Ettore Majorana meets his shadow (I)

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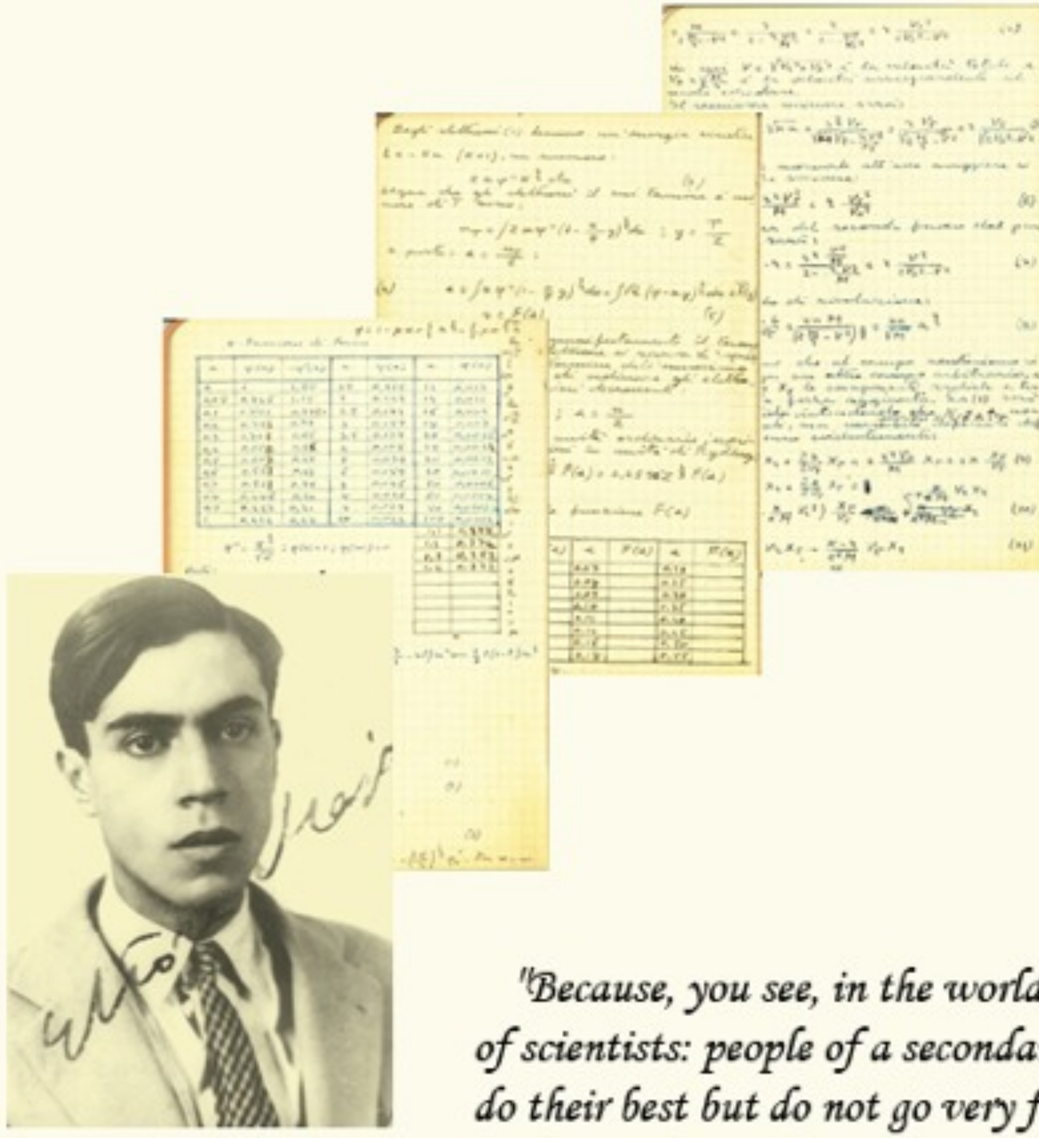
Meet the Genius

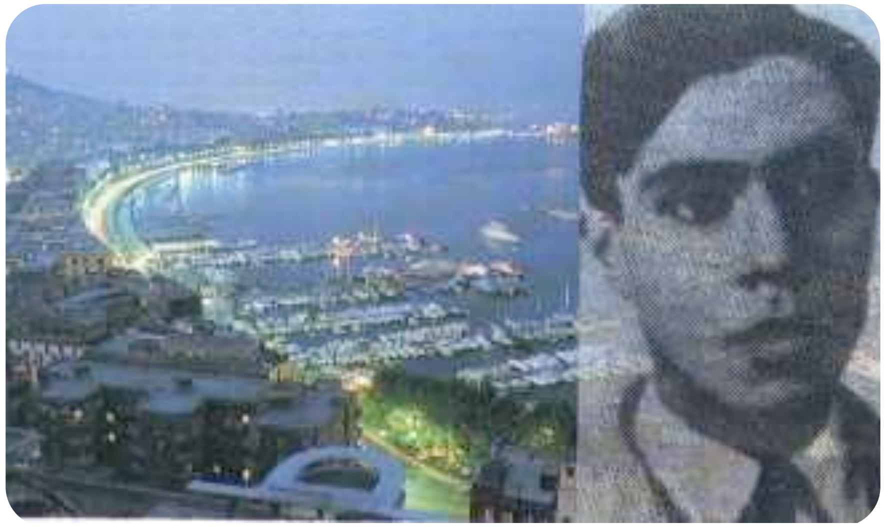


"Because, you see, in the world there are various categories of scientists: people of a secondary or tertiary standing, who do their best but do not go very far. There are also those of high standing, who come to discoveries of great importance, fundamental for the development of science.

But then there are geniuses like Galileo and Newton. Well, Ettore was one of them. Majorana had what no one else in the world had".

E. Fermi

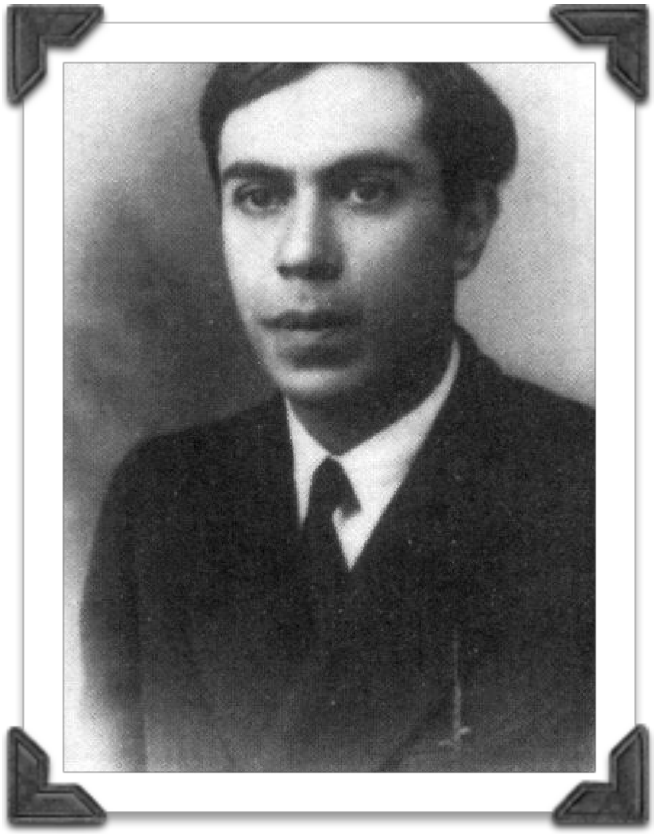
Science like poetry...



Nel marzo del 1938, prima di dissolversi e nel nulla, lo scienziato catanese Ettore Majorana, trentaduenne, fisico teorico di altissima, internazionale levatura, docente all'Università di Napoli, molto vicino al gruppo dei «ragazzi di via Panisperna», cioè i giovani ricercatori atomici guidati a Roma da Fermi, scrive due lettere, una ai familiari, una a un amico, nelle quali esprime il proposito di suicidarsi. Nella seconda annunzia addirittura il giorno, l'ora e il luogo del suicidio. Ma non vi tiene fede: non e solo il suo corpo non sarà ritrovato nel luogo indicato, e nemmeno in nessun altro luogo, ma le ricerche, attivate da Mussolini, approderanno a una mezza certezza e cioè che Majorana era ancora vivo qualche giorno dopo

La scienza, come la poesia, si sa che sta ad un passo dalla follia

Majorana Insight



The neutrino could be a true neutral particle

- Unlike the rest of the elementary fermions, the neutrino has no electric charge (or any other fundamental charge that cannot be violated).

