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Unitarity Violation in Low Scale Seesaw Schemes

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Neutrino mass in $SU(2)_L \times U(1)$ - The (n,m) models

Type-I and type-II Seesaw schemes

The quadratic part of the neutrino Lagrangian is:

$$\mathcal{L} = \sum_\alpha \left[-i\rho_\alpha^\dagger \sigma_\mu \partial_\mu \rho_\alpha - \frac{1}{2}\left(\rho_\alpha^T \sigma_2 M_{\alpha\beta}\,\rho_\beta + H.c\right)\right],$$

the symmetric neutrino mass matrix M_ν can be decomposed as:

$$M_\nu = \begin{pmatrix} M_1 & M_D \\ M_D^T & M_2 \end{pmatrix},$$

without Higgs triplet:

$$M_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \quad (\rho_n,\ \rho_m) \quad \Rightarrow \quad \text{type-I Seesaw.}$$

we need to transform to the physical states: $\boxed{\rho_i = \sum_j U_{ij}\nu_j}$

$$U^T M_\nu U = \text{real, diagonal} \quad (U \text{ Unitary})$$

Mass matrix diagonalization

Diagonalization, by making the ansatz:

$$U = \mathcal{U}\cdot V = \exp(iH)\cdot V \qquad H = \begin{pmatrix} 0 & S \\ S^\dagger & 0 \end{pmatrix}, \qquad V = \begin{pmatrix} V_1 & 0 \\ 0 & V_2 \end{pmatrix}.$$

$$U = \begin{pmatrix} (I - \frac{1}{2}S\,S^\dagger)\,V_1 & i\,S\,V_2 \\ i\,S\,V_1 & (I - \frac{1}{2}S^\dagger\,S)\,V_2 \end{pmatrix} + O(\epsilon^3),$$

where we defined the hierarchy parameter: $\boxed{\epsilon \equiv M_D\,M_R^{-1}}$, we obtain:

$$i\,S^* = -M_D\,M_R^{-1} \quad \Rightarrow$$

$$U = \begin{pmatrix} (I - \frac{1}{2}M_D^*(M_R^*)^{-1}M_R^{-1}M_D^T)\,V_1 & M_D^*(M_R^*)^{-1}V_2 \\ -M_R^{-1}M_D^T V_1 & (I - \frac{1}{2}M_R^{-1}M_D^T M_D^*(M_R^*)^{-1})\,V_2 \end{pmatrix} + O(\epsilon^3),$$

and finally, after block diagonalization:

$$m_\nu = -M_D M_R^{-1} M_D^T$$

Charged current in seesaw mechanism

$\mathcal{L} \supset i\frac{g}{\sqrt{2}}W_\nu \sum_{a=1}^n \overline{L}_a \gamma_\mu \rho_{aL} + h.c.$

$$\mathcal{L} \supset i\frac{g}{\sqrt{2}}W_\nu \bar{l}_b K_{b\alpha}\gamma_\mu \nu_{\alpha L} + h.c.$$

where $K_{b\alpha} = \sum_{c=1}^n \Omega_{cb}^* U_{c\alpha}$.

We could define:

$K \equiv (K_L, K_H)$,

$$\text{where} \quad K_L \sim \left(1 - \frac{1}{2}S\,S^\dagger\right)V_1 \equiv (1-\eta)V_1$$

$$K_H \sim (i\,S)\,V_2$$

$$\text{(type-I)} \quad \boxed{\eta \approx \frac{1}{2}S\,S^\dagger} \sim \frac{1}{2}\epsilon^*\epsilon^T.$$

$$M \backsim 10^{13}\,GeV \quad m_D \backsim 10^2\,GeV \quad \Rightarrow \quad \epsilon \approx 10^{-11} \Rightarrow \eta \approx 10^{-22}$$

Low scale seesaw schemes

Inverse seesaw

Introducing two sets of SU(3) $\otimes$ SU(2) $\otimes$ U(1) singlets ρ_m and S, the effective neutrino mass matrix is:

$$M_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M \\ 0 & M^T & 0 \end{pmatrix}, \quad (\rho_n,\ \rho_m,\ S)$$

where global lepton number conservation $U(1)_L$ was assumed.

