Neutrino Phenomenology

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Part 1
What Are Neutrinos Good For?

Energy generation in the sun starts with the reaction —

\[ p + p \rightarrow d + e^+ + \nu \]

Spin: \( \frac{1}{2} \quad \frac{1}{2} \quad 1 \quad \frac{1}{2} \quad \frac{1}{2} \)

Without the neutrino, angular momentum would not be conserved.

Uh, oh ……
The Neutrinos

Neutrinos and photons are by far the most abundant elementary particles in the universe. There are 340 neutrinos/cc.

The neutrinos are spin – 1/2, electrically neutral, leptons. The only known forces they experience are the weak force and gravity. This means that their interactions with other matter have very low strength. Thus, neutrinos are difficult to detect and study.

Their weak interactions are successfully described by the Standard Model.
The Neutrino Revolution
(1998 – …)

Neutrinos have nonzero masses!

Leptons mix!
These discoveries come from the observation of

\textit{neutrino flavor change}

(\textit{neutrino oscillation}).
The Physics of Neutrino Oscillation — Preliminaries
The Neutrino Flavors

We define the three known flavors of neutrinos, $\nu_e$, $\nu_\mu$, $\nu_\tau$, by W boson decays:

As far as we know, neither

nor any other change of flavor in the $\nu \rightarrow \ell$ interaction ever occurs. With $\alpha = e, \mu, \tau$, $\nu_\alpha$ makes only $\ell_\alpha$ ($\ell_e \equiv e$, $\ell_\mu \equiv \mu$, $\ell_\tau \equiv \tau$).
Neutrino Flavor Change

If neutrinos have masses, and leptons mix, we can have —

Give $\nu$ time to change character

$\nu_\mu \rightarrow \nu_\tau$

The last decade has brought us compelling evidence that such flavor changes actually occur.
Flavor Change Requires *Neutrino Masses*

There must be some spectrum of neutrino mass eigenstates $\nu_i$:

$$\text{Mass } (\nu_i) \equiv m_i$$
Flavor Change Requires *Leptonic Mixing*

The neutrinos $\nu_{e,\mu,\tau}$ of definite flavor 

$$(W \rightarrow e\nu_e \text{ or } \mu\nu_\mu \text{ or } \tau\nu_\tau)$$

must be *superpositions* of the mass eigenstates:

$$|\nu_\alpha> = \sum_i U^*_{\alpha i} |\nu_i>$$

Neutrino of flavor $\alpha = e, \mu, \text{ or } \tau$

PMNS Leptonic Mixing Matrix

There must be *at least 3* mass eigenstates $\nu_i$, because there are 3 orthogonal neutrinos of definite flavor $\nu_\alpha$. 
This mixing is easily incorporated into the Standard Model (SM) description of the $\ell\nu W$ interaction.

For this interaction, we then have —

$$L_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left( \bar{\ell}_{L\alpha} \gamma^\lambda \nu_{L\alpha} W_\lambda^- + \bar{\nu}_{L\alpha} \gamma^\lambda \ell_{L\alpha} W_\lambda^+ \right)$$

Taking mixing into account

If neutrino masses are described by an extension of the SM, and there are no new leptons, $U$ is unitary. Then —

$$\text{Amp}\left( W \rightarrow \ell_\alpha^+ + \nu_\alpha; \nu_\alpha \rightarrow \ell_\beta^- + W \right) \propto \sum_{i=1}^{3} U_{\alpha i}^* U_{\beta i} = \delta_{\beta\alpha}, \text{ as observed.}$$
The Meaning of $U$

$\ell^+_{\alpha} \rightarrow \frac{g}{\sqrt{2}} U^*_{\alpha i} \nu_i \rightarrow W^+$

$\nu_i \rightarrow \frac{g}{\sqrt{2}} U_{\beta i} \ell^-_{\beta}$

$U = \begin{bmatrix} \nu_1 & \nu_2 & \nu_3 \\ e & \frac{U_{e1}}{\sqrt{2}} & \frac{U_{e2}}{\sqrt{2}} & \frac{U_{e3}}{\sqrt{2}} \\ \mu & \frac{U_{\mu1}}{\sqrt{2}} & \frac{U_{\mu2}}{\sqrt{2}} & \frac{U_{\mu3}}{\sqrt{2}} \\ \tau & \frac{U_{\tau1}}{\sqrt{2}} & \frac{U_{\tau2}}{\sqrt{2}} & \frac{U_{\tau3}}{\sqrt{2}} \end{bmatrix}$

The $e$ row of $U$: The linear combination of neutrino mass eigenstates that couples to $e$.

