

# Introduction to accelerators: application to neutrino beams

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# Lecture goals/outline

**Goal:** present the basic concepts of accelerator physics to understand also the Nufact/Beta beam/Superbeam design

day 1-2)

**Introduction to accelerators**

**Beam transverse dynamics**

**Superconductivity for accelerators**

day 2-3)

**Longitudinal beam dynamics**

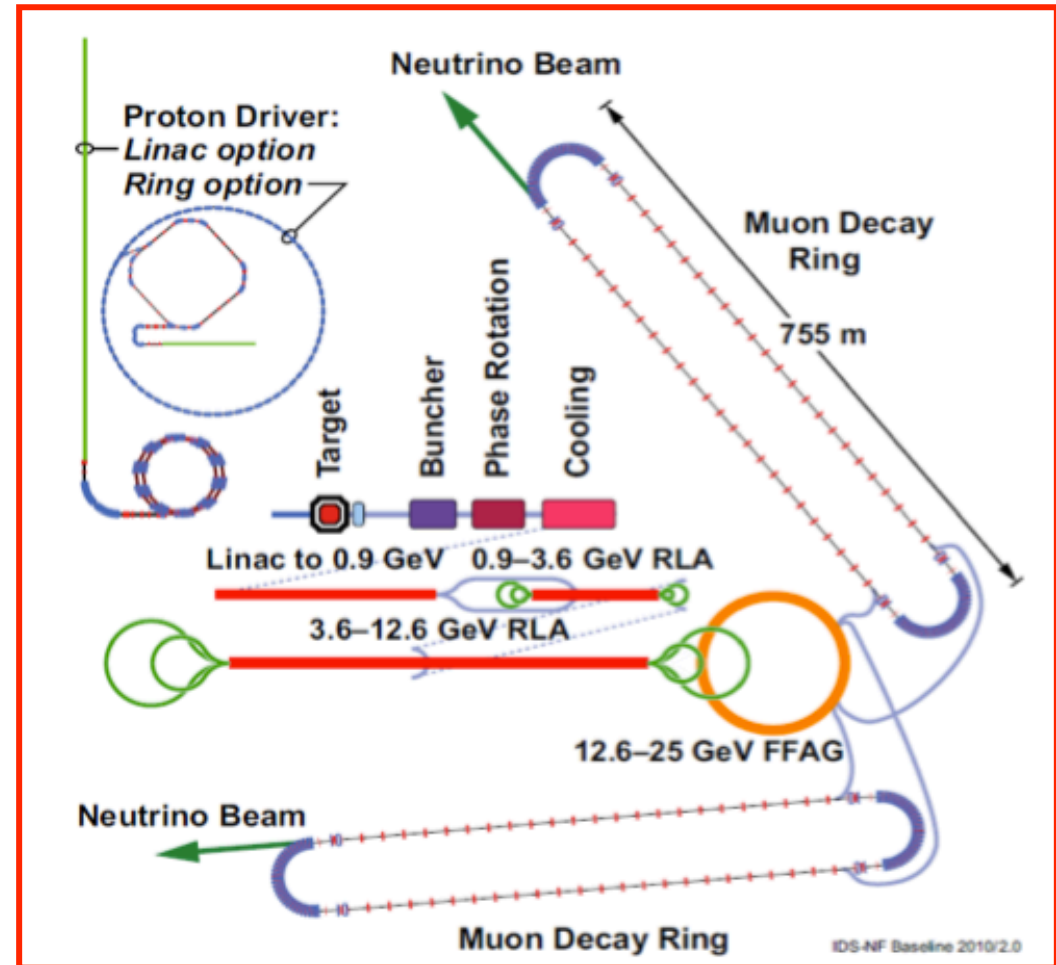
**Collective effects**

day 2-3)

**Superbeam design**

**Neutrino Factory design**

**Beta beams design**

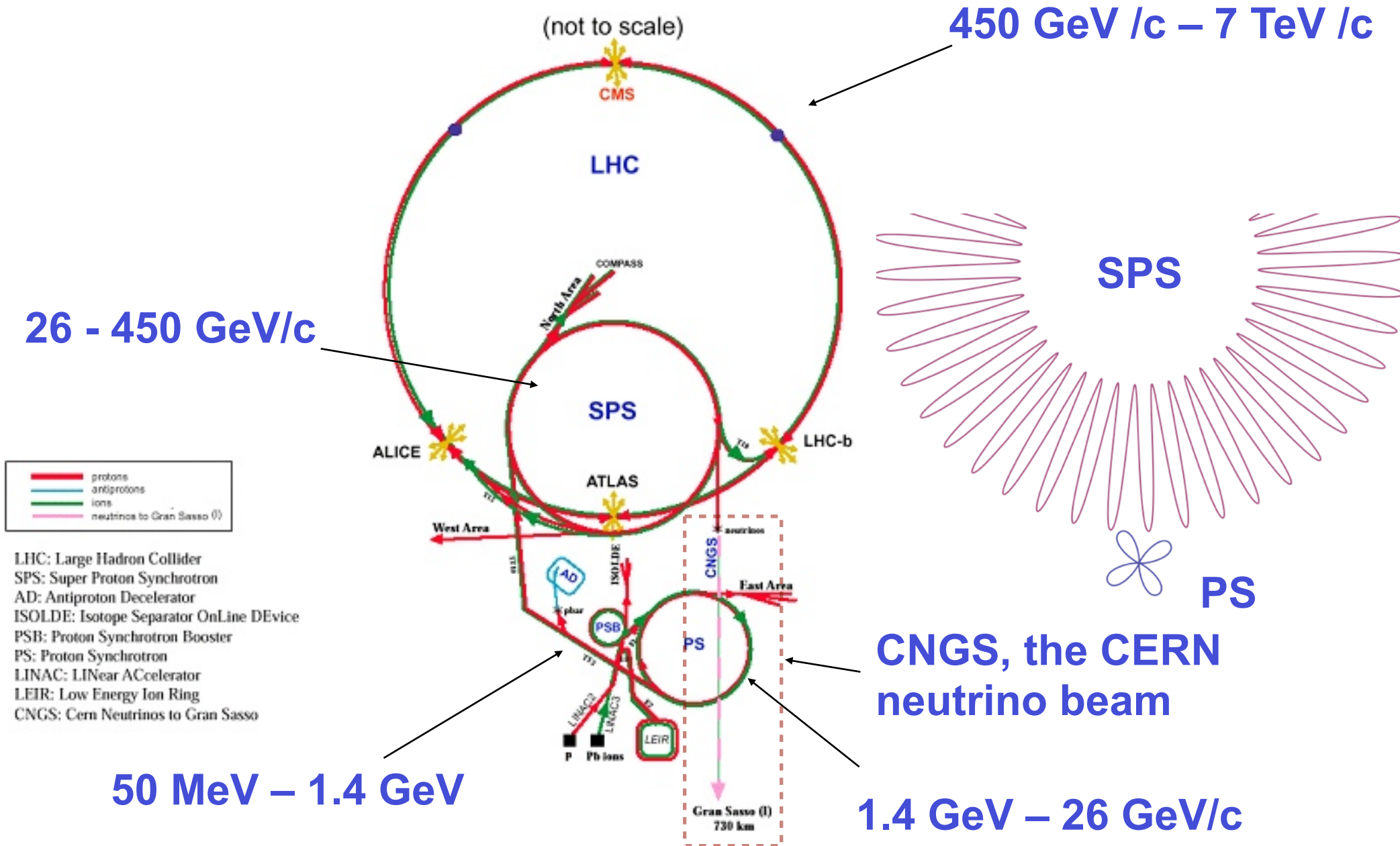


Whenever possible, example taken from existing machine, like LHC, CNGS.  
Attempt to mix reality with a bit of theory

**These lectures are not meant to be a MONOLOGUE**



# Existing facility as example: the CERN accelerator complex overview



# Mathematical approach of the lectures

**TEST:**

$$D_n = \beta_C \sin n\phi_C * \delta_{\text{supr}} * \sum_{i=1}^n \cos\left(i\phi_C - \frac{1}{2}\phi_C \pm \varphi_m\right) * \sqrt{\frac{\beta_m}{\beta_C}} -$$

$$- \cos n\phi_C * \delta_{\text{supr}} * \sum_{i=1}^n \sqrt{\beta_m \beta_C} * \sin\left(i\phi_C - \frac{1}{2}\phi_C \pm \varphi_m\right)$$

$$D_n = \sqrt{\beta_m \beta_C} * \sin n\phi_C * \delta_{\text{supr}} * \sum_{i=1}^n \cos\left((2i-1)\frac{\phi_C}{2} \pm \varphi_m\right) -$$

$$- \sqrt{\beta_m \beta_C} * \delta_{\text{supr}} * \cos n\phi_C * \sum_{i=1}^n \sin\left((2i-1)\frac{\phi_C}{2} \pm \varphi_m\right)$$

Remembering the trigonometric gymnastics shown above we get

$$D_n = \delta_{\text{supr}} * \sqrt{\beta_m \beta_C} * \sin n\phi_C * \sum_{i=1}^n \cos\left((2i-1)\frac{\phi_C}{2}\right) * 2 \cos \varphi_m -$$

$$- \delta_{\text{supr}} * \sqrt{\beta_m \beta_C} * \cos n\phi_C * \sum_{i=1}^n \sin\left((2i-1)\frac{\phi_C}{2}\right) * 2 \cos \varphi_m$$

$$D_n = 2\delta_{\text{supr}} * \sqrt{\beta_m \beta_C} * \cos \varphi_m \left\{ \sum_{i=1}^n \cos\left((2i-1)\frac{\phi_C}{2}\right) * \sin(n\phi_C) - \right.$$

$$\left. - \sum_{i=1}^n \sin\left((2i-1)\frac{\phi_C}{2}\right) * \cos(n\phi_C) \right\}$$

$$D_n = 2\delta_{\text{supr}} * \sqrt{\beta_m \beta_C} * \cos \varphi_m \sin(n\phi_C) \frac{\sin \frac{n\phi_C}{2} * \cos \frac{n\phi_C}{2}}{\sin \frac{\phi_C}{2}} -$$

$$- 2\delta_{\text{supr}} * \sqrt{\beta_m \beta_C} * \cos \varphi_m * \cos(n\phi_C) * \frac{\sin \frac{n\phi_C}{2} * \sin \frac{n\phi_C}{2}}{\sin \frac{\phi_C}{2}}$$

$$D_n = \frac{2\delta_{\text{supr}} * \sqrt{\beta_m \beta_C} * \cos \varphi_m}{\sin \frac{\phi_C}{2}} \left\{ 2 \sin \frac{n\phi_C}{2} \cos \frac{n\phi_C}{2} * \cos \frac{n\phi_C}{2} \sin \frac{n\phi_C}{2} - \right.$$

$$\left. - (\cos^2 \frac{n\phi_C}{2} - \sin^2 \frac{n\phi_C}{2}) \sin^2 \frac{n\phi_C}{2} \right\}$$

replace by ...

**“after some TLC transformations”**

... or ... **“after some beer”**

Method proposed by B. Holzer,  
see CERN summer student lectures

## Linus security blanket

*I'll use a minimum of math for the ones who fill better and to prove at least the basics principles*

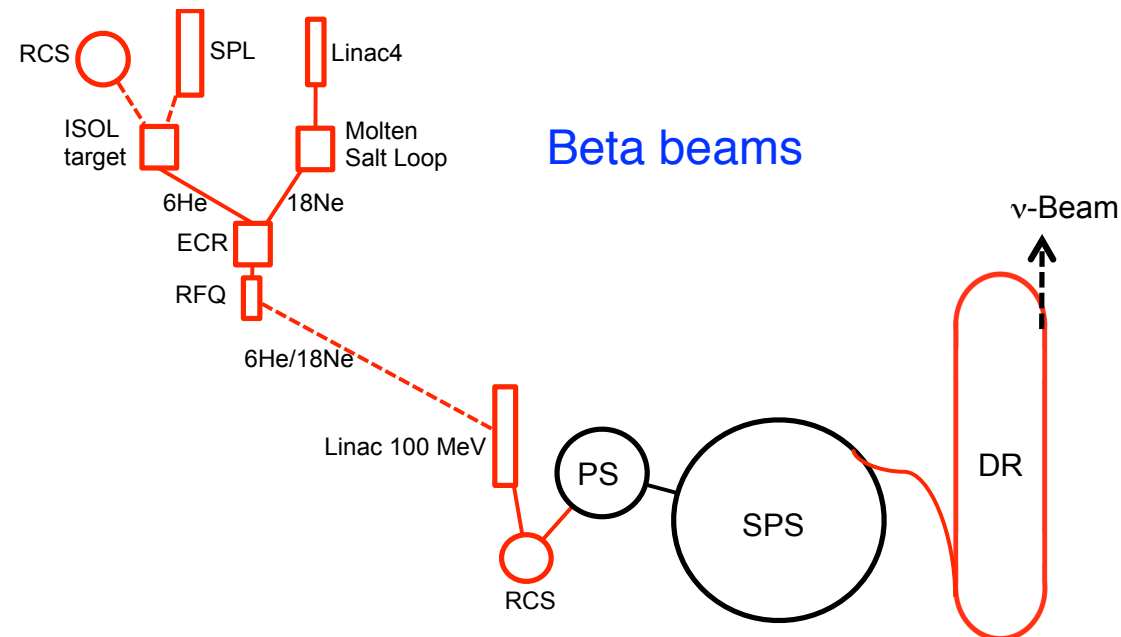
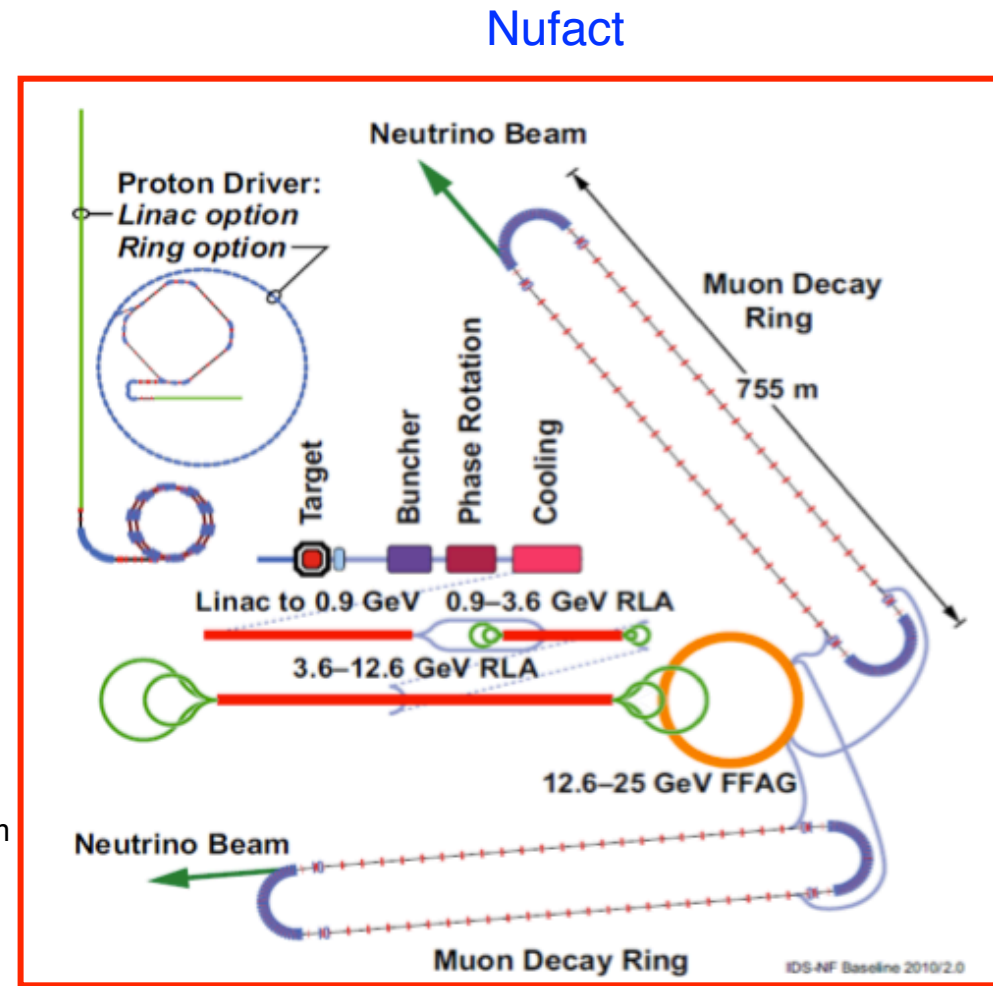
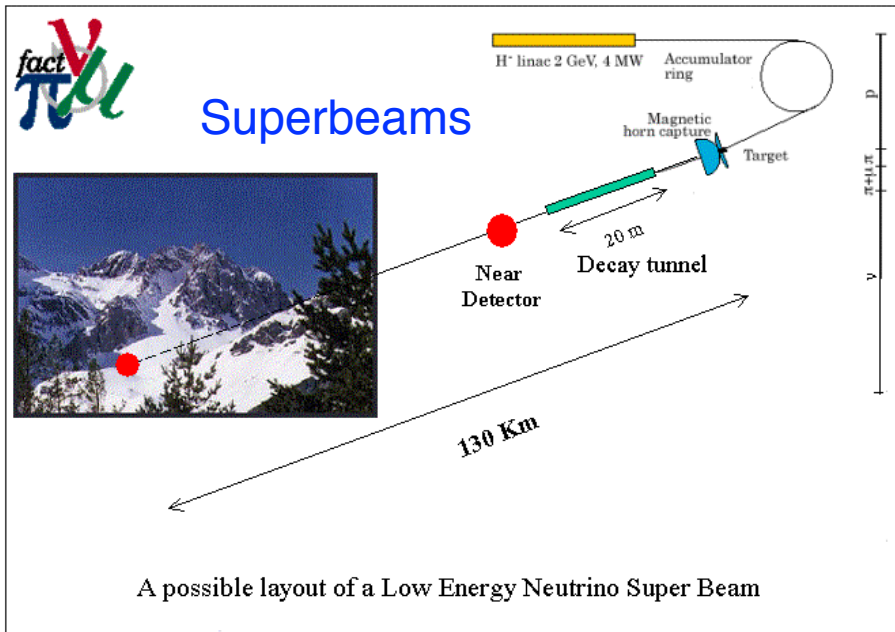


*I'll try to notice if you arrive at this stage during the lectures..*



*“Ipse dixit (he himself said it)” means: I have no time to explain the subject in details, I give you a reference in case, the details in that case are not fundamental for the understanding*

# What we would like to understand ...



# Interlude: a brief recall of energy scales

- **WARNING:** for purists or non-experts: Energy, Masses and Momentum have different units, which turn to be the same since  $c$  (speed of light) is considered equal to one.
- Energy[GeV], Momentum [GeV/c], Masses [GeV/c<sup>2</sup>]  
(Remember golden rule,  $E=mc^2$  has to be true also for units...)
- Just as a rule of thumb: **0.511 MeV/c<sup>2</sup>** (electron mass) corresponds to about **9.109 10<sup>-31</sup> kg**



An Example about energy scales: my cellular phone battery.

