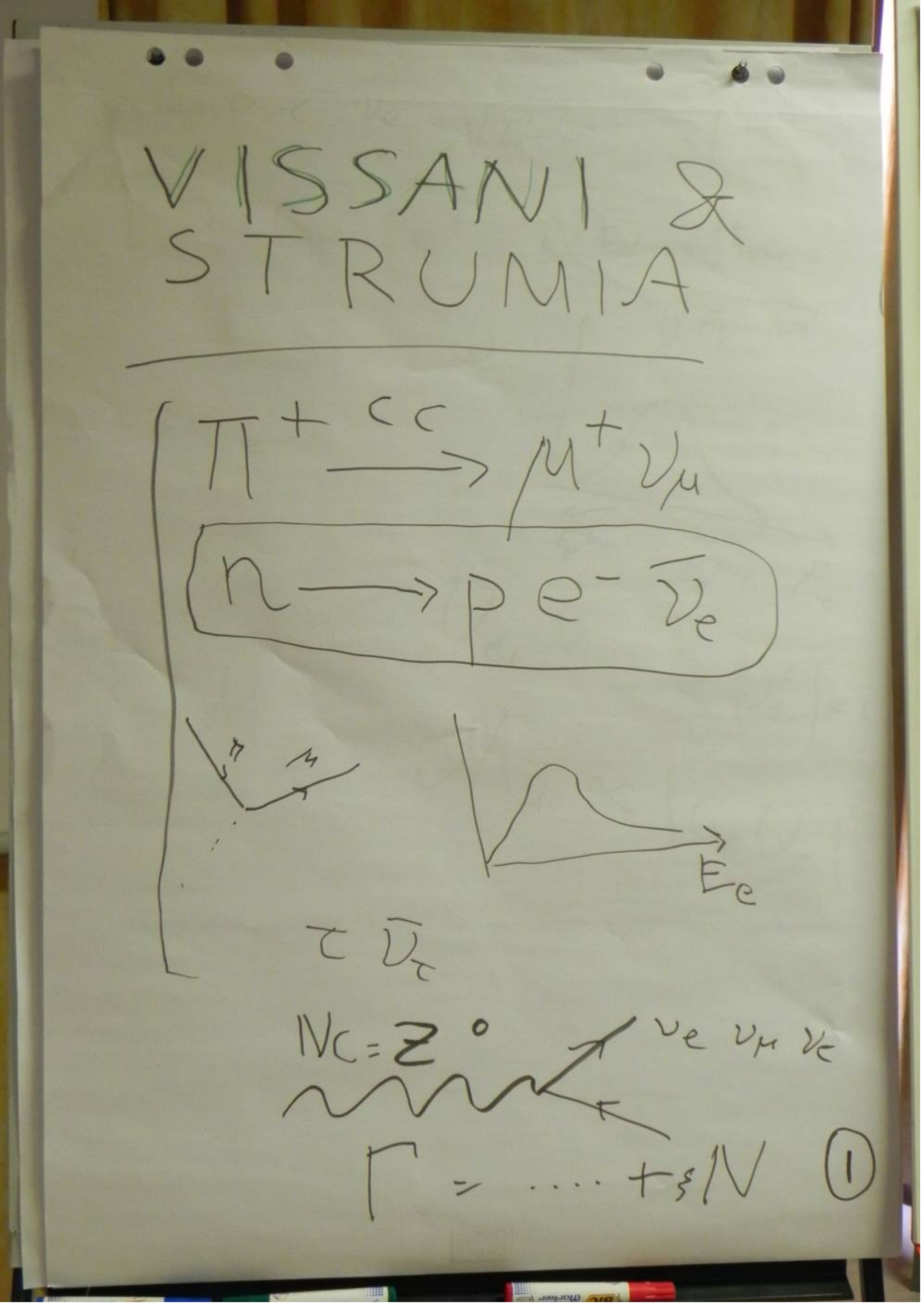
