# 0νββ Majorana Mass & CP-Violating Majorana Phases

## Group 9

International Neutrino Summer School 2011

26/07/11

# Question a):

- Find the expression for the effective Majorana mass for 0vββ under the following assumptions:
  - Inverted mass spectrum ($m_1 > m_3$)
  - $\theta_{13} = 0$
  - $m_1 = m_2$
- Experimentally, what is the smallest possible value for this effective Majorana mass?

# Effective Majorana Mass

- Assuming 3 flavours and that 0vββ is dominated by the light-neutrino exchange mechanism we find that:

$$\left[T_{1/2}^{0v}\right]^{-1} = G^{0v}(Q_{\beta\beta}, Z)\left|M^{0v}\right|^{2}\left\langle m_{\beta\beta}\right\rangle^{2}$$

where

$$\left\langle m_{\beta\beta}\right\rangle = \left|\sum_{i} m_i U_{ei}^2\right| = \left|m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\alpha} + m_3 s_{13}^2 e^{2i\beta}\right|$$

and

$$\alpha = \alpha_2 - \alpha_1 \qquad \beta = -\delta - \alpha_1$$

# Effective Majorana Mass

$$\langle m_{\beta\beta} \rangle = \left| m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\alpha} + m_3 s_{13}^2 e^{2i\beta} \right|$$

# Effective Majorana Mass

$$\langle m_{\beta\beta} \rangle = \left| m_1 c_{12}^2 \cancelto{1}{c_{13}^2} + m_2 s_{12}^2 \cancelto{1}{c_{13}^2} e^{2i\alpha} + m_3 \cancelto{0}{s_{13}^2} e^{2i\beta} \right|$$

$\theta_{13} = 0$ :

$$\langle m_{\beta\beta} \rangle = \left| m_1 c_{12}^2 + m_2 s_{12}^2 e^{2i\alpha} \right|$$

# Effective Majorana Mass

$$\langle m_{\beta\beta} \rangle = \left| m_1 c_{12}^2 \cancelto{1}{c_{13}^2} + m_2 s_{12}^2 \cancelto{1}{c_{13}^2} e^{2i\alpha} + m_3 \cancelto{0}{s_{13}^2} e^{2i\beta} \right|$$

$\theta_{13} = 0$ :

$$\langle m_{\beta\beta} \rangle = \left| m_1 c_{12}^2 + m_2 s_{12}^2 e^{2i\alpha} \right|$$

$m_1 = m_2$ :

$$\langle m_{\beta\beta} \rangle = m_1 \left| c_{12}^2 + s_{12}^2 e^{2i\alpha} \right|$$

# Smallest possible < $m_{\beta\beta}$ >

$$\langle m_{\beta\beta} \rangle = m_1 \left| c_{12}^2 + s_{12}^2 e^{2i\alpha} \right|$$

$\alpha = \pi / 2$ :

$$\langle m_{\beta\beta} \rangle = m_1 \left| c_{12}^2 - s_{12}^2 \right| = m_1 \left| 1 - 2 s_{12}^2 \right|$$

Inverted hierarchy:

$$m_1 = \sqrt{m_3 + \Delta m_{13}^2}$$

So for small $<m_{\beta\beta}>$ we want $m_3 = 0$, small $\Delta m^2{}_{13}$ and large $\sin^2\theta_{12}$

# Smallest possible < $m_{\beta\beta}$ >

| Measurement Level | $m_1$ / meV | $\sin^2\theta_{12}$ | $\langle m_{\beta\beta} \rangle$ / meV |
|---|---|---|---|
| Best Fit | 49.5 | 0.316 | 18.2 |
| 2σ | 47.7 | 0.35 | 14.3 |
| 3σ | 46.7 | 0.37 | 12.1 |

These should be compared to typical sensitivities of next-generation $0\nu\beta\beta$ experiments of ~ 30 - 100 meV.

# Question b):

- What would it take to establish the presence of a non-vanishing Majorana phase?

- Is it realistic to imagine we could actually do what is necessary?

