CP Violation

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Find the magnitudes of all |Uαi|

PMNS MATRIX

$$\begin{Bmatrix} \nu 1 \\ \nu 2 \\ \nu 3 \end{Bmatrix} = \begin{Bmatrix} Ue1 & U\mu 1 & U\tau 1 \\ Ue2 & U\mu 2 & U\tau 2 \\ Ue3 & U\mu 3 & U\tau 3 \end{Bmatrix} \times \begin{Bmatrix} \nu e \\ \nu \mu \\ \nu \tau \end{Bmatrix}$$

MATRIX IS UNITARY

$\sum_\alpha |U_{\alpha i}|^2 = 1, \alpha = e, \mu, \tau$

$\sum_i |U_{\alpha i}|^2 = 1, i=1,2,3$

GIVEN THAT

$$\begin{Bmatrix} Ue1 & U\mu 1 & U\tau 1 \\ Ue2 & \sqrt{0.33} & \sqrt{0.33} \\ Ue3 & 0.7 & 0.7 \end{Bmatrix}$$

AND USING UNITARITY RELATIONS

$|U_{e3}|^2 = 1 \quad - (|U_{\mu 3}|^2 + |U_{\tau 3}|^2)$

$|U_{e2}|^2 = 1 \quad - (|U_{\mu 2}|^2 + |U_{\tau 2}|^2)$

$|U_{\tau 1}|^2 = 1 \quad - (|U_{\tau 2}|^2 + |U_{\tau 3}|^2)$

$|U_{\mu 1}|^2 = 1 \quad - (|U_{\mu 2}|^2 + |U_{\mu 3}|^2)$

$|U_{e1}|^2 = 1 \quad - (|U_{e2}|^2 + |U_{e3}|^2)$

$|U_{e1}|^2 = 1 \quad - (|U_{\mu 1}|^2 + |U_{\tau 3}|^2)$

Find the magnitudes of all |Uαi|

Solving the equations

$|U_{e3}|^2 = 1 - (|U_{\mu 3}|^2 + |U_{\tau 3}|^2) = 0.02$

$|U_{e2}|^2 = 1 - (|U_{\mu 2}|^2 + |U_{\tau 2}|^2) = 0.34$

$|U_{\tau 1}|^2 = 1 - (|U_{\tau 2}|^2 + |U_{\tau 3}|^2) = 0.18$

$|U_{\mu 1}|^2 = 1 - (|U_{\mu 2}|^2 + |U_{\mu 3}|^2) = 0.18$

$|U_{e1}|^2 = 1 - (|U_{e2}|^2 + |U_{e3}|^2) = 0.64$

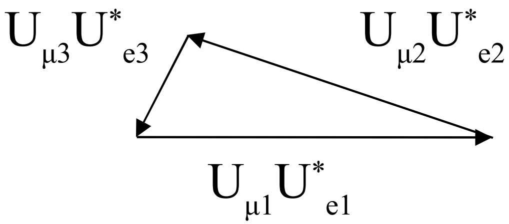
$|U_{e1}|^2 = 1 - (|U_{\mu 1}|^2 + |U_{\tau 3}|^2) = 0.64$

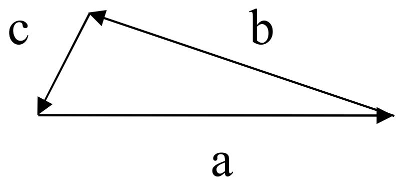
You get the matrix

PMNS MATRIX

$$\begin{Bmatrix} \nu 1 \\ \nu 2 \\ \nu 3 \end{Bmatrix} = \begin{Bmatrix} 0.8 & \sqrt{0.18} & \sqrt{0.18} \\ \sqrt{0.34} & \sqrt{0.33} & \sqrt{0.33} \\ \sqrt{0.02} & 0.7 & 0.7 \end{Bmatrix} \times \begin{Bmatrix} \nu e \\ \nu \mu \\ \nu \tau \end{Bmatrix}$$

Show that the area of the triangle is not zero and find its value





$a = |U_{\mu 1}U^*_{e1}| = 0.339$

$b = |U_{\mu 2}U^*_{e2}| = 0.335$

$c = |U_{\mu 3}U^*_{e3}| = 0.099$

We use the heron's formula

$A = (s * (s-a) * (s-b) * (s-c))^{1/2}$ where $s = (a+b+c)/2 = 0.3865$,

THEN

$|A_{\mu e}| = 0.016$

Show that the CP-violating difference is $\pm 32 \cdot |A_{\mu e}| \cdot \sin\Delta_{31} \cdot \sin\Delta_{21} \cdot \sin\Delta_{32}$

