Ettore Majorana meets his shadow (II)

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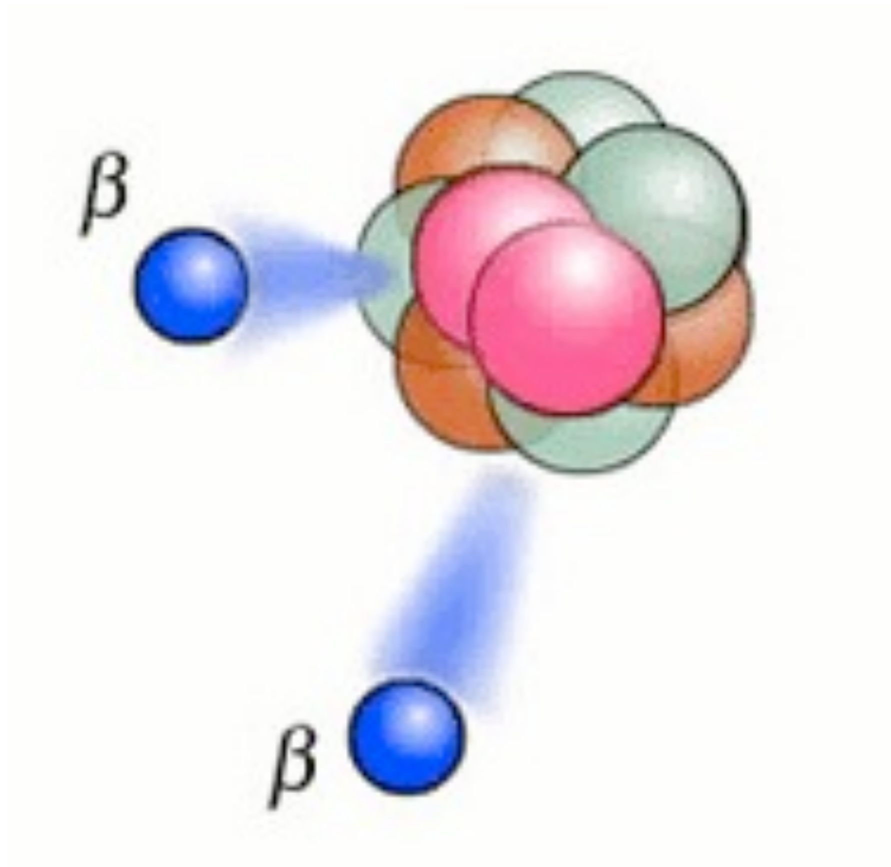
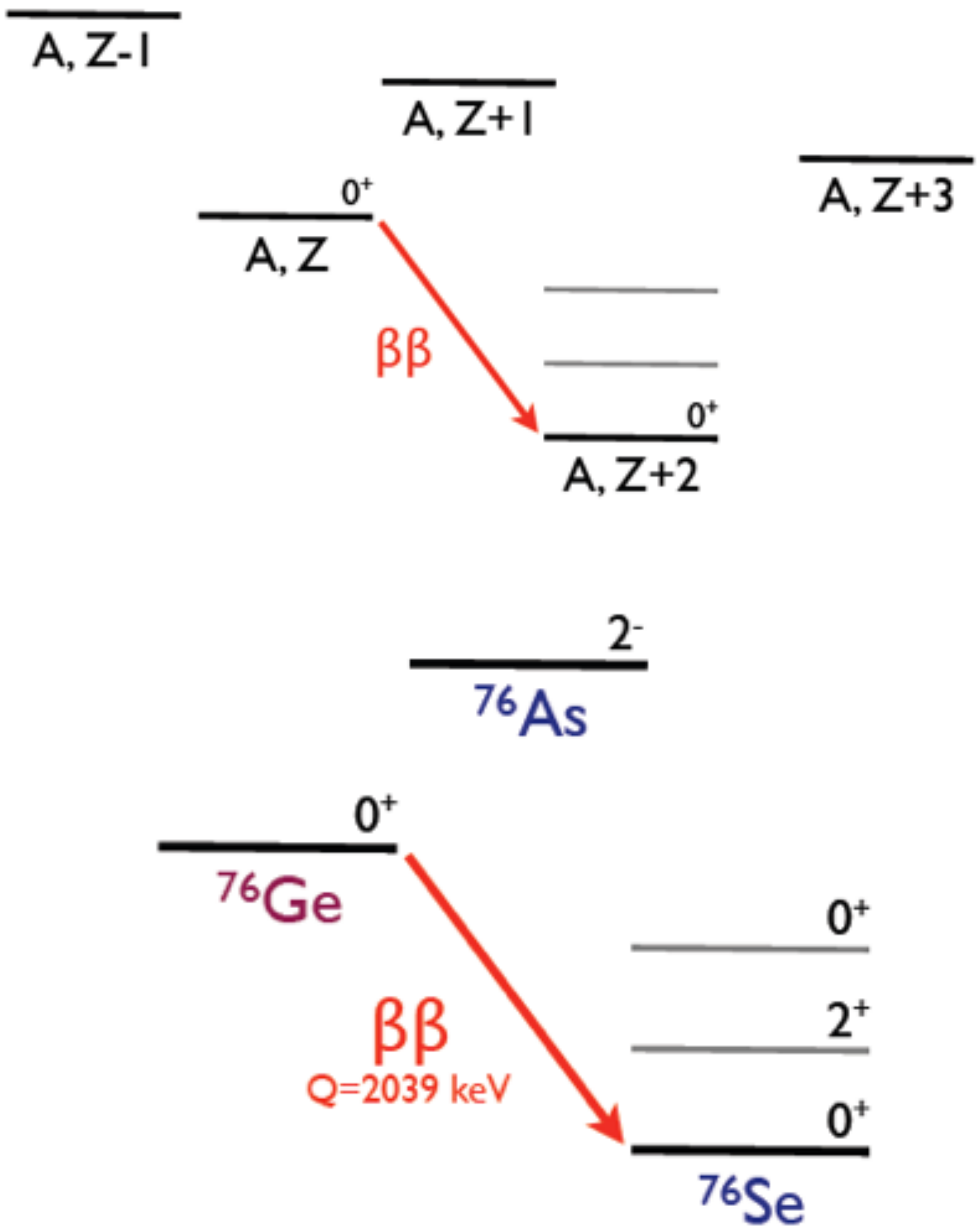
Neutrinoless double beta decay



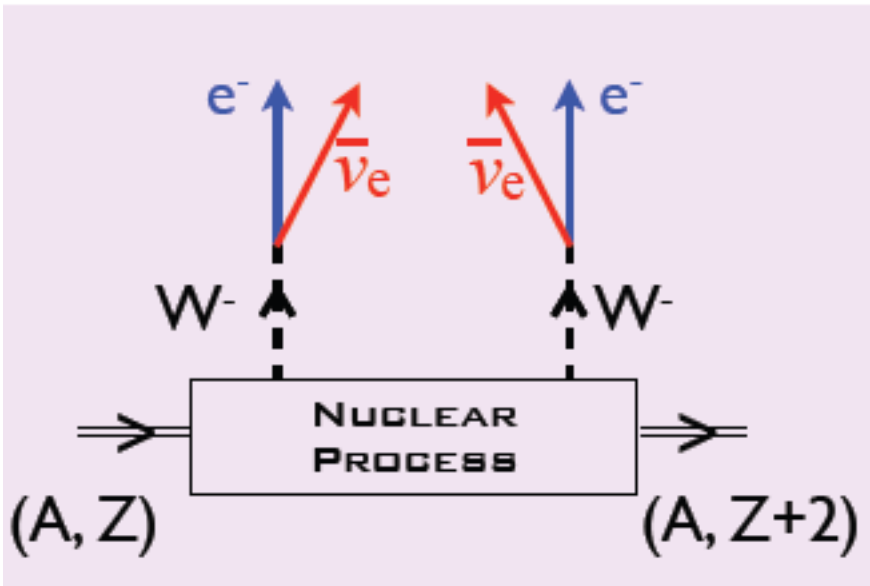
Double Beta Decay



- Some nuclei, otherwise quasi stable can decay by emitting two electrons and two neutrinos by a second order process mediated by the weak interaction.

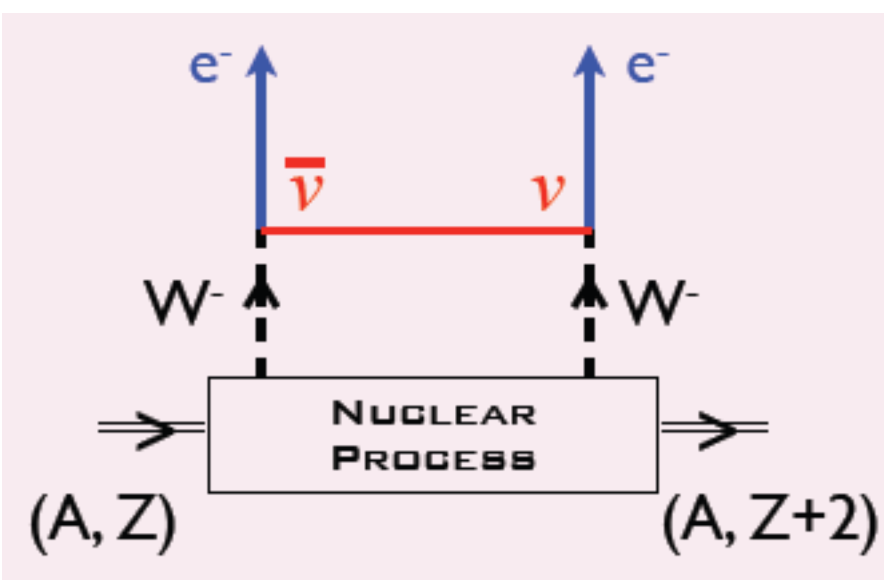
- This process exists due to nuclear pairing interaction that favors energetically the even-even isobars over the odd-odd ones.

Double Beta Decay



$$(Z, A) \to (Z+2, A) + 2\,e^- + 2\,\overline{\nu}_e,$$

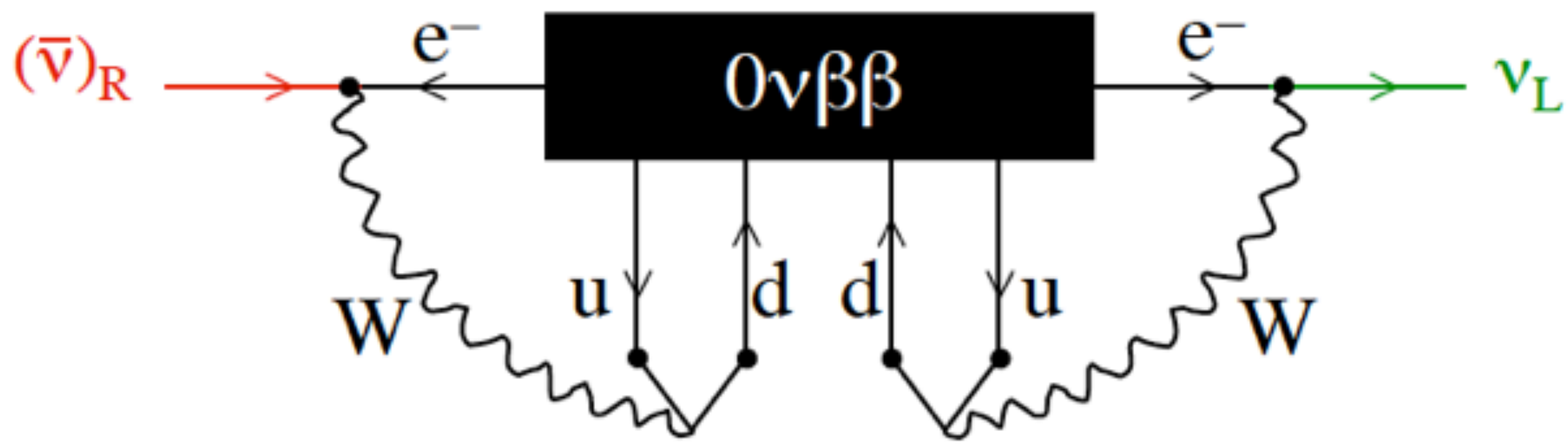
- Second order standard Model process. Long period (10^{18} y). Observed in many nuclei.



$$(Z, A) \to (Z+2, A) + 2\,e^-$$

- Forbidden unless there are massive Majorana neutrinos (necessary but not sufficient condition)

Black box theorem

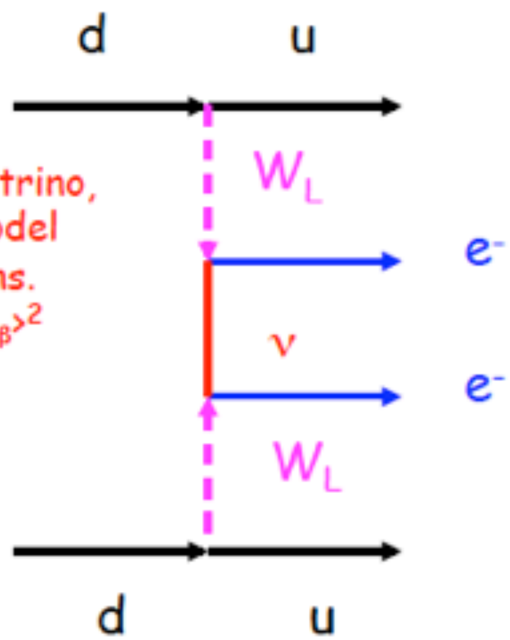


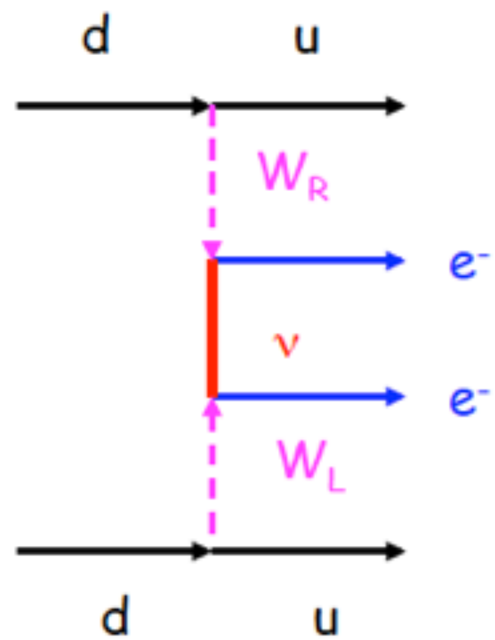
$(\bar{\nu})_R \rightarrow \nu_L$: A Majorana mass term