**Voltage: 3.7 V**

**Height: 4.5 cm**

**proton mass ~ 1 GeV**

To accelerate an electron to an energy equivalent to a proton mass:

**1 GeV/3.7 eV = 270 270 270 batteries**

**270 270 270 batteries \* 0.045 m ~ 12 000 000 m**

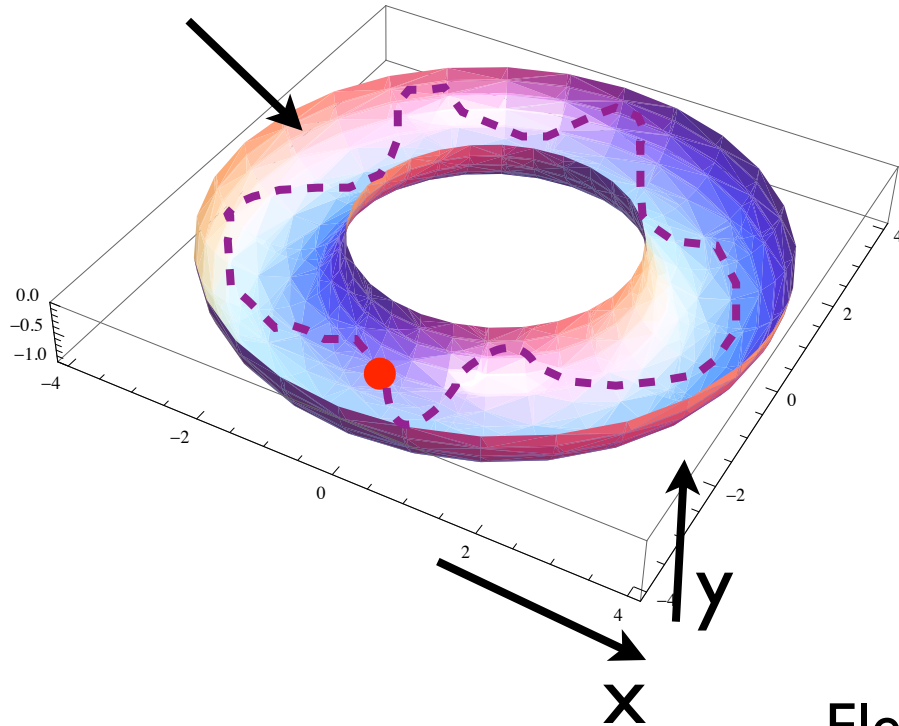
**12 000 000 m ~ THE EARTH DIAMETER**



Obviously one has to find a smarter way to accelerate particles to high energies instead of piling up cellular phone batteries ....

# How an accelerator works ?

## Accelerator



*Goal: keep enough particles confined in a well defined volume to accelerate them.*

*How ? Lorentz Force!*

$$\overline{F(t)} = q \left( \overline{E(t)} + \overline{v(t)} \otimes \overline{B(t)} \right)$$

Electric field  
accelerates particles

Particles of  
different energy  
(speed) behave differently

Magnetic field confines  
particles  
on a given trajectory

An **accelerator** is formed by a sequence (called **lattice**) of:

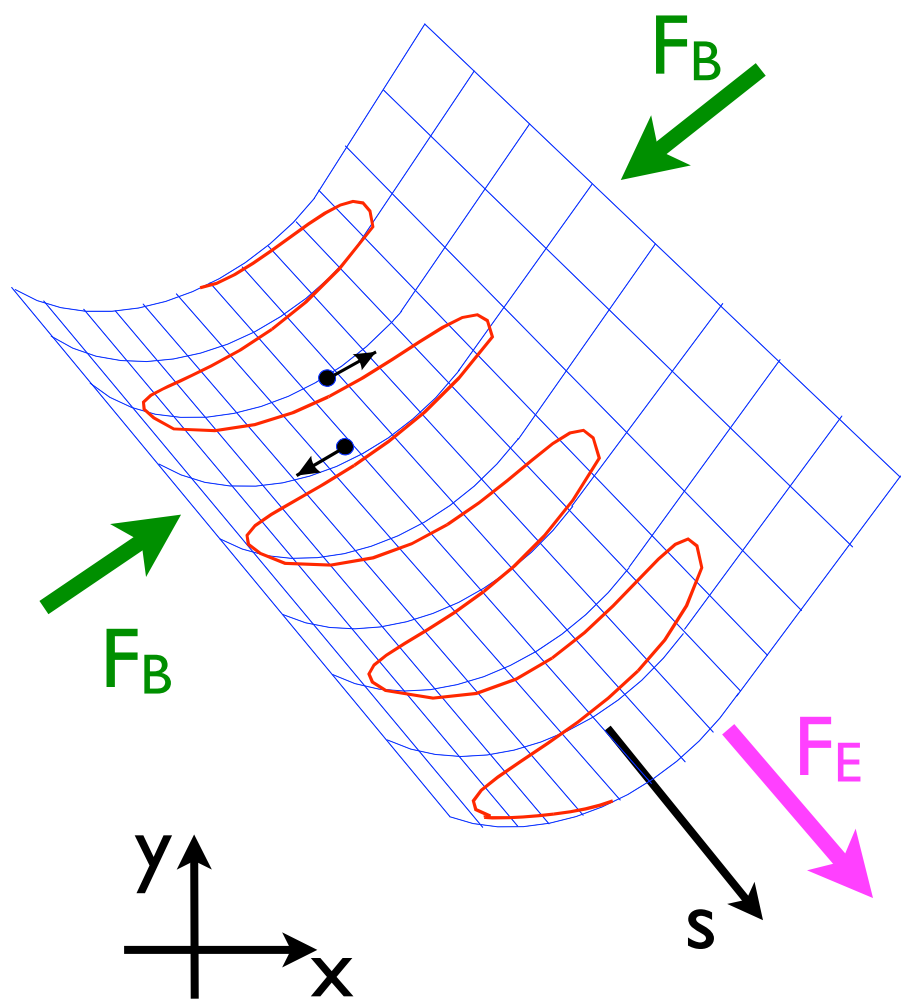
a) Magnets → Magnetic Field

b) Accelerating Cavity → Electric Field

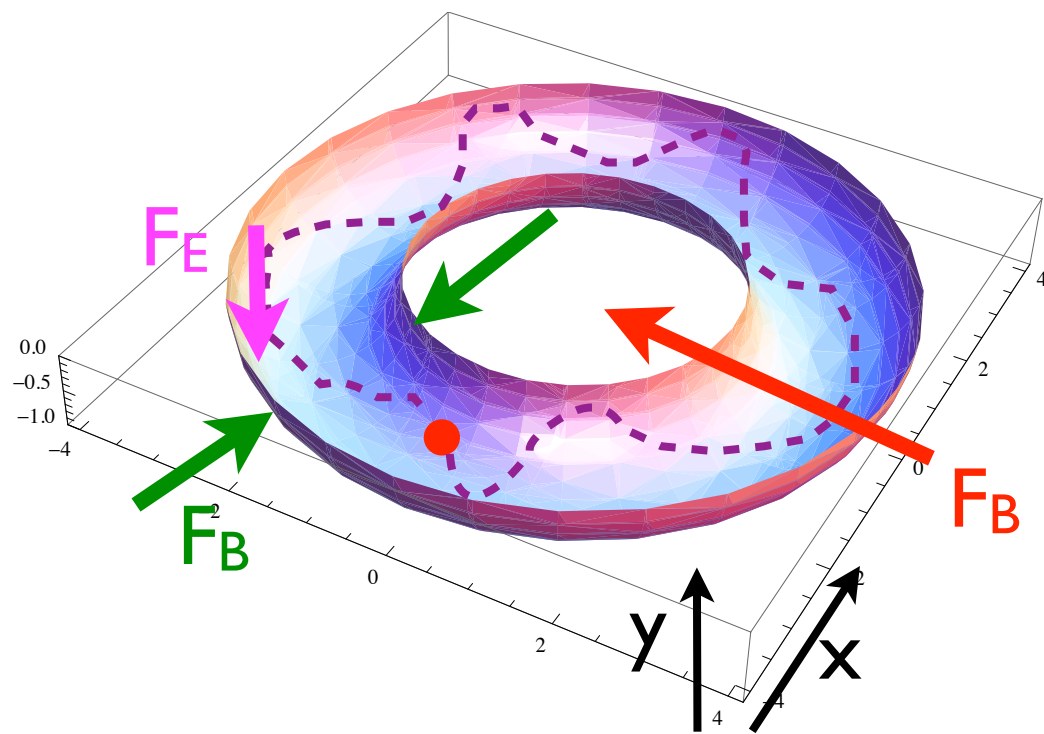


$$\overline{F}(t) = q \left( \underbrace{\overline{E}(t)}_{F_E} + \underbrace{\overline{v}(t) \otimes \overline{B}(t)}_{F_B} \right)$$

