$$F\left(q^2\right) = \frac{1}{\left(1-\frac{q^2}{m^2}\right)^2}$$

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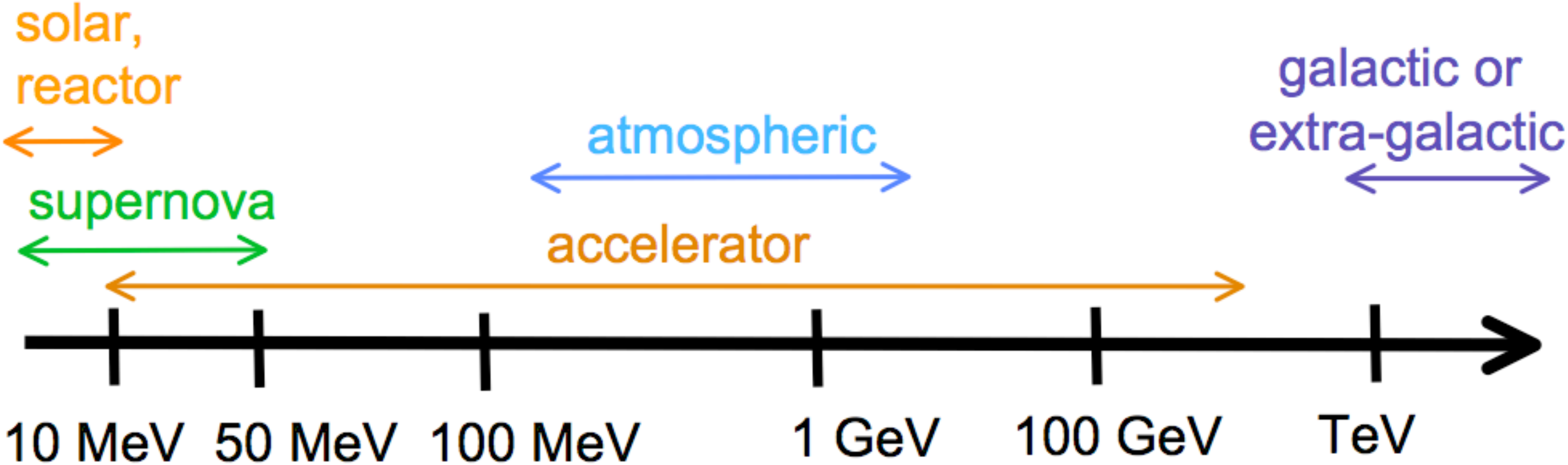
Energy dependence



- **target description** is different depending on the ν energy

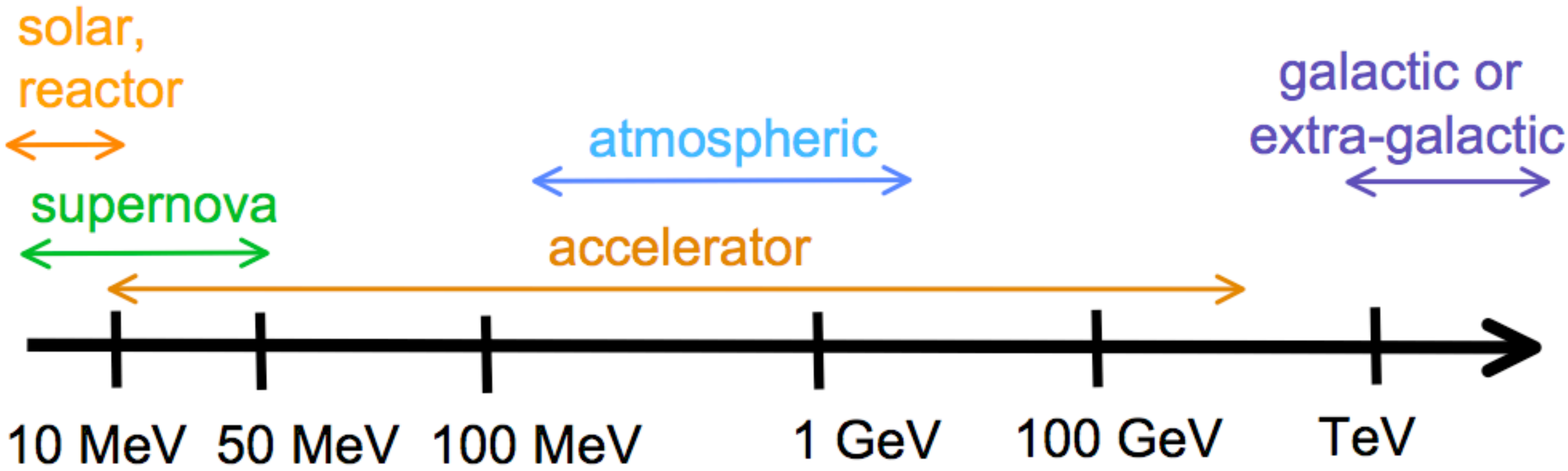
ν-nucleon ·····> **ν-quark**

elastic scattering
(nucleon form factors)

can also create
resonances
(another type of inelastic interaction)

inelastic scattering
(parton density functions)

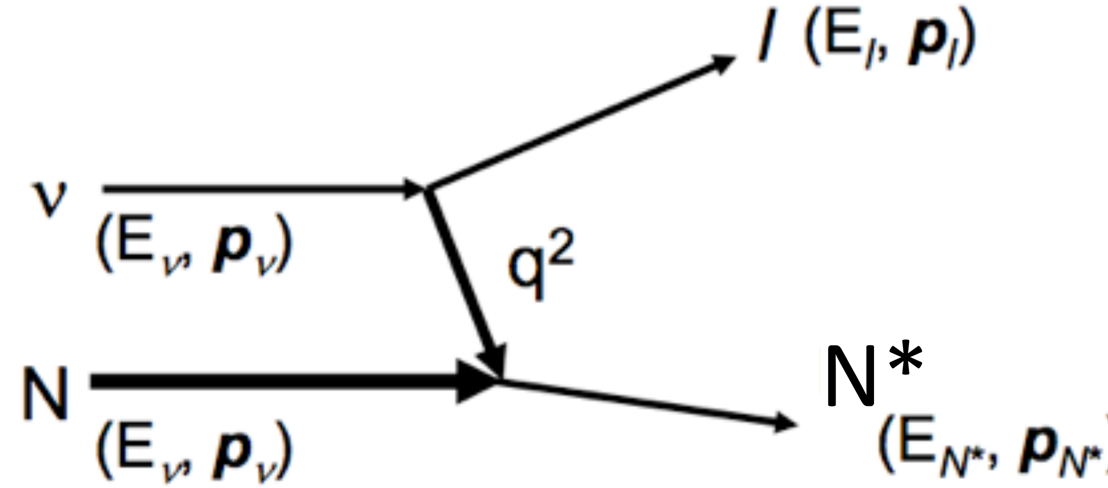
Energy dependence



- also, treatment of **nuclear effects** is energy dependent …

shell model, RPA, EFT → impulse approximation (Fermi Gas, spectral functions, etc.) → quark parton model

CCQE scattering



C.H. Llewellyn Smith, Phys. Rep. **3C**, 261 (1972)

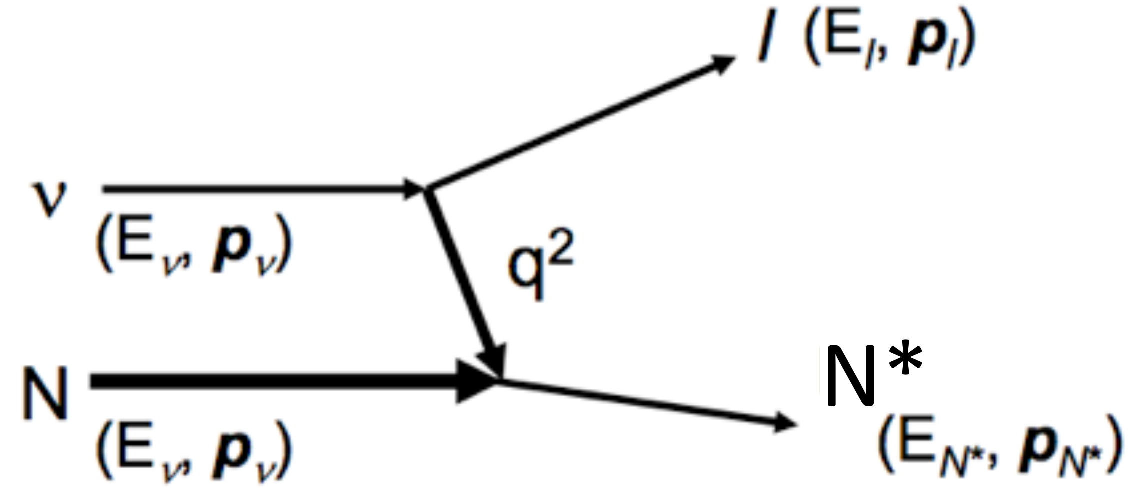
$$d\sigma = \frac{G_F^2 \cos^2\theta_C}{2} 2\pi L^{\mu\nu} W_{\mu\nu} \frac{d^3p}{(2\pi)^3}$$

leptonic tensor

$$L^{\mu\nu} = \frac{1}{2\varepsilon_i \varepsilon} \mathrm{Tr}\left[\gamma\cdot k\ \gamma^{\mu}\ (1 \mp \gamma^5)\ \gamma\cdot k_i\ \gamma^{\nu}\right]$$

easy to calculate, well-known

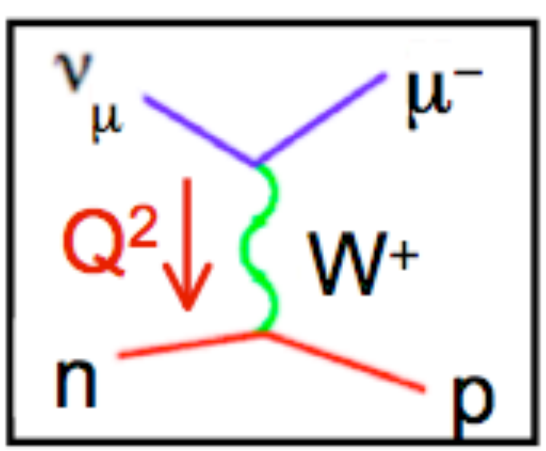
CCQE scattering



$$d\sigma = \frac{G_F^2 \cos^2\theta_C}{2} 2\pi L^{\mu\nu} W_{\mu\nu} \frac{d^3p}{(2\pi)^3}$$

hadronic tensor

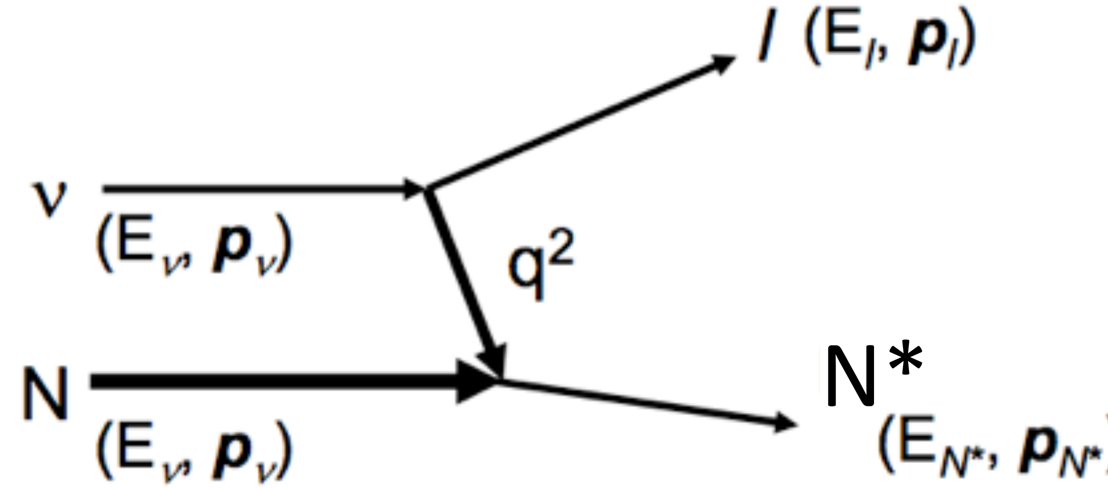
$$W^{\mu\nu}(\omega, q) = \sum\!\!\!\!\!\!\int_{\mathrm{f}} \langle \Psi_{\mathrm{f}} \mid J^\mu(\boldsymbol{q}) \mid \Psi_0 \rangle \times \langle \Psi_0 \mid J^{\nu\dagger}(\boldsymbol{q}) \mid \Psi_{\mathrm{f}} \rangle \, \delta(E_0 + \omega - E_{\mathrm{f}})$$



form factors
contains all of
the information
on the target

$$j^\mu = [F_1^{\mathrm{V}}(Q^2)\gamma^\mu + i\frac{\kappa}{2M}F_2^{\mathrm{V}}(Q^2)\sigma^{\mu\nu}q_\nu - \mathrm{F}_\mathrm{A}(Q^2)\gamma^\mu\gamma^5 + F_\mathrm{P}(Q^2)q^\mu\gamma^5]\tau^\pm$$

CCQE scattering

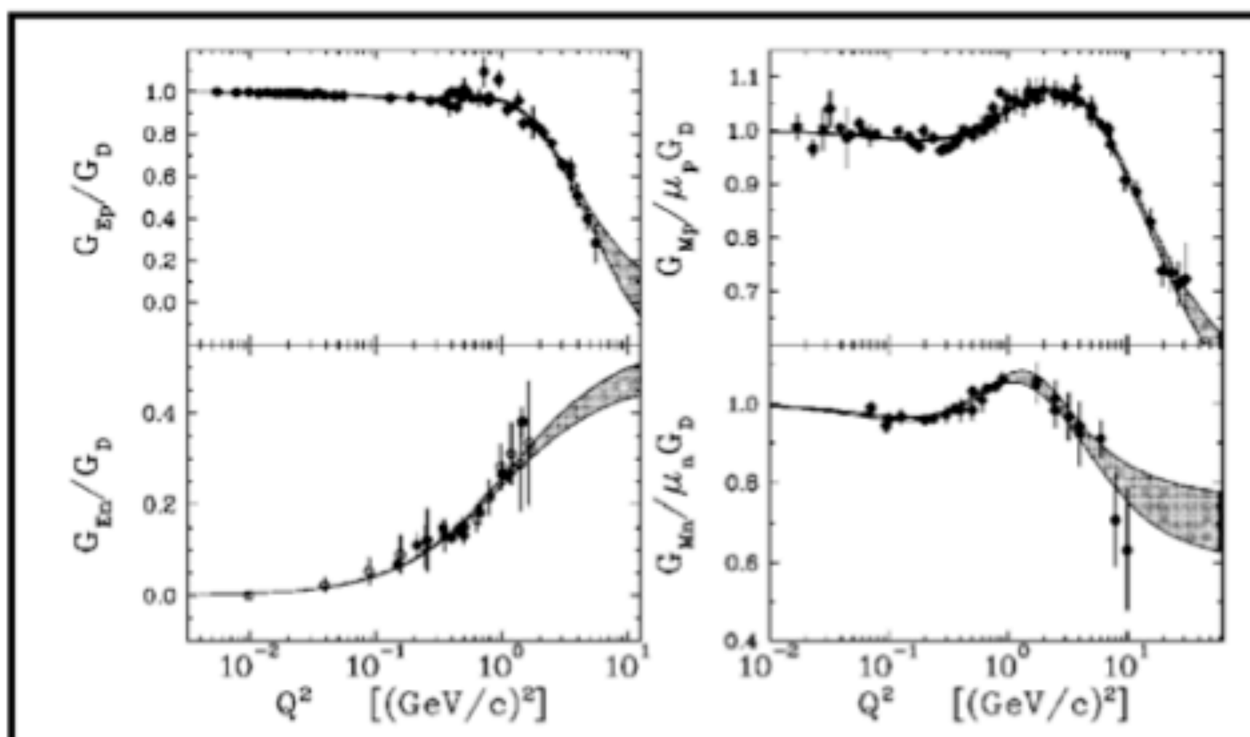


- FFs are not calculable, need to measure experimentally

$$
\begin{aligned}
j^\mu &= \left[F_1^{\rm V}(Q^2)\gamma^\mu + i\frac{\kappa}{2M}F_2^{\rm V}(Q^2)\sigma^{\mu\nu}q_\nu\right.\\
&\left. - F_A(Q^2)\gamma^\mu\gamma^5 + F_{\rm P}(Q^2)q^\mu\gamma^5\right]\tau^\pm
\end{aligned}
$$

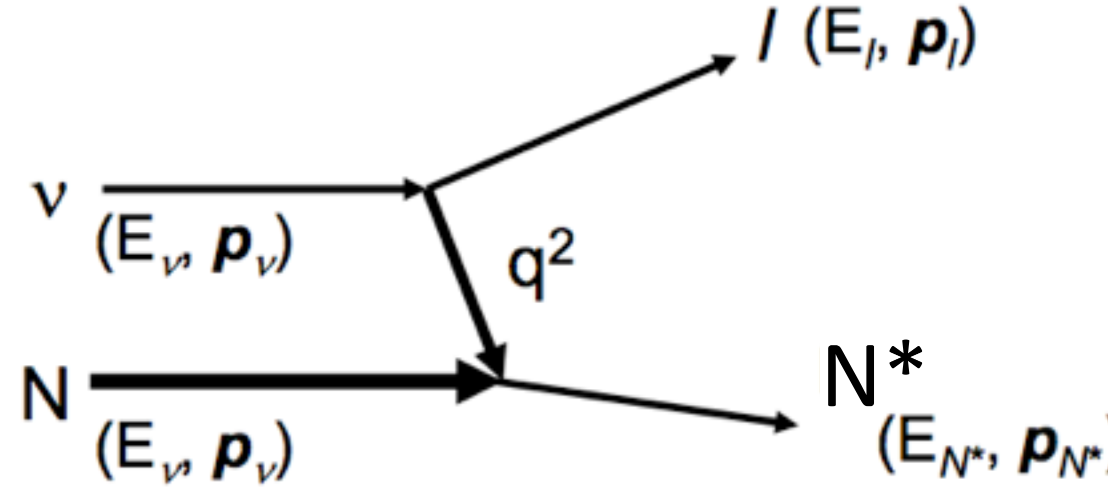
vector form factors

- proton is not point-like but is an extended object with some charge distribution
- vector part can be checked in e⁻ elastic scattering (well known, under control)



