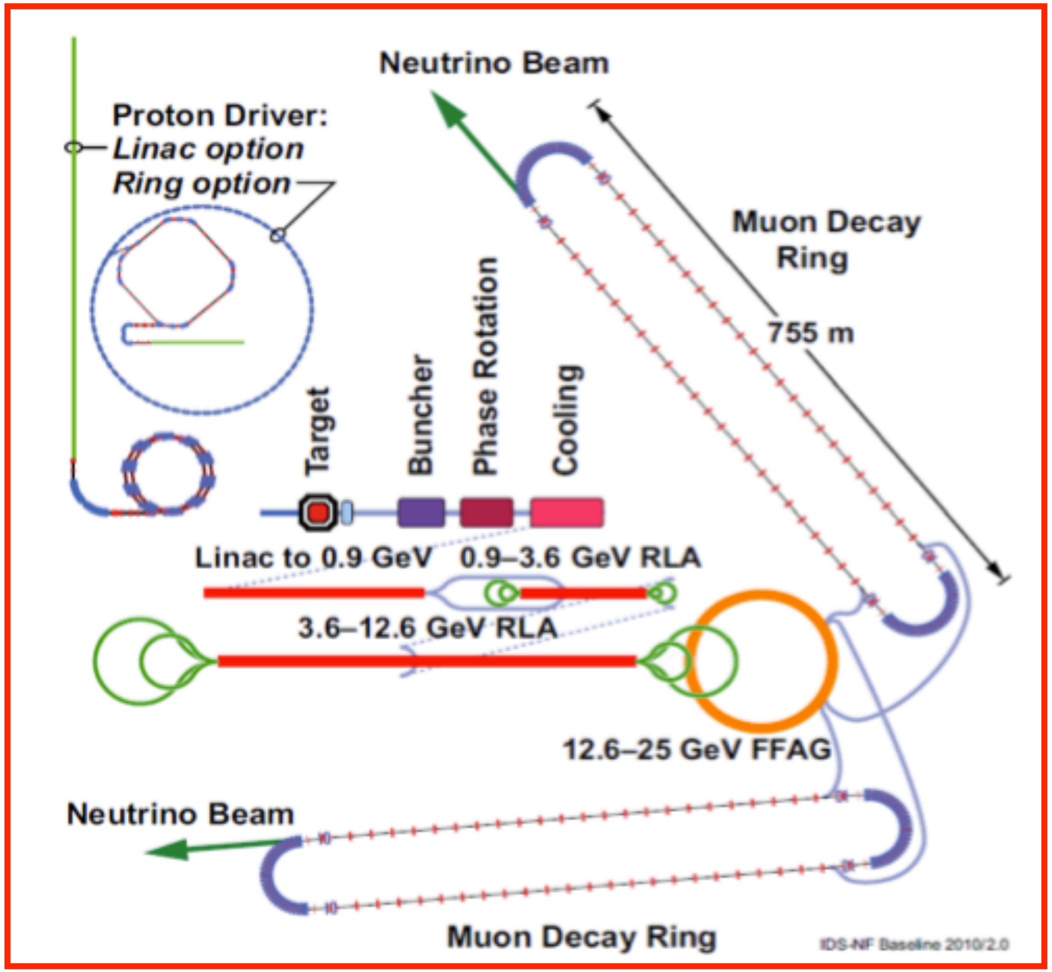
Lecture goals/outline

Goal: present the basic concepts of accelerator physics to understand also the Nufact/Beta beam/Superbeam design

day 1-2)
Introduction to accelerators
Beam transverse dynamics
Superconductivity for accelerators

day 2-3)
Longitudinal beam dynamics
Collective effects

day 2-3)
Superbeam design
Neutrino Factory design
Beta beams design



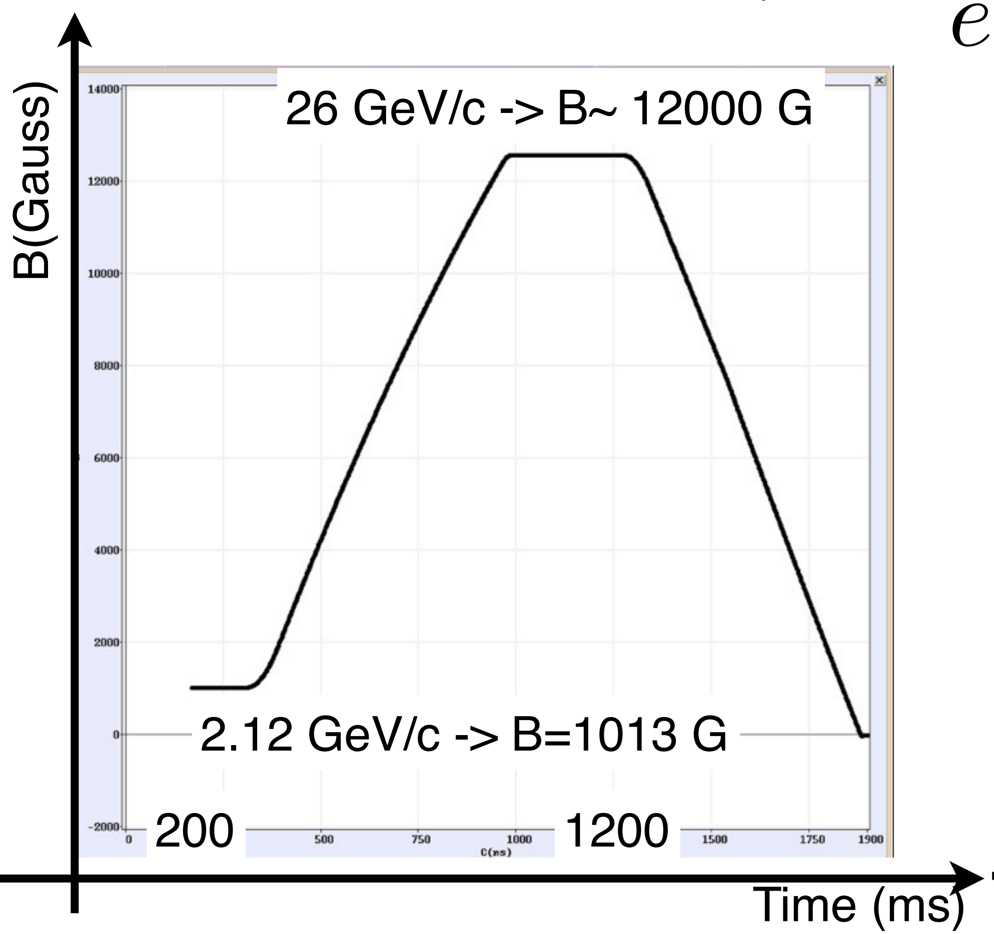
Whenever possible, example taken from existing machine, like LHC, CNGS.
Attempt to mix reality with a bit of theory

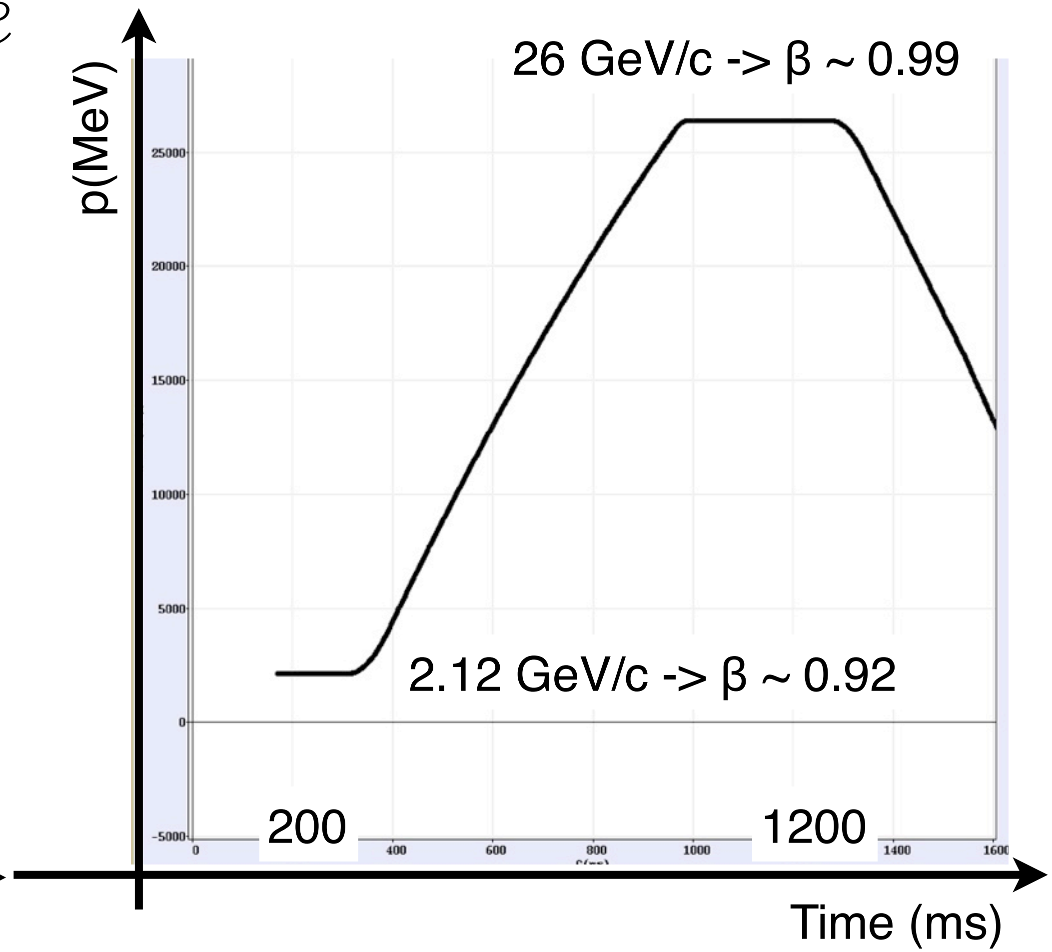
These lectures are not meant to be a MONOLOGUE

Reference particle motion

Def.: The reference particle is a "theoretical" particle whose energy (momentum) is exactly matched to the magnetic field according to:

$$B\rho = \frac{p}{e}$$



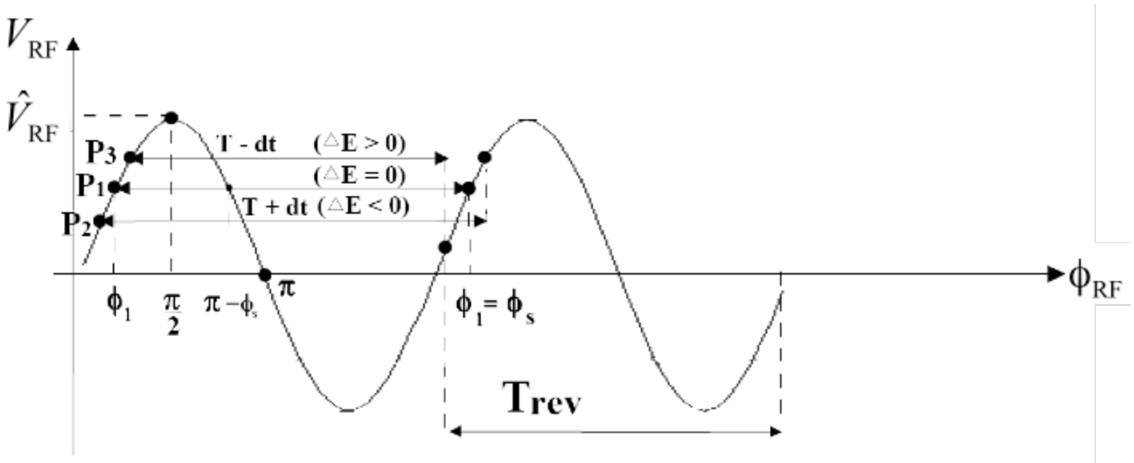


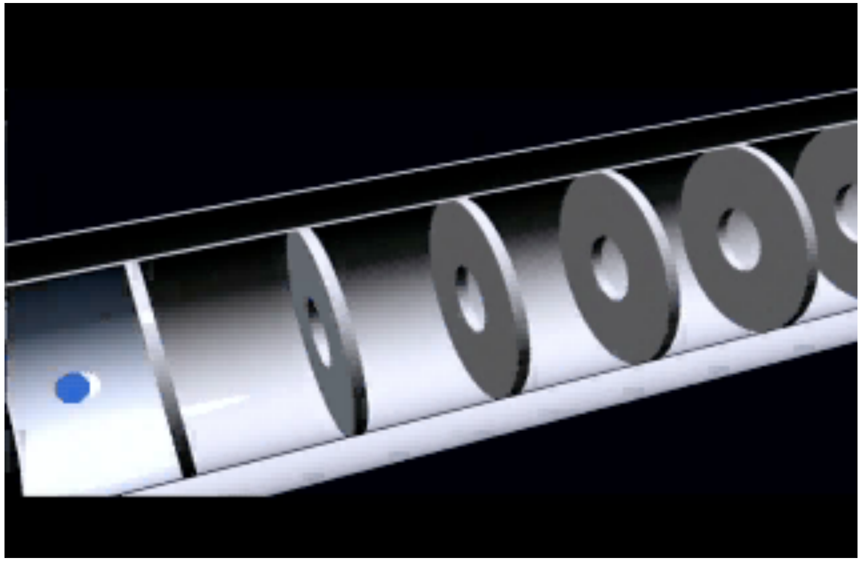
Let's try to accelerate .... (above an example of the PS)

Acceleration

- Particles are accelerated by an **RF (radio frequency) electric field which is confined in cavities.**
- **The electric field varies in time as a sinus wave in such a way, that at each revolution, the particle comes back at the RF to see the acceleration.**

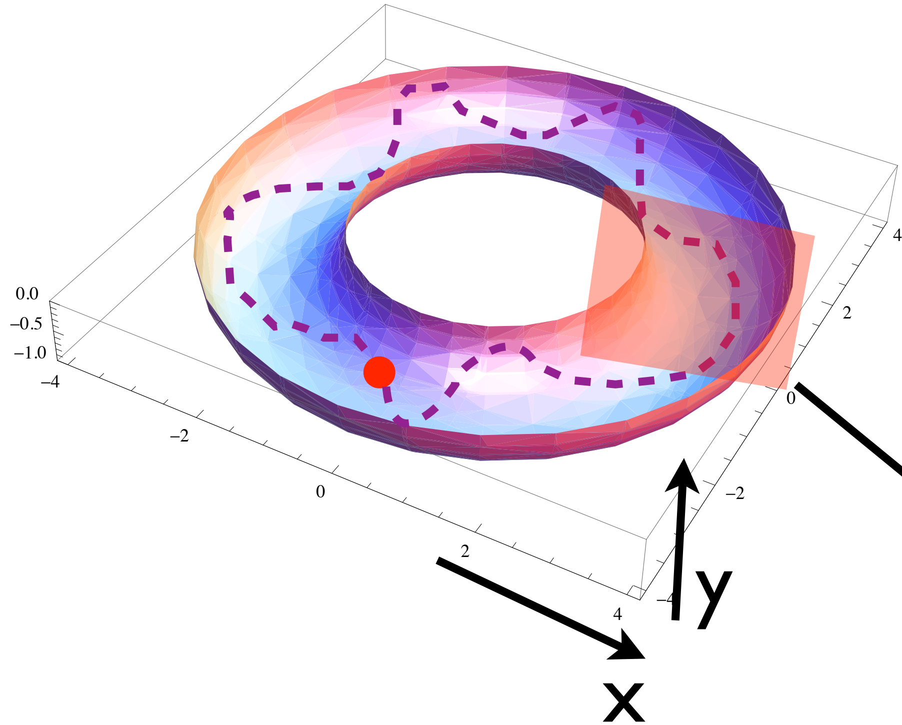
$$V_{RF} = \hat{V}_{RF} \sin\phi_{RF}(t) \Rightarrow \Delta E_1 = e\,\hat{V}_{RF} \sin\phi_1$$

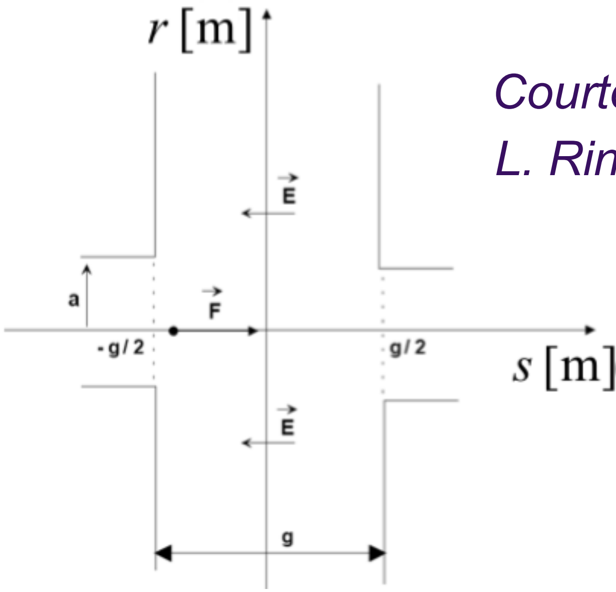




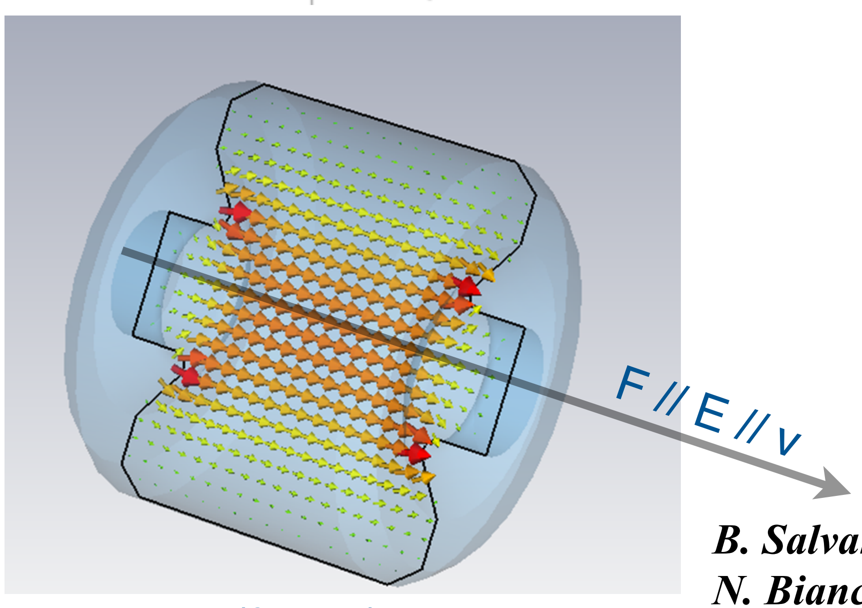
Acceleration I

Acceleration again with Lorentz force: $\overline{F(t)} = q\left(\overline{E(t)} + \overline{v(t)} \otimes \overline{B(t)}\right)$





Courtesy
L. Rinolfi

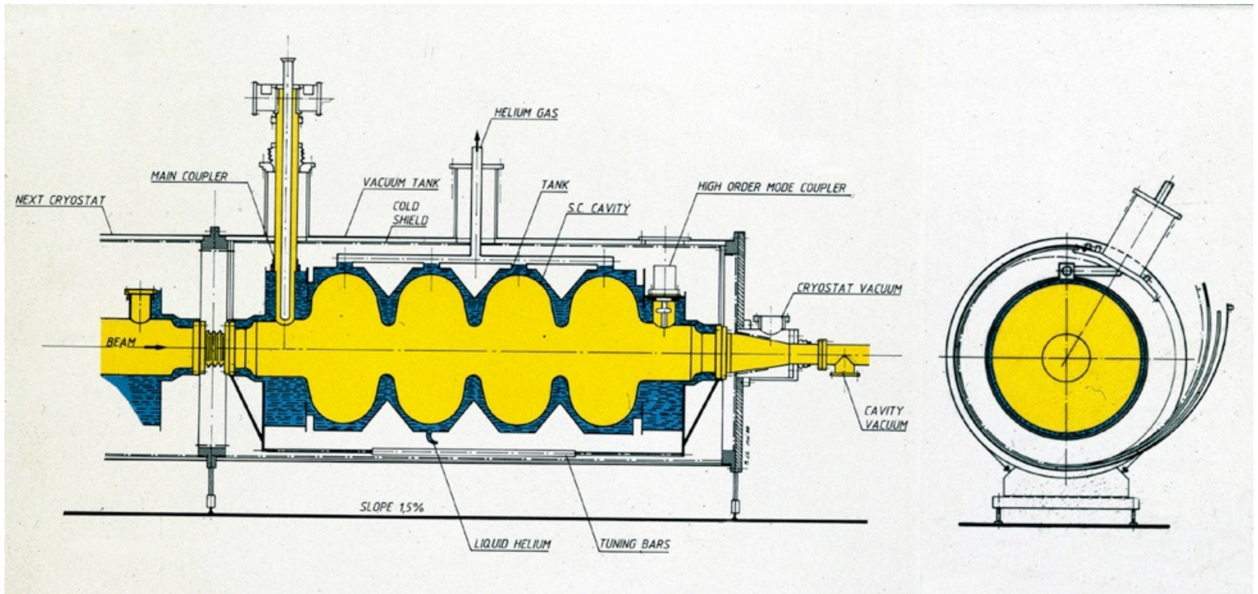


B. Salvant
N. Biancacci

In a well define part of the accelerator,
a RF (radio frequency) cavity generates an electric field parallel to the velocity of a zero divergence particle.
The cavity itself acts as a resonator.

Obs: The magnetic field associated to the RF wave is negligible (for us).

RF systems, LEP, LHC



SUPERCONDUCTING CAVITY WITH ITS CRYOSTAT

Example for LHC:

485 keV gain per turn
ACCELERATION TAKES TIME

How long is a wave?
fcav= 400 MHz
λ= c / fcav ~ 75 cm

A typical cavity can provide from few kV/m few MV/m

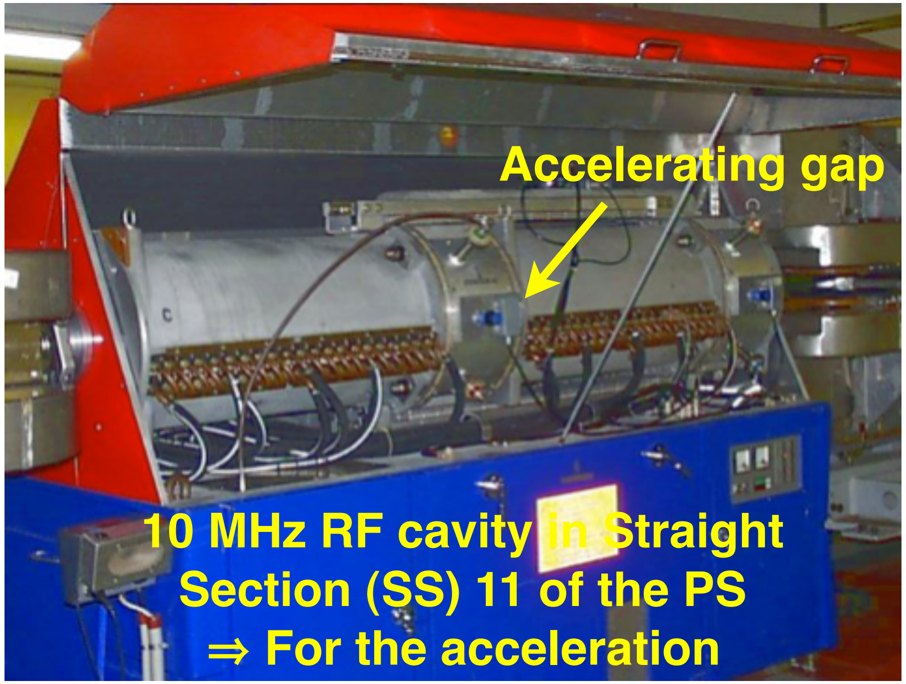
Example for LEP:

120 cavities (room temperature) at 352 MHz,
provided over **300 MV circumferential voltage**
(! that's why we do not bend with E fields...)