# Measuring Majorana CP Phases

$$\langle m_{\beta\beta} \rangle = \left| m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\alpha} + m_3 s_{13}^2 e^{2i\beta} \right|$$

- 0vββ experiments can be sensitive to these phases.
- However, from separate measurements we also need to know:
  - Mixing angles
  - Mass splittings
  - A different measure of absolute neutrino mass (e.g. from KATRIN)

# Measuring Majorana CP Phases

- To answer this question, we considered the four CP-conserving values of $\langle m_{\beta\beta} \rangle$ (assuming 3ν flavours):

$$\left\langle m_{\beta\beta} \right\rangle_1 = \left| m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 + m_3 s_{13}^2 \right|$$

$$\left\langle m_{\beta\beta} \right\rangle_2 = \left| m_1 c_{12}^2 c_{13}^2 - m_2 s_{12}^2 c_{13}^2 + m_3 s_{13}^2 \right|$$

$$\left\langle m_{\beta\beta} \right\rangle_3 = \left| m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 - m_3 s_{13}^2 \right|$$

$$\left\langle m_{\beta\beta} \right\rangle_4 = \left| m_1 c_{12}^2 c_{13}^2 - m_2 s_{12}^2 c_{13}^2 - m_3 s_{13}^2 \right|$$

- We also considered the case $\delta \neq 0$ and found that it had little effect on the following analysis...