- Many new physics processes can induce bb0nu decay. But all of them imply Majorana neutrinos.

bb0nu putative mechanisms

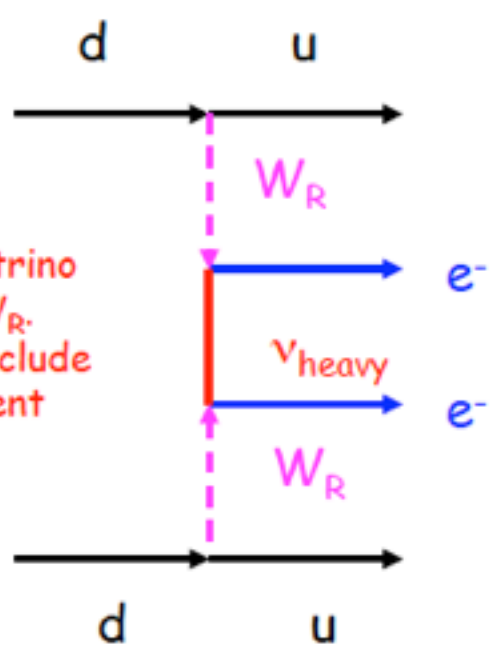
Light Majorana neutrino, only Standard Model weak interactions. Decay rate $\sim \langle m_{\beta\beta} \rangle^2$

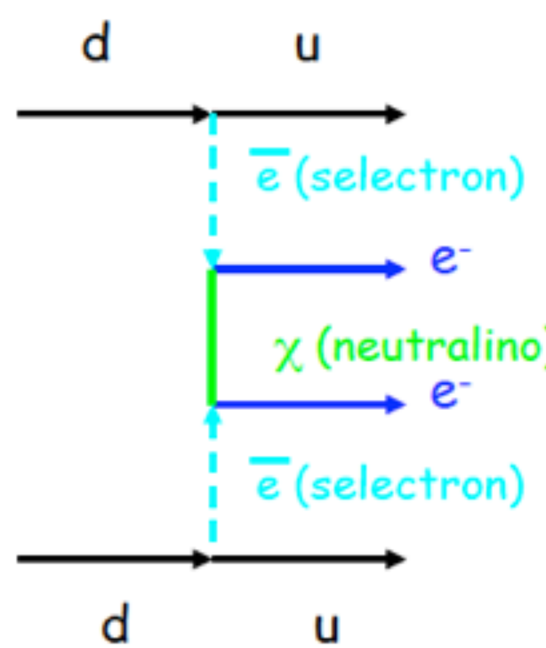




Light Majorana neutrinos. Model extended to include right-handed W_R. Mixing extended between the left and right-handed neutrinos. This is the mode where the rate $\sim\lambda^2$ or η^2

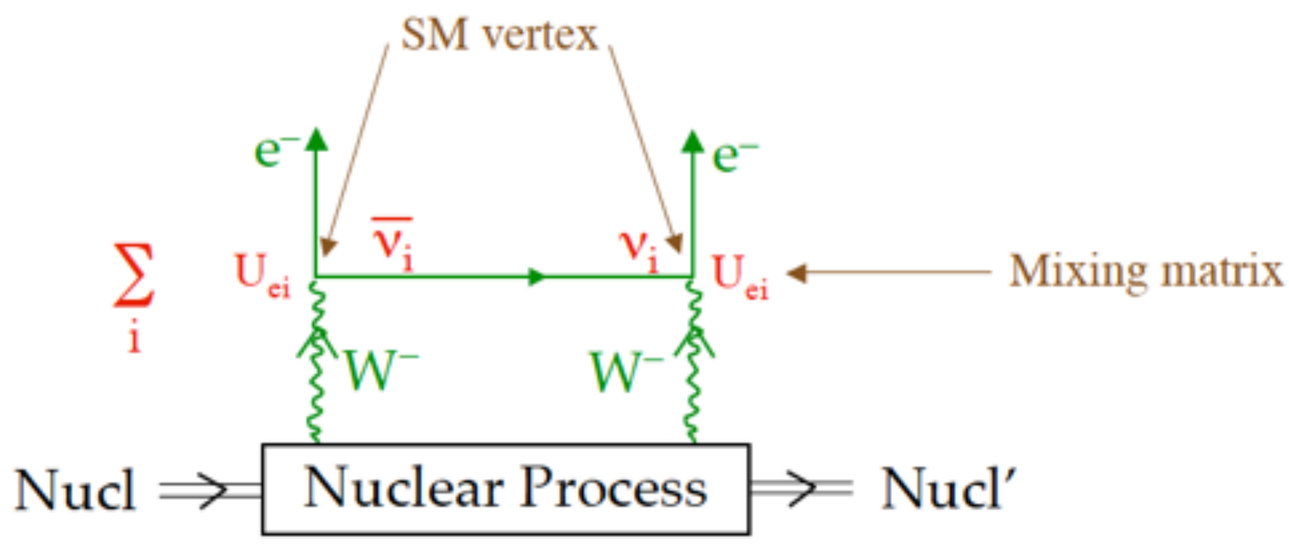
Heavy Majorana neutrino interacting with W_R. Model extended to include right-handed current interactions.





Supersymmetry with R-parity violation. Many new particles invoked. Light Majorana neutrinos exist also.

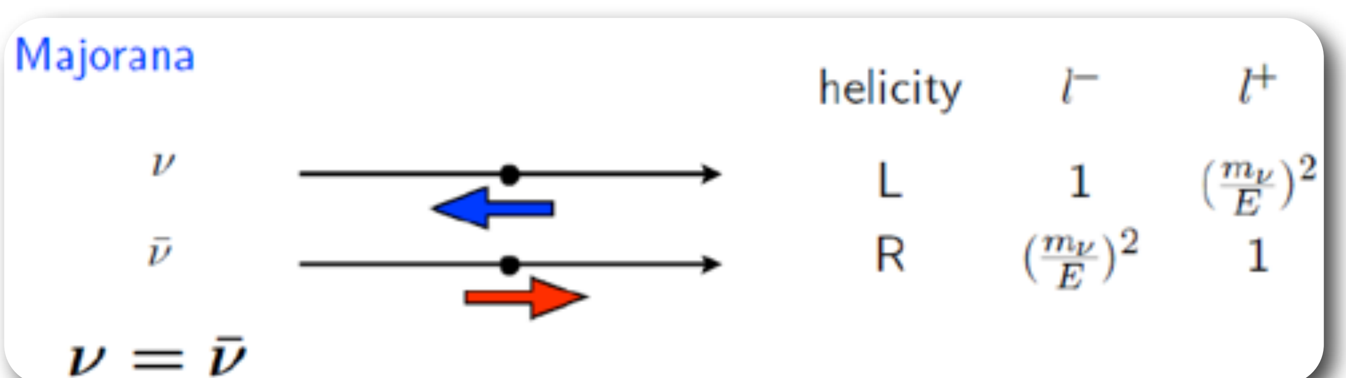
The simplest case



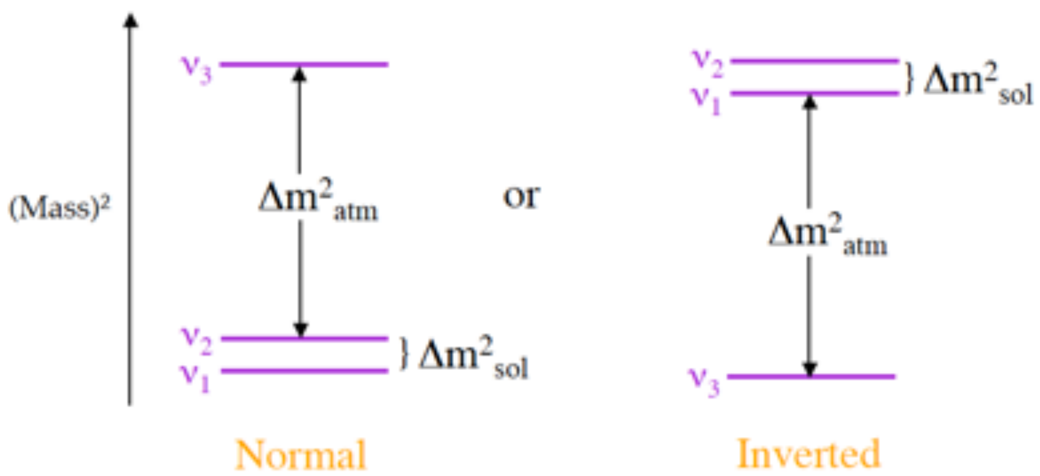
- The emitted antineutrino is RH + O(m/E) LH.
- The amplitude of each contribution goes like m.
- The process requires an helicity flip.

- To connect mass eigenstates (ν_i) to flavor eigenstates (SM vertex) we use the mixing matrix.

$$\mathrm{Amp}[0\nu\beta\beta] \propto \left|\sum_i m_i U_{ei}^2\right| \equiv m_{\beta\beta}$$



$m_{\beta\beta}$



$\Delta m^2_{sol} \cong 8 \times 10^{-5}\ \mathrm{eV}^2$, $\Delta m^2_{atm} \cong 2.7 \times 10^{-3}\ \mathrm{eV}^2$

$$\begin{array}{ccccccc} & \text{Atmospheric} & & \text{Cross-Mixing} & & \text{Solar} \\ U = & \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} & \times & \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} & \times & \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$c_{ij} \equiv \cos\theta_{ij}$
$s_{ij} \equiv \sin\theta_{ij}$

$$\times \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Majorana ~~CP~~ phases

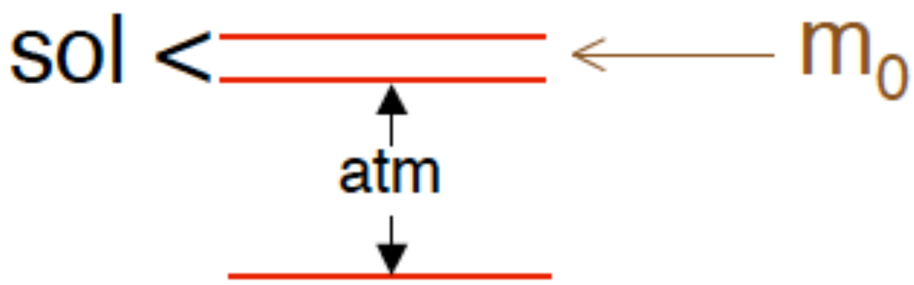
$\theta_{12} \approx \theta_{sol} \approx 34°$, $\theta_{23} \approx \theta_{atm} \approx 37\text{-}53°$, $\theta_{13} \lesssim 10°$

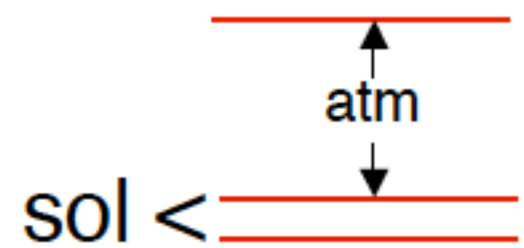
$$m_{\beta\beta} = \left| \sum_j m_j |U_{ej}|^2 \right|$$

$$= \left| |U_{e1}|^2\ m_1 + |U_{e2}|^2\ m_2\ e^{i\alpha_1} + |U_{e3}|^2\ m_3\ e^{i\alpha_2} \right|$$

$$= \left| \cos^2\theta_{13}(|m_1|\cos^2\theta_{12} + |m_2|e^{2i\alpha_1}\sin^2\theta_{12}) + |m_3|e^{2i(\alpha_2-\delta)}\sin^2\theta_{13} \right|$$

Guess the neutrino mass



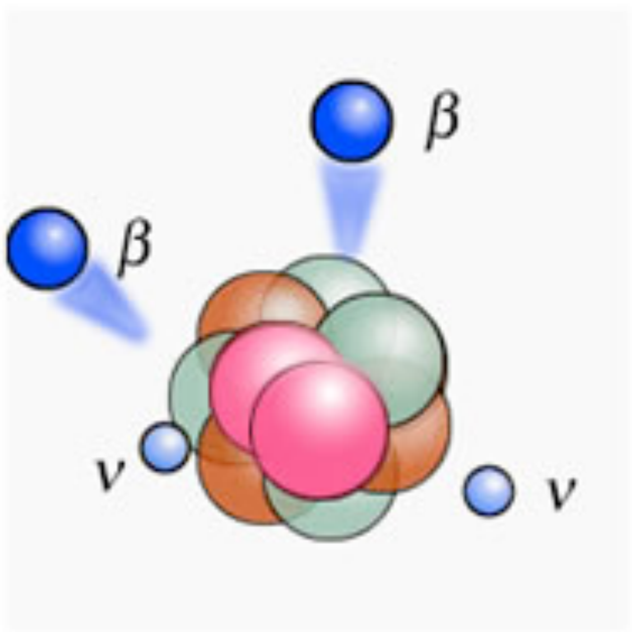


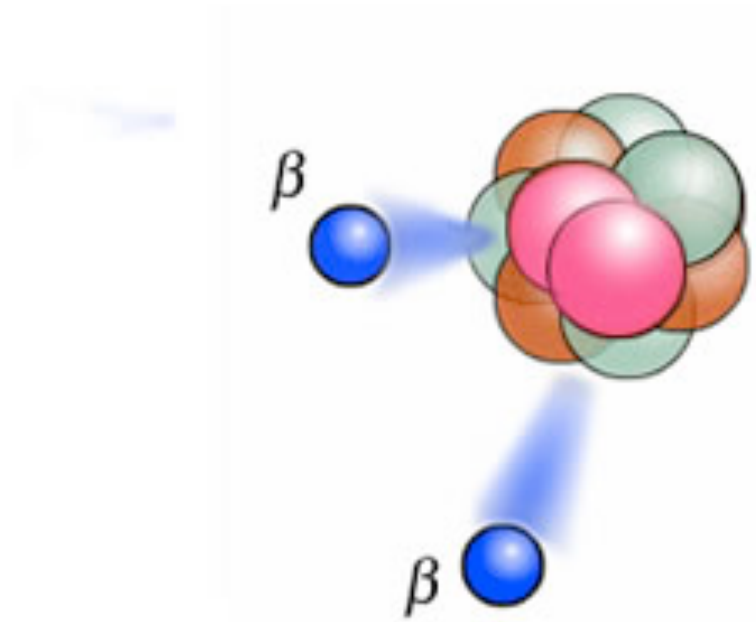
- Consider the normal and inverted hierarchies.