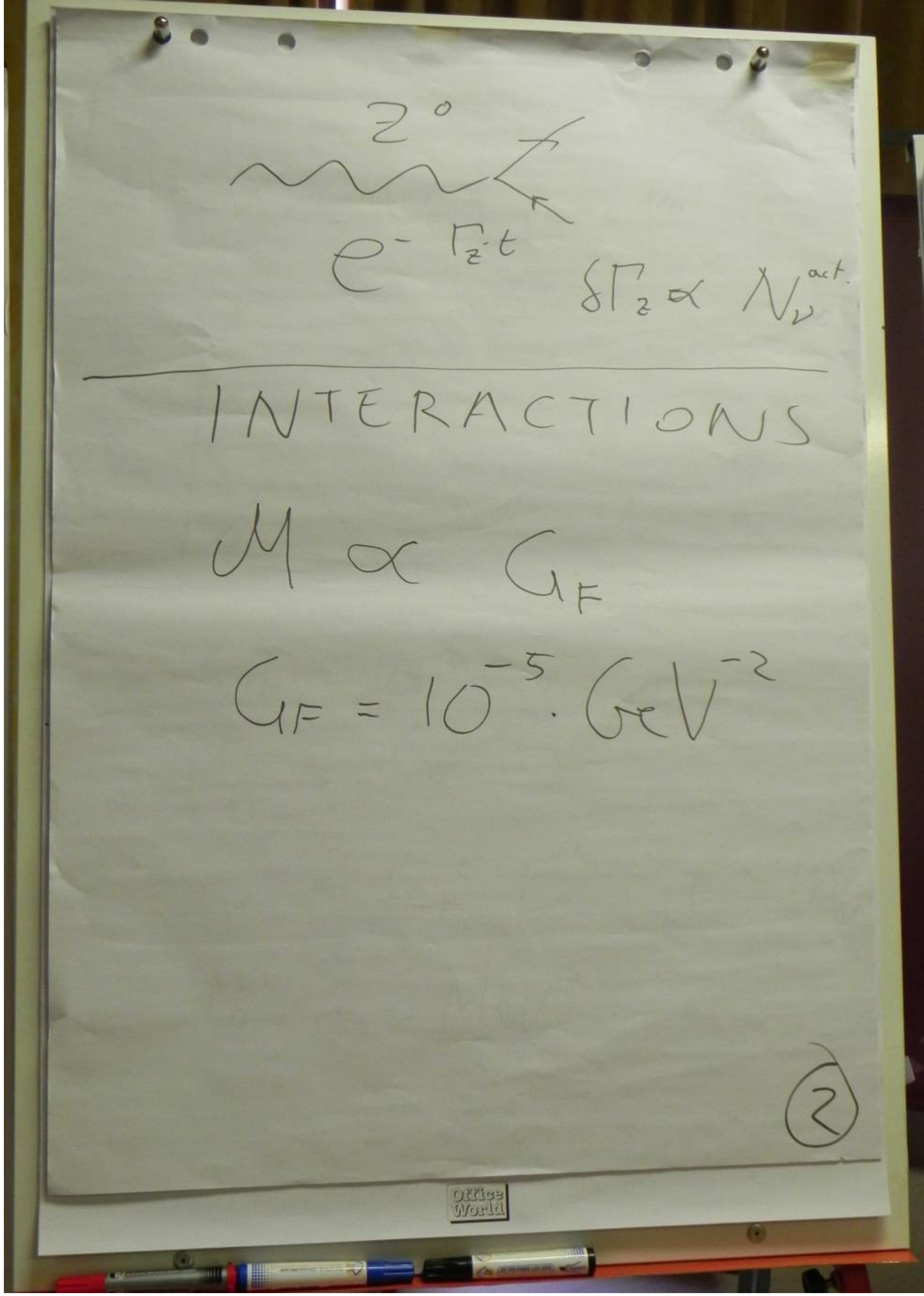