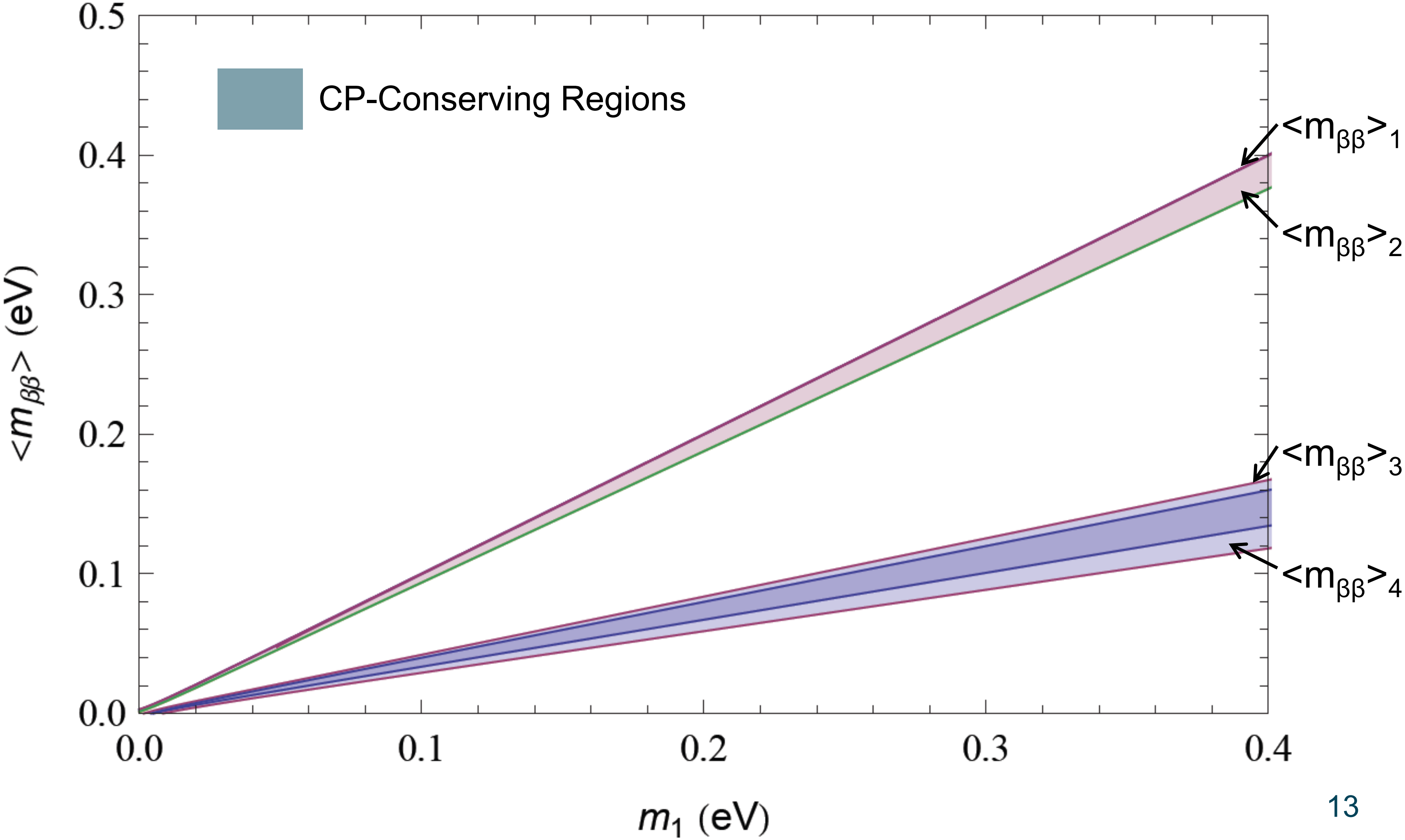
# Measuring Majorana CP Phases

- In the following we assumed:
  - No experimental error on $\Delta m^2$
  - Experimental error from mixing angles.

- Then, plotting the four CP-conserving values as a function of lightest neutrino mass with variation of mixing angles yields a graphical representation of the measurement potential for CP-violation...