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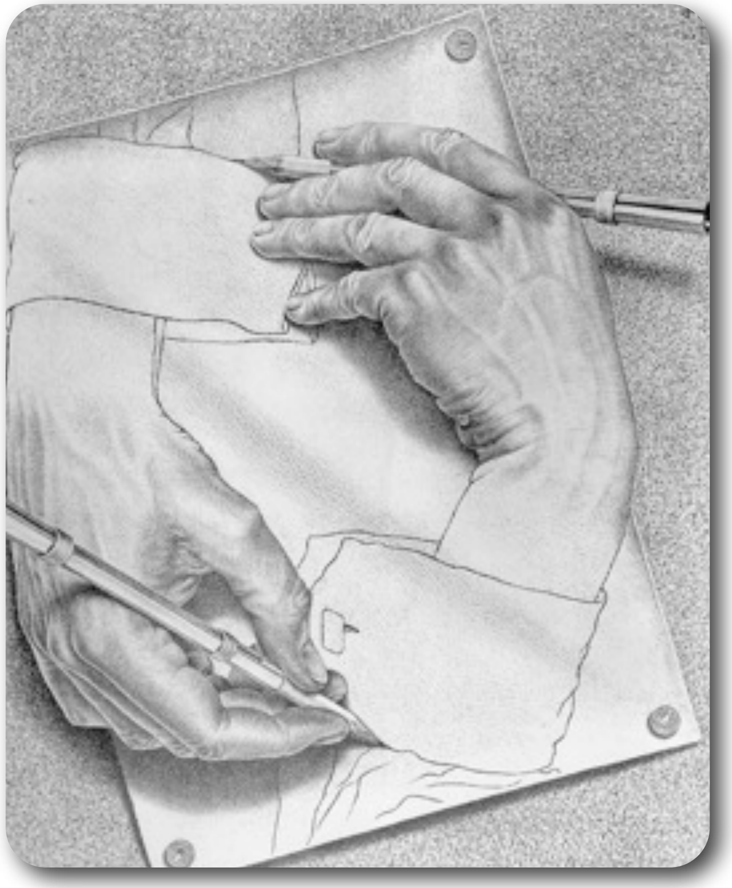

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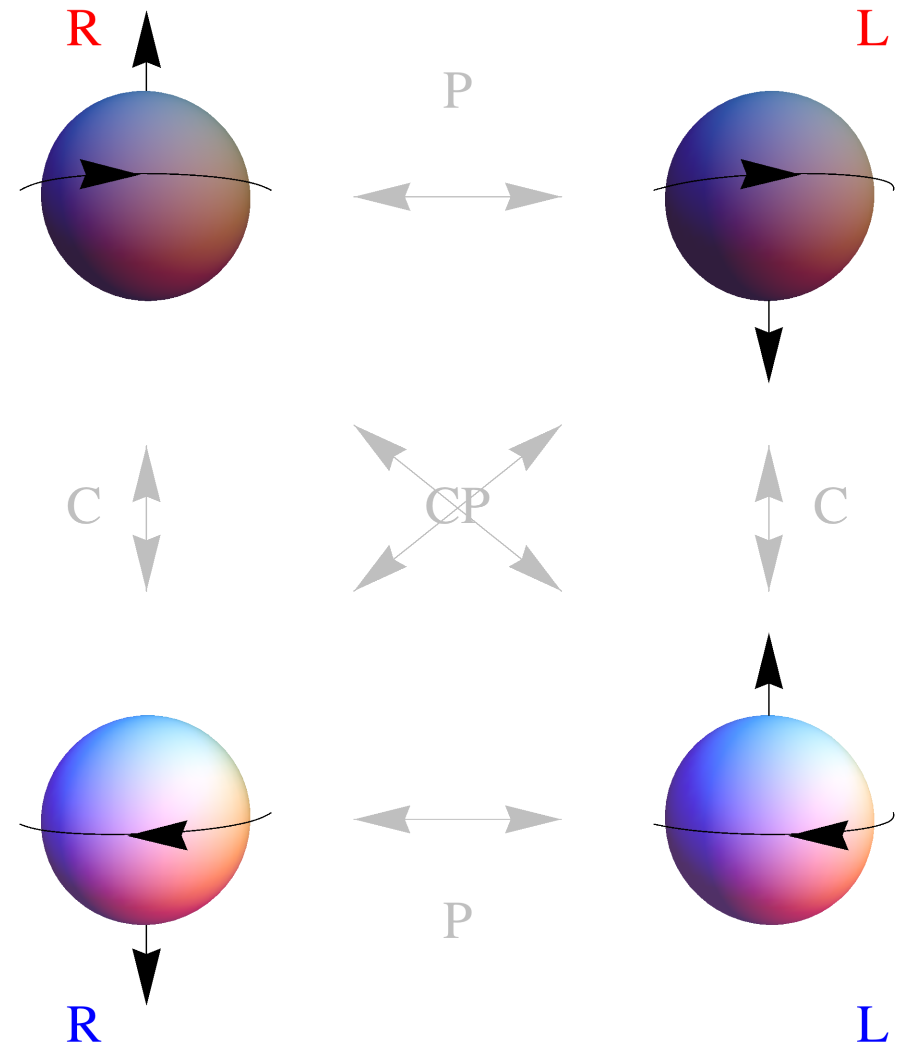
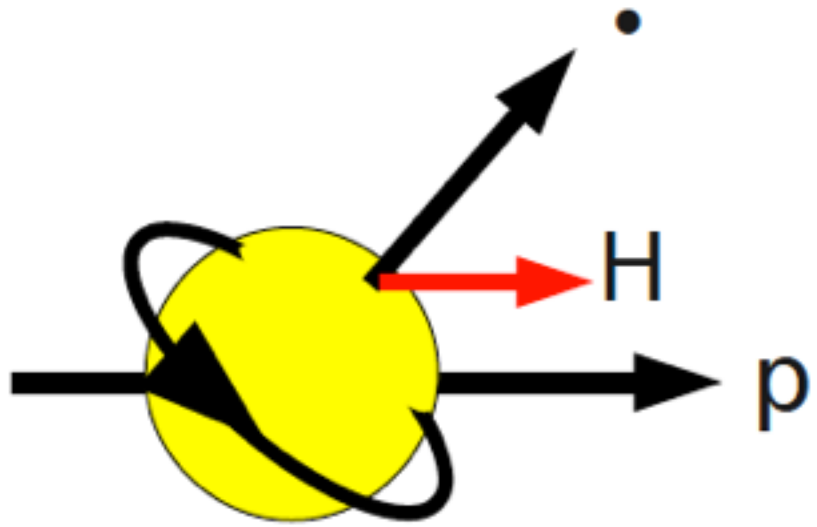
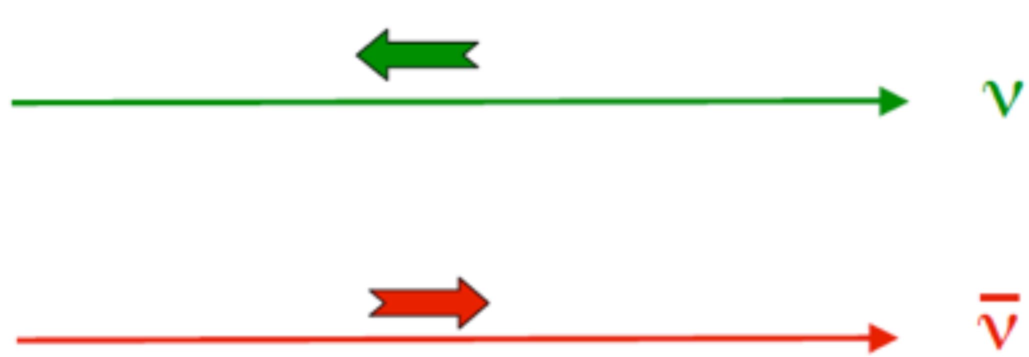
Neutrinos & antineutrinos



Helicity



$$\hat{H} = \frac{\vec{\sigma}\cdot\vec{p}}{|\vec{p}|}$$



- Helicity is the projection of the spin along the particle direction.

- It is not a Lorentz invariant unless the particle is massless.

- In the SM neutrinos are massless, therefore:
  - neutrinos are left-handed (helicity in the opposite direction of movement)
  - antineutrinos are right-handed (helicity in the same direction of movement)

Chirality

- Something is chiral if it cannot be superimposed in its mirror image.

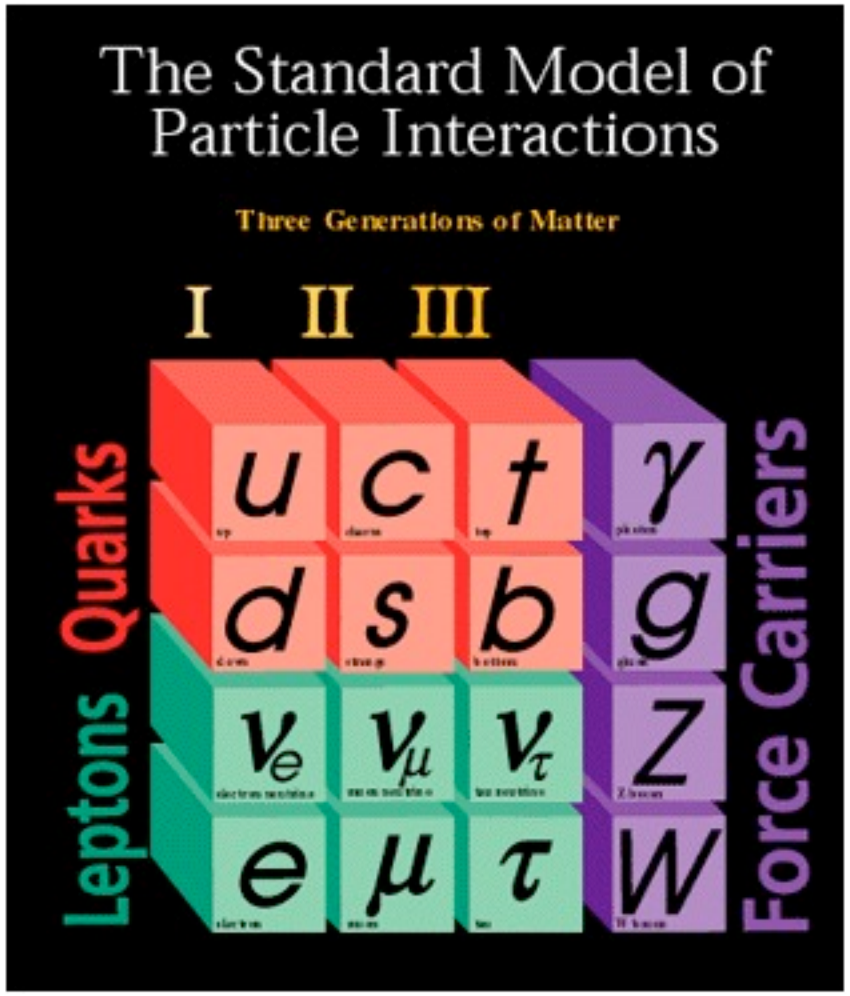
- Chiral left and right handed fermions satisfy the equations.