*Linear Accelerator*

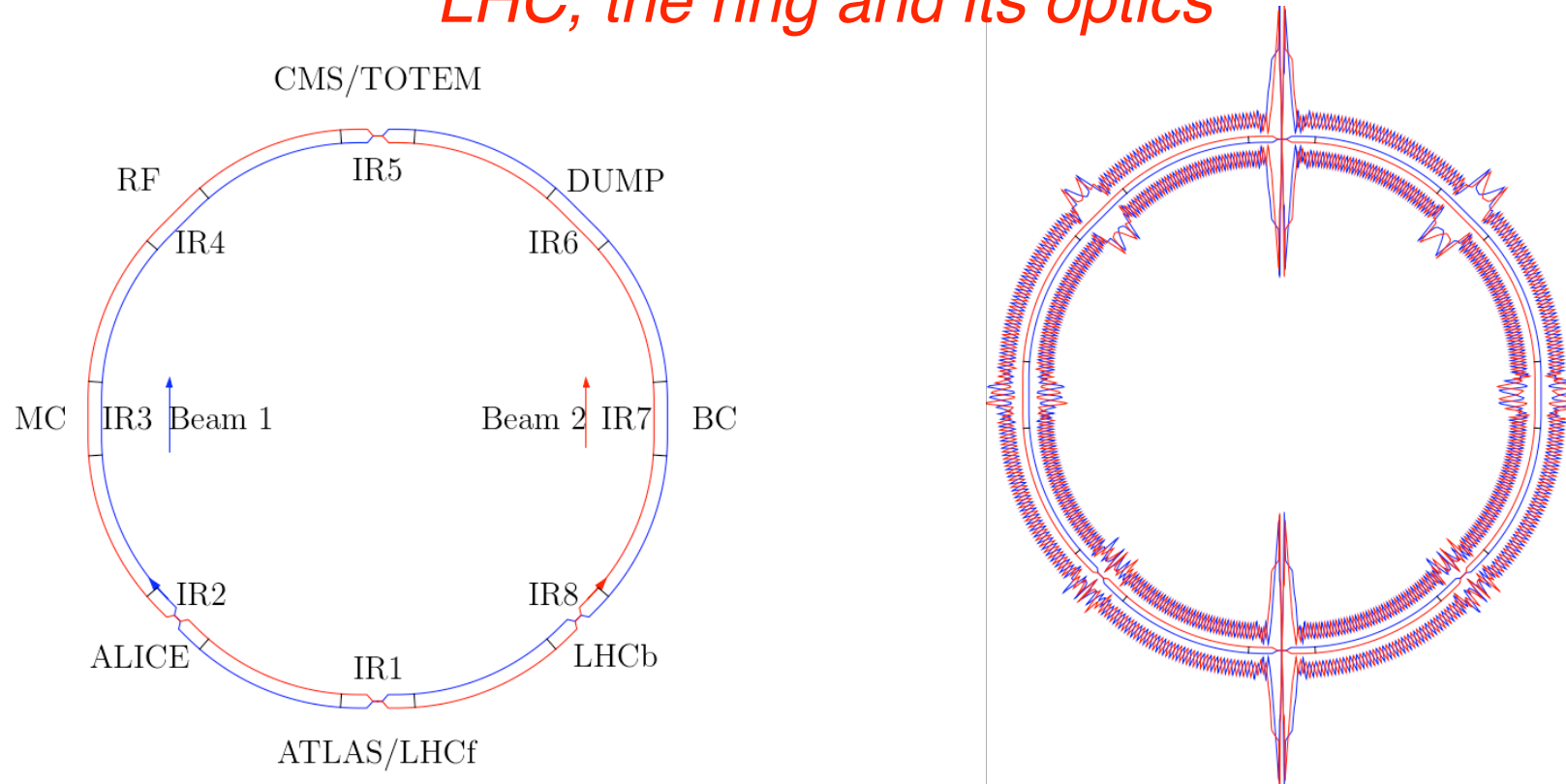


*Circular Accelerator*

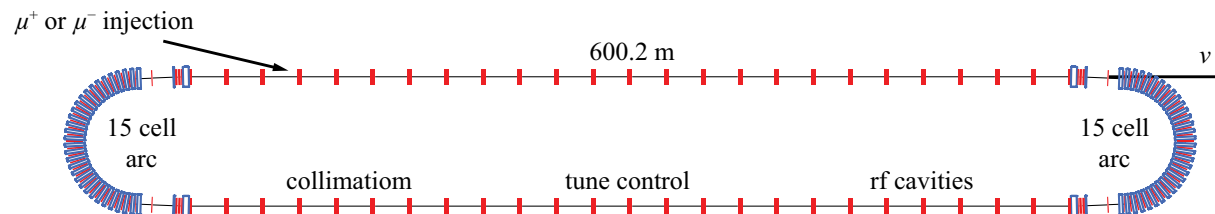


# Today lectures... let's go circular

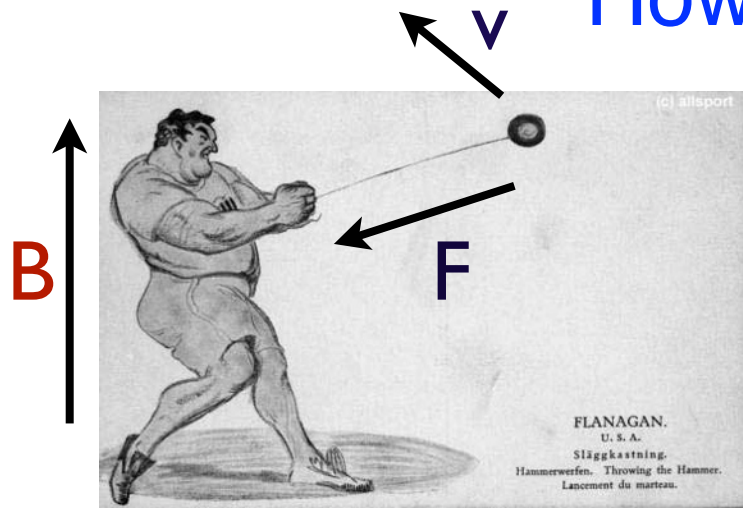
## *LHC, the ring and its optics*



## *Nufact storage ring*

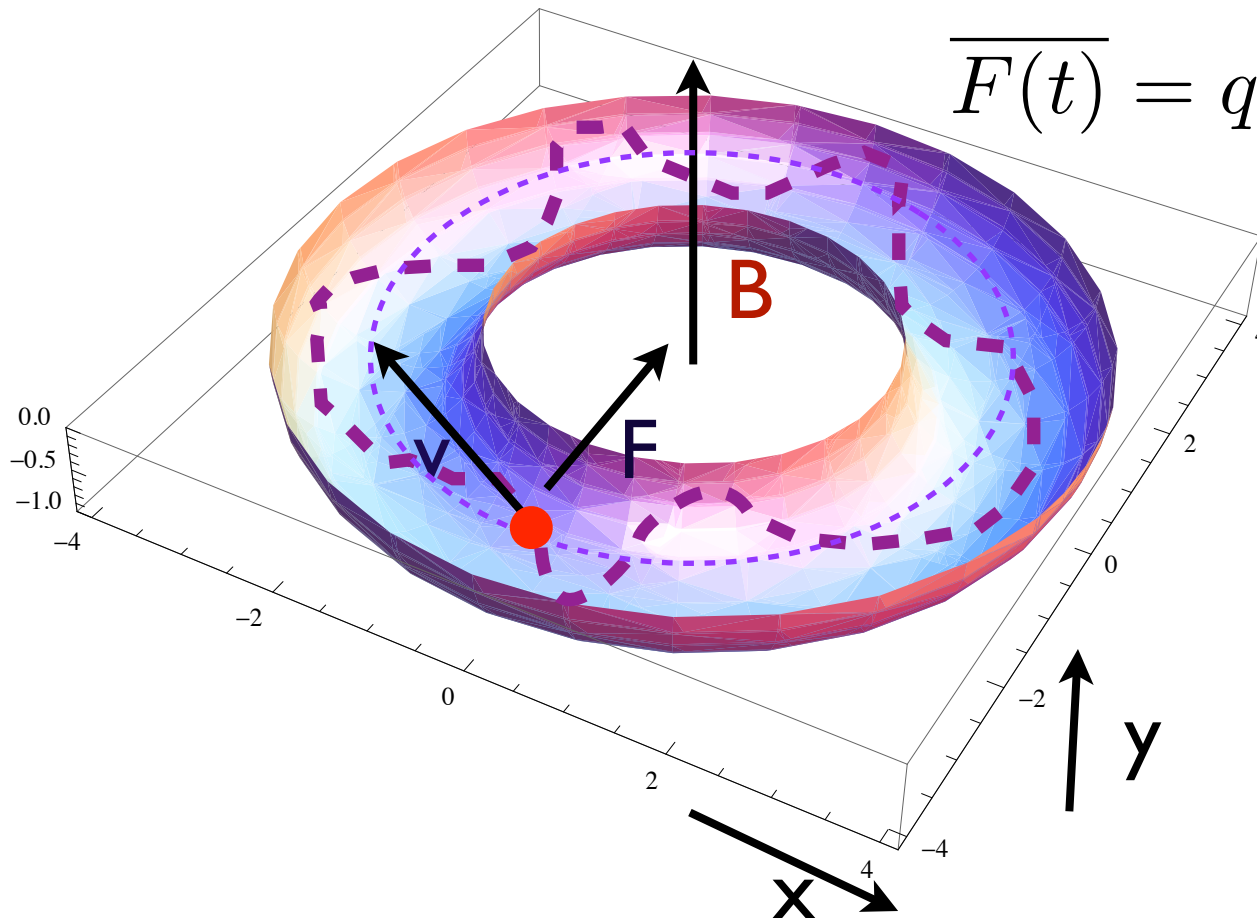


# How a circular accelerator works ?



*Goal: keep enough particles confined in a well defined volume to accelerate them.*

*How ? Lorentz Force!*



$$\overline{F(t)} = q \left( \overline{E(t)} + \overline{v(t)} \otimes \overline{B(t)} \right)$$

↓  
Particles of different energy (speed) behave differently

↓  
Magnetic field confines particles on a given trajectory

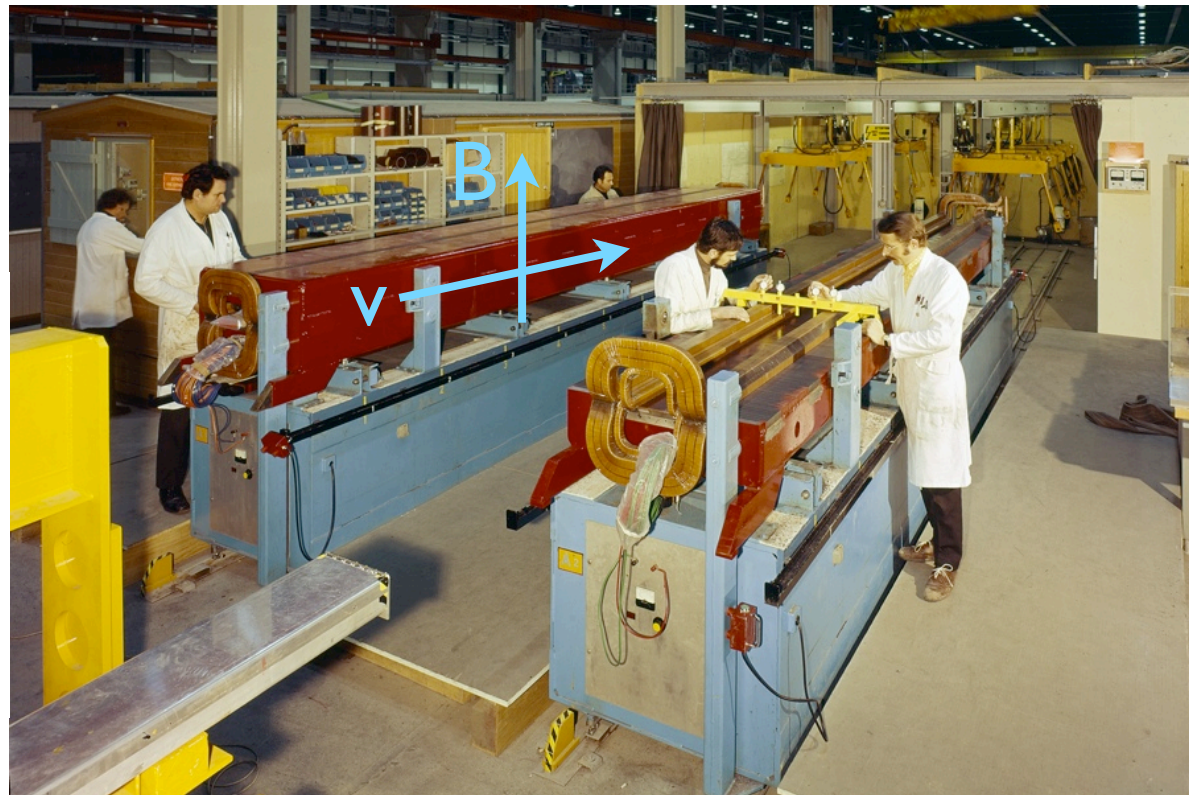
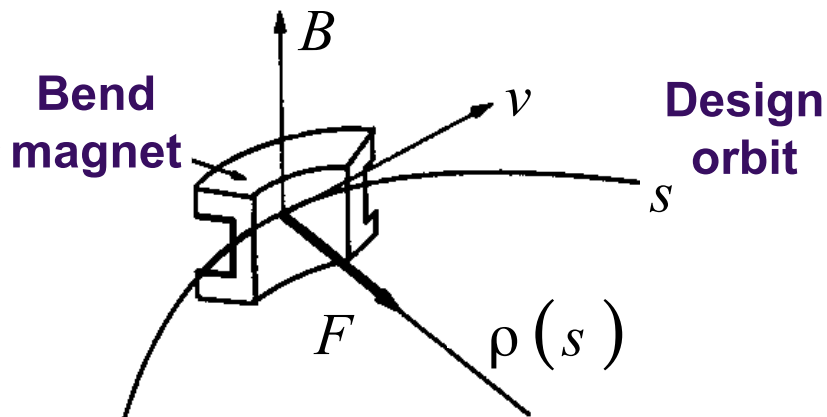
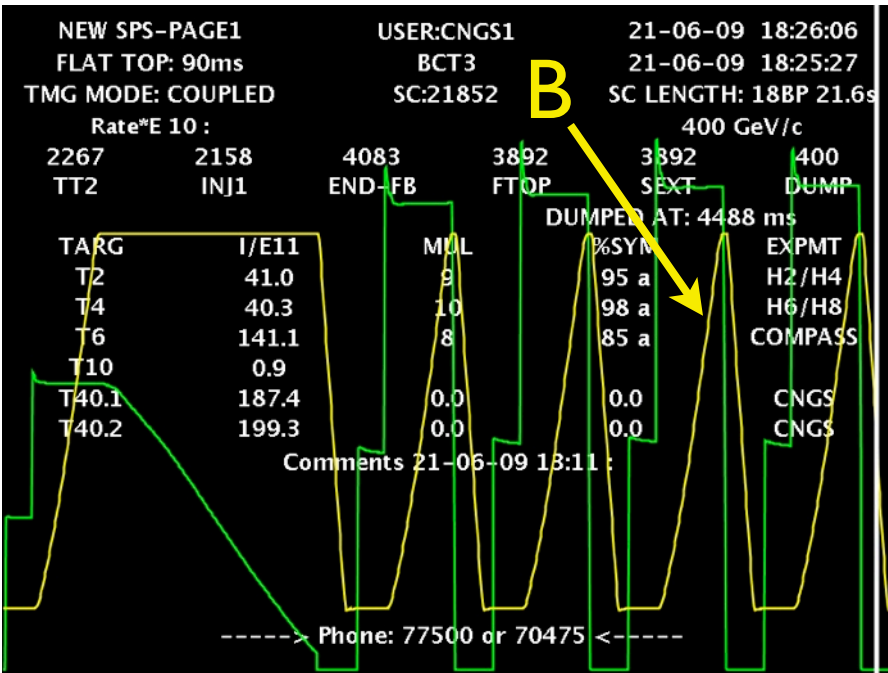


# Dipole

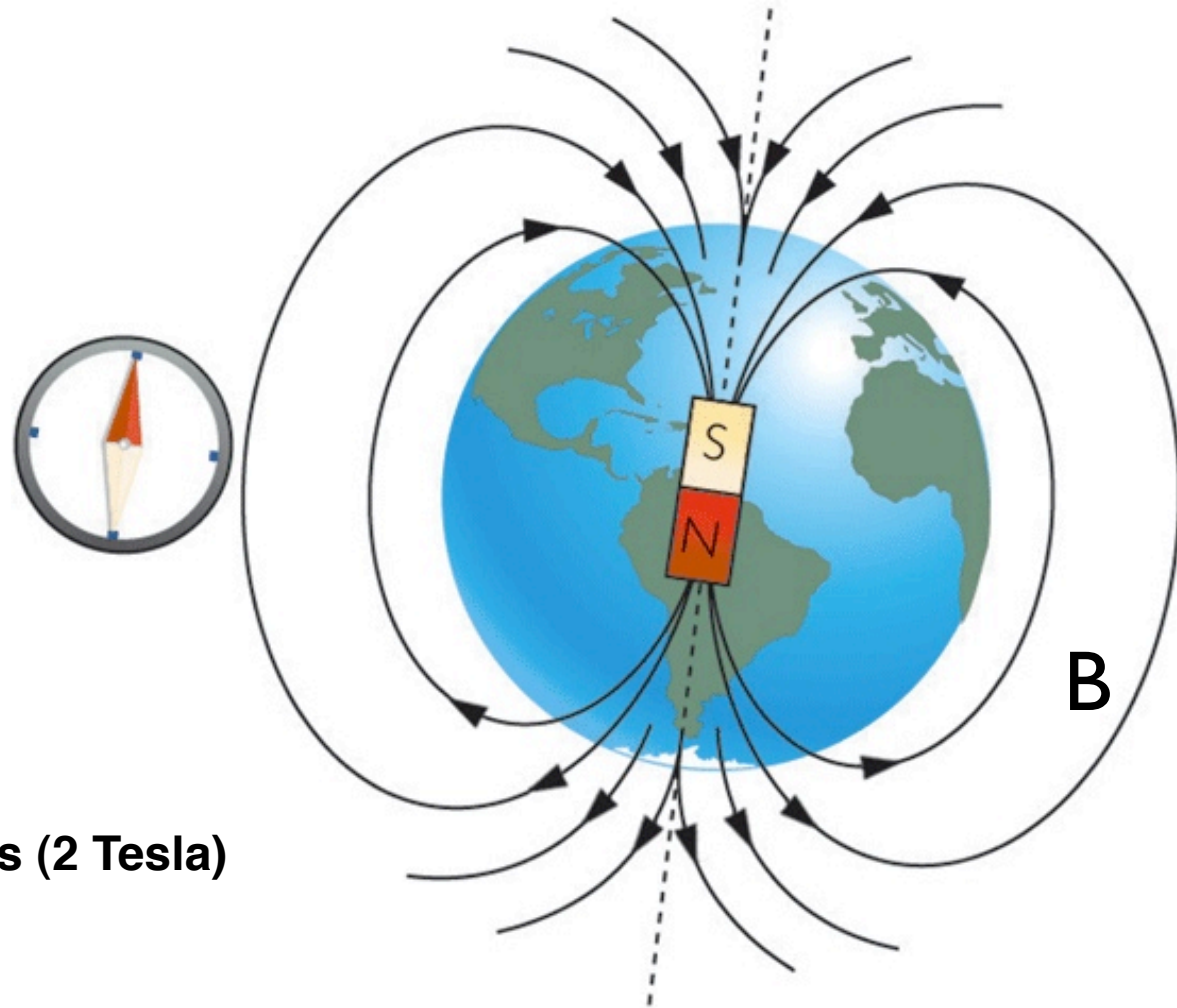
Force given by the vertical magnetic field compensates the centrifugal force to keep the particles on the central trajectory, i.e. in the center of the beam pipe.

Once the beam accelerates, the magnetic field is increased synchronously

**SPS dipoles, in total about 500**



# Two dipoles you should know we well



**Earth Magnetic Field : ~ 0.6 Gauss**

**Typical SPS dipole field: ~ 20000 Gauss (2 Tesla)**

**Typical magnet current required: 1 kA**

# Synchrotron (1952, 3 GeV, BNL)

New concept of circular accelerator. The magnetic field of the bending magnet varies with time.

**As particles accelerate, the B field is increased proportionally.**

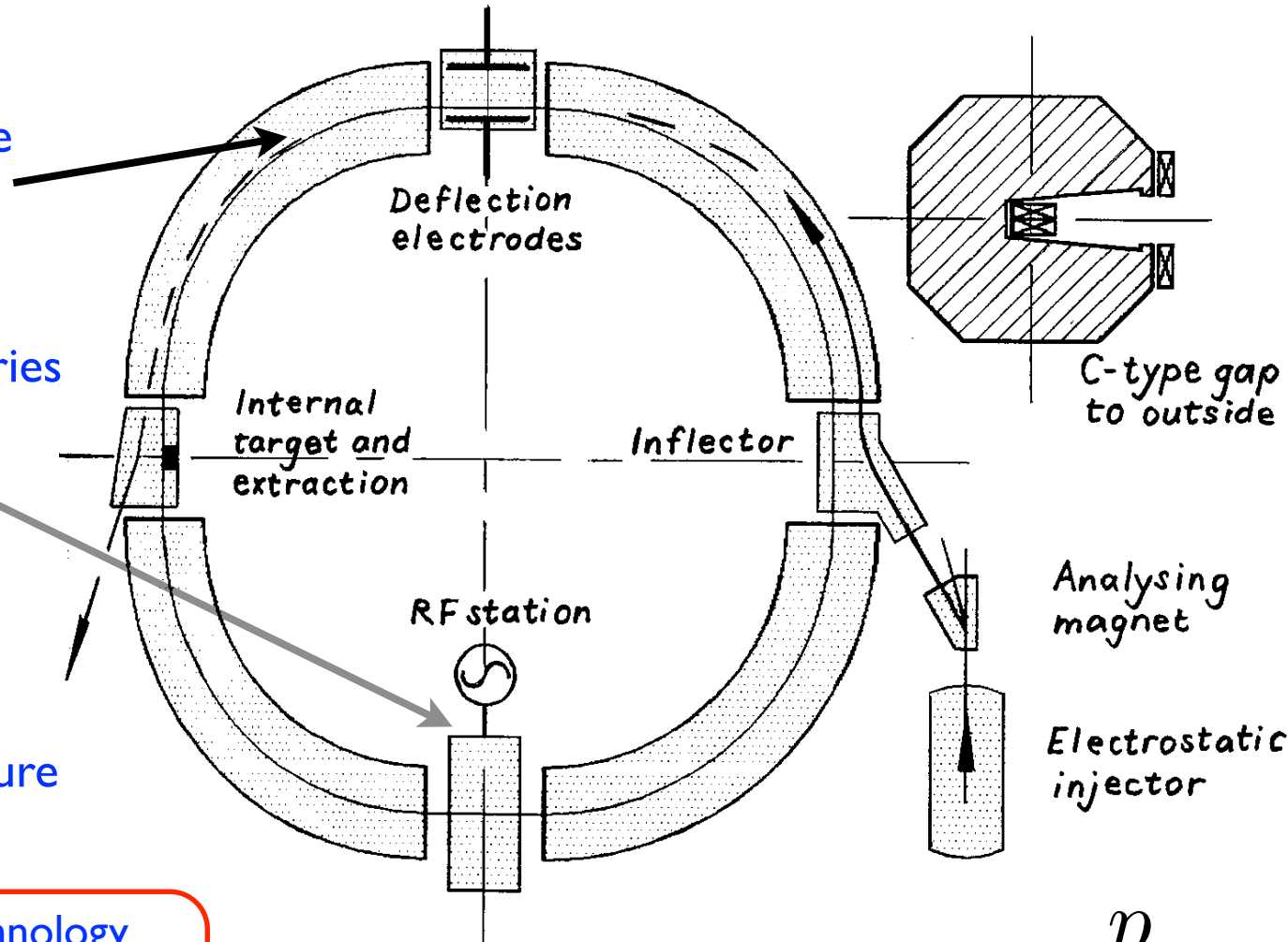
The frequency of the accelerating cavity, used to accelerate the particles, has also to change.