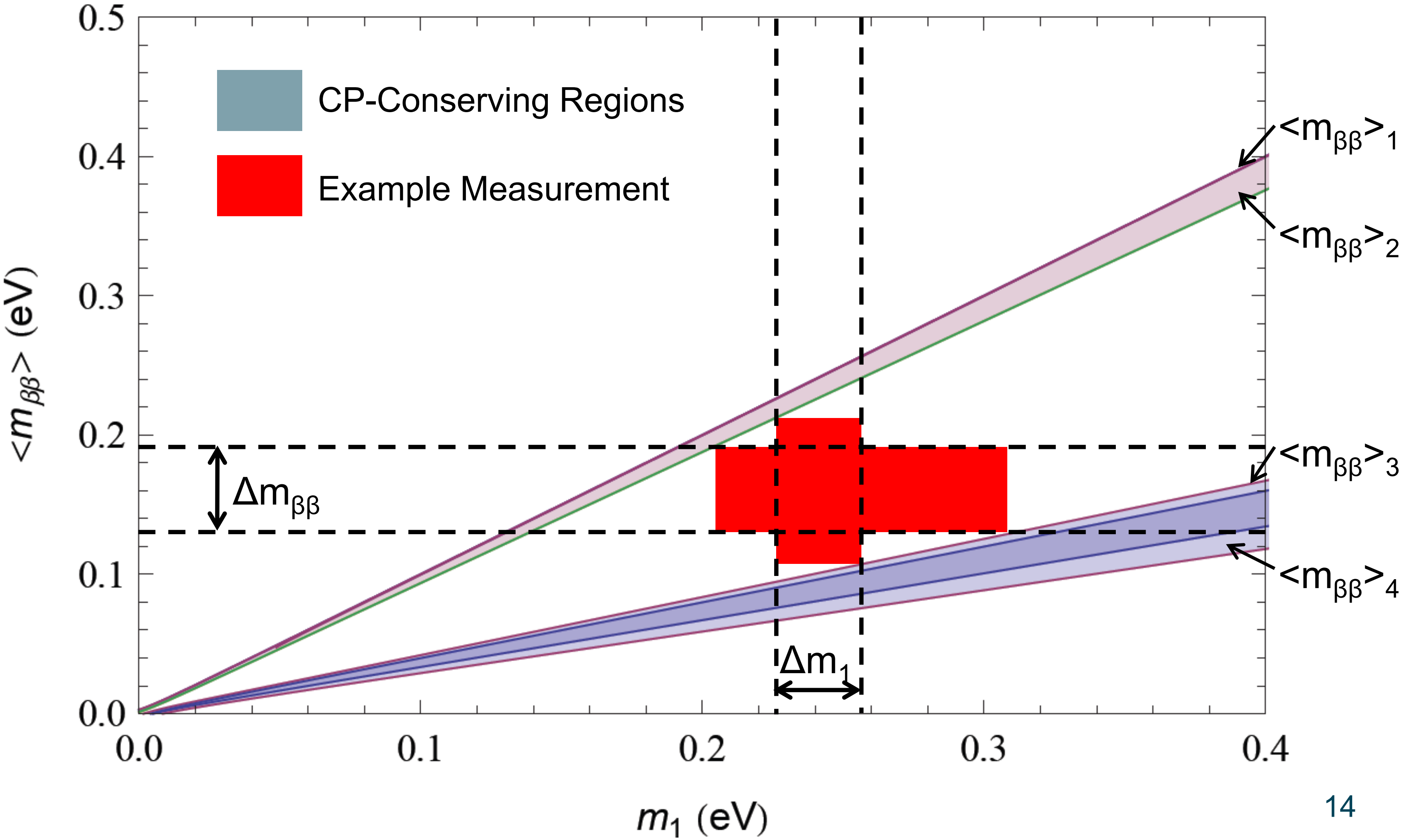
# Measuring Majorana CP Phases

- **Normal Hierarchy** with $\sigma(\Delta m^2) = 0$



# Measuring Majorana CP Phases

- **Normal Hierarchy** with $\sigma(\Delta m^2) = 0$



# Extracting α and β from a measurement

- So it looks like we should be able to measure non-vanishing Majorana phases via 0νββ in the future as long as $m_1$ is large enough

- But is it possible to extract α and β?

- Consider the following:

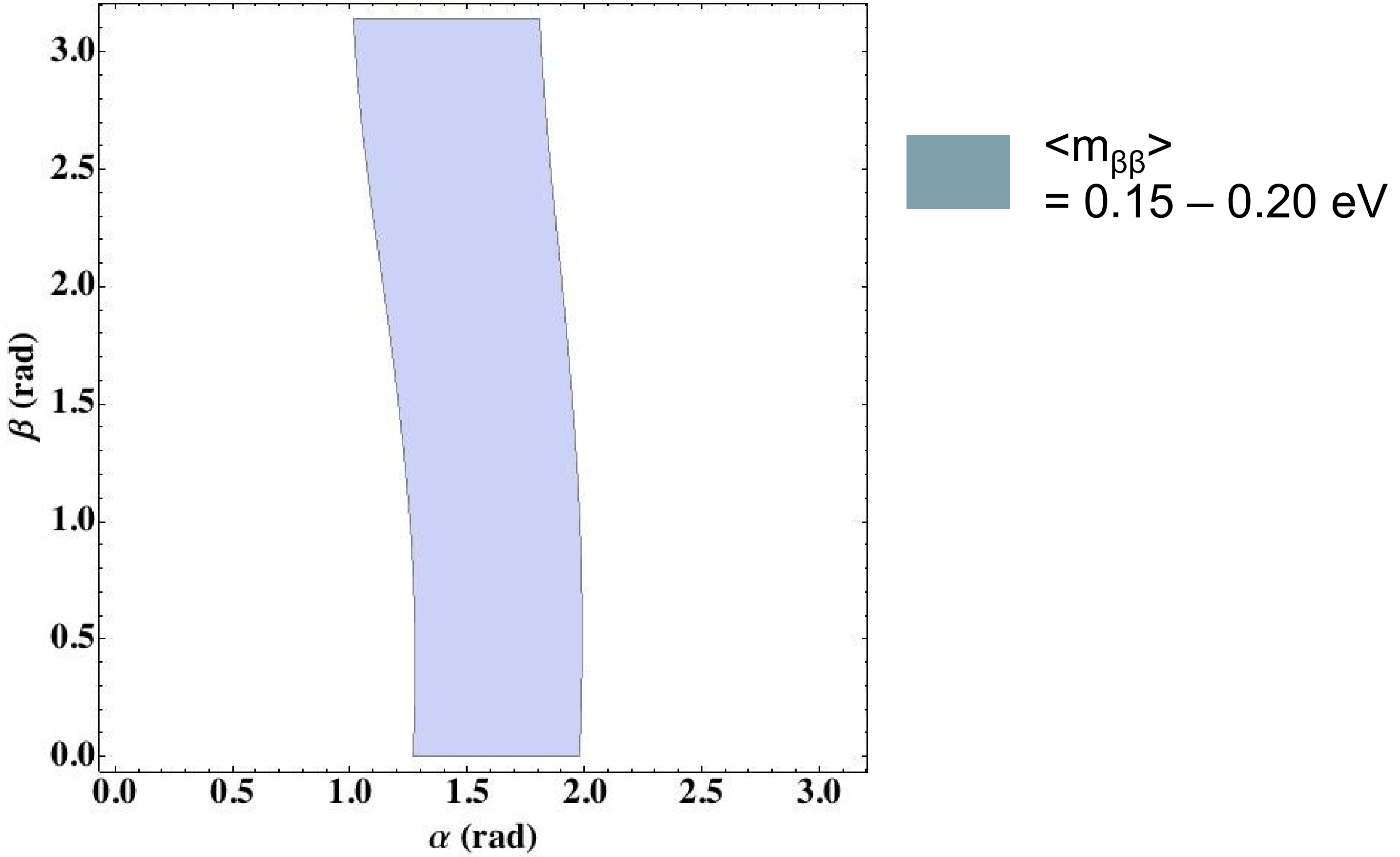
$$\langle m_{\beta\beta} \rangle^2 = A^2 + B^2 + C^2 + AB\cos(\alpha) + AC\cos(\beta) + BC\cos(\alpha - \beta)$$