Then, the new superconducting RF provided
2000 MV circumferential voltage
(LEP was 27 km circumference, basically filled by RF cavities)

Example of RF cavities in the PS

The dimension of the cavity changes with the RF wave length



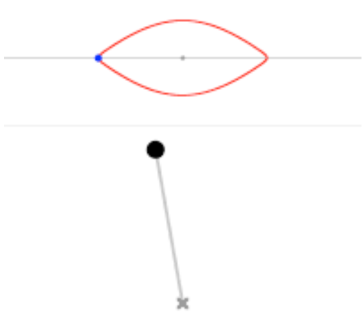
World Radio Switzerland: 88.4 MHz

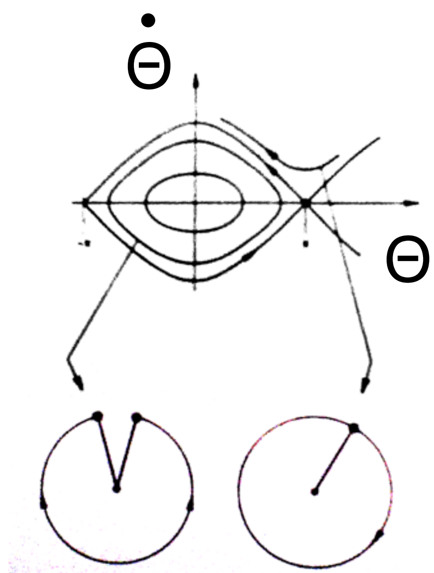
Longitudinal dynamics
Classical mechanics.... oscillating pendulum

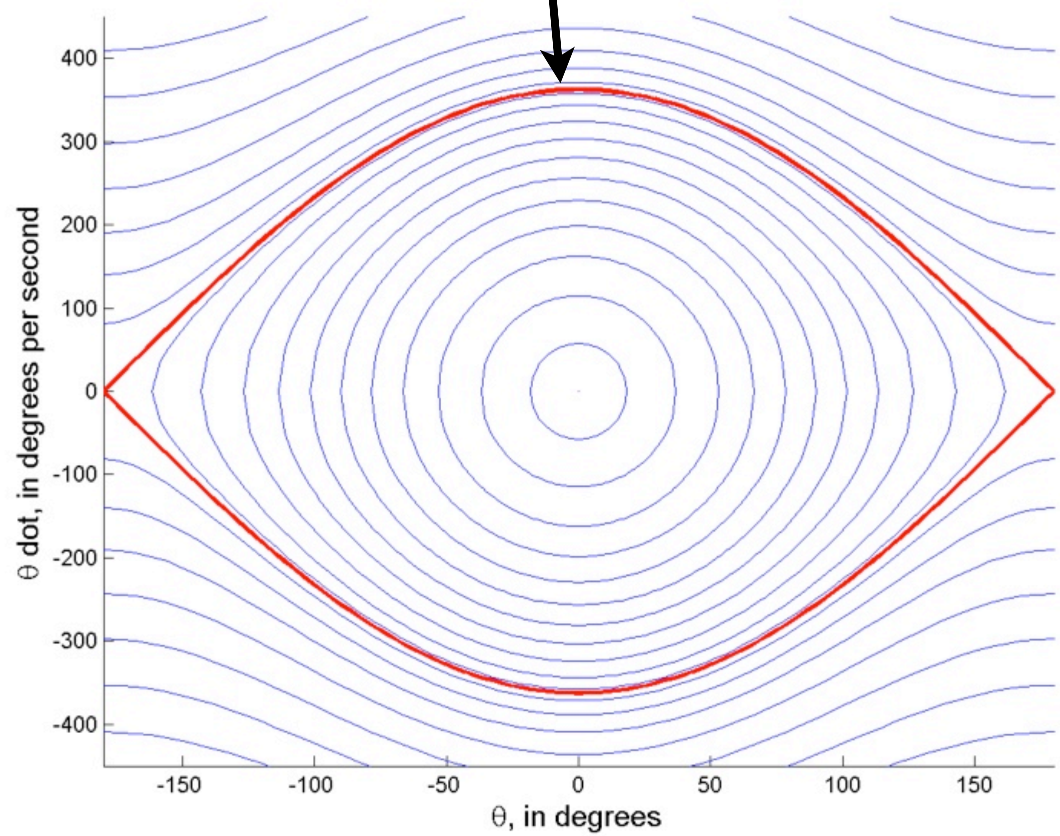
$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$$

Constant force:
g = acceleration of gravity
l = pendulum length

The movement is stable only inside the separatrix

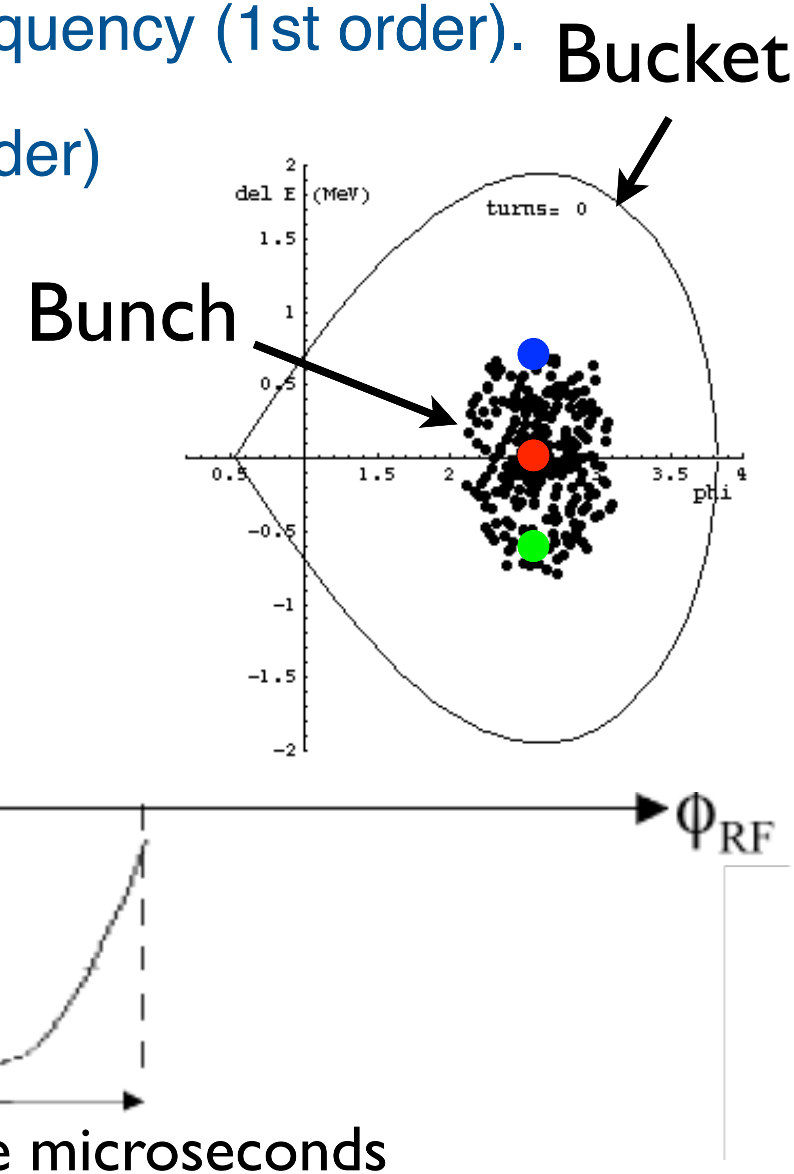


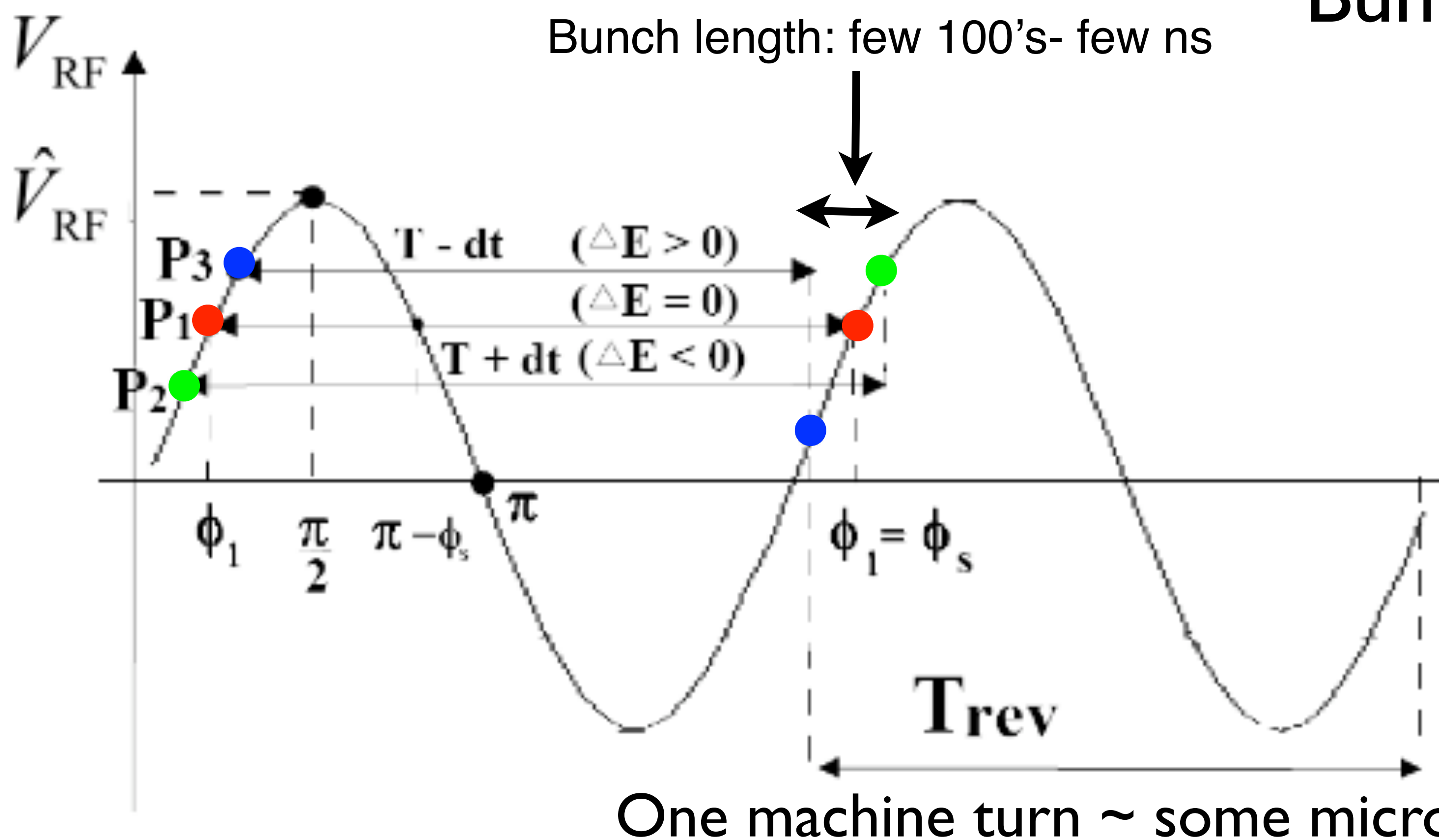




Longitudinal focusing, as the pendulum ...

- Particles are confined within a range in phase and energy called **BUCKET** and are grouped into **bunches by the electric field**.
- The bunch length depends on the RF frequency (1st order).
- The energy spread by RF voltage (1st order)





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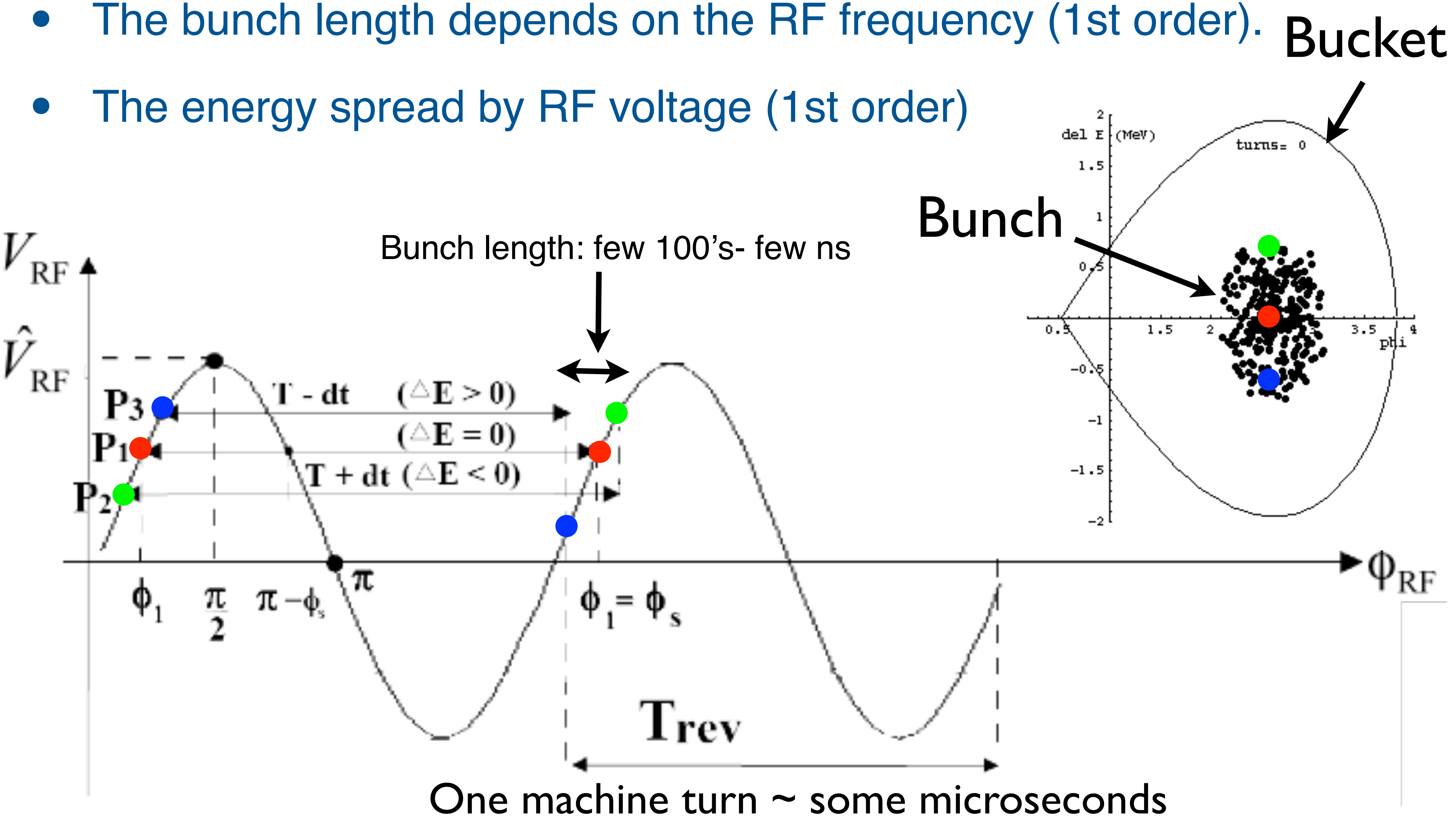
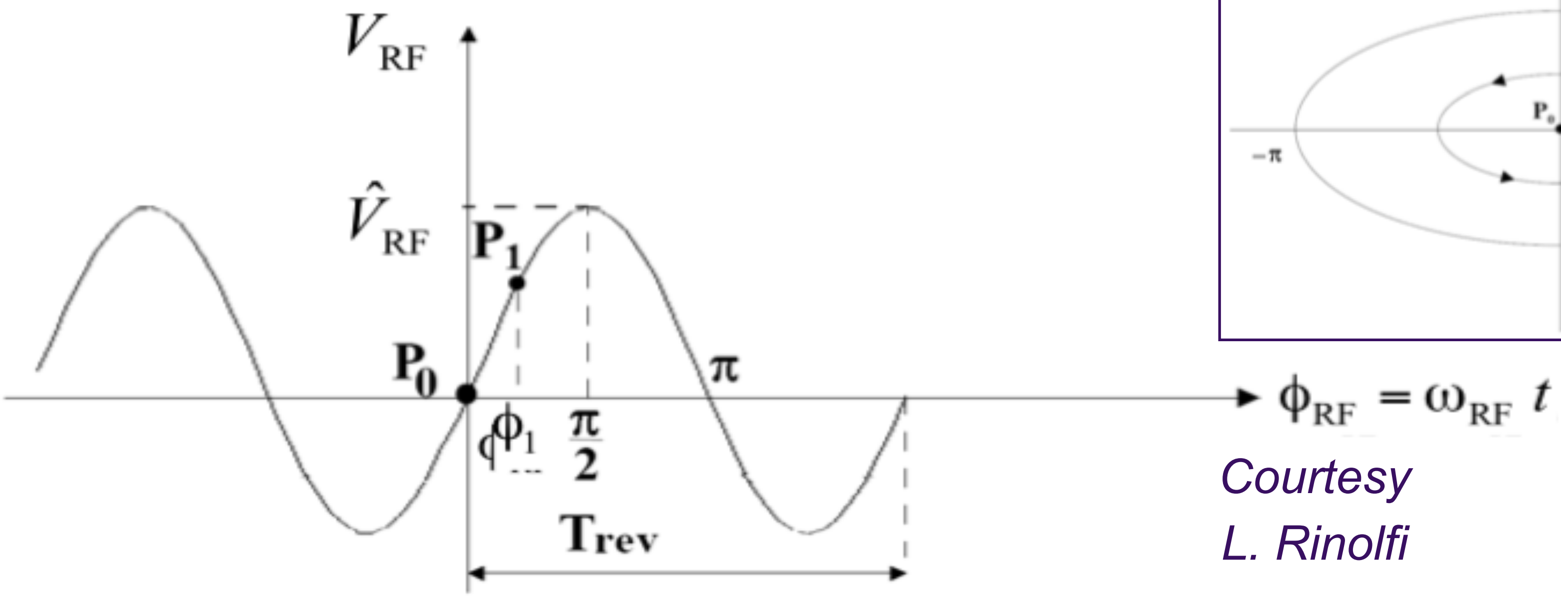
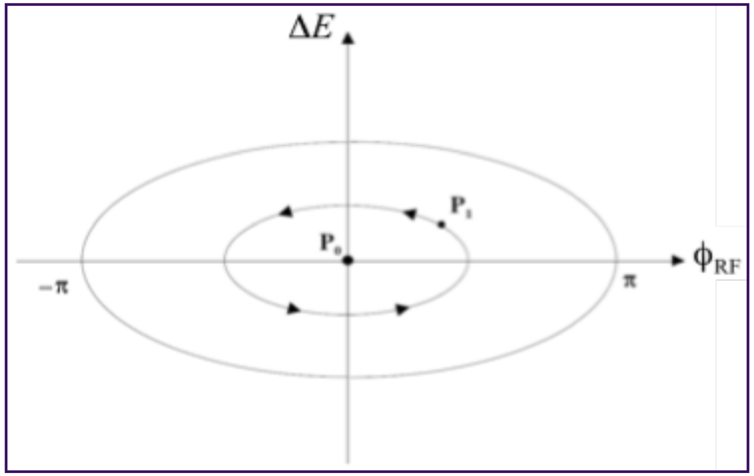


No, acceleration, only Longitudinal Focusing

$$V_{RF} = \hat{V}_{RF} \sin\phi_{RF}(t) \Rightarrow \Delta E_1 = e\,\hat{V}_{RF} \sin\phi_1$$





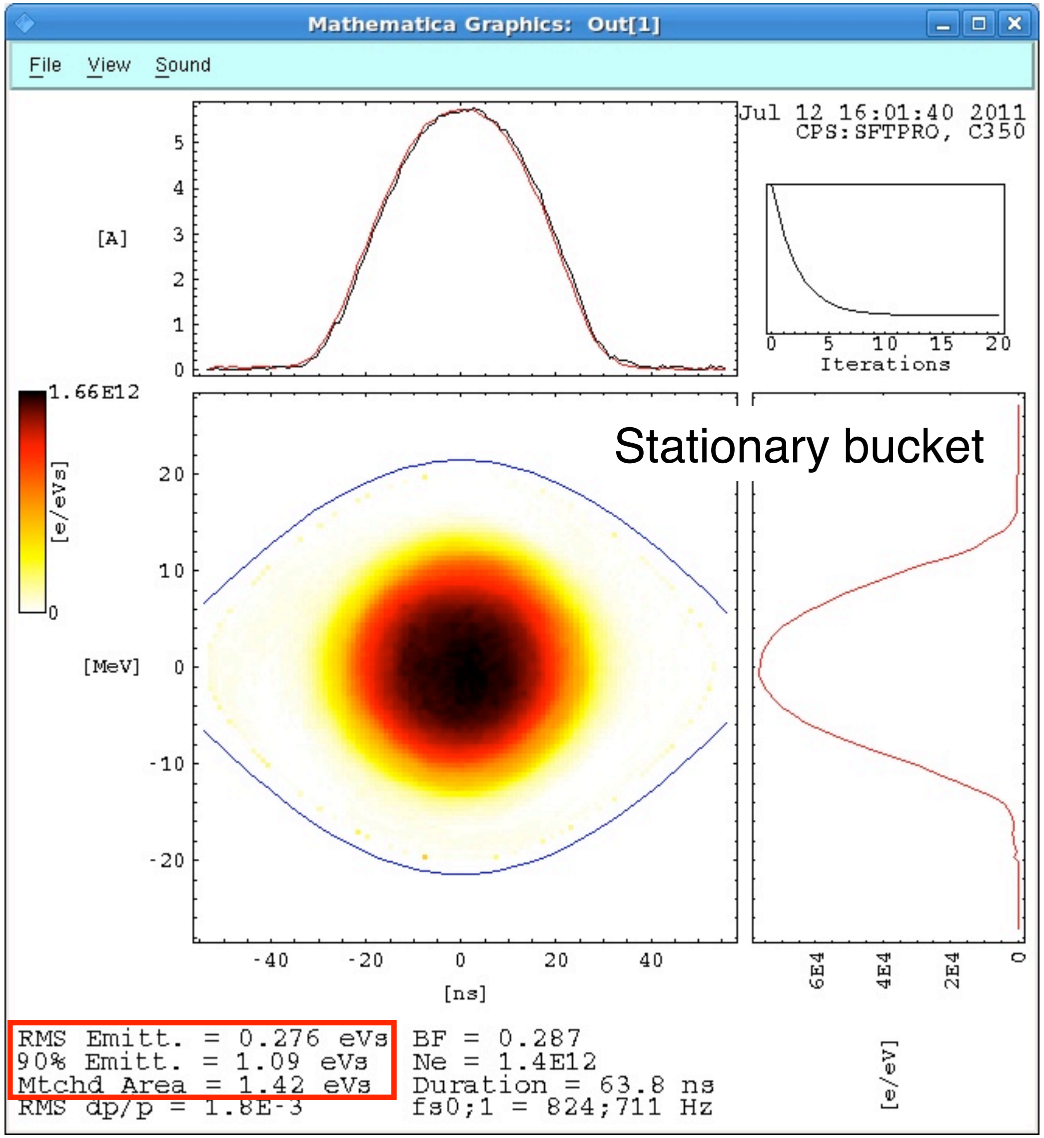
Courtesy
L. Rinolfi

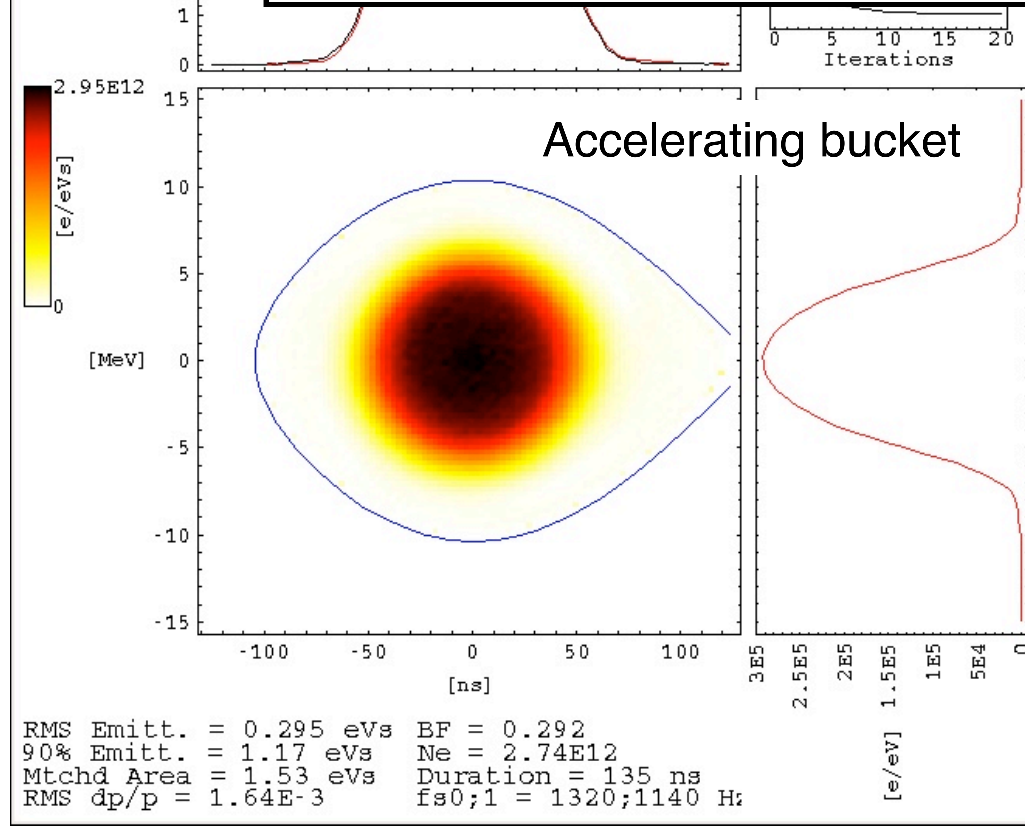
The RF cavities can act as focusing elements, as quadrupoles, but in the phase space described by (dp/p, phase). Actually they can confine the motion around the reference particle with the separatrix defining the bucket. The bucket is asymmetric wrt the reference particle when accelerating due to the sinusoidal function of the

