Neutrino Phenomenology

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Part 1

What Are Neutrinos Good For?

Energy generation in the sun starts with the reaction —

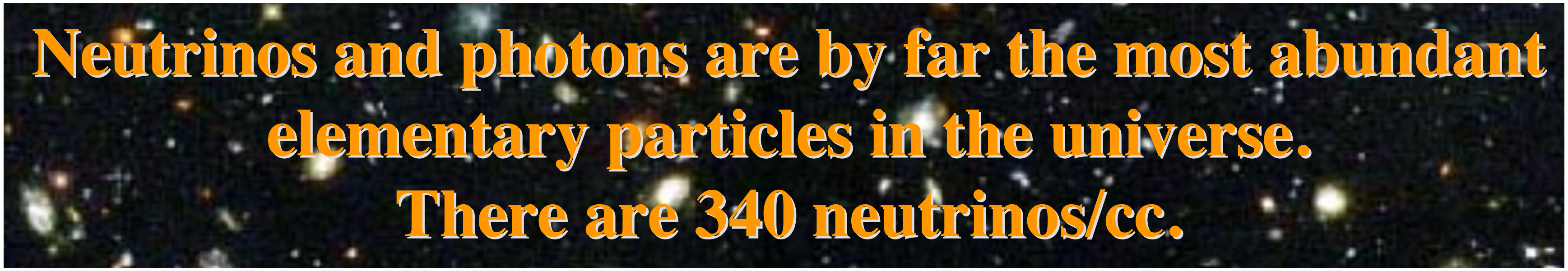
$$p + p \rightarrow d + e^{+} + \nu$$

Spin:	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{2}$

Without the neutrino, angular momentum would not be conserved.

Uh, oh ……

The Neutrinos



The neutrinos are spin – 1/2, electrically neutral, leptons.

The only known forces they experience are the weak force and gravity.

This means that their interactions with other matter have very low strength.
Thus, neutrinos are difficult to detect and study.

Their weak interactions are successfully described by the Standard Model.

The Neutrino Revolution
(1998 – …)

Neutrinos have nonzero masses!

Leptons mix!

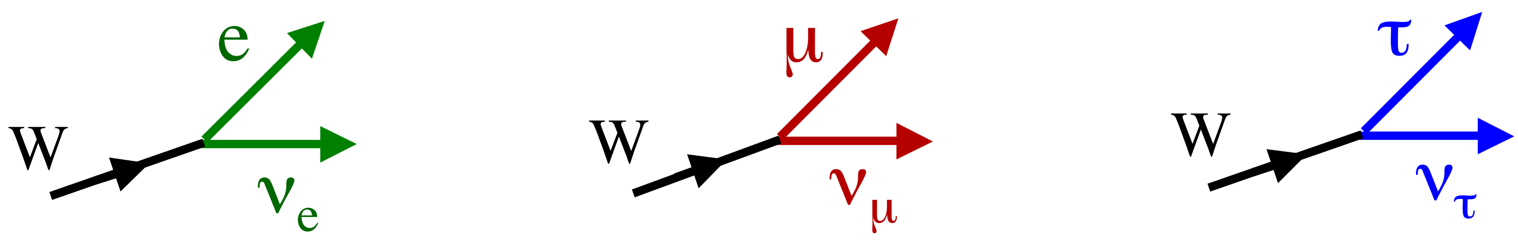
These discoveries come from the observation of ***neutrino flavor change (neutrino oscillation)***.

The Physics of Neutrino Oscillation

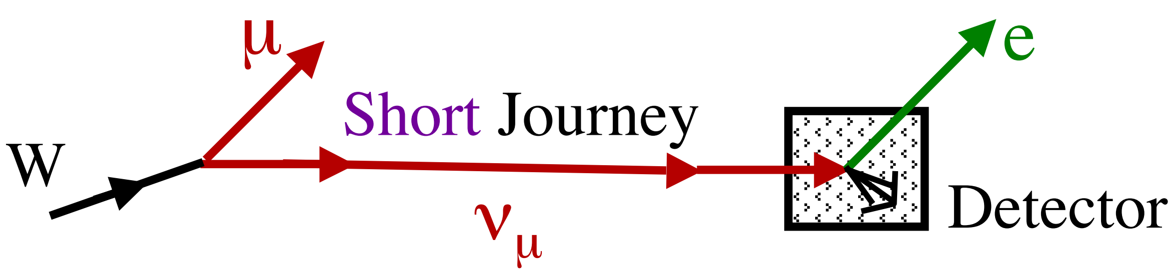
— Preliminaries

The Neutrino Flavors

We *define* the three known flavors of neutrinos, ν_e, ν_μ, ν_τ, by W boson decays:



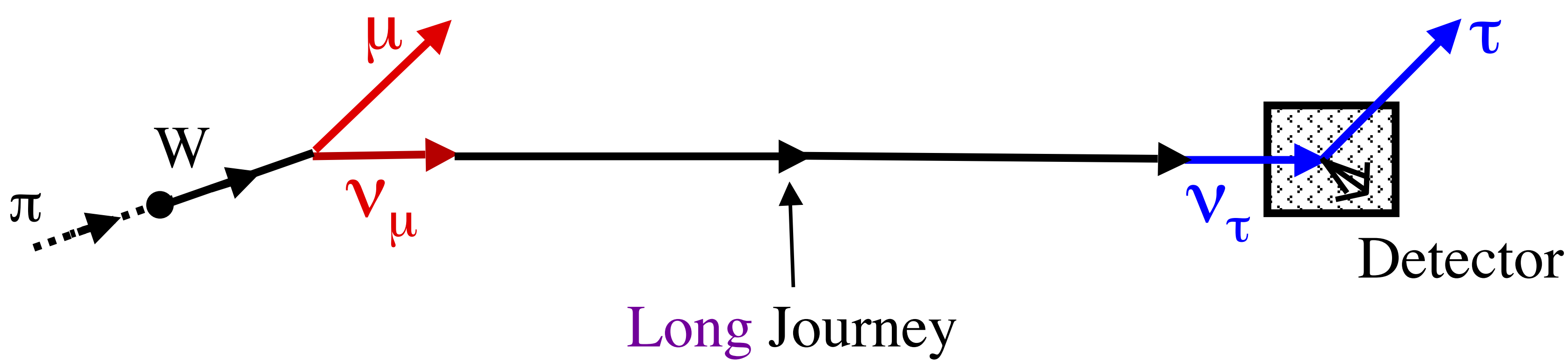
As far as we know, neither



nor any other change of flavor in the $\nu \rightarrow \ell$ ***interaction*** ever occurs. With $\alpha = e, \mu, \tau$, ν_α makes only ℓ_α ($\ell_e \equiv e$, $\ell_\mu \equiv \mu$, $\ell_\tau \equiv \tau$).