where

$$A = m_1 c_{12}^2 c_{13}^2 \qquad B = m_2 s_{12}^2 c_{13}^2 \qquad C = m_3 s_{13}^2$$

# Extracting α and β from a measurement

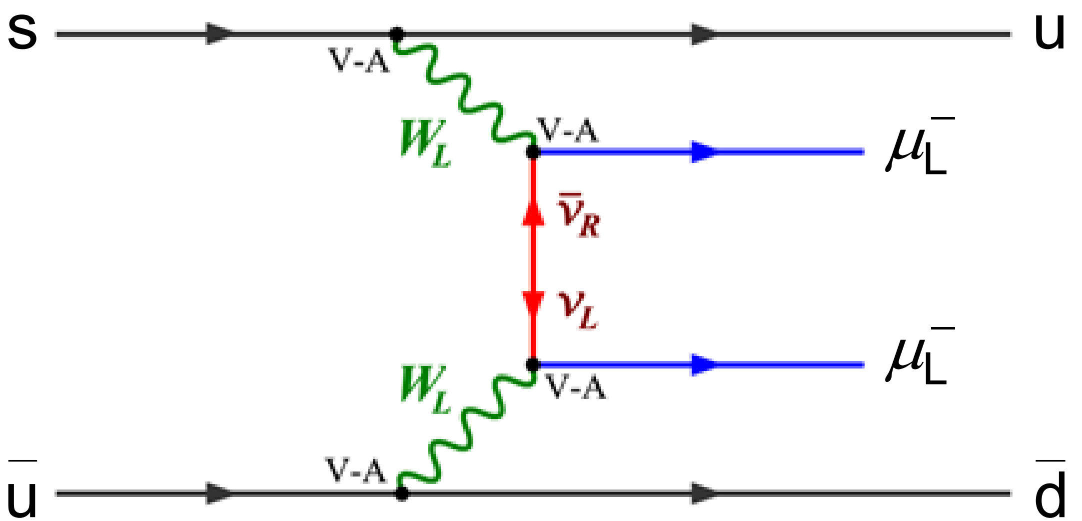
- Example plot of α and β for a measurement of <$m_{\beta\beta}$>




- We need more information to disentangle α and β

# Extracting α and β from measurements

- Consider $K^{-} \rightarrow \pi^{+} + \mu^{-} + \mu^{-}$ :




- By comparing with 0vββ we can construct

$$\langle m_{\mu\mu} \rangle = \left| \sum_i m_i U_{\mu i}^2 \right| = \left| m_1 U_{\mu 1}^2 + m_2 U_{\mu 2}^2 + m_3 U_{\mu 3}^2 \right|$$

# Extracting α and β from measurements

- As with 0vββ, we now find:

$$\langle m_{\mu\mu} \rangle^2 = A^2 + B^2 + C^2 + AB\cos(\alpha) + AC\cos(\beta) + BC\cos(\alpha - \beta)$$