Use the measured parameters of the mixing matrix (assume $\theta 13 \sim 10^0$.

- Do the calculation for the extreme values of the phases (e.g, consider the cases $\delta=\alpha_1=\alpha_2=0$, and then π).

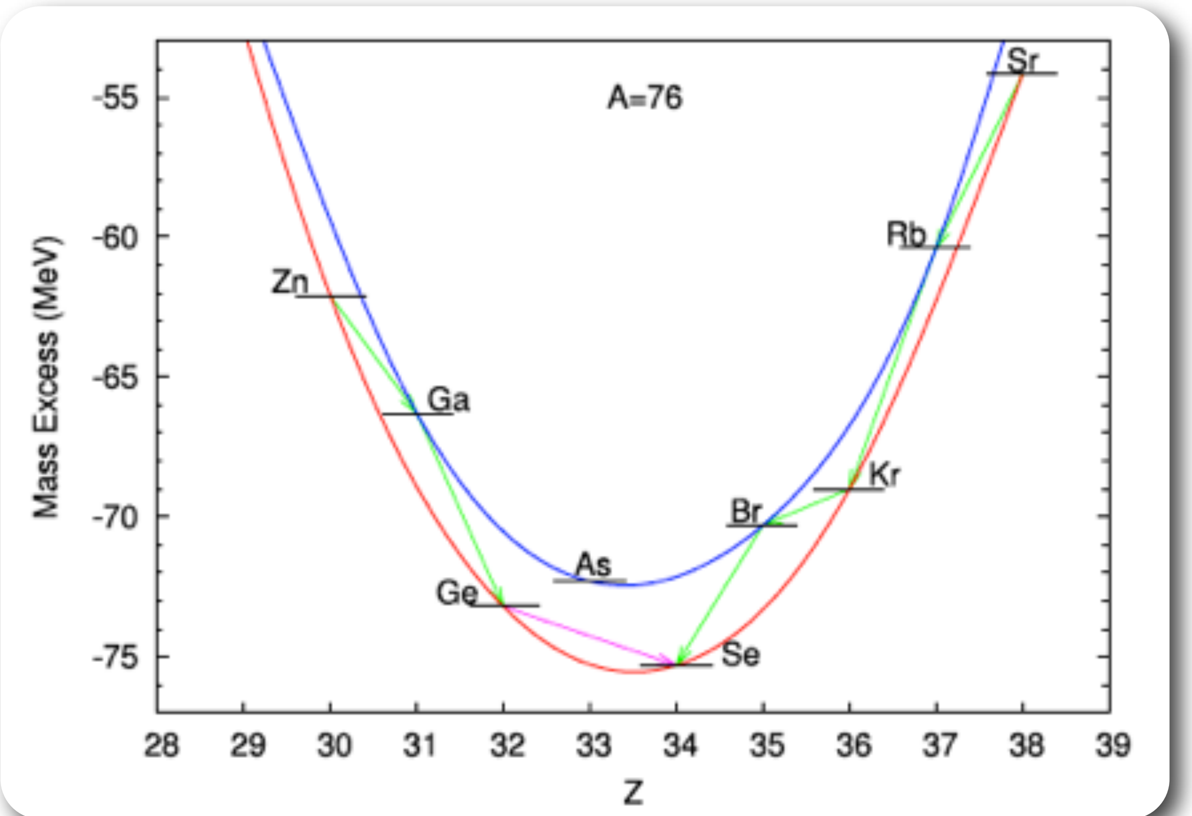
Nuclear physics and dbd





Mass parabola

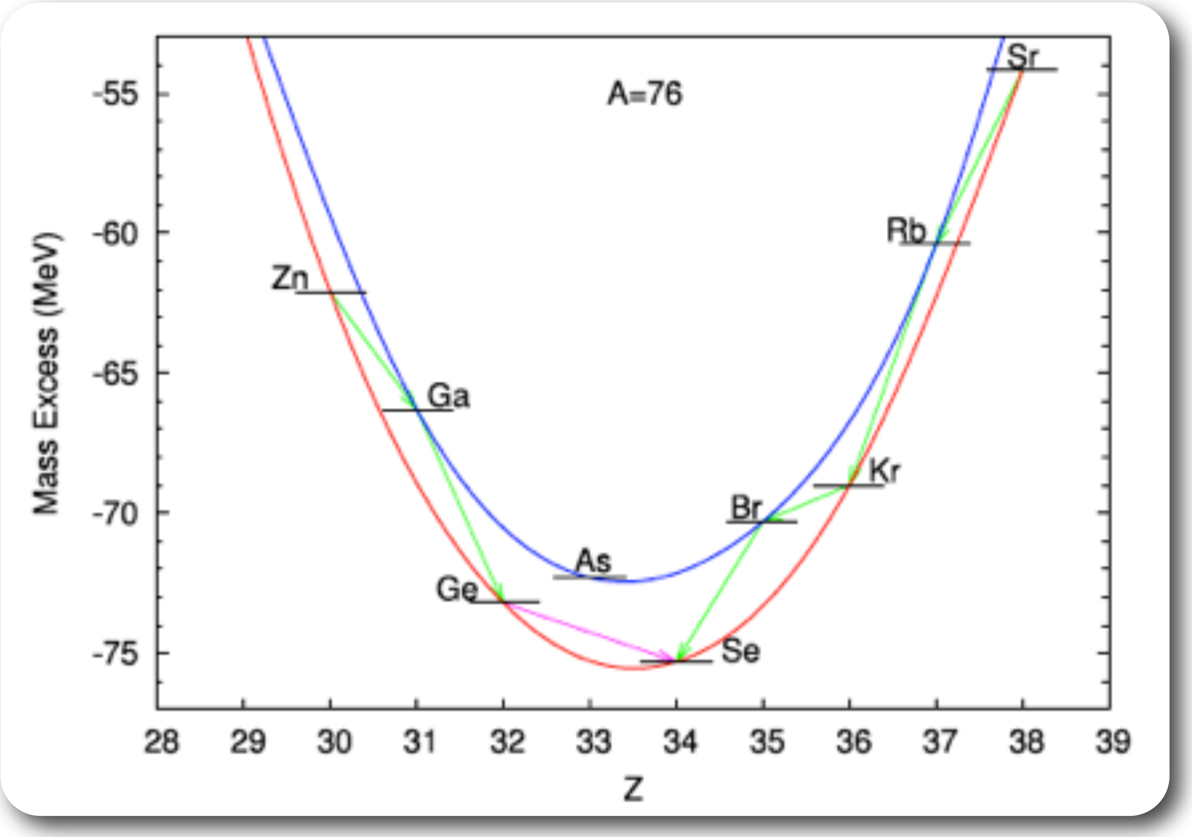
$$M_A(Z,A) = const + 2b_{sym}\frac{(A/2-Z)^2}{A^2} + b_{Coul}\frac{Z^2}{A^{1/3}} + m_e Z + \delta$$



- For Even A nuclei one gets two curves, corresponding to odd-odd (blue curve) and even-even nuclei

- $M_A(Z,A)$ is a parabola as a function of Z.
- δ describes nuclear paring the increase in binding as pairs of like nucleons couple to angular momentum zero.
- $\delta = \pm 12/A^{1/2}$ for odd N and odd Z, or even N and even Z.
- δ = 0 for odd A.
- For odd A nuclei, typically only one isotope is stable.

DBD is even-even business



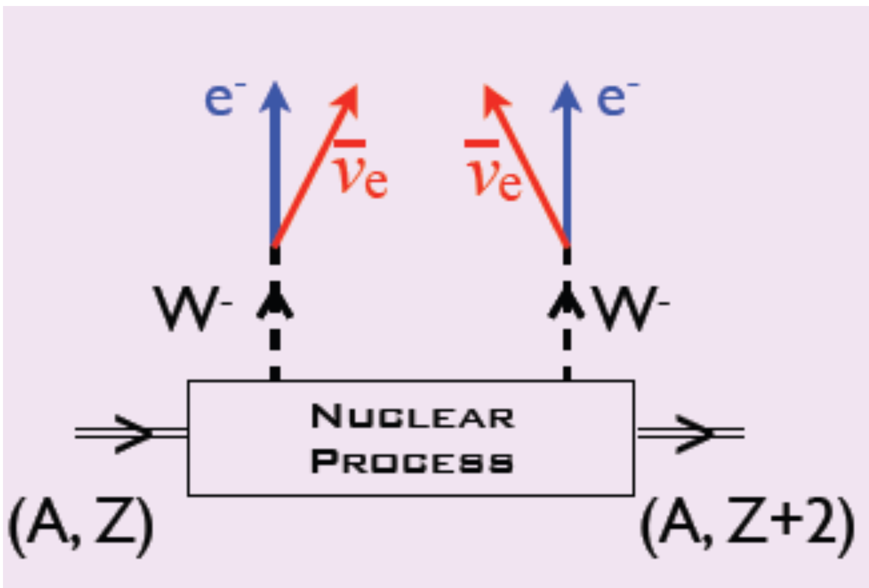
- Near the bottom of the parable one finds that the isotope Ge is stable against β decay (it cannot decay to As which has larger mass)
- However, it can decay to Se which is also even-even via second order process.
- This is the DBD process.

All ground states of even-even nuclei have spin and parity 0+ and thus transitions $0^+ \rightarrow 0^+$ are expected in all cases.

DBD nuclei

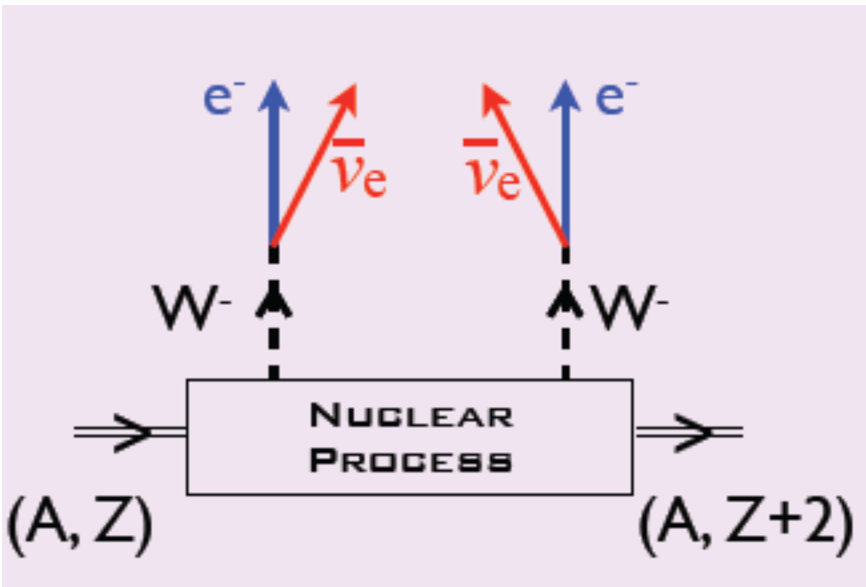
	Q (MeV)	Abund.(%)
$^{48}Ca \rightarrow {}^{48}Ti$	4.271	0.187
$^{76}Ge \rightarrow {}^{76}Se$	2.040	7.8
$^{82}Se \rightarrow {}^{82}Kr$	2.995	9.2
$^{96}Zr \rightarrow {}^{96}Mo$	3.350	2.8
$^{100}Mo \rightarrow {}^{100}Ru$	3.034	9.6
$^{110}Pd \rightarrow {}^{110}Cd$	2.013	11.8
$^{116}Cd \rightarrow {}^{116}Sn$	2.802	7.5
$^{124}Sn \rightarrow {}^{124}Te$	2.228	5.64
$^{130}Te \rightarrow {}^{130}Xe$	2.533	34.5
$^{136}Xe \rightarrow {}^{136}Ba$	2.479	8.9
$^{150}Nd \rightarrow {}^{150}Sm$	3.367	5.6

Decay rate for bb2n



- Fermi golden rule for second order weak decay.
- Sum over m includes all relevant virtual states in the intermediate odd-odd nucleus
- β labels the different Dirac structures of the weak interaction Hamiltonian.