Neutrino Flavor Change

If neutrinos have masses, and leptons mix, we can have —



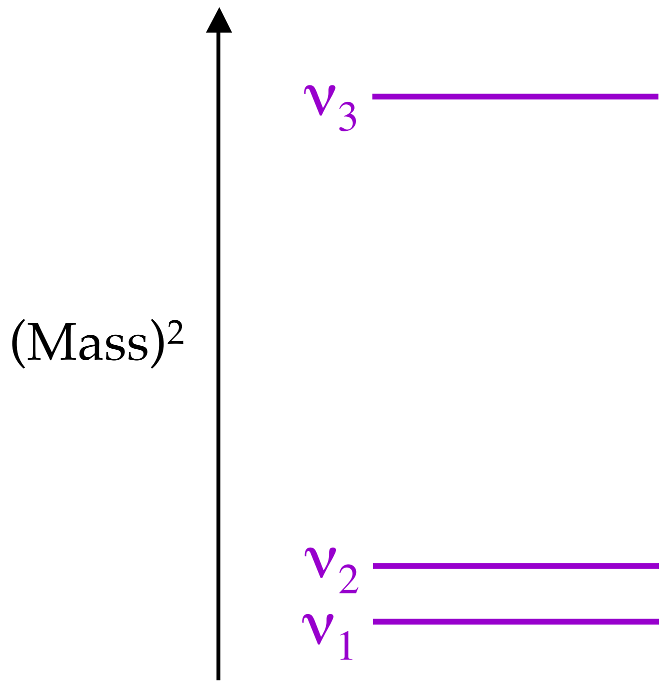
Give ν time to change character

$$\nu_\mu \longrightarrow \nu_\tau$$

The last decade has brought us compelling evidence that such flavor changes actually occur.

Flavor Change Requires ***Neutrino Masses***

There must be some spectrum
of neutrino mass eigenstates ν_i:



Mass (ν_i) $\equiv m_i$

Flavor Change Requires ***Leptonic Mixing***

The neutrinos $\nu_{e,\mu,\tau}$ of definite flavor

$(W \rightarrow e\nu_e \text{ or } \mu\nu_\mu \text{ or } \tau\nu_\tau)$

must be superpositions of the mass eigenstates:

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i}\, |\nu_i\rangle \quad .$$

Neutrino of flavor $\alpha = e, \mu, \text{ or } \tau$

PMNS Leptonic Mixing Matrix

Neutrino of definite mass m_i

There must be ***at least 3*** mass eigenstates ν_i, because there are 3 orthogonal neutrinos of definite flavor ν_α.

This *mixing* is easily incorporated into the Standard Model (SM) description of the $\ell\nu W$ interaction.

For this interaction, we then have —

Semi-weak coupling

Left-handed

$$L_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left(\overline{\ell}_{L\alpha} \gamma^{\lambda} \nu_{L\alpha} W_{\lambda}^{-} + \overline{\nu}_{L\alpha} \gamma^{\lambda} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

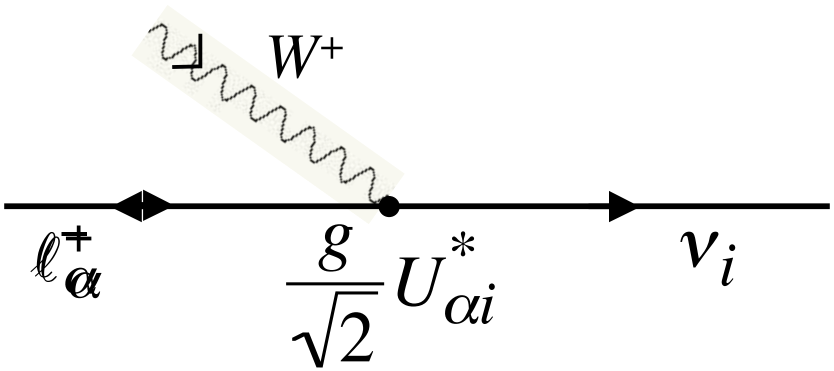
$$= -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\overline{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} + \overline{\nu}_{Li} \gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

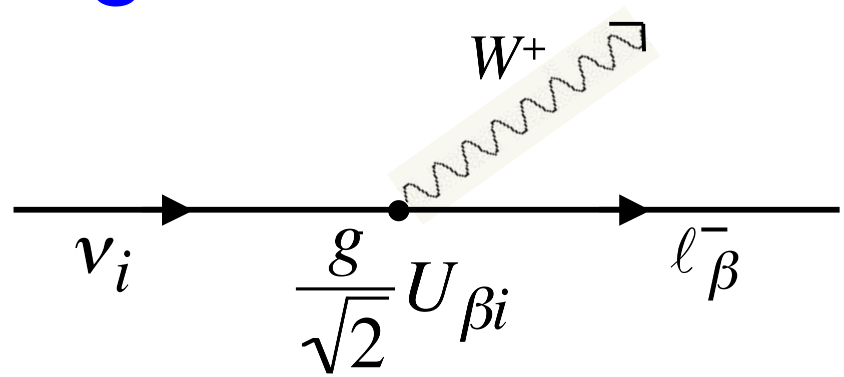
Taking mixing into account

If neutrino *masses* are described by an extension of the SM, and there are no new leptons, U is *unitary*. Then —

$$\mathrm{Amp}\left(W \to \ell_{\alpha}^{+} + \nu_{\alpha}; \nu_{\alpha} \to \ell_{\beta}^{-} + W \right) \propto \sum_{i=1}^{3} U_{\alpha i}^{*} U_{\beta i} = \delta_{\beta\alpha}, \text{ as observed.}$$

The Meaning of *U*





$$
U \;=\; \begin{array}{c c}
 & \begin{array}{ccc} \nu_1 & \nu_2 & \nu_3 \end{array} \\
\begin{array}{c} e \\ \mu \\ \tau \end{array} & \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}
\end{array}
$$

The e row of U: The linear combination of neutrino mass eigenstates that couples to e.

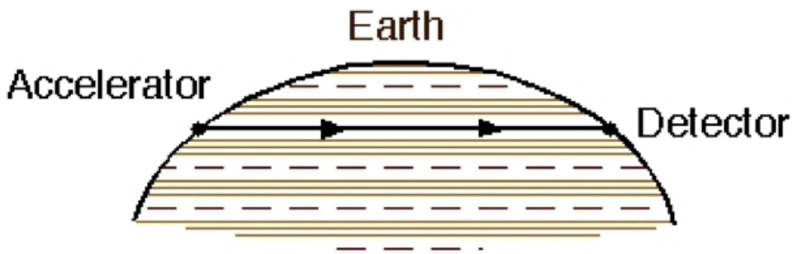
The ν_1 column of U: The linear combination of charged-lepton mass eigenstates that couples to ν_1 .

Slides on

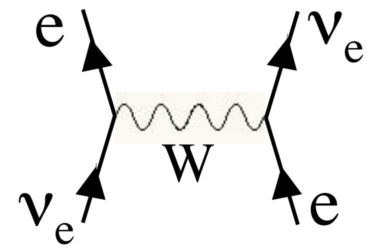
The Physics of Neutrino Oscillation

go here.