$$P_L f_L \equiv \frac{(1-\gamma_5)}{2} f_L = f_L \quad \text{and} \quad P_R f_R \equiv \frac{(1+\gamma_5)}{2} f_R = f_R$$

- For a massless fermion helicity=chirality

- Massive fermions, instead may have both helicity states

SM neutrino

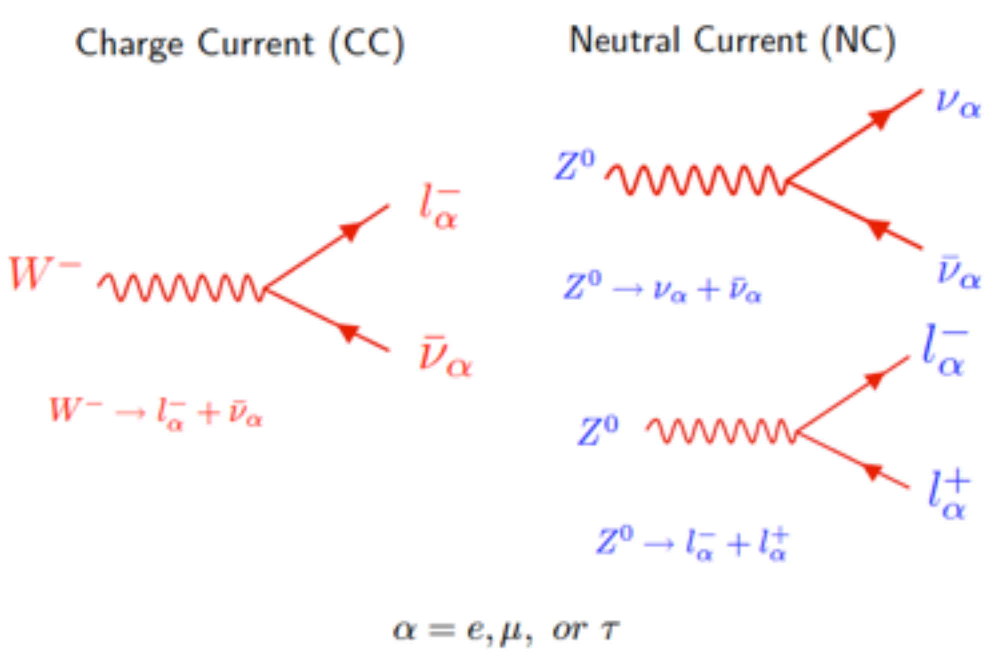


Lepton SU(2) doublets

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

Charged lepton SU(2) singlets

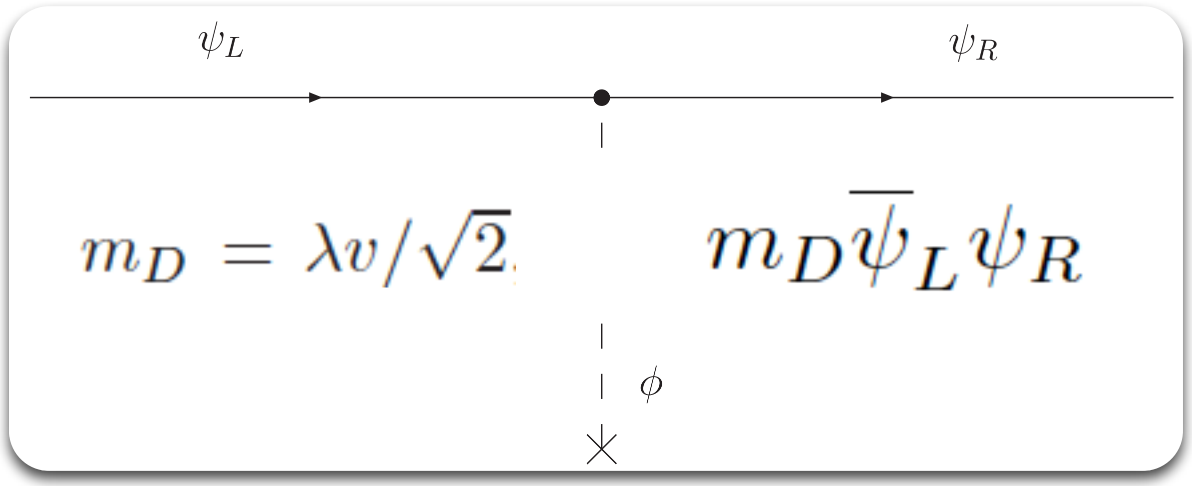
$$e_R, \mu_R, \tau_R$$



Lepton number is conserved

$$L(\nu) = L(\ell^-) = -L(\bar{\nu}) = -L(\ell^+) = 1.$$

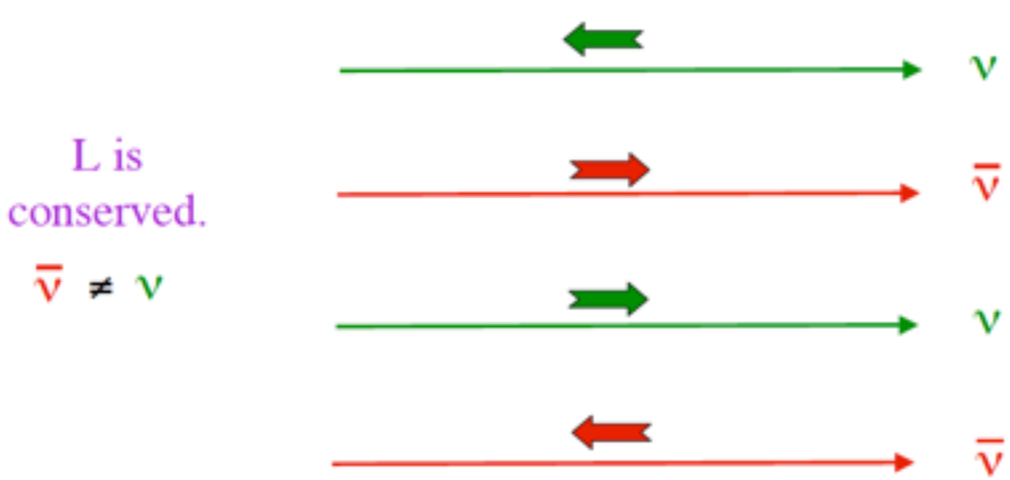
Dirac fermions in the SM



$$\lambda \bar{\psi}_R \phi \psi_L \xrightarrow{\text{SSB}} \lambda v \bar{\psi}_R \psi_L$$

ν is the mass eigenstate, and has mass m_D.

We have 4 mass-degenerate states:



This collection of 4 states is a Dirac neutrino plus its antineutrino.

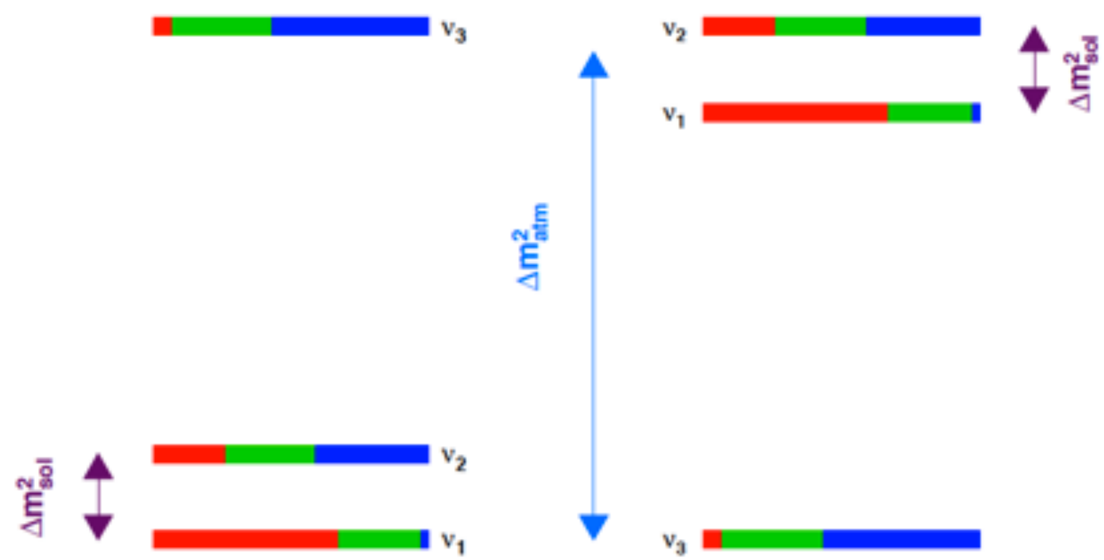
- In the SM charged leptons get their masses by coupling left- and right- (chiral) fields to the Higgs field.