J.J. Kelly, Phys. Rev. **C70**, 068202 (2004)

CCQE scattering



- FFs are not calculable, need to measure experimentally

$$j^\mu = [F_1^V(Q^2)\gamma^\mu + i\frac{\kappa}{2M}F_2^V(Q^2)\sigma^{\mu\nu}q_\nu$$
$$- F_A(Q^2)\gamma^\mu\gamma^5 + F_P(Q^2)q^\mu\gamma^5]\tau^\pm$$

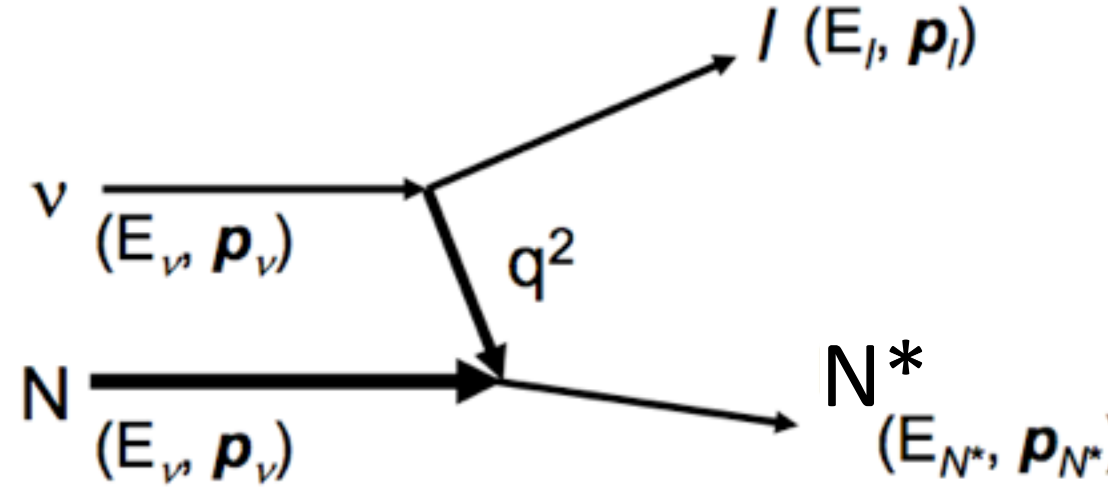
pseudoscalar form factor

contribution enters as $(m_l/M)^2$

small for ν_e, ν_μ

- since F_P is small and know F_V from e^- scattering, σ is then determined at these energies … except for F_A …

CCQE scattering



- FFs are not calculable, need to measure experimentally

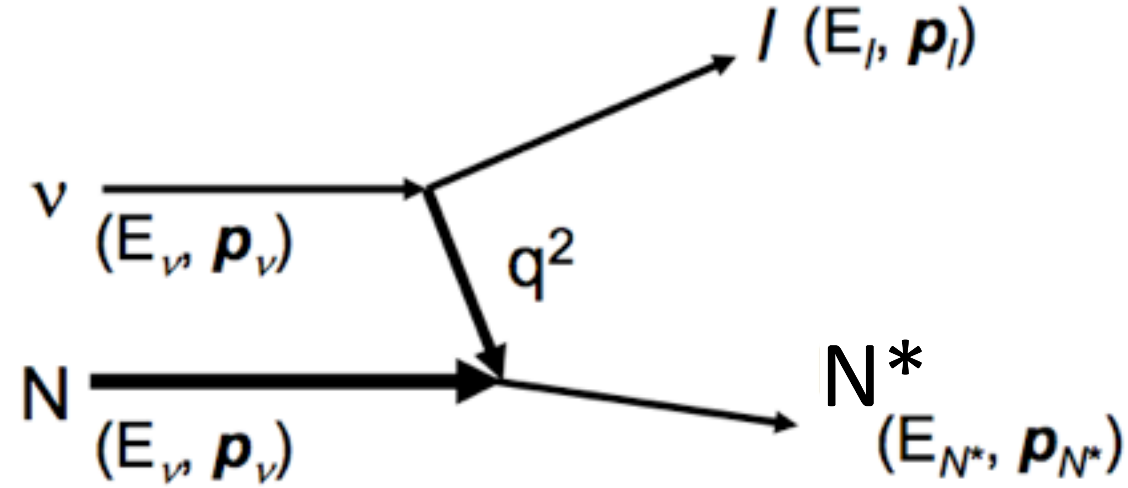
$$j^\mu = [F_1^V(Q^2)\gamma^\mu + i\frac{\kappa}{2M}F_2^V(Q^2)\sigma^{\mu\nu}q_\nu - F_A(Q^2)\gamma^\mu\gamma^5 + F_P(Q^2)q^\mu\gamma^5]\tau^\pm$$

axial form factor

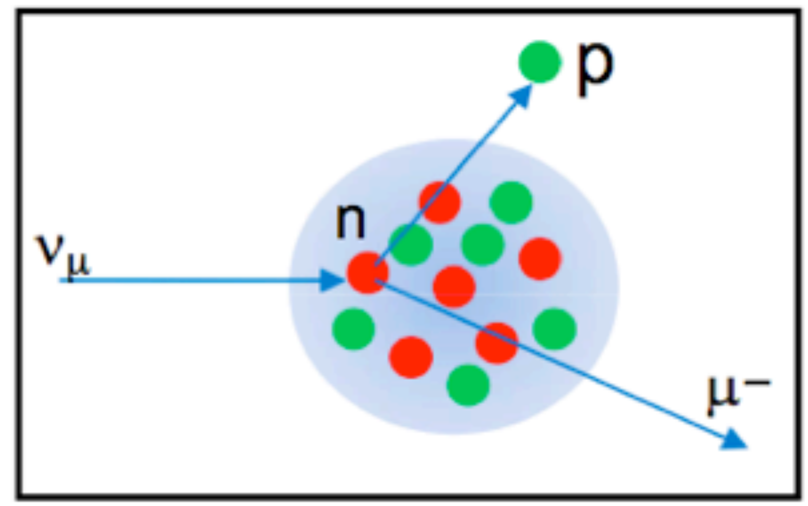
$$F_A(Q^2) = \frac{1.267}{(1+Q^2/M_A^2)^2}$$

$F_A(Q^2=0)$ determined from β decay
(same value saw earlier for IBD)

CCQE scattering



- in a nucleus, target nucleon has some initial momentum which modifies the observed scattering

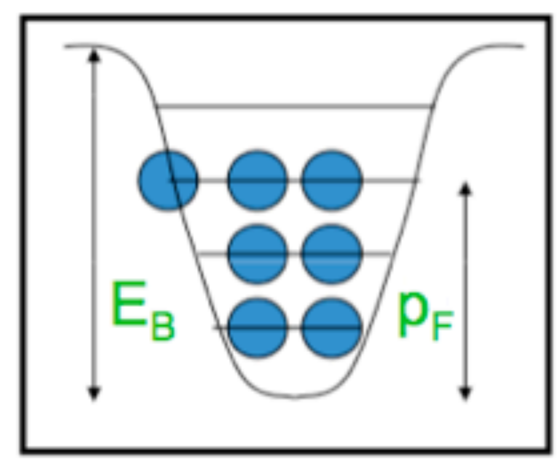


- hadronic tensor now an integral over initial nucleon states

$$W_A^{\mu\nu} = \frac{1}{2}\int d^3p\, dE\, P(\mathbf{p}, E) \frac{1}{4\, E_{|\mathbf{p}|}\, E_{|\mathbf{p}+\mathbf{q}|}} W^{\mu\nu}(\tilde{p}, \tilde{q})$$

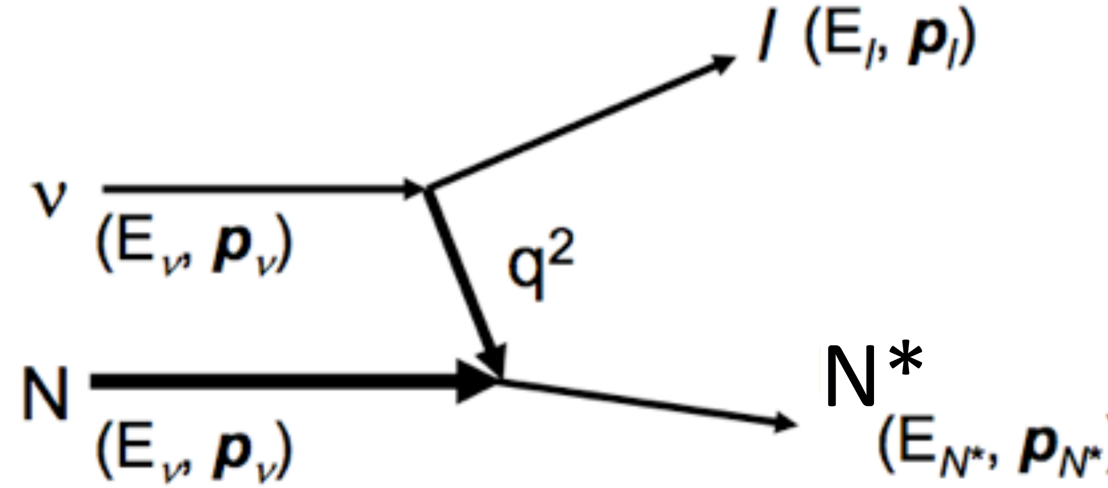
- simplest: **Fermi Gas model**
  (2 free parameters)

  p_F=220 MeV/c (^{12}C)
  E_B=25 MeV

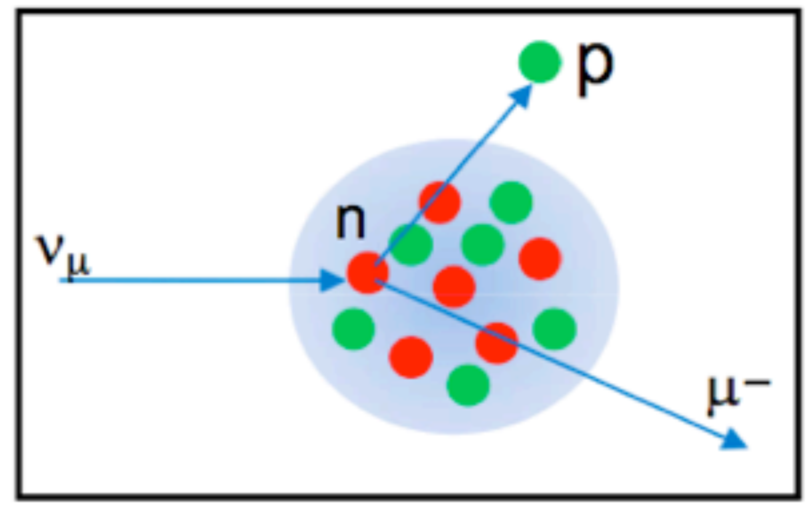


$$P_{RFGM}(\mathbf{p}, E) = \left(\frac{6\,\pi^2\, A}{p_F^3}\right) \theta(p_F - \mathbf{p})\, \delta(E_\mathbf{p} - E_B + E)$$

CCQE scattering

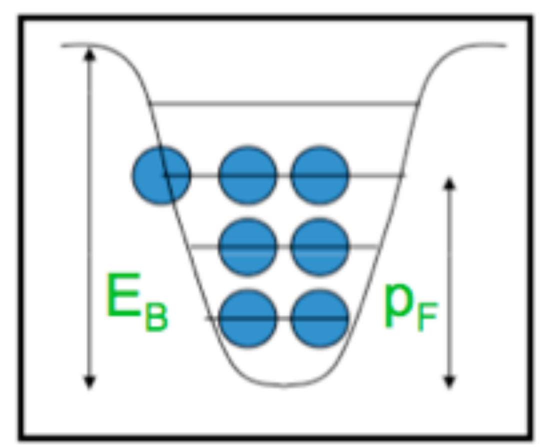


- in a nucleus, target nucleon has some initial momentum which modifies the observed scattering



- hadronic tensor now an integral over initial nucleon states

$$W_A^{\mu\nu} = \frac{1}{2}\int d^3p\, dE\, P(\mathbf{p}, E) \frac{1}{4\, E_{|\mathbf{p}|}\, E_{|\mathbf{p+q}|}} W^{\mu\nu}(\tilde{p}, \tilde{q})$$

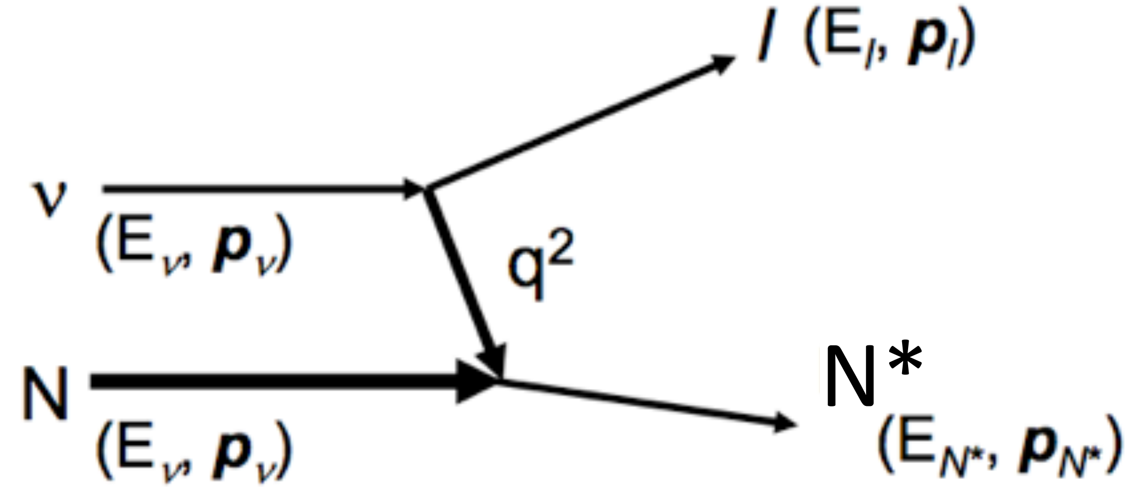


- simplest: **Fermi Gas model** (2 free parameters)

p_F=220 MeV/c (^{12}C)
E_B =25 MeV

- energy transfer > E_B
- final state: p_p > p_F (Pauli blocking)

CCQE scattering



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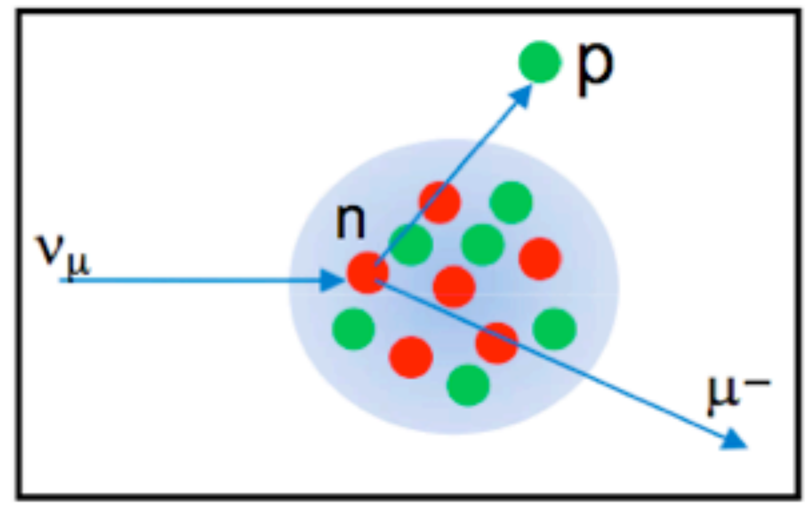

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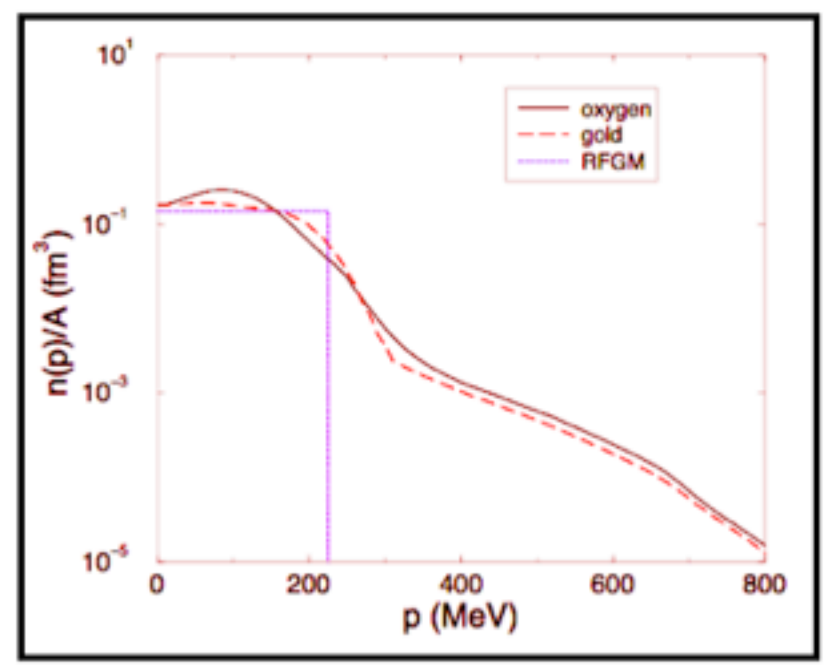
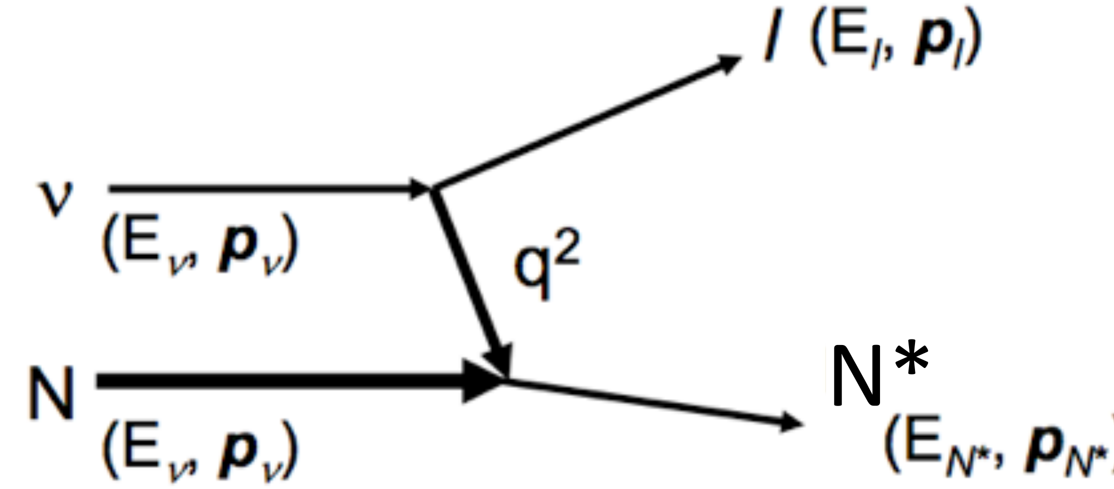
$$W_A^{\mu\nu} = \frac{1}{2}\int d^3p\, dE\, P(\mathbf{p}, E) \frac{1}{4\, E_{|\mathbf{p}|}\, E_{|\mathbf{p}+\mathbf{q}|}} W^{\mu\nu}(\tilde{p}, \tilde{q})$$