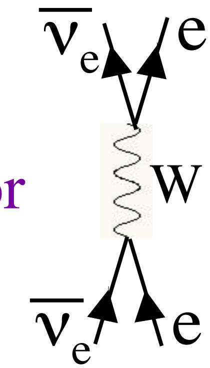
Neutrino Flavor Change In Matter



involves



or



Coherent forward scattering via this
W-exchange interaction leads to
an extra interaction potential energy —

$$V_W = \begin{cases} +\sqrt{2}G_F N_e, & \nu_e \\ -\sqrt{2}G_F N_e, & \overline{\nu_e} \end{cases}$$

Fermi constant

Electron density

This raises the effective mass of ν_e, and lowers that of $\overline{\nu_e}$.

The fractional importance of matter effects on an oscillation involving a vacuum splitting Δm^2 is —

$$\underbrace{[\sqrt{2}G_F N_e]}_{\text{Interaction energy}} / \underbrace{[\Delta m^2/2E]}_{\text{Vacuum energy}} \equiv x \; .$$

The matter effect —

— Grows with neutrino energy E

— Is sensitive to Sign(Δm^2)

— Reverses when ν is replaced by $\bar{\nu}$

This last is a "fake CP violation", but the matter effect is negligible when $x << 1$.

Evidence For Flavor Change

Neutrinos	Evidence of Flavor Change
Solar	Compelling
Reactor (L ~ 180 km)	Compelling
Atmospheric	Compelling
Accelerator (L = 250 and 735 km)	Compelling
Stopped μ^+ Decay $\left(\begin{array}{c}\text{LSND}\\ L \approx 30\text{ m}\end{array}\right)$	Does MiniBooNE see this too??

Very recent evidence to be discussed soon.

KamLAND Evidence for Oscillatory Behavior

The KamLAND detector studies $\overline{\nu}_e$ produced by Japanese nuclear power reactors ~ 180 km away.

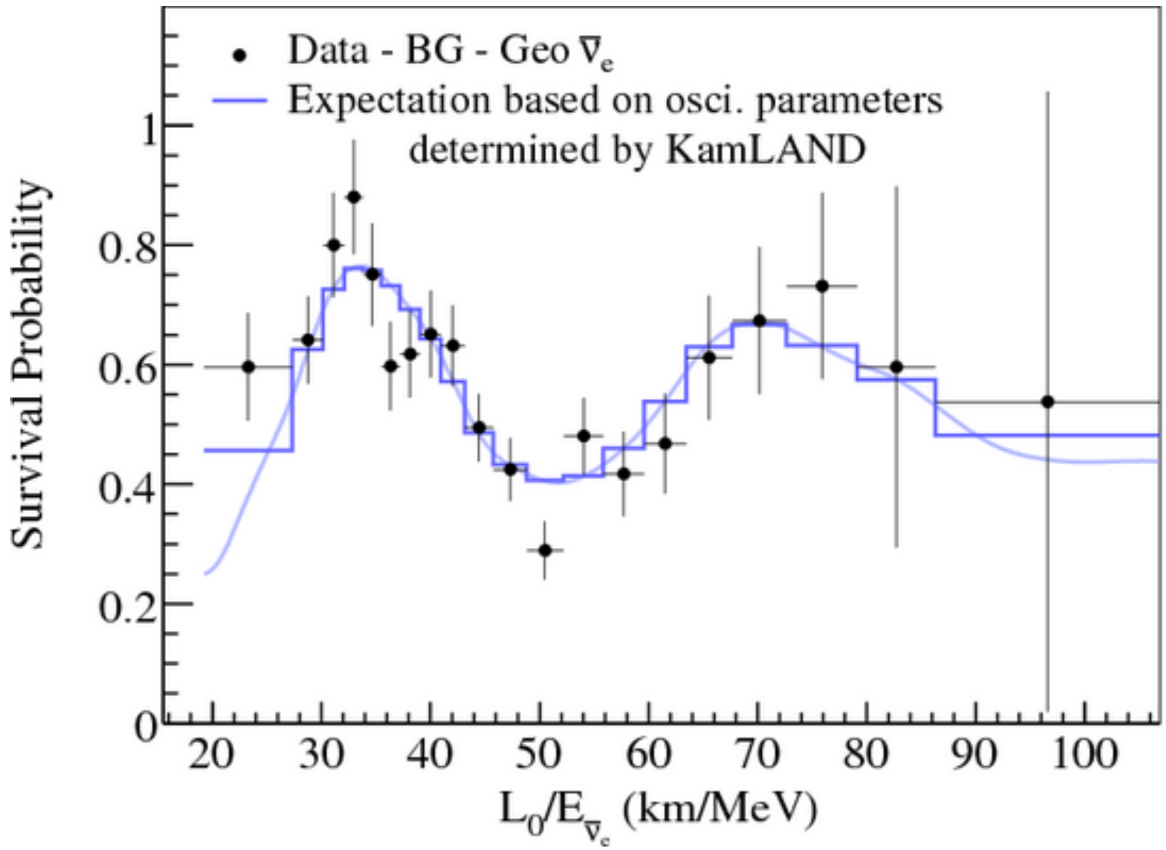
For KamLAND, $x_{Matter} < 10^{-2}$. Matter effects are negligible.

The $\bar{\nu}_e$ survival probability, $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$, should oscillate as a function of L/E following the vacuum oscillation formula.