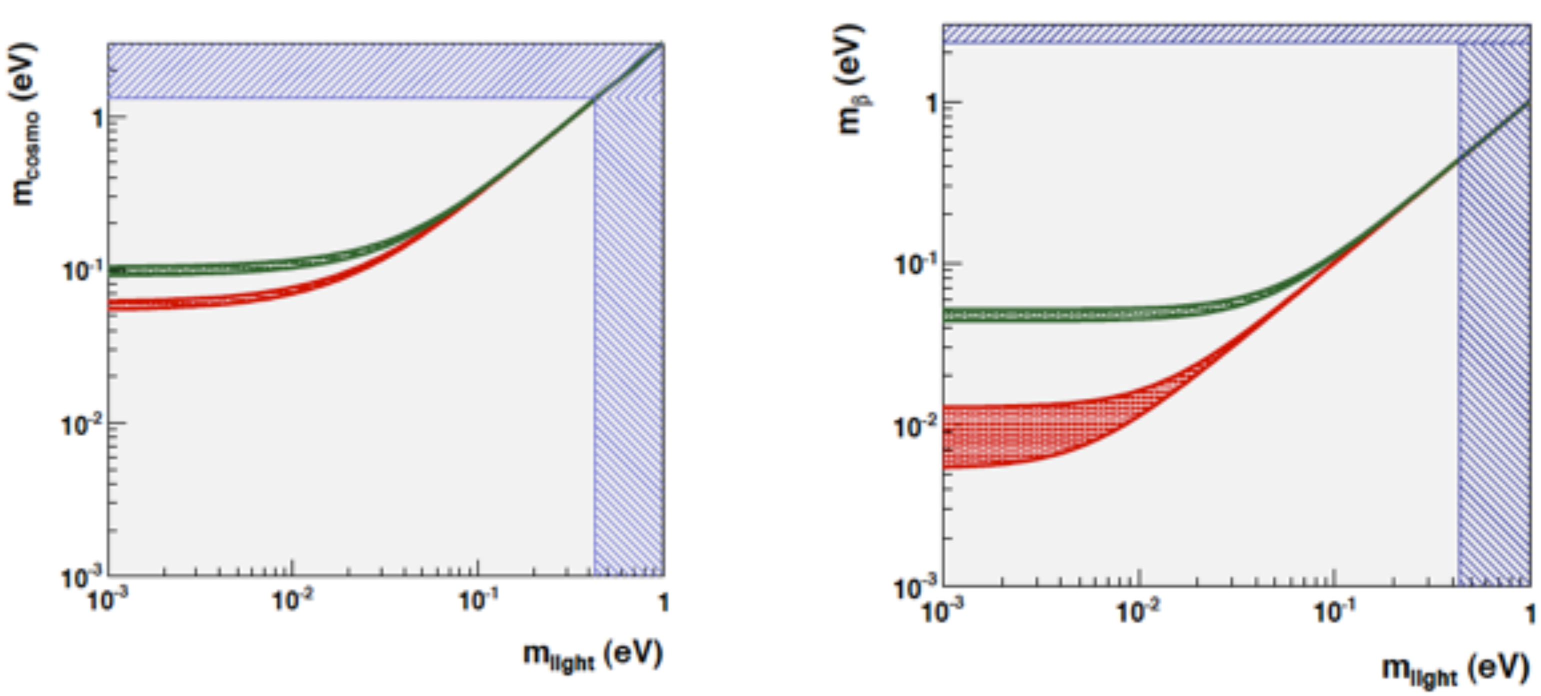
- It always involves two different fields with opposite quantum numbers (e.g, electric charge)

- Four d.o.f. with the same mass (Dirac fermions). We call two of them the particle the other two the antiparticle.

- There is no ν_R in the SM. Therefore the neutrino is massless

But neutrinos are massive

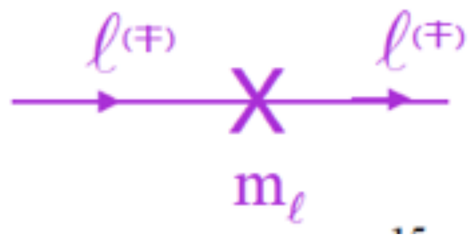




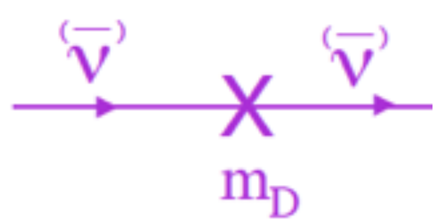
- Neutrino oscillation measure two mass splits.
- Two mass hierarchies are possible.
- The absolute scale of the mass is not fixed.
- Direct neutrino mass measurements and cosmology set the scale to about 1 eV

Dirac Mass term for the neutrino?

$m_\ell \bar{\ell}_L \ell_R$



$m_D \bar{\nu}_L \nu_R$

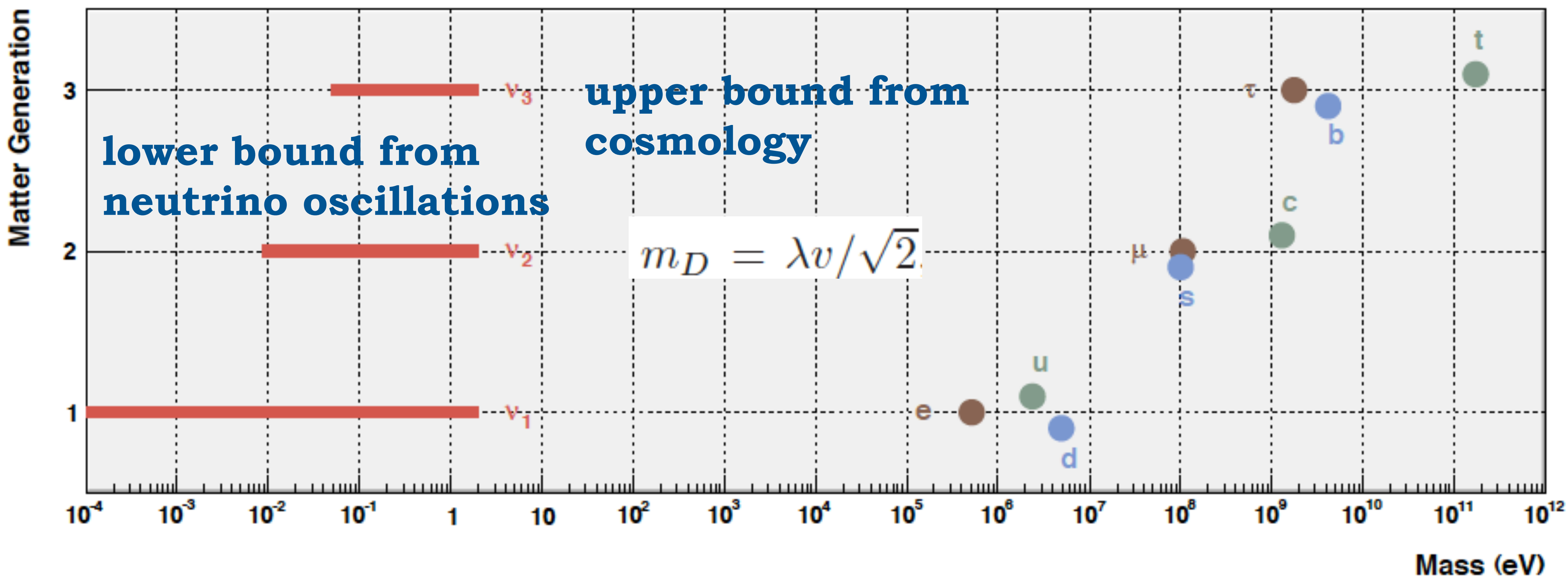


- Nothing prevents us from treating the neutrino as a “neutral electron”.

- So, we add a ν_R to the SM and treat the neutrino as a Dirac particle (4 degrees of freedom)

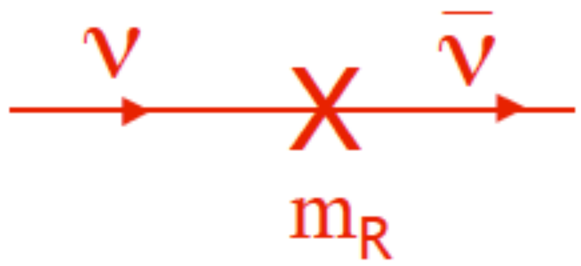
- Lepton number is still conserved.

Why are neutrino masses so small?