The $\nu_1$ column of $U$: The linear combination of charged-lepton mass eigenstates that couples to $\nu_1$. 

Slides on The Physics of Neutrino Oscillation go here.
Neutrino Flavor Change In Matter

Coherent forward scattering via this $W$-exchange interaction leads to an extra interaction potential energy —

$$V_W = \begin{cases} +\sqrt{2}G_F N_e, & \nu_e \\ -\sqrt{2}G_F N_e, & \bar{\nu}_e \end{cases}$$

Fermi constant | Electron density

This raises the effective mass of $\nu_e$, and lowers that of $\bar{\nu}_e$. 
The fractional importance of matter effects on an oscillation involving a vacuum splitting $\Delta m^2$ is —

$$ \left[ \sqrt{2} G_F N_e \right] / \left[ \Delta m^2 / 2E \right] \equiv x . $$

The matter effect —

— Grows with neutrino energy $E$
— Is sensitive to $\text{Sign}(\Delta m^2)$
— Reverses when $\nu$ is replaced by $\bar{\nu}$

This last is a “fake CP violation”, but the matter effect is negligible when $x \ll 1$. 
**Evidence For Flavor Change**

**Neutrinos**
- Solar Reactor
  - (L \(\sim\) 180 km)
- Atmospheric Accelerator
  - (L = 250 and 735 km)
- Stopped \(\mu^+\) Decay
  - LSND
  - (L \(\approx\) 30 m)

**Evidence of Flavor Change**
- Compelling
- Compelling
- Compelling

Does MiniBooNE see this too??

Very recent evidence to be discussed soon.
KamLAND Evidence for Oscillatory Behavior
The KamLAND detector studies $\bar{\nu}_e$ produced by Japanese nuclear power reactors $\sim 180$ km away.

For KamLAND, $x_{\text{Matter}} < 10^{-2}$. Matter effects are negligible.

The $\bar{\nu}_e$ survival probability, $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$, should oscillate as a function of $L/E$ following the vacuum oscillation formula.

In the two-neutrino approximation, we expect —

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta \sin^2 \left[ 1.27 \Delta m^2 (eV^2) \frac{L(km)}{E(GeV)} \right].$$
$L_0 = 180 \text{ km}$ is a flux-weighted average travel distance.

$P(\bar{\nu}_e \rightarrow \nu_e)$ actually oscillates!
What We Have Learned
The (Mass)$^2$ Spectrum

\[ \Delta m^2_{\text{atm}} \]

or

\[ \Delta m^2_{\text{sol}} \]

\[ \Delta m^2_{\text{sol}} \approx 7.5 \times 10^{-5} \text{ eV}^2, \quad \Delta m^2_{\text{atm}} \approx 2.3 \times 10^{-3} \text{ eV}^2 \]
The Absolute Scale of Neutrino Mass

How far above zero is the whole pattern?

Oscillation Data $\Rightarrow \sqrt{\Delta m^2_{\text{atm}}} < \text{Mass}[\text{Heaviest } \nu_i]$
The Upper Bound From Cosmology

(See Concha Gonzalez-Garcia)

Neutrino mass affects large scale structure.

Cosmological Data + Cosmological Assumptions ⇒

\[ \sum m_i < (0.17 - 1.0) \text{ eV} \]

If there are only 3 neutrinos,

\[ 0.04 \text{ eV} \leq \text{Mass}[\text{Heaviest } \nu_i] < (0.07 - 0.4) \text{ eV} \]

\[ \sqrt{\Delta m^2_{\text{atm}}} \]

Cosmology

Seljak, Slosar, McDonald
Hannestad; Pastor
The Upper Bound From Tritium

Cosmology is wonderful, but there are known loopholes in its argument concerning neutrino mass.

The absolute neutrino mass can in principle also be measured by the kinematics of $\beta$ decay.

**Tritium decay**: $^{3}H \rightarrow ^{3}He + e^{-} + \bar{\nu}_{i} ; \; i = 1, 2, \text{or} \; 3$

$$BR\left(^{3}H \rightarrow ^{3}He + e^{-} + \bar{\nu}_{i}\right) \propto |U_{ei}|^{2}$$

In $^{3}H \rightarrow ^{3}He + e^{-} + \bar{\nu}_{i}$, the bigger $m_{i}$ is, the smaller the maximum electron energy is.

*There are 3 separate thresholds in the $\beta$ energy spectrum.*
The $\beta$ energy spectrum is modified according to —

\[
(E_0 - E)^2 \Theta[E_0 - E] \Rightarrow \sum_i |U_{ei}|^2 (E_0 - E) \sqrt{(E_0 - E)^2 - m_i^2} \Theta[(E_0 - m_i) - E]
\]

Maximum $\beta$ energy when there is no neutrino mass

Present experimental energy resolution is insufficient to separate the thresholds.