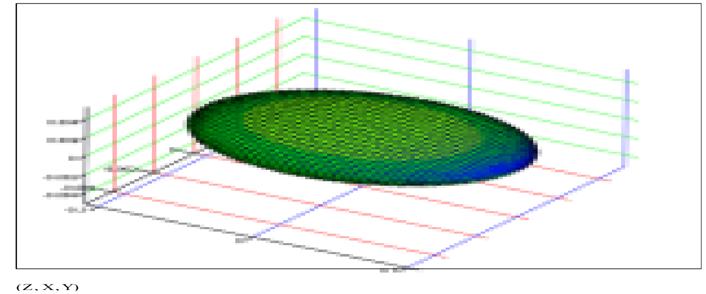
Def: the motion in the phase space described by (dp/p, phase) or (dp/p, time) is called **Synchrotron motion, following the Synchrotron frequency.**

The bucket height depends on the maximum cavity voltage, the bunch length on the frequency

Example of two real bunches/buckets





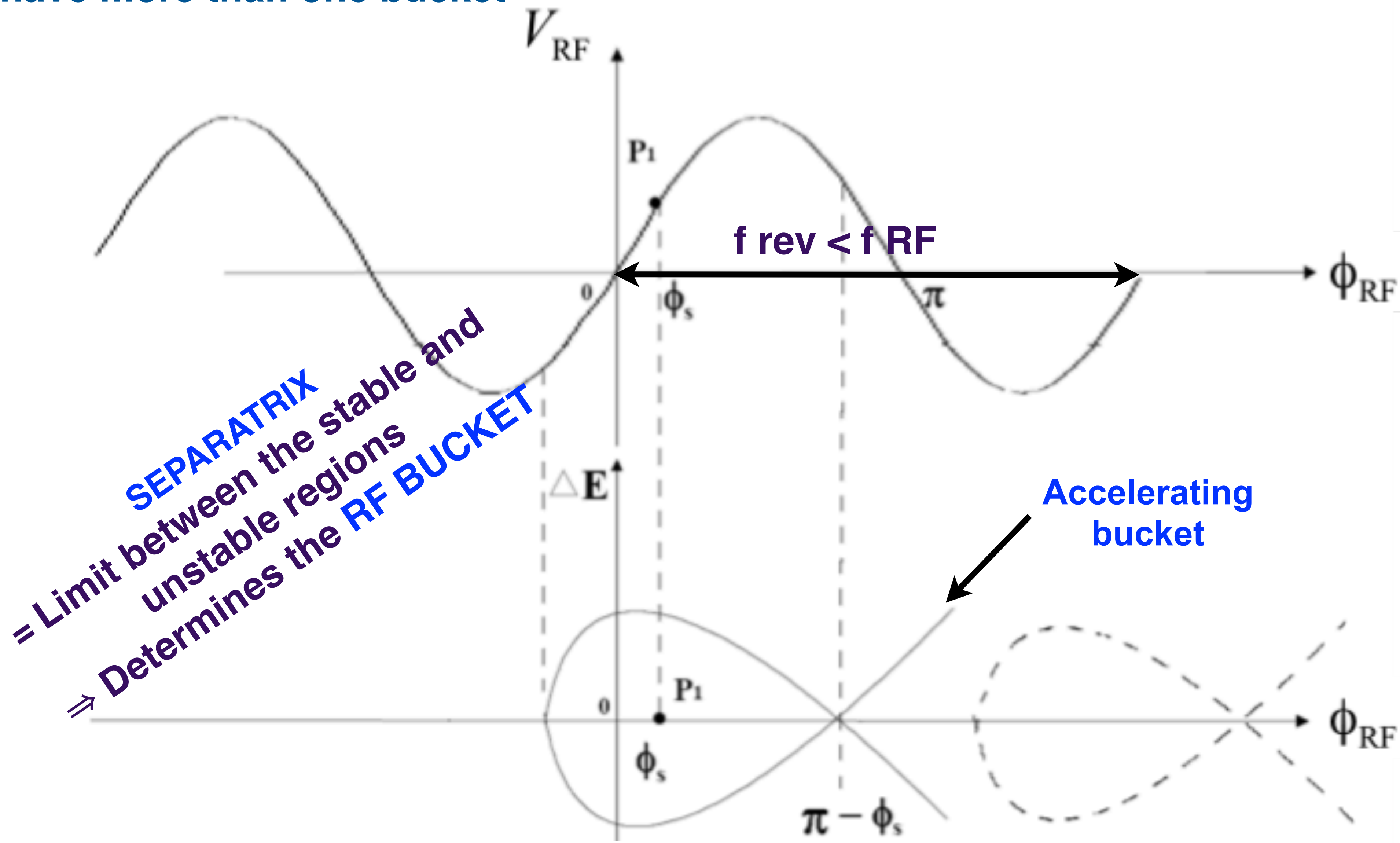


a proton bunch: focused longitudinal by the RF field

Def: one can define a **longitudinal emittance**, exactly as in the transverse plane.
The emittance is measured in *eVs*
One can also define the **Synchrotron tune** as the number of oscillations in the longitudinal plane per machine turn. Typically, the Synchrotron tune is <<1.

How many buckets ?

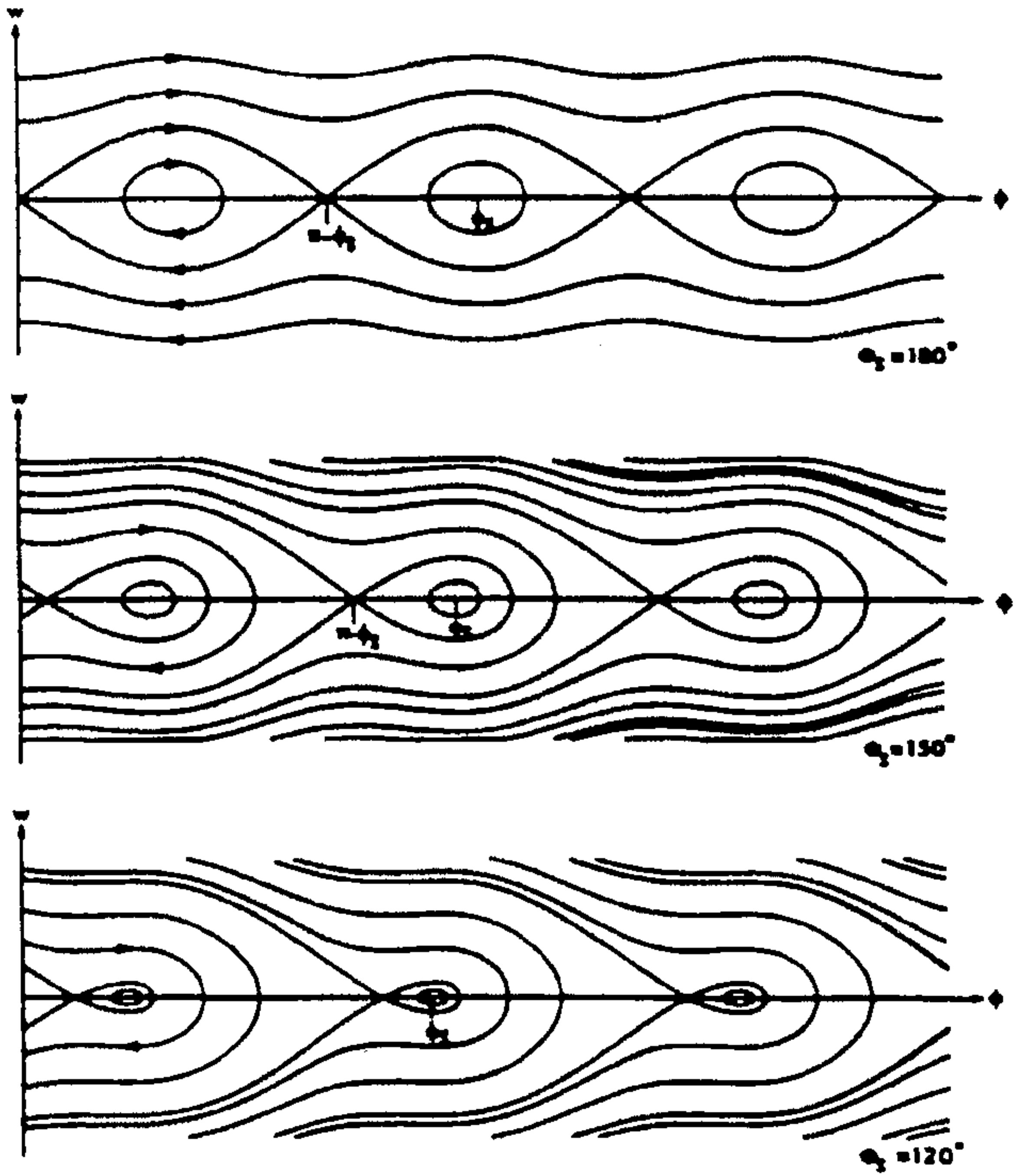
The revolution frequency can be smaller than the RF frequency, so one can have more than one bucket



The number of buckets is given by *h, the harmonic number, how many buckets can be accommodated in a machine turn*

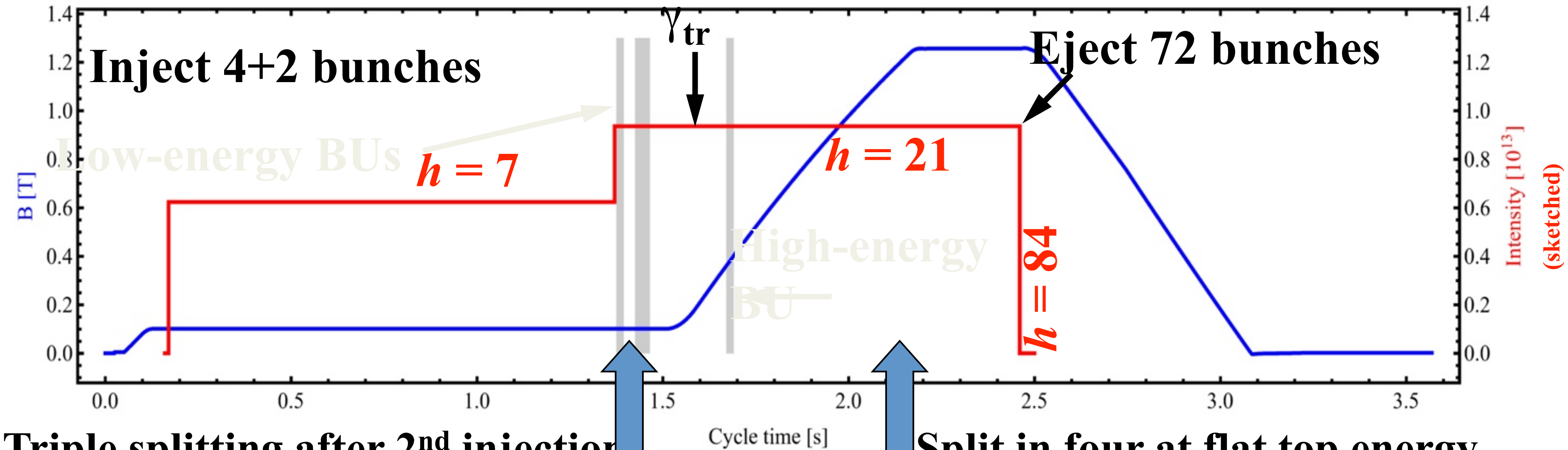
Courtesy E. Metral

A chain of buckets

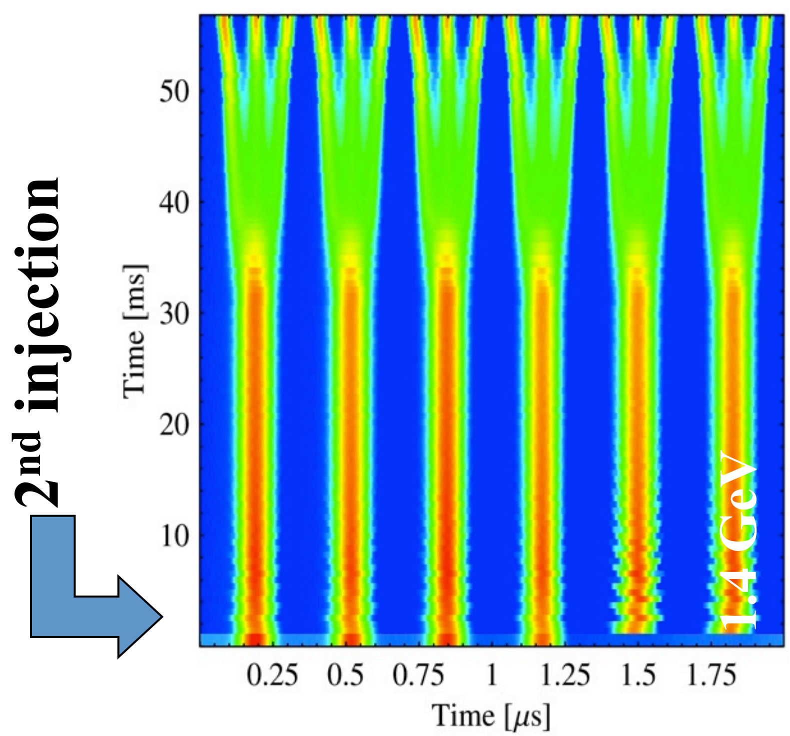


Courtesy E. Wilson

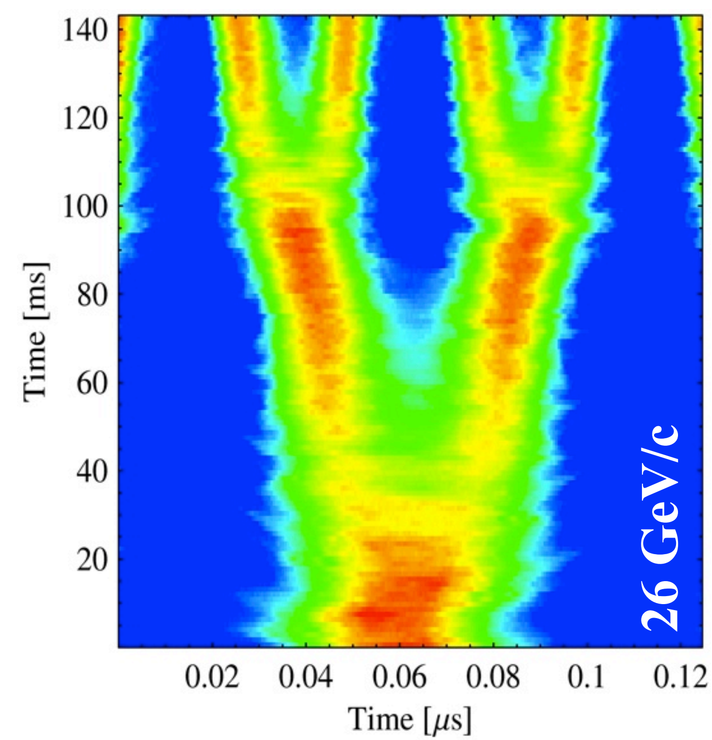
The LHC25 (ns) cycle in the PS. Double batch



Triple splitting after 2nd injection



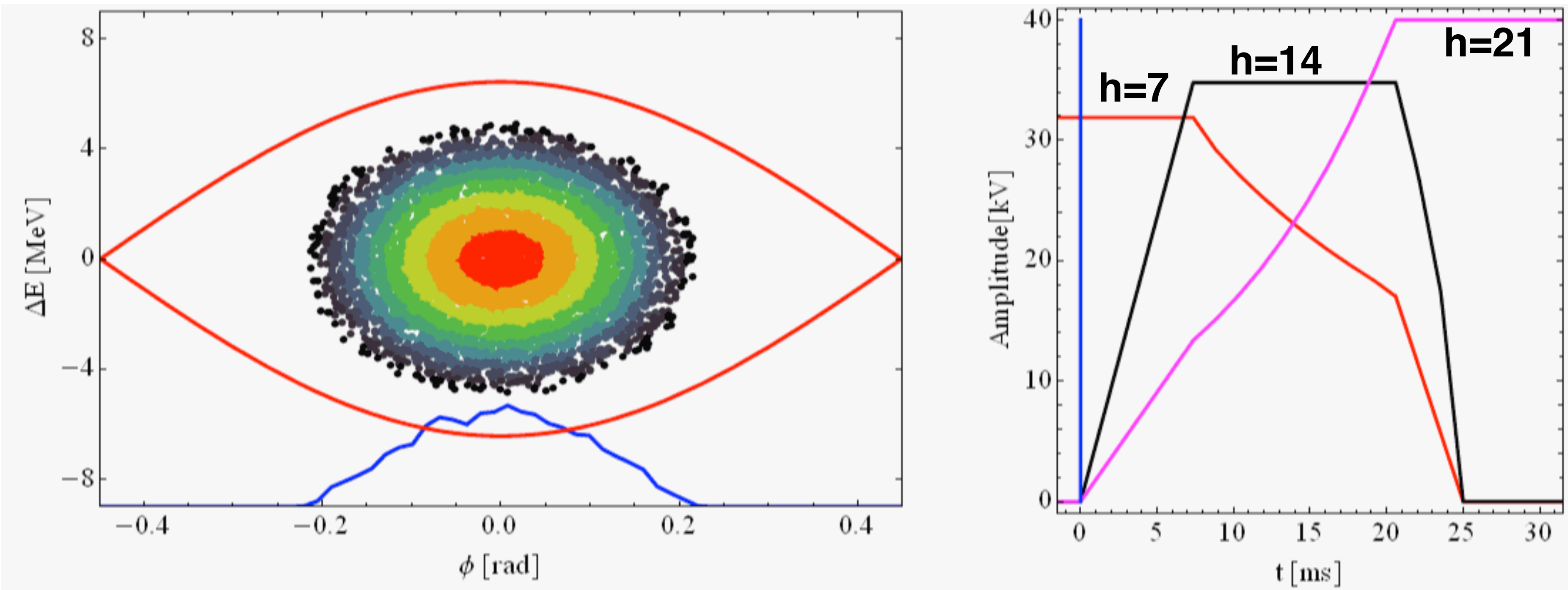
Split in four at flat top energy



→ Each bunch from the Booster divided by 12 → 6 × 3 × 2 × 2 = 72

Example of RF gymnastics: Triple bunch-splitting (LHC25, LHC50)

Split bunches in three similar parts (*h*7 → *h*21)



Three RF harmonics at the same time h=7→ h=14→ h=21.

The Voltage of different group of cavities at different harmonics are "adiabatically" increases and decreased to obtain a bunch splitting with stationary buckets

If we would be looking to the Hemiltonian of the system, we would be creating extra fix points by introducing the high field harmonics *(ipse dixit...)*

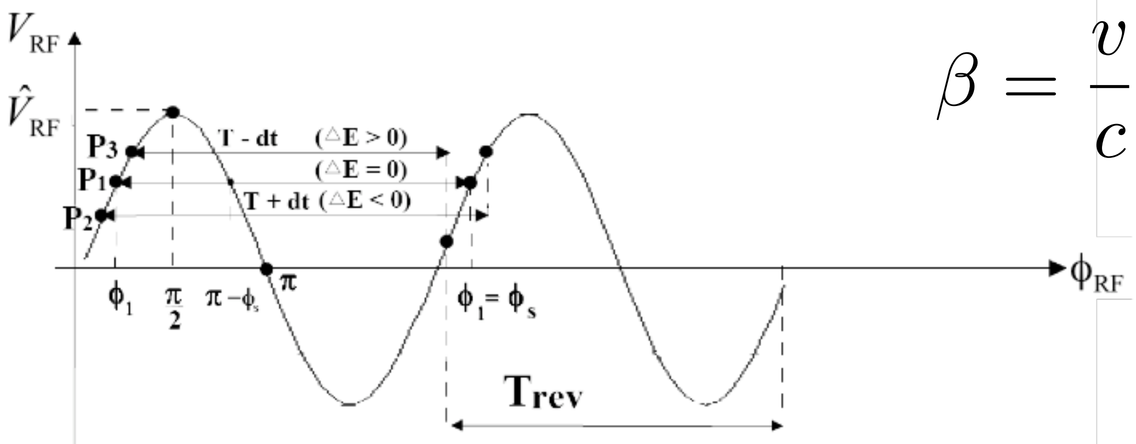
Is the cavity frequency really constant?

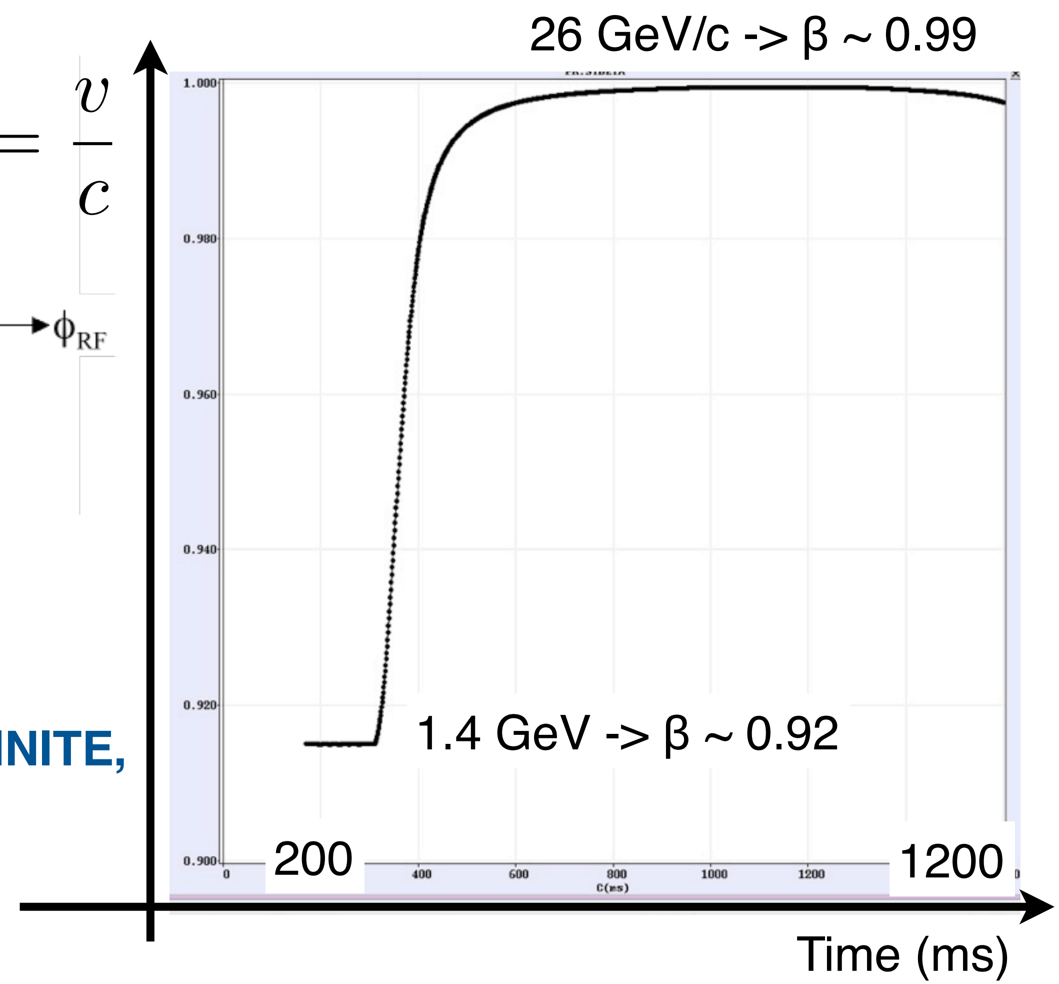
In a low energy machine, the variation of the beta (relativistic), can be still pretty large

Accelerating still means an energy increase, but also velocity increase.