- If neutrinos acquire their mass by coupling to the same vev than the charged leptons, why neutrino masses are so much smaller than charged lepton masses?
- One needs to fine tune: choose arbitrarily an small λ for neutrinos

Majorana Mass terms



$$m_R \overline{\nu_R}^c \nu_R$$

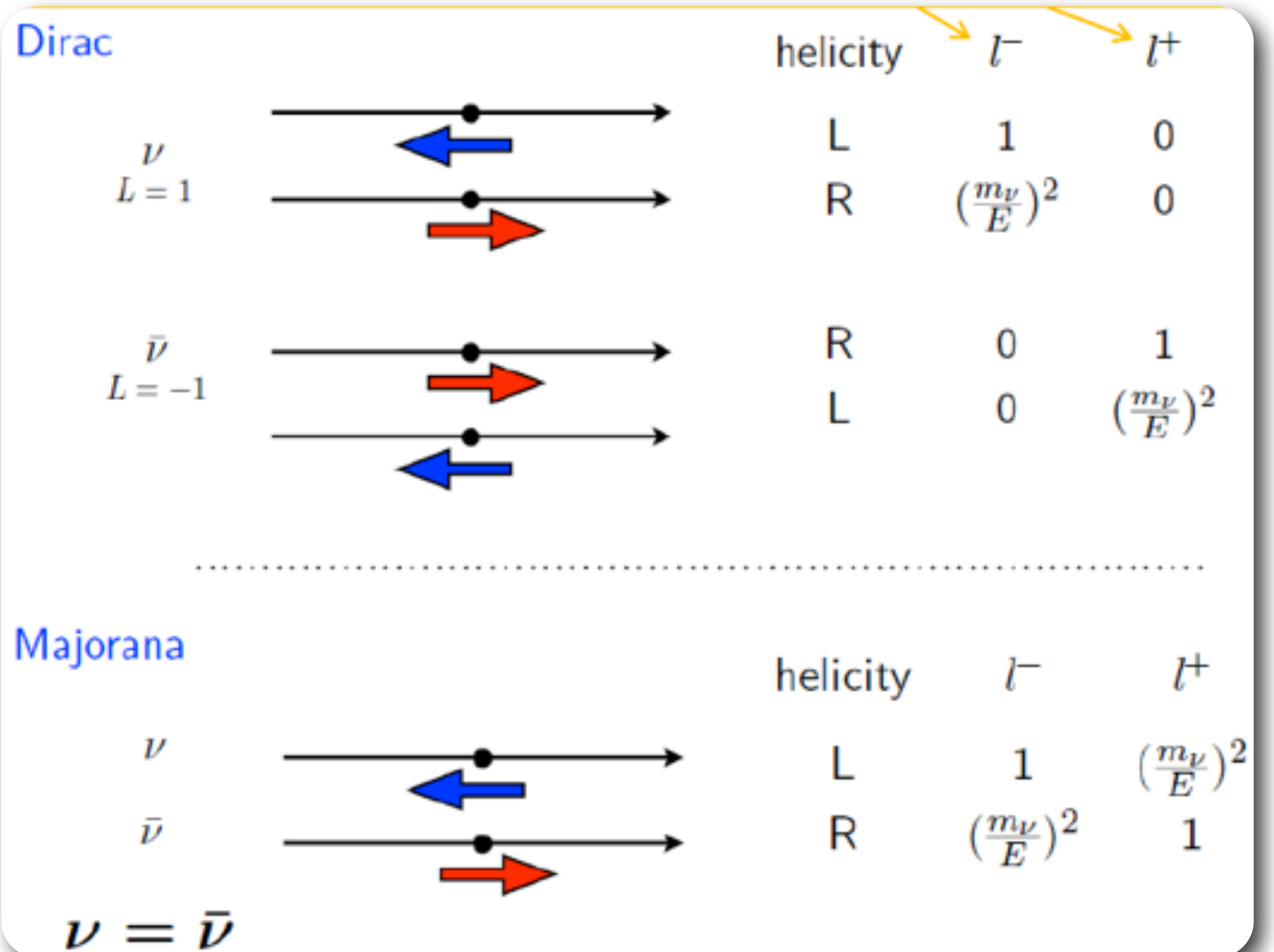
We have only 2 mass-degenerate states:

- Since we have a ν_R singlet, nothing prevents us from forming opposite-chirality fields by charge-conjugating the ν_R field (we could do the same with ν_L field but then we would be violating EW charge)

- But this does not conserve lepton number! (answer: so what? nothing sacred about this)

- We have only two d.o.f. with the same mass now.

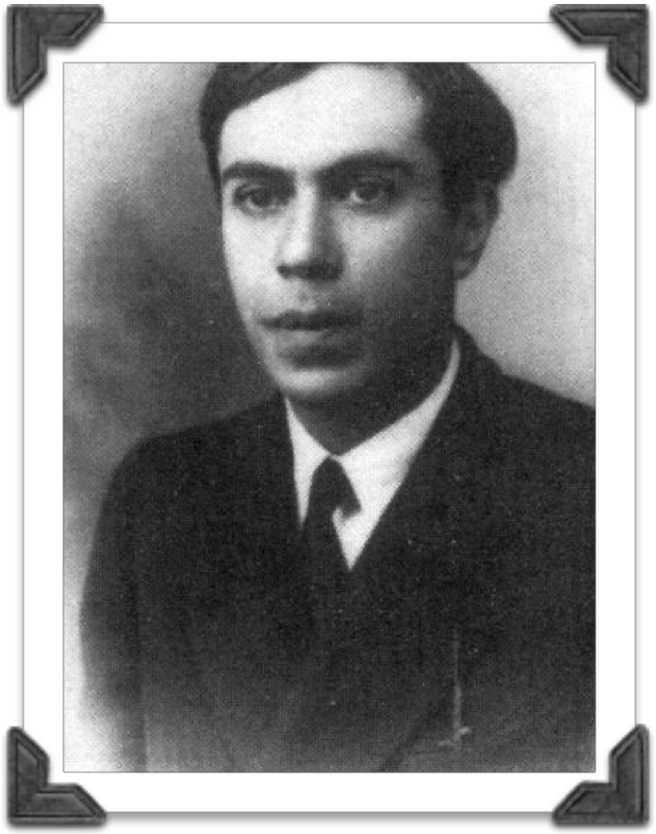
Dirac and Majorana neutrinos



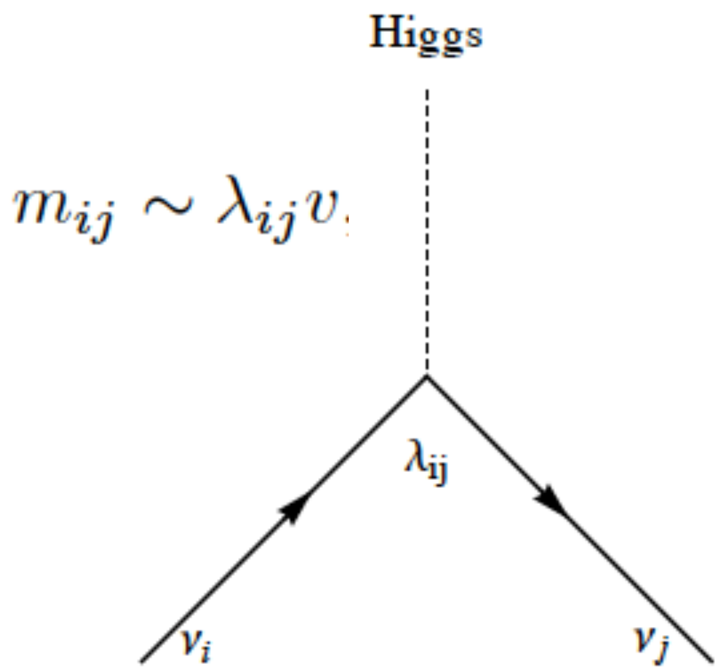
- Unlike electrons (electric charge) and Dirac neutrinos (lepton number) Majorana neutrinos do not have quantum numbers that allow us to say wether they are particles or antiparticles.

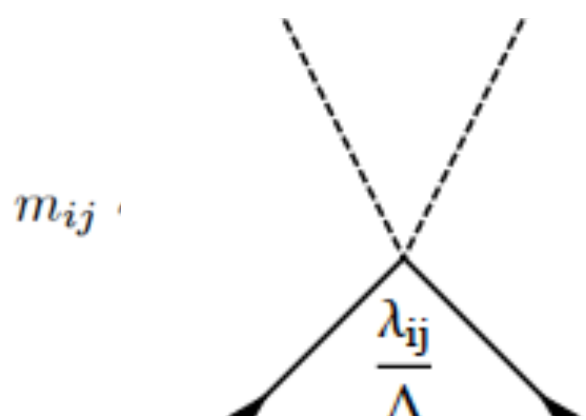
- Thus the Majorana neutrino can behave as a particle or antiparticle. Transitions between both states imply an helicity flip of order $(m/E)^2$

Why Majorana insight was important?