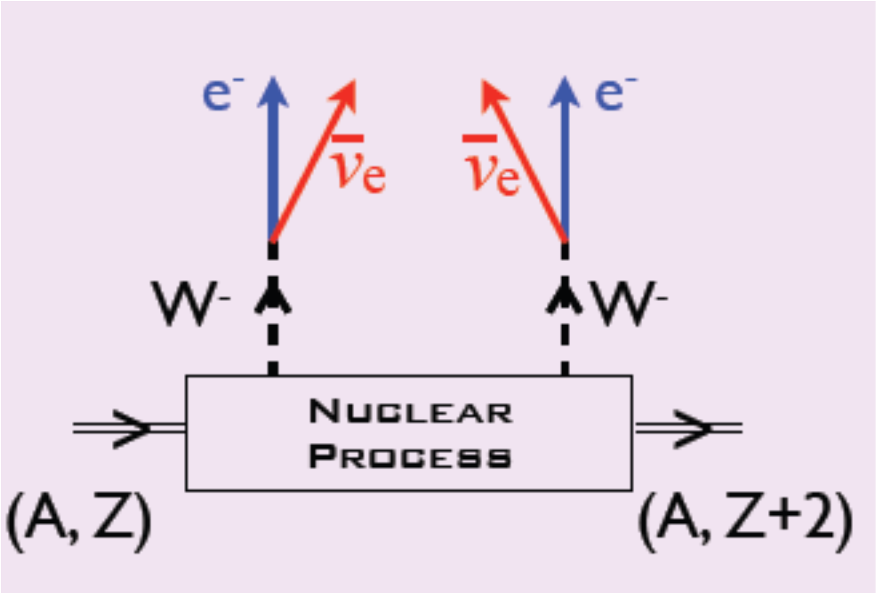
$$\frac{1}{\tau} = 2\pi\delta(E_0 - \Sigma_f E_f)\left[\Sigma_{m,\beta}\frac{\langle f|H_\beta|m\rangle\langle m|H^\beta|i\rangle}{E_i - E_m - p_\nu - E_e}\right]^2$$



- For $0^+ \to 0^+ \Rightarrow$ Long wavelength approximation: replace lepton energies by average value.

$$E_e + p_\nu \sim E_0/2, \qquad E_0 = M_i - M_f$$

- E0 total energy of the decay.
- Rate formula separates into a product of the nuclear and lepton parts



- Lepton part is just a phase space integral.

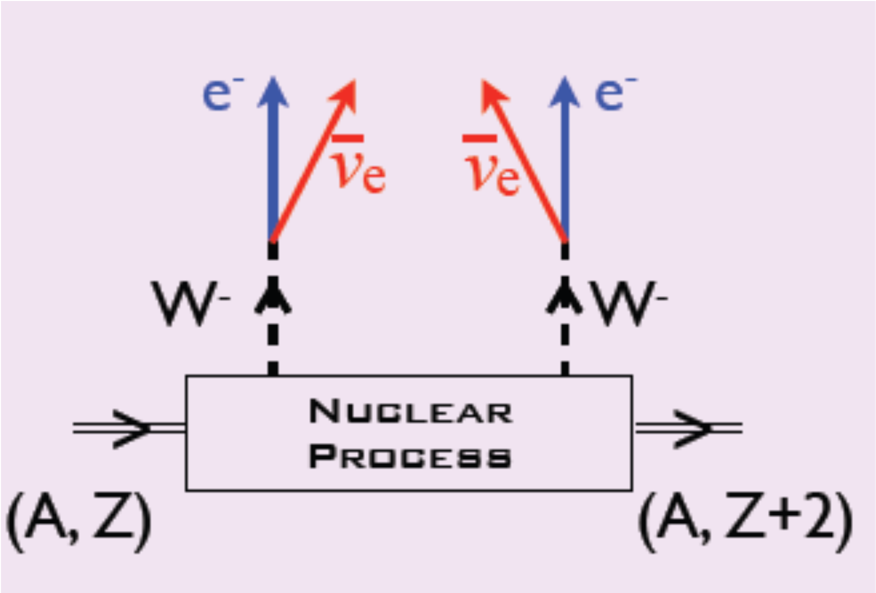
$$\int_{m_e}^{E_0-m_e} F(Z,E_{e1})p_{e1}E_{e1}dE_{e1}\int_{m_e}^{E_0-E_1} F(Z,E_{e2})p_{e2}E_{e2}dE_{e2}(E_0-E_{e1}-E_{e2})^5/30$$

$$F(Z,E)=\frac{E}{p}\frac{2\pi Z\alpha}{1-e^{-2\pi Z\alpha}}$$

Non relativistic Coulomb (Primakoff-Rossen)

$$\frac{dN}{dK}\sim K(T_0-K)^5\left(1+2K+\frac{4K^2}{3}+\frac{K^3}{3}+\frac{K^4}{30}\right)$$

- Sum electron spectrum
- K is the sum of the kinetic energies of both electrons in units of the electron mass.



- The nuclear structure information is contained in the nuclear matrix element

$$M^{2\nu} = \Sigma_m \frac{\langle 0_f^+ | \vec{\sigma}_i \tau_i^+ | m \rangle \langle m | \vec{\sigma}_k \tau_k^+ | 0_i^+ \rangle}{E_m - (M_i + M_f)/2}$$

$\langle m | \vec{\sigma}_k \tau_k^+ | 0_i^+ \rangle$ **β^- strength initial nucleus ⇒ Explore in exchange reactions such as (p,n) and (^{3}He,t)**

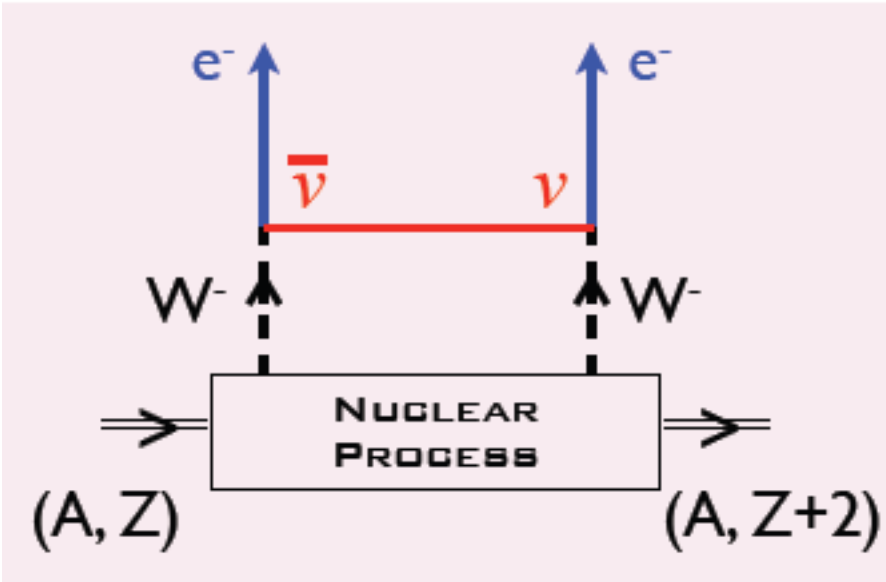
$\langle 0_f^+ | \vec{\sigma}_i \tau_i^+ | m \rangle$ **β^+ strength initial nucleus ⇒ Explore in exchange reactions such as (n,p) and (d^2,He)**

Measured Decay rates for bb2n

Isotope	$T^{2\nu}_{1/2}$ (y)	$M^{2\nu}_{GT}$ (MeV^{-1})
^{48}Ca	$(3.9 \pm 0.7 \pm 0.6) \times 10^{19}$	0.05 ± 0.01
^{76}Ge	$(1.7 \pm 0.2) \times 10^{21}$	0.13 ± 0.01
^{82}Se	$(9.6 \pm 0.3 \pm 1.0) \times 10^{19}$	0.10 ± 0.01
^{96}Zr	$(2.0 \pm 0.3 \pm 0.2) \times 10^{19}$	0.12 ± 0.02
^{100}Mo	$(7.11 \pm 0.02 \pm 0.54) \times 10^{18}$	0.23 ± 0.01
^{116}Cd	$(2.8 \pm 0.1 \pm 0.3) \times 10^{19}$	0.13 ± 0.01
^{128}Te$^{(1)}$	$(2.0 \pm 0.1) \times 10^{24}$	0.05 ± 0.005
^{130}Te	$(7.6 \pm 1.5 \pm 0.8) \times 10^{20}$	0.032 ± 0.003
^{136}Xe	$> 1.0 \times 10^{22}$ (90% CL)	<0.01
^{150}Nd	$(9.2 \pm 0.25 \pm 0.73) \times 10^{18}$	0.06 ± 0.003
^{238}U$^{(2)}$	$(2.0 \pm 0.6) \times 10^{21}$	0.05 ± 0.01

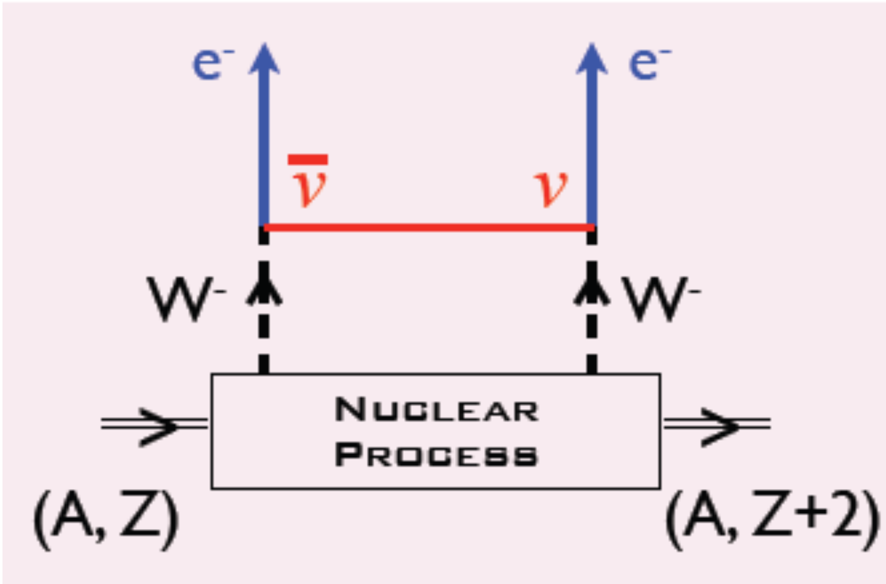
Age of the universe: ~14 × 10^9 y

Decay rate for bb0n



- The amplitude $Z_{0\nu}$ is second order in the weak interaction and depends on the currents in the effective low-energy semileptonic Hamiltonian H

$$[T^{0\nu}_{1/2}]^{-1} = \sum_{\text{spins}} \int |Z_{0\nu}|^2 \delta(E_{e1} + E_{e2} - Q_{\beta\beta}) \frac{d^3p_1}{2\pi^3} \frac{d^3p_2}{2\pi^3}$$



- The lepton part of the amplitude is written as a product of two left-handed currents.
- ν_j, ν_k represent neutrino mass eigenstates

$$\bar{e}(x)\gamma_\rho \frac{1}{2}(1-\gamma_5)\nu_j(x)\bar{e}(y)\gamma_\sigma \frac{1}{2}(1-\gamma_5)\nu_k(y)$$

- Transform to flavor basis.

$$\sum_k \overline{e}(x)\gamma_\mu(1-\gamma_5)U_{ek}\phi_k(x)\ \overline{e}(y)\gamma_\nu(1-\gamma_5)U_{ek}\phi_k(y) \quad = \qquad \phi_k^c = \phi_k$$

$$-\sum_k \overline{e}(x)\gamma_\mu(1-\gamma_5)U_{ek}\phi_k(x)\ \overline{\phi_k^c}(y)\gamma_\nu(1+\gamma_5)U_{ek}e^c(y)\ ,$$

- $\phi_k(x)\ \overline{\phi_k^c}(y)$ $\Rightarrow$ Lepton propagator

Only if neutrinos are Majorana particles

$$-\frac{i}{4}\int\sum_k \frac{d^4q}{(2\pi)^4}e^{-iq\cdot(x-y)}\overline{e}(x)\gamma_\mu(1-\gamma_5)\frac{q^\rho\gamma_\rho+m_k}{q^2-m_k^2}\gamma_\nu(1+\gamma_5)e^c(y)\;U_{ek}^2\;;$$

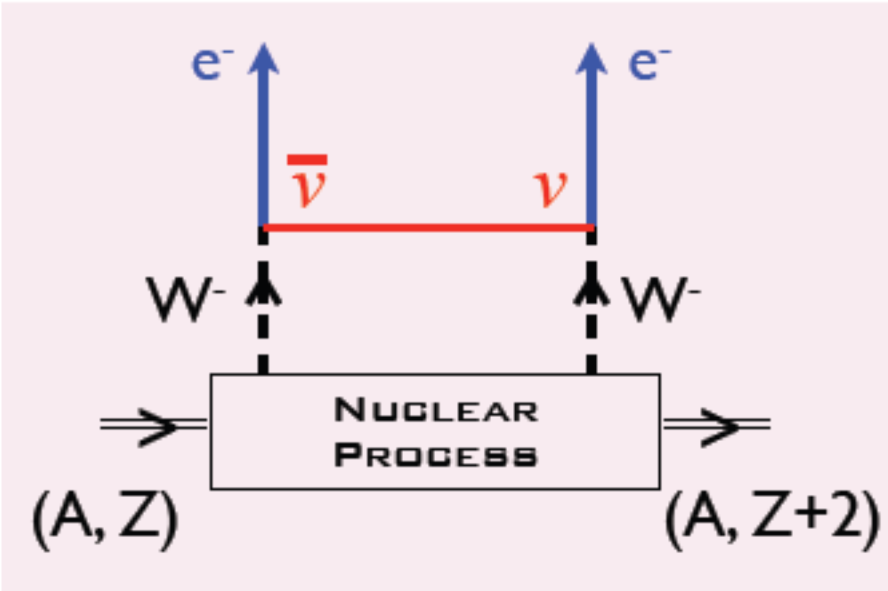
• Lepton part of the amplitude after contracting wave functions. q is the momentum transfer four-vector

$$\frac{\cancel{q^\rho\gamma_\rho}+m_k}{q^2-\cancel{m_k^2}} \longrightarrow \frac{1}{2}(1-\gamma_5)(\cancel{q^\mu\gamma_\mu}+m_j)\frac{1}{2}(1-\gamma_5)=m_j\frac{1}{2}(1-\gamma_5)$$

for light neutrinos

• Thus the decay amplitude for purely left-handed lepton currents is proportional to the neutrino Majorana mass m_j.