The Trev is not longer a constant, the cavity has a certain band with or the resonant frequency has to be tunable!

$$\beta = \frac{v}{c}$$





The RF frequency depends on the geometry of the cavity.

The mechanical "tunability" of the cavity is FINITE, can be at max few %:

a) more than one RF system

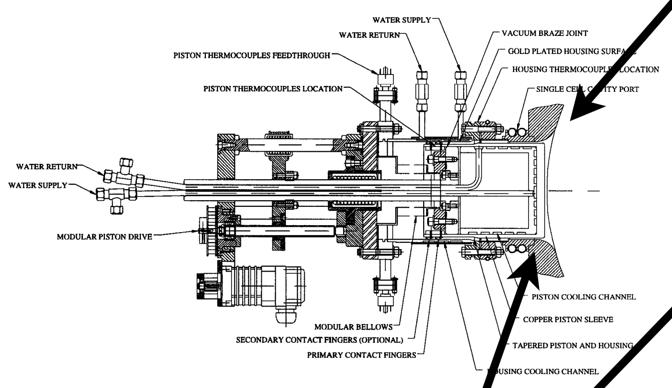
b) more than one ring/accelerator to reach the final energy

How a cavity is tuned ?

It the frequency required by the beam is outside the cavity range, the cavity shape can be adjusted by **mechanical tuners, i.e PISTONS, or electrical tuners.**

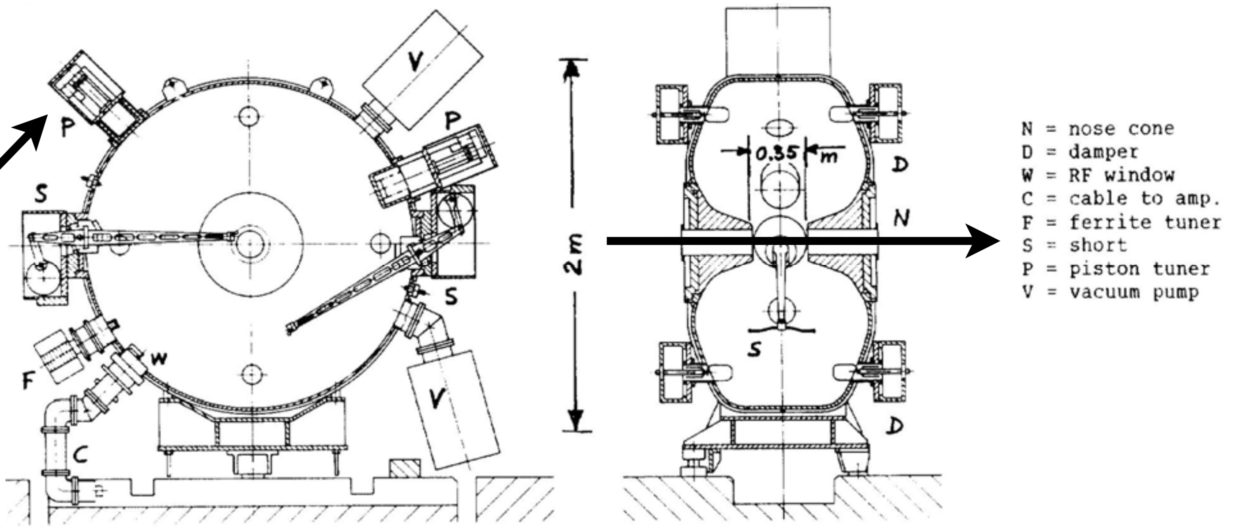
Obs: Being mechanical, there is no mean to use these devices to adjust the frequency of the cavity during the acceleration, that take place in only few hundreds ms, or few seconds.

J. Jones et al.



Inner surface of the cavity

Piston



Why do I care? If you have a very long accelerator with a very large number of cavities they have to be tuned as in an Orchestra

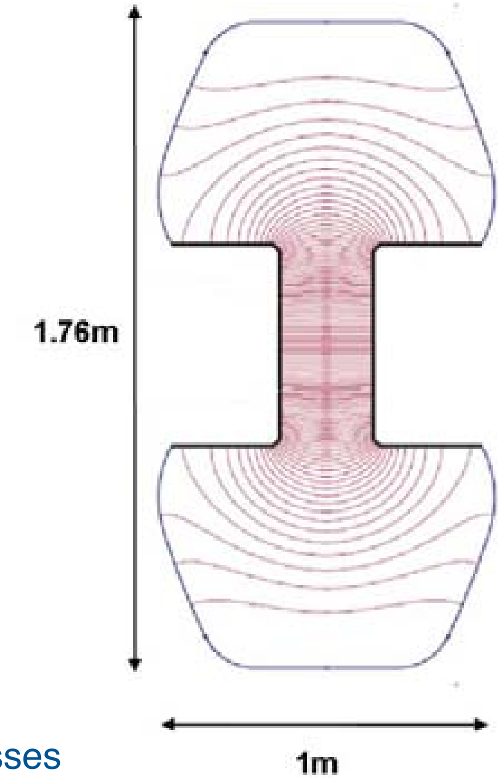
A real cavity .... prototype for CERN-Nufact

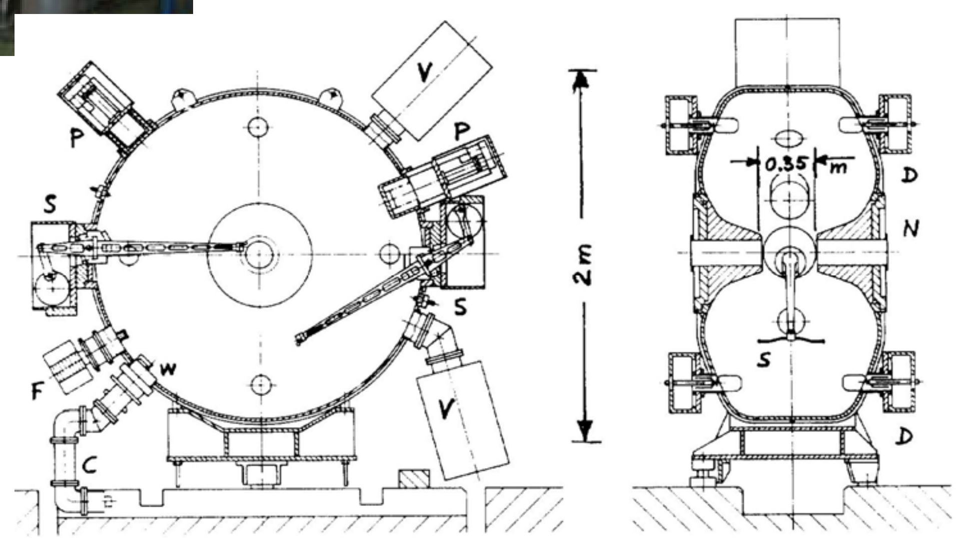
80 MHz cavity

E-field gradient	4.1	MV/m
Gap field	14.7	MV/m
Peak surface field	25.9	MV/m
Kilpatrick factor	2.4	Kilp.
Output power	2.65	MW
Repetition frequency	1	Hz
Pulse length	270	µs

A cavity has to be be filled by the electric field and the power is consumed by the beam and by Joule losses

E field





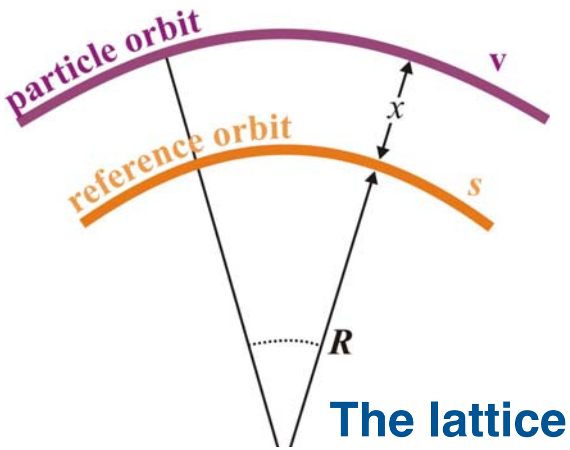
N = nose cone
D = damper
W = RF window
C = cable to amp.
F = ferrite tuner
S = short
P = piston tuner
V = vacuum pump

from CERN-AB-2006-025

A bit of theory.... be patient ....

The road to observe some physical effects during acceleration and justify what just described.... (slide “stolen” from H. Hillert)

The total path length is given by



$$L = \int_{s_0}^{s} \left[\frac{R(\tilde{s}) + x_D(\tilde{s})}{R(\tilde{s})} \right] d\tilde{s} = L_0 + \int_{s_0}^{s} \frac{x_D(\tilde{s})}{R(\tilde{s})} d\tilde{s}$$

With $\delta = \Delta p / p$, and $x_D(s) = D(s) \cdot \delta$ we obtain

$$L - L_0 = \Delta L = \delta \int_{s_0}^{s} \frac{D(\tilde{s})}{R(\tilde{s})} d\tilde{s}$$

The lattice is here

and define the **momentum compaction factor** by

$$\boxed{\alpha_c = \frac{\Delta L / L_0}{\Delta p / p} = \frac{1}{L_0} \cdot \int_{s_0}^{s} \frac{D(\tilde{s})}{R(\tilde{s})} d\tilde{s}}$$

The travel time is given by $\tau = L / (\beta c)$, and

$$\Delta \ln \tau = \frac{\Delta \tau}{\tau} = \frac{\Delta L}{L} - \frac{\Delta \beta}{\beta} = \left(\alpha_c - \frac{1}{\gamma^2} \right) \cdot \delta = -\eta \cdot \delta,$$ giving $$\boxed{\eta = \frac{1}{\gamma^2} - \alpha_c}$$

which is called the **slip factor** η.

The momentum compaction factor related the variation of the path length wrt an energy error... important to understand why the MOON affects a storage ring ...

Transition crossing

- **TRANSITION ENERGY**: The increase of energy has 2 contradictory effects
  - An increase of the particle's velocity
  - An increase of the length of the particle's trajectory
- According to the variations of these 2 parameters, the revolution frequency evolves differently
  - (1) **Below transition energy:** The velocity increases faster than the length
    ⇒ The revolution frequency increases
  - (2) **Above transition energy:** It is the opposite case
    ⇒ The revolution frequency decreases
- **At transition energy:** A variation of energy does not modify the frequency
  - The variation of the velocity is compensated by the variation of the trajectory
    => The momentum compaction is ZERO..... **so what ????**

a) Exactly transition energy there is no longitudinal focusing

b) The particle stable phase has to be put to the other side of the accelerating sinus to maintain the longitudinal focusing (see (2))

A bit of more theory, ref. for equation of movement

The longitudinal motion takes place in the phase space described by Energy error and phase error wrt the reference particle.

Period

$$\frac{\Delta T}{T_0} = -\frac{\Delta\omega}{\omega_0} = -\eta \cdot \frac{\Delta p}{p_0}$$

Def.

Ref. particle

$$\Delta\varphi = \varphi - \varphi_0 \quad \rightarrow \quad \Delta\varphi = \omega_{RF} \cdot \Delta T = h \cdot \omega_0 \cdot \Delta T$$

harmonics

from Def. η

$$(\Delta\varphi)_{rev} = -\eta h \omega_0 T_0 \frac{\Delta p}{p_0} = -2\pi h \eta \frac{\Delta p}{p_0} = -\frac{2\pi h \eta}{\beta^2}\frac{\Delta E}{E_0}$$

velocity

Def.

$$\Delta\dot{\varphi} = {}^{1}\!/_{T_0} \cdot (\Delta\varphi)_{rev}$$

$$\Delta\dot{E} = {}^{1}\!/_{T_0} \cdot (\Delta E)_{rev}$$

beer

$$\frac{d^2\Delta\varphi}{dt^2} + \frac{2\pi h \eta}{\beta^2 T_0^{\,2} E_0} \cdot (\Delta E)_{rev} = 0$$

Def.

E gain per turn

$$(\Delta E)_{rev} = eU(\varphi) - W(E) = eU_0 \sin\varphi - W(E)$$

$$\frac{d^2\Delta\varphi}{dt^2} + \frac{2\pi h \eta}{\beta^2 T_0^2 E_0} \cdot \left[eU_0 \sin(\varphi_0 + \Delta\varphi) - W(E)\right] = 0$$

W(E$_0$)=eU(φ$_0$), energy loss per turn,

a bit of beer...

Trick, Taylor expansion of $\frac{d}{dt}\Delta E$

(slide “stolen” from H. Hillert)

A bit of more theory, ref. for equation of motion

Def. of synchrotron period →

$$\underbrace{\left(\frac{2\pi h\eta e}{\beta^2 T_0^2 E_0}\cdot U_0\cos\varphi_0\right)}_{=\Omega_S^2}$$

We brilliantly find →

$$\Delta\ddot{\varphi}+\Omega_S^2\cdot\Delta\varphi = 0$$

With solution

Harmonic oscillator $\Delta\varphi = \widehat{\Delta\varphi}\cdot\cos\left(\Omega_S t+\phi\right)$

"ipse dixit" line....

Bunch length

$$l_b = 2\cdot\frac{c\sqrt{2\pi}}{\beta\omega_0}\cdot\sqrt{\frac{\eta E_0}{heU_0\cos\varphi_0}}\cdot\left(\frac{\Delta E}{E_0}\right)$$

What the formula wants to tell

a) if η -> 0 the bunch length -> 0 (Transition)
b) $\omega_0 = h\,\omega_{rf}$, high RF frequency -> short bunches
c) low voltage -> long bunches

Max energy error

$$\left(\frac{\Delta E}{E_0}\right)_{\max} = \beta\sqrt{\frac{2eU_0\sin\varphi_0}{\pi h\eta E_0}\cdot\left(\sqrt{q^2-1}-\arccos\frac{1}{q}\right)}$$

a) if η -> 0, Energy error goes to ∞ (Transition)
b) An increase of the RF voltage gives a moderate increase of the bucket hight

$$q = \frac{\text{maximum RF voltage}}{\text{desired energy gain}} = \frac{eU_0}{eU_0\sin\varphi_0} = \frac{1}{\sin\varphi_0}$$

Synchrotron frequency

$$\underbrace{\left(\frac{2\pi h\eta e}{\beta^2 T_0^2 E_0}\cdot U_0\cos\varphi_0\right)}_{=\Omega_S^2}$$

a) if η -> 0, there is no longitudinal motion
b) a change in voltage, changes the frequency
c) a change in h, as for the bunch splittings, changes the synchrotron frequency

And if we go linear?

btw... before this .... it is clear that:

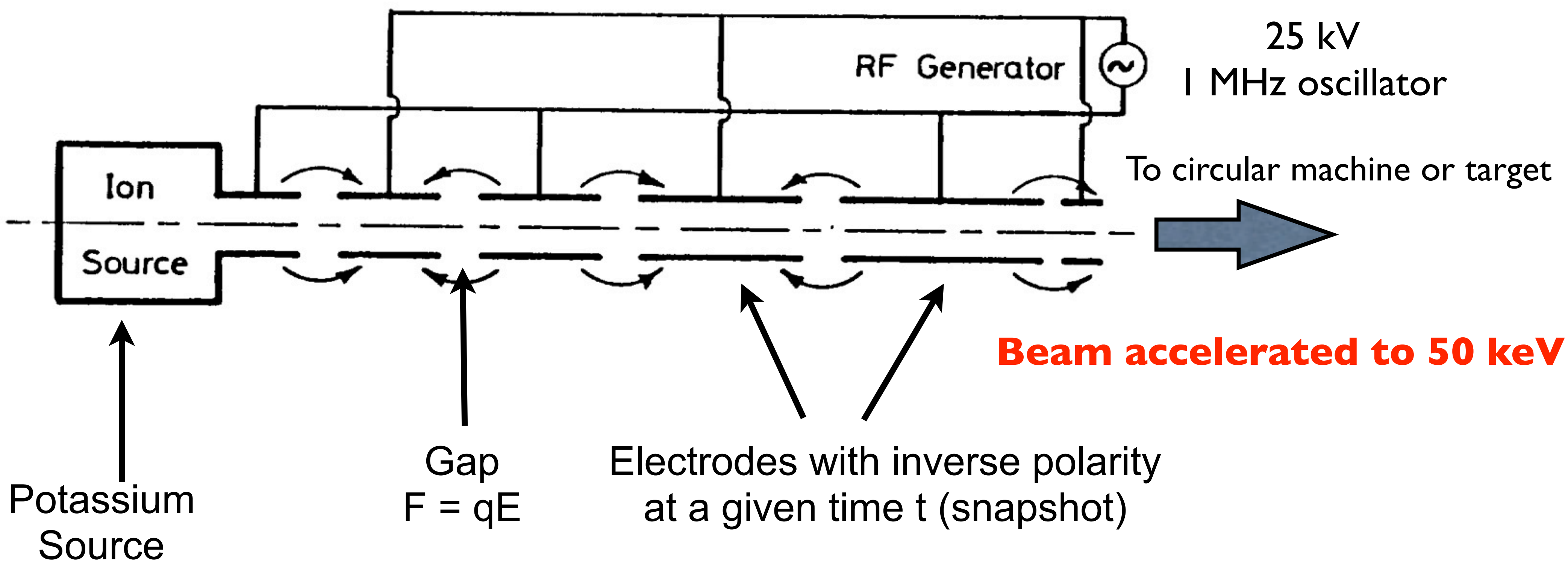
a) in a bunch+separatrix plot the acceleration cannot be see except by looking to the shape of the separatrix.
The (0,0) is the reference particle, with changing energy.
The reference particle has a SYNCHRONOUS PHASE $\phi_s=\phi_0$ wrt the RF

b) to accelerate the beam MUST be bunched

c) Transition in a ring is a bad moment.... actually, typically we try to inject far away from transition energy.... above typically

c) Referendum: Why we use RF fields and not static fields?

Wideroe linac: the first linear accelerating structure

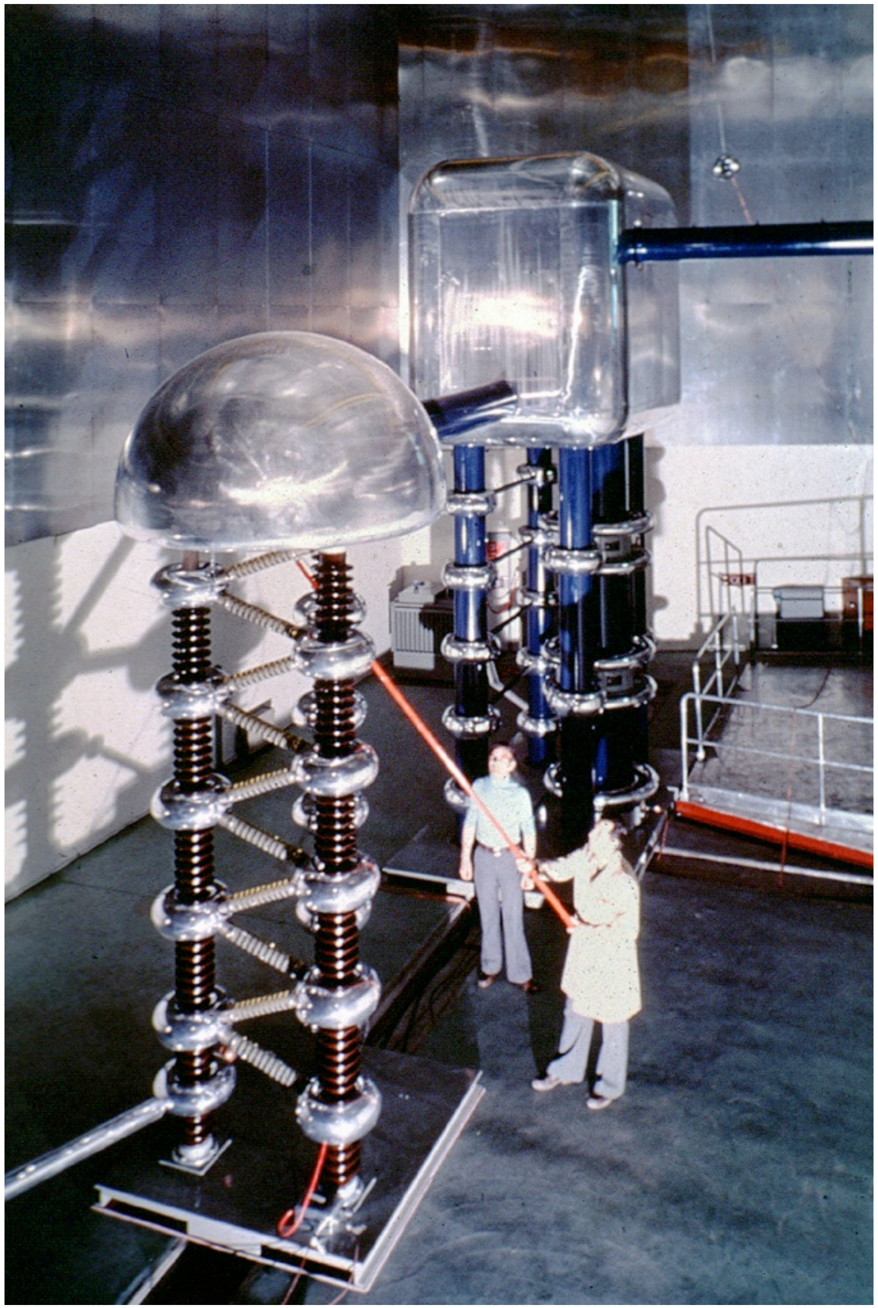


First linac composed by drift tubes interleaved by acceleration gaps powered by an RF generator. (1928)

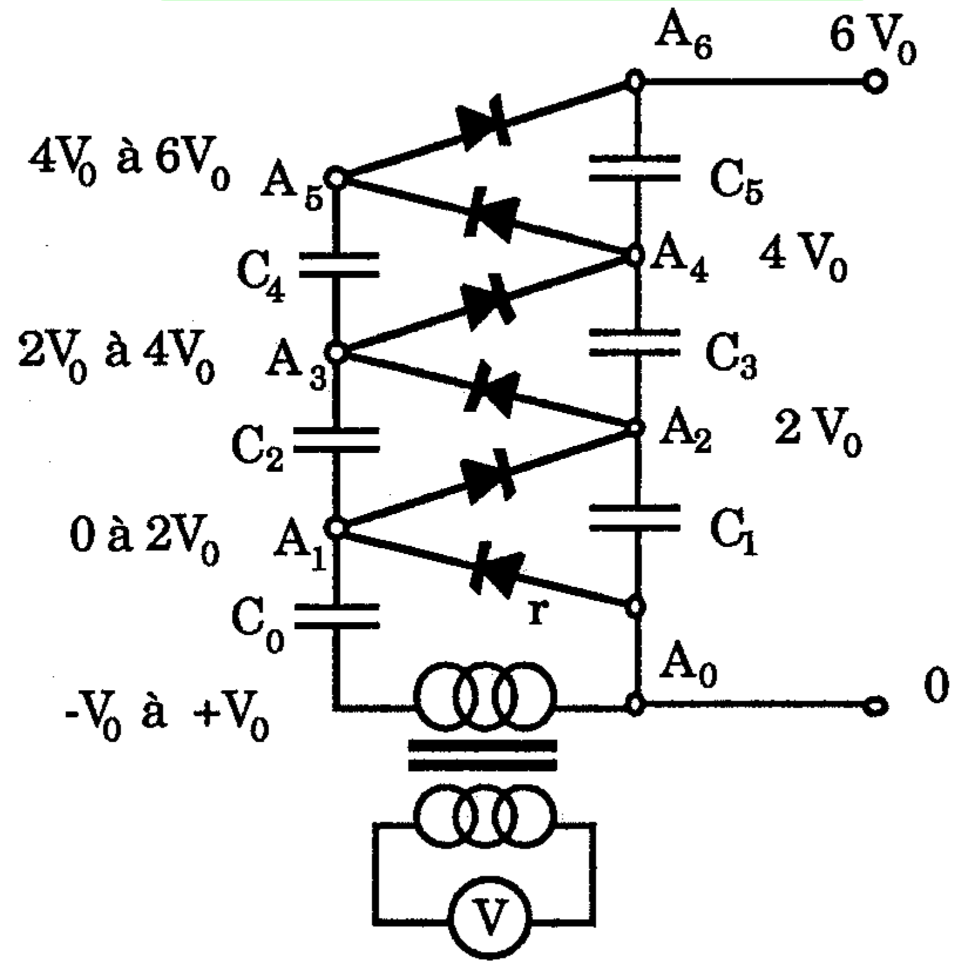
Obs: the drift tube length has to increase because particles are not yet relativistic. To an energy increase corresponds a speed increase, and the particle has to travel more in the shielded region to be in phase with the accelerating field.