VISSANI &
STRUMIA
cc
$\pi^+ \longrightarrow \mu^+ \nu_\mu$
$n \longrightarrow p\, e^- \bar{\nu}_e$
E_e
$\tau\ \bar{\nu}_\tau$
$NC = Z^0$
$\nu_e\ \nu_\mu\ \nu_\tau$
$\Gamma = \dots + 3N$
1


Z^0
$e^{-\Gamma_Z \cdot t}$
$\delta\Gamma_Z \propto N_\nu^{act.}$
INTERACTIONS
$\mathcal{M} \propto G_F$
$G_F = 10^{-5} \cdot GeV^{-2}$
2

$$\boxed{\hbar = c = 1}$$

$$\Gamma_n \propto |M|^2 = G_F^2$$

$$G_F^2\, d^3p_p\, d^3p_e\, d^3p_\nu$$

$$\cdot\, \delta^4(p_n - p_p - p_e - p_e)$$

$$\Gamma = \frac{1}{T}$$

$$[M] = M^{-4}\ M^9\ \frac{1}{M^4}$$

$$G_F^2\, d^3p_e\, d^3p_\nu\, \delta(E_n - E_p - E_\nu - E_e)$$

$$\Gamma_n \propto G_F^2\ \cancel{d}\ p_e E_e dE_e \cdot p_\nu E_\nu dE_\nu$$

$$\delta(Q - E_\nu - E_e)$$

5

$$\Delta p\, \Delta x \sim \hbar \qquad mc^2 = E$$

$$[p] = \frac{1}{[x]} \qquad [E] = [p] = [m]$$

$$c = \frac{L}{T} \quad [L] = [T \qquad = \frac{1}{[L]} = \frac{1}{[T]}$$

$$d^3p = p^2 dp\, d\Omega$$

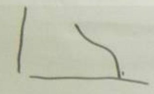
$$= p^2 dp = p E dE$$

$$p^2 + m^2 = E^2$$

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 \cdot M_W^2}$$

4

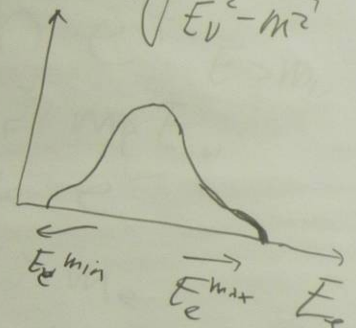
$n \to p\, e^- \bar{\nu}_e$



$$\Gamma_n \sim G_F^2\, \underbrace{p_e}_{\sqrt{E_e^2 - m^2}} E_e\, \underbrace{p_\nu}_{\sqrt{E_\nu^2 - m^2}} E_\nu \, dE_e$$

$E_\nu + E_e = Q$

Fermi



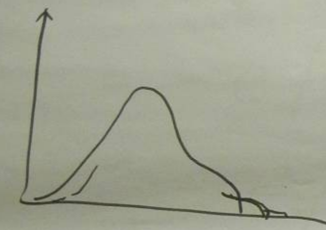
$$\bar{\nu}_e = \sum_i U_{ei}\, \bar{\nu}_i \qquad \sum_i |U_{ei}^2| = 1$$

$$\mathcal{M}_{\bar{\nu}_e} \qquad \mathcal{M}_{\bar{\nu}_i} \propto |G_F U_{ei}|^2$$

5

$$\frac{d\Gamma_n}{dE_e} \propto \sum |U_{ei}|^2\, p_e E_e\, \underbrace{p_{\nu_i}}_{\sqrt{E_\nu^2 - m_i^2}} E_{\nu_i}$$

