

# A novel emittance measurement technique for the first analysis of the MICE muon beams

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on behalf of the international Muon Ionization Cooling Experiment

## Demonstrating the fast emittance reduction required at a Neutrino Factory

International Neutrino Summer School 2011

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### Ionization Cooling

- Reduce  $(p_x, p_y, p_z)$  in Liquid Hydrogen
- Replace lost  $p_z$  to maintain bunching

Absorber      RF Cavities

### Emittance (focus):

$$\epsilon_n = \frac{1}{mc} \sigma_x \sigma_{p_x} \propto \langle A \rangle$$

### Cooling:

$$\frac{d\epsilon_n}{dz} = -\frac{\epsilon_n}{\beta^2 E} \frac{dE}{dz} + \frac{\beta_\perp (13.6 \text{ MeV})^2}{2\beta^3 E m X_0}$$

### Larmor frame:

$$x(z) = \sqrt{A\beta(z)} \sin\psi(z)$$

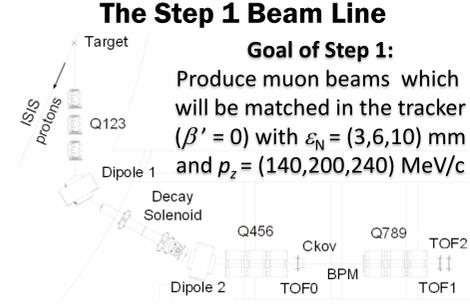
### The Scintillating Fibre Trackers

Reconstructs transverse phase space to 1/2 mm and 1 MeV by fitting helical muon trajectories

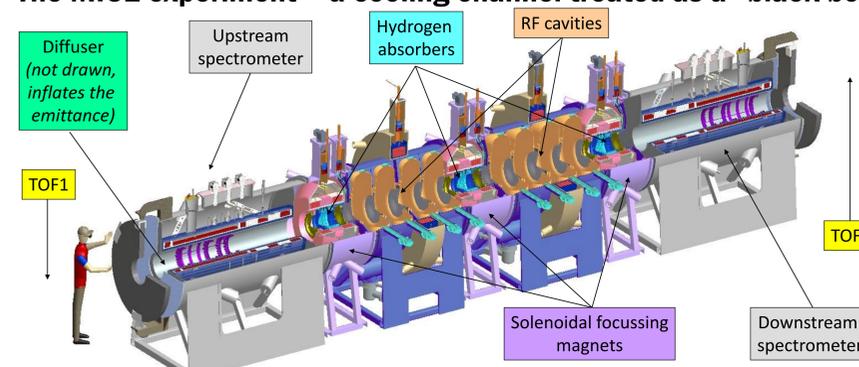


### The Step 1 Beam Line

Goal of Step 1: Produce muon beams which will be matched in the tracker ( $\beta' = 0$ ) with  $\epsilon_N = (3, 6, 10)$  mm and  $p_z = (140, 200, 240)$  MeV/c

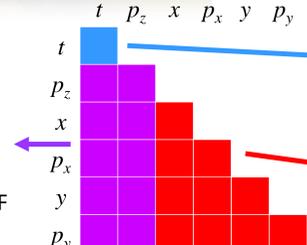


### The MICE experiment – a cooling channel treated as a “black box”



Diffuser (not drawn, inflates the emittance)      Upstream spectrometer      Hydrogen absorbers      RF cavities      Solenoidal focusing magnets      Downstream spectrometer

### The beam's covariance matrix from measurements of individual muons



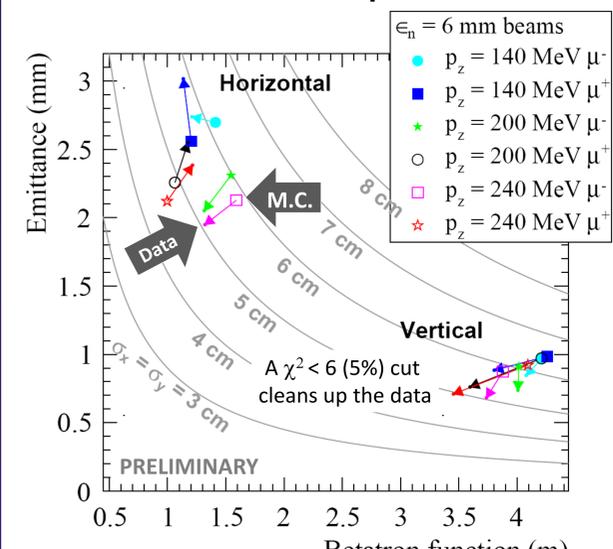
Longitudinal-transverse coupling: Matches the beam in the RF