Measurements of the spectrum bound the average neutrino mass —

\[
\langle m_\beta \rangle = \sqrt{\sum_i |U_{ei}|^2 m_i^2}
\]

Presently: $\langle m_\beta \rangle < 2 \text{ eV}$
Leptonic Mixing

This has the consequence that —

\[ |\nu_i\rangle = \sum_{\alpha} U_{\alpha i} |\nu_\alpha\rangle. \]

Flavor-\(\alpha\) fraction of \(\nu_i\) = \(|U_{\alpha i}|^2\).

When a \(\nu_i\) interacts and produces a charged lepton, the probability that this charged lepton will be of flavor \(\alpha\) is \(|U_{\alpha i}|^2\).
The spectrum, showing its approximate flavor content, is

\[ \nu_3 \]

\[ \Delta m^2_{\text{atm}} \]

\[ \nu_2 \]

\[ \nu_1 \]

\[ (\text{Mass})^2 \]

\[ \Delta m^2_{\text{sol}} \]

or

\[ \nu_3 \]

\[ \Delta m^2_{\text{atm}} \]

\[ \nu_2 \]

\[ \nu_1 \]

\[ \Delta m^2_{\text{sol}} \]

Normal

\[ \nu_e [\left| U_{ei} \right|^2] \]

\[ \nu_\mu [\left| U_{\mu i} \right|^2] \]

\[ \nu_\tau [\left| U_{\tau i} \right|^2] \]

Inverted

\[ \sin^2 \theta_{13} \]
[\Delta m^2_{\text{atm}}] \quad \text{Bounded by reactor exps. with } L \sim 1 \text{ km}

\text{From max. atm. mixing, } \nu_3 \approx \frac{\nu_\mu + \nu_\tau}{\sqrt{2}}

\begin{cases}
\text{From } \nu_\mu (\text{Up}) \text{ oscillate but } \nu_\mu (\text{Down}) \text{ don’t}
\end{cases}

\begin{cases}
\text{In LMA–MSW, } P_{\text{sol}}(\nu_e \to \nu_e) = \nu_e \text{ fraction of } \nu_2
\end{cases}

\begin{cases}
\Delta m^2_{\text{sol}} \quad \text{From distortion of } \bar{\nu}_e (\text{reactor}) \\
\text{and } \nu_e (\text{solar}) \text{ spectra}
\end{cases}

\begin{cases}
\text{From max. atm. mixing, } \nu_1 + \nu_2 \\
\text{includes } (\nu_\mu - \nu_\tau) / \sqrt{2}
\end{cases}

\begin{align*}
\nu_e [|U_{ei}|^2] & \quad \nu_\mu [|U_{\mu i}|^2] & \quad \nu_\tau [|U_{\tau i}|^2]
\end{align*}
The 3 X 3 Unitary Mixing Matrix

**Caution**: We are assuming the mixing matrix $U$ to be 3 x 3 and unitary.

$$L_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left( \bar{\ell}_{L\alpha} \gamma^\lambda \nu_{L\alpha} W^\lambda_- + \bar{\nu}_{L\alpha} \gamma^\lambda \ell_{L\alpha} W^\lambda_+ \right)$$

$$= -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \sum_{i=1,2,3} \left( \bar{\ell}_{L\alpha} \gamma^\lambda U_{\alpha i} \nu_{L i} W^\lambda_- + \bar{\nu}_{L i} \gamma^\lambda U_{\alpha i}^* \ell_{L\alpha} W^\lambda_+ \right)$$

$$(CP)\left( \bar{\ell}_{L\alpha} \gamma^\lambda U_{\alpha i} \nu_{L i} W^\lambda_- \right)(CP)^{-1} = \bar{\nu}_{L i} \gamma^\lambda U_{\alpha i} \ell_{L\alpha} W^\lambda_+$$

Phases in $U$ will lead to CP violation, unless they are removable by redefining the leptons.
$U_{\alpha i}$ describes —

$$U_{\alpha i} \sim \langle \ell_\alpha W^+ | H | \nu_i \rangle$$

When $|\nu_i\rangle \rightarrow |e^{i\phi} \nu_i\rangle$, $U_{\alpha i} \rightarrow e^{i\phi} U_{\alpha i}$

When $|\ell^-_\alpha\rangle \rightarrow |e^{i\phi} \ell^-_\alpha\rangle$, $U_{\alpha i} \rightarrow e^{-i\phi} U_{\alpha i}$

Thus, one may multiply any column, or any row, of $U$ by a complex phase factor without changing the physics.