$E_e + E_{\nu_i} = Q = m_n - m_p$



$U_{e1} \quad m_1$

$U_{e2} \quad m_2$

$G_F = 10^{-5}\, \mathrm{GeV}^{-2}$

$\Gamma_n \approx G_F^2\, Q^5$

$\sigma \approx G_F^2\, E^2 \qquad E < m_N$

$\Gamma_n / \sigma \sim \mathrm{MeV}^3$

6

$$n \to p\, e^-\, \bar{\nu}_e$$

$$\bar{\nu}_e\, p \to n\, e^+ \qquad E > m_n$$

$$\sigma \sim G_F^2\, m_p\, E_\nu$$

$$\nu_e\, e^- \to \nu_e\, e^-$$

$$E > m_e$$

$$\sigma \sim G_F^2\, m_e\, E_\nu$$

$$c = 3\; 10^{10}\ \mathrm{cm/s}$$

$$\hbar c = 200\ \mathrm{MeV \cdot fm}$$

$$1\ \mathrm{fm} = 10^{-13}\ \mathrm{cm}$$

7

$$\overset{1}{S} = \int \overset{M^{-4}}{d^4x}\ \overset{M^4}{\mathcal{L}(\psi, A)}$$

$$\delta S = 0 \qquad [S] = [\hbar] = 1$$

$$\mathcal{L}_{int} = -e A^\mu (\bar{\psi} \gamma_\mu \psi)$$

$$= -e A^\mu J_\mu$$

$$\mathcal{H}_{int} = -\mathcal{L}_{int} = e A^\mu J_\mu$$

$$= e\left[\Phi \rho - \vec{A} \cdot \vec{J}\right]$$

$$H = \int d^3x\ \cancel{E}\,\cancel{V}(e \phi \rho)$$

8

$$H = \frac{p^2}{2m}$$

$$H = e\phi + \frac{(\vec{p} - e\vec{A})^2}{2m}$$

$$(\vec{B}\ \vec{\mu})$$

$$H - e\phi = \frac{(\vec{p} - e\vec{A})^2}{2m}$$

$$\vec{p} = -i\vec{\nabla}$$

$$(E\ \ \vec{p}) \to (E - e\phi, \vec{p} - e\vec{A})$$

$$(\phi, \vec{A})$$

9

$$(i\partial\!\!\!/ - m)\psi = 0$$

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi$$

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - e\underline{A\!\!\!/} - m)\psi$$

$$\partial^\mu = \frac{\partial}{\partial x_\mu} \qquad \partial\!\!\!/ = \partial^\mu \gamma_\mu$$

$$\delta S = 0 \qquad \frac{\delta \mathcal{L}}{\delta \bar{\psi}} = \partial_\mu \frac{\delta \mathcal{L}}{\delta \partial_\mu \bar{\psi}}$$

$$\boxed{(i\partial\!\!\!/ - eA\!\!\!/ - m)\psi = 0}$$

10

$$(i\not{\partial} - m)\,\psi = 0$$

$$(i\gamma^\mu \partial_\mu - m)\,\psi = 0$$

$$H^2 = \hat{\vec{p}} + m^2 \qquad E^2 = p^2 + m^2$$

$$\hat{\vec{p}} = -i\vec{\nabla}$$

$$E = \pm\sqrt{\vec{p}^2 + m^2}$$

$$H = \vec{\alpha}\,\hat{\vec{p}} + \beta m$$

$$H^2 = \alpha_i \alpha_j \hat{p}_i \hat{p}_j + \beta^2 m^2 + (\alpha_i \beta + \beta \alpha_i)\,\hat{p}_i m = \hat{\vec{p}}^2 + m^2$$

11

$$\begin{cases} \alpha_i \beta + \beta \alpha_i = 0 \\ \beta^2 = 1 \\ \alpha_i \alpha_j + \alpha_j \alpha_i = \delta_{ij} \end{cases}$$

$$\sigma_i \sigma_j + \sigma_j \sigma_i = \delta_{ij}$$

$$\beta = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \qquad \vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$$