Main limitation: after a certain energy, the length of the drift tube is too long. The RF frequency has increase to some 10 MHz, need to enclose the structure in a resonator to avoid field losses.

Cockroft-Walton. Old CERN proton pre-injector



High voltage unit composed by a multiple rectifier system



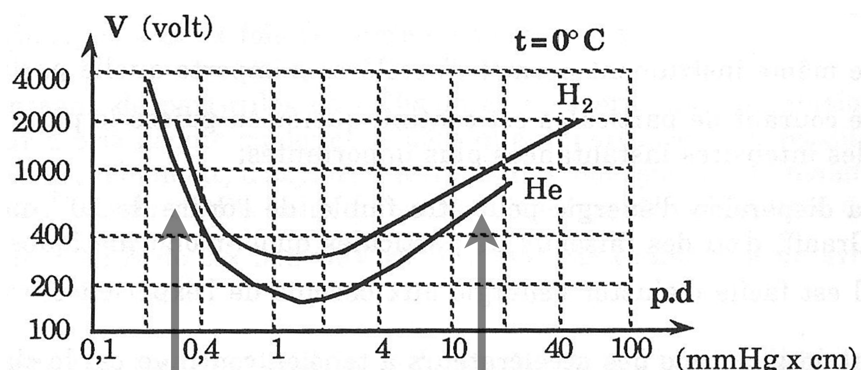
CERN: 750 kV, used until 1993

Bits an pieces are in the garden outside the Microcosm

Main limitation

Main limitation:
electric discharge due to too high Voltage.
Maximum limit: 1 MV

Limit set by Paschen law:
the breaking Voltage between two parallel electrodes depends only on the pressure of the gas between the electrodes and their distance



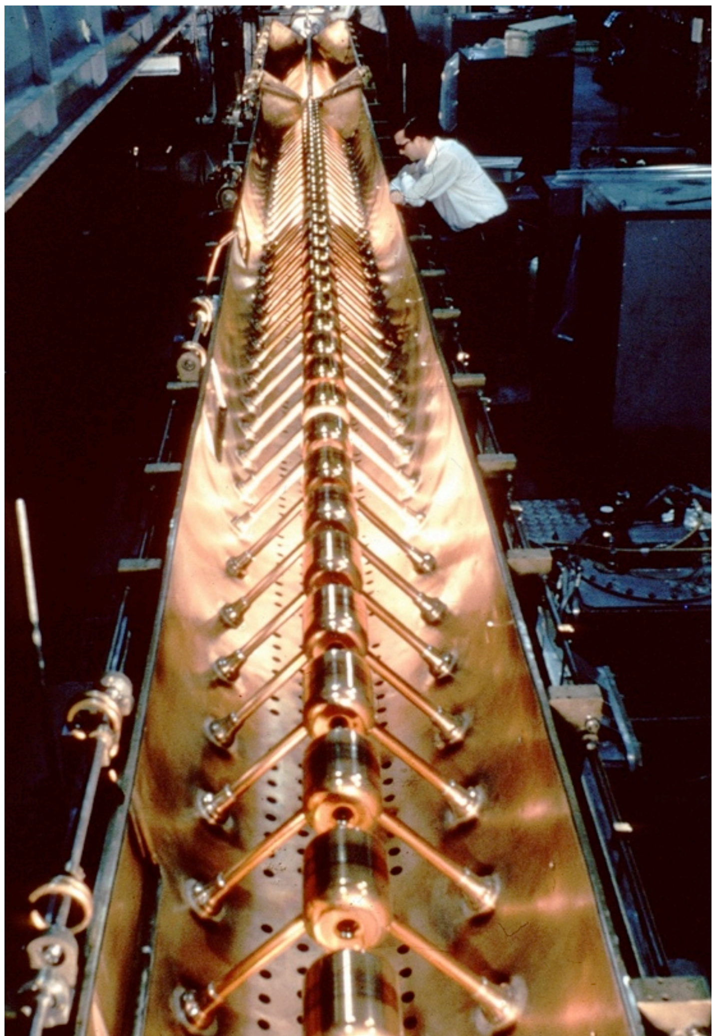
Low pressure: gas not too dense, long mean average path of electrons

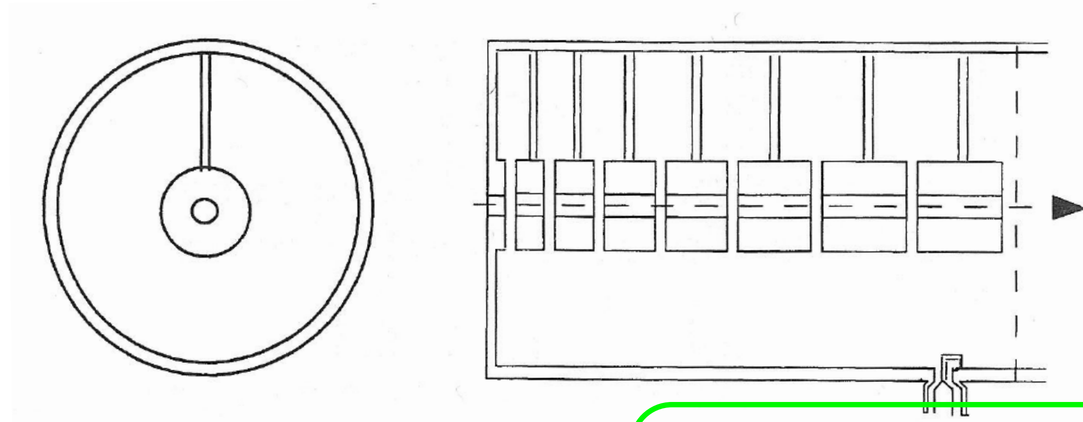
High pressure: dense gas, large Voltage needed for gas ionisation

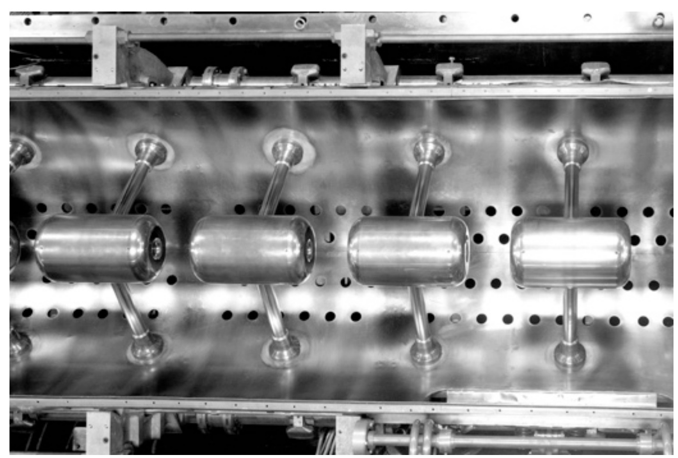
Alvarez drift tube linac

Linac composed by ***drift tubes*** interleaved by ***acceleration gaps*** as Wideroe linac, but field generated in a ***resonant cavity***. The frequency of the field can go up to 200 MHz.

Currently we have ***two Linacs at CERN*** with Alvaretz structure, ***for protons and ions***.





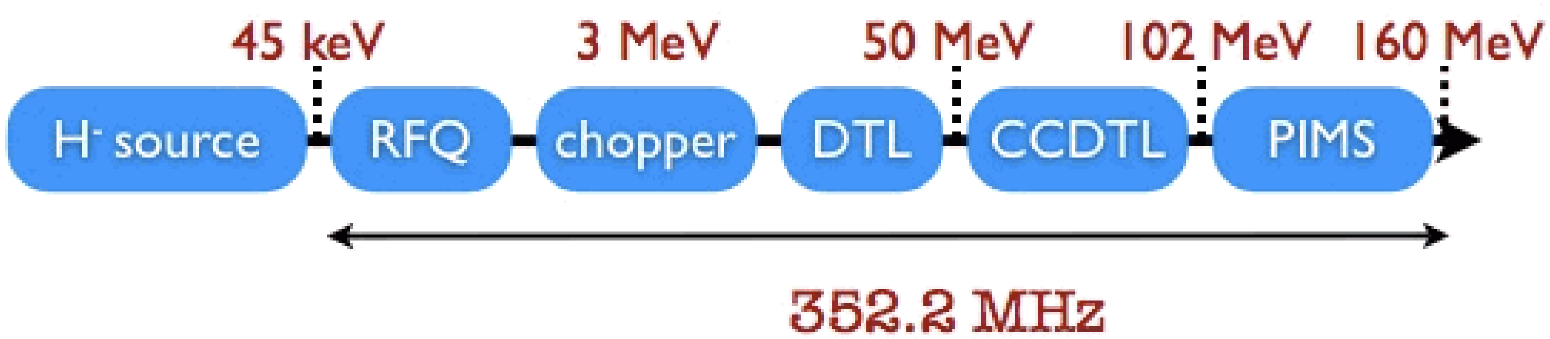


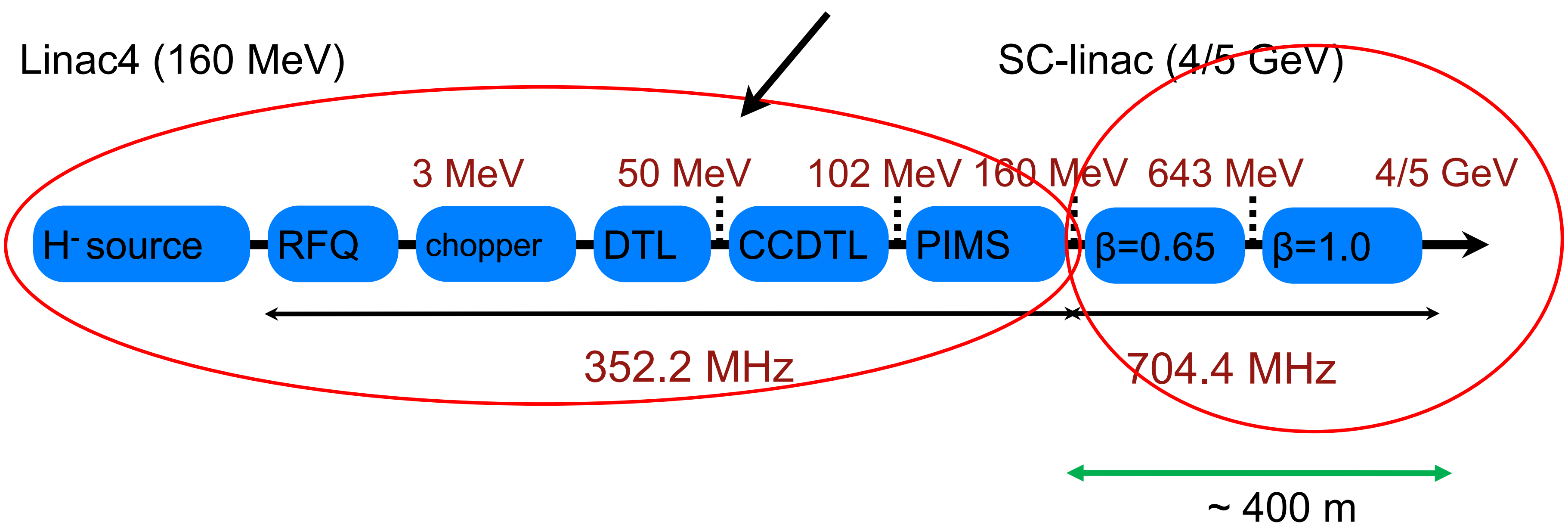
Inner structure of Linac 1 (Alvarez type). **The drift tubes are supported on stems, through which the current for the quadrupole magnets** (located inside the tubes) and the cooling water are supplied.

Linac 1 accelerated protons to 50 MeV.

Proton RF linac: LINAC4

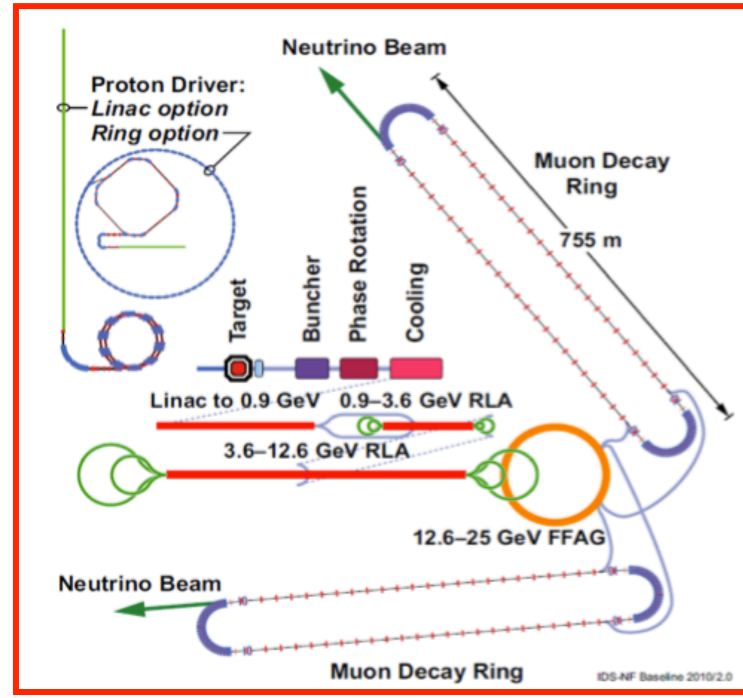
Proton Linacs used as low energy injectors. L4 in construction at CERN as been conceived as the low energy part of the Linac for the Nufact



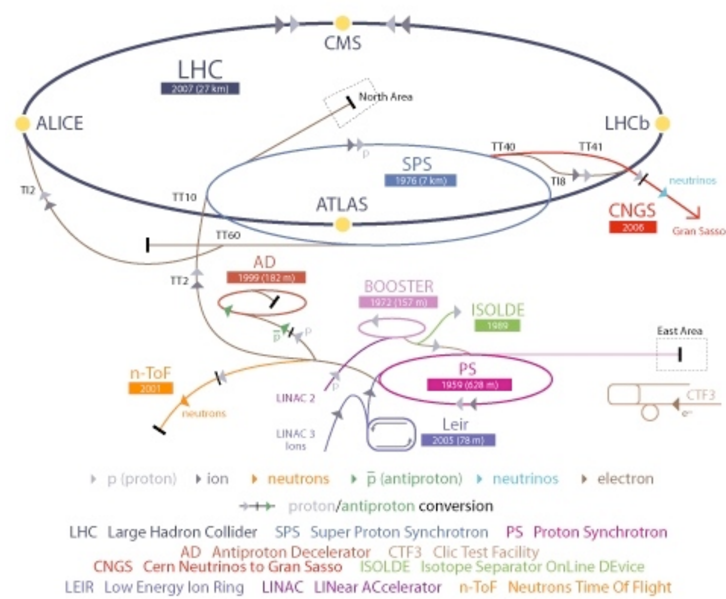


Why not linear all the time...

- Accelerating by linear structure is fine, but:
  - not so efficient... every cavity is used just for a single passage if very high energy required
  - the total length determine the maximum energy
    - very expensive in term of tunnel and infrastructure for a given energy
    - High energy part must be done with rings (not true for electrons... for synchrotron radiation issues...)
    - **Low energy part can be done by re-circulating through the same linear structure**



CERN Accelerator Complex

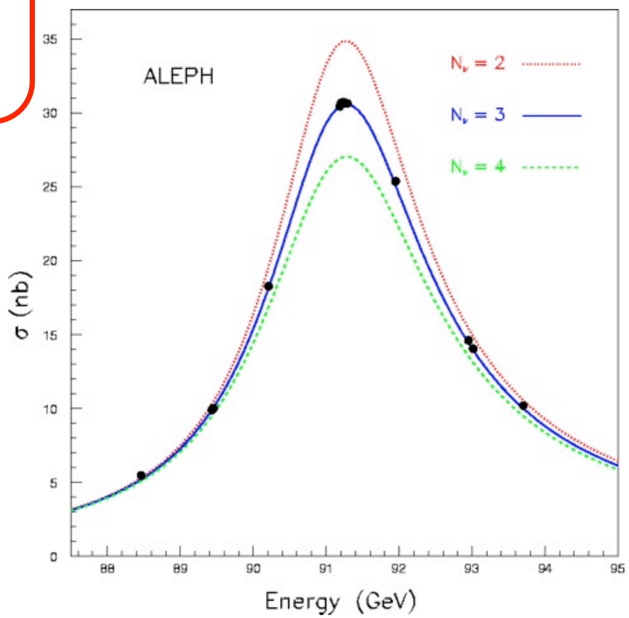


What can influence an accelerator?

The physics case:
the Z mass at LEP has been measured with an error of 2 MeV.
Energy of the accelerator has to be know better than 20 ppm.

Energy measurements obtained by during last years of LEP operation

Nominal (GeV)	E_{CM} (LEP) (GeV)
181	180.826 ± 0.050
182	181.708 ± 0.050
183	182.691 ± 0.050
184	183.801 ± 0.050
Combined	182.652 ± 0.050



What can influence the energy of a collider?

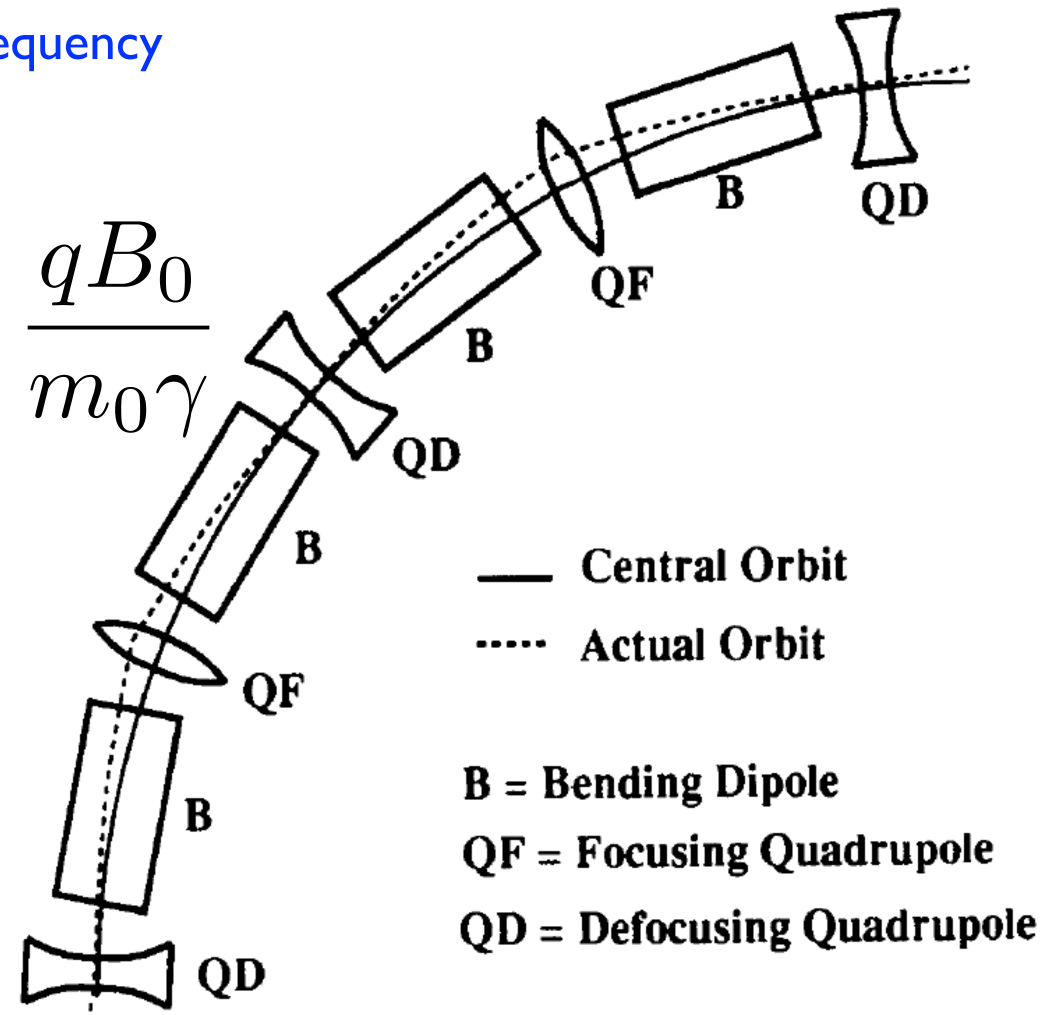
“Rappel”of strong focusing synchrotron optics

Stable orbit is bent by the main dipoles, centered in the quadrupoles, no field

Energy fixed by bending strength and cavity frequency

$$f_{RF} = h \cdot f_{rev}$$

$$f_{rev} = \frac{v}{C_c} = \frac{v}{2\pi\rho} = \frac{1}{2\pi} \cdot \frac{qB_0}{m_0\gamma}$$



A variation of the Circumference C induces changes in the energy proportional to α, the momentum compaction factor.

$$\frac{\Delta E(t)}{E_0} = -\frac{1}{\alpha} \frac{\Delta C(t)}{C_c}$$

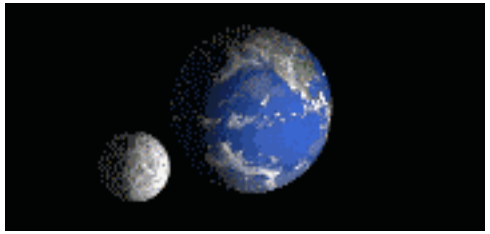
In LEP $\alpha = 1.86\ 10^{-4}$ a small variation the circumference induces a large variation in energy

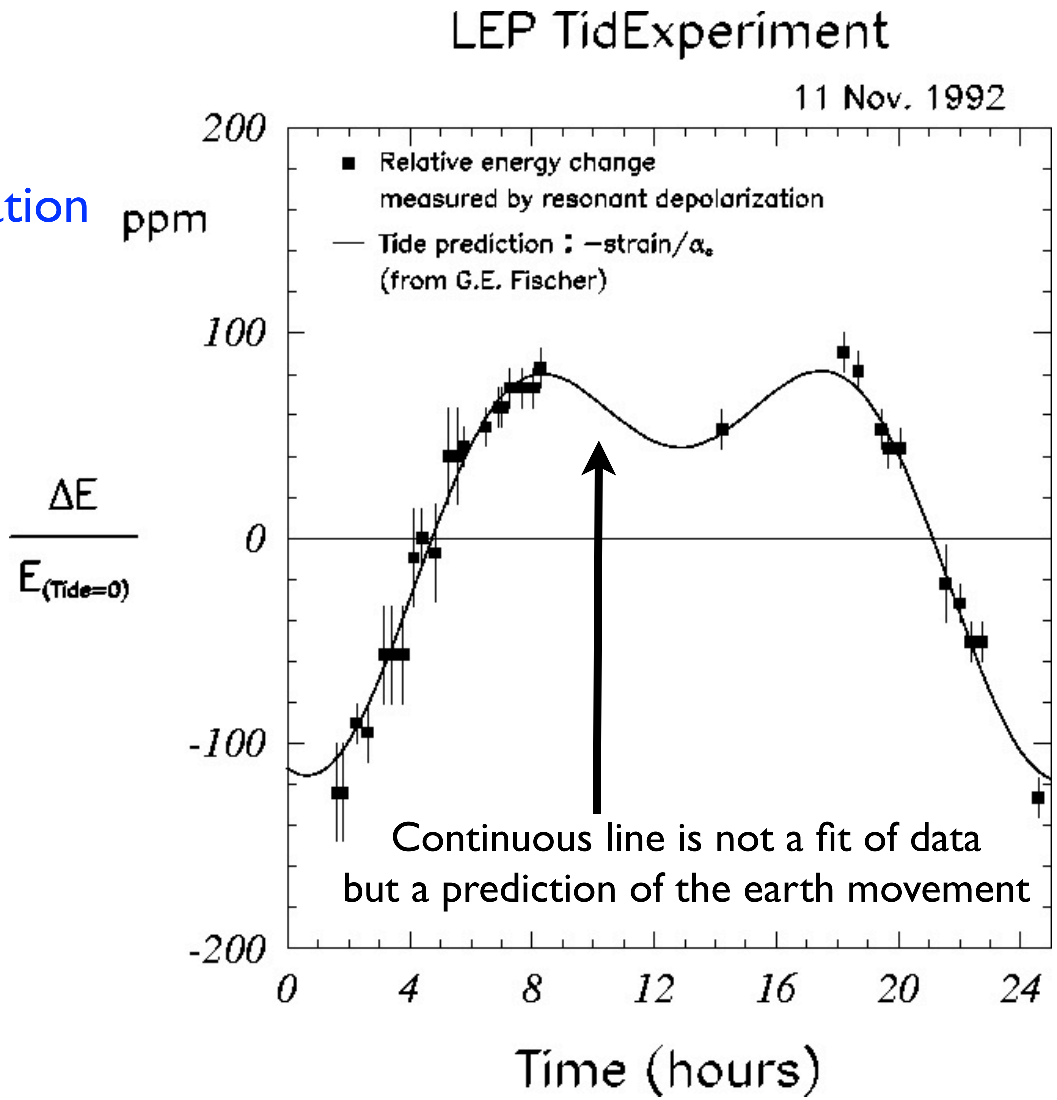
Moon tides can change earth geometry

Moon induces a earth deformation similar to water tide.