In the two-neutrino approximation, we expect —

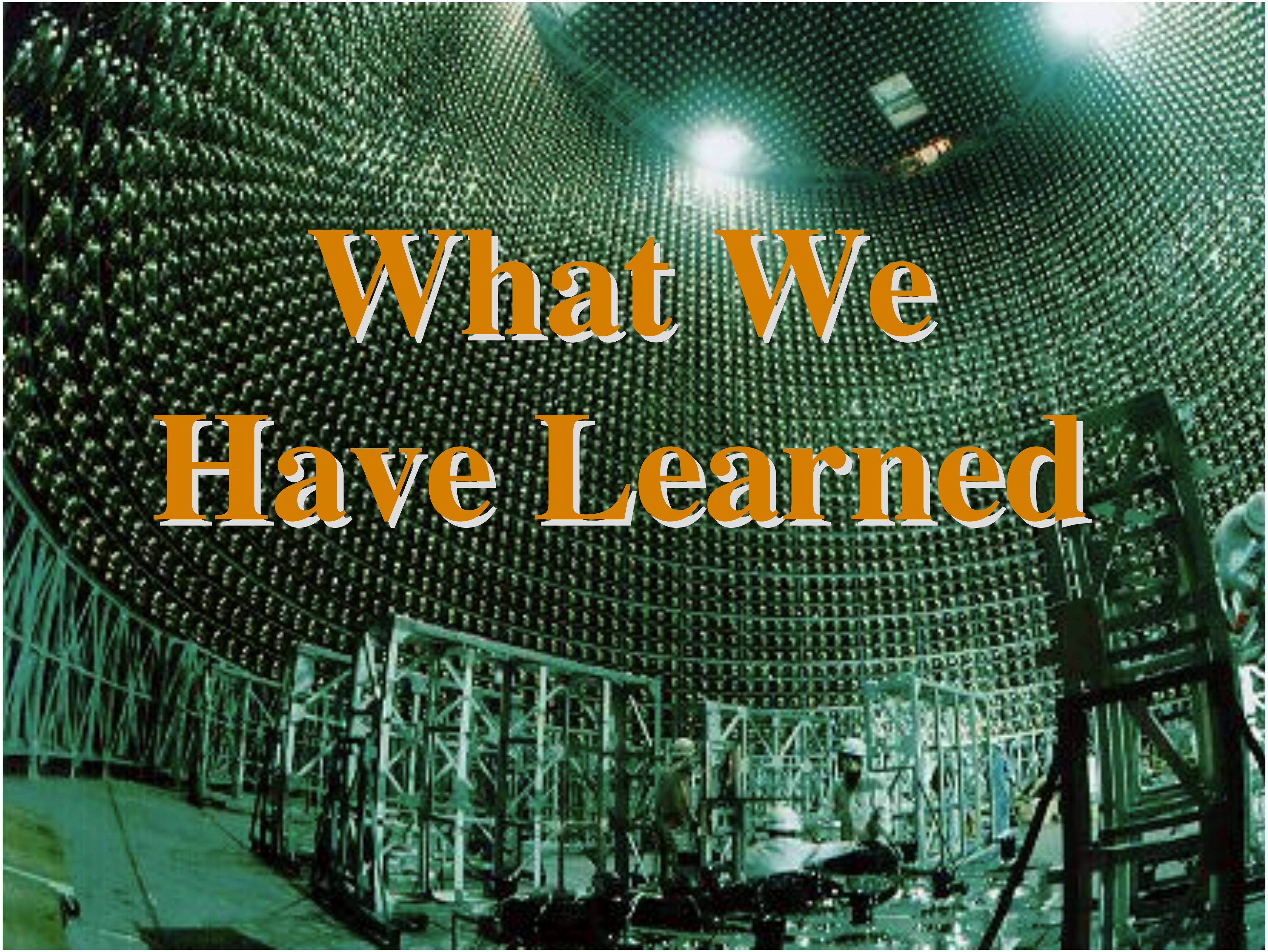
$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \sin^2 2\theta \sin^2\left[1.27\Delta m^2 \left(eV^2\right)\frac{L(km)}{E(GeV)}\right].$$

Survival probability $P(\bar{\nu}_e \to \bar{\nu}_e)$ of reactor $\bar{\nu}_e$

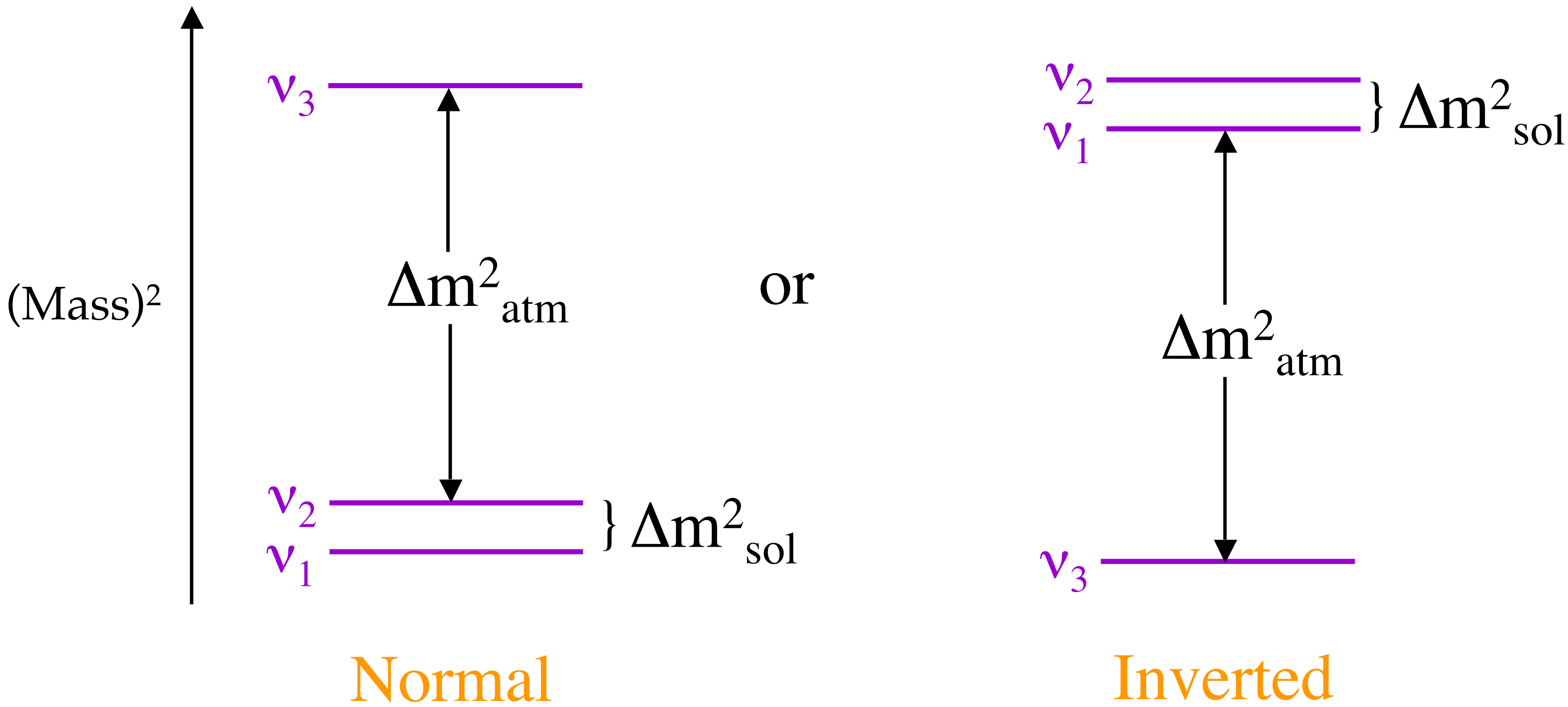


L_0 = 180 km is a flux-weighted average travel distance.

$P(\bar{\nu}_e \to \bar{\nu}_e)$ actually oscillates!