$$\langle m_{\beta\beta}\rangle \equiv \left|\sum_k m_k U_{ek}^2\right| = \left|m_1|U_{e1}|^2+m_2|U_{e2}|^2e^{i(\alpha_2-\alpha_1)}+m_3|U_{e3}|^2e^{i(-\alpha_1-2\delta)}\right|$$



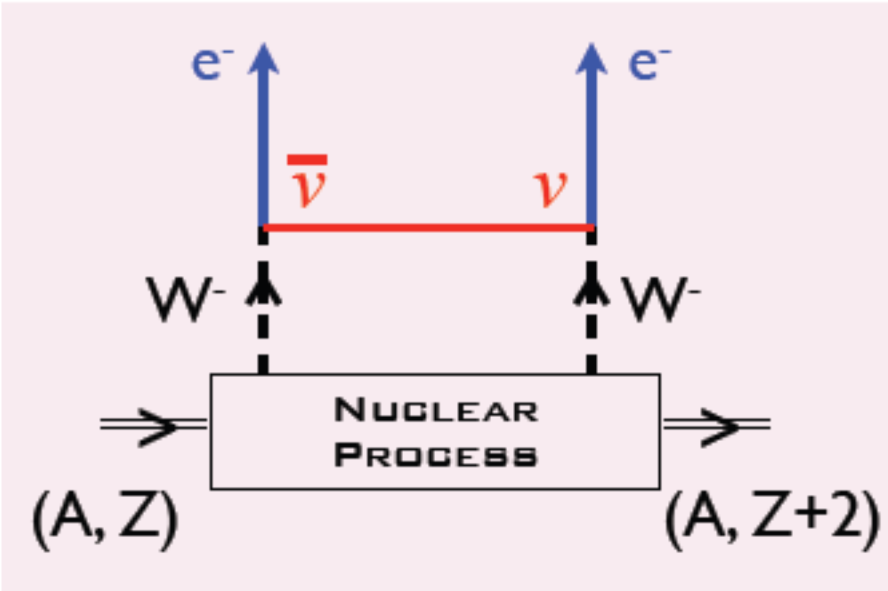
- To complete the calculation⇒ multiply the lepton part of the amplitude by the nuclear matrix element and integrate.

|n> complete set of intermediate nuclear states, En corresponding energies

$$\langle f|J_L^\mu(x)J_L^\nu(y)|i\rangle = \sum_n \langle f|J_L^\mu(\vec{x})|n\rangle\langle n|J_L^\nu(\vec{y})|i\rangle e^{-i(E_f-E_n)x_0}e^{-i(E_n-E_i)y_0}$$

Ei and Ef are the energies of the initial and final nuclei

- Integrate over time coordinates x_0 and y_0,
- Combine the exponential with a similar factor from the q-dependence of the neutrino propagator
- Ignore neutrino masses (assumed small)



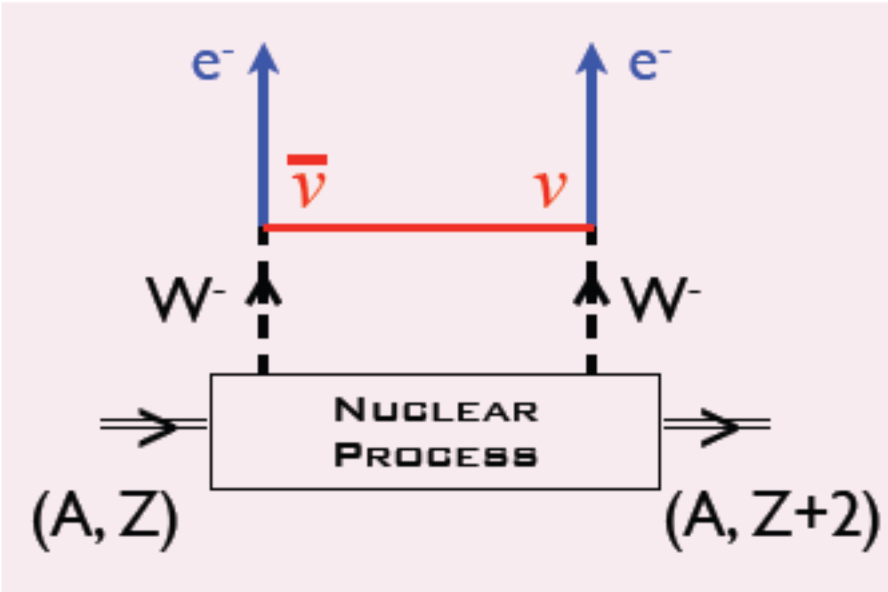
• q_0 = q is the energy of the virtual neutrino, to be integrated over along with the virtual momenta, and E_{e1}, E_{e2} are the energies of the outgoing electrons.

$$2\pi\delta(E_f + E_{e1} + E_{e2} - E)\sum_n \left[\frac{\langle f|J_L^{\mu}(\vec{x})|n\rangle\langle n|J_L^{\nu}(\vec{y})|i\rangle}{q^0(E_n + q^0 + E_{e2} - E_i)} + \frac{\langle f|J_L^{\nu}(\vec{x})|n\rangle\langle n|J_L^{\mu}(\vec{y})|i\rangle}{q^0(E_n + q^0 + E_{e1} - E_i)}\right]$$

• q0 ⇒average inverse spacing between nucleons ~100 MeV.

• q0 is much larger than the excitation energy of states contributing to the decay amplitude ⇒ replace the intermediate-state energies by an estimate E of their average value.

• Closure approximation: replaces the sum over intermediate states by 1 (15 % error at most)



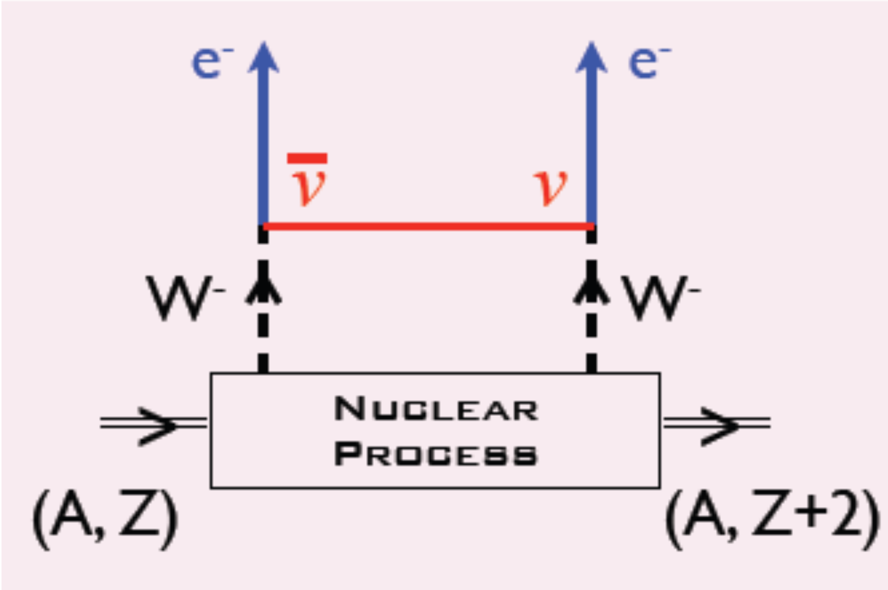
- Closure approximation

$$2\pi\delta(E_f + E_{e1} + E_{e2} - E_i)\left[\frac{\langle f|J_L^\mu(\vec{x})J_L^\nu(\vec{y})|i\rangle}{q^0(\overline{E} + q^0 + E_{e2} - E_i)} + \frac{\langle f|J_L^\nu(\vec{x})J_L^\mu(\vec{y})|i\rangle}{q^0(\overline{E} + q^0 + E_{e1} - E_i})\right]$$

- Next we need an expression for the hadronic current
- Impulse approximation:⇒ the hadronic current is obtained from that of free nucleons.

charge-changing hadronic current for a nucleon (i.e. the proton-neutron matrix element of the current)

$$\langle p|J_L^\mu(x)|p'\rangle = e^{iqx}\overline{u}(p)\left(g_V(q^2)\gamma^\mu - g_A(q^2)\gamma_5\gamma^\mu - ig_M(q^2)\frac{\sigma^{\mu\nu}}{2m_p}q_\nu + g_P(q^2)\gamma_5 q^\mu\right)\overline{u}(p')$$



$$q = p' - p,\ g_V \equiv g_V(0) = 1$$

$$g_A \equiv g_A(0) = 1.26$$

$$g_P(q^2) = 2m_p g_A(q^2)/(q^2 + m_\pi^2)$$

$$\langle p|J_L^\mu(x)|p'\rangle = e^{iqx}\overline{u}(p)\left(g_V(q^2)\gamma^\mu - g_A(q^2)\gamma_5\gamma^\mu - ig_M(q^2)\frac{\sigma^{\mu\nu}}{2m_p}q_\nu + g_P(q^2)\gamma_5 q^\mu\right)\overline{u}(p')$$

$$g_V(q^2) = \frac{g_V}{(1+q^2/\Lambda_V^2)^2}\ ,\quad g_A(q^2) = \frac{g_A}{(1+q^2/\Lambda_A^2)^2}\ ,\quad \Lambda_V^2 = 0.71\ (\mathrm{GeV})^2 \text{ and } \Lambda_A^2 = 1.09\ (\mathrm{GeV})^2$$

•Integrate the rate over electron phase space making the long-wavelength approximation. Only 0+ $\rightarrow$ 0+ decay is considered.

$$[T_{1/2}^{0\nu}]^{-1} = G_{0\nu}(Q_{\beta\beta}, Z)\,|M_{0\nu}|^2\,\langle m_{\beta\beta}\rangle^2$$

$$[T^{0\nu}_{1/2}]^{-1} = G_{0\nu}(Q_{\beta\beta}, Z)\,|M_{0\nu}|^2\,\langle m_{\beta\beta}\rangle^2$$

$$M_{0\nu} \simeq \left(\frac{g_A}{1.25}\right)^2 \left(M^{GT}_{0\nu} - \frac{g_V^2}{g_A^2} M^F_{0\nu}\right)$$

Neutrino potential

$$M^F_{0\nu} = \langle f|\sum_{a,b} H(r_{ab}, \overline{E})\tau_a^+\tau_b^+|i\rangle \text{ , and}$$

$$M^{GT}_{0\nu} = \langle f|\sum_{a,b} H(r_{ab}, \overline{E})\vec{\sigma}_a\cdot\vec{\sigma}_b\tau_a^+\tau_b^+|i\rangle$$

$$H(r, \overline{E}) \approx \frac{2R}{\pi r}\int_0^\infty dq \frac{\sin qr}{q + \overline{E} - (E_i + E_f)/2}$$

- The summation $\Sigma_{a,b}$ is over all nucleons, $\sigma_{a(b)}$ are spin Pauli matrices, and |fi> |i> are the final (initial) nuclear states.

- In contrast to ββ2ν, which involves only Gamov-Teller transitions through intermediate 1+ states (because of low momentum transfer), the nuclear matrix element for ββ0ν contains both a Fermi (F) and a Gamov-Teller (GT) part

Phase space for bb0n

$$G^{0\nu}(Q,Z) \sim \int F(Z,E_{e1})F(Z,E_{e2})p_{e1}p_{e2}E_{e1}E_{e2}\delta(E_0 - E_{e1} - E_{e2})dE_{e1}dE_{e2}$$

$$\frac{dN}{dT_e} \sim (T_e+1)^2(T_0 - T_e + 1)^2$$

Primakoff-Rosen. Energy spectrum for each individual electron

$$G^{0\nu}_{PR} \sim \left(\frac{E_0^5}{30} - \frac{2E_0^2}{3} + E_0 - \frac{2}{5}\right)$$

Dependence of phase space for bb0nu

$$E_0 = M_i - M_f$$

$$\frac{dN}{dK} \sim K(T_0 - K)^5\left(1 + 2K + \frac{4K^2}{3} + \frac{K^3}{3} + \frac{K^4}{30}\right)$$