where now

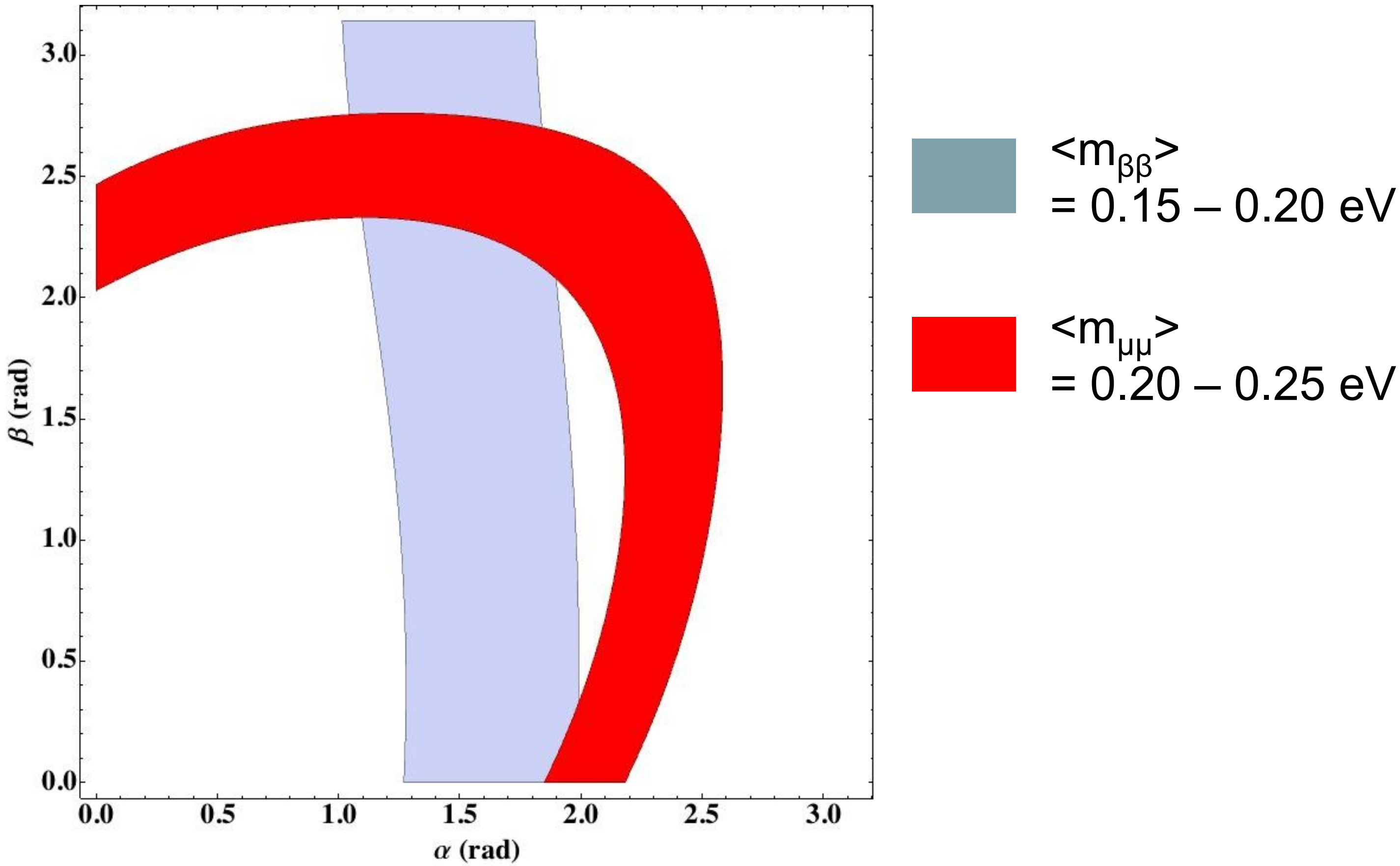
$$A = m_1 (s_{12}c_{23} - c_{12}s_{23}s_{13})^2$$

$$B = m_2 (c_{12}c_{23} - s_{12}s_{23}s_{13})^2$$

$$C = m_3 s_{23}^2 c_{13}^2$$

# Extracting α and β from measurements

- Example plot of α and β for measurements of $\langle m_{\beta\beta} \rangle$ and $\langle m_{\mu\mu} \rangle$




- It is now possible to disentangle values of α and β for certain combinations of $\langle m_{\beta\beta} \rangle$ and $\langle m_{\mu\mu} \rangle$