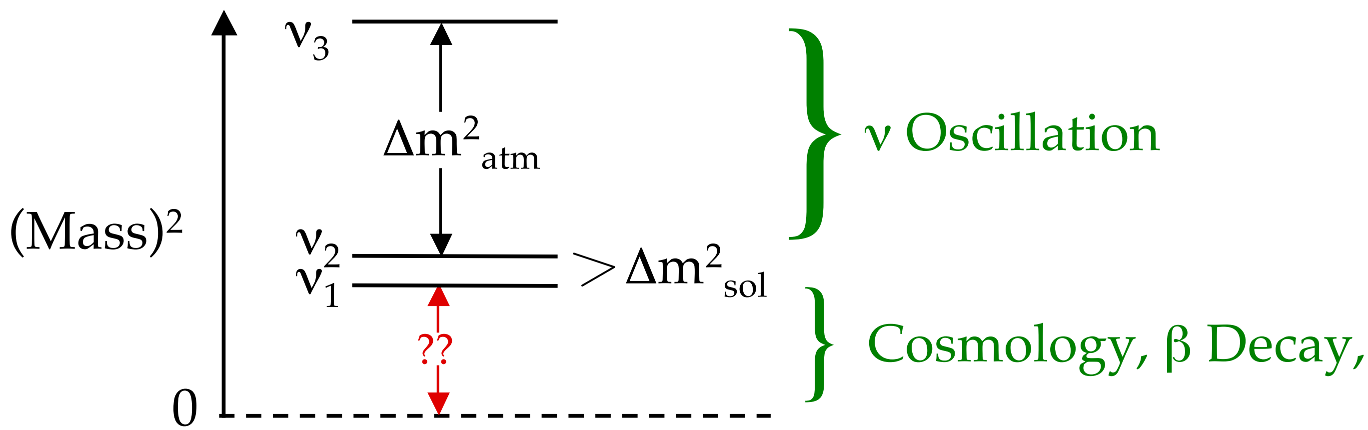

What We
Have Learned

The (Mass)^2 Spectrum



$\Delta m^2_{sol} \cong 7.5 \times 10^{-5}\ eV^2$, $\Delta m^2_{atm} \cong 2.3 \times 10^{-3}\ eV^2$

The Absolute Scale of Neutrino Mass



How far above zero is the whole pattern?

Oscillation Data $\Rightarrow \sqrt{\Delta m^2_{atm}} <$ Mass[Heaviest ν_i]

The Upper Bound From Cosmology

(See Concha Gonzalez-Garcia)

Neutrino mass affects large scale structure.

Cosmological Data + Cosmological Assumptions $\Rightarrow$

$$\underbrace{\Sigma\, m_i}_{\text{Mass}(\nu_i)} < (0.17 - 1.0)\ \text{eV}\ .$$

(Seljak, Slosar, McDonald
Hannestad; Pastor)

If there are only 3 neutrinos,

$$\underbrace{0.04\ \text{eV}}_{\sqrt{\Delta m^2_{atm}}} \lesssim \text{Mass[Heaviest } \nu_i] < \underbrace{(0.07 - 0.4)\ \text{eV}}_{\text{Cosmology}}$$

The Upper Bound From Tritium

Cosmology is wonderful, but there are known loopholes in its argument concerning neutrino mass.

The absolute neutrino mass can in principle also be measured by the kinematics of β decay.

Tritium decay: $\quad {}^3H \to {}^3He + e^- + \overline{\nu_i}\ ;\ i = 1, 2, \text{or } 3$

$$BR\left({}^3H \to {}^3He + e^- + \overline{\nu_i}\right) \propto \left|U_{ei}\right|^2$$

In ${}^3H \to {}^3He + e^- + \overline{\nu_i}$, the bigger m_i is, the smaller the maximum electron energy is.

There are 3 separate thresholds in the β energy spectrum.

The β energy spectrum is modified according to —

$$(E_0 - E)^2 \Theta[E_0 - E] \Rightarrow \sum_i |U_{ei}|^2 (E_0 - E)\sqrt{(E_0 - E)^2 - m_i^2}\, \Theta[(E_0 - m_i) - E]$$

Maximum β energy when there is no neutrino mass

β energy

Present experimental energy resolution is insufficient to separate the thresholds.