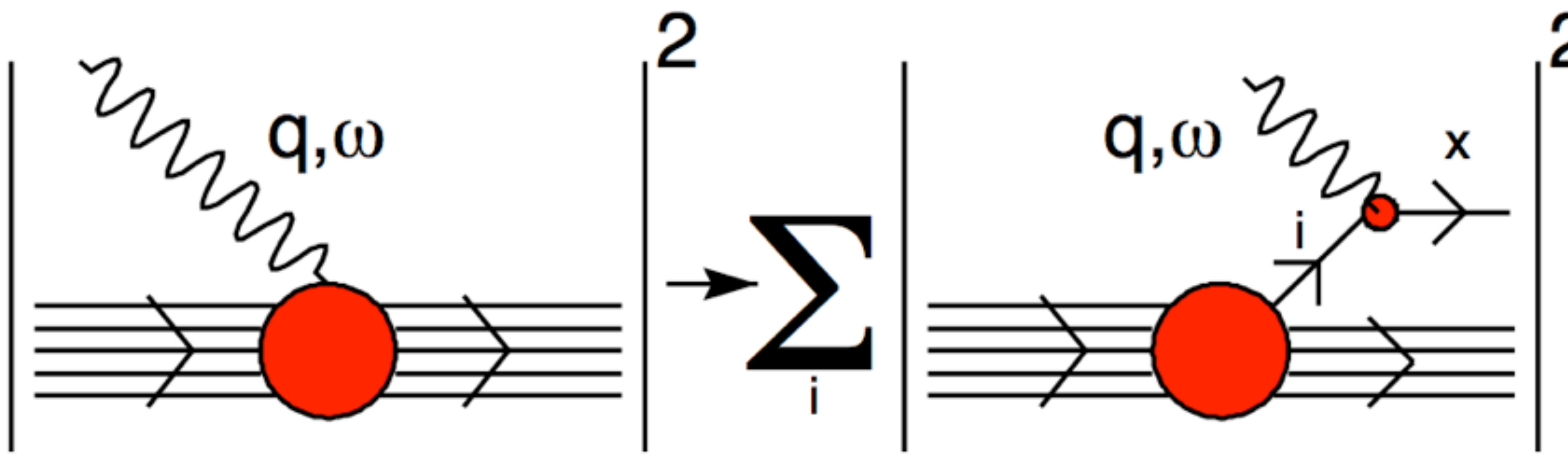
- simplest: **Fermi Gas model**
- more realistic: **spectral functions**
  superscaling



CCQE scattering



Impulse approximation

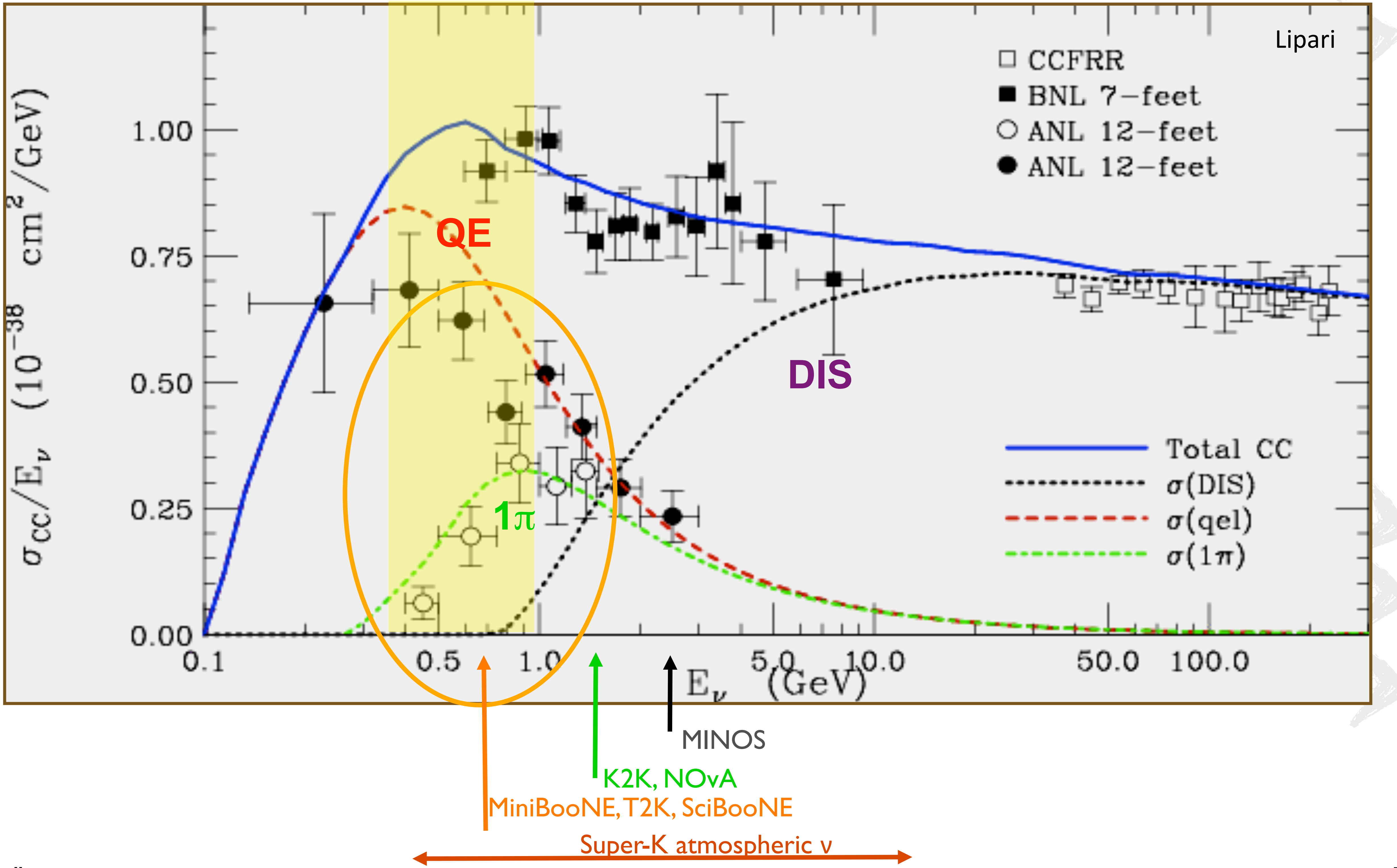


CCQE Summary

- Neutrino-nucleus cross sections are complicated by hadronic vertex
- Use form factors to parameterise distributions of nucleonic (and nuclear) "charge"
- No theory predicts these, so they must be measured
- We use several approximations in modelling rates and kinematics
  - Simplistic nuclear model
  - Impulse approximation

Energy range

Neutrino oscillation searches drive need for better cross section knowledge

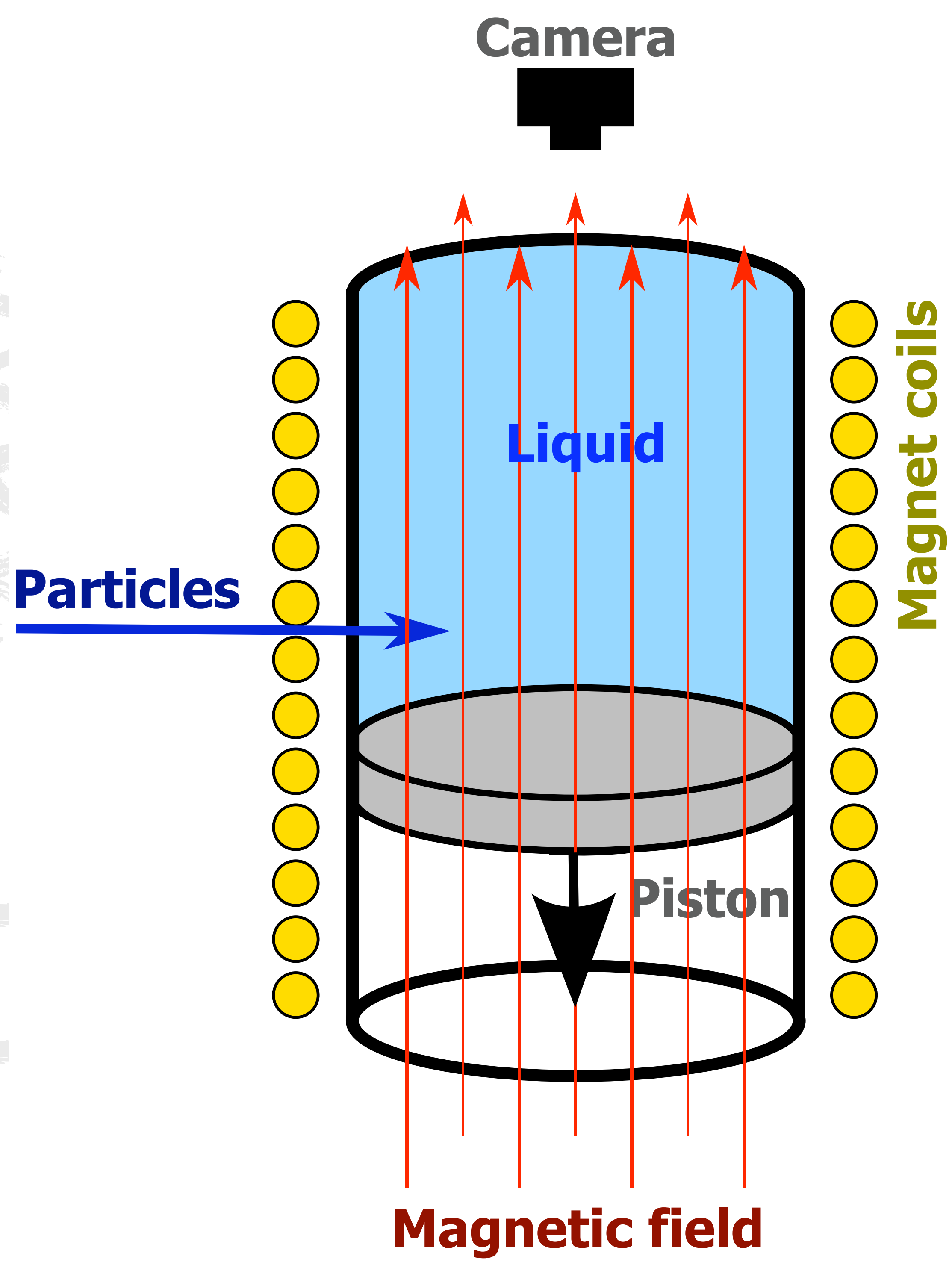


Past Experiments

- Bubble chambers
  - First measurements
- Conventional neutrino beams

Bubble Chambers

- Super heated liquid
  - Ionisation creates bubbles
- External trigger
  - Cameras
- Very good position resolutions
- Slow reconstruction - human hand scanning
  - Extremely limited statistics



Bubble Chambers

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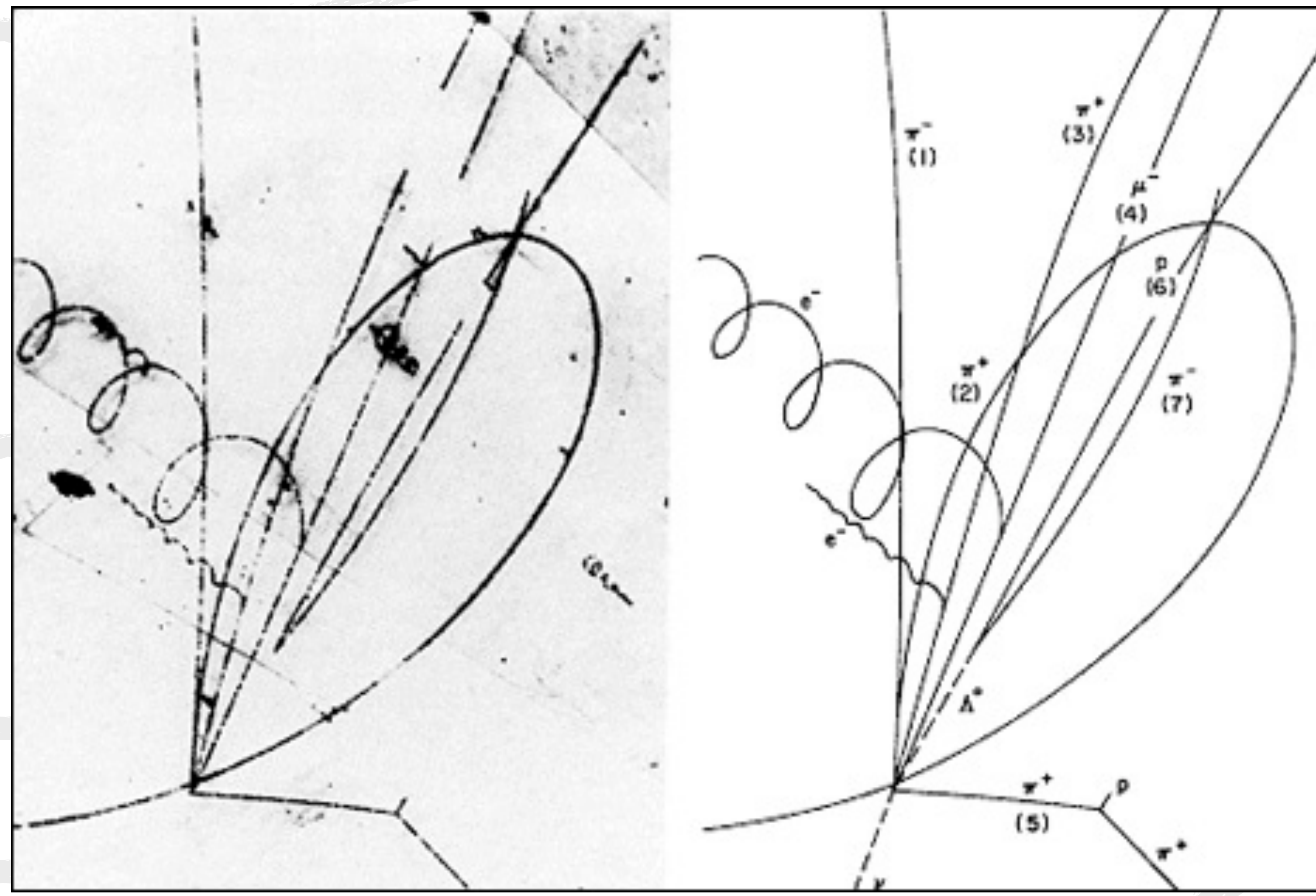
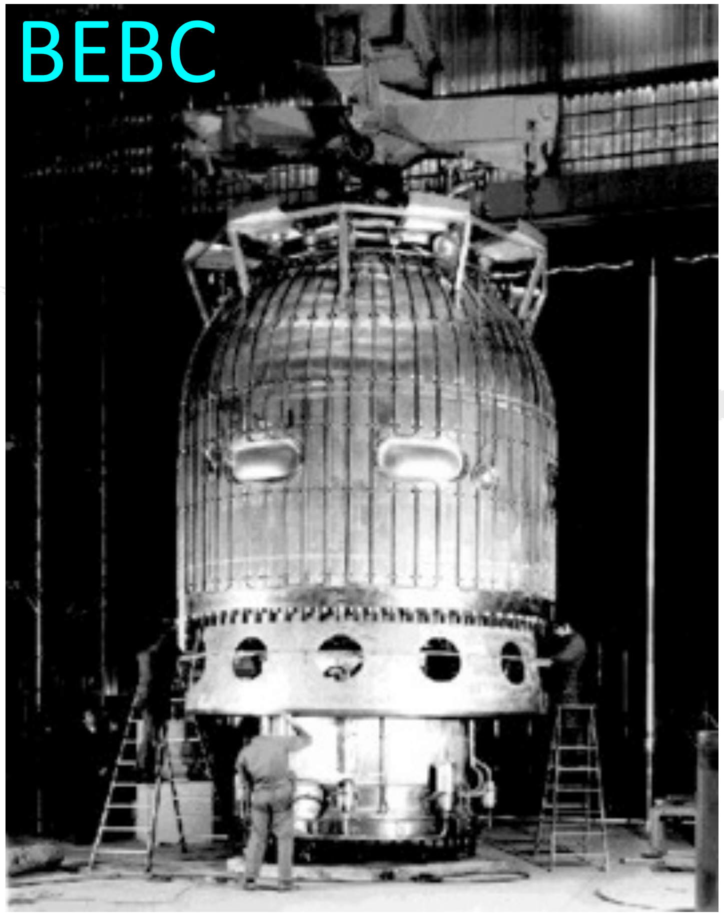