Dependence of phase space for bb2nu

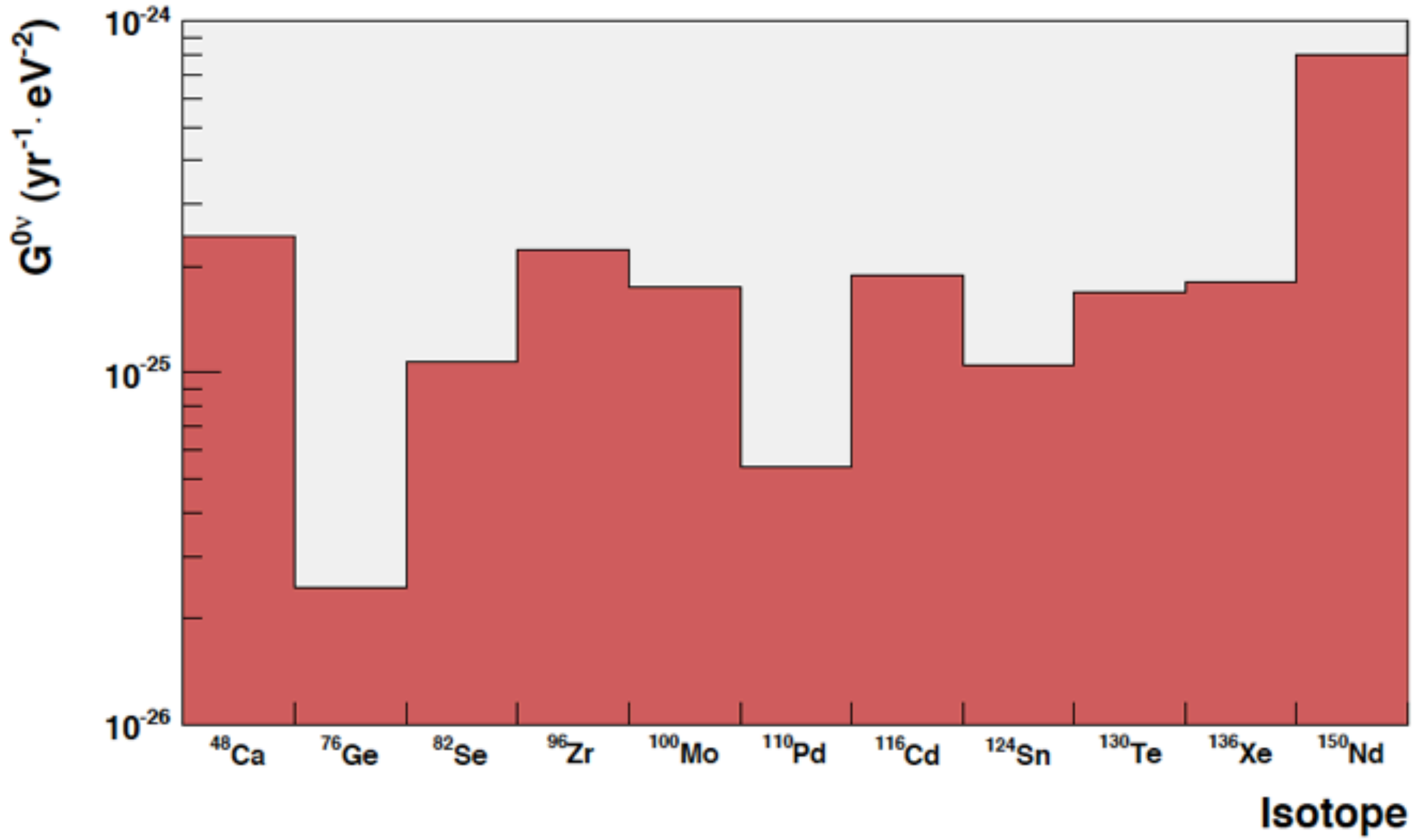
Phase space: bb0n vs bb2n

- bb0n mode has the advantage of the two-lepton final state: E^5 dependence compared to the four-lepton final state which has E^{11} dependence.

- In addition, the large average momentum of the virtual neutrino, compared with the typical nuclear excitation energy also makes the bb0n decay faster.

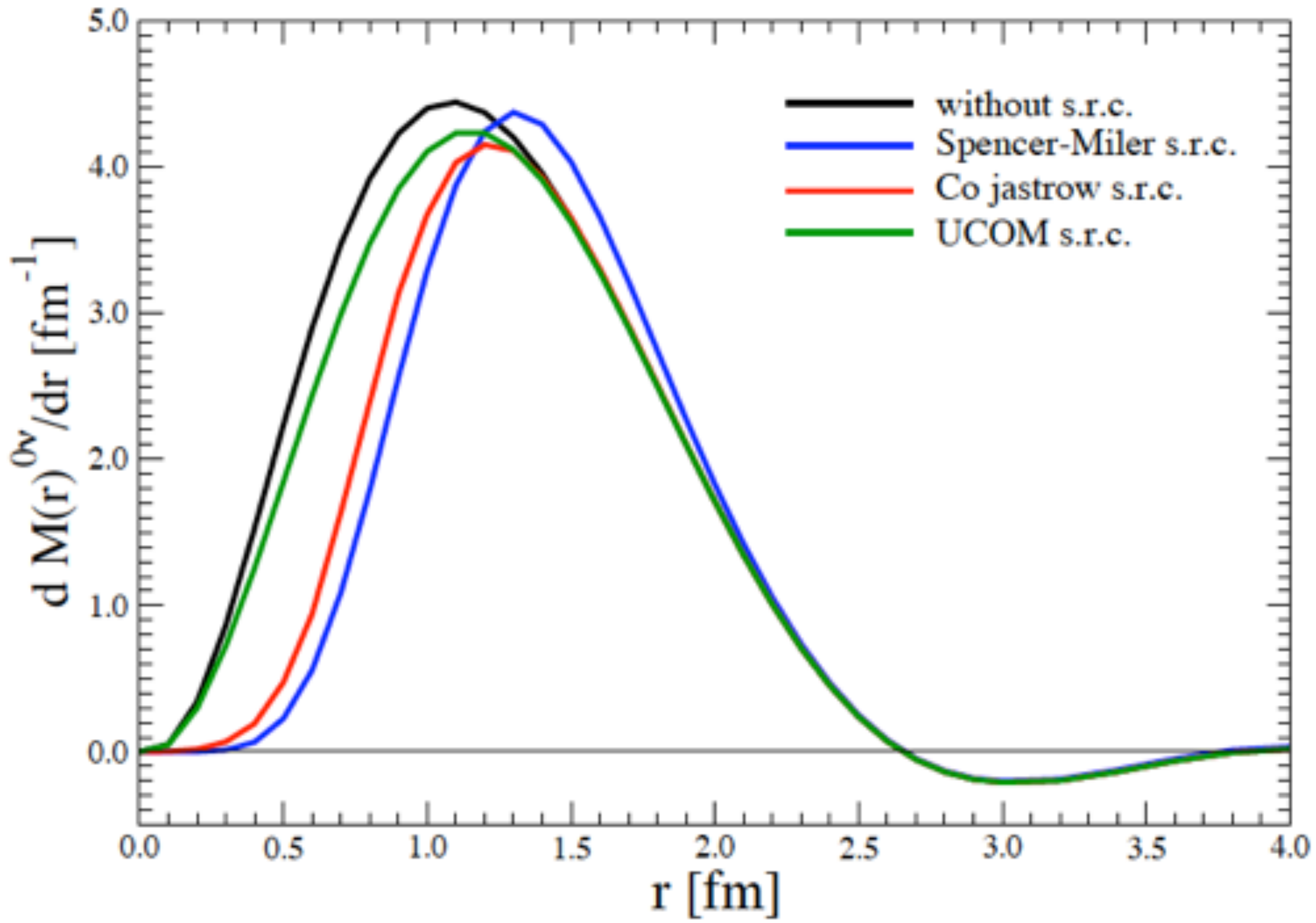
- Thus for $m_{\beta\beta} \sim m_e$, the bb0nu decay would be 105 times faster than the bb2n decay. It is this phase space advantage which makes the bb0n decay a sensitive probe for Majorana neutrino mass.

Phase space for diferrent istopes



- Quite similar for most isotopes except for Ge-76 (x 10 smaller than average), Pd-110 (x5 smaller) and Nd-150 (x5 larger)

Dependence of NME with r



- Typical distances among the two decaying nucleons that contribute the most to NME. Only relatively short distances, r < 2 – 3 fm →only the nearest neighbor neutrons undergo ββ0ν transitions. The fact that two nucleons strongly repel each other for distances r < 0.5 – 1.0 fm should be taken into account (e.g, UCOM method)

Evaluation of nuclear matrix elements

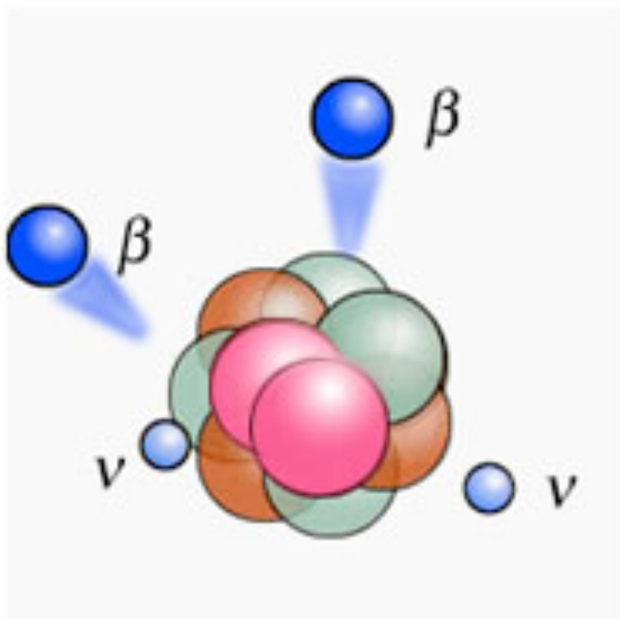
$$M^{F}_{0\nu} = \langle f| \sum_{a,b} H(r_{ab}, \overline{E})\tau^{+}_{a}\tau^{+}_{b} |i\rangle \text{ , and}$$

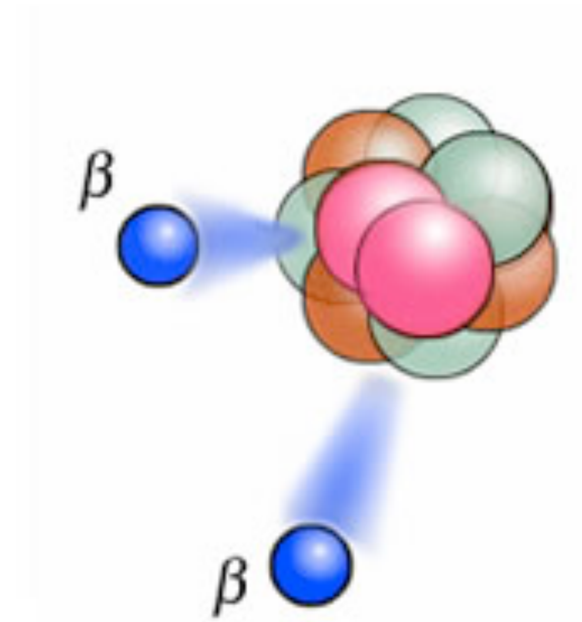
$$M^{GT}_{0\nu} = \langle f| \sum_{a,b} H(r_{ab}, \overline{E})\vec{\sigma}_a \cdot \vec{\sigma}_b \tau^{+}_{a}\tau^{+}_{b} |i\rangle$$

- To compute ββ0ν decay rates f ⇒ need to evaluate the initial and final state wavefunctions |i> and |f>, and the nuclear matrix elements connecting the two.

- Complicated nuclear many-body problem ⇒ calculation cannot be done exactly, and some approximations need to be introduced. ⇒ NME industry

Nuclear matrix elements





Major techniques

- The Interacting Shell Model (ISM). A microscopic calculation

- The Quasiparticle Random Phase Approximation (QRPA). Uses the fact that the most important part of the residual interaction among nucleons is the pairing force

- The Interacting Boson Model (IBM).

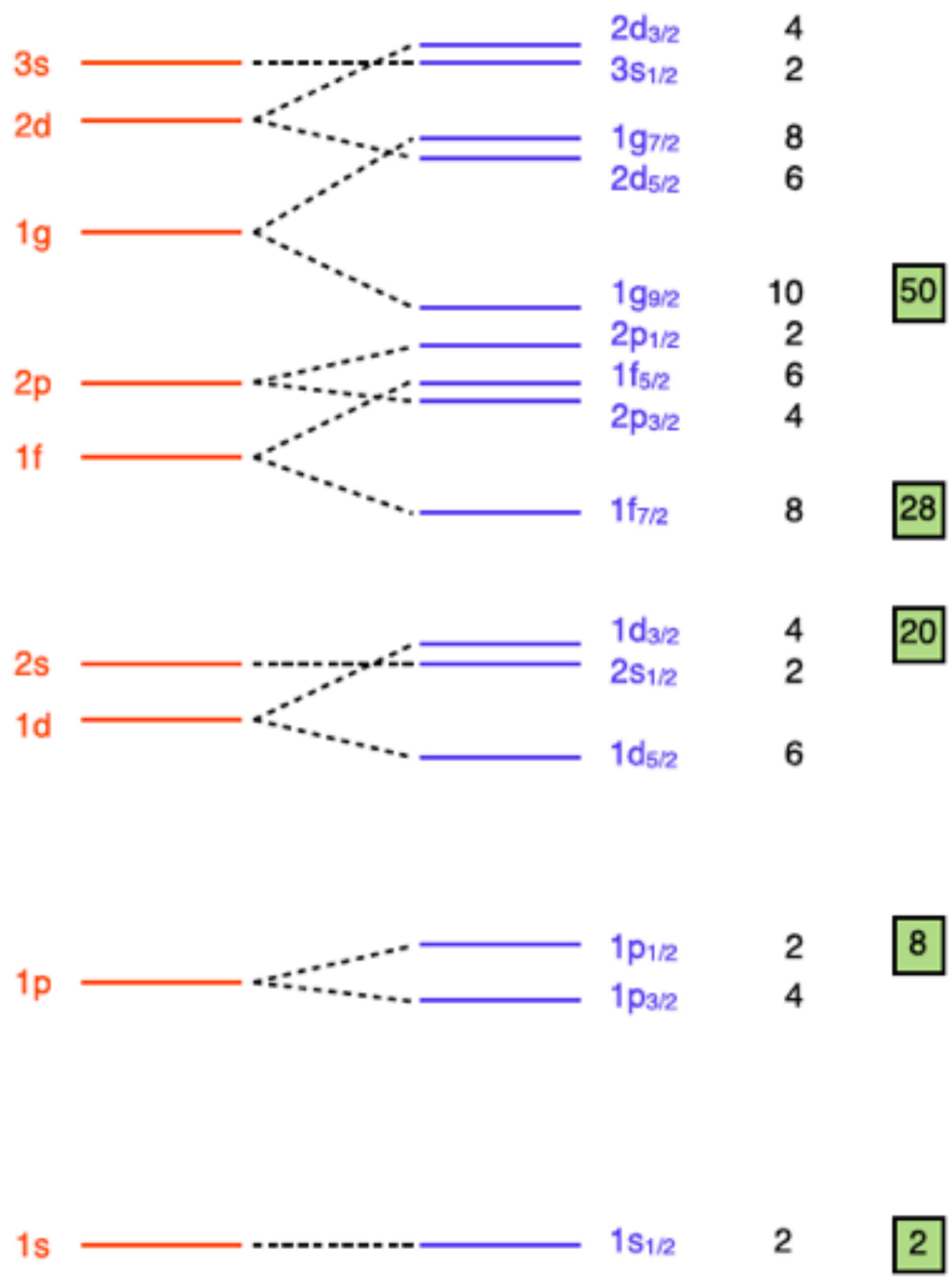
ISM

$$U(r) = \frac{1}{2}\,\hbar\omega\, r^2 + D\,\vec{l}^{\,2} + C\,\vec{l}\cdot\vec{s}.$$

- Based in Independent particle model. Nucleons move independently in a mean field with a strongly attractive spin-orbit term

- $\frac{1}{2}\,\hbar\omega\, r^2 + D\,\vec{l}^{\,2}$ the harmonic oscillator plus the surface correction describe the bound nucleon nature of the problem.