$\mathbf{B} = \mathbf{B}(t)$  magnetic field from the bending magnets

$\mathbf{p} = \mathbf{p}(t)$  particle momentum varies by the RF cavity

$e$  electric charge

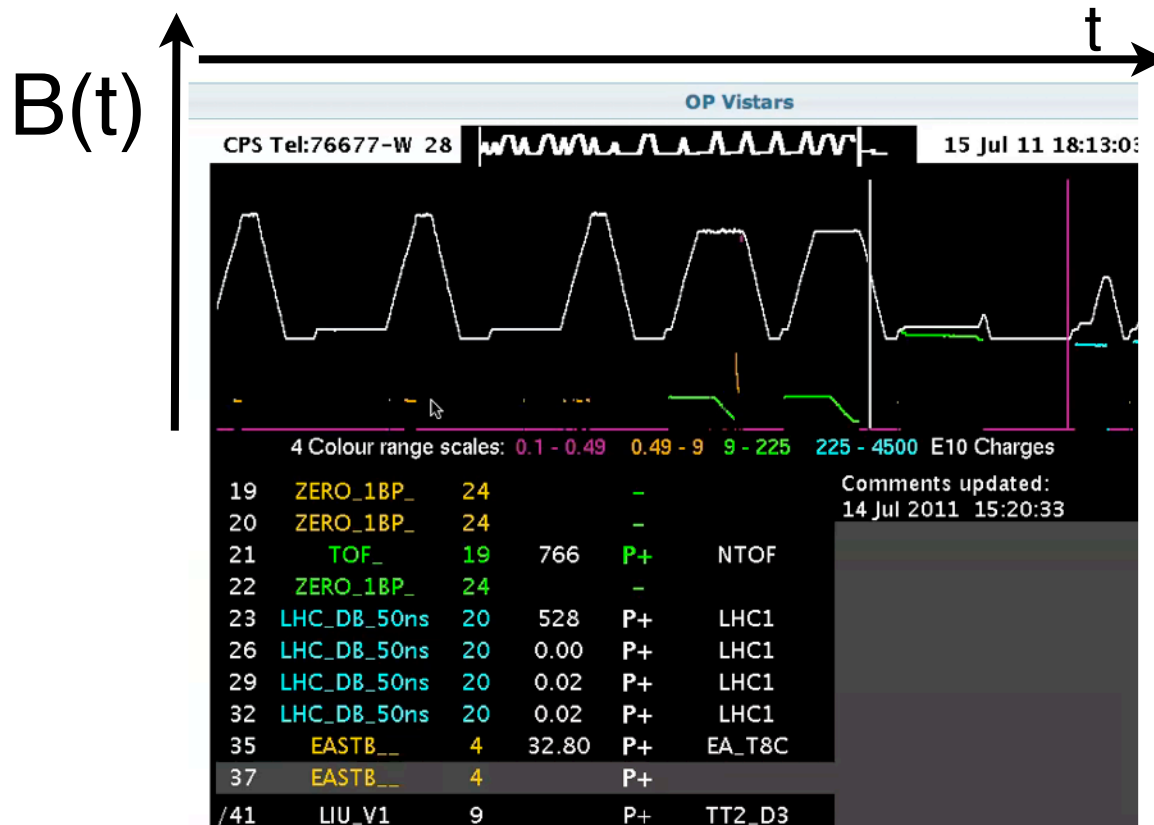
$\rho$  constant radius of curvature



Bending strength limited by used technology to max  $\sim 1$  T for room temperature conductors

Particle rigidity: 
$$B\rho = \frac{p}{e}$$

# An example of cycling machine: the CERN-PS (Proton Synchrotron)



PS: first synchrotron ever build with Quadrupoles

**PS is a slow synchrotron: pulses every 1.2 s (or multiples)**

Inj. field: 1013 G, extraction 12000 G

$$\frac{dB}{dt} = 24 \text{ G/ms}$$

RCS: Rapid cycling synchrotron, few 100 ms to ramp the magnets

# Beam rigidity vs E and B fields

$$\vec{F} = e \cdot (\vec{E} + \vec{v} \times \vec{B}) \quad \text{Lorentz Force}$$

$$\vec{F}_\perp = \gamma m_0 c \dot{\vec{\beta}}_\perp = e \cdot (\vec{E}_\perp + \vec{\beta} c \times \vec{B}) \stackrel{!}{=} \gamma m_0 \frac{v^2}{R}$$

↑
↑  
 Lorentz Force                      Centrifugal force

Why bend particles with magnetic fields and not electric fields at high energy ?

$$e R B_\perp = p = e \frac{1}{v} R E_\perp$$

Ultrarelativistic particles:  $v \approx c \Rightarrow R B_\perp = 1 \text{ Tm} \Leftrightarrow R E_\perp \approx 300 \text{ MV/m}$

↑  
Simple magnet

↑  
Very high voltage  
(max few MV/m)



# Van De Graaf electrostatic generator (1928)

A flavor how to get 10 MV/m ....

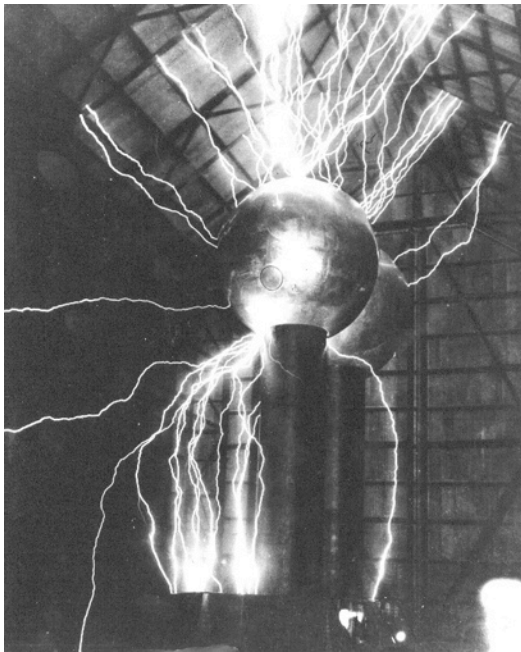
A rotating belt charges a top terminal up to the maximum voltage before sparking.

Maximum accelerating Voltage: **10 MV**

Typical speed: 20 m/s

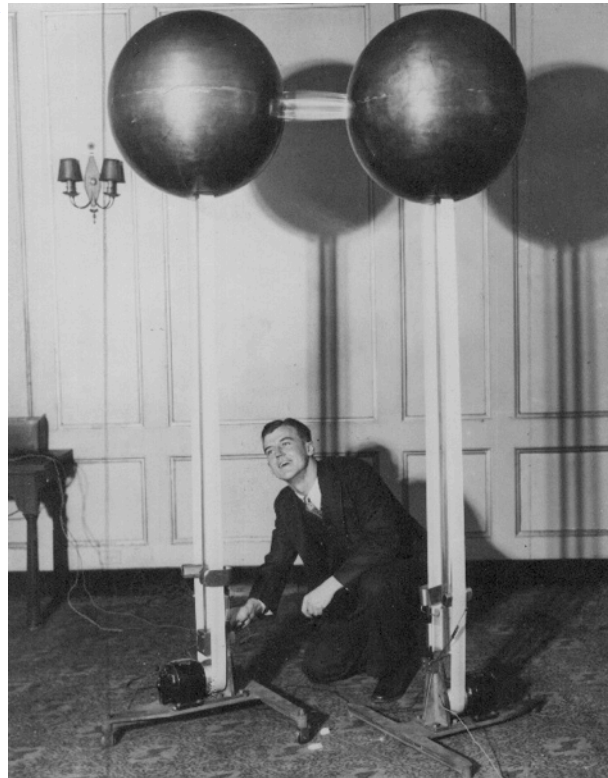
Height: 0.5 m

Top terminal: 1 MV - 10 MV



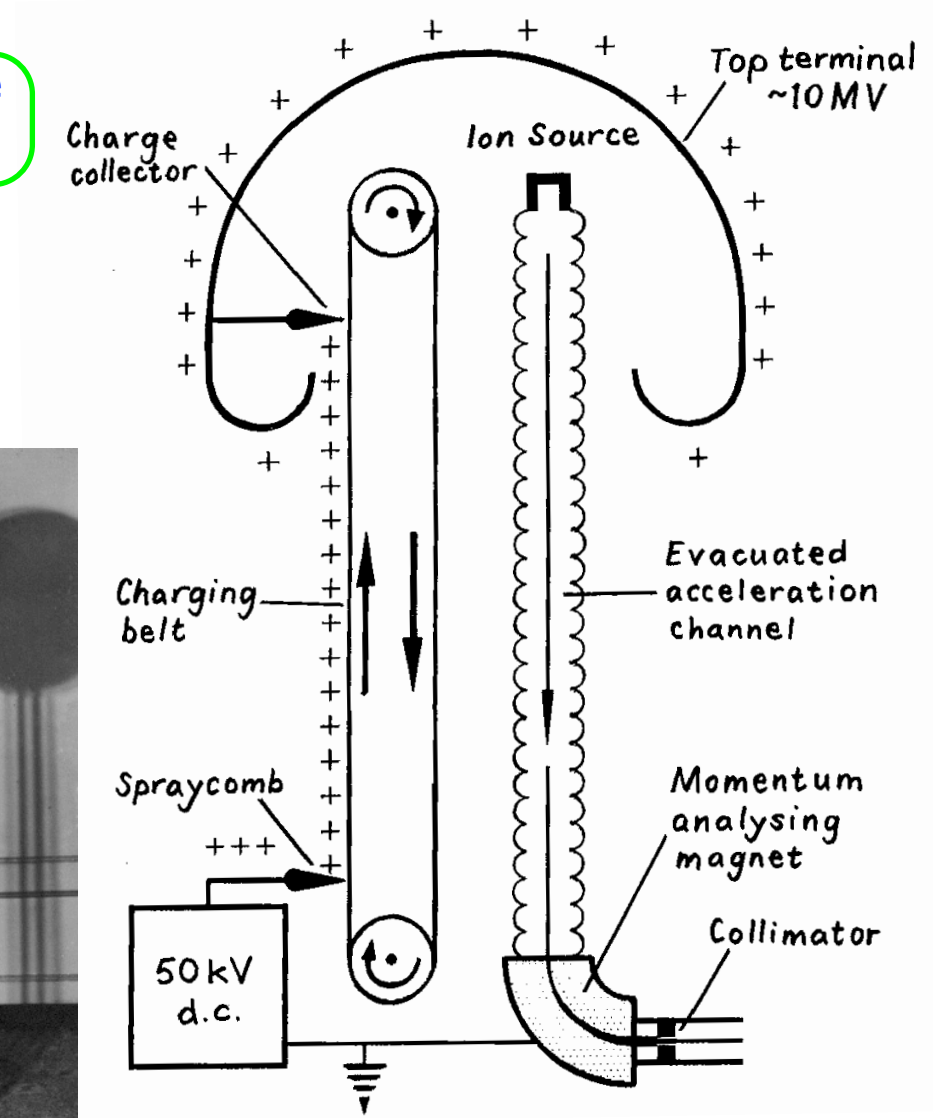
AT ROUND HILL SPARKING TO HANGAR (LONG EXPOSURE)

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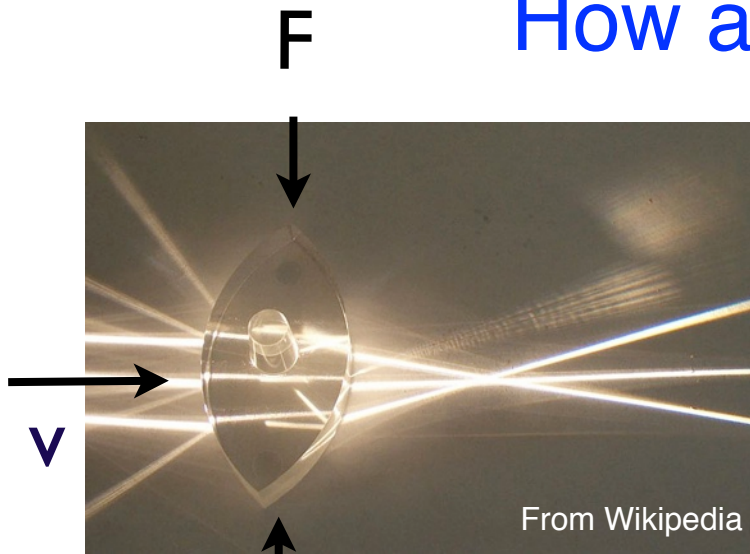


R. J. VAN DE GRAAFF WITH FIRST GENERATOR

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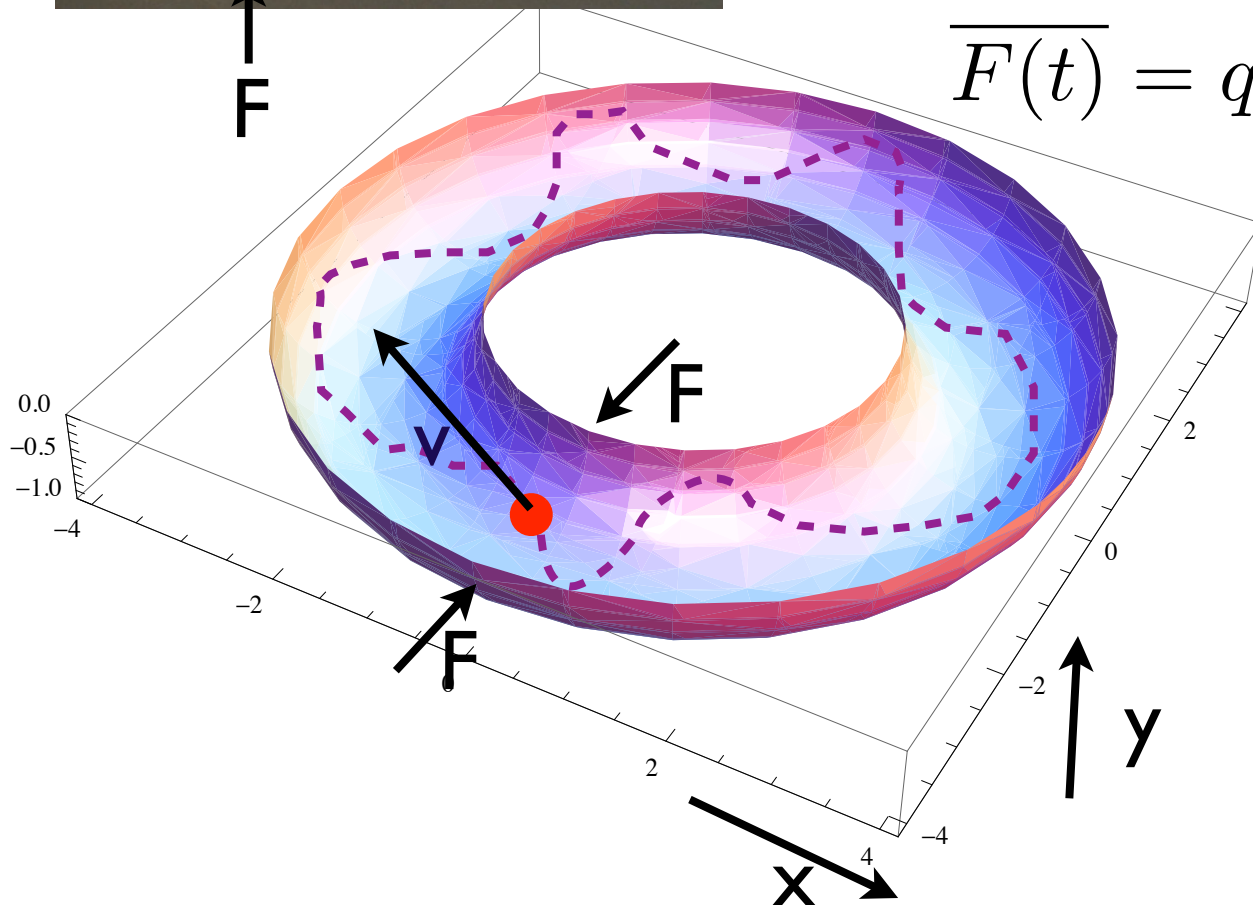


# How an accelerator works ?



Goal: keep enough particles confined in **a well defined volume** to accelerate them.