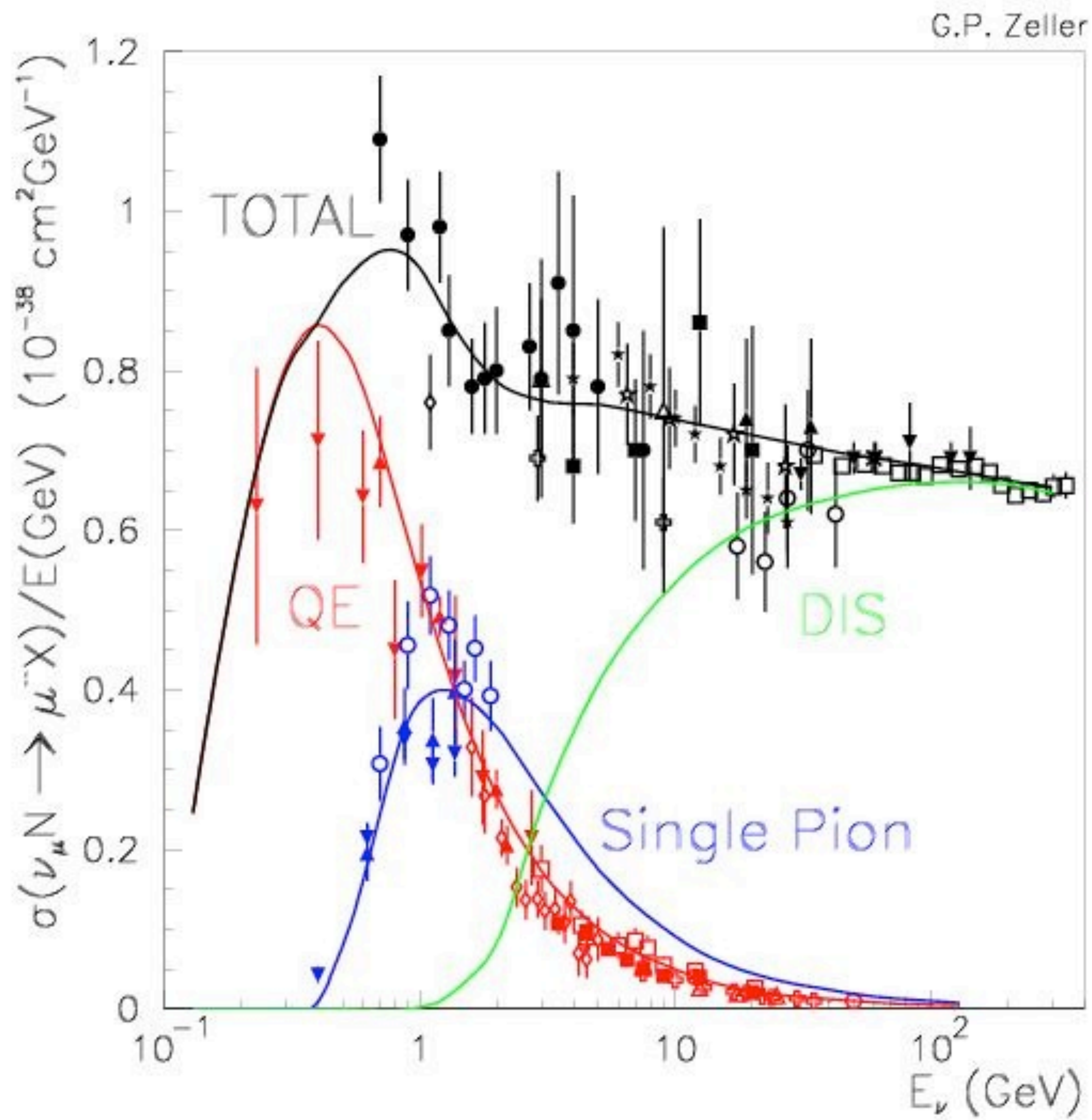
More bubble chamber photos



The 80-inch Bubble Chamber

Bubble Chambers 2

- Bubble chambers used with neutrinos at CERN, ANL, BNL, FNAL
- Neutrino
- Antineutrino
- Slow data processing motivates use of electronic readout



Bubble Chambers 2

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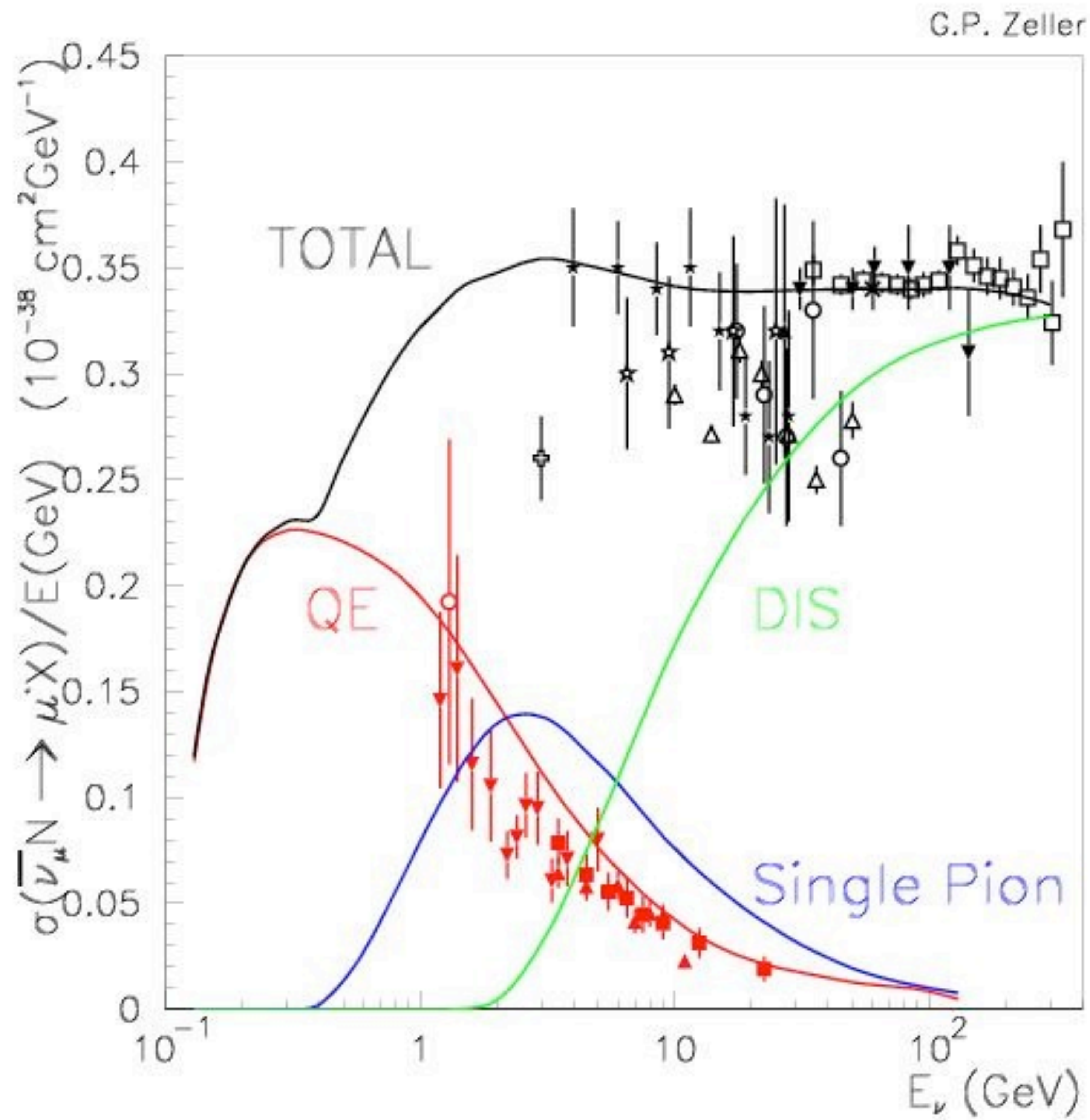
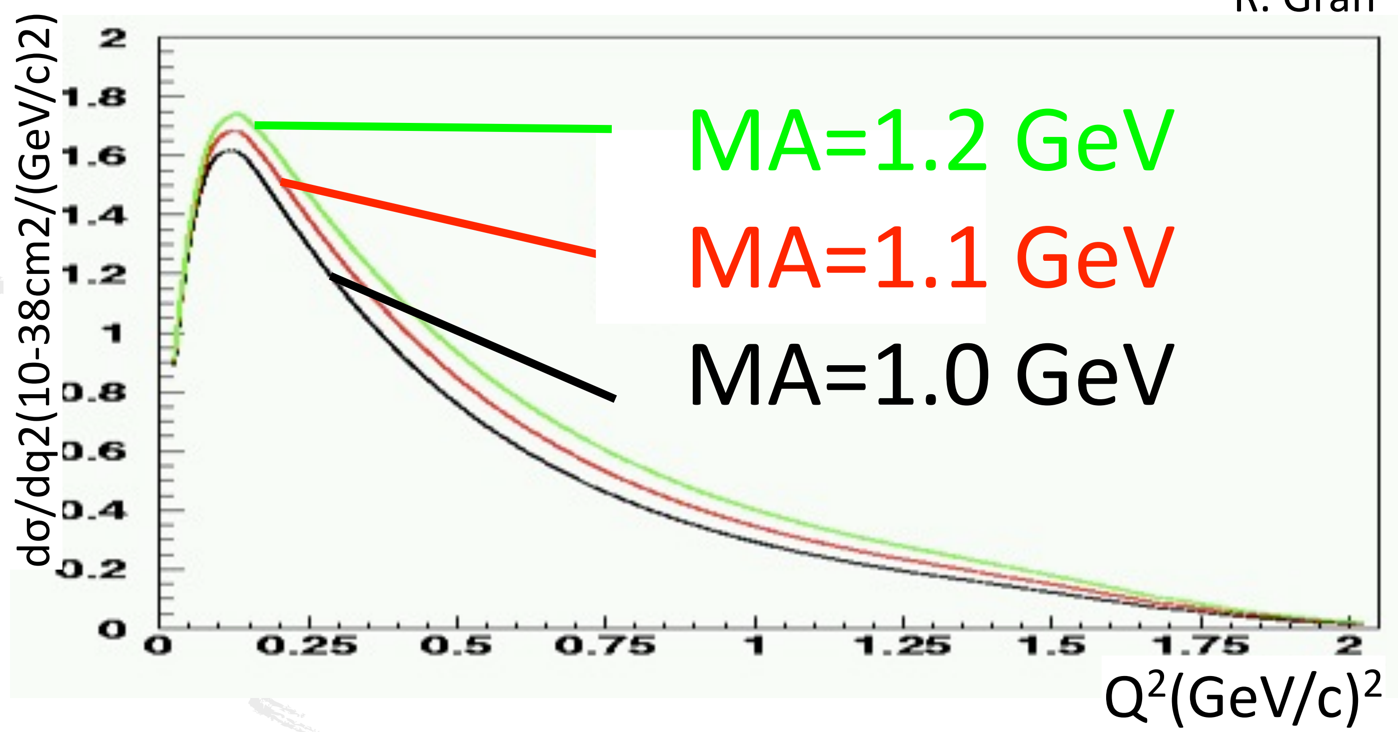


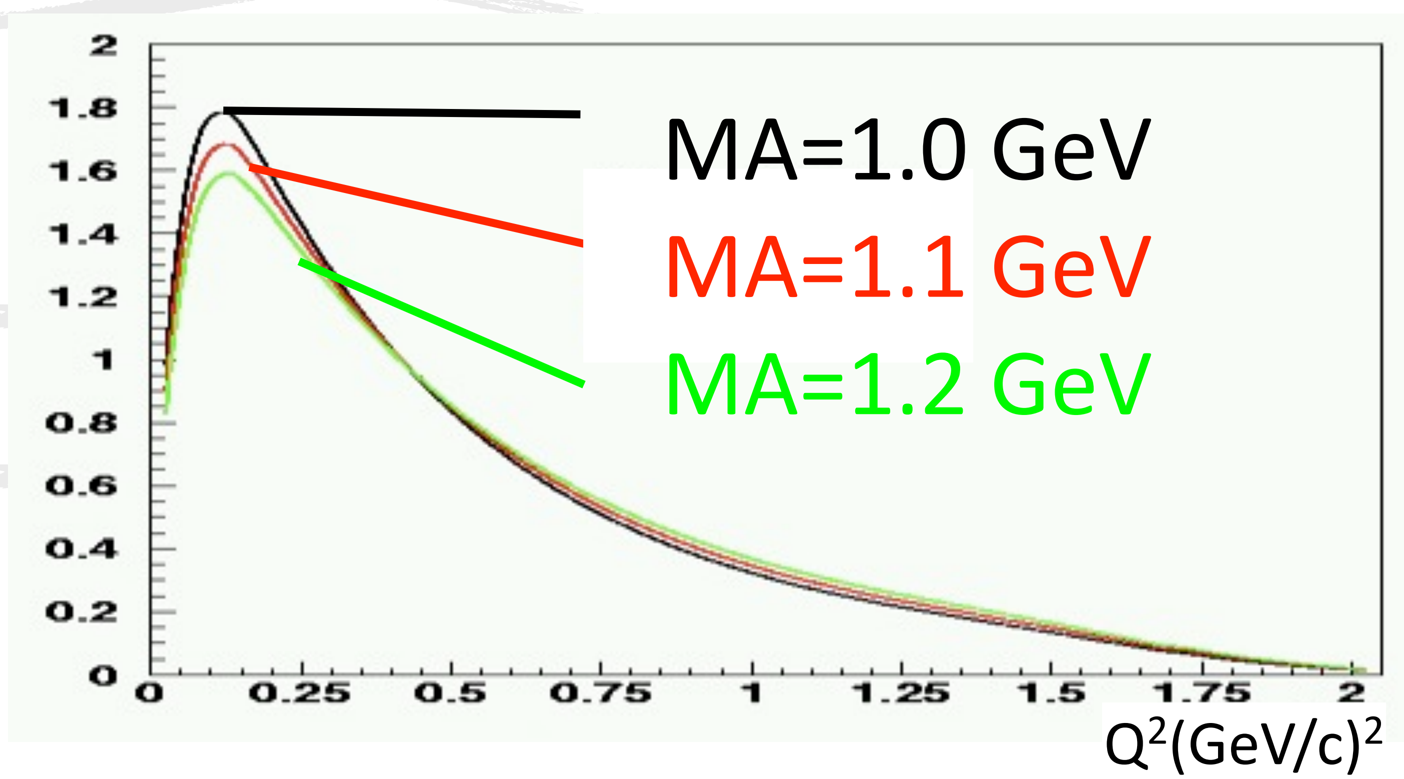
Effect of M_A

Absolute Cross-section (includes normalisation)

R. Gran



Shape only



Cheaters!

- Compare shapes of Q^2 distributions and find best fit for M_A
- Use LLewelyn-Smith theory to infer total cross section
- Use total cross section to calculate total flux
- Use this flux to "measure" cross-sections!

- Modern experiments are finding the problems created by this.

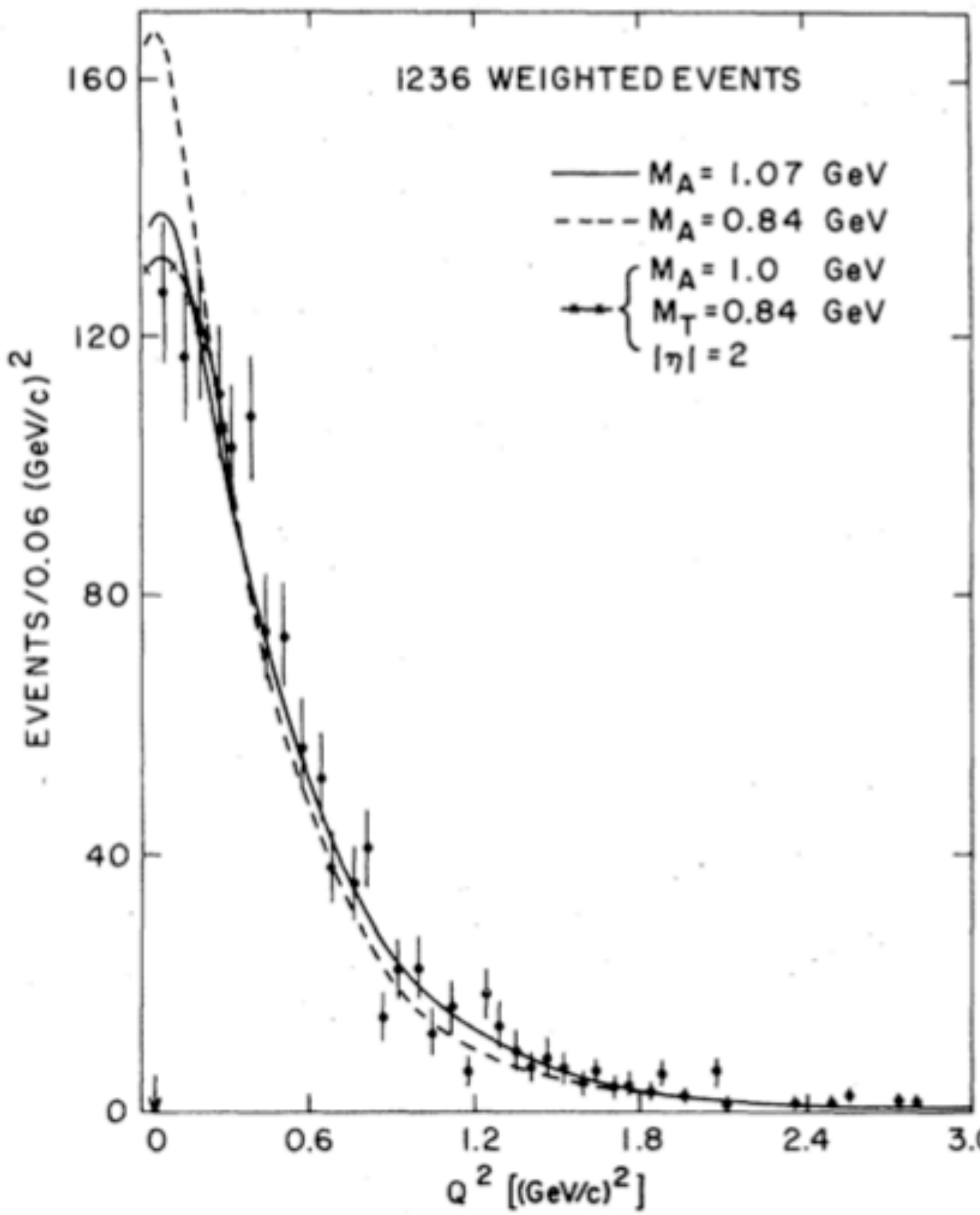
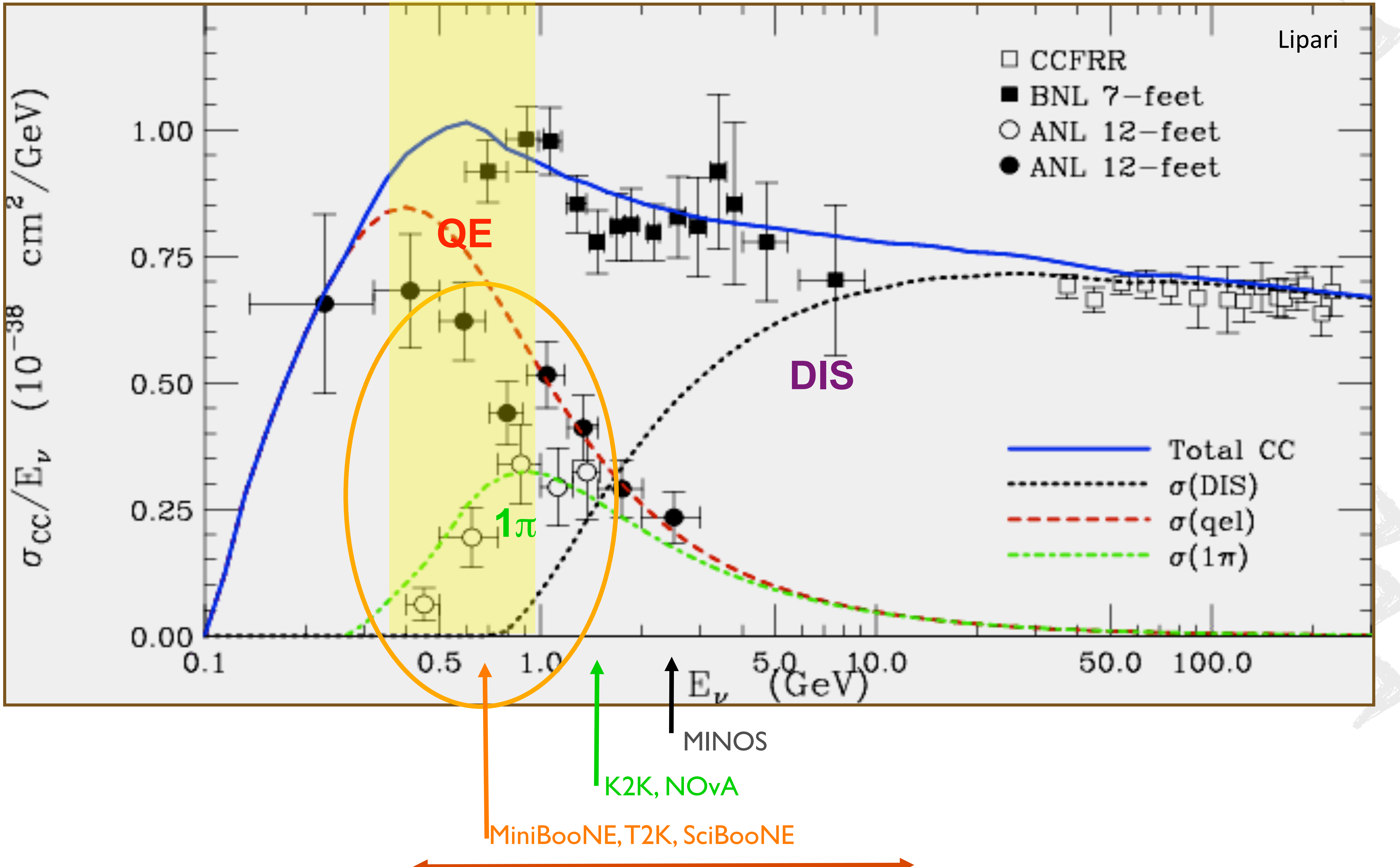