12

$$H = \vec{\alpha}\,\vec{p} + \beta m$$

$$(\beta p^0 - \beta\vec{\alpha}\,\vec{p} - m)\psi = 0$$

$$(\gamma^0 p^0 - \vec{\gamma}\,\vec{p} - m)\psi = 0$$

$$(\not{p} - m)\psi = 0$$

$$S = \int \bar{\psi}(i\not{\partial} - m)\psi$$

$$\boxed{\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}\cdot 1_{4\times 4}}$$

$$g^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$$

13

HALZEN-MARTIN

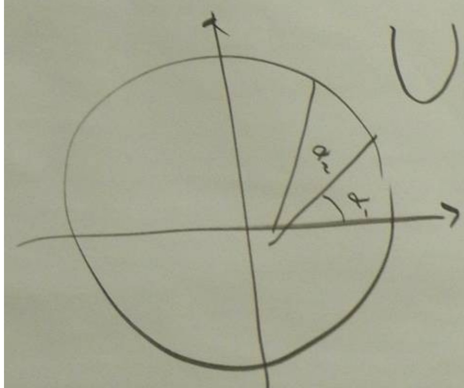
$$\mathcal{L} = \bar{\psi}(i\not{\partial} - m)\psi$$

$$S = \int d^4x\,\mathcal{L} \qquad p^\mu = i\partial^\mu \longrightarrow p^\mu - eA^\mu$$

$$\mathcal{L} = \bar{\psi}(\not{p} - e\not{A} - m)\psi = 0$$

$$\begin{cases} A_\mu \longrightarrow A_\mu + \partial_\mu\alpha(x) \\ \psi \longrightarrow e^{ie\alpha(x)}\psi \end{cases}$$

$$U(1) = \{e^{i\alpha}\}$$



$$e^{i\alpha_1}\cdot e^{i\alpha_2} = e^{i\alpha_{1+}}$$

14

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\partial_\mu(\partial_\nu \alpha) - \partial_\nu(\partial_\mu \alpha) = 0$$

SU(2) [YANG MILLS]

$$\begin{cases} \psi \longrightarrow e^{i \frac{\vec{\sigma}\vec{\alpha}(x)}{2}} \psi \\ A \longrightarrow \ ?\ \ldots \end{cases}$$

$$\psi = \begin{pmatrix} \nu \\ e \end{pmatrix}$$

15

HEISEMBERG

$$\psi = \begin{pmatrix} p \\ n \end{pmatrix}$$

$$\mathcal{L}_{YM} = \bar{\psi}\left(i\not\gamma - g\frac{\vec{\sigma}}{2}\vec{\not A}\right)\psi$$

$$\mathcal{L}_{QED} = \bar{\varphi}\left(i\not\gamma - e\not A\right)\varphi$$

ELECT. FIELD

PHOTON

$$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$

16

$$\frac{g}{2}\begin{pmatrix} \not{A}_3 & \not{A}_1 - i\not{A}_2 \\ \not{A}_1 + i\not{A}_2 & -\not{A}_3 \end{pmatrix}$$ OUT OF DIAG ↓

$$(\bar{\nu}, \bar{e})\,\frac{g}{2}\begin{pmatrix} \not{A}_3 & \not{A}_1 - i\not{A}_2 \\ \not{A}_1 + i\not{A}_2 & -\not{A}_3 \end{pmatrix}\begin{pmatrix} \nu \\ e \end{pmatrix}$$

$$-\frac{g}{2}\,\bar{\nu}\left(\not{A}_1 - i\not{A}_2\right)e = -\frac{g}{\sqrt{2}}\,\bar{\nu}\gamma^\mu e \cdot W^+_\mu$$

$$W^+ = \frac{A_1 - A_2}{\sqrt{2}}$$

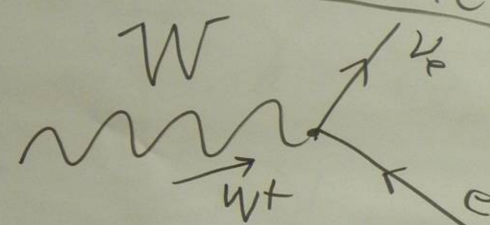
17

$$\mathcal{L}_{W} = -\frac{g}{\sqrt{2}}\left[\left(\bar{\nu}_e \gamma^\mu e\right)\cdot W^+_\mu\right]$$

(1) $e \to e_L$, $\nu_e \to \nu_{eL}$

[LATER EXPL.]