Measurements of the spectrum bound the average neutrino mass —

$$\langle m_\beta \rangle = \sqrt{\sum_i |U_{ei}|^2 m_i^2}$$

$$\boxed{\text{Presently: } \langle m_\beta \rangle < 2 \text{ eV}}$$

Mainz & Troitzk

Leptonic Mixing

This has the consequence that —

Mass eigenstate

Flavor eigenstate

$$|\nu_i\rangle = \sum_\alpha U_{\alpha i}\, |\nu_\alpha\rangle \;.$$

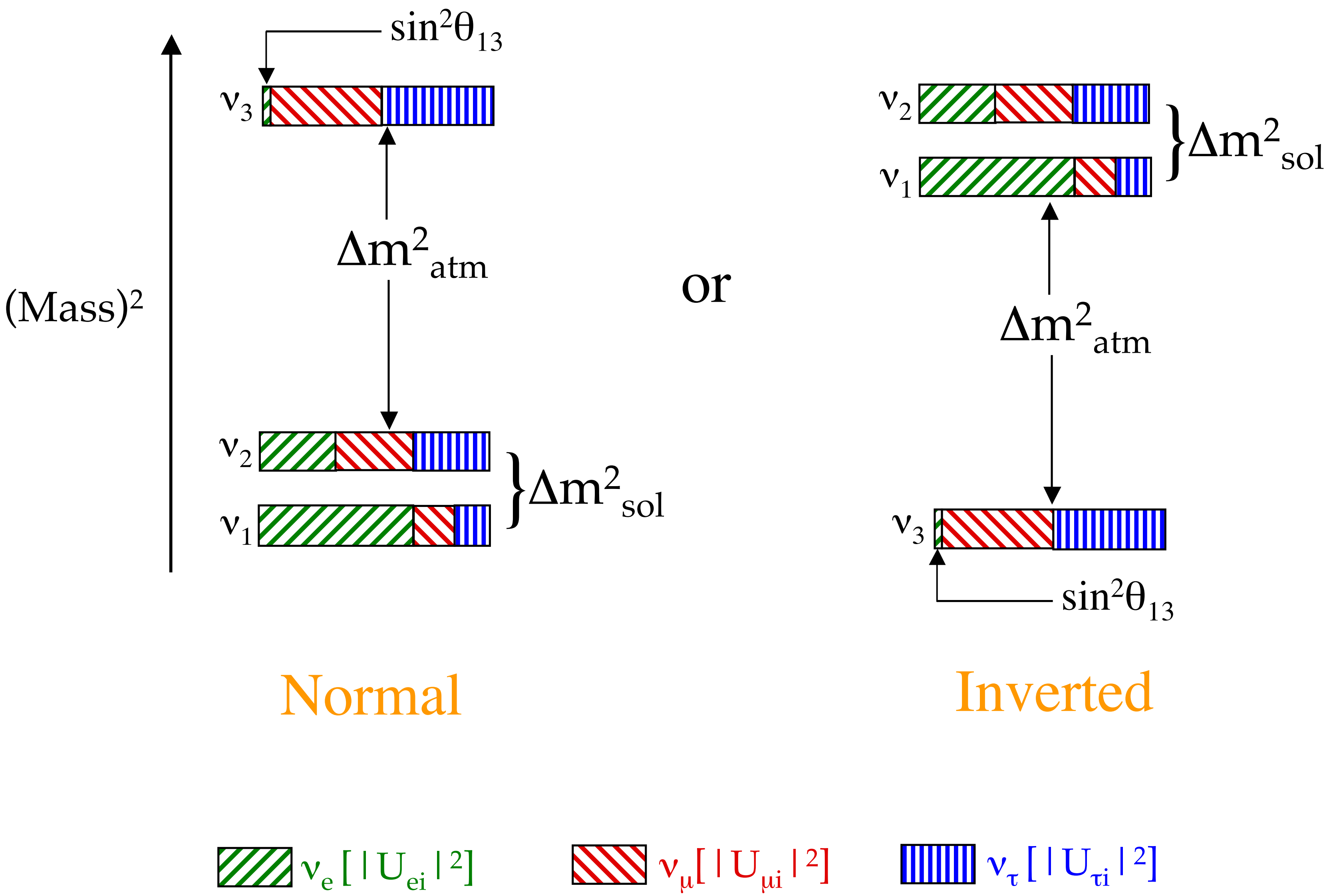
e, μ, or τ

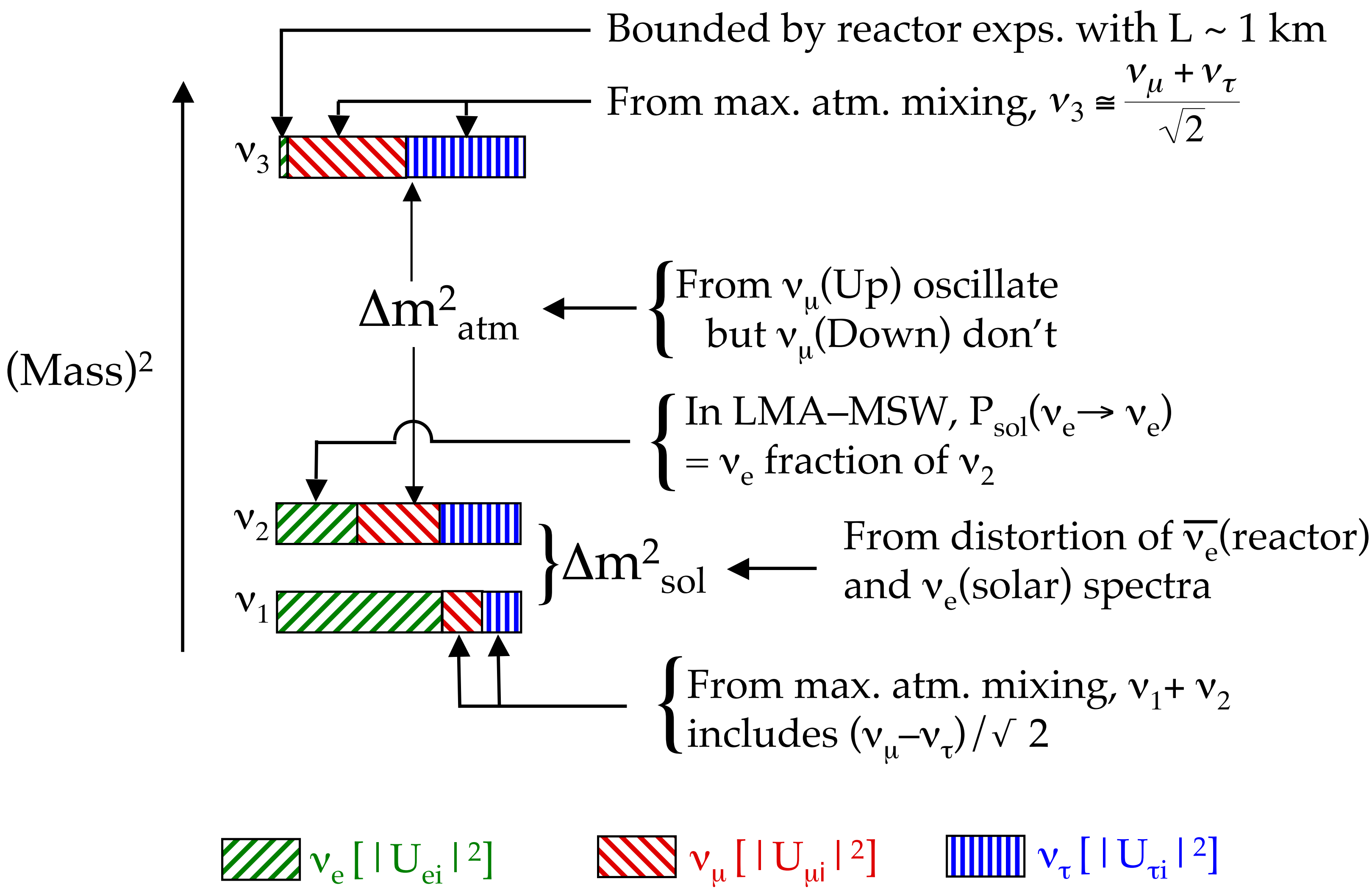
Leptonic Mixing Matrix

Flavor-α fraction of $\nu_i = |U_{\alpha i}|^2$.

When a ν_i interacts and produces a charged lepton, the probability that this charged lepton will be of flavor α is $|U_{\alpha i}|^2$.

The spectrum, showing its approximate flavor content, is




Bounded by reactor exps. with L ~ 1 km
From max. atm. mixing, $\nu_3 \cong \frac{\nu_\mu + \nu_\tau}{\sqrt{2}}$
ν_3
Δm^2_{atm}
From ν_μ(Up) oscillate but ν_μ(Down) don't
(Mass)^2
In LMA–MSW, $P_{sol}(\nu_e \to \nu_e)$ = ν_e fraction of ν_2
ν_2
ν_1
Δm^2_{sol}
From distortion of $\overline{\nu_e}$(reactor) and ν_e(solar) spectra
From max. atm. mixing, $\nu_1 + \nu_2$ includes $(\nu_\mu - \nu_\tau)/\sqrt{2}$
ν_e [$|U_{ei}|^2$]
ν_μ [$|U_{\mu i}|^2$]
ν_τ [$|U_{\tau i}|^2$]

The 3 X 3 Unitary Mixing Matrix

Caution: We are *assuming* the mixing matrix U to be 3 x 3 and unitary.