Majorana neutrinos and smallness of neutrino masses





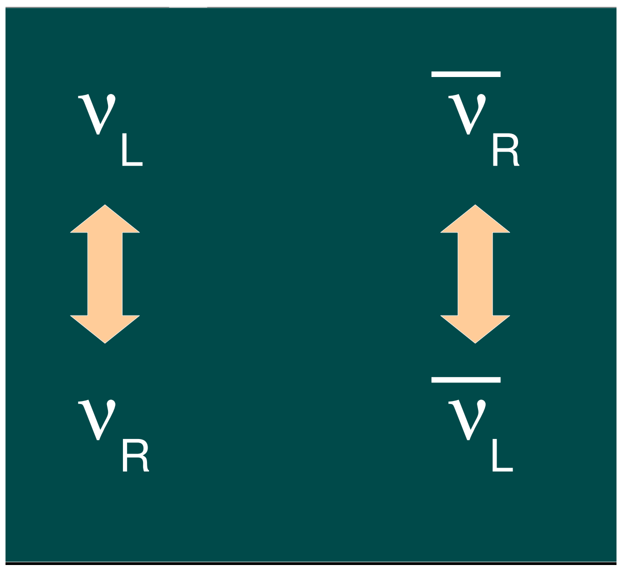
- Dirac masses are obtained by direct coupling to v

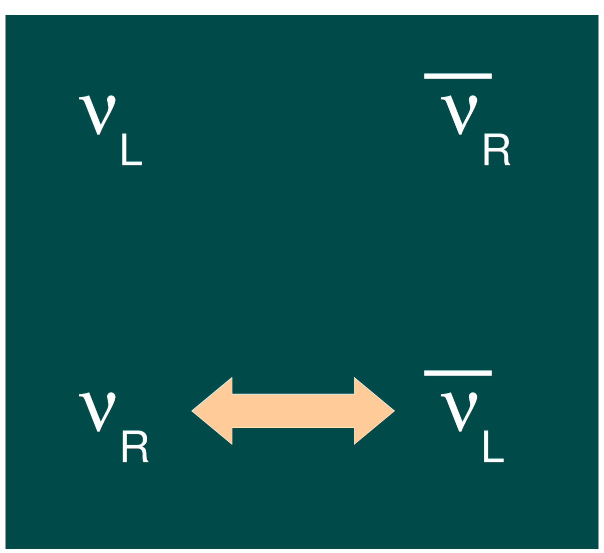
- Majorana masses couple to v^2 and therefore require an extra constant Λ which has the dimension of the mass.

- By making Λ large one can produce small masses for a Majorana lepton without fine-tuning the coupling constant λ.

See-saw mechanism

Dirac





Majorana

- We include in the Lagrangian both Majorana en Dirac terms

$$-\mathcal{L}_D = -m_D(\overline{\nu_L}\nu_R + \overline{\nu_R}\nu_L)$$

$$-\mathcal{L}_R = \frac{1}{2} m_R(\overline{\nu_R}(\nu_R)^c + \overline{(\nu_R)^c}\nu_R)$$

$$-\mathcal{L}_{\mathrm{D+R}} = \frac{1}{2}\overline{\mathcal{N}_L^c}\, M\, \mathcal{N}_L + \mathrm{h.c.}$$

$$M = \begin{pmatrix} 0 & m_{\mathrm{D}} \\ m_{\mathrm{D}} & m_R \end{pmatrix}$$

$$\mathcal{N}_L = \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$$

- ν_L and ν_R do not have a definite mass since they are coupled by the Dirac mass term

- We ned to find the states ν_{1L} and N_{1L} with definite masses m_1 and M.

Diagonalization of M

$$U^T M U = \begin{pmatrix} m_1 & 0 \\ 0 & M_1 \end{pmatrix} \qquad \mathcal{N}_L = U\, n_L\,, \qquad \text{with} \qquad n_L = \begin{pmatrix} \nu_{1L} \\ N_{1L} \end{pmatrix}$$

- Diagonalized mass terms

$$-\mathcal{L}_{D+R} = \frac{1}{2}\left(m_1\,\overline{(\nu_{1L})^c}\,\nu_{1L} + M_1\,\overline{(N_{1L})^c}\,N_{1L}\right) + \text{h.c.}\,,$$

- Pure Majorana term

$$-\mathcal{L}_R = \frac{1}{2} m_R(\overline{\nu_R}(\nu_R)^c + \overline{(\nu_R)^c}\nu_R)$$

- **So, we end up with two Majorana neutrinos**! The insertion of a Dirac mass term and a right handed Majorana mass term in the Lagrangian for massive neutrinos has resulted in Majorana particles

See-saw hypothesis

- No SM principle prevents m_R from being large
But we expect m_D to be of the same order of the mass of the quarks and charged leptons-

- Nothing prevents us from assuming $m_R >> m_D$

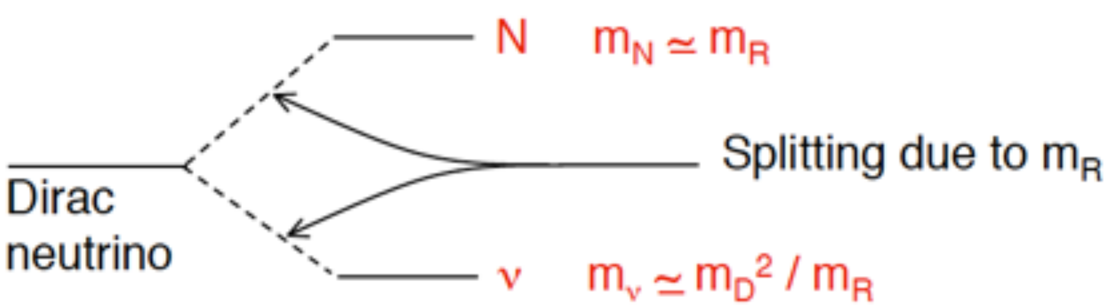
The see-saw limit

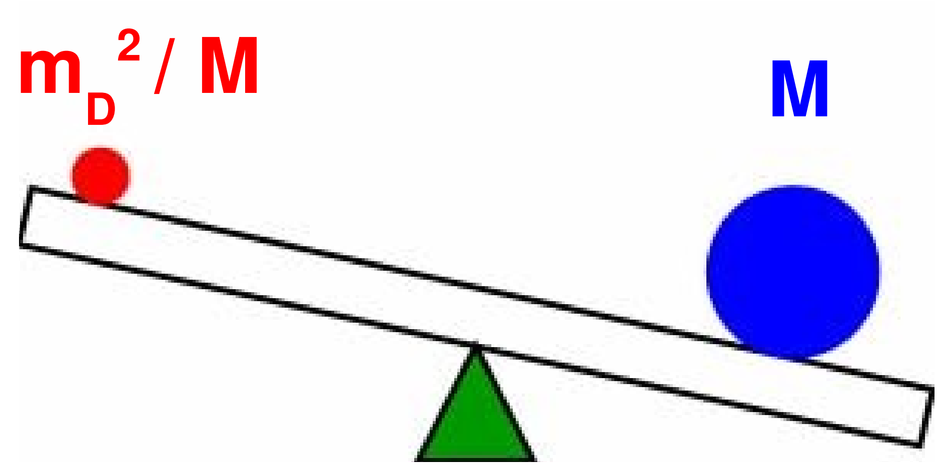
$$D_\nu \cong \begin{bmatrix} m_D{}^2/m_R & 0 \\ 0 & m_R \end{bmatrix}$$