- $C\,\vec{l}\cdot\vec{s}$ the spin-orbit part gives the proper separation of the subshells and explain the nuclear magic number.



• Low-lying energy levels in a single-particle shell model with an oscillator potential
⇒without spin-orbit (left)
⇒with spin-orbit (right)
interaction.

• The number to the right of a level indicates its degeneracy, (2j+1). The boxed integers indicate the magic numbers.

$$\mathcal{H} = \sum_{ij} \mathcal{K}_{ij}\, a_i^\dagger a_j - \sum_{i\leq j\, k\leq l} \mathcal{V}_{ijkl}\, a_i^\dagger a_j^\dagger a_k a_l$$

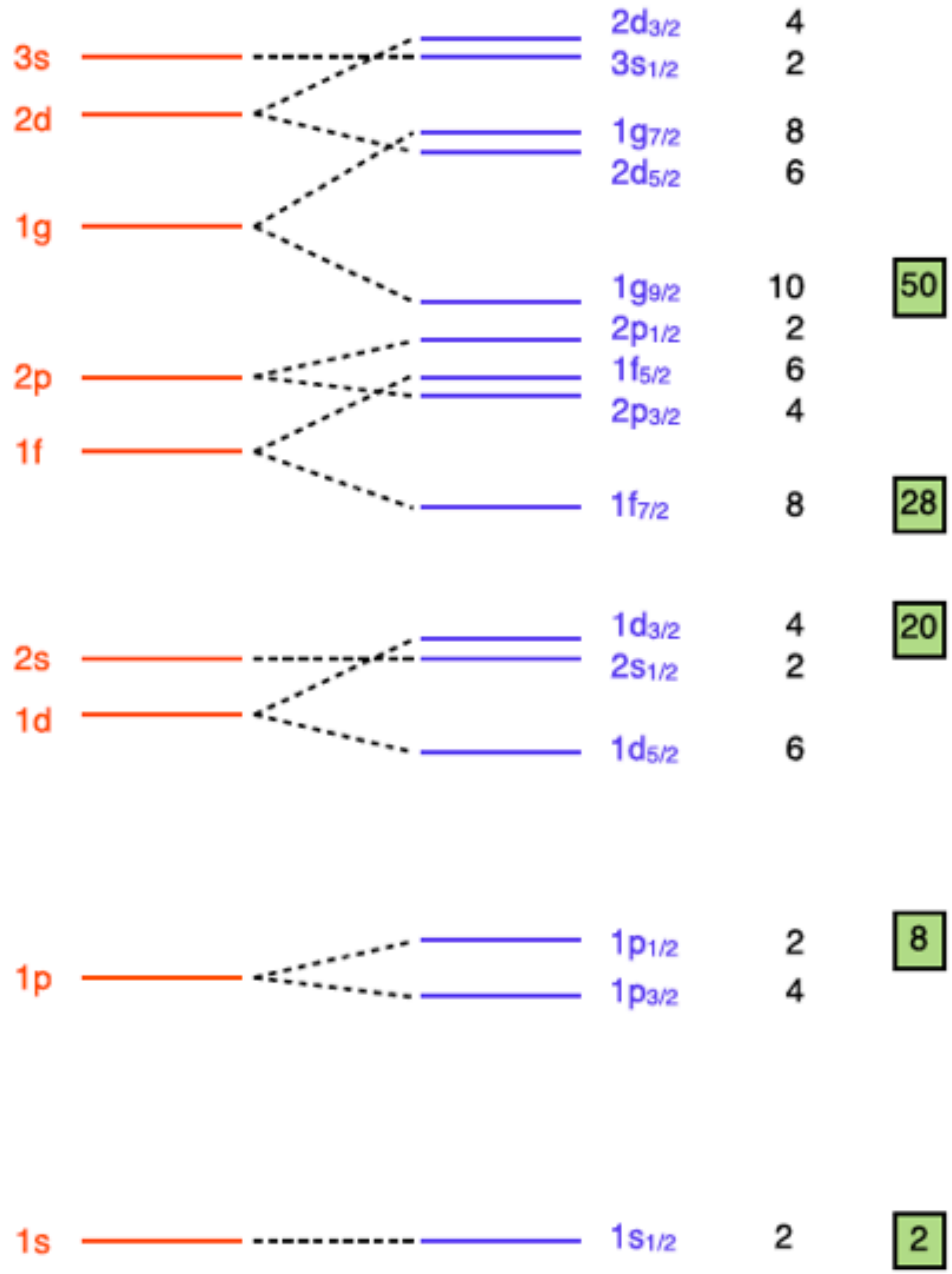
•As the number of protons and neutrons depart from the magic numbers ⇒ must include the “residual” two-body nucleon interaction among nucleons: IPM ⇒ ISM.

•Contains a kinetic (K) and a potential (V) term that adds one or two particles in orbits of total angular momentum i, j and removes one or two from orbits k, l, subject to the Pauli principle ($\{a^\dagger, a_j\} = \delta_{ij}$).

•To solve the problem⇒ diagonalize the matrix $<\phi_I\ |H|\ \phi_I'>$ with off-diagonal elements being either 0 or ± V_{ijkl}

Conceptually simple. However, the dimension of the matrices os so large as to make

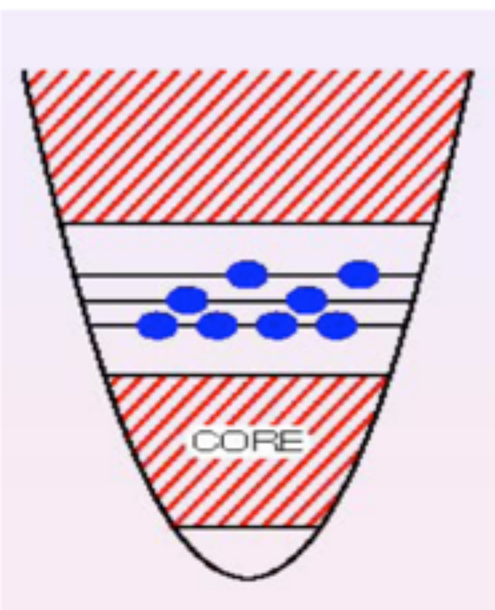
the problem intractable, except for the lightest nuclei.



- Due to the high dimensionality of the problem one cannot include in the valence space too many single-particle orbits.
- Even the most advanced evaluations include just one oscillator shell.
- In most decay candidate nuclei the valence space usually omits important spin-orbit partners.
- For Ge-76 the valence space
consists of $p_{1/2}$ $p_{3/2}$ $f_{5/2}$ and $g_{9/2}$ orbits while omitting the essentially occupied $f_{7/2}$ and empty $g_{7/2}$ spin-orbit partners

Interacting Shell Model

Basic procedures: Treat the nucleus as a collection of protons and neutrons bound in a potential well, and interacting through an effective interaction. The procedure consists of several steps:



1) Define the valence space
2) Derive the effective hamiltonian H_{eff} using the nucleon-nucleon interaction plus some empirical nuclear data.
3) Solve the equations of motion to obtain the ground state wave functions.

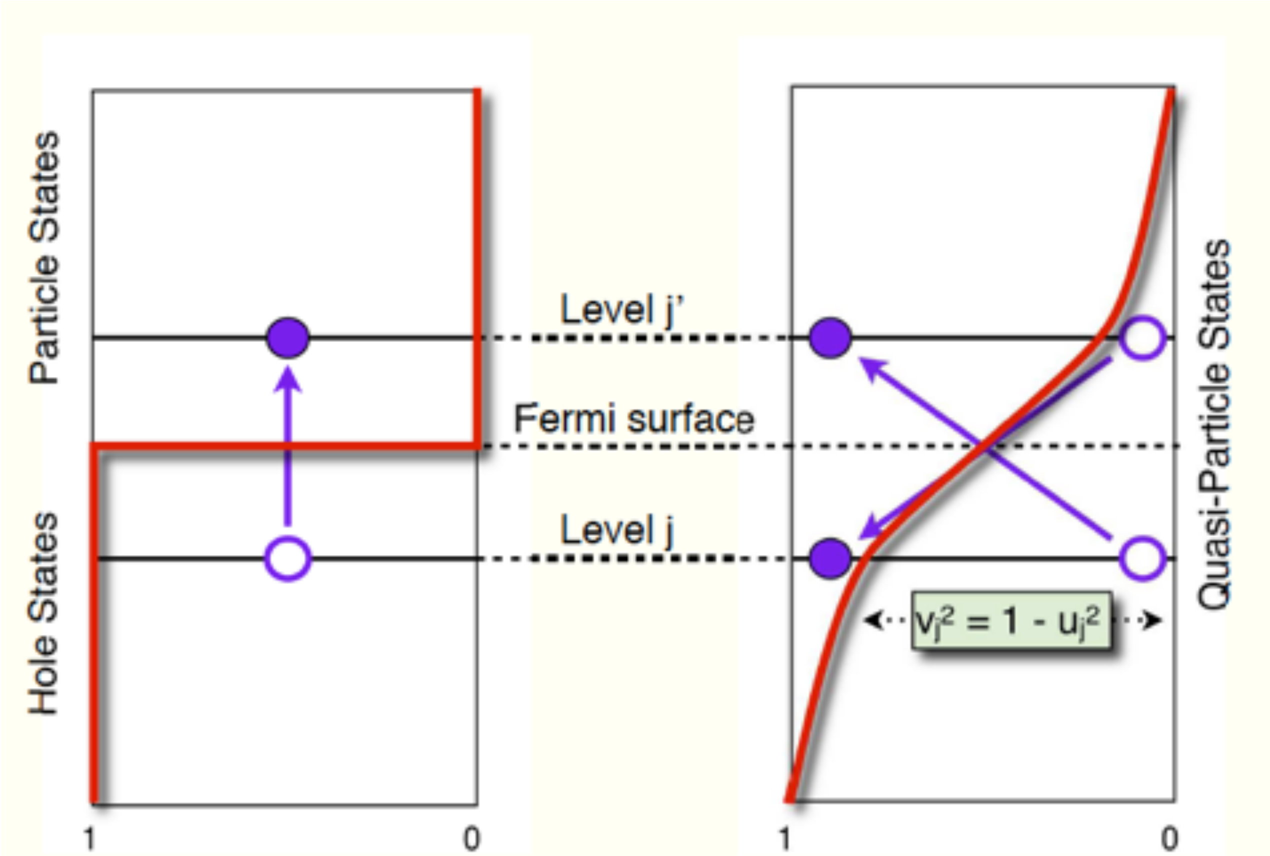
Note: Completely full or completely empty subshells in both the initial and final nuclei will not participate in the ββ decay.

- ISM conceptually simple, but complicated in practice.
- Given a good enough residual interaction the problem is reduced to diagonalizing a matrix in a sufficiently large basis ("valence space").
- In the ISM a limited valence space is used but all configurations of valence nucleons are included.

QRPA

- The pairing force accounts for the tendency of nucleons to couple pairwise to especially stable configurations, i.e. into nuclei with even N, even Z.
- This force favors the coupling of neutrons with neutrons, and protons with protons, so that the orbital angular momentum and spin of each couple adds to zero.
- As the result of the pairing force, the nuclear ground state is mainly composed of Cooper-like pairs of neutrons and protons coupled to J = 0+ total angular momentum.
- In QRPA, the nucleon pairing is introduced via the BCS theory of superconductivity. A unitary transformation is first performed to change from a particle to a quasiparticle basis.

QRPA



- Quasiparticles are generalized fermions which are partly particles (with probability u^2_j and partly holes with probability v^2_j.
- j is the single-particle orbital the quasiparticle belongs to

- Quasiparticles are just a mathematical construct to account for pairing between like nucleons while retaining the simplicity of the independent particle model (quasi particles are kept as independent particles)
- The transformation smears out the nuclear Fermi surface over several orbitals, for both protons and neutrons,

• Once the problem has been transformed into the simpler quasiparticle basis, the QRPA goal is to evaluate the transition amplitudes associated with charge changing one-body operator T^{JM} connecting the 0+ vacuum of quasiparticles in the even-even nucleus with any of the J^{π} excited states in the neighboring odd-odd nuclei.

• Such states are described as harmonic oscillations above this vacuum. QRPA also takes into account
that the ground state is not of purely independent quasiparticle character, but may contain correlations.

• Two-particle, two-hole excitations are included
in the QRPA vacuum state, as opposed to the BCS vacuum.
excitation

• The transition amplitude needs to take into account that the creation of a particle-hole pair from the BCS vacuum (the so-called forward-going amplitude X) can lead to the same final state J^{π} as the destruction of a particle-hole pair from a two-particle, two-hole excitation (the backward going amplitude Y).