The global lepton number could be broken everywhere the zero entries are. If we introduce a mass $\mu_{ij}S_iS_j$ term, we break L in two units:

$$M_\nu = \begin{pmatrix} 0 & M_D & 0 \\ M_D^T & 0 & M \\ 0 & M^T & \mu \end{pmatrix}, \quad (\rho_n,\ \rho_m,\ S)$$

$$m_\nu = M_D {M^T}^{-1} \mu\, M^{-1} M_D^T$$

Because the smallness of the light neutrino mass m_ν is due to the smallness of the μ mass, which is different in the type-I seesaw, this mechanism is named as inverse.

When $\mu \to 0$ a global number symmetry is recovered and neutrinos are massless. The smallness of μ is natural, in 't Hooft sense.

Linear seesaw

In the simplest LR model:

$$\text{SU(2)}_L \otimes \text{SU(2)}_R \otimes \text{U(1)}_{B-L}$$

with additional gauge singlet fermion $S_L \sim (1,1,0)$ (and the right-handed antiparticle $(S_L)^c = S_R^c$):

$$-\mathcal{L}^{Yukawa}_{Leptons} = [\bar{\Psi}_L(Y_1\phi + Y_2\tilde{\phi})\Psi_R + h.c] + [Y_3(\bar{\Psi}_L\chi_L S_R^c + \bar{\Psi}_R\chi_R S_L) + h.c]$$

where $\phi \sim (2,2,0)$, $\tilde{\phi} \equiv \tau_2\phi^*\tau_2$, $\chi_L \sim (2,1,-1)$, $\chi_R \sim (1,2,-1)$.

After R breaking by $<\chi_R>= v_R$, and EW breaking by:

$$<\phi>= \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix},$$

and possibly of $<\chi_L>= v_L$, and defining: $M_L \equiv Y_3 v_L$, $M \equiv Y_3 v_R$ and $M_D = Y_1\kappa + Y_2\kappa'$, we find:

$$M_\nu = \begin{pmatrix} 0 & M_D & M_L \\ M_D^T & 0 & M \\ M_L^T & M^T & 0 \end{pmatrix}$$

and finally, after block diagonalization the light neutrino matrix is:

$$m_\nu = M_D(M_L M^{-1})^T + (M_L M^{-1})M_D^T.$$

Diagonalization

In an approximate way we could calculate the block diagonalizing matrix as:

$$\mathcal{U} = R_{23}(\pi/2)\,R_{13}(S)\,R_{12}(S)$$

where R are rotations. Using the *Schechter-Valle* parameterization for the last two rotations, we have:

$$\mathcal{U} = \begin{pmatrix} I - S\,S^\dagger & iS & iS \\ 0 & \frac{1}{\sqrt{2}}(I + \frac{1}{2}S^\dagger S) & -\frac{1}{\sqrt{2}}(I - \frac{1}{2}S^\dagger S) \\ i\sqrt{2}S^\dagger & \frac{1}{\sqrt{2}}(I - \frac{3}{2}S^\dagger S) & \frac{1}{\sqrt{2}}(I - \frac{1}{2}S^\dagger S) \end{pmatrix},$$

$$\text{where} \quad iS^* = -\frac{1}{\sqrt{2}}m_D(M^T)^{-1}.$$

and we define:

$\boxed{\eta^{I,L} \approx S\,S^\dagger.}$

$$M \backsim 10^3 GeV \quad m_D \backsim 10^2 GeV \quad \Rightarrow \quad \epsilon \approx 10^{-1} \Rightarrow \eta^{I,L} \approx \textit{Percent!}$$

Finally. the neutrino mixing matrix is:

$$U = \mathcal{U}\cdot \begin{pmatrix} V_1 & 0 & 0 \\ 0 & V_2 & 0 \\ 0 & 0 & V_3 \end{pmatrix},$$

where V_i diagonalizes each block and we assumed that $V_1, V_2 \sim I$.