FIG. 6. The Q^2 distribution for selected quasielastic events. The smooth line shows the best fit for $M_A = 1.07$ GeV.

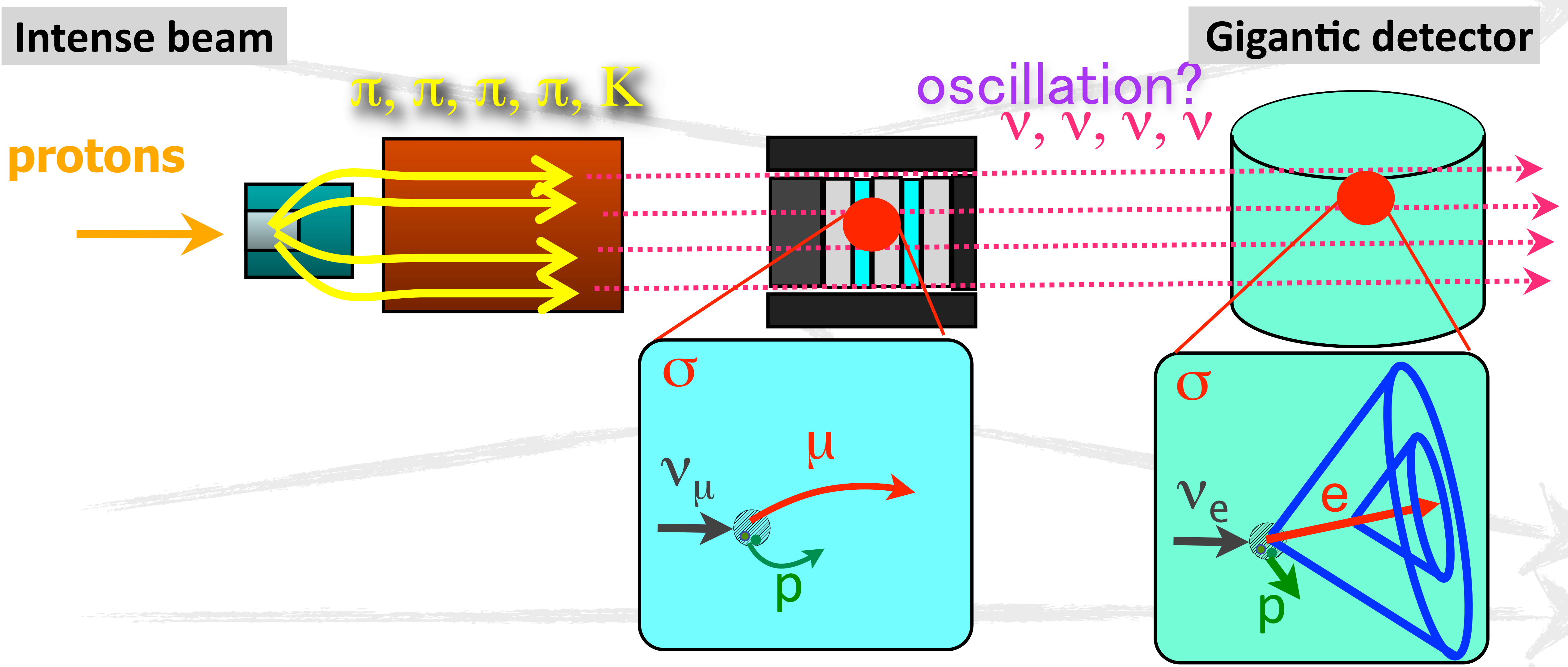
Energy range

Neutrino oscillation searches drive need for better cross section knowledge



Accelerator ν Experiments

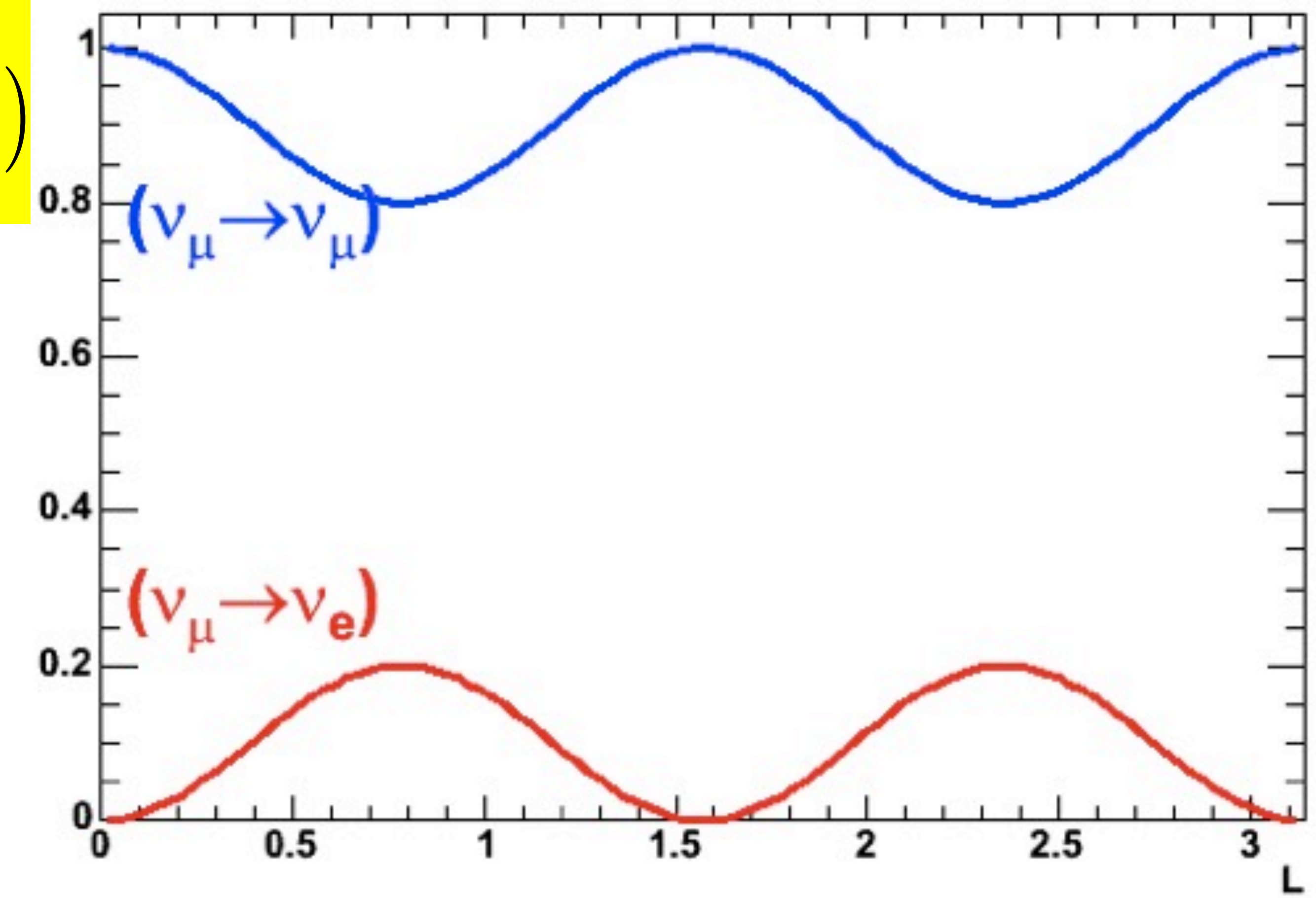
Searching for last mixing angle, θ_{13}, via $\nu_\mu \rightarrow \nu_e$.



$$\Phi_\nu^{near}(E)\cdot\sigma^{near}(E)\cdot\varepsilon^{near}(E) \Leftrightarrow \Phi_\nu^{far}(E)\cdot\sigma^{far}(E)\cdot\varepsilon^{far}(E)$$

$$P(\nu_\mu \to \nu_e) = \sin^2 2\theta_{12} \sin^2(1.27\Delta m_{12}^2 \frac{L}{E})$$

- 2 fundamental parameters
  - $\Delta m^2 \leftrightarrow$ period
  - $\theta_{12} \leftrightarrow$ magnitude
- 2 experimental parameters
  - L = distance travelled
  - E = neutrino energy
- Choose L&E to target ranges of Δm^2 and θ
- Neutrinos disappear and appear



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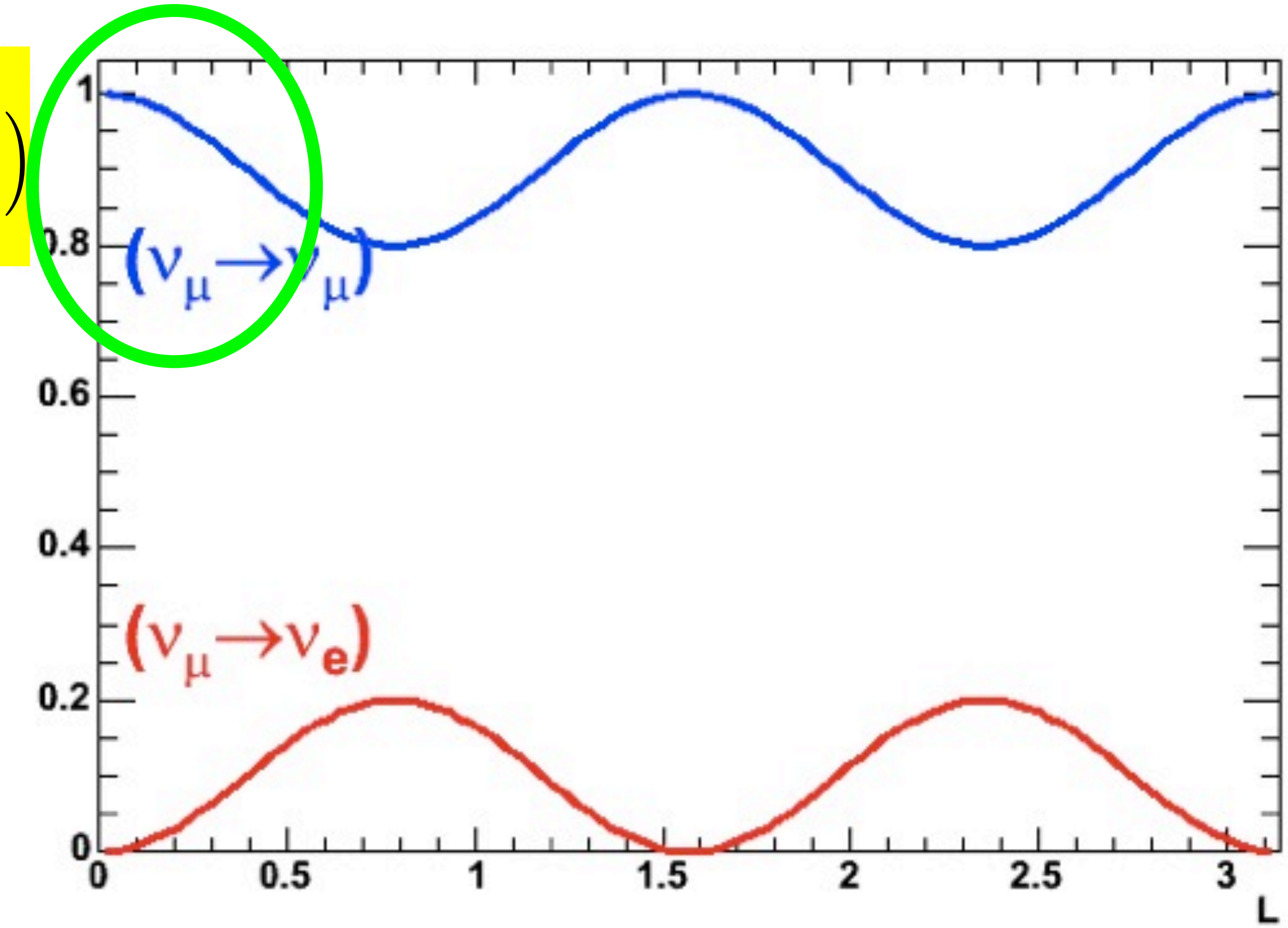