How ? Lorentz Force!



$$\overline{F}(t) = q \left( \overline{E}(t) + \overline{v}(t) \otimes \overline{B}(t) \right)$$

Particles of different energy (speed) behave differently

Magnetic field confines particles on a given trajectory

# Synchrotrons: strong focusing machine

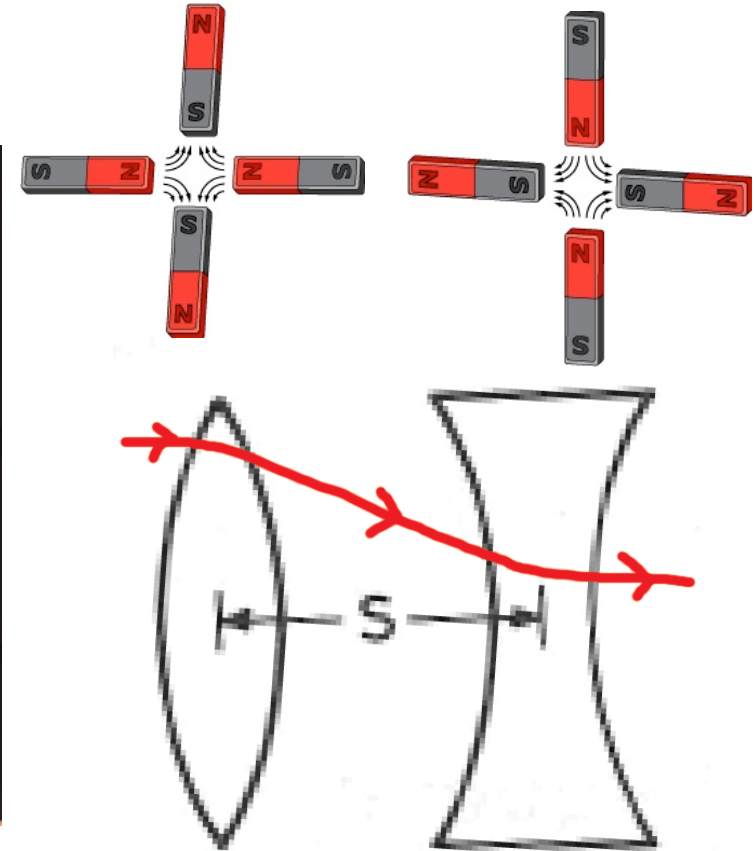
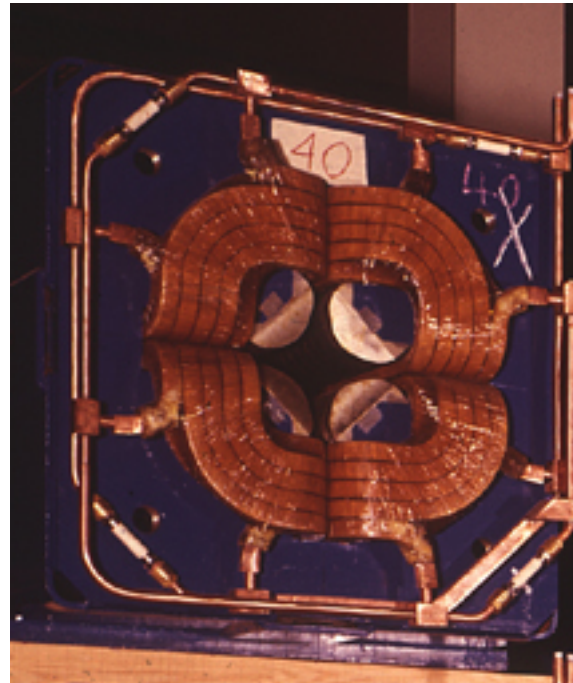
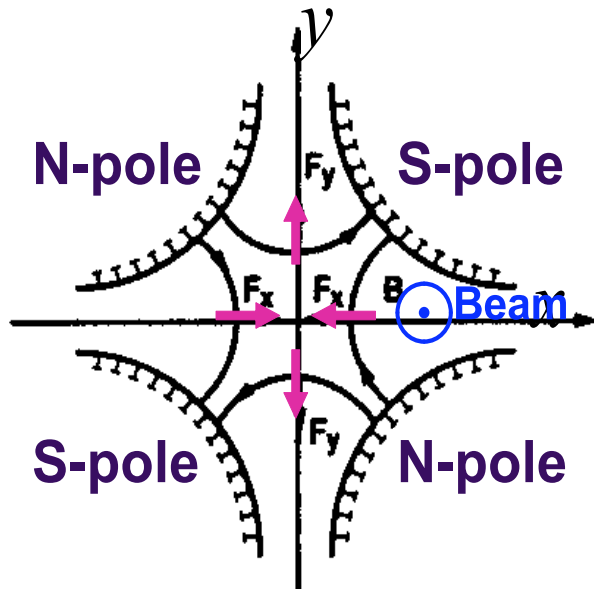
Dipoles are interleaved with quadrupoles to focus the beam.

**Quadrupoles act on charged particles as lens for light.**

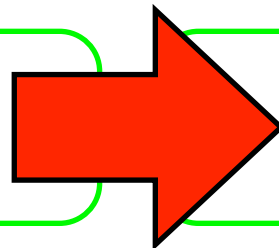
**By alternating focusing and defocusing lens**

**(Alternating Gradient quadrupoles) the beam dimension is kept small (even few  $\mu\text{m}^2$ ).**

## QUADRUPOLES



B field is focusing in one plane  
but defocusing in the other.



Typical lattice is FODO,  
focusing-drift-defocusing

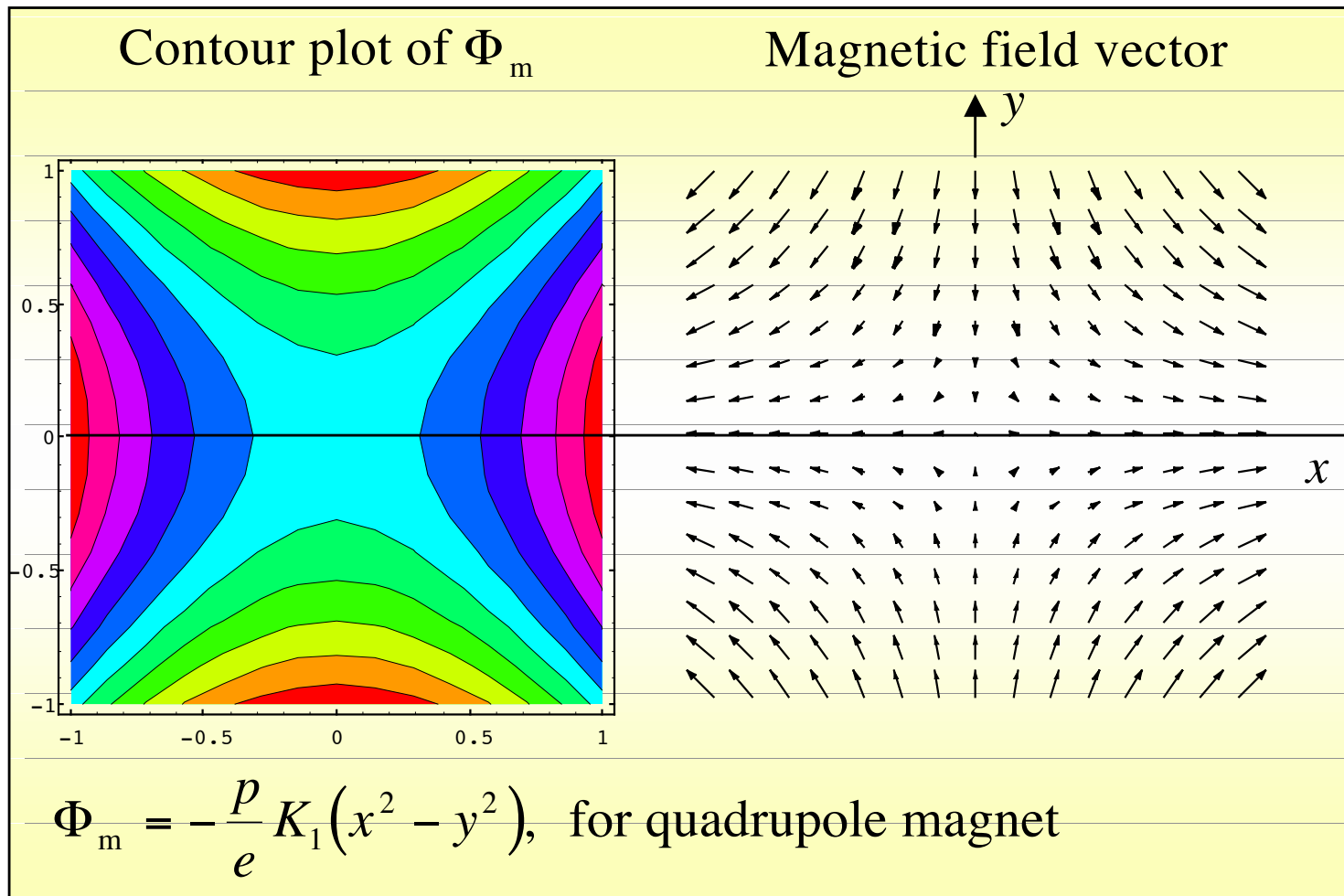


# Quadrupole field

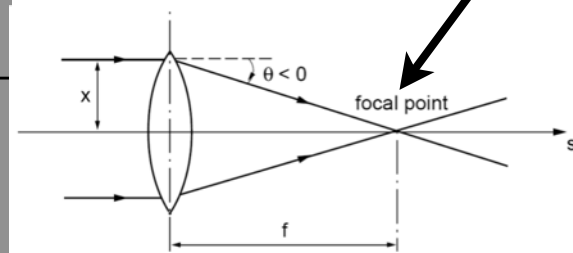
$$B = -\nabla \Phi_m \quad \Phi_m = \text{Vector potential}$$

The field increases linearly with the distance from the center of the magnet

Obviously,  $K$ , the gradient, has a sign. By convention  $+$  means focusing quadrupole in the horizontal plane.



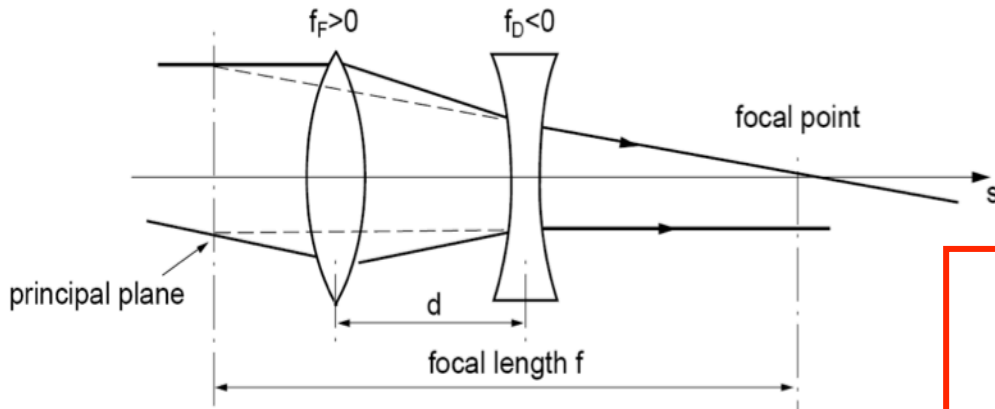
$$l K = -\frac{l}{B\rho} \frac{\partial B}{\partial x} = \frac{1}{f}$$



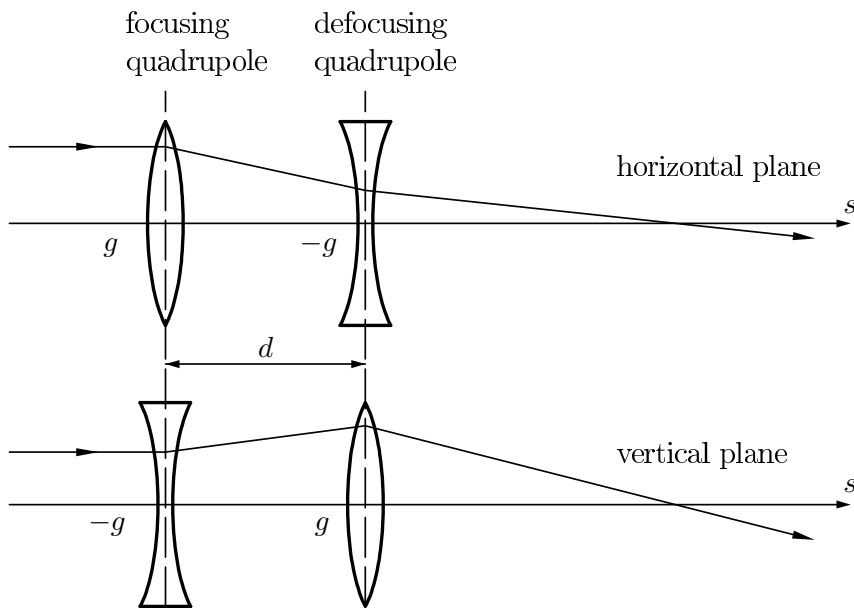
$$\tan \theta = -\frac{x}{f}$$

# FODO structure

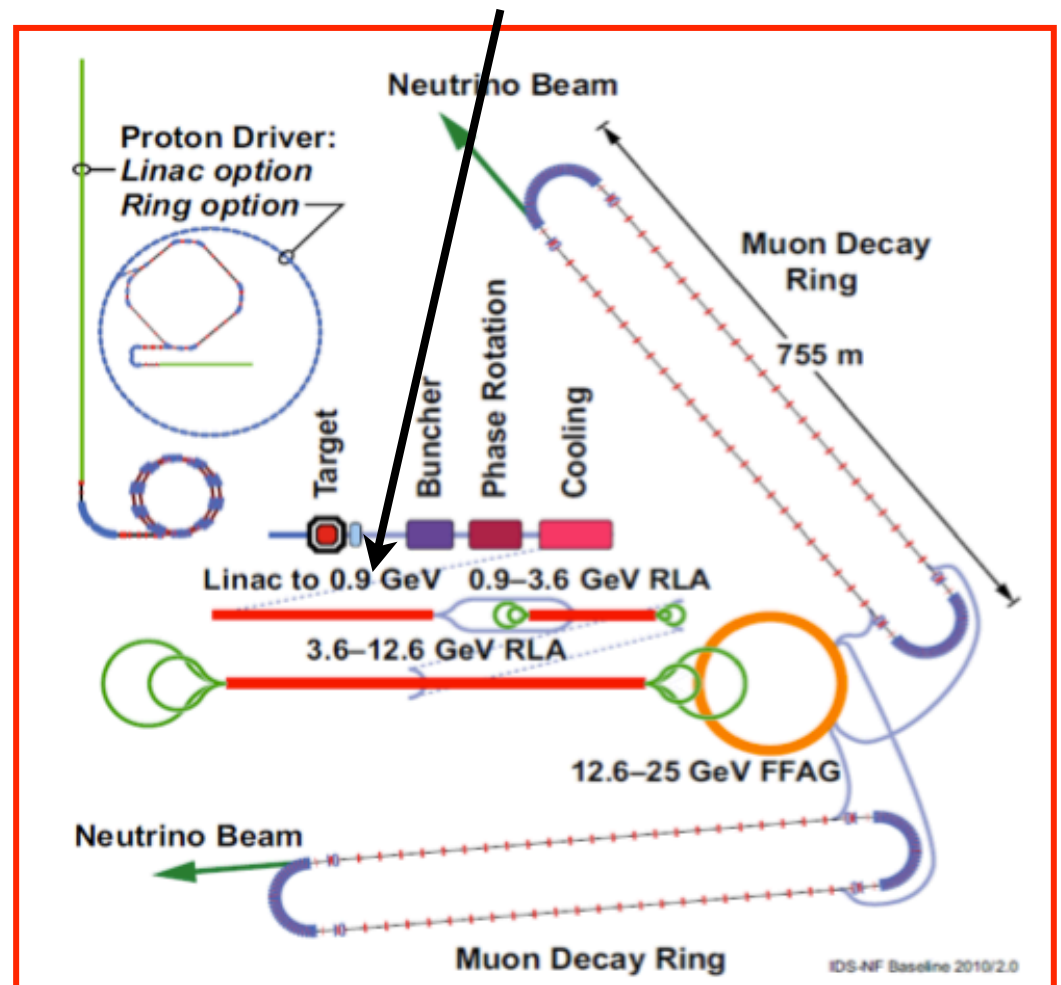
The FODO structure allows the focussing in both plane as in classical optics.