$$m_1 \simeq m_D^2/m_R \ll |m_D|$$

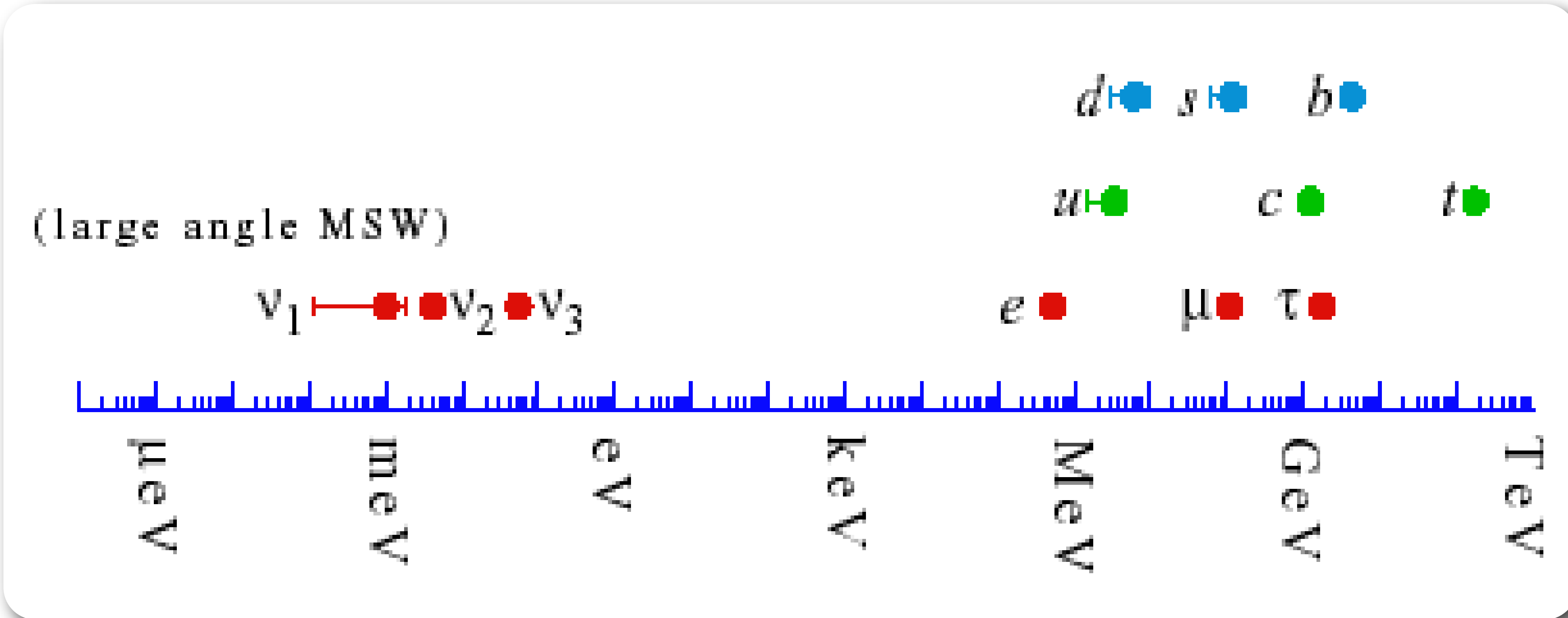
$$M_1 \simeq m_R.$$

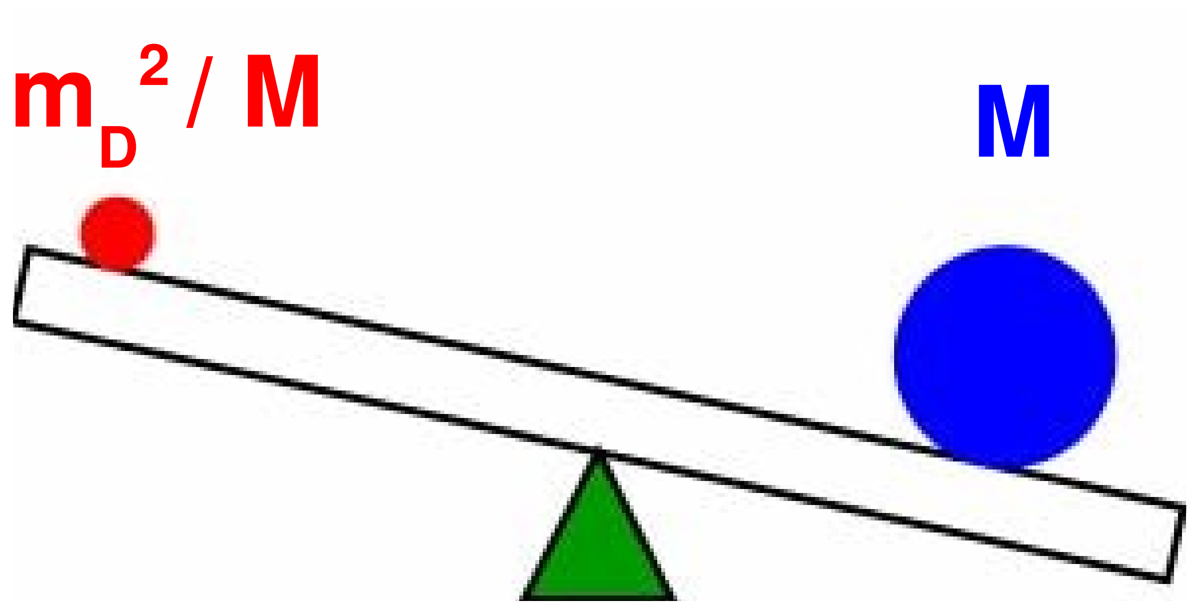
The Majorana mass term split a Dirac neutrino into two Majorana neutrinos.





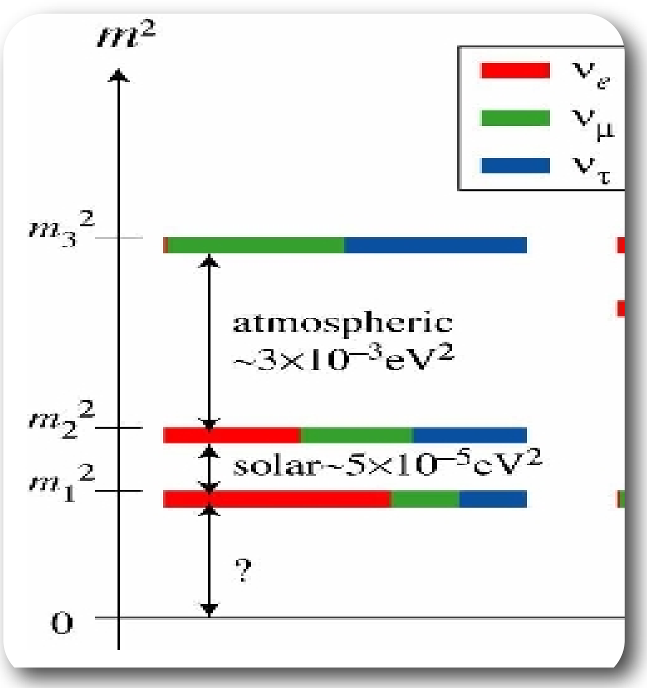
Majorana neutrinos explain smallness of neutrino masses





Do your research

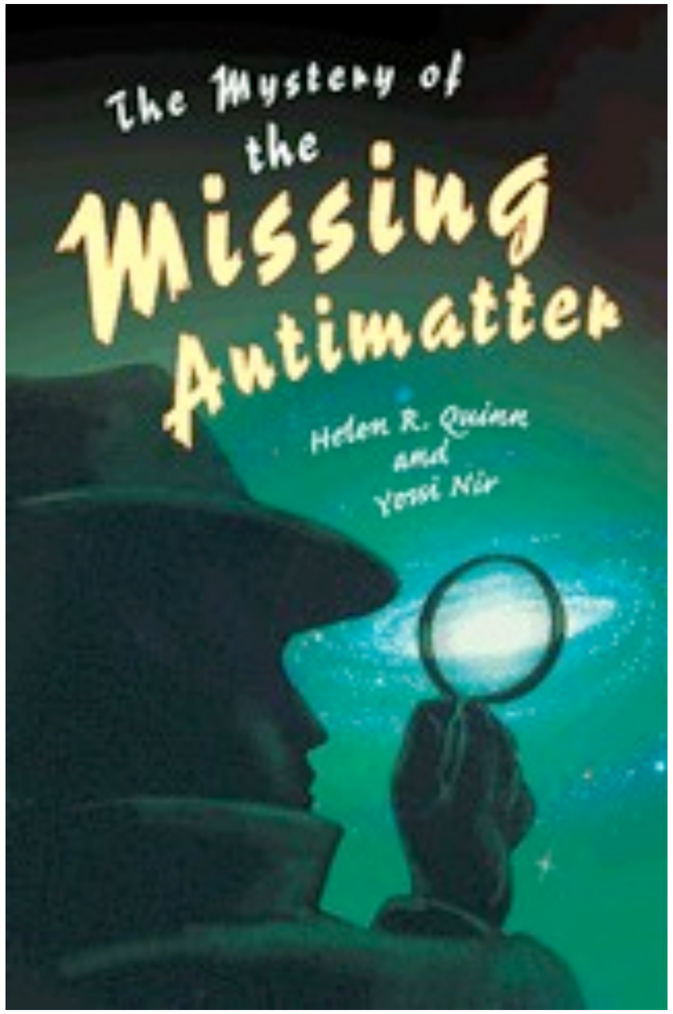
• The oscillation experiments tell you something about the mass of at least two neutrinos. Assuming that the lightest neutrino (ν1) has zero or close to zero mass, the solar split measures a very light mass for ν2 and the atmospheric split measures a mass of ~50 meV for ν3.