Total deformation of the LEP about 4 mm
Energy variation of 100 ppm

The 12 h cycle is due to the earth deformation





The effect is modulated by the different tide intensities and by the SUN tides

[redacted]

Beam collective effects and instabilities

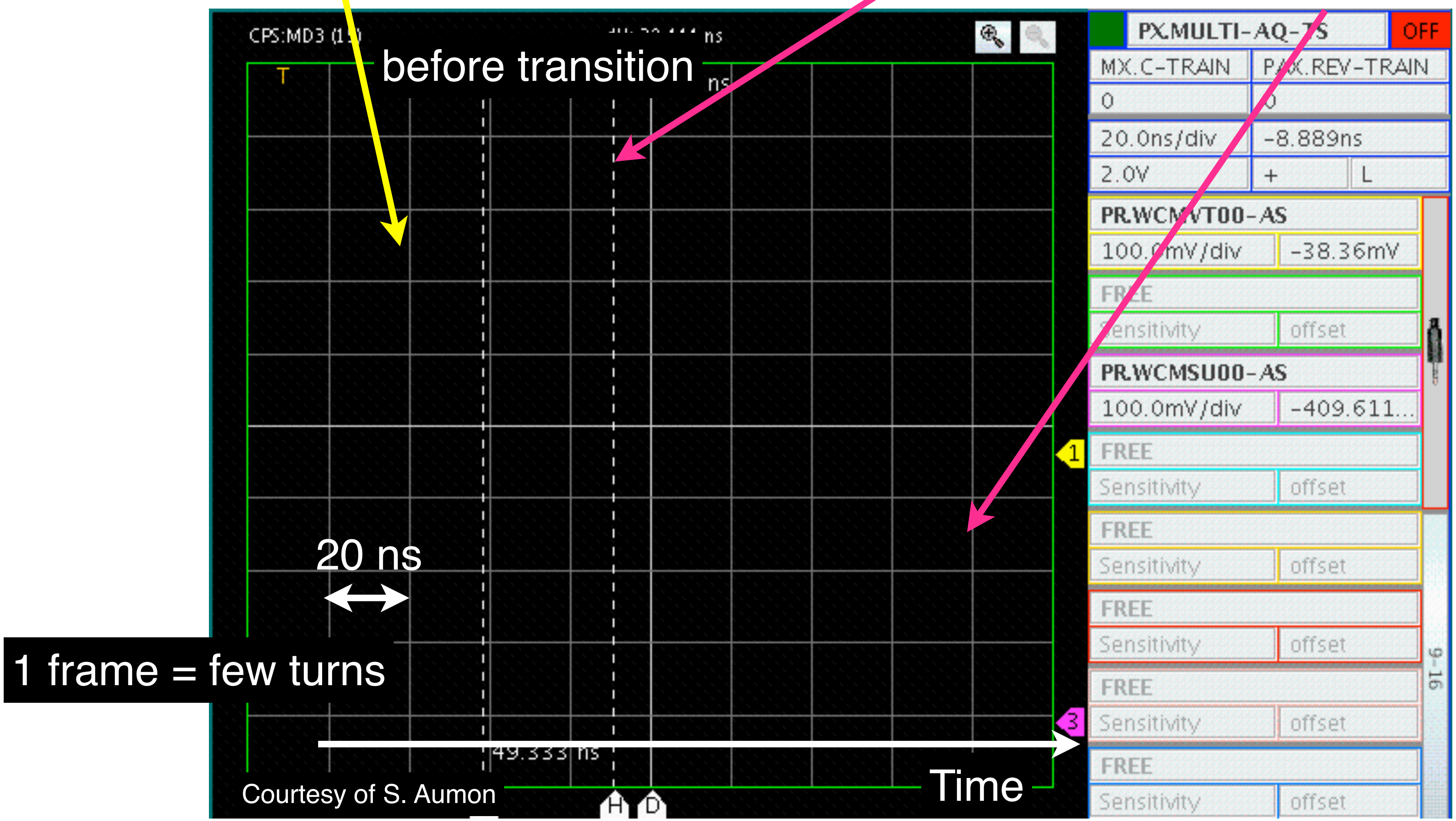
So far, beam living either in “single particle” world: every particle do not see all the others in terms of Coulomb interaction or the surrounding world...

Actually, this is a bit too optimistic... see example below

Beam losses
A fraction of the beam touched the vertical aperture

Vertical position of the beam within a bunch

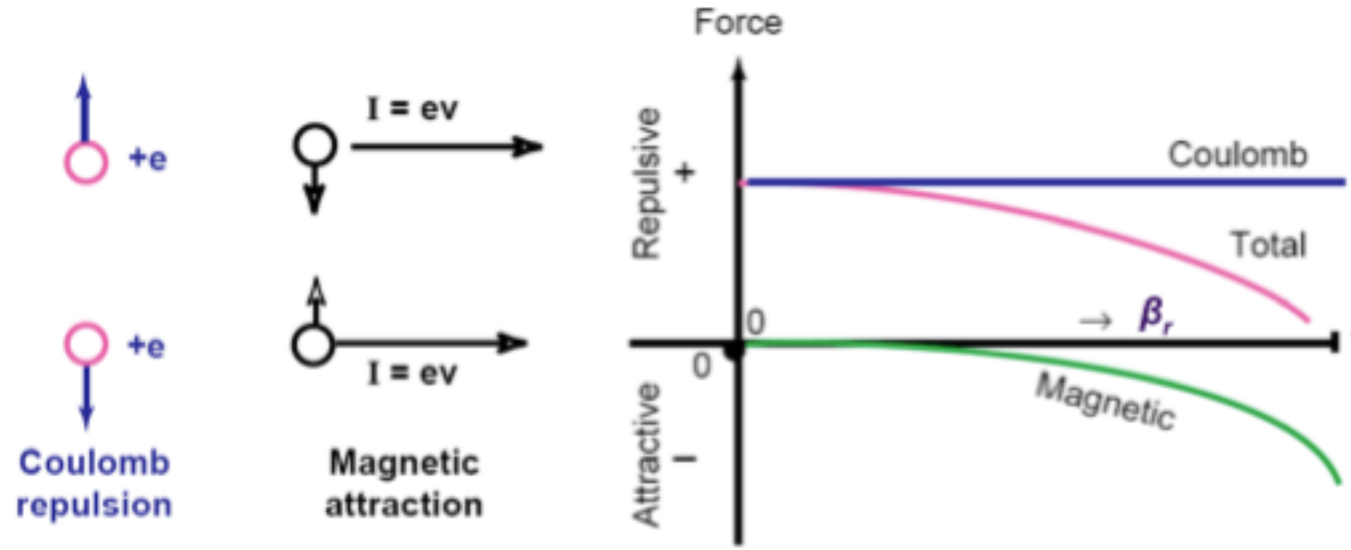
Bunch intensity



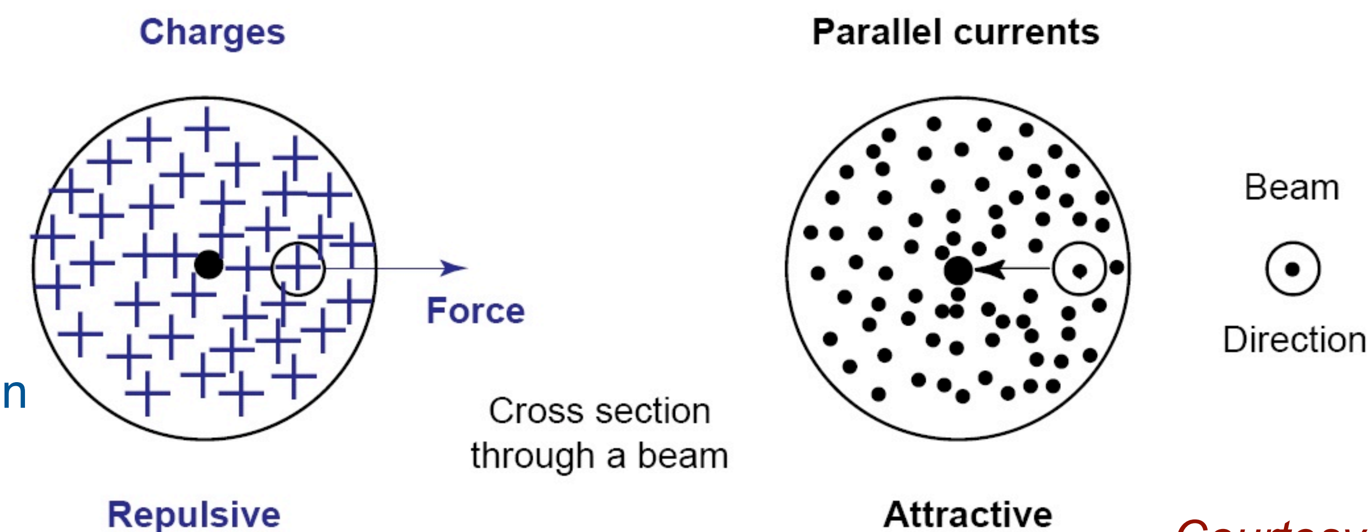
Courtesy of S. Aumon

First effect: Space charge

Two particles in free space

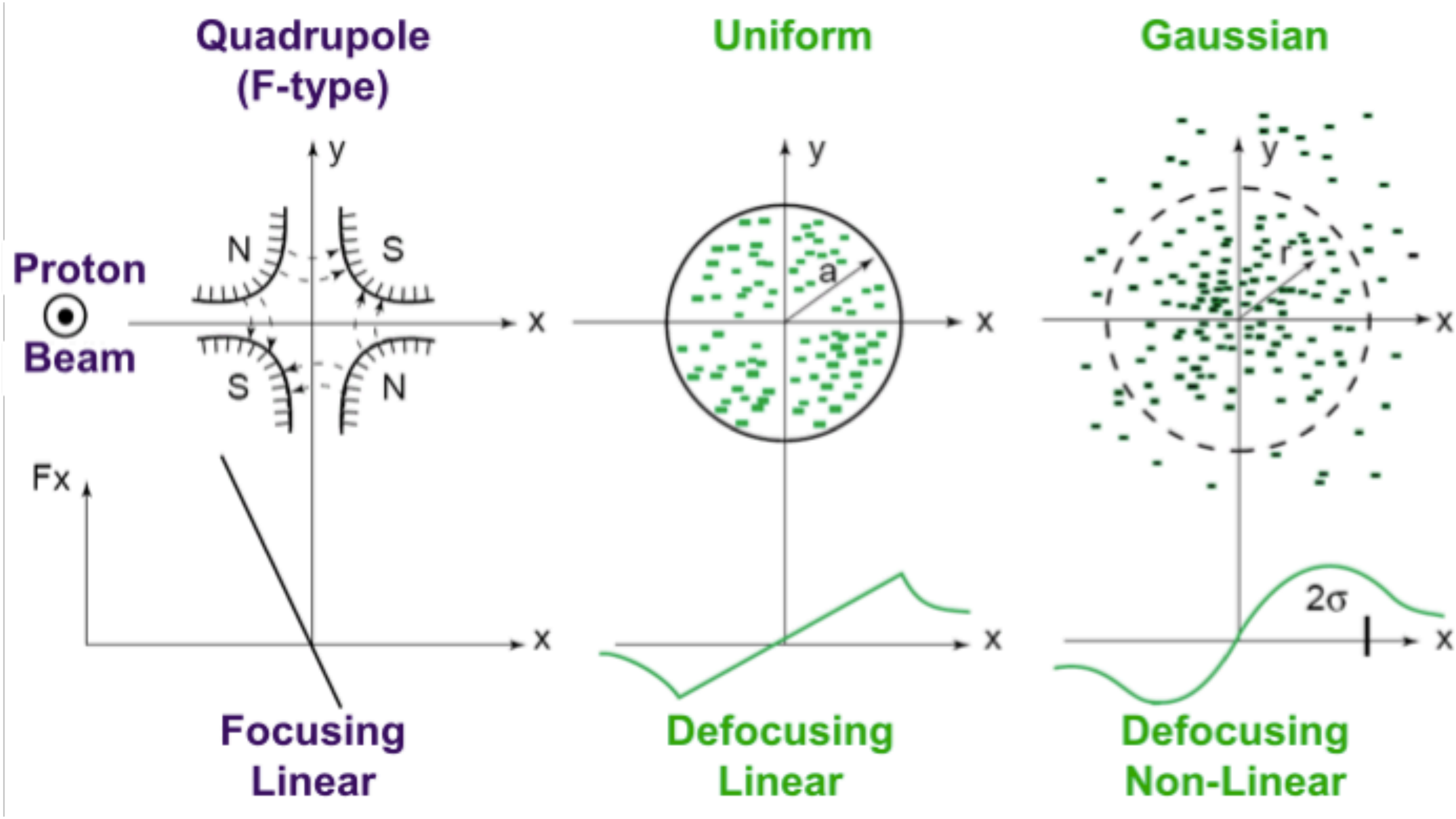


Many charged particles traveling in an de-bunched beam with circular cross-section feel the average field due to the many charges grouped together



Courtesy K. Schindl

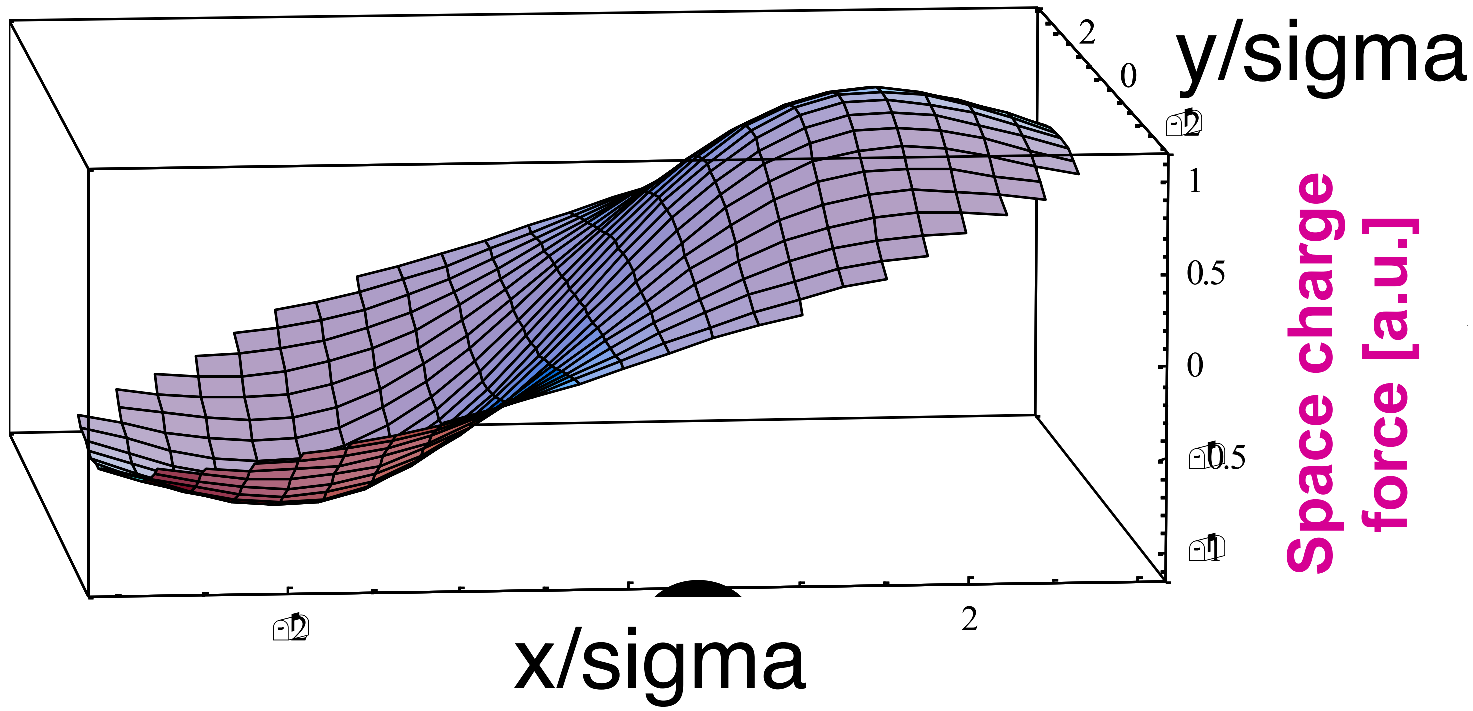
Space charge effect



The interaction between the particles within the same bunch acts as a defocusing quadrupole. This is because the **INCOHERENT SPACE CHARGE FORCE is generated by the repulsion between particles with the same charge**

The center of the distribution feels zero force due to the symmetry of the distribution

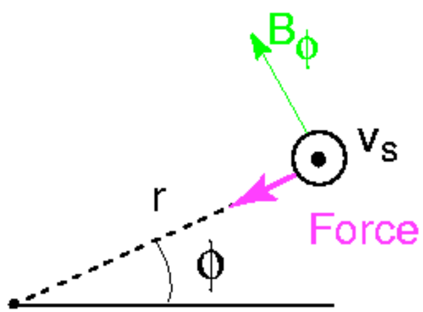
Space charge force on a bunch



Example of a beam of a transverse size equals to sigma extending up to about 3.5 sigmas

Proof that space charge has a deleterious effect

Force felt by a test particle within a bunch



From Maxwell equations

(beer or look spare slides..)

$$F_r = e(E_r - v_s B_\varphi)$$

$$F_r = \frac{eI}{2\pi\varepsilon_0 \beta c}(1-\beta^2)\frac{r}{a^2} = \frac{eI}{2\pi\varepsilon_0 \beta c^2}\frac{1}{\gamma^2}\frac{r}{a^2}$$

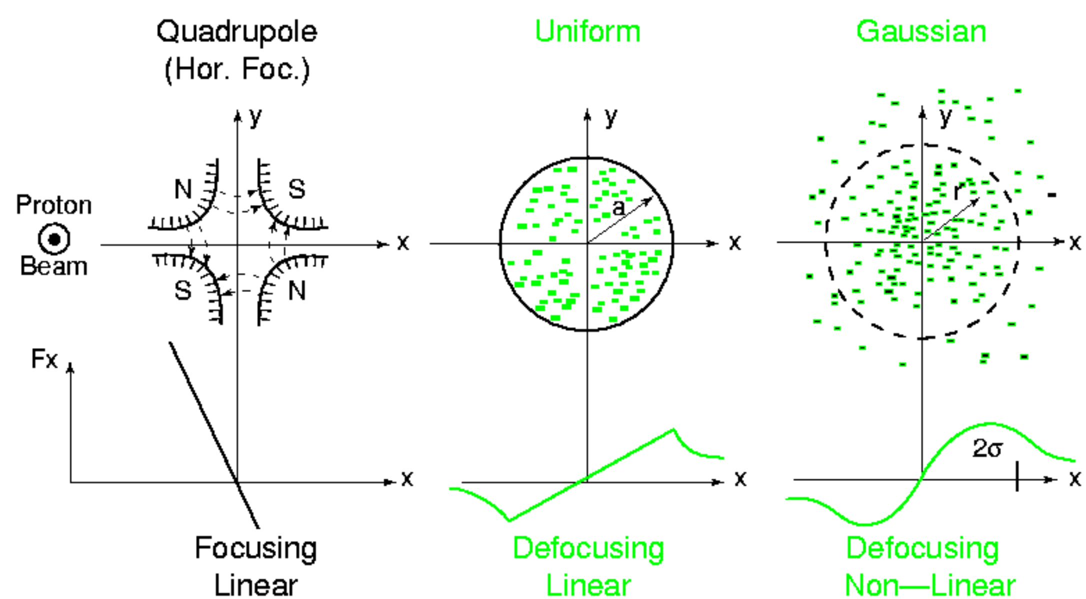
Electric magnetic

$$F_x = \frac{eI}{2\pi\varepsilon_0 \beta c \gamma^2 a^2} x$$

$$F_y = \frac{eI}{2\pi\varepsilon_0 \beta c \gamma^2 a^2} y$$

Space charge force

- ❑ circular beam
- ❑ uniform charge density
- ❑ F_x, F_y linear in x, y
- ❑ force → 0 for $\gamma \gg 1$ $(\beta \to 1)$
- ❑ defocusing lens in either plane



from E. Wilson

It is not like a simple quadrupole, so it cannot be compensated!

Applying the SC gradient to Hill's equation

Here used to start understanding

- Beam not bunched (so no acceleration)
- Uniform density in the circular x-y cross section (not very realistic)

$$x'' + \left(K(s) + K_{SC}(s)\right)x = 0 \quad \Rightarrow Q_{x0} \text{ (external)} + \Delta Q_x \text{ (space charge)}$$

For small "gradient errors" k_x

$$\Delta Q_x = \frac{1}{4\pi}\int_0^{2R\pi} k_x(s)\beta_x(s)ds = \frac{1}{4\pi}\int_0^{2R\pi} K_{SC}(s)\beta_x(s)ds$$

$$\Delta Q_x = -\frac{1}{4\pi}\int_0^{2R\pi}\frac{2r_0 I}{e\beta^3\gamma^3 c}\frac{\beta_x(s)}{a^2}ds = -\frac{r_0 RI}{e\beta^3\gamma^3 c}\left\langle\frac{\beta_x(s)}{a^2(s)}\right\rangle = -\frac{r_0 RI}{e\beta^3\gamma^3 cE_x}$$

$$\boxed{\Delta Q_{x,y} = -\frac{r_0 N}{2\pi E_{x,y}\beta^2\gamma^3}}$$

using $I = (Ne\beta c)/(2R\pi)$ with
N…number of particles in ring
$E_{x,y}$….emittance containing 100% of particles

- "Direct" space charge, unbunched beam in a synchrotron
- Vanishes for $\gamma \gg 1$
- Important for low-energy machines
- Independent of machine size $2\pi R$ for a given N

from E. Wilson

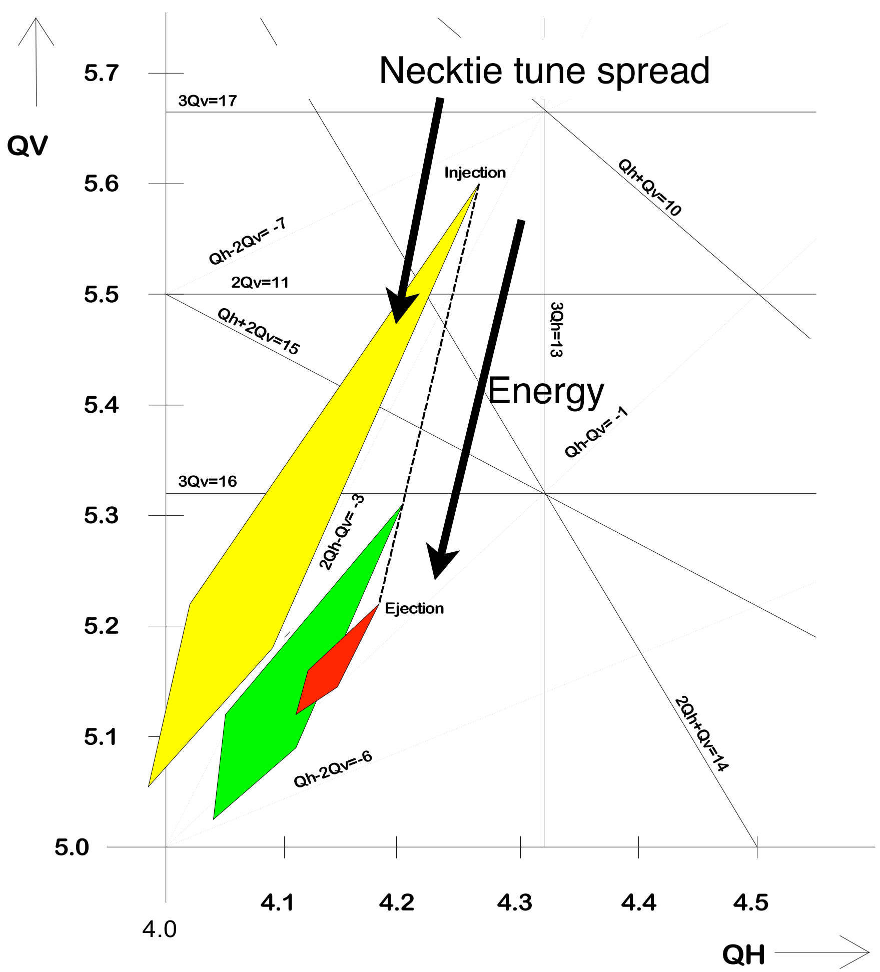
Tune footprint

Depending on the position of the particle within the bunch, the tune is not the one given by the quadrupoles but it contains a contribution from space charge.