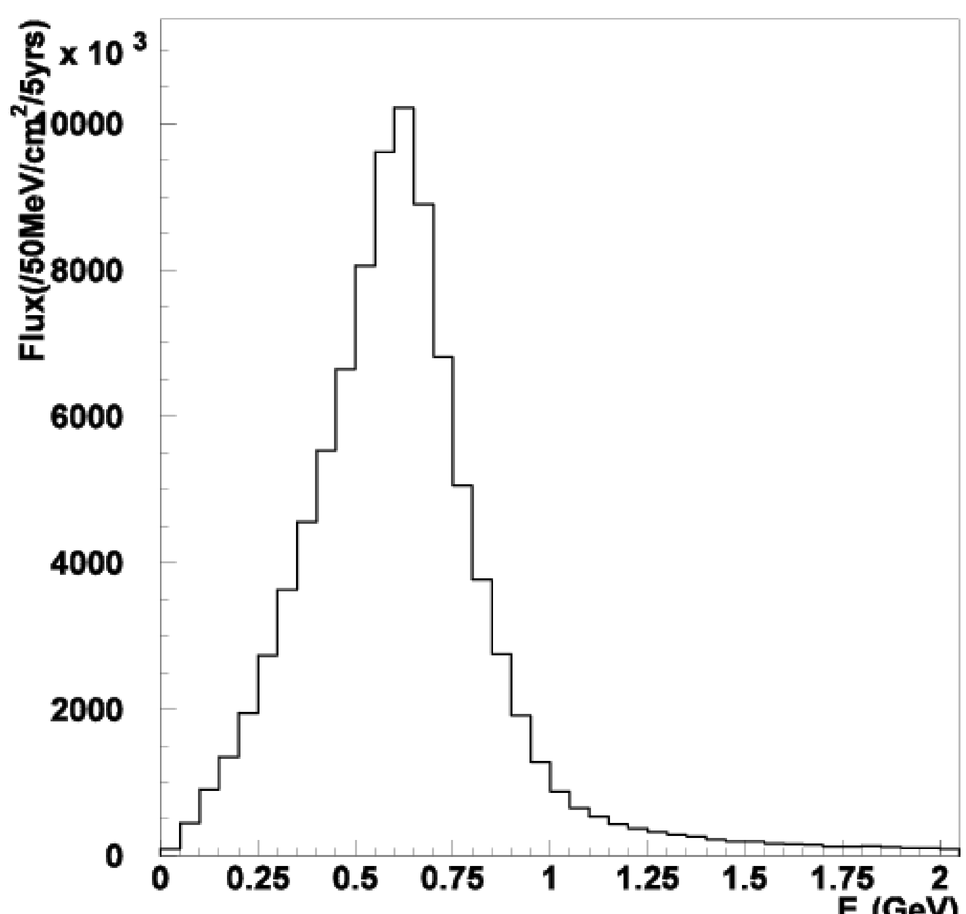

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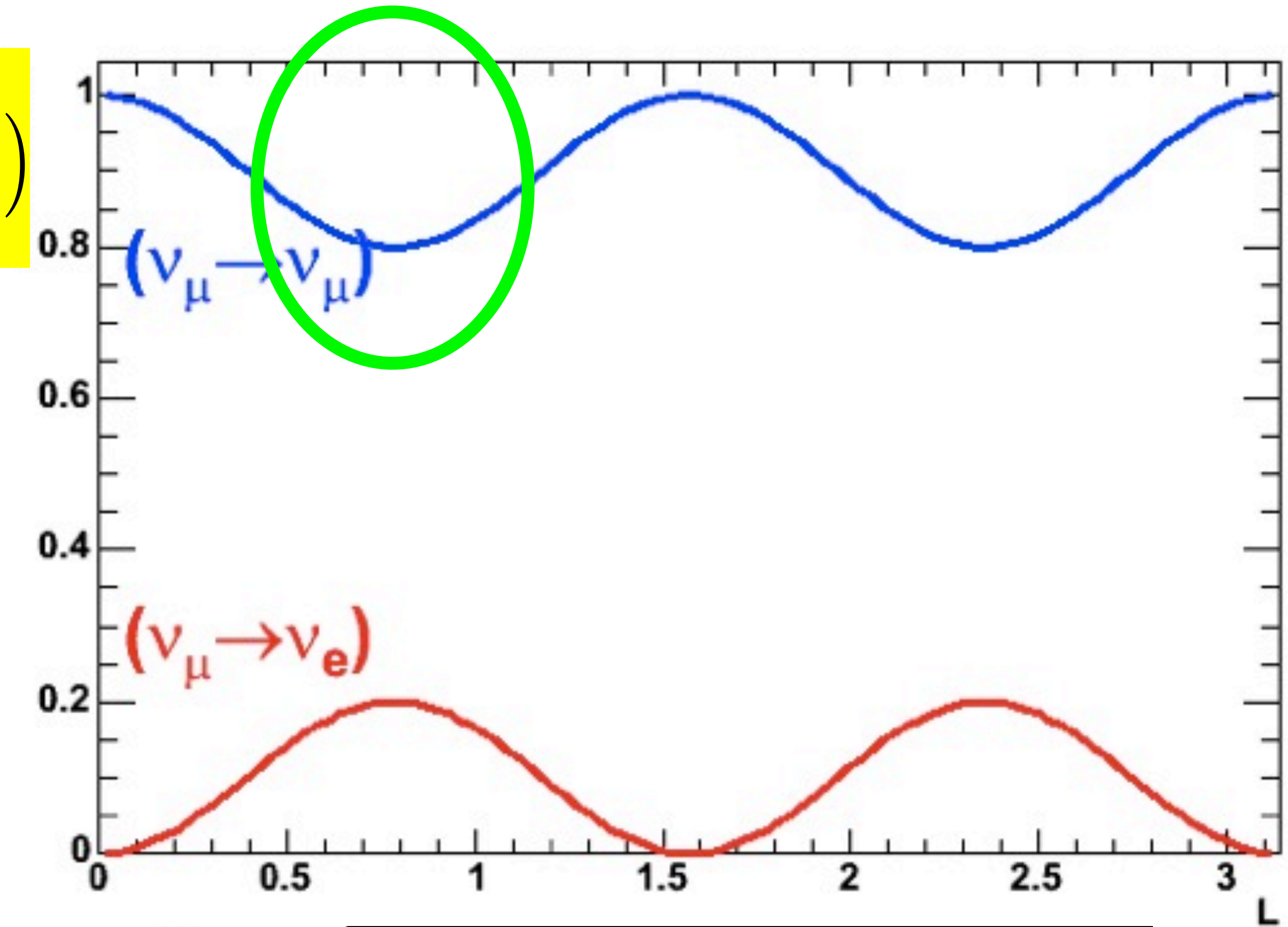

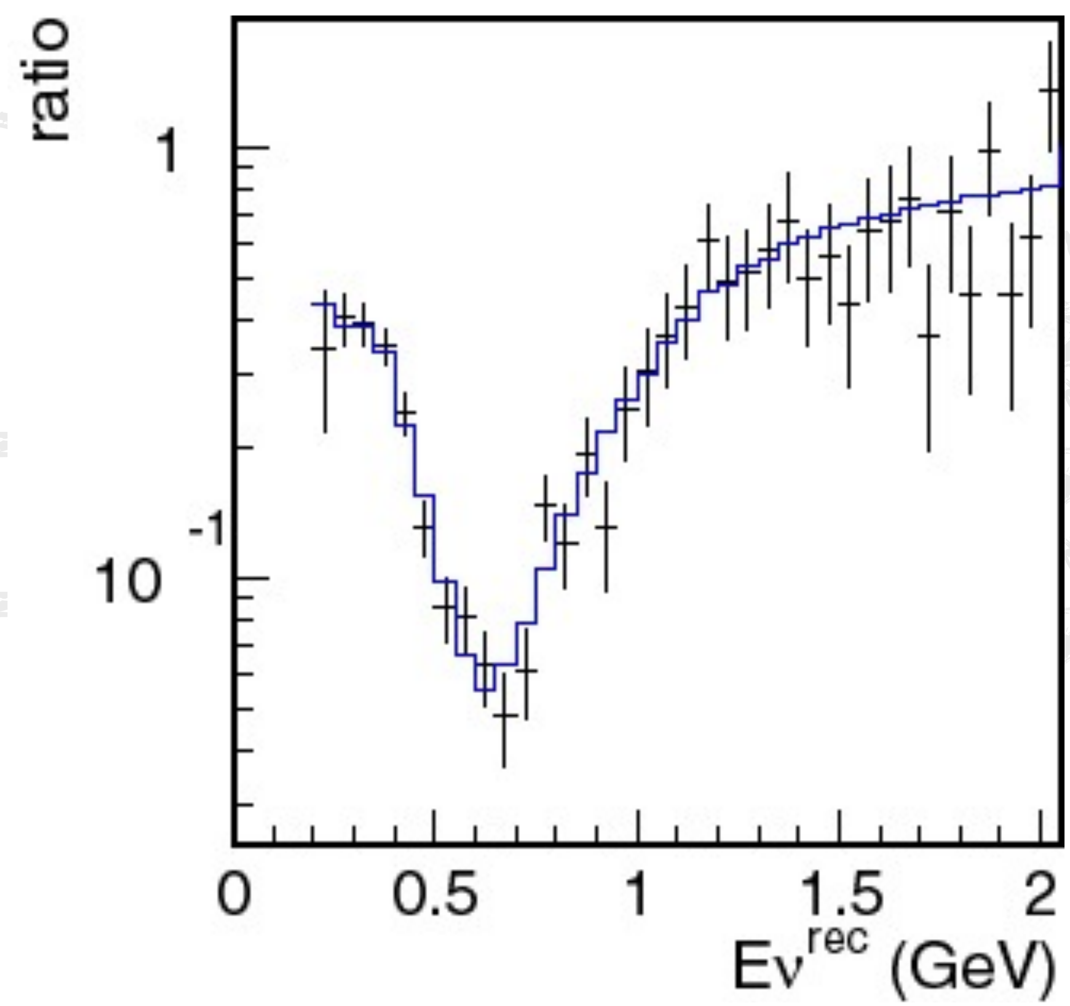

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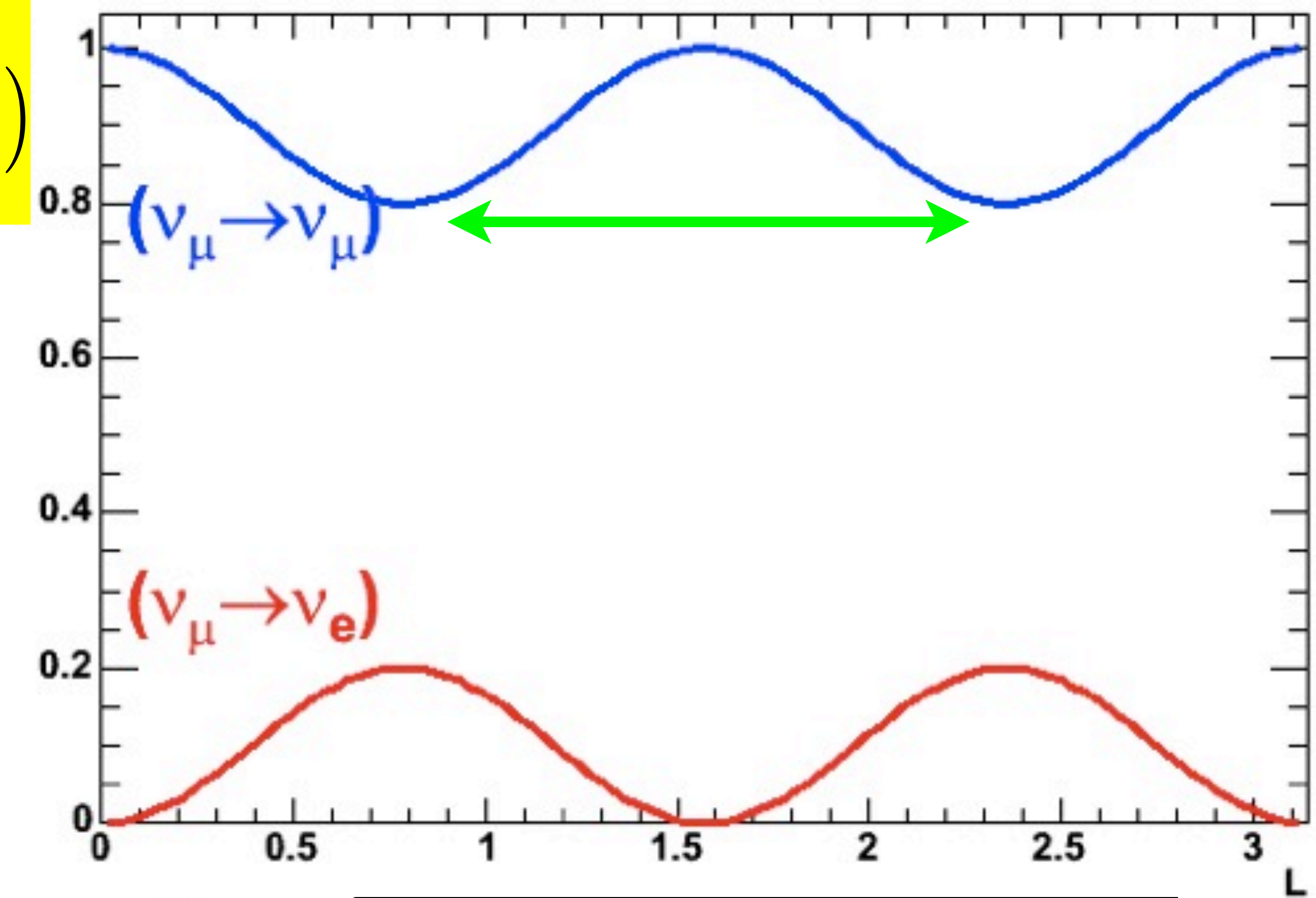

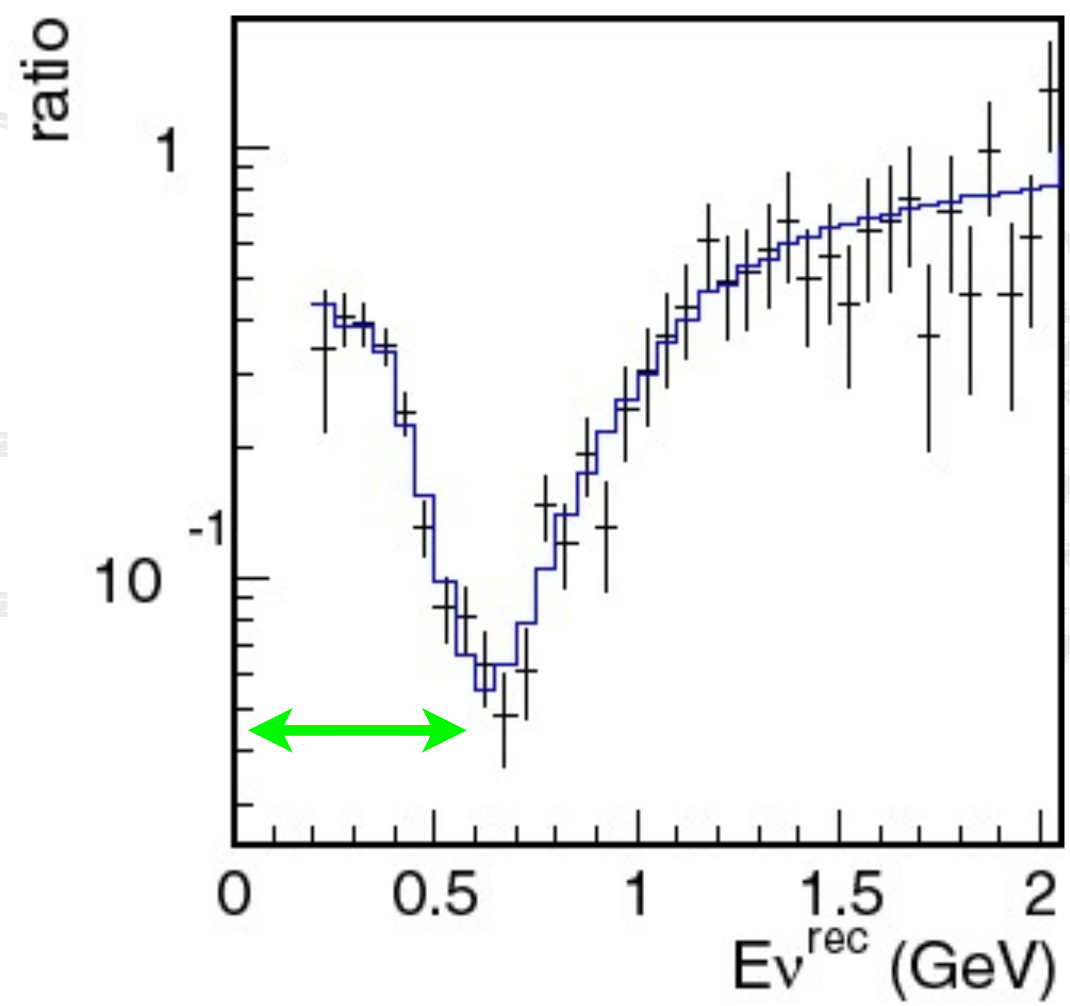

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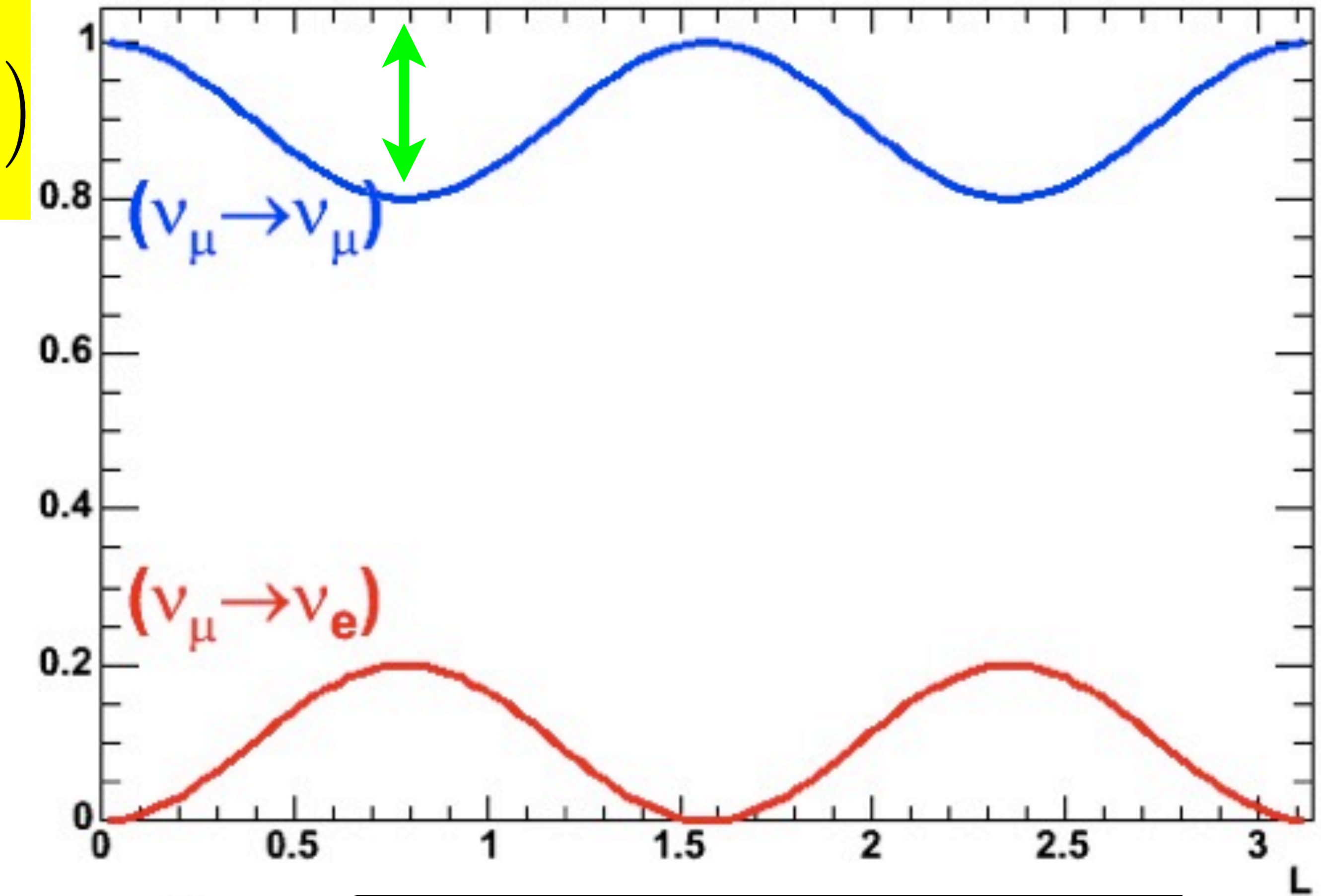