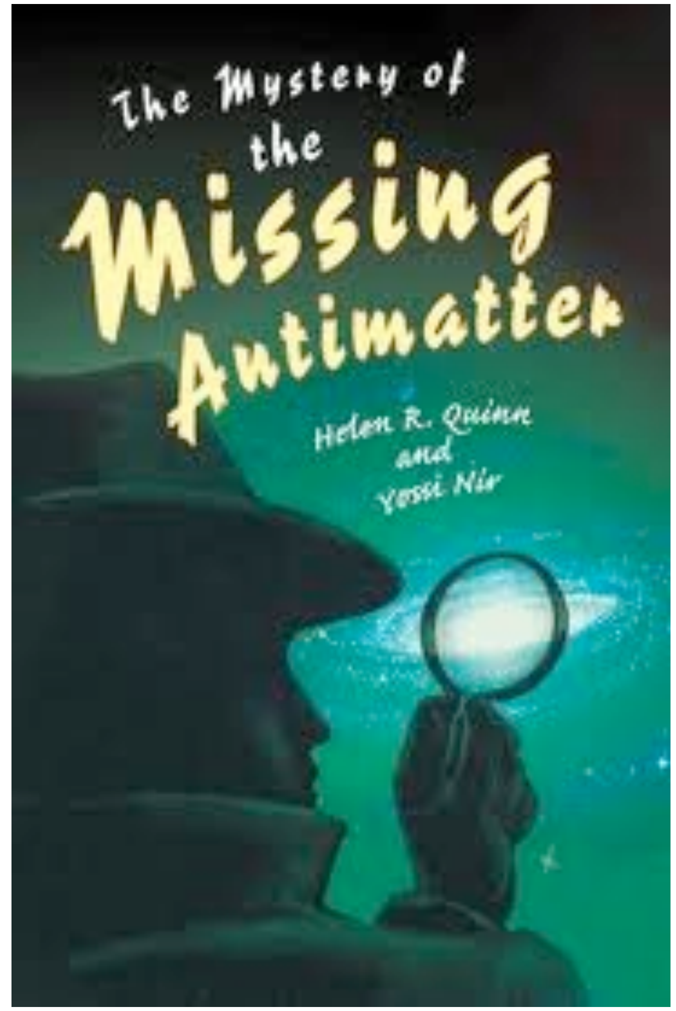


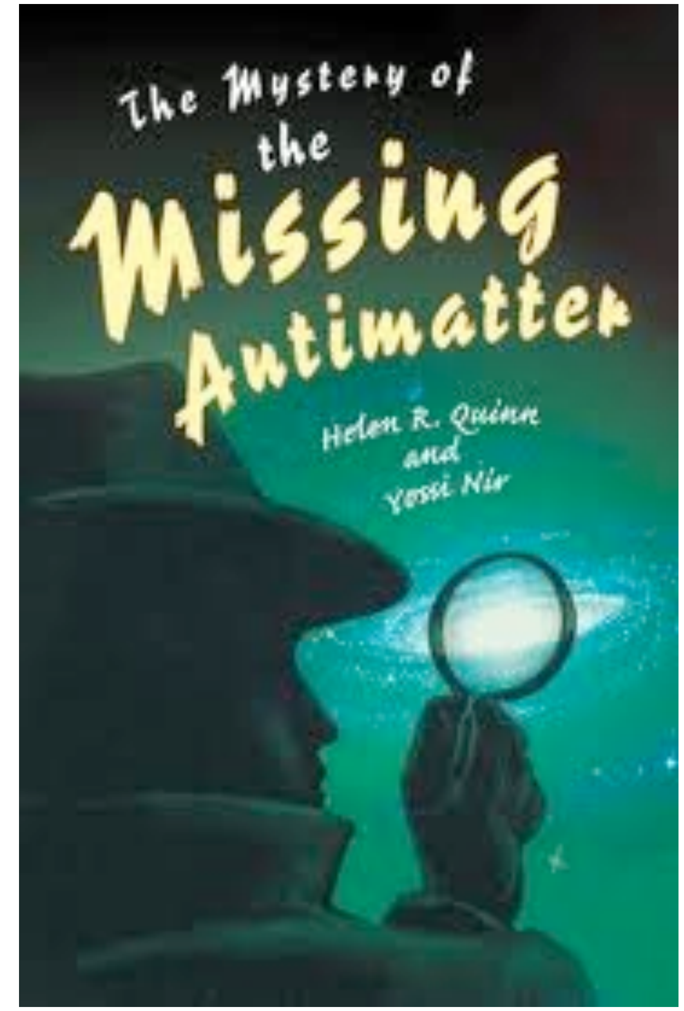
• What is the mass that see-saw predicts for N=νR in this case? what is the scale?

1) If you assume ml = me
2) If you assume ml = mtop

What happened to antimatter?







- The Big-Bang theory of the origin of the Universe requires matter and antimatter to be equally abundant at the very hot beginning

CP violation and Majorana neutrinos

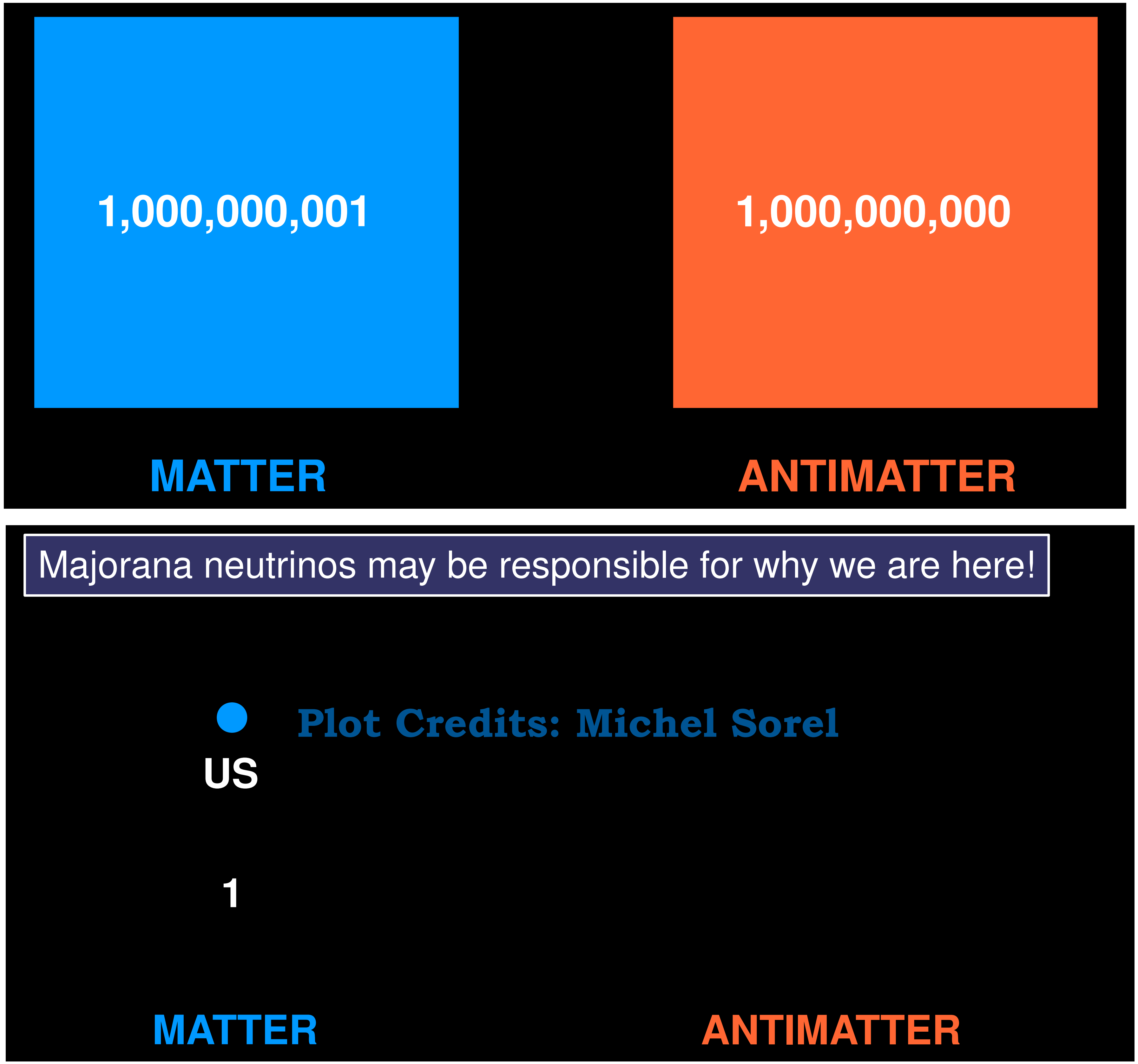
$$N \rightarrow e^- + H^+ \quad \text{and} \quad N \rightarrow e^+ + H^-$$

Standard-Model Higgs

- If there is CP violation in the lepton sector, the heavy Majorana neutrino N can violate CP too and decay with different rates to electrons and positrons. This results in an unequal number of leptons and antileptons in the early universe

- Leptonic asymmetry is later transferred to baryons, resulting in...

The reason why we are here...



CP violation + Majorana neutrinos

+ =

=Universe

CP violation + Majorana neutrinos

+  =

=Universe

CP violation + Majorana neutrinos

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CP violation + Majorana neutrinos

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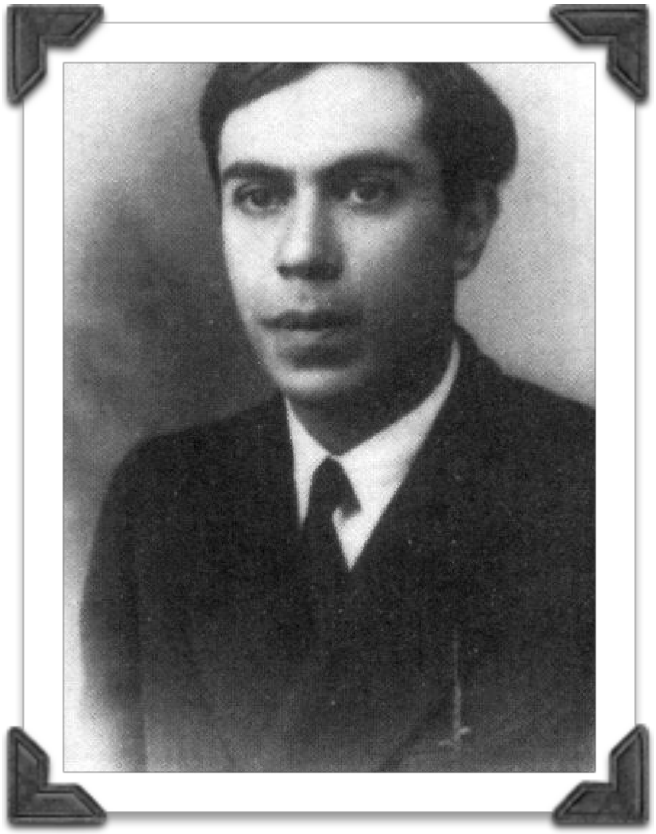
Can we demonstrate Majorana's insight to be true?



Do your research

- In this lectures I will sell you my favorite way of testing Majorana hypothesis (meaning: demonstrating that Majorana was right, because of course, he was...)

- But do you have any other ideas? How could you explore the Majorana nature of the neutrinos? Propose experiments.