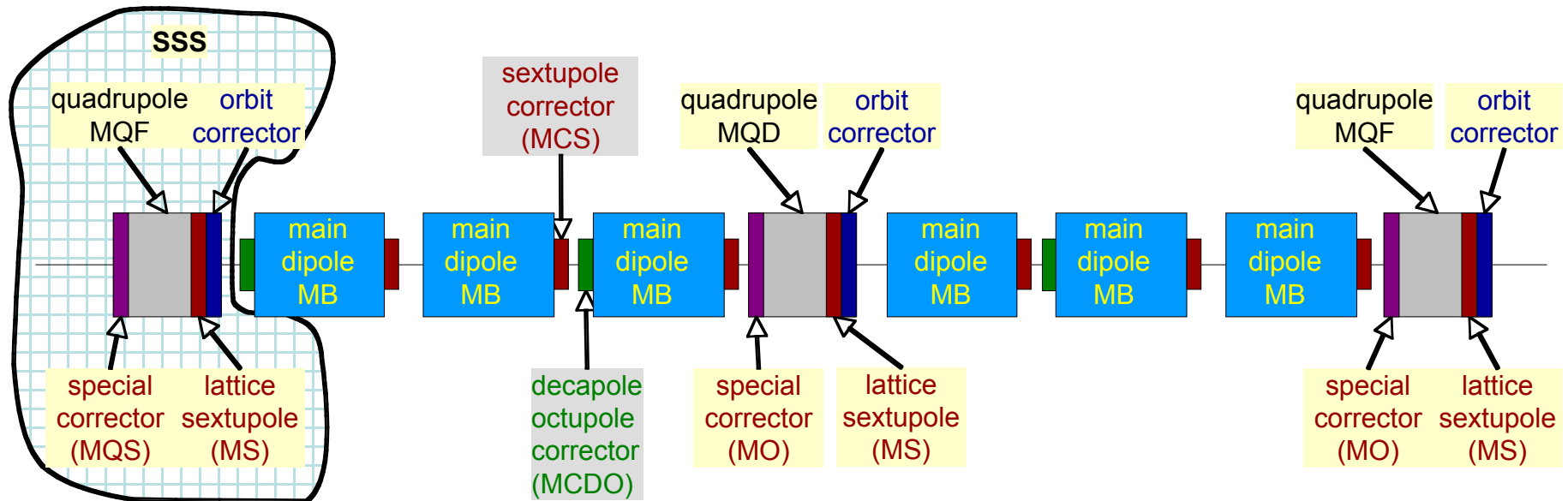
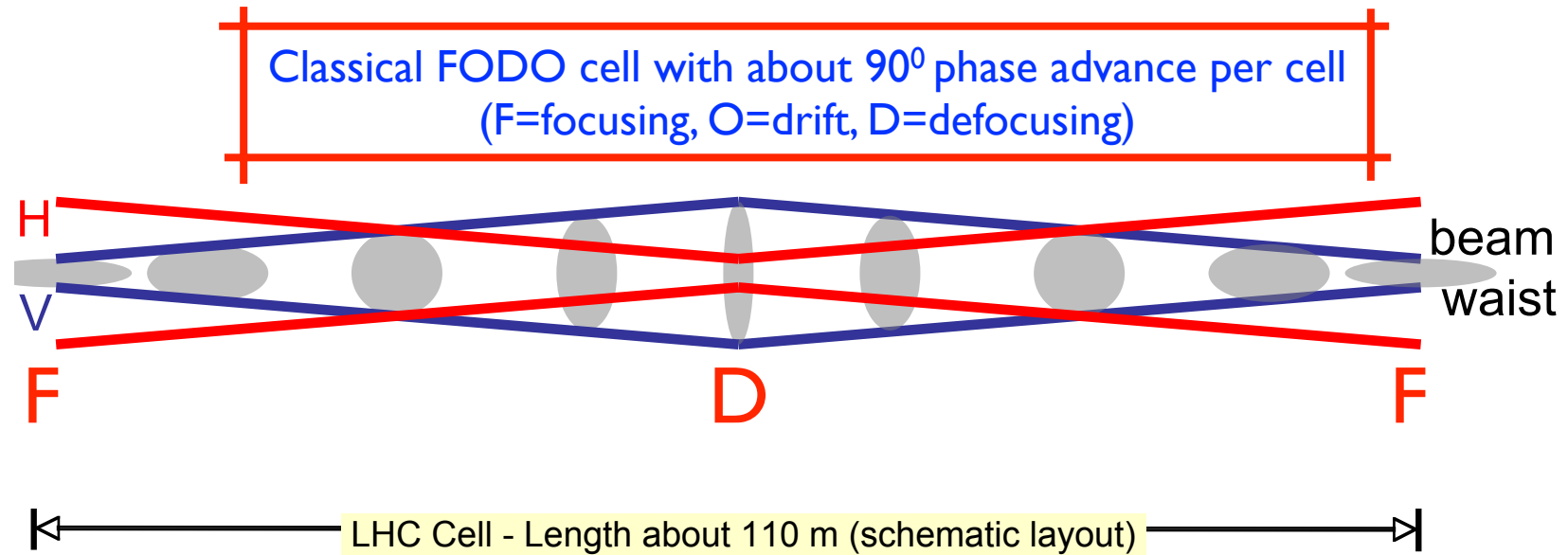
$$f = \frac{f_F^2}{d} > 0 \quad (\text{when } f_D = -f_F)$$



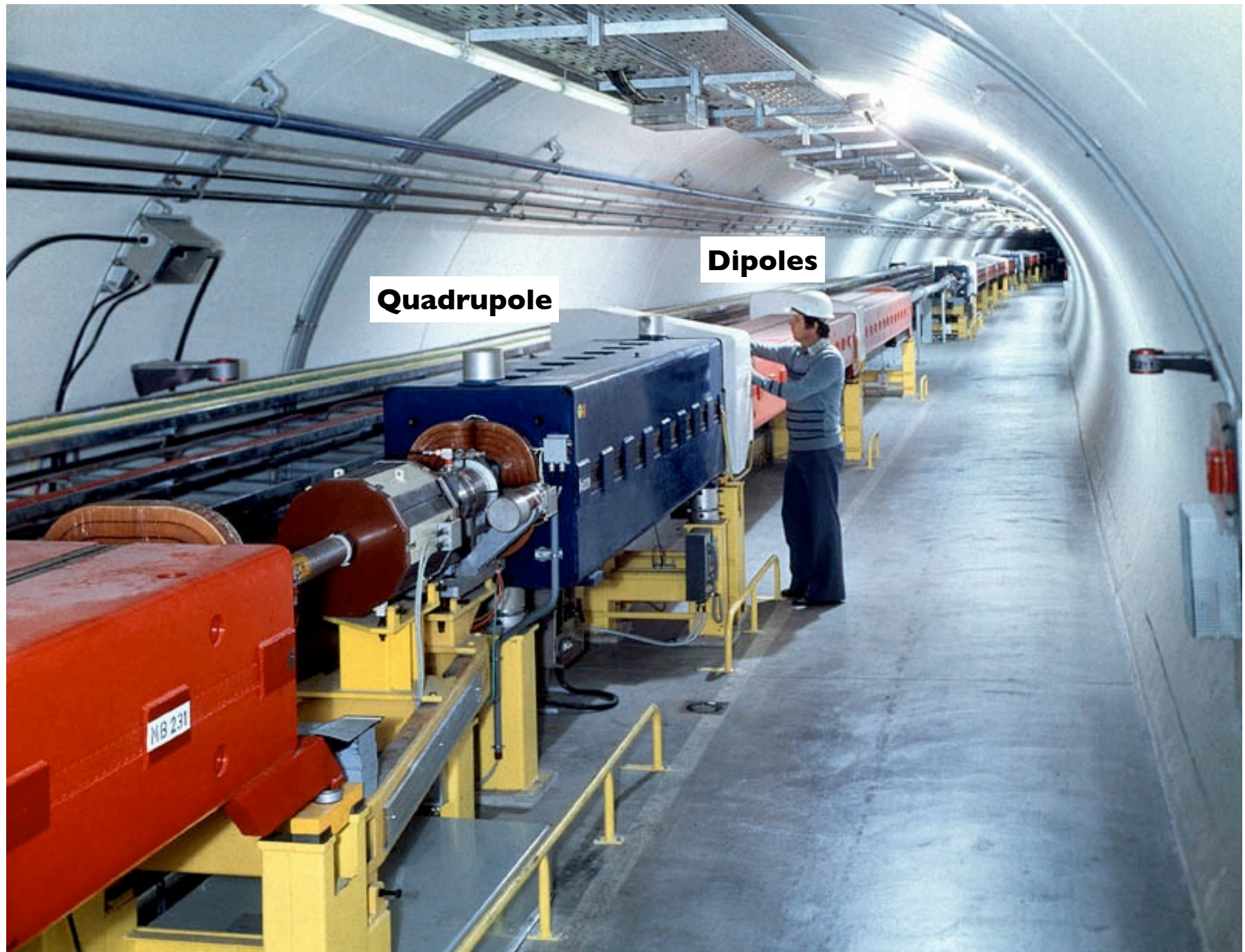
Quadrupolar focusing basically everywhere after this point...



# An example of a lattice: LHC cell

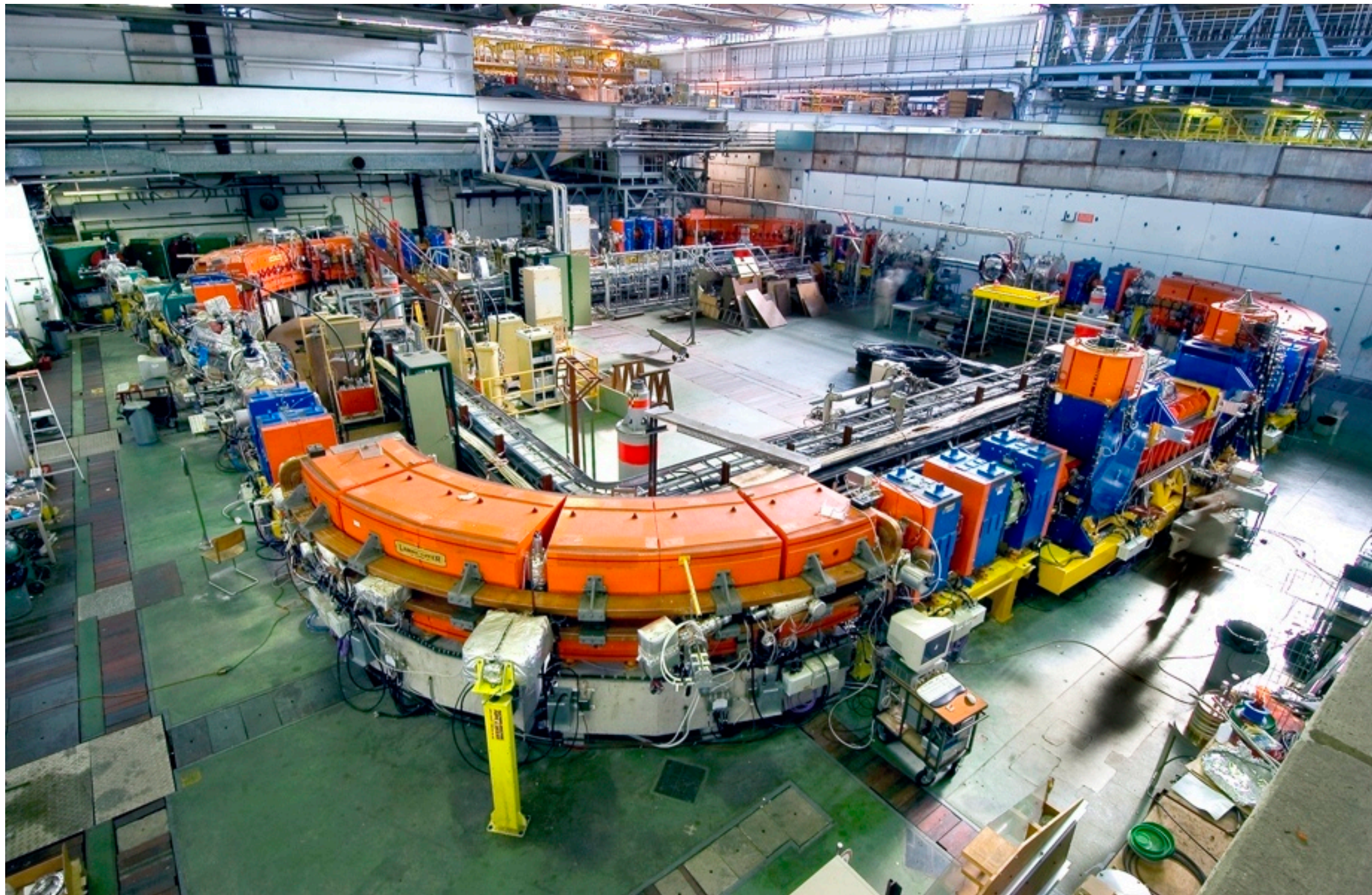


# The SPS tunnel





# A synchrotron in a view: LEIR (Low Energy Ion Ring)

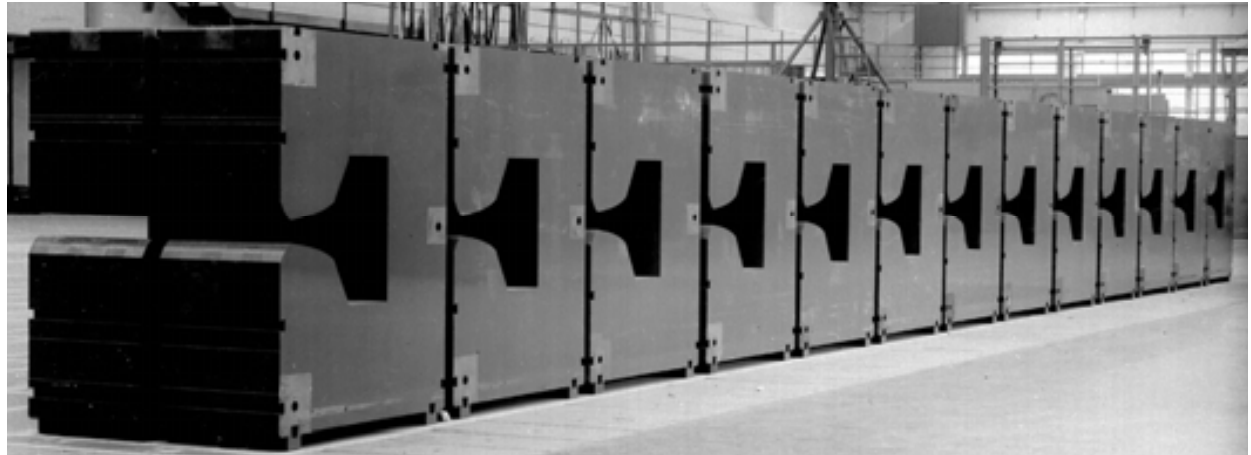
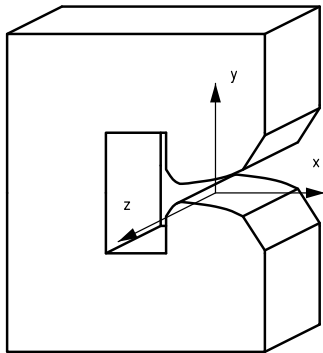




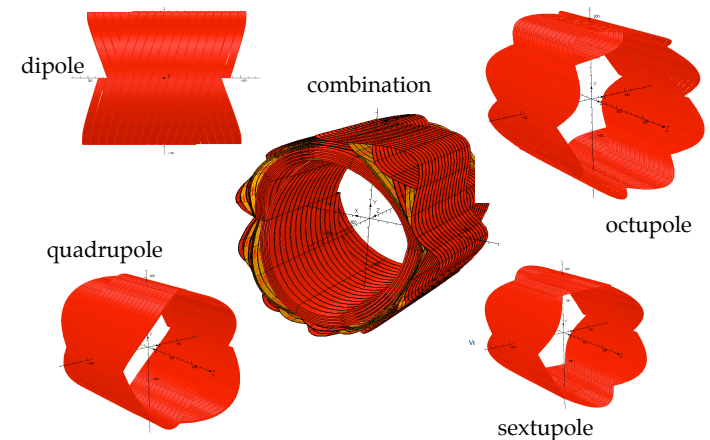
# Combined function magnets

Some accelerators, like the PS have combined function magnets, i.e., quadrupolar and dipolar component is generated in a single magnets