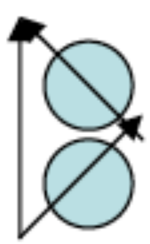
• The amplitudes X and Y as well as the corresponding energy eigenvalues ω_m are determined by solving the QRPA equations of motion for each J^{π}.

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ Y \end{pmatrix}$$

• The terms A and B depend on the interaction matrix elements between quasi-particle configurations. They can be written in terms of particle-hole (p-h) and particle-particle (p-p) matrix elements.

IBM

J=0 s-boson

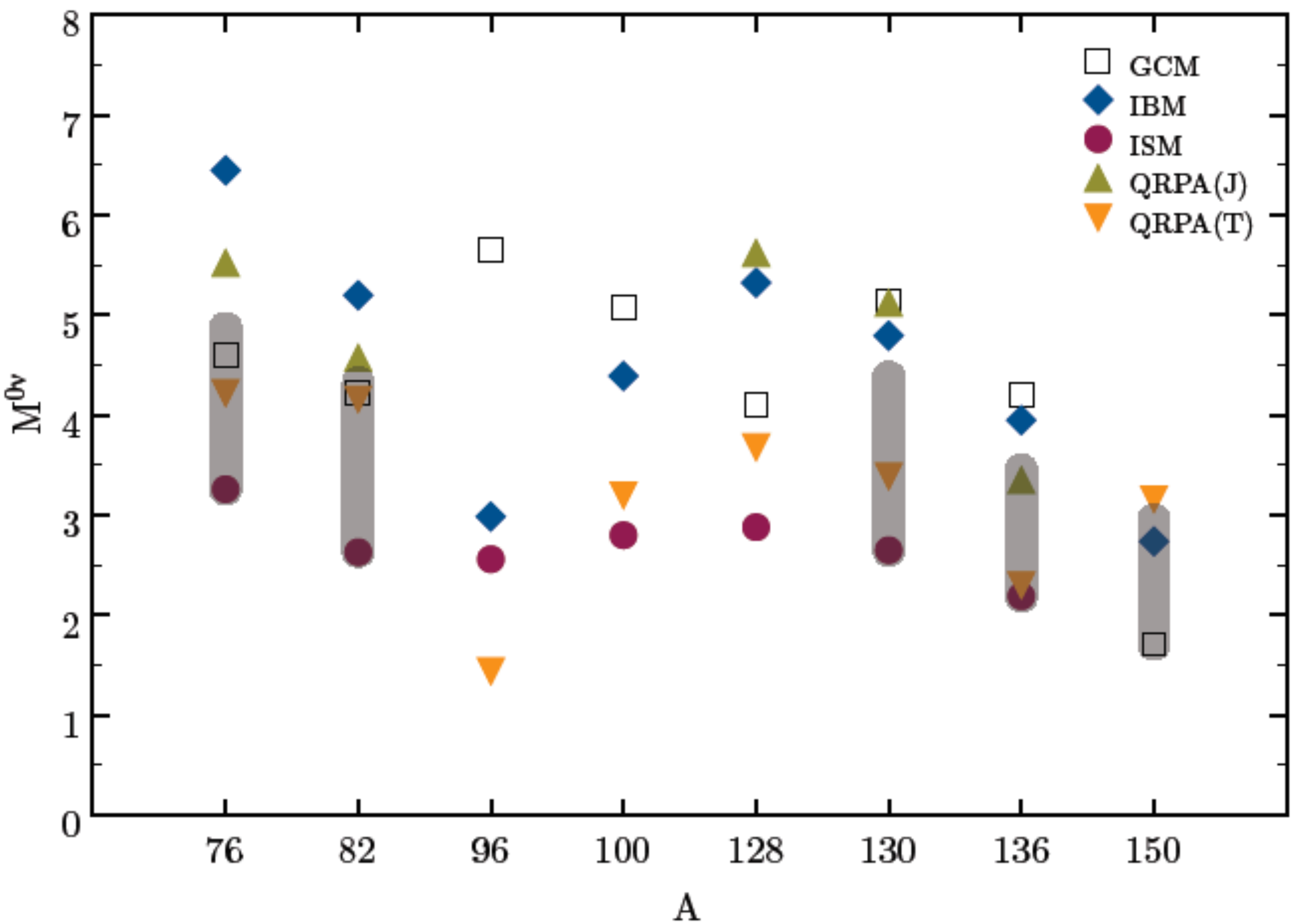


J=2 d-boson

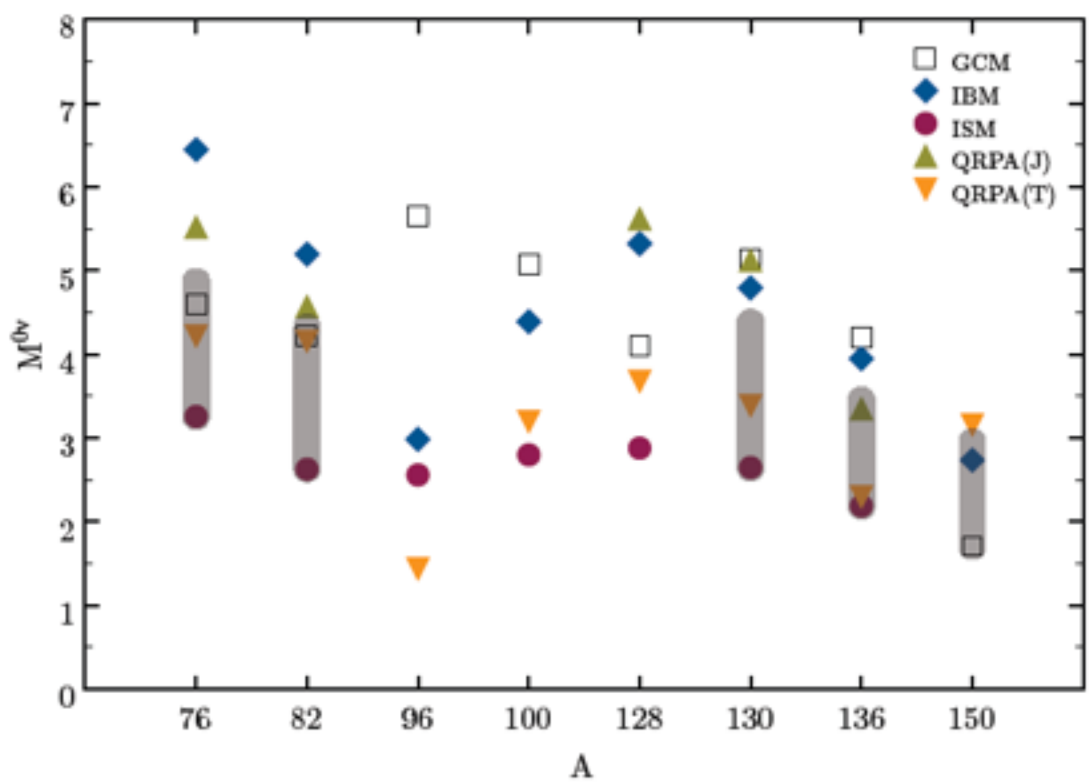
Unpaired fermions

- Main problem with ISM: The larger the number of nucleons becomes, the more shells have to be taken into account, and the number of nuclear states soon becomes so colossal that the shell model becomes intractable.
- IBM postulates that identical nucleon pairs are represented by bosons with angular momenta J = 0 or J = 2.
- The multitude of shells which appears in the shell model is then reduced to the simple s-shell (J = 0) and the d-shell (J = 2).

A Physics Motivated Range of NMES



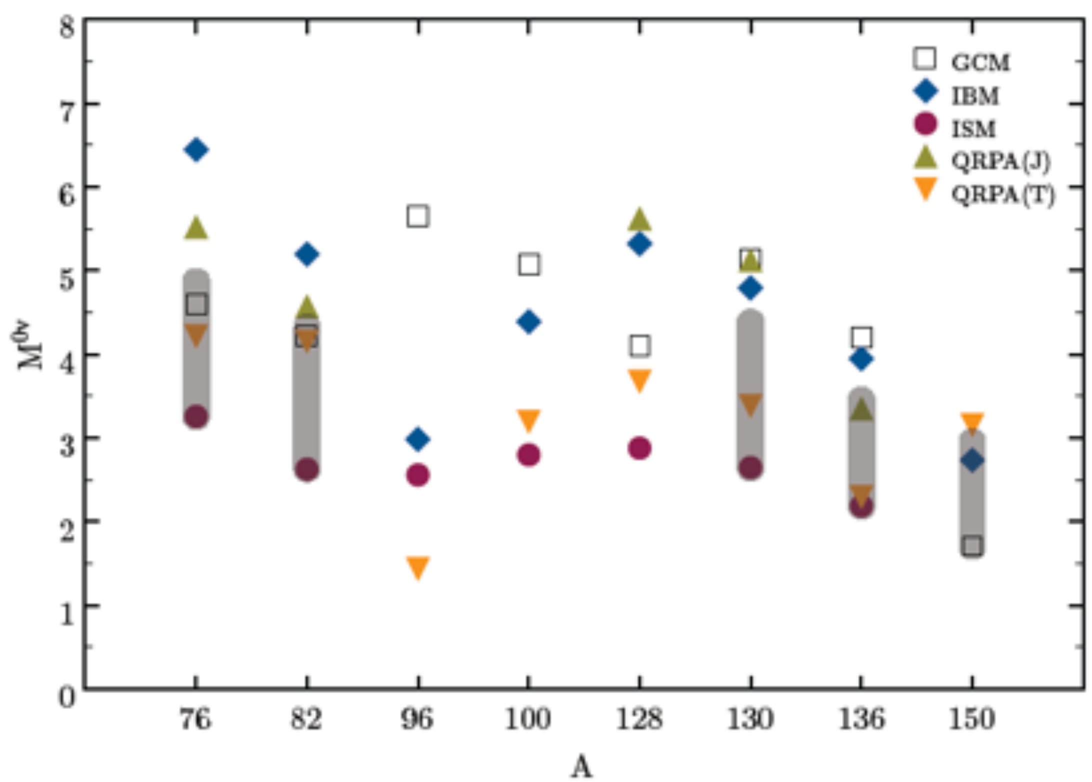
ISM



- The Clear advantage of the ISM calculations is their full treatment of the nuclear correlations, while their drawback is that they may underestimate the NMEs due to the limited number of orbits in the affordable valence spaces.

- The effect can be of the order of reducing the NME by 25%.

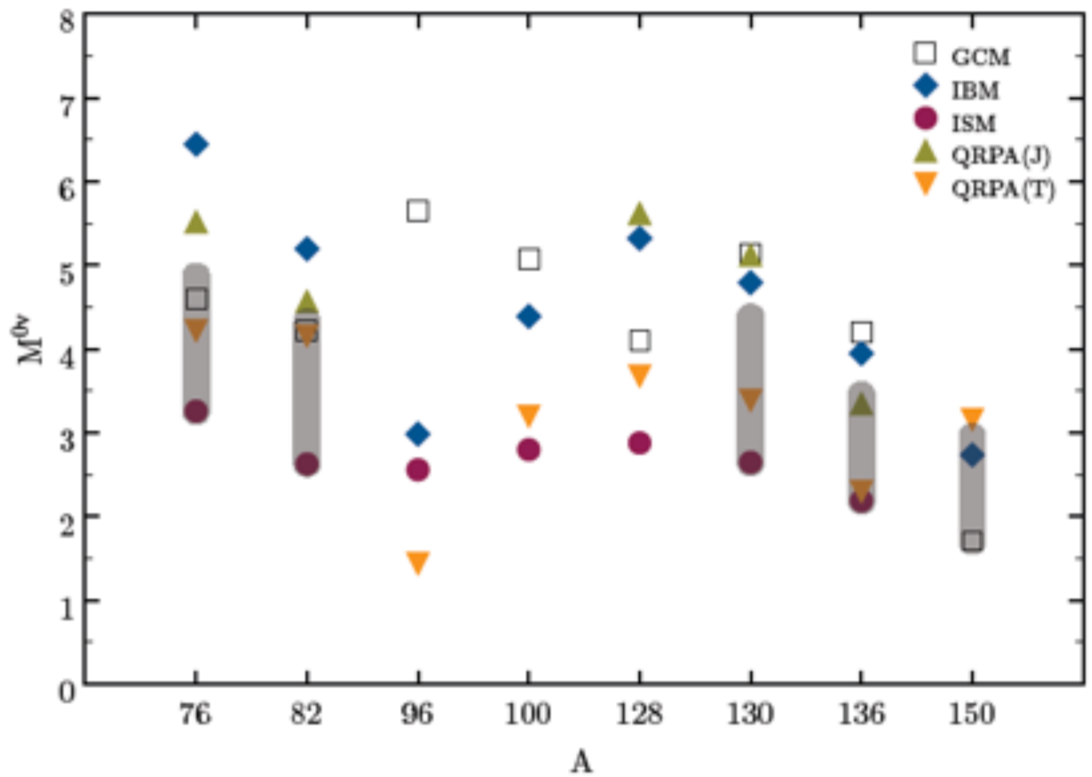
QRPA and IBM



- On the contrary, the QRPA variants and the IBM are bound to underestimate the multipole correlations in one or another way.

- As it is well established that these correlations tend to diminish the NMEs, these methods should tend to overestimate them.

PMR



•Take out all elements in common.
• Compute a range between models

• Example: For Xe-136 ISM value defines the lower range. For the upper one average the NMEs from the QRPA and IBM. The resulting interval is [2:19-3:45], and the most probable value is 2.82
• PMR permits to use a single range (or even average) rather than quantifying the differences between the methods as error bars (as discussed that would be false)