(2) USE THIS WITH FEY. RULES



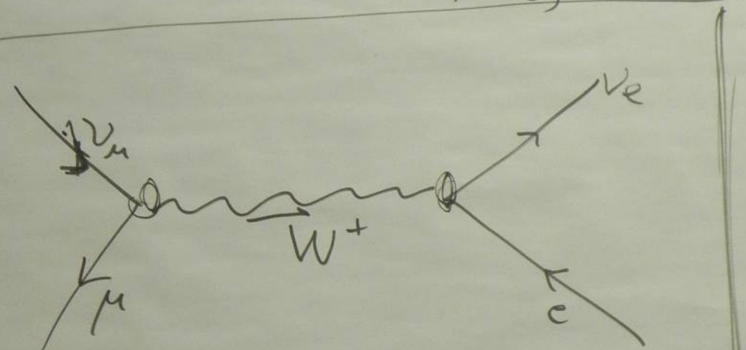
$$= -i\,\frac{g}{\sqrt{2}}\,\gamma^\mu P_L$$

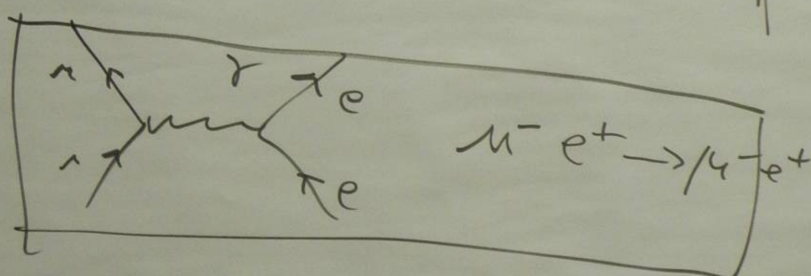
18

UNIVERSALITY

$$+ \cancel{g}(\bar{\nu}_\mu \gamma^\mu \mu) + (\bar{\nu}_\tau \gamma^\mu \tau)\Big] W_\mu$$

$$+ \text{“}\bar{u}\gamma^\mu d\text{”} \qquad \frac{|N_{ij}|^2}{E_i - E_j}$$





(19)

$$\mu^+ \longrightarrow \bar{\nu}_\mu \, e^+ \, \nu_e$$

$\mathcal{M} \propto G_F$

PHENOM.

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 M_W^2}$$

THEORY

$g = ?$

$$\frac{e^2}{4\pi} = \frac{1}{137}$$

(20)

$$\mathcal{L} = \bar{\psi}_e \left(i\not\partial - g\,\frac{\vec{\sigma}\cdot\vec{\not A}}{2} \right)\psi_e$$

$$\bar{\psi}_\mu \left(i\not\partial - g\,\frac{\vec{\sigma}\cdot\vec{\not A}}{2} \right)\psi_\mu$$

$$\bar{e}\left(i\not\partial - Q_e \not A \right) e$$

$$\bar{\mu}\left(i\not\partial - Q_\mu \not A \right) \mu$$

$$\bar{\nu}_e\left(i\not\partial - Q_{\nu_e} \not A \right) \nu_e$$

$$n \to p\, e\, \nu$$

21

SU(5)

$$5 \quad \begin{pmatrix} d^c_1 \\ d^c_2 \\ d^c_3 \\ \nu \\ e \end{pmatrix} \qquad \begin{array}{c} 1/3 \\ 1/3 \\ 1/3 \\ 0 \\ -1 \\ \hline 0 \end{array}$$

$$\left(\begin{array}{ccc|cc} 1/3 & & & & \\ & 1/3 & & & \\ & & 1/3 & & \\ \hline & & & 0 & \\ & & & & -1 \end{array}\right)$$

22