Constraining $|\eta_{ij}|$ in the low scale seesaw schemes

Lepton Flavor Violating processes

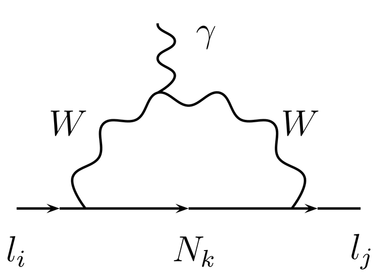
The mixing of light neutrino with heavy neutrino gives at one loop LFV effects that are related to the unitarity deviation. The resulting branching ratio is given by:

$$BR(l_i \to l_j\gamma) = \frac{\alpha_W^3 s_W^2}{256\pi^2}\frac{m_{l_i}^5}{M_W^4}\frac{1}{\Gamma_{l_i}}|G_{ij}^W|^2$$

where

$$G_{ij}^W = \sum_{k=4}^{9} K_{ik}^* K_{jk} G_\gamma^W \left(\frac{m_{N_k}^2}{M_W^2}\right)$$

$$G_\gamma^W(x) = -\frac{2x^3 + 5x^2 - x}{4(1-x)^3} - \frac{3x^3}{2(1-x)^4}\ln x$$



as $K_{b\alpha} \sim U_{b\alpha}(S)$ Then $\boxed{\eta^{I,L} \leftrightarrow BR(l_i \to l_j\gamma)}$

Numerical results – Inverse seesaw

We have always the freedom to $\mu = \text{diag}\{\mu_i\}$ then we can use the Casas-Ibarra parameterization for m_D:

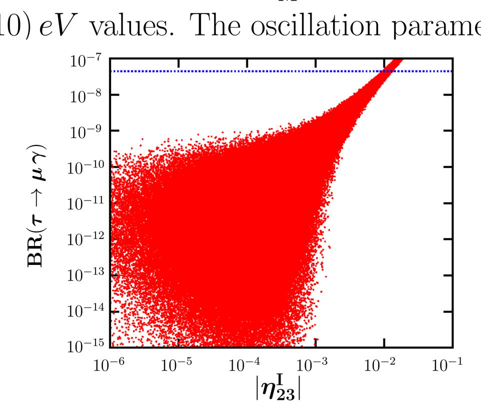
$$m_D = V_1\,diag\{\sqrt{m_i}\}\,R^T\,diag\{\sqrt{\mu_i^{-1}}\}\,M^T \quad \text{where} \quad R^T = R^{-1}$$

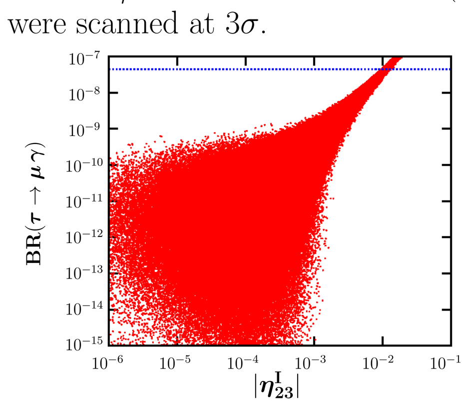
For real m_D matrix and assuming $M = \text{diag}\{M_i\}$ we finally have 9 free parameters.

To easily find points that fulfills $(m_D) < 175\,GeV$ we scan over the remaining free parameters, in the following way:

$$\mu_{ii} = v_\mu\,(1 + \varepsilon_{ii})$$
$$M_{ii} = v_M\,(1 + \varepsilon_{ii}) \quad \text{where} \quad |\varepsilon| \sim 5\times 10^{-1}.$$

The M matrix scale v_M was fixed to $1\,TeV$ while v_μ scale was scanned over $(0.1 - 10)\,eV$ values. The oscillation parameters were scanned at 3σ.