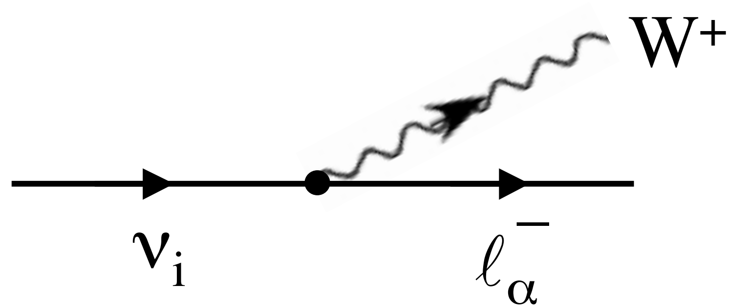
$$L_{SM} = -\frac{g}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \left(\bar{\ell}_{L\alpha} \gamma^{\lambda} \nu_{L\alpha} W_{\lambda}^{-} + \bar{\nu}_{L\alpha} \gamma^{\lambda} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

$$= -\frac{g}{\sqrt{2}} \sum_{\substack{\alpha=e,\mu,\tau \\ i=1,2,3}} \left(\bar{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} + \bar{\nu}_{Li} \gamma^{\lambda} U_{\alpha i}^{*} \ell_{L\alpha} W_{\lambda}^{+} \right)$$

$$(CP)\left(\bar{\ell}_{L\alpha} \gamma^{\lambda} U_{\alpha i} \nu_{Li} W_{\lambda}^{-} \right)(CP)^{-1} = \bar{\nu}_{Li} \gamma^{\lambda} U_{\alpha i} \ell_{L\alpha} W_{\lambda}^{+}$$

Phases in U will lead to CP violation, unless they are removable by redefining the leptons.

$U_{\alpha i}$ describes —



$$U_{\alpha i} \sim \langle \ell_\alpha^- W^+ | H | \nu_i \rangle$$

$$\text{When } |\nu_i\rangle \rightarrow |e^{i\varphi}\nu_i\rangle, \; U_{\alpha i} \rightarrow e^{i\varphi} U_{\alpha i}$$

$$\text{When } |\ell_\alpha^-\rangle \rightarrow |e^{i\varphi}\ell_\alpha^-\rangle, \; U_{\alpha i} \rightarrow e^{-i\varphi} U_{\alpha i}$$

Thus, one may multiply any column, or any row, of U by a complex phase factor without changing the physics.

Some phases may be removed from U in this way.

Exception: If the neutrino mass eigenstates are their own antiparticles, then —

Charge conjugate

$$\nu_i = \nu_i^{\,c} = C\overline{\nu_i}^{\,T}$$

One is no longer free to phase-redefine ν_i without consequences.

U can contain additional CP-violating phases.

How Many Mixing Angles and $\not{CP}$ Phases Does U Contain?

Real parameters before constraints:	18
Unitarity constraints — $\sum_i U_{\alpha i}{}^* U_{\beta i} = \delta_{\alpha\beta}$	
Each row is a vector of length unity:	– 3
Each two rows are orthogonal vectors:	– 6
Rephase the three ℓ_α :	– 3
Rephase two ν_i , if $\overline{\nu}_i \neq \nu_i$:	– 2
Total physically-significant parameters:	4
Additional (Majorana) $\not{CP}$ phases if $\overline{\nu}_i = \nu_i$:	2

How Many Of The Parameters Are Mixing Angles?

The *mixing angles* are the parameters in U when it is *real*.

U is then a three-dimensional rotation matrix.

Everyone knows such a matrix is described in terms of 3 angles.

Thus, U contains 3 mixing angles.

Summary

Mixing angles	$\not{CP}$ phases if $\bar{\nu}_i \neq \nu_i$	$\not{CP}$ phases if $\bar{\nu}_i = \nu_i$
3	1	3

The Mixing Matrix

$$U = \overset{\text{Atmospheric}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix}} \times \overset{\text{Cross-Mixing}}{\begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix}} \times \overset{\text{Solar}}{\begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$\times \underset{\text{Majorana } \cancel{CP} \text{ phases}}{\begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$c_{ij} \equiv \cos\theta_{ij}$
$s_{ij} \equiv \sin\theta_{ij}$

Hints??

$\theta_{12} \approx \theta_{sol} \approx 34°, \ \theta_{23} \approx \theta_{atm} \approx 39\text{-}51°, \ \theta_{13} \lesssim 12°$

δ would lead to $P(\overline{\nu}_\alpha \rightarrow \overline{\nu}_\beta) \neq P(\nu_\alpha \rightarrow \nu_\beta)$. $\cancel{CP}$

But note the crucial role of $s_{13} \equiv \sin\theta_{13}$.

Recent Evidence For Non-Zero θ_{13}

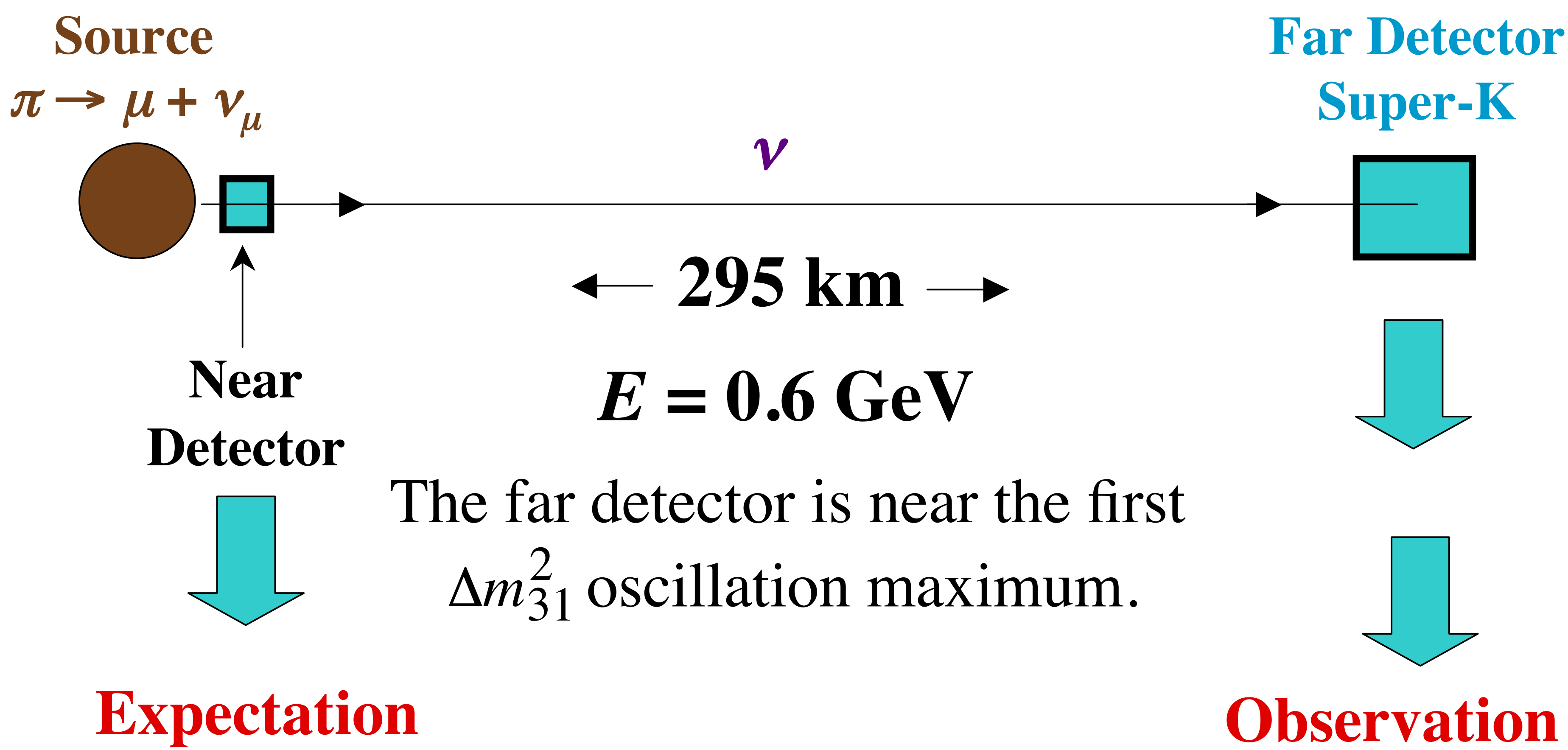
In an experiment where L/E is too small for the small splitting $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$ to be seen,