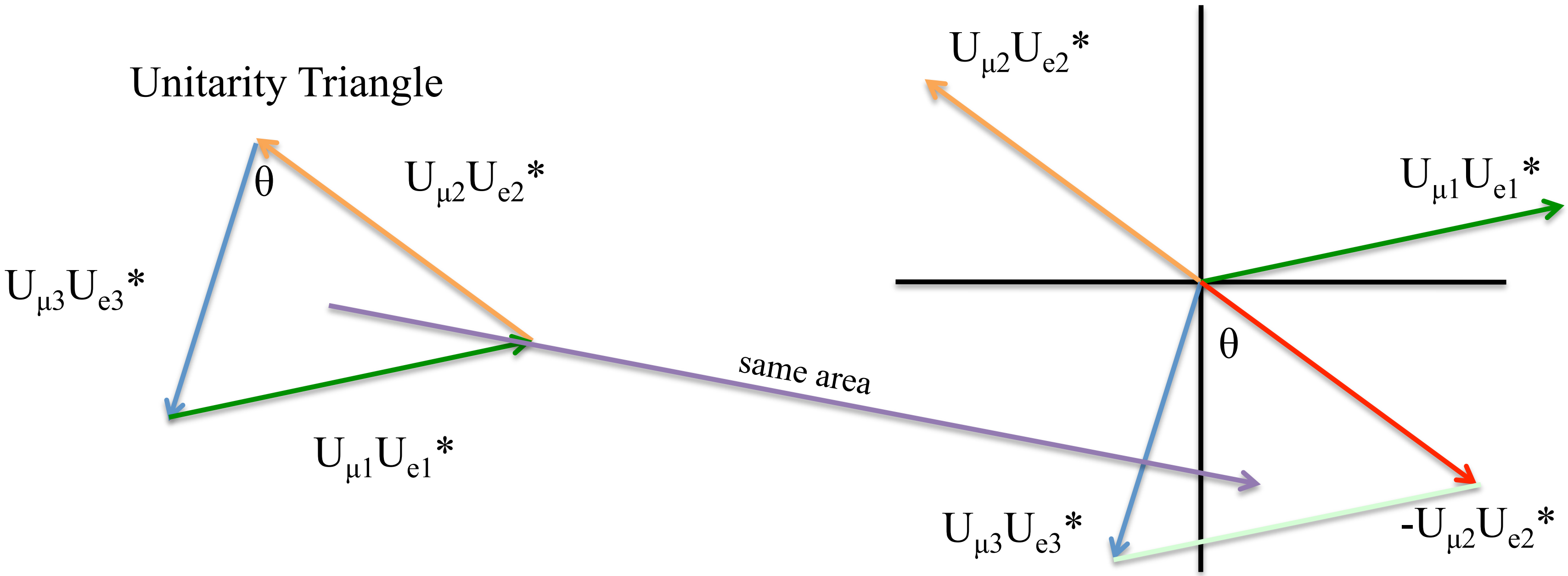
$A = P(\nu_\mu \rightarrow \nu_e) = 4[|U_{\mu 3}U_{e3}|^2 \sin^2\Delta_{31} + |U_{\mu 2}U_{e2}|^2 \sin^2\Delta_{21} + 2||\sin\Delta_{31}\sin\Delta_{21}\cos(\Delta_{32}+\delta_{32})]$

$B = P(\overline{\nu}_\mu \rightarrow \overline{\nu}_e) = 4[|U_{\mu 3}U_{e3}|^2 \sin^2\Delta_{31} + |U_{\mu 2}U_{e2}|^2 \sin^2\Delta_{21} + 2||\sin\Delta_{31}\sin\Delta_{21}\cos(\Delta_{32}-\delta_{32})]$

$A - B = \pm 8 \cdot |U_{\mu 3}U_{e3}^*| \cdot |U_{\mu 2}U_{e2}^*| \cdot \sin\Delta_{31} \cdot \sin\Delta_{21} \cdot [\cos(\Delta_{32} + \delta_{32}) - \cos(\Delta_{32} - \delta_{32})]$