Spread in time: Defined by TOF measurements of extrapolated RF phase

Transverse phase space: Measured upstream and downstream by the scintillating fibre trackers

## The first MICE beam measurements and a glimpse into the future

### The measured beams are quite well matched



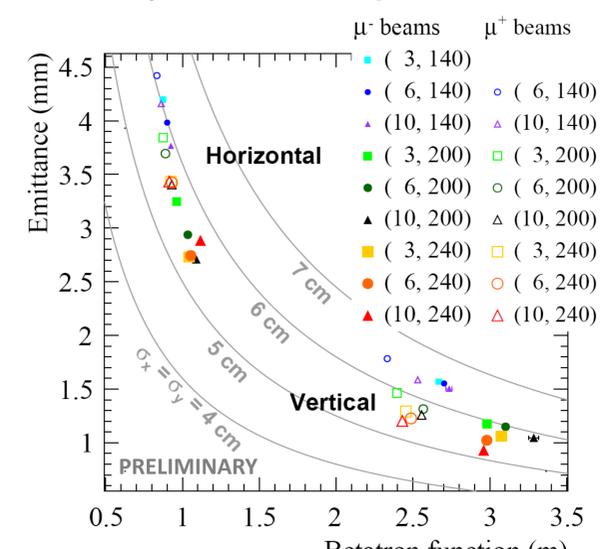
Horizontal      Vertical

$\epsilon_n = 6$  mm beams

- $p_z = 140$  MeV  $\mu^-$
- $p_z = 140$  MeV  $\mu^+$
- $p_z = 200$  MeV  $\mu^-$
- $p_z = 200$  MeV  $\mu^+$
- $p_z = 240$  MeV  $\mu^-$
- $p_z = 240$  MeV  $\mu^+$

A  $\chi^2 < 6$  (5%) cut cleans up the data

### A summary of the MICE Step 1 beam line data

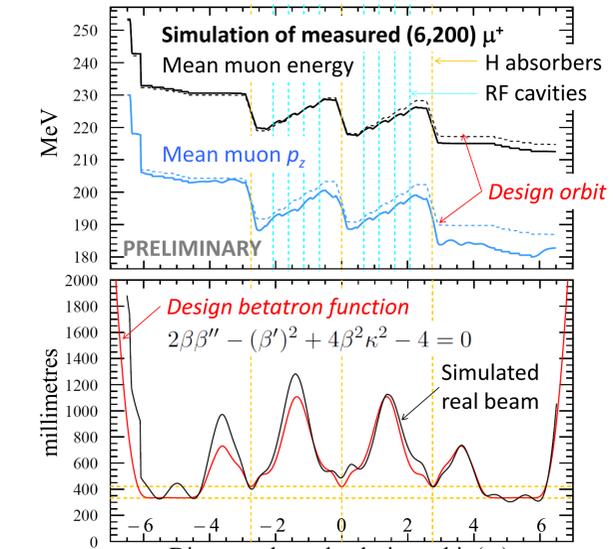


Horizontal      Vertical

$\mu^-$  beams       $\mu^+$  beams

- (3, 140)
- (6, 140)
- (10, 140)
- (3, 200)
- (6, 200)
- (10, 200)
- (3, 240)
- (6, 240)
- (10, 240)