$$P\left(\nu_\mu \to \nu_e\right) \cong 4\left|U_{\mu 3} U_{e3}\right|^2 \sin^2\left(\Delta m_{31}^2 \frac{L}{4E}\right)$$

$$= \boxed{\sin^2 2\theta_{13}} \sin^2 \theta_{23} \sin^2\left(\Delta m_{31}^2 \frac{L}{4E}\right)$$

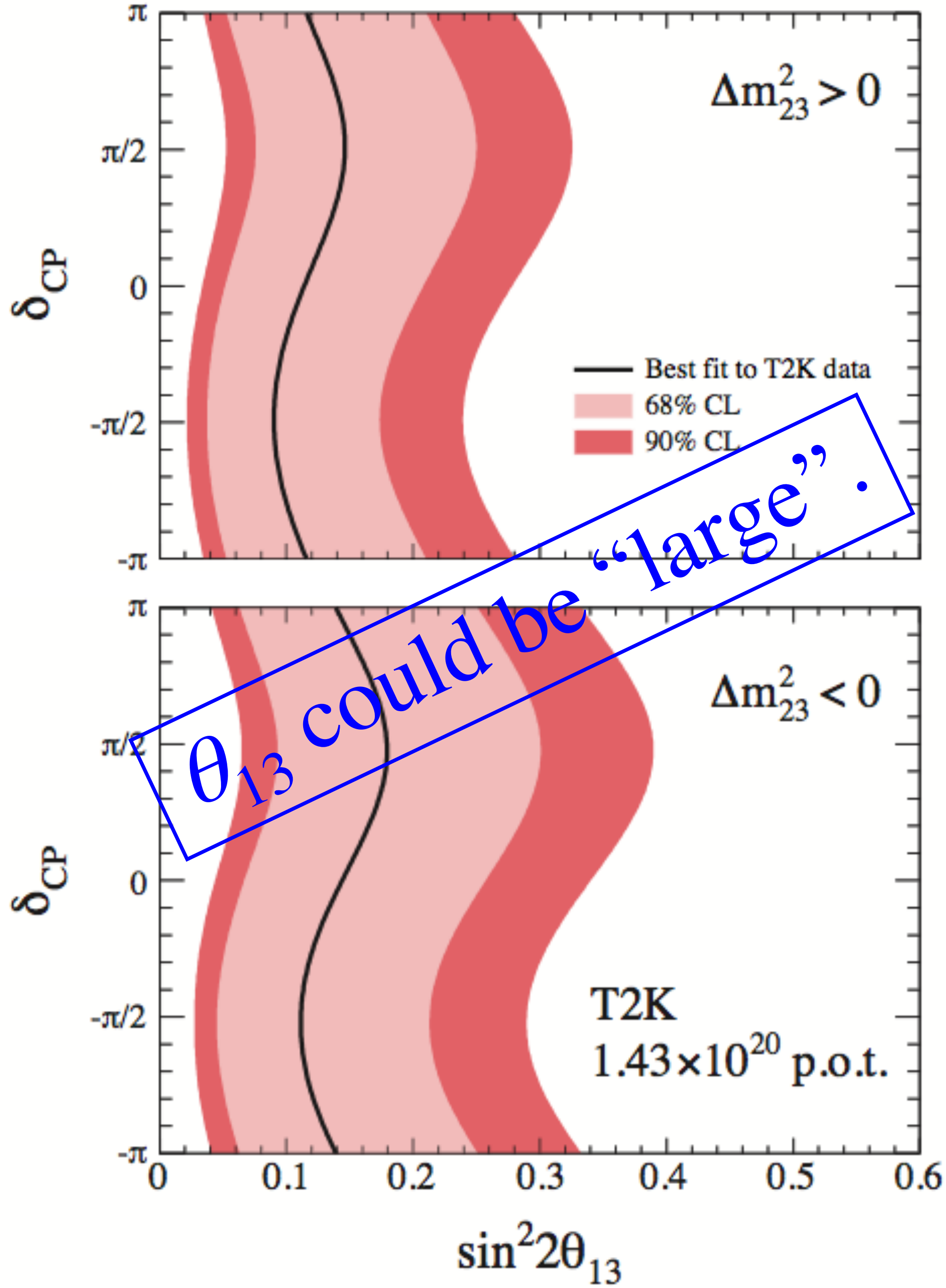
T2K has looked for $\nu_\mu \to \nu_e$ in a long-baseline experiment:

The T2K experiment



The far detector is near the first Δm^2_{31} oscillation maximum.

T2K sees 6 ν_e candidate events in the far detector, whereas 1.5 are expected if $\theta_{13} = 0$.



These take the $\Delta m^2{}_{21}$ contributions and matter effects into account.

MINOS, not designed to look for $\nu_\mu \to \nu_e$, sees 62 candidate events where 50 are expected if $\theta_{13} = 0$.

While not highly significant by itself, this result is consistent with that from T2K.

There Is Nothing Special About θ_{13}

All mixing angles must be nonzero for $\cancel{CP}$ in oscillation.

For example —

$$P\left(\bar{\nu}_\mu \to \bar{\nu}_e\right) - P\left(\nu_\mu \to \nu_e\right) = 2\cos\theta_{13}\sin 2\theta_{13}\sin 2\theta_{12}\sin 2\theta_{23}\sin\delta$$
$$\times \sin\left(\Delta m^2{}_{31}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{32}\frac{L}{4E}\right)\sin\left(\Delta m^2{}_{21}\frac{L}{4E}\right)$$

In the factored form of U, one can put δ next to θ_{12} instead of θ_{13}.

The Majorana ~~CP~~ Phases

The phase α_i is associated with neutrino mass eigenstate ν_i:

$U_{\alpha i} = U^0{}_{\alpha i} \exp(i\alpha_i/2)$ for all flavors α.

$$\mathrm{Amp}(\nu_\alpha \rightarrow \nu_\beta) = \sum_i U_{\alpha i}{}^* \exp(-\, i m_i{}^2 L/2E)\, U_{\beta i}$$

is insensitive to the Majorana phases α_i.

Only the phase δ can cause CP violation in neutrino oscillation.