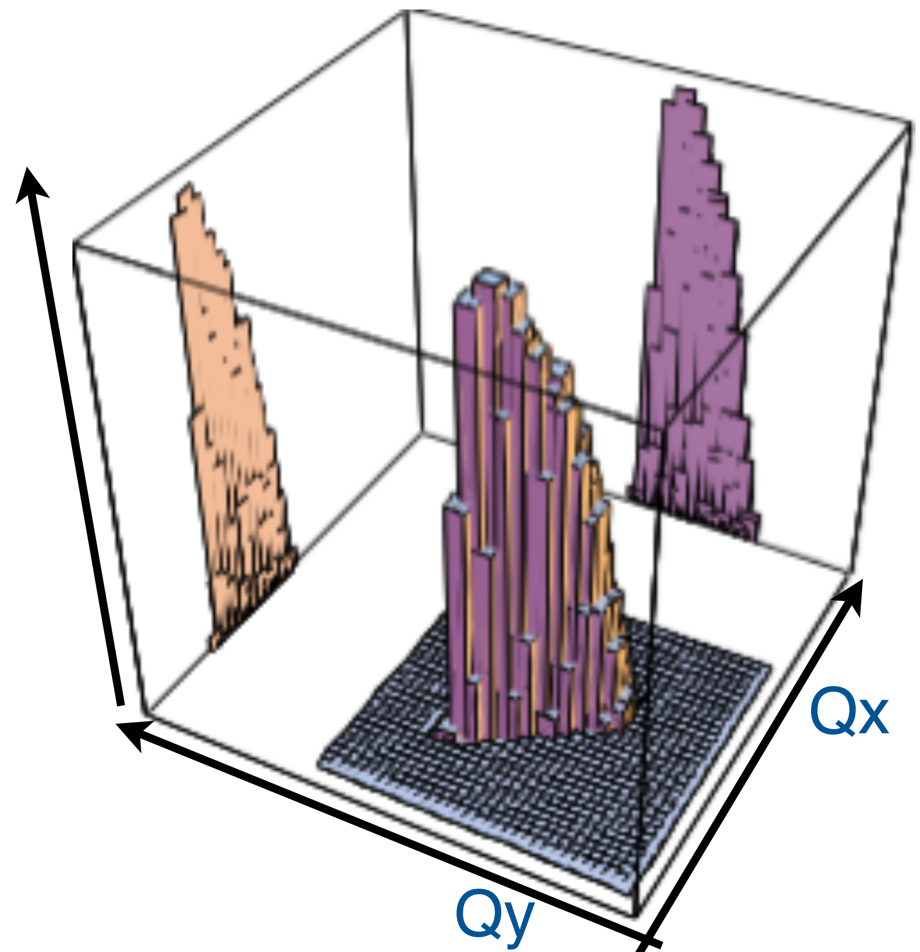
Imp: The tune shift decreases with the energy

$$\Delta Q_{x,y} = -\frac{r_0 N}{2\pi E_{x,y}\beta^2\gamma^3}$$

Evolution of the tune shift in the PSB during the magnetic cycle

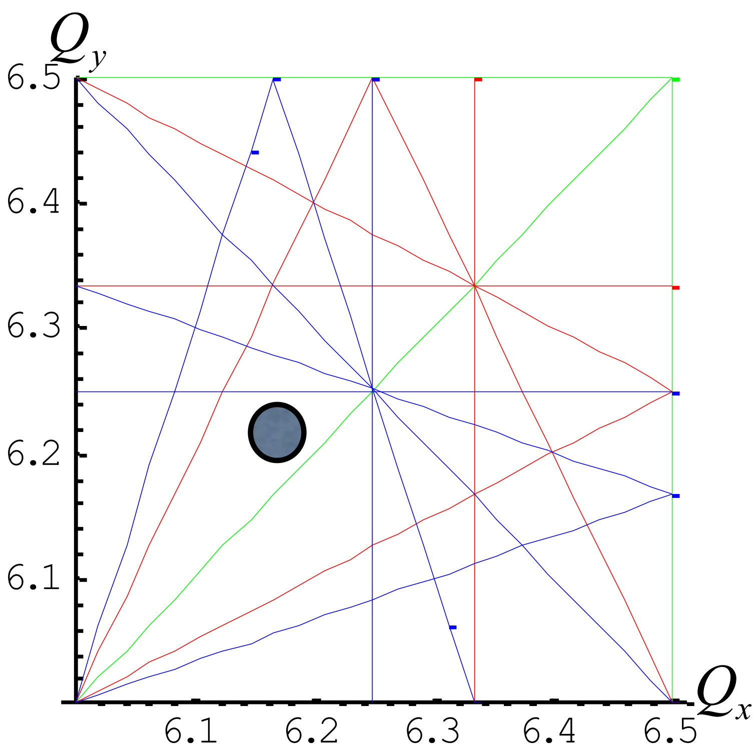


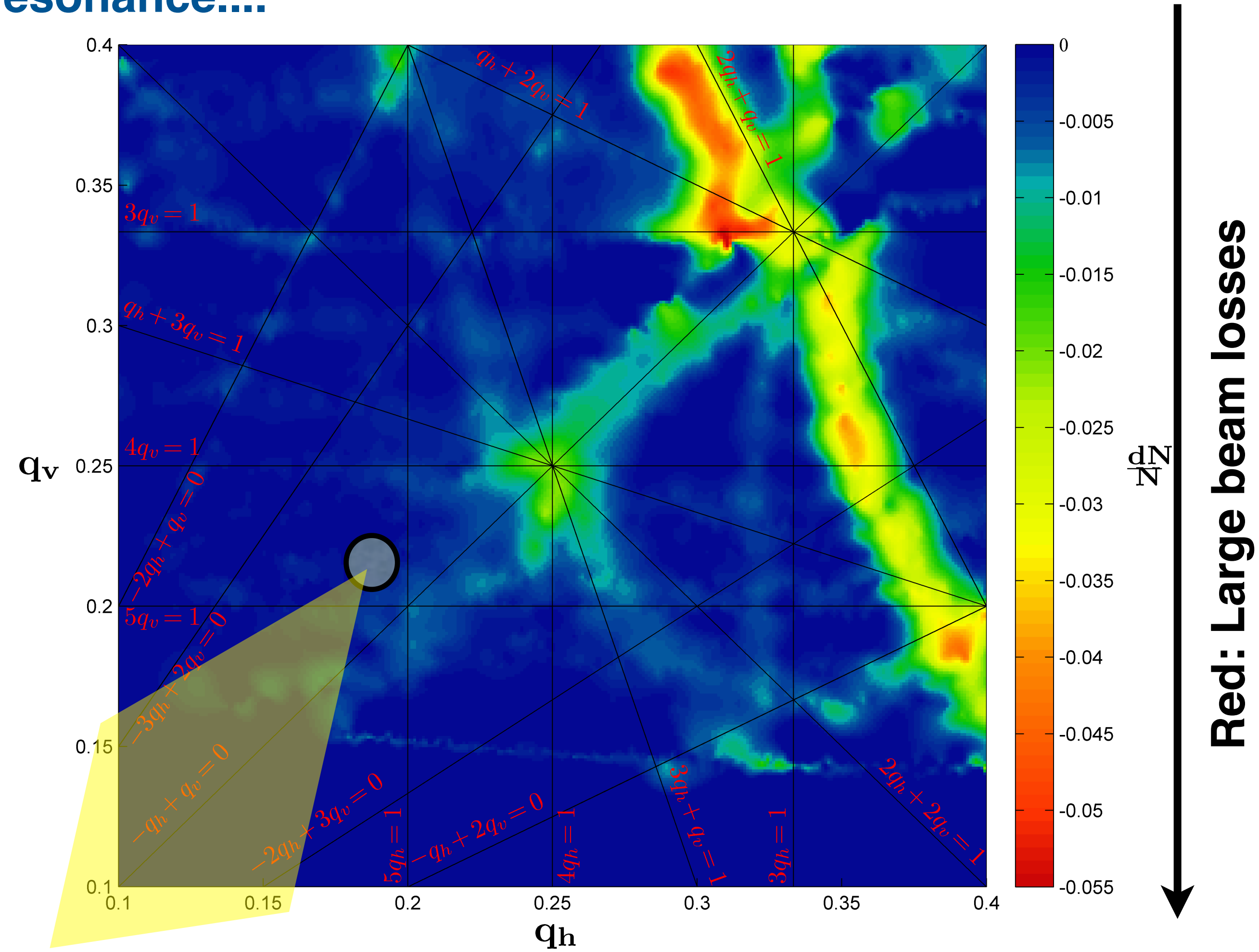
3D footprint



A practical example ... the PS

Typical working point choose for the PS: with a space charge of -0.3 we cross the integer resonance....





Courtesy of A. Huschauer

Why is important? Maximum intensity that can be injected in every machine depends on the maximum tune shift before ending up on a resonance (see PS and Beta Beam)

How to cure space charge?

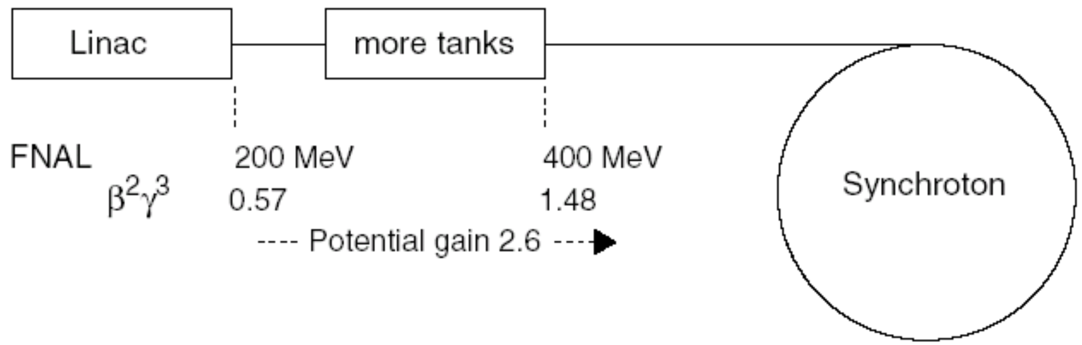
Direct space charge

$$\Delta Q_y \approx \frac{N}{E_y \beta^2 \gamma^3} \frac{\hat{I}}{\bar{I}}$$

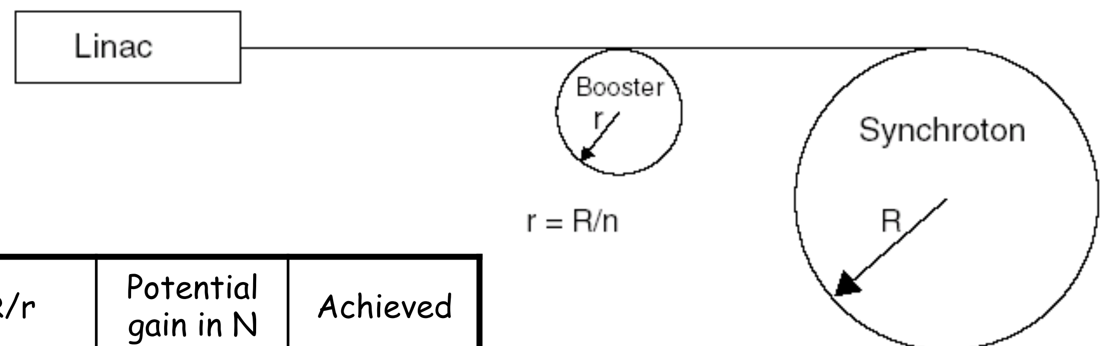
Problem: A large proton synchrotron is limited in N because ΔQ_y reaches 0.3 … 0.5 when filling the (vertical) acceptance.

Solution: Increase N by raising the injection energy and thus $\beta^2\gamma^3$ while keeping to the same ΔQ. Ways to do this:

Make a longer (higher-energy) Linac (by adding tanks as has been done in Fermilab)



Add a small "Booster" synchrotron of radius r = R/n with n the number of batches (BNL) or rings (CERN)

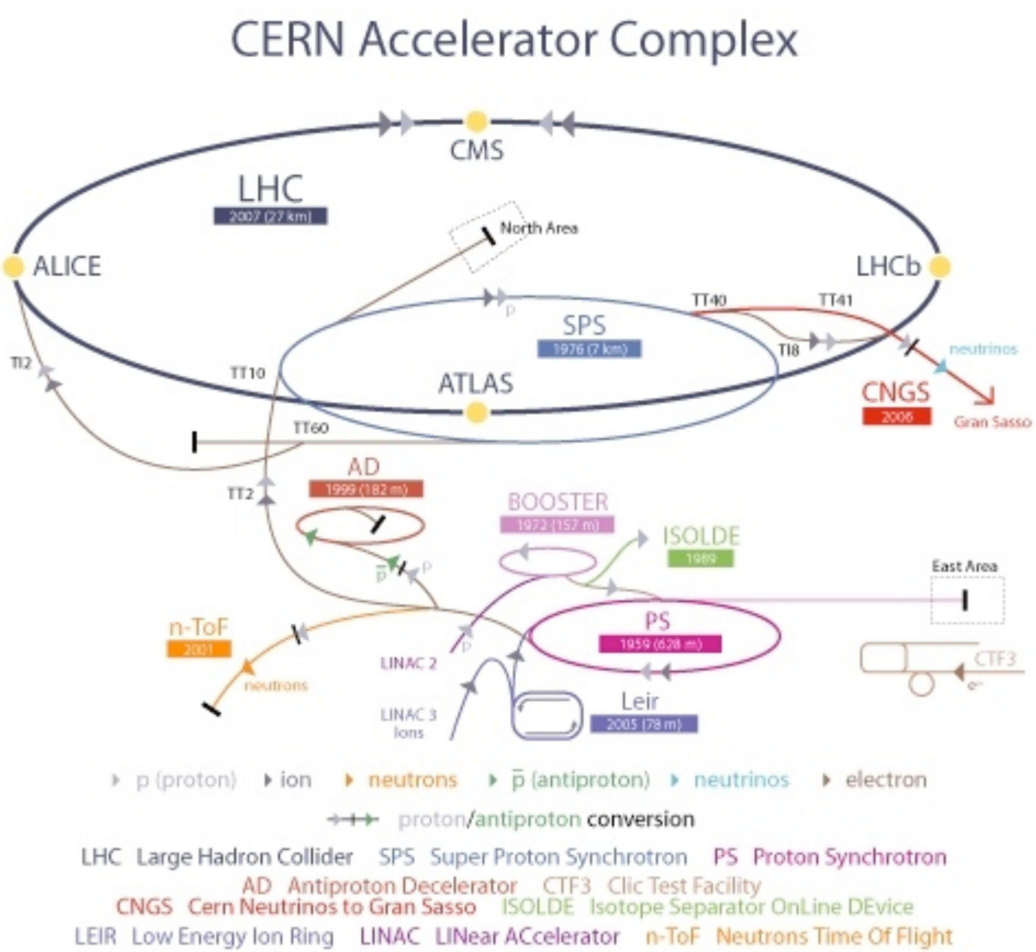


	Linac (MeV)	Booster (GeV)	n=R/r	Potential gain in N	Achieved
CERN PS	50	1	4(rings)	59	~15
BNL AGS	200	1.5	4(batches)	26	~8

I could not produce a better one than from E. Wilson

The solution: SPEND MONEY TO INJECT ALWAYS BEYOND THE SPACE CHARGE LIMIT i.e., AT THE BEST POSSIBLE ENERGY FOR THE INTENSITY AND EMITTANCE DESIRED

Brief excursus... now we know why an injector chain



The same arguments apply for the Nufact and Beta Beam accelerator complex

The LHC injection energy is 450 GeV/c, collision energy will be 7 TeV per beam (today 3.5 TeV)

Question: Why I cannot have just one accelerator from a proton source up to collision energies?

Amongst less obvious ones:

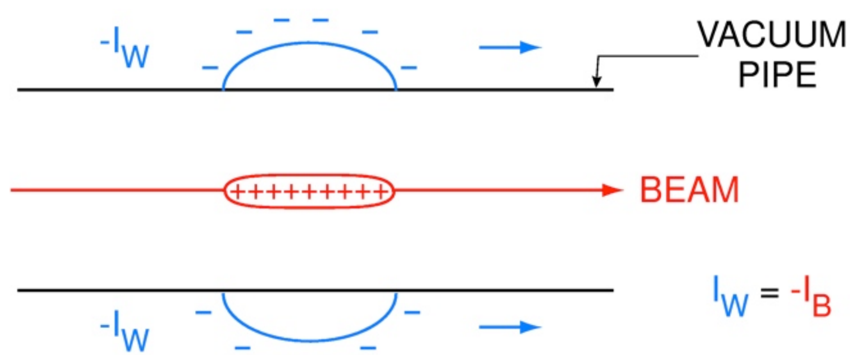
a) The dipoles (magnets) should be able to go to from practically zero field to very high ones

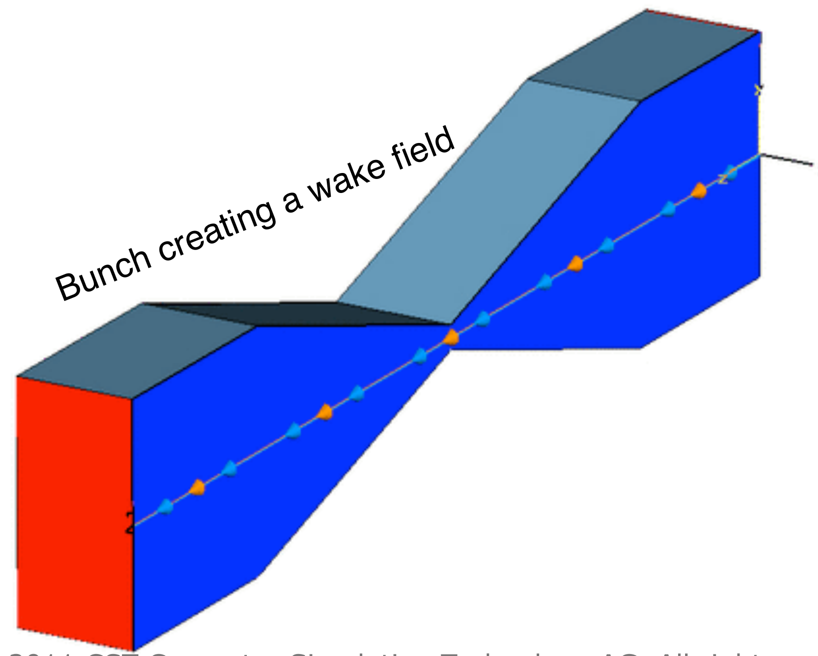
b) There should be a very large number of RF cavities with different frequency for the very large change in revolution frequency

c) The space charge effect would risk to blow up the beam emittance by crossing forests of resonances during acceleration creating losses or spoiling the beam quality

Impedance: beam coupling with surroundings

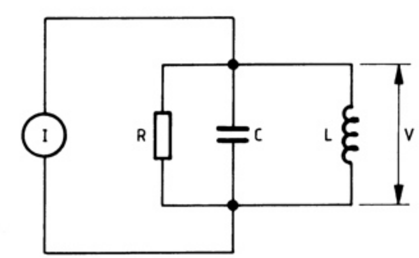
- **Wake fields** = Electromagnetic fields generated by the beam interacting with its surroundings (vacuum pipe, etc.) creating
  - Energy loss
  - Beam instabilities
  - Excessive heating





- For a collective instability to occur, the beam environment must not be a perfectly conducting smooth pipe.

- Impedance (Sessler&Vaccaro) = Fourier transform of the wake field. **The coupling between surrounding element of the machine and the beam is described in term of Impedance Z, as the one of an equivalent electrical circuit (R, C, L)**



An example of instability: TMCI

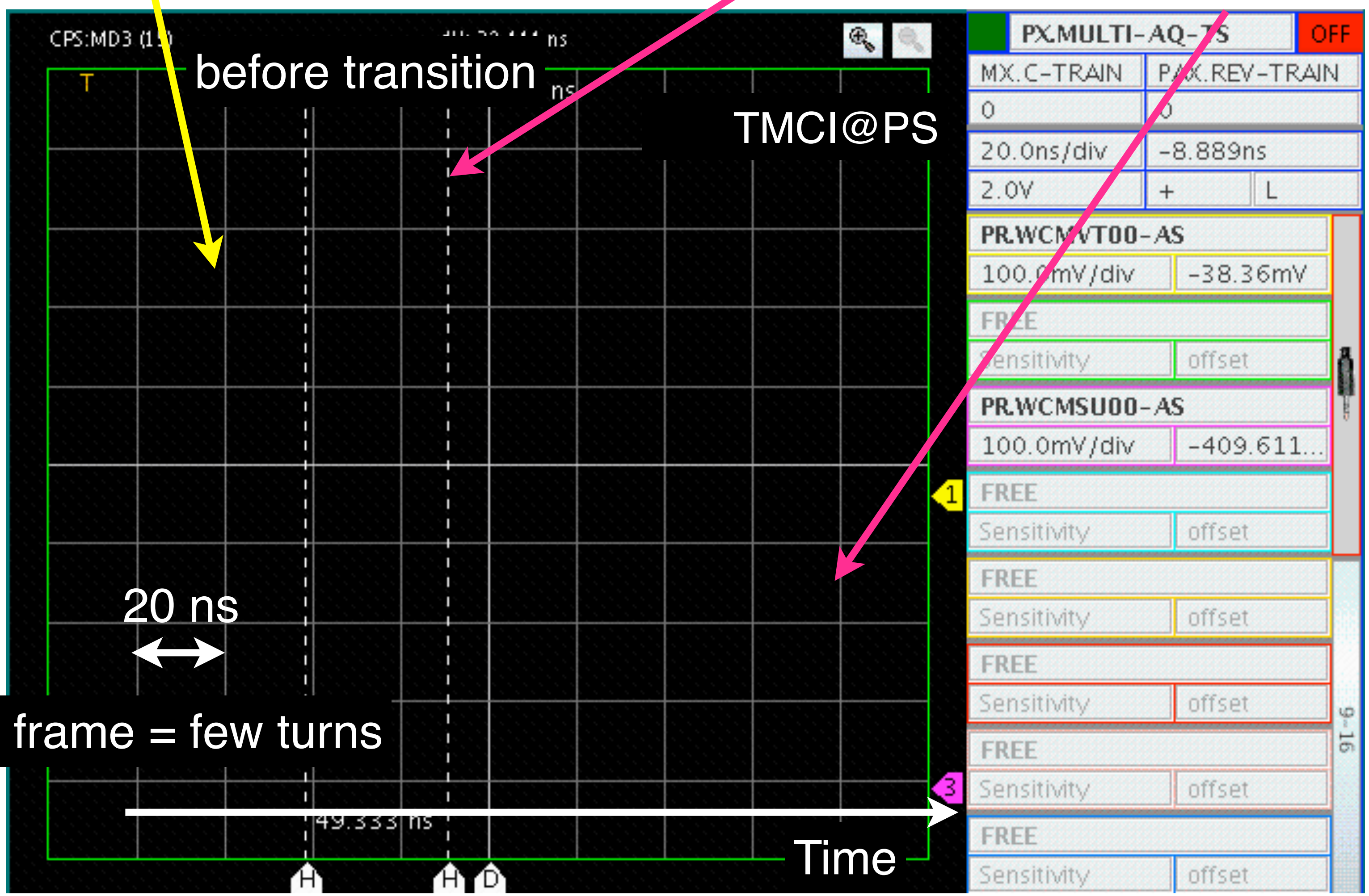
The head of the bunch create a wakefield in the **vertical plane** which excites the tail of the bunch. **This instability is defined as TMCI: Transverse Mode Coupled Instability because the wave on the bunch is not stationary.**

Why I care? Limitation of the SPS for the Beta beams

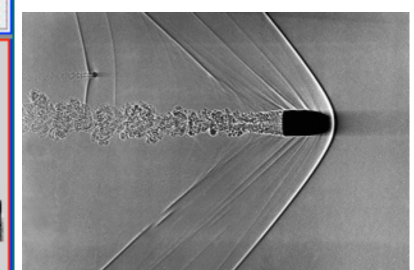
Vertical position of the beam within a bunch

Bunch intensity

Beam losses
A fraction of the beam touched the vertical aperture



Bullet...