### The beams should perform well in Step 6



Simulation of measured (6,200)  $\mu^+$

Mean muon energy      H absorbers      RF cavities

Mean muon  $p_z$

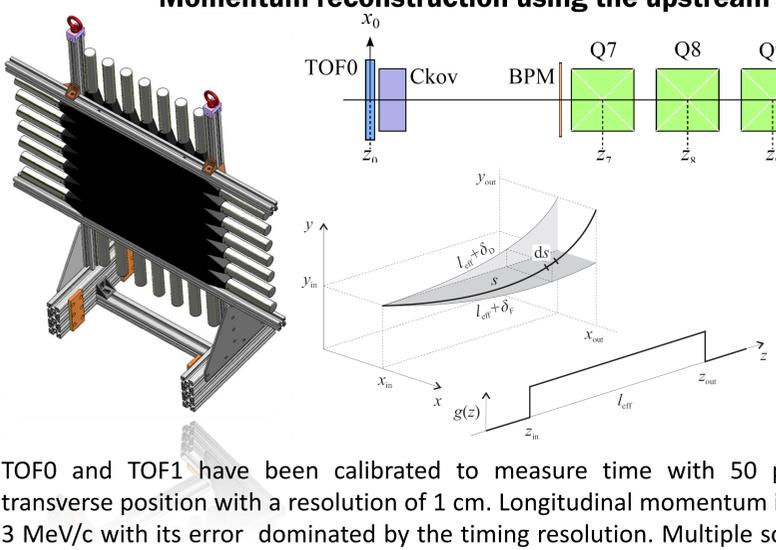
Design orbit

Design betatron function:  $2\beta\beta'' - (\beta')^2 + 4\beta^2\kappa^2 - 4 = 0$

Simulated real beam

## Reconstructing longitudinal momentum from the time of flight

### Momentum reconstruction using the upstream time of flight detectors



TOF0      Ckov      BPM      Q7      Q8      Q9      TOF1

Assume the path length  $s \approx z_{\text{TOF1}} - z_{\text{TOF0}}$

Estimate the momentum  $p/E = s/t$

10 iterations

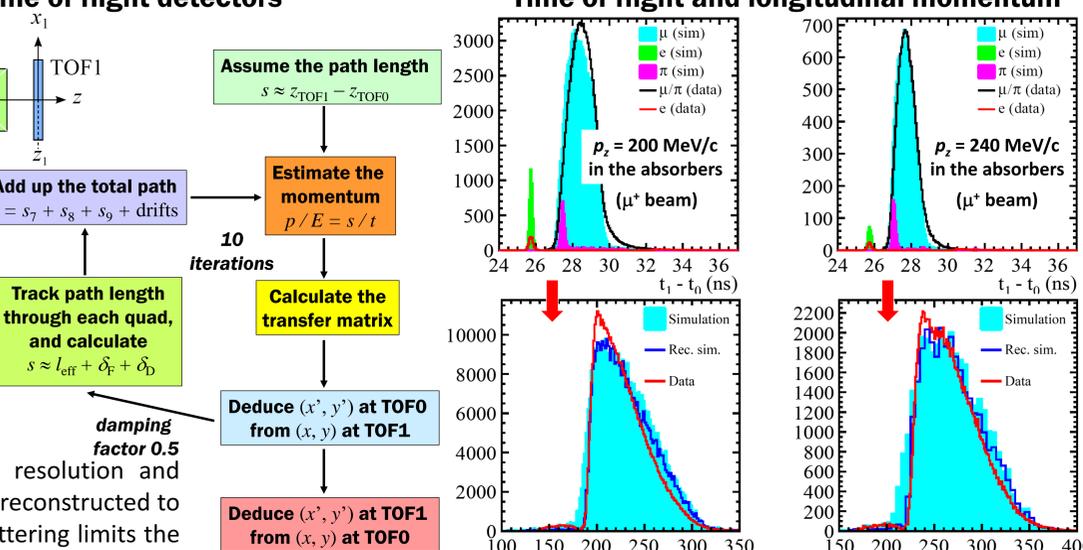
Calculate the transfer matrix

Deduce  $(x', y')$  at TOF0 from  $(x, y)$  at TOF1

Deduce  $(x', y')$  at TOF1 from  $(x, y)$  at TOF0

damping factor 0.5

### Time of flight and longitudinal momentum



$p_z = 200$  MeV/c in the absorbers ( $\mu^+$  beam)

