Problem #8

Group 1

When there are only 3 flavors, there is only one independent CP-violating term.

- CPT invariance implies that

$$P(\nu_\alpha \rightarrow \nu_\beta) = P(\overline{\nu_\beta} \rightarrow \overline{\nu_\alpha}).$$

- The CP violating difference between corresponding neutrino & anti neutrino oscillation

$$\Delta_{\alpha\beta} = P(\nu_\alpha \rightarrow \nu_\beta) - P(\overline{\nu_\alpha} \rightarrow \overline{\nu_\beta}).$$

- CPT invariance implies $\Delta_{\alpha\alpha} = 0$,i.e., there can not be a CP violating difference between the probabilities that a neutrino and its anti neutrino survive with their original flavors.
- So, CP violation is possible only in $\Delta_{\alpha\beta}$ with $\alpha \neq \beta$,i.e. , in the appearance of a new flavor of neutrino ν_β in a beam of neutrinos born as ν_α.

Task: To show that the six possible non zero $\Delta_{\alpha\beta}$ ($\alpha\neq\beta$) are all equal apart from a sign:

$$\Delta_{e\mu} = \Delta_{\mu\tau} = \Delta_{\tau e} = -\Delta_{\mu e} = -\Delta_{\tau\mu} = -\Delta_{e\tau} \equiv \Delta$$

- $|\nu_\alpha (0) > = \Sigma_{j=1}{}^{3} [\ U_{\alpha j}|\nu_j\ >]$
- $|\nu_\alpha (t) > = \Sigma_{j=1}{}^{3} [\ U_{\alpha j}|\nu_j (t) >]$

  $= \Sigma_{j=1}{}^{3}[U_{\alpha j}|\nu_j\ > e^{-i m_j{}^2 L/2E}\ ]$ (propagating in time)
- Projection on flavor eigen state $|\nu_\beta >$ is

  $$< \nu_\beta|\nu_\alpha (t) > = \Sigma_{j=1}{}^{3}[U_{\beta j}{}^{*}\ U_{\alpha j}\ e^{-i m_j{}^2 L/2E}]$$
- The probability of appearance of ν_β from initial ν_α is,

$$P_{\alpha\beta} = |< \nu_\beta|\nu_\alpha (t) > |^2$$

$$= \Sigma_{j=1}{}^{3}[\ |U_{\beta j}{}^{*}\ U_{\alpha j}\ |^2]$$

$$+ [|U_{\beta 1}{}^{*}\ U_{\alpha 1}\ U_{\beta 2}\ U_{\alpha 2}{}^{*} e^{-i[m_1{}^2 - m_2{}^2]L/2E} + \text{complex conjugate}]$$

$+ \ldots$

- $P_{\alpha\beta}$ can be simplified to :

$$P_{\alpha\beta} = \delta_{\alpha\beta} - 4\sum_{i>j} Re\ (\square_{\alpha\beta ij}) sin^2(\Delta_{ji}) - 2\sum_{i>j} Im(\square_{\alpha\beta ij}) \sin(2\Delta_{ji})$$

Where, $\square_{\alpha\beta ij} = U_{\alpha i}\, U_{\beta j} U_{\alpha j}{}^{*} U_{\beta i}{}^{*}$ (plaquette)

$$\Delta_{ji} = [m_j{}^2 - m_i{}^2] L/4E$$

- So, $\Delta_{\alpha\beta} = P_{\alpha\beta} - \overline{P_{\alpha\beta}}$

$$= P_{\alpha\beta} - P_{\beta\alpha} \text{ (Assuming CPT invariance)}$$

$$= \underbrace{4\sum_{i>j}[\mathrm{Re}(\square_{\beta\alpha ij}) - Re\ (\square_{\alpha\beta ij})] sin^2(\Delta_{ji})}_{=0} +$$

$$2\sum_{i>j}[\mathrm{Im}(\square_{\beta\alpha ij}) - Im(\square_{\alpha\beta ij})] \sin(2\Delta_{ji})$$

The CP violating term!

The U_{PMNS} matrix:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} e^{i\chi_1} & 0 & 0 \\ 0 & e^{i\chi_2} & 0 \\ 0 & 0 & e^{i\chi_3} \end{pmatrix} \cdot \widetilde{U}_{PMNS} \cdot \begin{pmatrix} e^{i\phi_1} & 0 & 0 \\ 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

- If we rotate the phases of flavor eigen states to zero,
- If we rotate the phases of mass eigen states to zero.
- Then the U_{PMNS} matrix is ,

$$\widetilde{U}_{PMNS} \equiv R_{23}(\theta_{23}) U_{13}(\theta_{13}, \delta) R_{12}(\theta_{12}) =$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13} \end{pmatrix}$$

$c_i \equiv \cos\theta_i,\ s_i \equiv \sin\theta_i$

Calculation of $\Delta_{\alpha\beta}$: $\Delta_{e\mu}$

- $\square_{e\mu 12} = c_{12}\, s_{12} c_{13}{}^2 [-c_{12}\, s_{12} c_{23}{}^2 - c_{12}{}^2\, c_{23}\, s_{23} s_{13} e^{-i\delta} + s_{12}{}^2\, c_{23}\, s_{23} s_{13} e^{i\delta} + c_{12}\, s_{12} s_{23}{}^2 s_{13}{}^2$
- $\square_{e\mu 13} = c_{12}\, s_{13} c_{13}{}^2 s_{23} [-s_{12} c_{23} e^{i\delta} - c_{12}\, s_{13} s_{23}]$
- $\square_{e\mu 23} = s_{12}\, s_{13} c_{13}{}^2 s_{23} [c_{12} c_{23} e^{i\delta} - c_{12}\, s_{13} s_{23}]$

- $$\square_{\mu e 1\,2} = s_{12} c_{13}^2 c_{12} [-c_{12} c_{23}^2 s_{12} + s_{12}^2 c_{23} s_{23} s_{13} e^{-i\delta}$$
  $$- c_{12}^2 c_{23} s_{23} s_{13} e^{i\delta} + c_{12} s_{23}^2 s_{13}^2 s_{12}]$$
- $$\square_{\mu e 1\,3} = s_{23} c_{12} c_{13}^2 s_{13} (-s_{12} c_{23} e^{-i\delta} - c_{12} s_{23} s_{13})$$
- $$\square_{\mu e 2\,3} = c_{13}^2 s_{12} s_{23} s_{13} (c_{12} c_{23} e^{-i\delta} - s_{12} s_{23} s_{13})$$

- We get,

$\Delta_{e\mu}$ = - $\Delta_{\mu e}$

$= -2s_{12}\ s_{13}c_{13}{}^{2}s_{23}c_{12}\ c_{23}\sin\delta\ [\sin(2\Delta_{21}) - \sin(2\Delta_{31})$

$+\sin(2\Delta_{32})] \equiv \Delta$

Proceeding similarly,

$\Delta_{\mu\tau} = -\Delta_{\tau\mu} \equiv \Delta$

& $\Delta_{\tau e} = -\Delta_{e\tau} \equiv \Delta$.