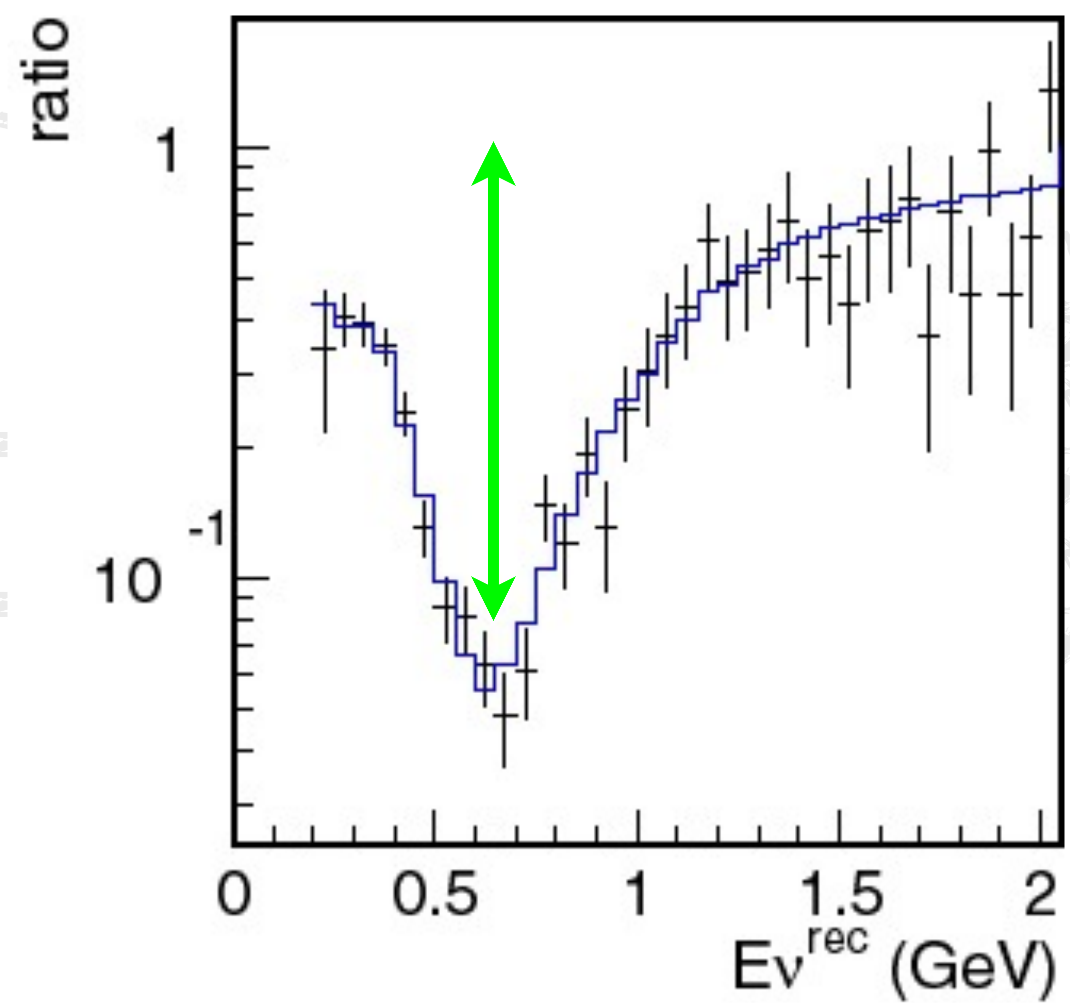

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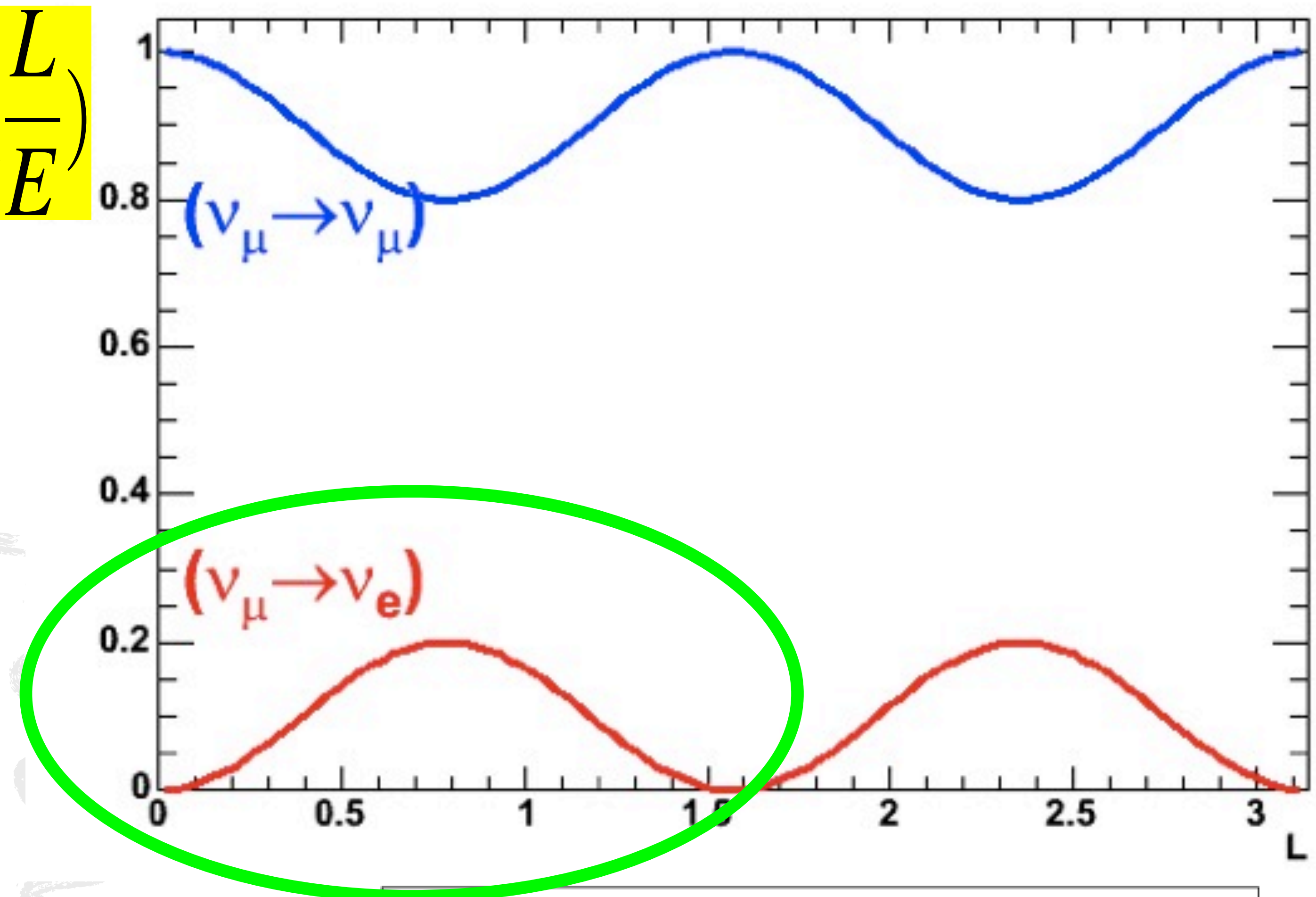
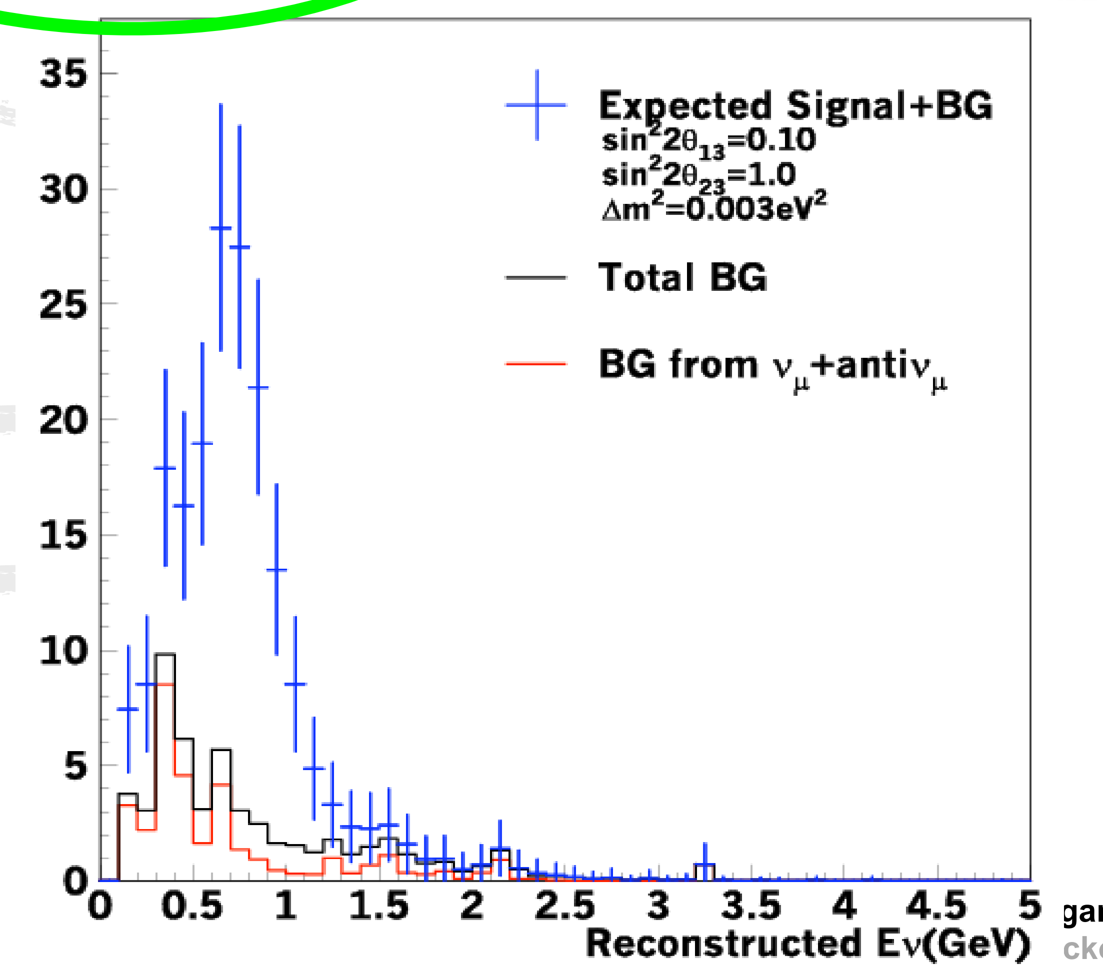
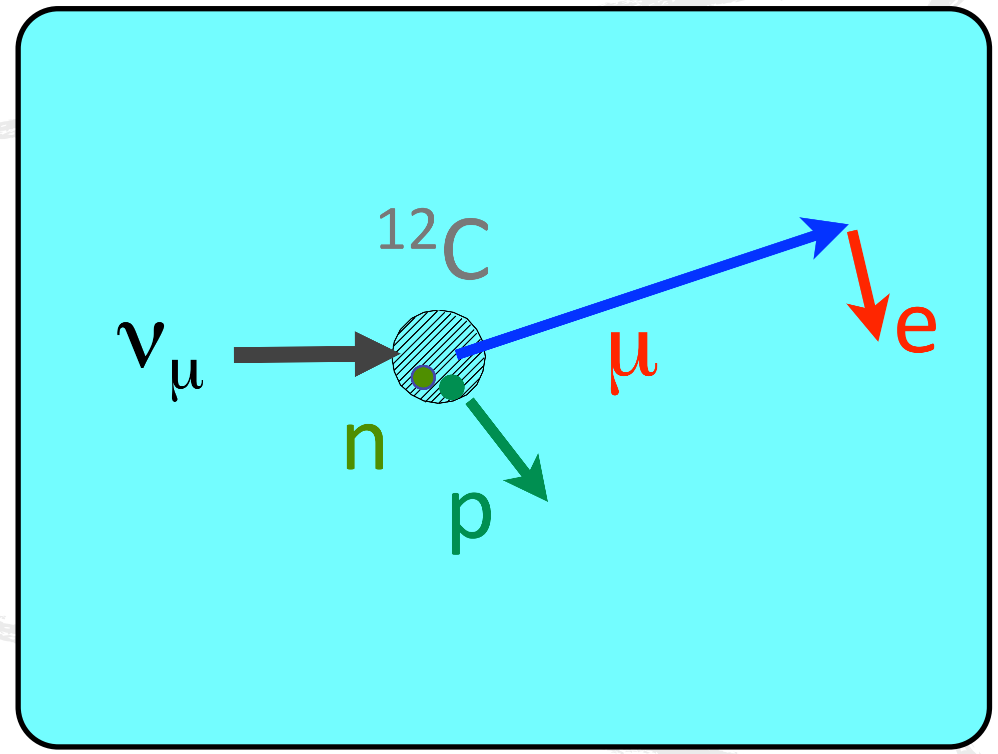




CCQE Topology

- Final state particles in detector:
  - Outgoing lepton
    - Key to measurement
    - Muons can be tagged with penetration, PID or decay electron
  - Recoil nucleon
    - Usually below Cherenkov threshold
  - Recoil nucleus
    - Effectively invisible

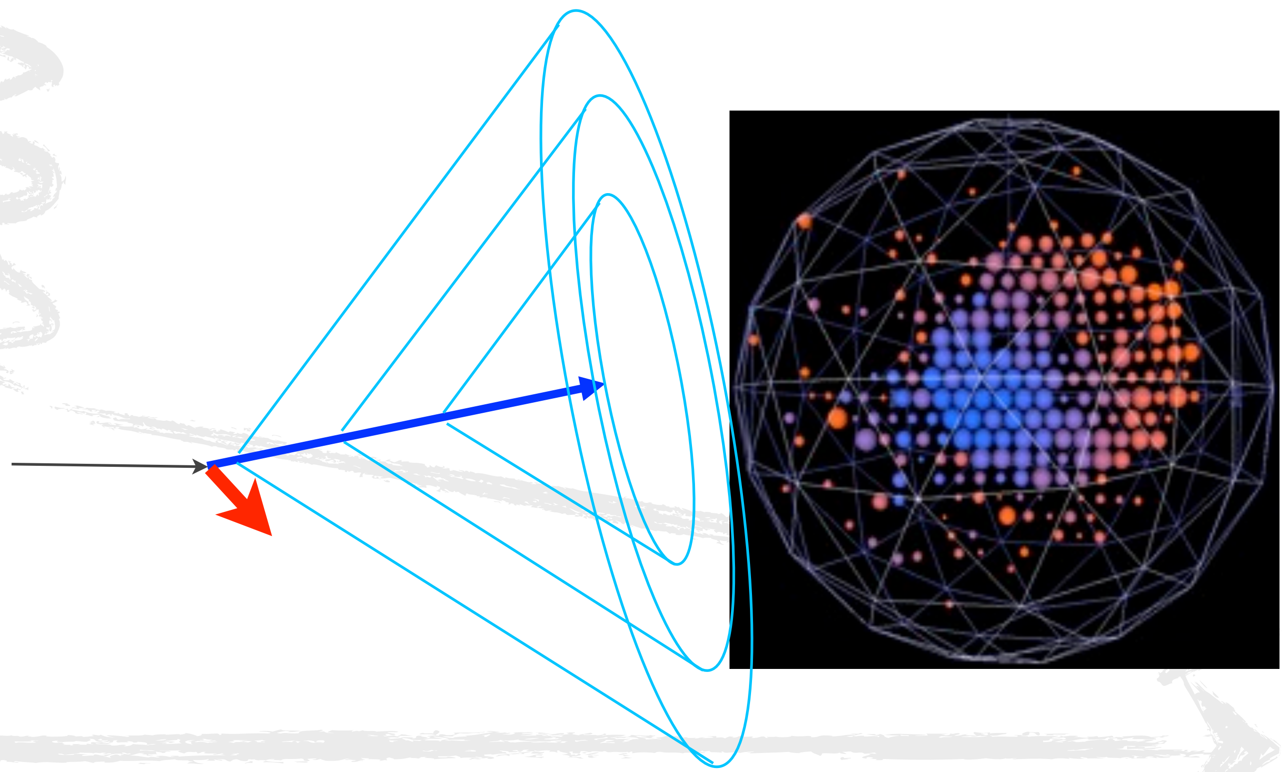


Main background comes from CC1π+ with unobserved pion

Using CCQE events

$$E_\nu^{QE} = \frac{1}{2}\frac{2M_pE_\mu - m_\mu^2}{M_p - E_\mu + \sqrt{(E_\mu^2 - m_\mu^2)}\cos\theta_\mu}$$

- Determine flux (in near detector)
- Reconstruct neutrino energy using outgoing lepton
- Energy reconstruction is important for neutrino oscillation measurements

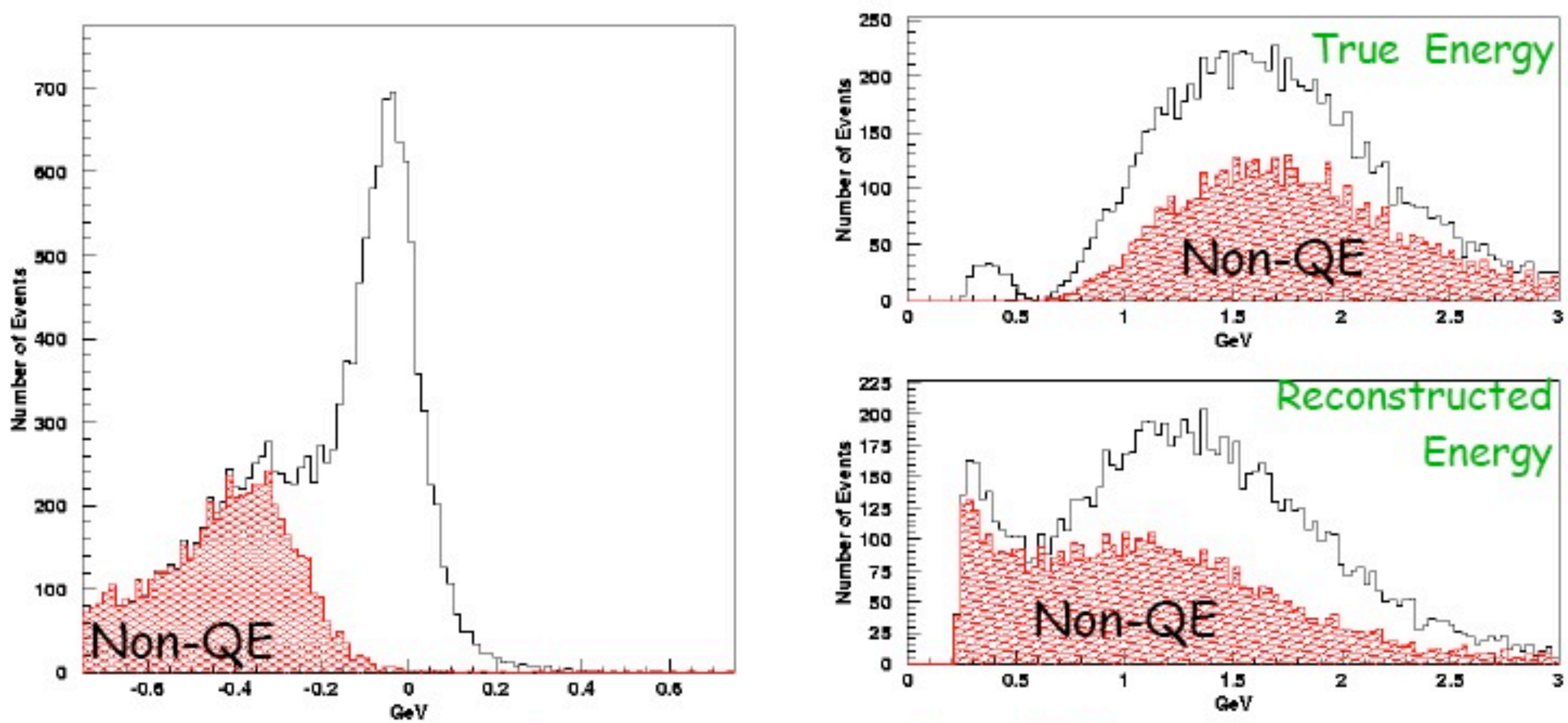


$$P_{osc}(\nu_\mu \to \nu_e) = \sin^2 2\theta \sin^2\left(\frac{1.27\Delta m^2(\mathrm{eV}^2)L(\mathrm{km})}{E_\nu(\mathrm{GeV})}\right)$$