$p_z = 240$  MeV/c in the absorbers ( $\mu^+$  beam)

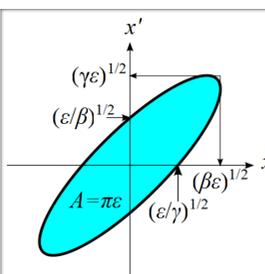
Simulation      Rec. sim.      Data

TOF0 and TOF1 have been calibrated to measure time with 50 ps resolution and transverse position with a resolution of 1 cm. Longitudinal momentum is reconstructed to 3 MeV/c with its error dominated by the timing resolution. Multiple scattering limits the reconstruction of the angles  $x' = dx/dz$  and  $y'$  to 10 mrad.

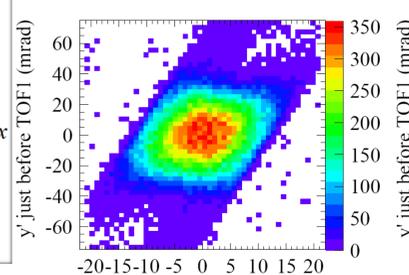
## A new single particle technique for reconstructing transverse trace space

### The covariance matrix is measured by reconstructing the trajectories of individual muons

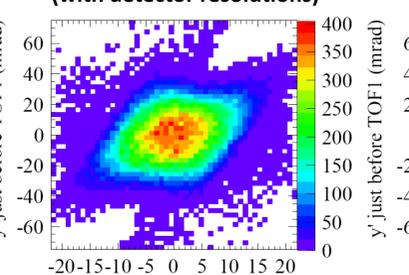
Emittance is traditionally measured using observations of a beam's RMS size in three or more profile monitors. A beam's covariance matrix  $\Sigma = \text{Cov}[x, x']$  is transformed as  $\Sigma_1 = M\Sigma_0M^T$  in the same fashion that position  $x$  and angle  $x' = dx/dz$  transform as  $(x_1, x_1') = M(p_z) \cdot (x_0, x_0')$ . One may solve for the  $\sigma_{xx'}$  and  $\sigma_{x'x'}$  given three or more measurements of  $\sigma_{xx}$  at different positions in the focusing channel. The emittance is given by  $\epsilon = (\det \Sigma)^{1/2}$ . This technique is not possible for beams with wide momentum spectra, as the transfer matrices vary too much in the quadrupoles. Instead,  $M(p_z)$  can be reconstructed individually for each muon, based on the a momentum measurement, and the angles deduced from the position measurements in TOF0 and TOF1.



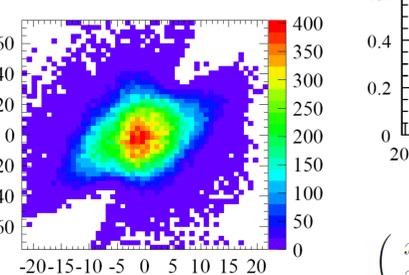
Simulation



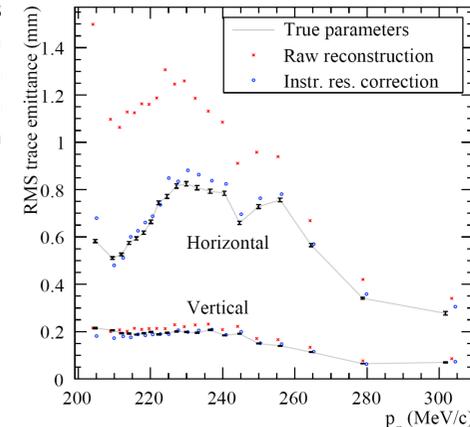
Reconstructed simulation (with detector resolutions)



Reconstructed data



### Emittance varies as a function of $p_z$



RMS trace emittance (mm)

True parameters      Raw reconstruction      Instr. res. correction

Horizontal      Vertical

The key formula

$$\begin{pmatrix} x_0' \\ x_1' \end{pmatrix} = \frac{1}{M_{12}} \begin{pmatrix} -M_{11} & 1 \\ -1 & M_{22} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \end{pmatrix}$$