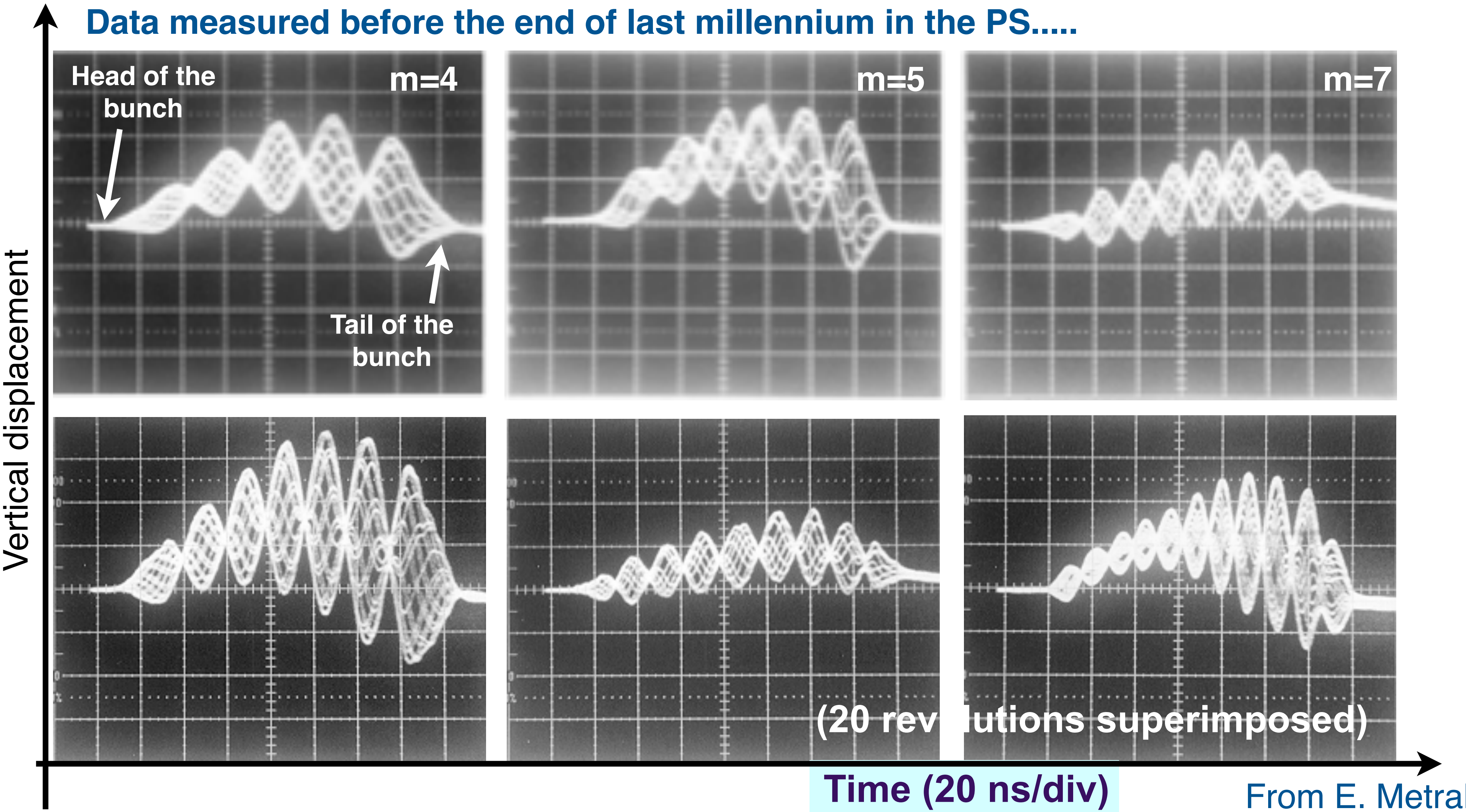
Nasa

Head-tail instability

In the HEADTAIL instability, a standing wave pattern is created along the batch

The mode of the instability is determined by counting the number of nodes

Data measured before the end of last millennium in the PS.....



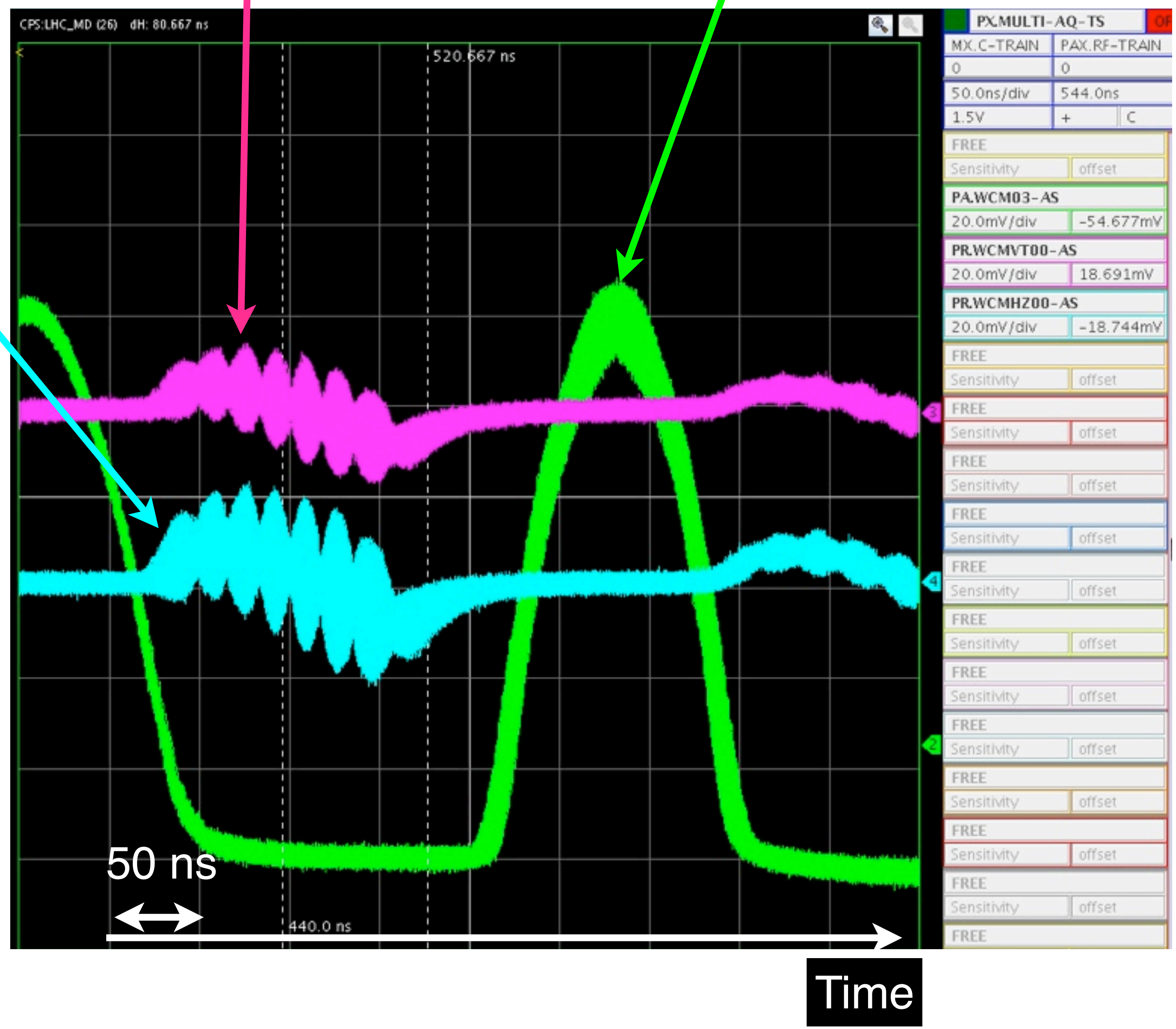
From E. Metral

“Last Sunday-night” Head-tail

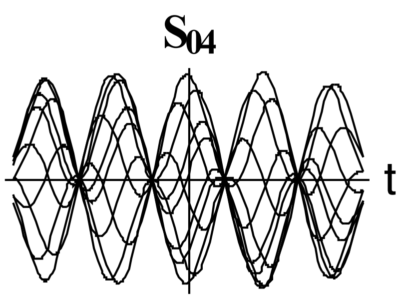
Horizontal position of the beam within a bunch

Vertical position of the beam within a bunch

Bunch intensity



Mode *mq* ⇒ *q* nodes

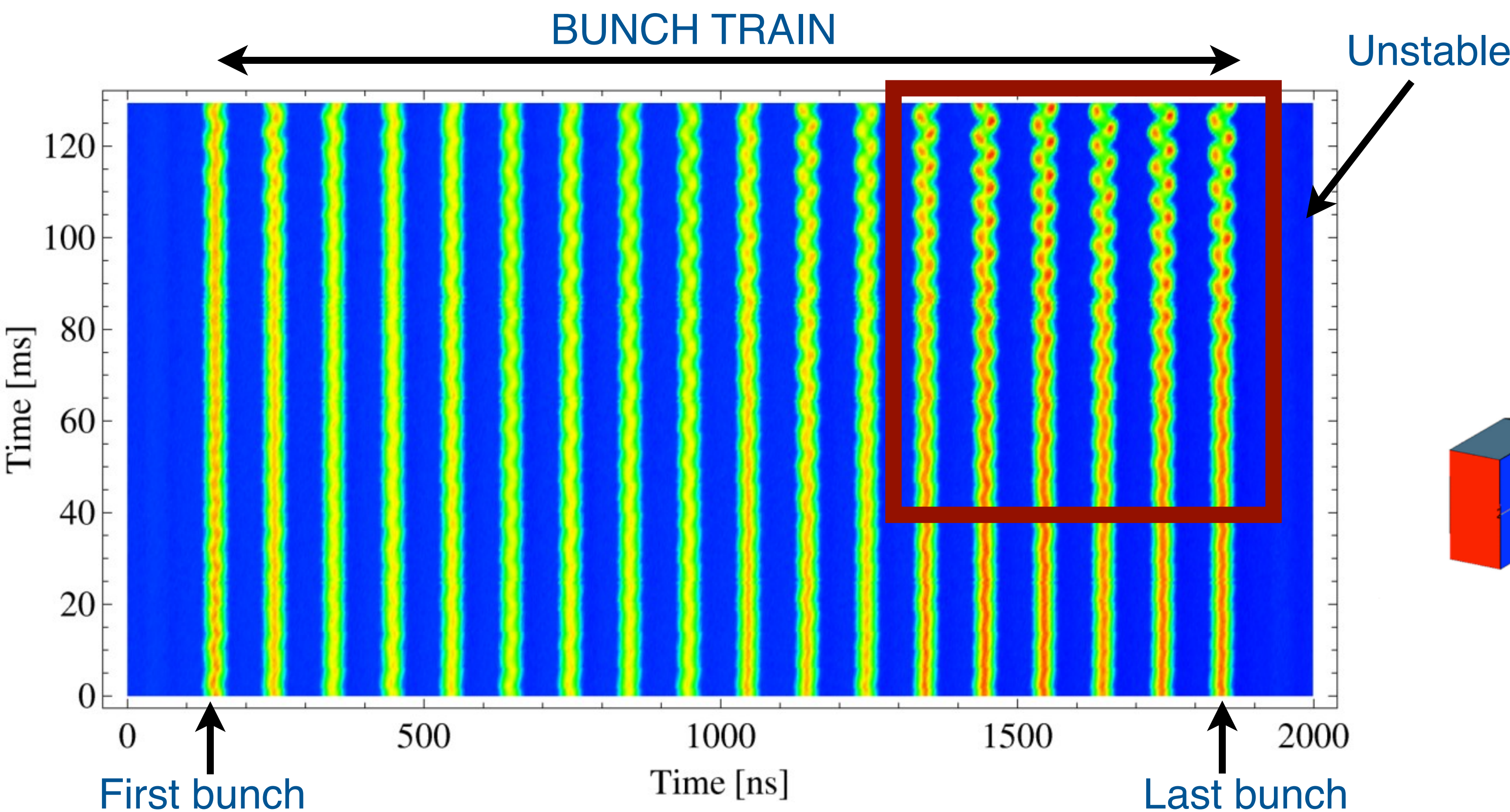


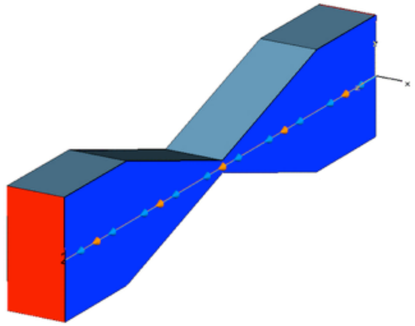
In the data, q=6

A second example: longitudinal instability

The head of the bunch trains create a wake making the rest of the bunch train oscillating longitudinally, i.e., the beam starts to oscillates in the bucket.

This instability is called COUPLED BUNCH, since is generated between bunches

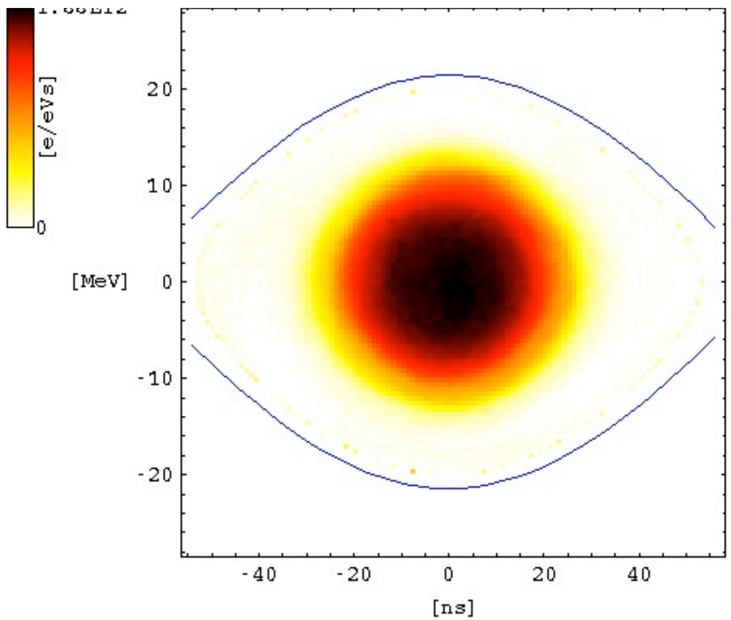




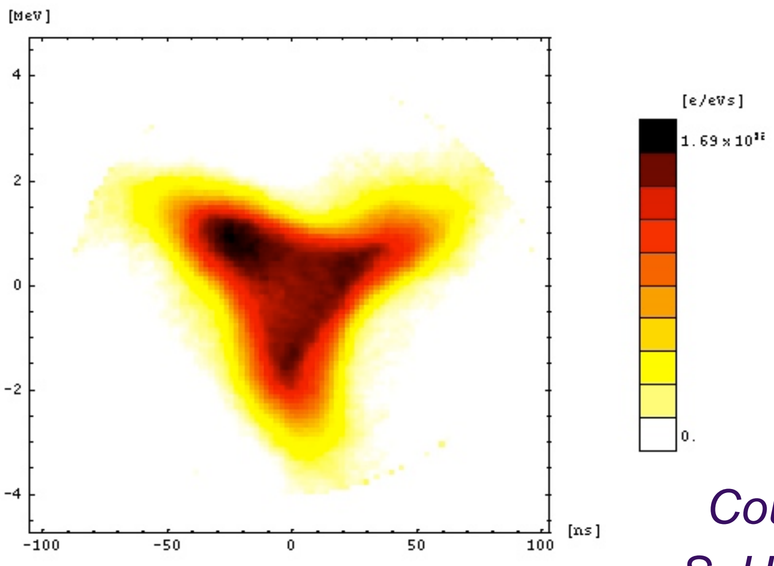
From H. Damerau

How to completely screw up a bunch

a) This is a normal bunch



b) This is an unstable bunch (sextupolar instability)

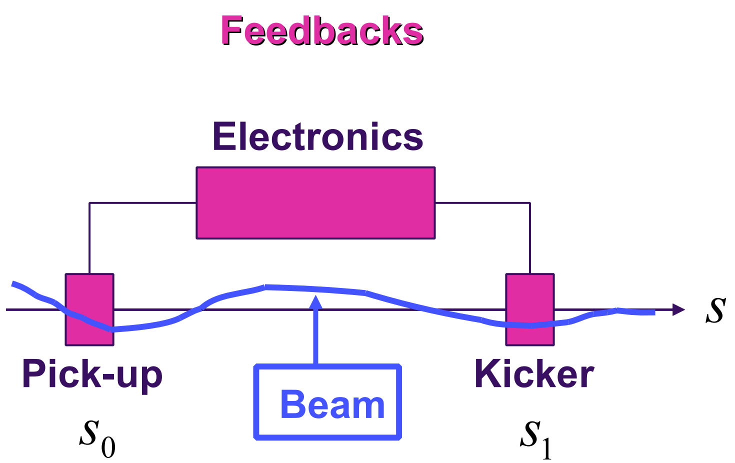


Courtesy S. Hancock

Everyone can understand that b) risks to be more difficult to threat than a).....

How to cure the instabilities, longit. or transverse

- Main sources of impedances:
  - abrupt changes in machine aperture, causing eletromagnetic field trapping
  - bad material conductivity, high resistance...
- For new machines a total impedance budget is allocated to reduce beam instabilities for a given intensity
- For existing machines, any optimisation to reduce impedance sources is pretty wise...
  - plus installation of feedback using fast magnets or dedicated RF cavities
  - plus distributing energy between the planes (the Fermat Riddle...)

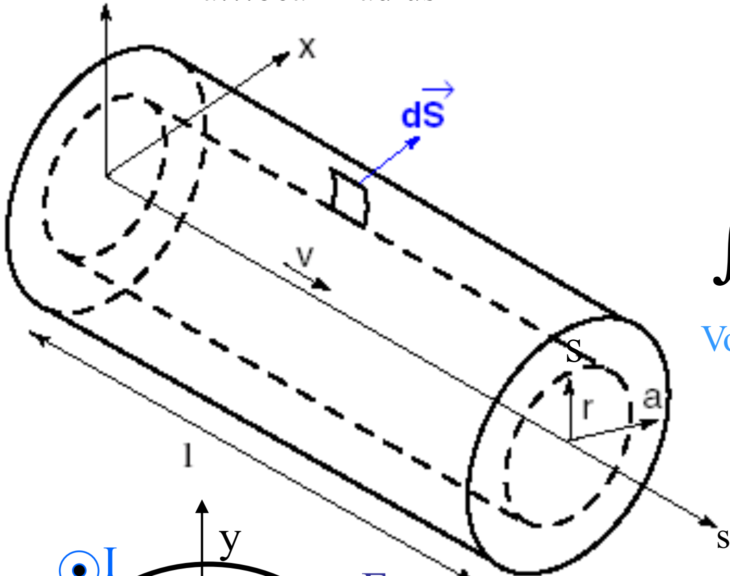


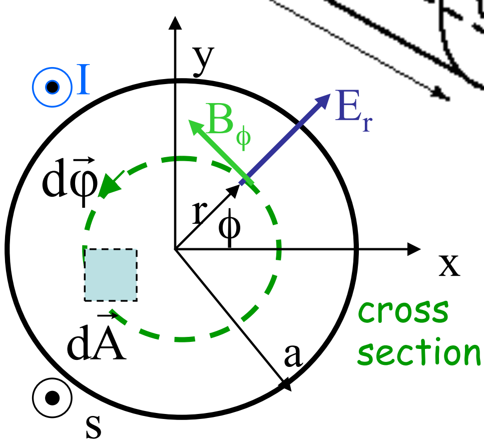
Thanks for your attention!!!

Spare for space charge...

Direct Space Charge - Fields

η…charge density in Cb/m^3
λ... constant line charge $\pi a^2 \eta$
I…constant current $\lambda\beta c = \pi a^2 \eta \beta c$
a…beam radius





Electric

$$\vec{E} = E_r$$

$$\mathrm{div}\,\vec{E} = \frac{\eta}{\varepsilon_0}$$

Magnetic

$$\vec{B} = B_\varphi$$

$$\mathrm{curl}\,\vec{B} = \mu_0 \vec{J}$$

Current density $(\beta c \eta)$

$$\iiint \mathrm{div}\,\vec{E}\, dV = \iint \vec{E}\, d\vec{S} \qquad \oint \vec{B}\, r\, d\vec{\varphi} = \iint \mathrm{curl}\,\vec{B}\, d\vec{A}$$

Volume element

Apply these integrals over

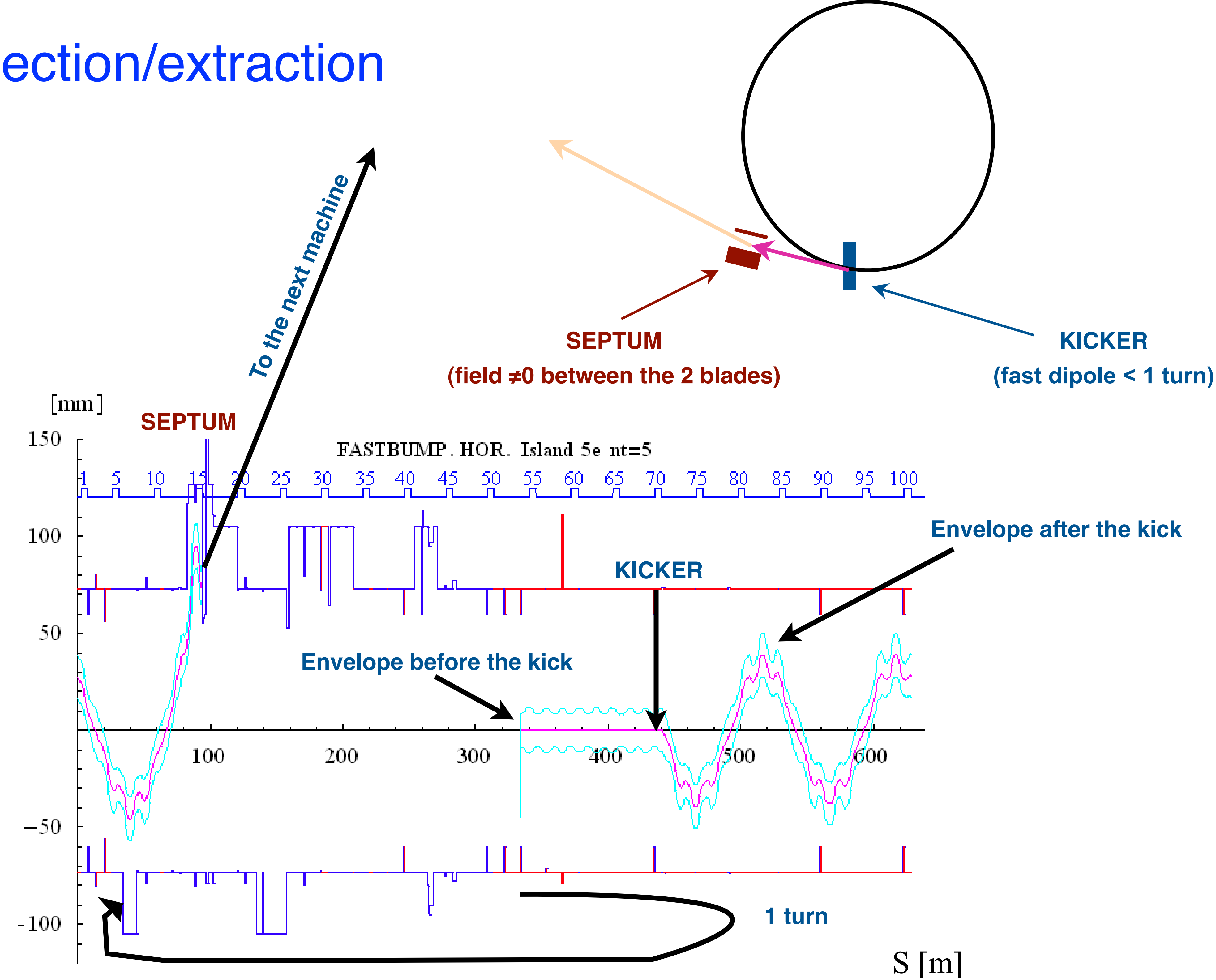
cylinder radius r
length l

cross section
radius r

$$r^2 \pi l \frac{\eta}{\varepsilon_0} = E_r 2r\pi l \qquad B_\varphi 2r\pi = \mu_0 r^2 \pi \beta c \eta$$

$$E_r = \frac{I}{2\pi\varepsilon_0 \beta c} \frac{r}{a^2} \qquad B_\varphi = \frac{I}{2\pi\varepsilon_0 c^2} \frac{r}{a^2}$$

Injection/extraction



Matching beam parameters

The Twiss parameters between two consecutive machines must be matched to avoid any emittance blow-up due to filamentation

Courtesy D. Möhl