NH $\boxed{|\eta^I_{23}| < 1.3\times 10^{-2}}$ IH $\boxed{|\eta^I_{23}| < 1.3\times 10^{-2}}$

Numerical results – Linear seesaw

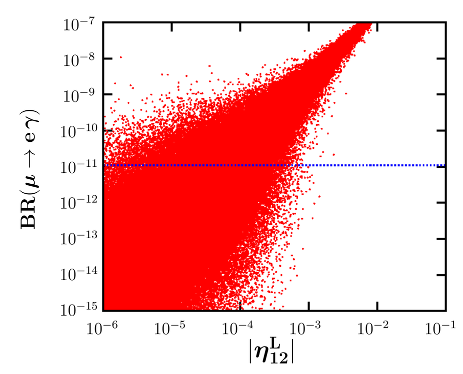
$$m_D = V_1\,diag\{\sqrt{m_i}\}\,A^T diag\{\sqrt{m_i}\}\,V_1^T\left(M_L^T\right)^{-1}M^T$$

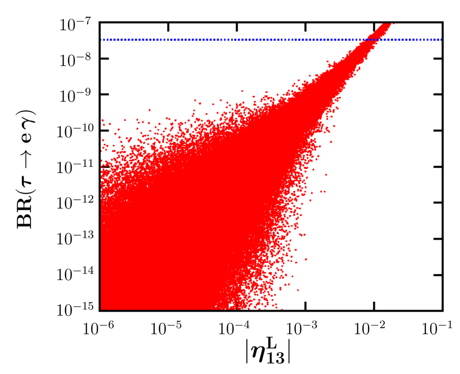
Where $A^T + A = I$. We have always the freedom to choose $M_L = \text{diag}\{M_{L_i}\}$.

For real m_D matrix and assuming $M = \text{diag}\{M_i\}$ we finally have 9 free parameters.

The scan over the parameters satisfies the same conditions and procedure like we did for the inverse scheme.

The M matrix scale v_M was fixed to $1\,TeV$ while v_L scale was scanned over $(0.1 - 10)\,eV$ values.





IH $\boxed{|\eta^L_{12}| < 1.4\times 10^{-3}}$ IH $\boxed{|\eta^L_{13}| < 1.1\times 10^{-2}}$

Summary

- Unitarity violation in the low scale seesaw mechanism is not strongly constrained by the charged lepton flavor violating processes. The results are:

Proc.	$\mu \to e\gamma$		$\tau \to e\gamma$		$\tau \to \mu\gamma$	
H	NH	IH	NH	IH	NH	IH
$\lvert\eta^I_{12}\rvert <$	1.5×10^{-3}	1.5×10^{-3}	2.8×10^{-2}	2.8×10^{-2}	2.8×10^{-2}	2.8×10^{-2}
$\lvert\eta^I_{13}\rvert <$	2.0×10^{-2}	2.1×10^{-2}	1.2×10^{-2}	1.2×10^{-2}	3.1×10^{-2}	3.2×10^{-2}
$\lvert\eta^I_{23}\rvert <$	2.6×10^{-2}	2.7×10^{-2}	6.3×10^{-2}	4.3×10^{-2}	**1.3×10^{-2}**	**1.3×10^{-2}**
$\lvert\eta^L_{12}\rvert <$	2.2×10^{-3}	**1.4×10^{-3}**	5.2×10^{-2}	5.2×10^{-2}	5.3×10^{-2}	5.7×10^{-2}
$\lvert\eta^L_{13}\rvert <$	3.6×10^{-2}	4.2×10^{-2}	1.2×10^{-2}	**1.1×10^{-2}**	4.8×10^{-2}	4.8×10^{-2}
$\lvert\eta^L_{23}\rvert <$	2.8×10^{-2}	3.4×10^{-2}	5.5×10^{-2}	5.4×10^{-2}	**1.3×10^{-2}**	**1.3×10^{-2}**

The more constraining η_{ij} bounds are in red.

- Since the bounds on η are not strongly constrained, they could be probed at future Neutrino Factories.