To find the CP-violating difference, we need to first find $|A_{\mu e}|$…

*In the relations above, $\Delta_{ij} = \Delta m_{ij}^2 L/4E$ and $\delta_{32} = \arg(U_{\mu 3}U_{e3}^* U_{\mu 2}^* U_{e2})$



In polar form:

$$U_{\mu 2}U_{e2}^* = |U_{\mu 2}U_{e2}^*| \cdot \exp\{i \cdot \arg(U_{\mu 2}U_{e2}^*)\}$$

$$-U_{\mu 2}U_{e2}^* = |-U_{\mu 2}U_{e2}^*| \cdot \exp\{i \cdot \arg(-U_{\mu 2}U_{e2}^*)\} = |U_{\mu 2}U_{e2}^*| \cdot \exp\{i \cdot [\pi + \arg(U_{\mu 2}U_{e2}^*)]\}$$

$$U_{\mu 3}U_{e3}^* = |U_{\mu 3}U_{e3}^*| \cdot \exp\{i \cdot \arg(U_{\mu 3}U_{e3}^*)\}$$

The angle θ is then:

$|z| = |-z|$, $\arg(-z) = \pi + \arg(z)$, $\arg(z^*) = -\arg(z)$, $|z^*| = |z|$, $|zw| = |wz|$

$$\theta = \arg(-U_{\mu 2}U_{e2}^*) - \arg(U_{\mu 3}U_{e3}^*)$$

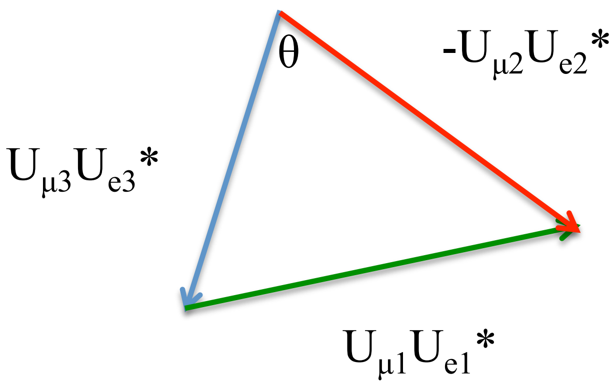
$$\theta = \pi + \arg(U_{\mu 2}U_{e2}^*) - \arg(U_{\mu 3}U_{e3}^*)$$

$$\theta = \pi - \arg(U_{e2}U_{\mu 2}^*) - \arg(U_{\mu 3}U_{e3}^*) = \pi - \arg(U_{\mu 3}U_{e3}^*) - \arg(U_{\mu 2}^*U_{e2})$$

$$\theta = \pi - \arg(U_{\mu 3}U_{e3}^*U_{\mu 2}^*U_{e2})$$

$$\theta = \pi - \delta_{32}$$

Unitarity Triangle



Now that we have the side lengths and angle θ, we get the area of the unitarity triangle as…

$$|A_{\mu e}| = \tfrac{1}{2}\,|U_{\mu 3}U_{e3}^*| \cdot |-U_{\mu 2}U_{e2}^*| \cdot |\sin(\pi - \delta_{32})|$$
$$= \tfrac{1}{2}\,|U_{\mu 3}U_{e3}^*| \cdot |-U_{\mu 2}U_{e2}^*| \cdot \sin\delta_{32}$$

$\cos(a + b) - \cos(a - b) = -2 \cdot \sin a \cdot \sin b$

The CP violating difference is then

$$A - B = \pm 8 \cdot |U_{\mu 3}U_{e3}^*| \cdot |U_{\mu 2}U_{e2}^*| \cdot \sin\Delta_{31} \cdot \sin\Delta_{21} \cdot [\cos(\Delta_{32} + \delta_{32}) - \cos(\Delta_{32} - \delta_{32})]$$
$$= \pm 16 \cdot |U_{\mu 3}U_{e3}^*| \cdot |U_{\mu 2}U_{e2}^*| \cdot \sin\Delta_{31} \cdot \sin\Delta_{21} \cdot \sin\Delta_{32} \cdot \sin\delta_{32}$$
$$= \pm 32 \cdot |A_{\mu e}| \cdot \sin\Delta_{31} \cdot \sin\Delta_{21} \cdot \sin\Delta_{32},$$

where the sign of this quantity depends on which of the oscillations is more likely.

Using the numbers given in (a), $|A_{\mu e}|$ is nonzero, suggesting CP violation.