*Some* phases may be removed from $U$ in this way.
**Exception:** If the neutrino mass eigenstates are their own antiparticles, then —

\[ \nu_i = \nu_i^c = C\bar{\nu}_i^T \]

One is no longer free to phase-redefine \( \nu_i \) without consequences.

U can contain additional CP-violating phases.
How Many Mixing Angles and $\mathbb{CP}$ Phases Does $U$ Contain?

Real parameters before constraints: 18

Unitarity constraints — $\sum_i U_{\alpha i}^* U_{\beta i} = \delta_{\alpha\beta}$

Each row is a vector of length unity: – 3

Each two rows are orthogonal vectors: – 6

Rephase the three $\ell_\alpha$: – 3

Rephase two $\nu_i$, if $\bar{\nu}_i \neq \nu_i$: – 2

Total physically-significant parameters: 4

Additional (Majorana) $\mathbb{CP}$ phases if $\bar{\nu}_i = \nu_i$: 2
How Many Of The Parameters Are Mixing Angles?

The *mixing angles* are the parameters in $U$ when it is *real*.

$U$ is then a three-dimensional rotation matrix.

Everyone knows such a matrix is described in terms of 3 angles.

Thus, $U$ contains 3 mixing angles.

**Summary**

<table>
<thead>
<tr>
<th>Mixing angles</th>
<th>$\mathcal{CP}$ phases if $\bar{\nu}_i \neq \nu_i$</th>
<th>$\mathcal{CP}$ phases if $\bar{\nu}_i = \nu_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>
The Mixing Matrix

\[ U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \times \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \times \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \]

\[ c_{ij} \equiv \cos \theta_{ij} \]
\[ s_{ij} \equiv \sin \theta_{ij} \]

\[ \theta_{12} \approx \theta_{\text{sol}} \approx 34^\circ, \; \theta_{23} \approx \theta_{\text{atm}} \approx 39-51^\circ, \; \theta_{13} \lesssim 12^\circ \]

\[ \delta \] would lead to \( P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta). \]

But note the crucial role of \( s_{13} \equiv \sin \theta_{13}. \)

Hints??
Recent Evidence For Non-Zero $\theta_{13}$

In an experiment where $L/E$ is too small for the small splitting $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$ to be seen,

$$P(\nu_\mu \to \nu_e) \approx 4|U_{\mu 3}U_{e 3}|^2 \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right)$$

$$= \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 \left( \frac{\Delta m_{31}^2 L}{4E} \right)$$

T2K has looked for $\nu_\mu \to \nu_e$ in a long-baseline experiment:
The T2K experiment

$\pi \rightarrow \mu + \nu_\mu$

Near Detector

ν

$295 \text{ km}$

$E = 0.6 \text{ GeV}$

The far detector is near the first $\Delta m^2_{31}$ oscillation maximum.

Expectation

T2K sees 6 $\nu_e$ candidate events in the far detector, whereas 1.5 are expected if $\theta_{13} = 0$. 

Far Detector

Super-K

Observation
These take the $\Delta m^2_{21}$ contributions and matter effects into account.

$\theta_{13}$ could be "large".
MINOS, not designed to look for $\nu_\mu \rightarrow \nu_e$, sees 62 candidate events where 50 are expected if $\theta_{13} = 0$.

While not highly significant by itself, this result is consistent with that from T2K.
There Is Nothing Special About $\theta_{13}$

*All* mixing angles must be nonzero for $CP$ in oscillation.

For example —

$$P(\bar{\nu}_\mu \to \bar{\nu}_e) - P(\nu_\mu \to \nu_e) = 2 \cos \theta_{13} \sin 2 \theta_{13} \sin 2 \theta_{12} \sin 2 \theta_{23} \sin \delta$$

$$\times \sin \left( \Delta m^2_{31} \frac{L}{4E} \right) \sin \left( \Delta m^2_{32} \frac{L}{4E} \right) \sin \left( \Delta m^2_{21} \frac{L}{4E} \right)$$

In the factored form of $U$, one can put $\delta$ next to $\theta_{12}$ instead of $\theta_{13}$.
The Majorana $CP$ Phases

The phase $\alpha_i$ is associated with neutrino mass eigenstate $\nu_i$:

$$U_{\alpha_i} = U^0_{\alpha_i} \exp(i\alpha_i/2) \text{ for all flavors } \alpha.$$ 

$$\text{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i U_{\alpha_i}^* \exp(-im_i^2L/2E) U_{\beta_i}$$

is insensitive to the Majorana phases $\alpha_i$.

Only the phase $\delta$ can cause CP violation in neutrino oscillation.