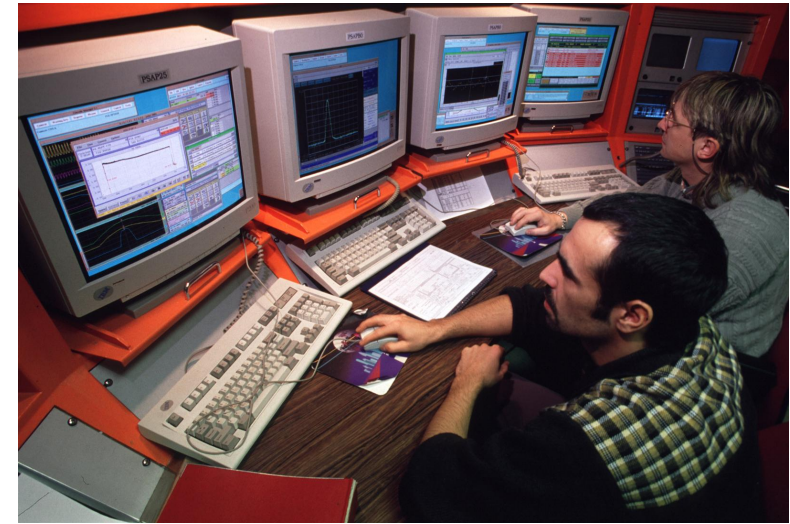
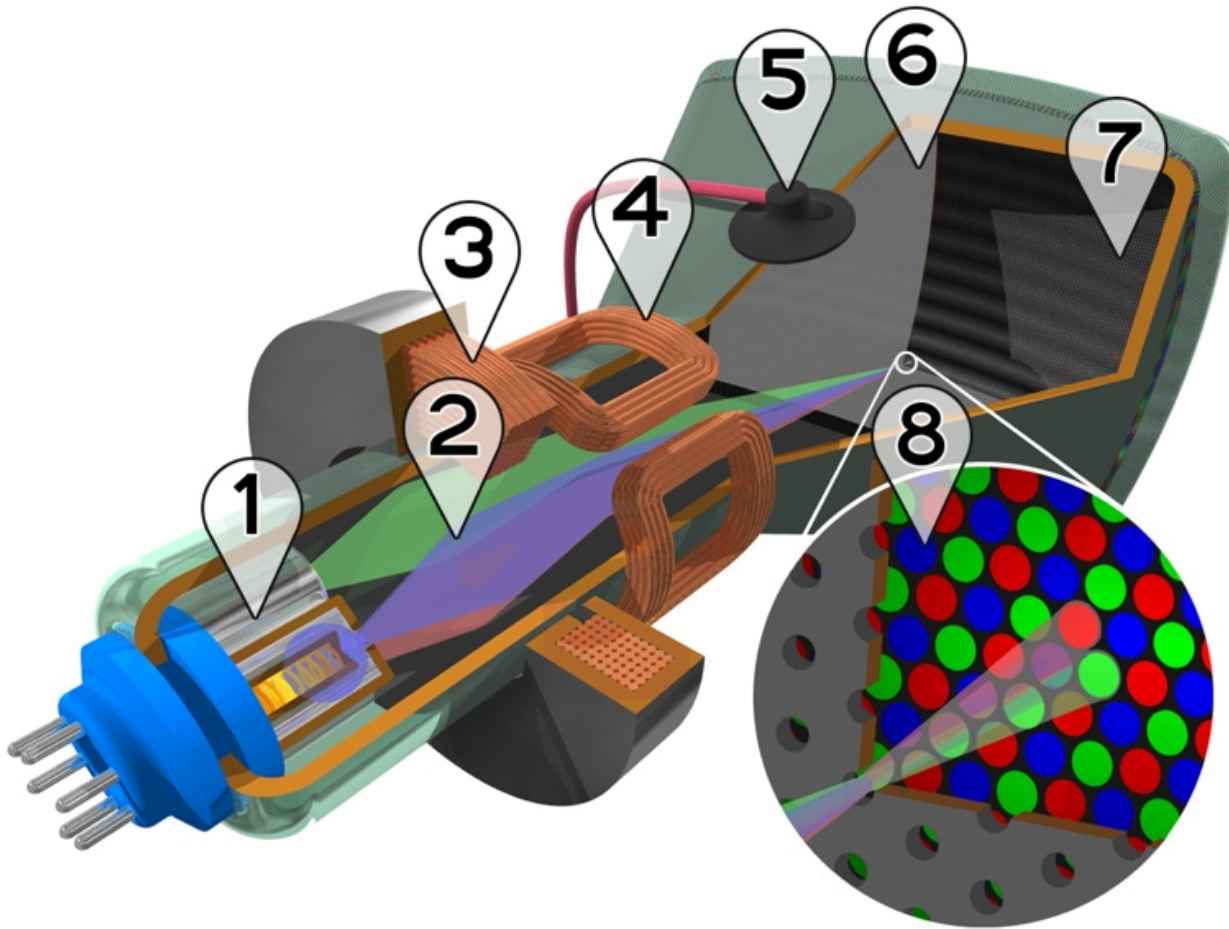
Open block



See later for FFAGs (Fixed Field Alternating Gradients accelerators)



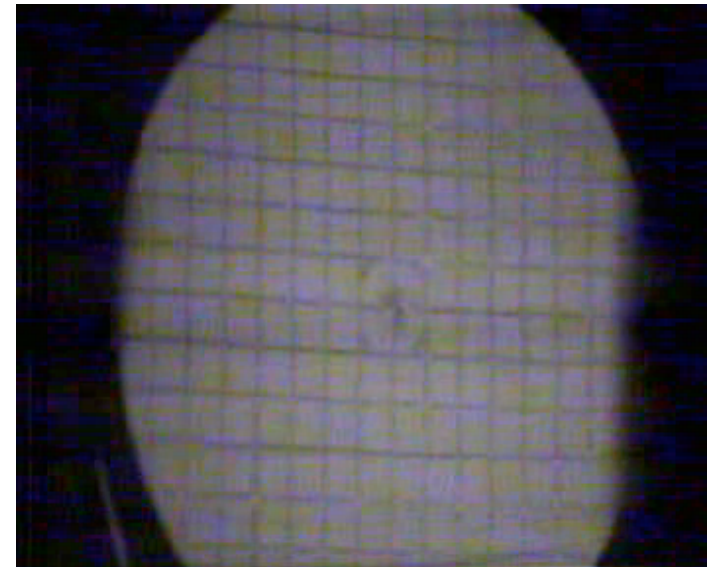
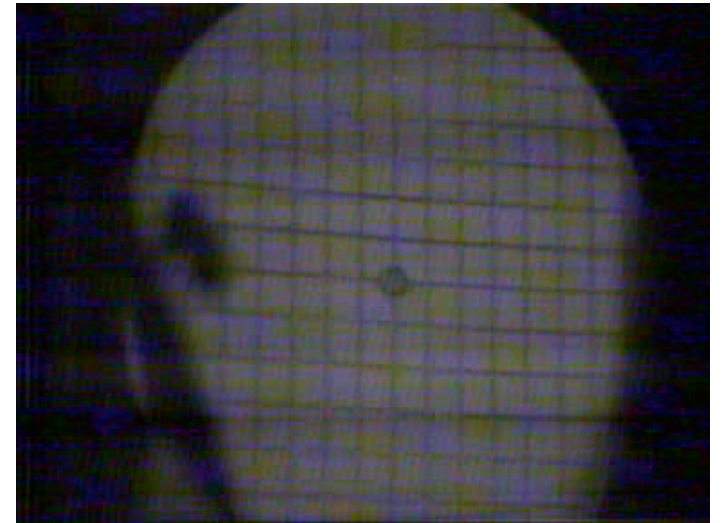
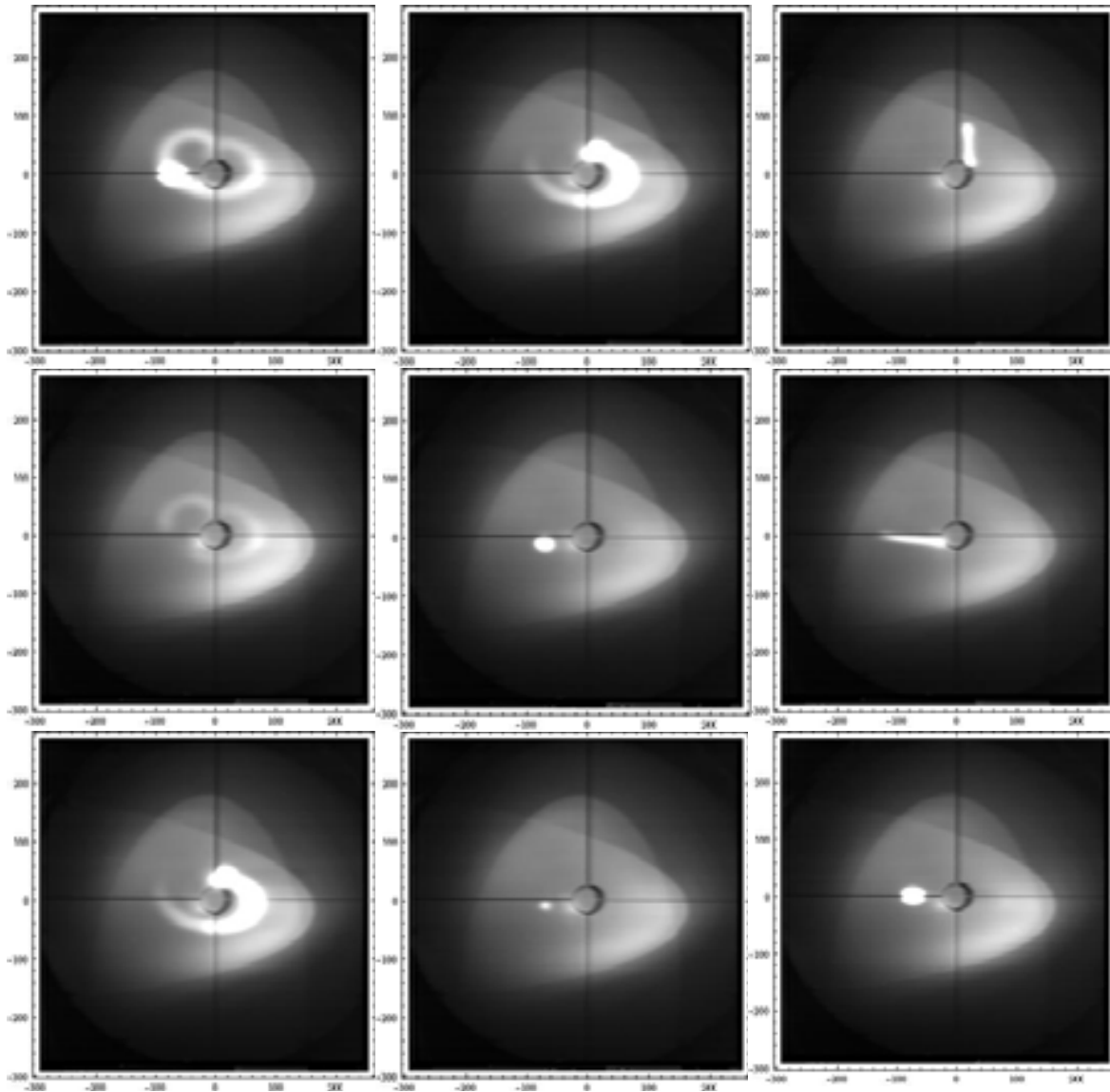
# An accelerator that you know very well



1. **Three Electron guns** (for red, green, and blue phosphor dots)
2. **Electron beams**
3. **Focusing coils**
4. **Deflection coils**
5. **Anode connection**
6. **Mask** for separating beams for red, green, and blue part of displayed image
7. **Phosphor layer** with red, green, and blue zones
8. **Close-up** of the phosphor-coated inner side of the screen



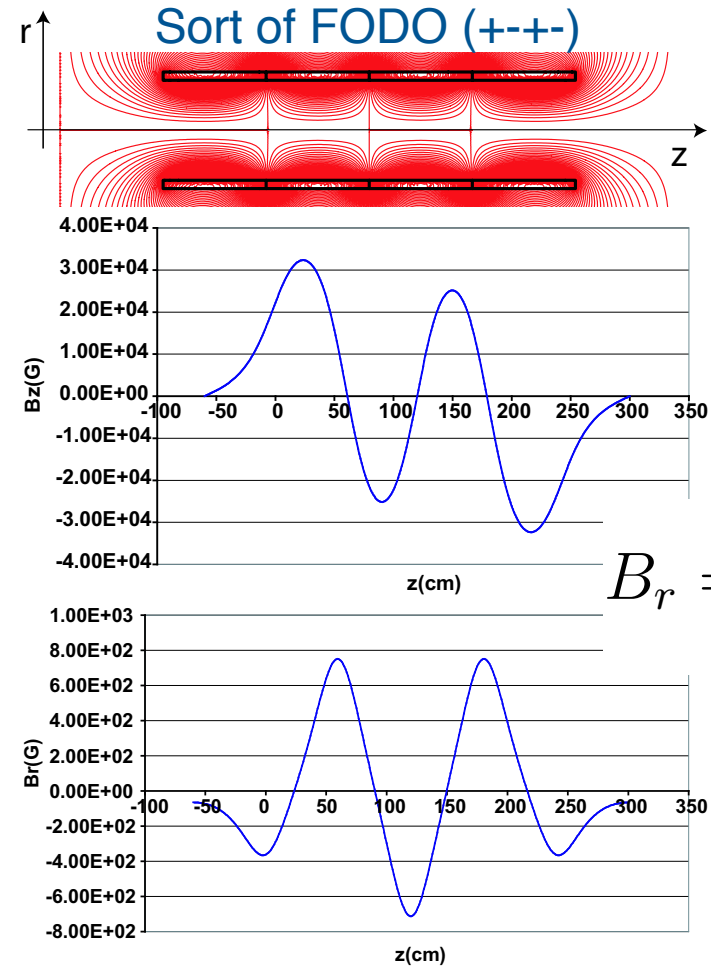
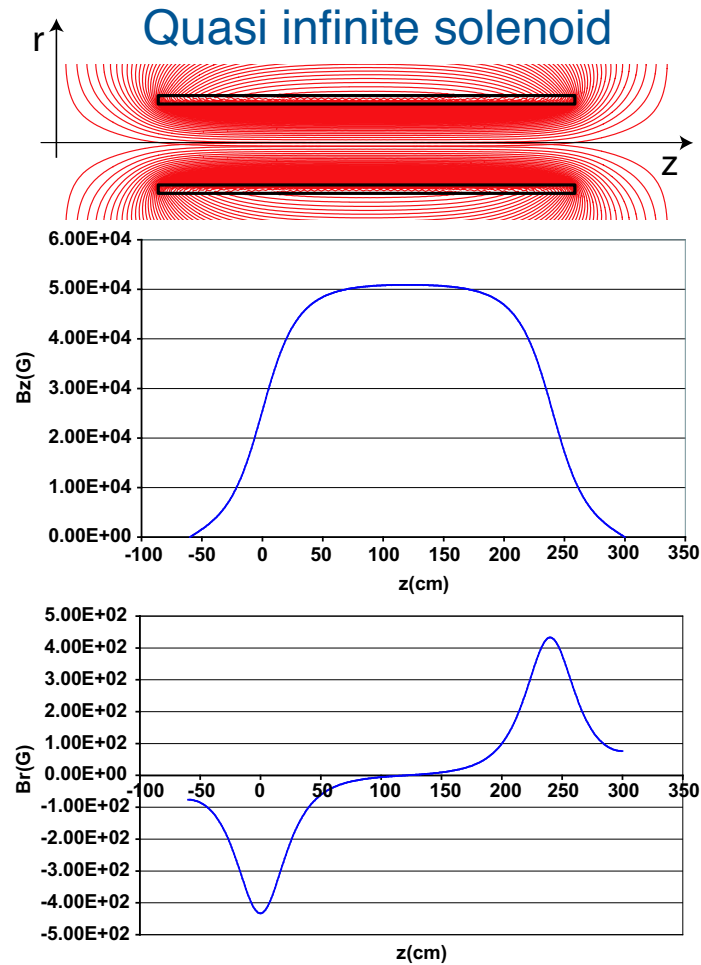
# Real beam images