CC-nonQE and Oscillations

Non-QE interactions and E_ν Reconstruction

Example: K2K Flux MC

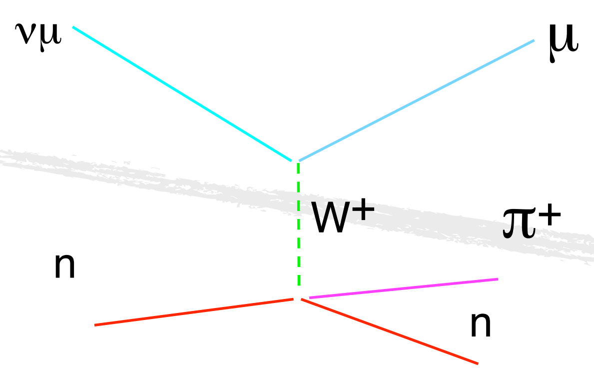


True - Reconstructed Energy

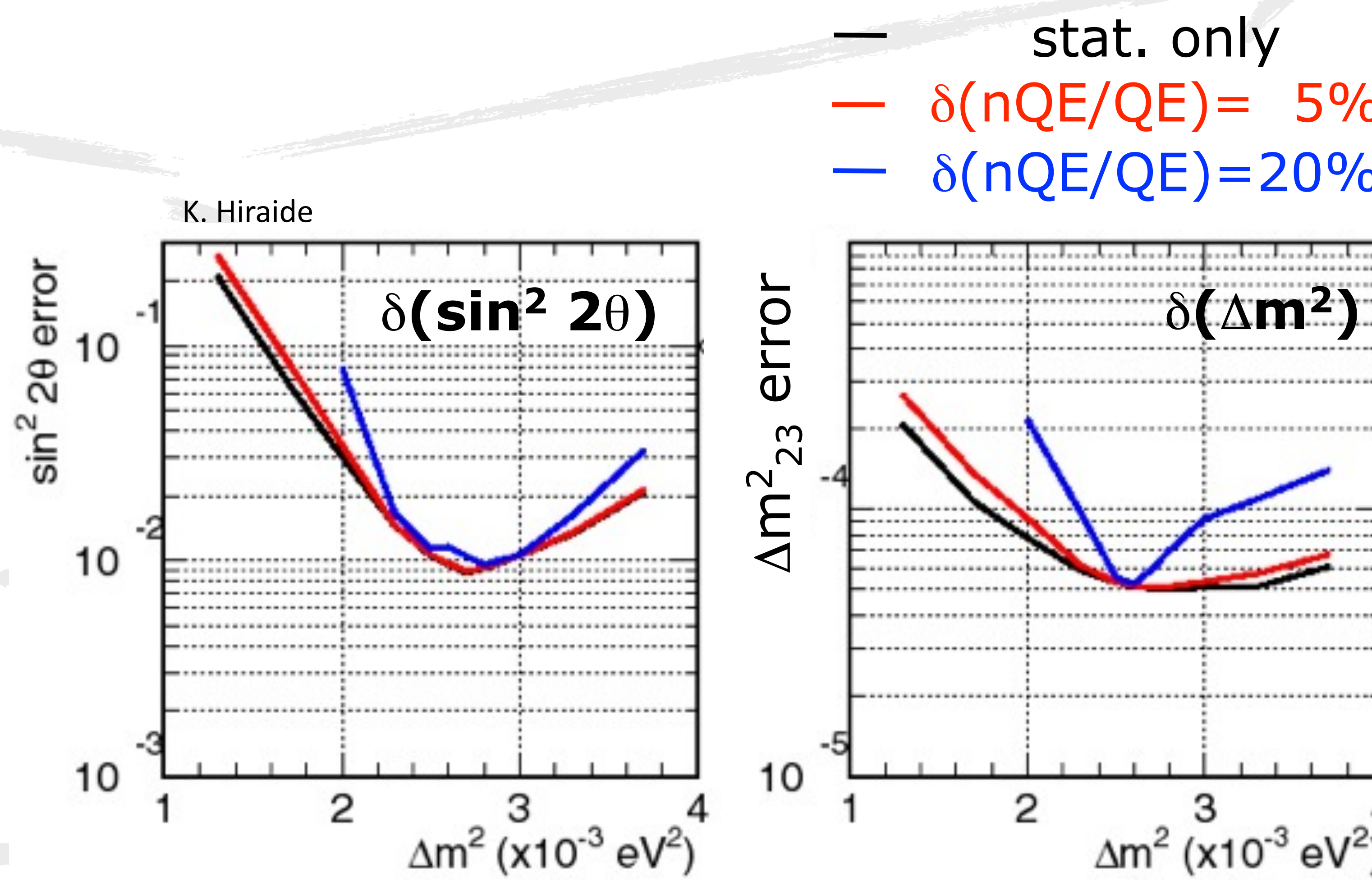
<u>Non-QE reconstructs at low-energy in the oscillation dip!</u>

C. Walter, NuInt07

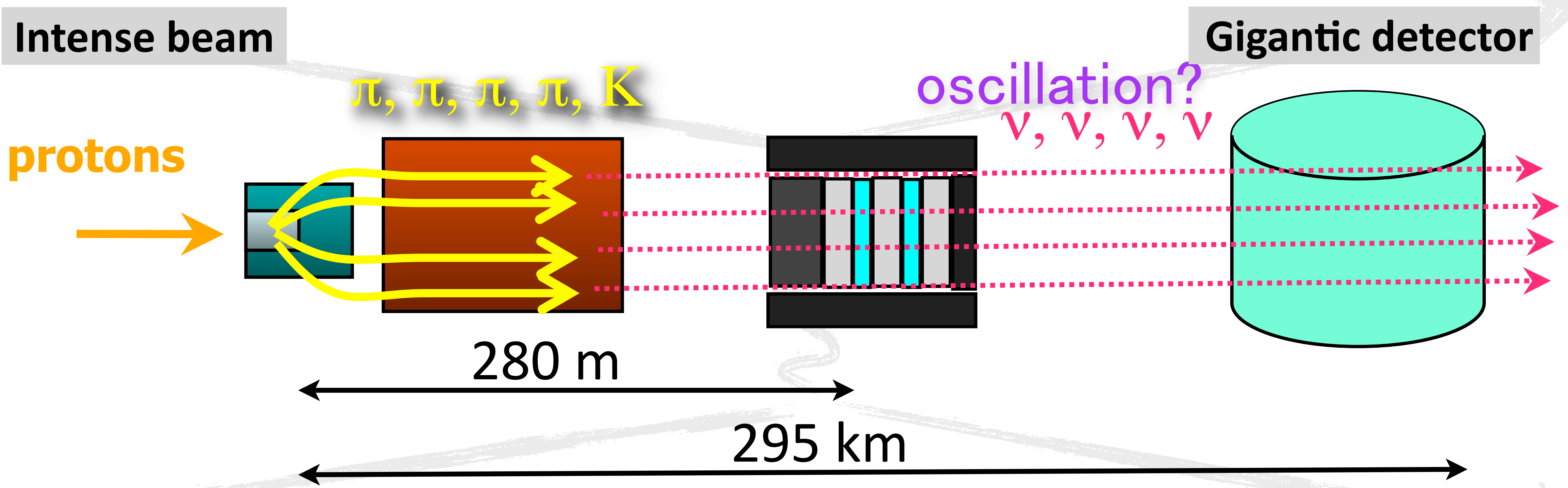
CC-nonQE and Oscillations



- CC1π^+ events can create *bias* in oscillation parameter extraction
- Must reduce uncertainty in σ(CC1π^+) from 20% to 5% for T2K

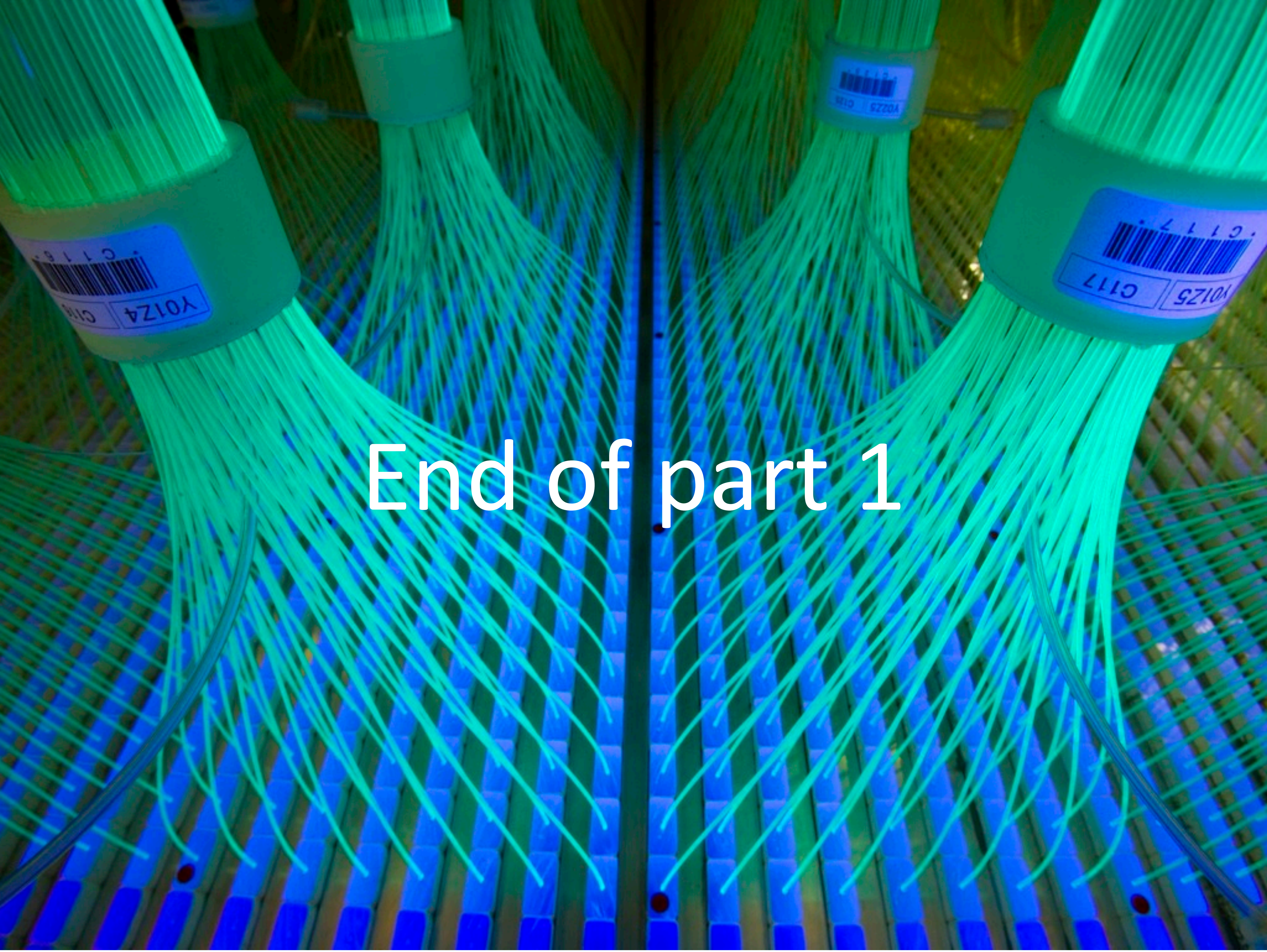


Identical detectors?



1/r2 law means near flux ~1e6 times bigger than far flux!

Detector occupancy too high for open volume detectors.


End of part 1

More definitions

$\nu = \frac{q \cdot P}{M} = E - E'$ is the lepton's energy loss in the nucleon rest frame (in earlier literature sometimes $\nu = q \cdot P$). Here, E and E' are the initial and final lepton energies in the nucleon rest frame.

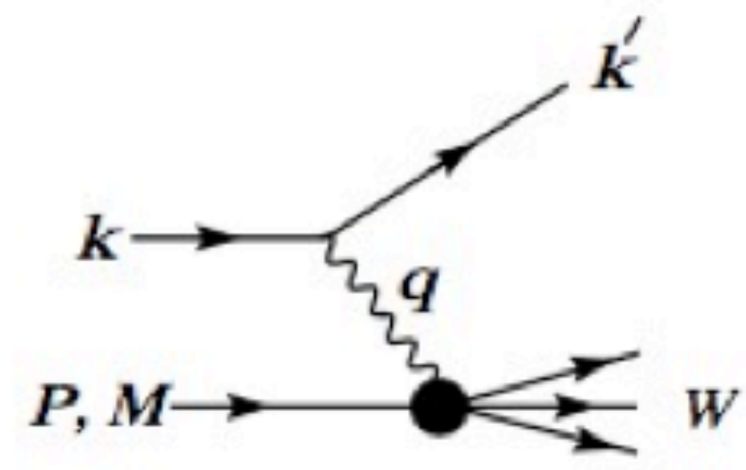
$Q^2 = -q^2 = 2(EE' - \vec{k} \cdot \vec{k}') - m_\ell^2 - m_{\ell'}^2$ where $m_\ell (m_{\ell'})$ is the initial (final) lepton mass. If $EE' \sin^2(\theta/2) \gg m_\ell^2, m_{\ell'}^2$, then

$\approx 4EE' \sin^2(\theta/2)$, where θ is the lepton's scattering angle with respect to the lepton beam direction.

$x = \frac{Q^2}{2M\nu}$ where, in the parton model, x is the fraction of the nucleon's momentum carried by the struck quark.

$y = \frac{q \cdot P}{k \cdot P} = \frac{\nu}{E}$ is the fraction of the lepton's energy lost in the nucleon rest frame.

$W^2 = (P+q)^2 = M^2 + 2M\nu - Q^2$ is the mass squared of the system X recoiling against the scattered lepton.



$$H_W^{eff} = \frac{G_F}{\sqrt{2}}\left\{\left[\bar{e}\gamma_\mu(1-\gamma_5)\nu_e\right]\left[\bar{\nu}_e\gamma^\mu(1-\gamma^5)e\right] + 2\rho\left[\bar{\nu}_l\gamma_\mu\frac{1}{2}(1-\gamma_5)\nu_l\right]\left[\bar{e}\gamma^\mu(g_V - g_A\gamma_5)e\right]\right\}$$

$$= \frac{G_F}{\sqrt{2}}\left[\bar{\nu}_l\gamma_\mu\frac{1}{2}(1-\gamma_5)\nu_l\right]\left[\bar{e}\gamma^\mu(c_V - c_A\gamma_5)e\right]$$

- $c_V = \rho g_V + 1$ $\nu_e e^-$ ($\nu_\mu e^-$: no -1: NC only)
- $c_A = \rho g_A + 1$ $\nu_e e^-$ ($\nu_\mu e^-$: no -1: NC only)
  - $c_L = ½*(c_V + c_A) = g_L + 1$ for ν_e
    $= g_L$ for ν_μ
  - $c_R = ½*(c_V - c_A) = g_R$

$$\sin^2\theta_W = 0.223$$

	c_L	c_R	$c_L^2 + \frac{1}{3}c_R^2$	$\frac{1}{2}c_L c_R$
$\nu_e e^-$	$1/2 + \sin^2\theta_W$	$\sin^2\theta_W$	0.5525	0.0845
$\bar{\nu}_e e^-$	$\sin^2\theta_W$	$1/2 + \sin^2\theta_W$	0.2317	0.0845
$\nu_\mu e^-$	$-1/2 + \sin^2\theta_W$	$\sin^2\theta_W$	0.0901	0.0311
$\bar{\nu}_\mu e^-$	$\sin^2\theta_W$	$-1/2 + \sin^2\theta_W$	0.0775	0.0311

- The matrix element (same as β decay) is

$$\frac{1}{2}|T|^2 = \frac{G_F^2 \cos^2\theta_C}{2} 8 E_e E_\nu \left[(1 + \beta\cos\theta) + 3 g_A^2 \left(1 - \frac{\beta}{3}\cos\theta \right) \right]$$

$$\sigma(\bar{\nu}_e p \to e^+ n) = \frac{1}{2E_\nu} \int \frac{d^3 p_e}{2E_e (2\pi)^3} (2\pi) \delta(E_\nu - E_e + m_p - m_n) \frac{1}{2} |T|^2$$

$$= \frac{G_F^2 E_e E_p}{\pi} \cos^2\theta_C (1 + 3 g_A^2)$$

$$\approx 9.30 \times 10^{-42} \left(\frac{E_\nu}{10\ MeV} \right)^2 cm^2$$

- NOTE: $\sigma(\bar{\nu}_e p \to e^+ n) = \sigma(\nu_e n \to e^- p)$

No free neutron

$$\cos\theta_C = 0.974,\ g_A = 1.267$$

$$d\sigma / d\theta \propto 1 - 0.104 \cos\theta$$

Well known process and reliable calculation for the low energy anti-neutrino (~MeV)!

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Many thanks to T. Nakaya, KS McFarland, G Zeller, from whose slides I borrowed heavily!