# Extracting α and β from measurements

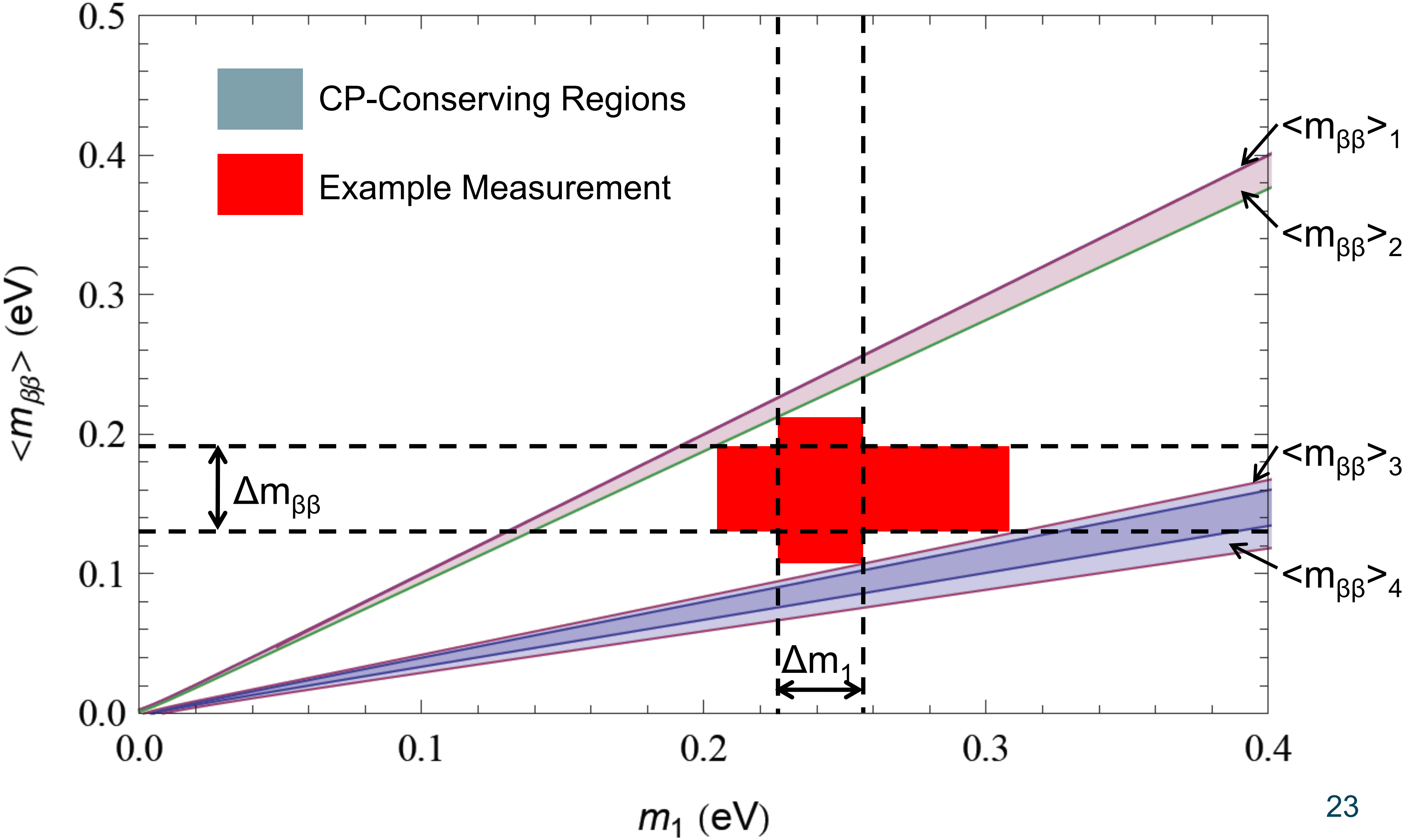
- So it seems that all is OK for a measurement.
- BUT in a typical kaon beam we might have ~ $10^{11}$ particles
- In a 0νββ experiment, we might have 100kg or ~ $10^{26}$ atoms
- For 0νββ we expect $N_{sig} \sim N_{atm}\Gamma_{\beta\beta}$ and a similar order of magnitude for $\Gamma_{\mu\mu}$
- Therefore we'll see ~ $10^{15}$ less signal for $K \rightarrow \pi\mu\mu$
- Back to the drawing board...

# FIN

# Backup Slides

# Measuring Majorana CP Phases

- **Normal Hierarchy** with $\sigma(\Delta m^2) = 0$



# Measuring Majorana CP Phases

- **Inverted Hierarchy** with $\sigma(\Delta m^2) = 0$