Courtesy of B. Goddard



# Beam transport with solenoids



$$B_r = -\frac{r}{2} \frac{\partial B_z(z)}{\partial z}$$

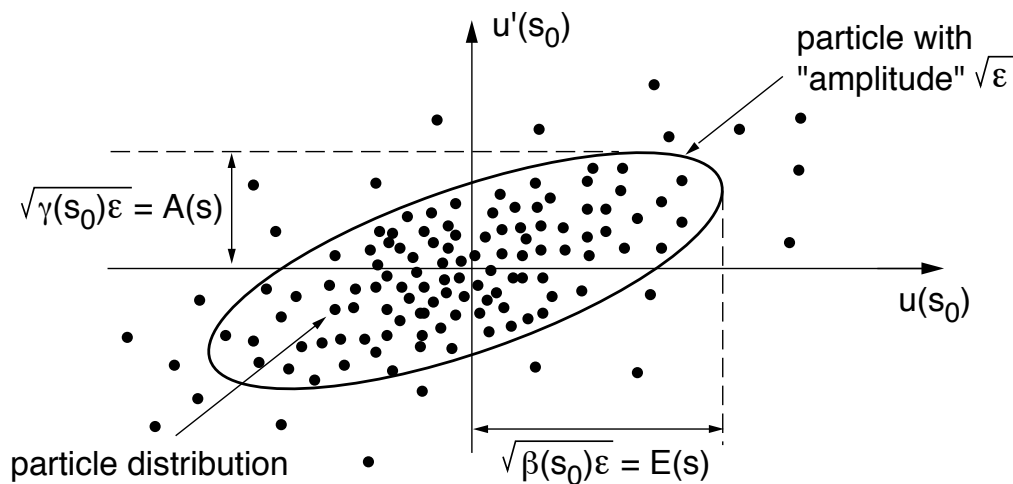
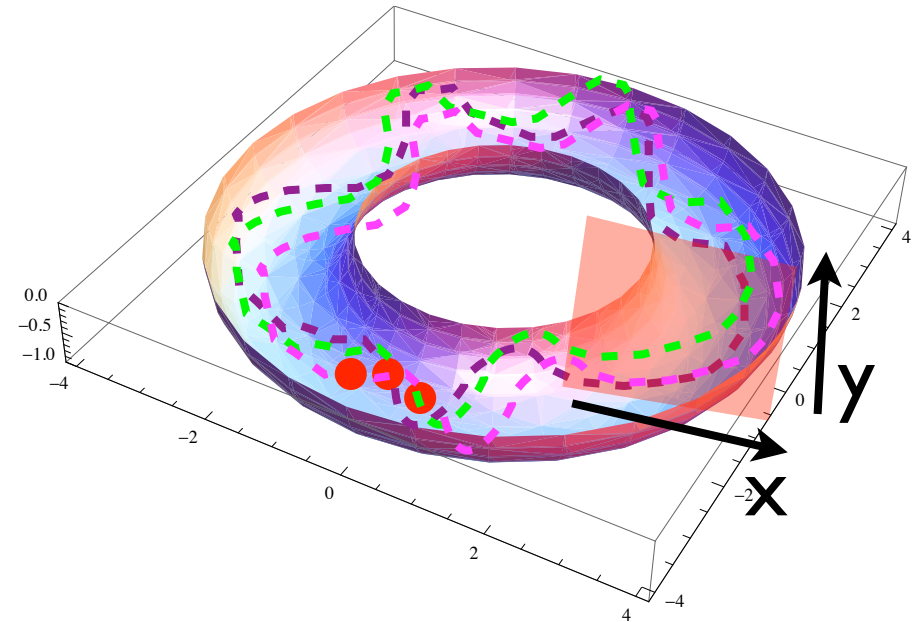
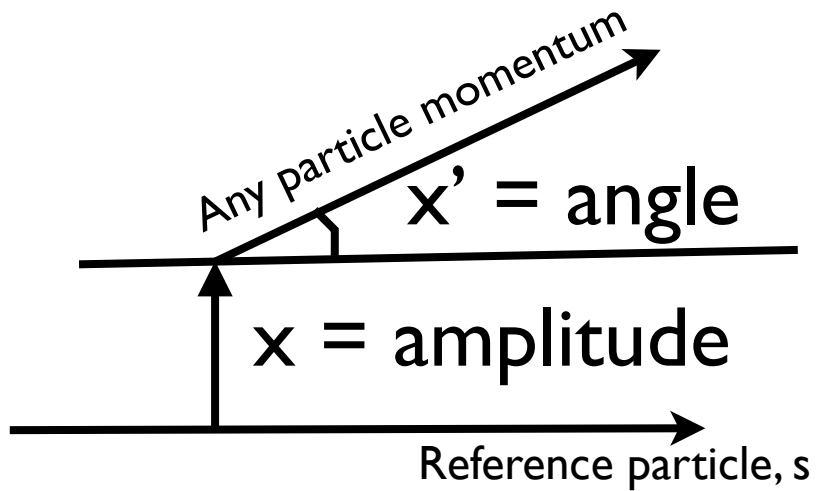
For low energy beams, beam transport can be done using solenoids  
(remember for low energy part of the Neutrino Factory)

In the transport, the transverse momentum is conserved  
for the zone at constant field:

$$B\rho^2 = \text{const}$$

$$\frac{B}{p_T^2} = \text{const}$$

# Our reference frame: $xx'$ , the phase space



The space occupied in the  $xx'$  (or  $yy'$ ) plane by the beam at a given position in the machine is defined as **Emittance**

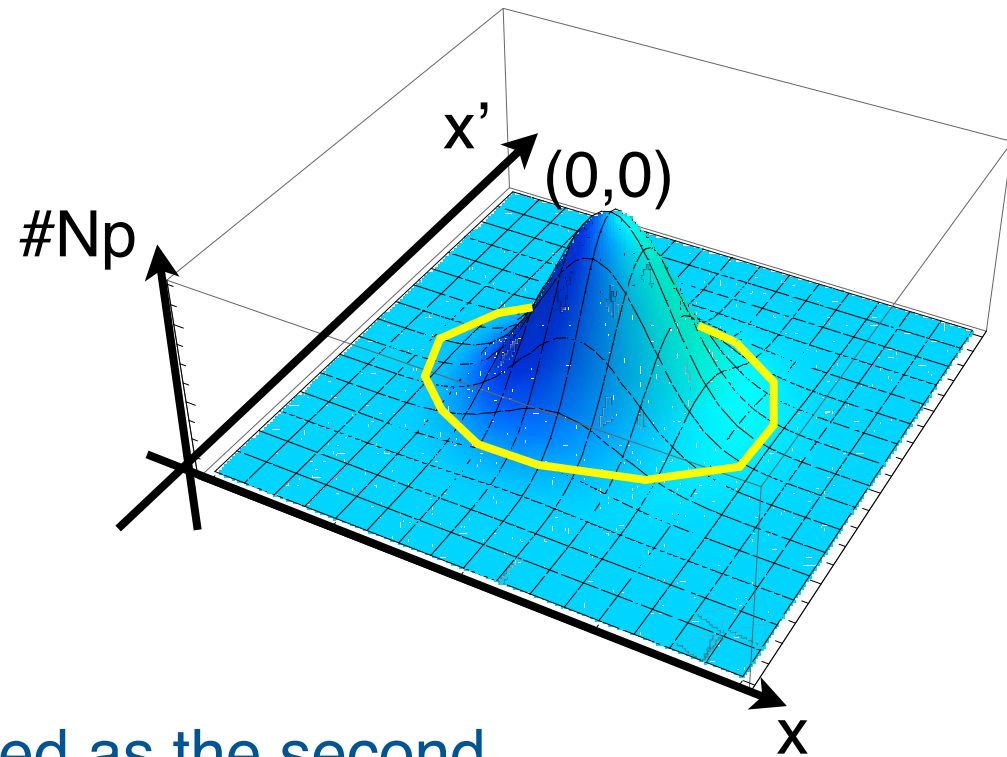
**The occupied phase space is an ellipse**

# Emittance: statistical definition

Let's consider a centered beam:

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i = 0$$

$$\bar{x}' = \frac{1}{N} \sum_{i=1}^N x_i' = 0$$



Def: The physical emittance is defined as the second order momentum of the particle distribution

$$\varepsilon = \frac{1}{N} \sqrt{\frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N (x_i x_j' - x_j x_i')^2} = \frac{1}{N} \sqrt{2 \sum_{i=1}^N \sum_{j=1}^N A_{ij}^2}$$

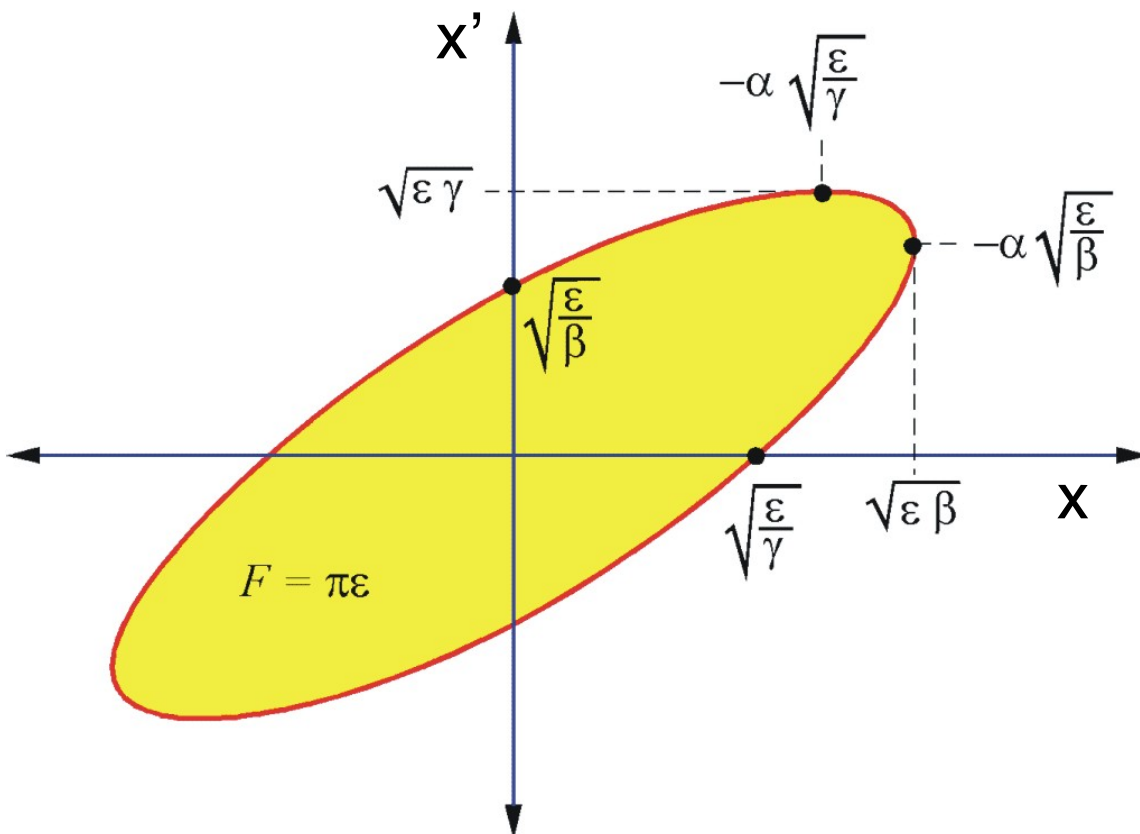
Units of the emittance: [mm mrad] (typically)

# The beam ellipse

The beam ellipse is described by convention by 3 parameters  
the **TWISS PARAMETERS** :  $\alpha, \beta, \gamma$

The ellipse equation takes the form:

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \epsilon$$



**Beta is NOT v/c**

**Gamma is NOT E/m**

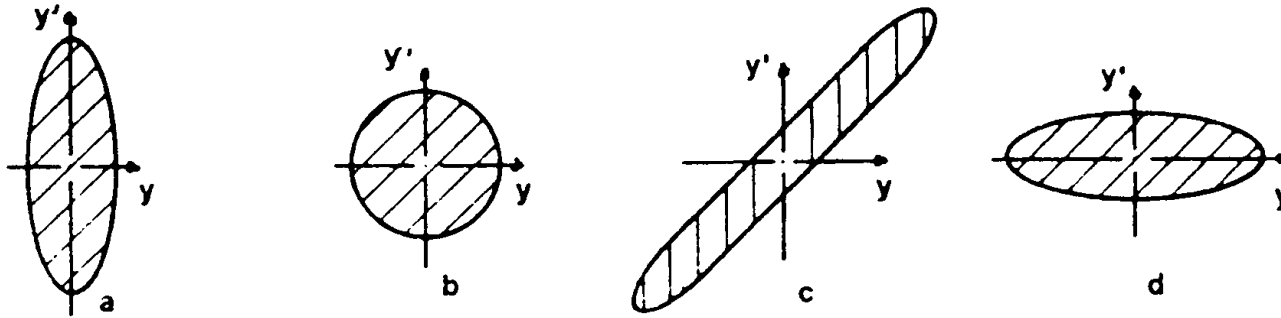
**This is an unfortunate choice  
of our predecessors for which  
I cannot be blamed ....**

# THE LAW: Liouville theorem

**Theorem:** In the vicinity of a particle, the particle density in phase space is a constant if the particle move in an external magnetic field or in a general field which the force do not depend upon velocity (*ipse dixit...*), i.e., **the beam is like an incompressible fluid in phase space**

## Implications:

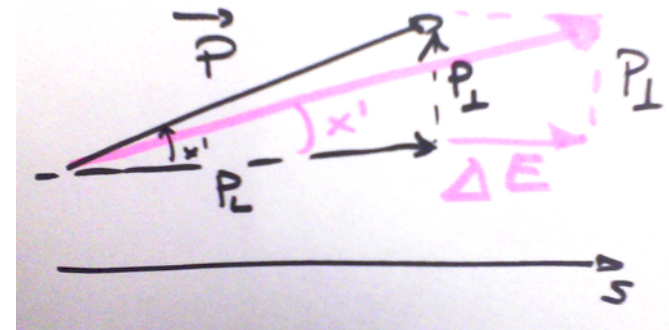
a) the emittance is conserved when the beam is transported via a magnetic system



**The ellipse is distorted/stretched but the surface is conserved.**

b) the emittance is **NOT** conserved if we accelerate, except if we normalize the emittance wrt to  $\beta\gamma$  (relativistic).  **$x'$  is reduced by the acceleration.**

$$\epsilon_{norm} = \epsilon_{phys} * \beta_{rel} * \gamma_{rel}$$



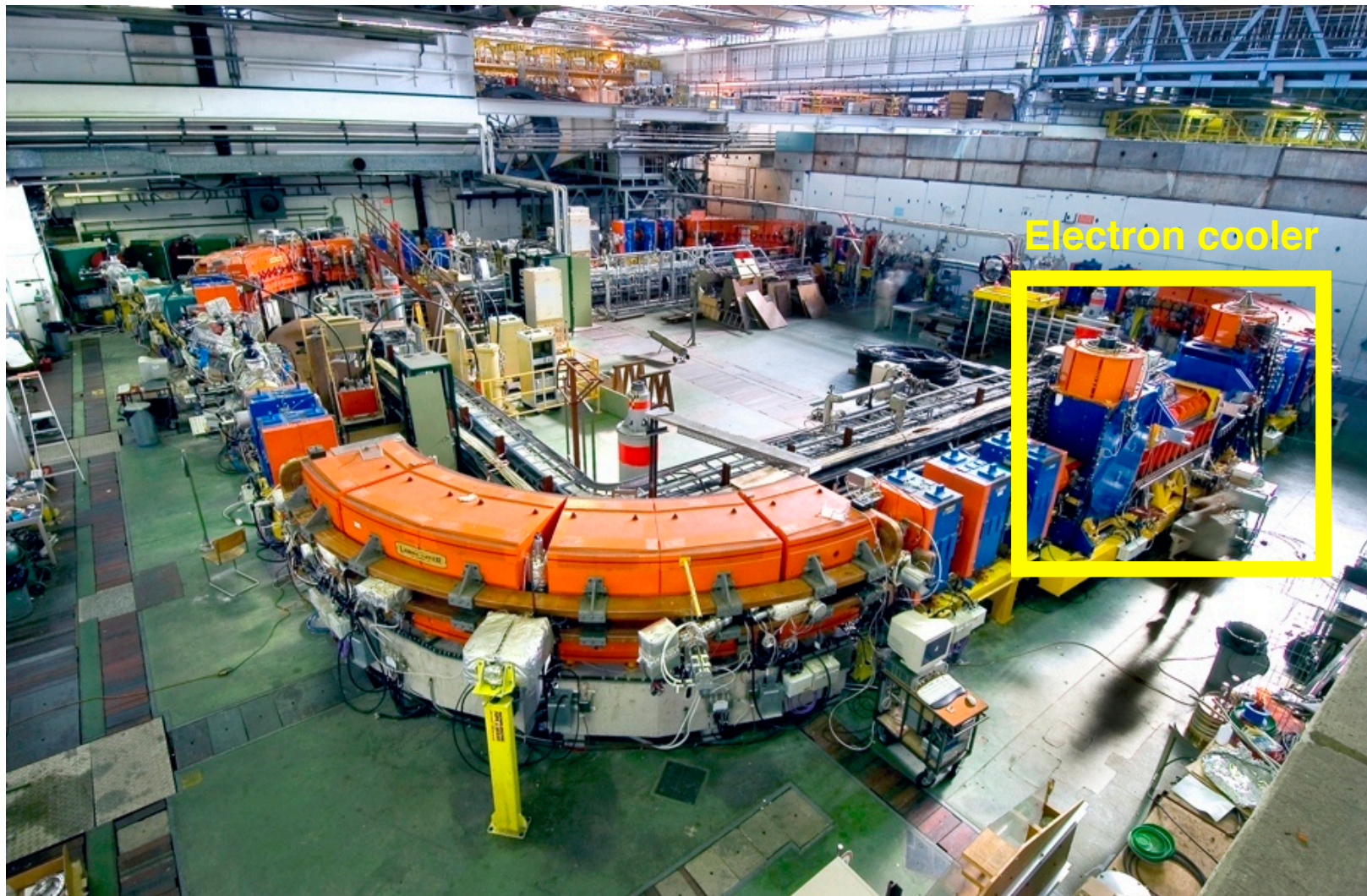
c) if we want to reduce emittance at constant energy, we have to “cheat”: **BEAM COOLING**



# Beam cooling

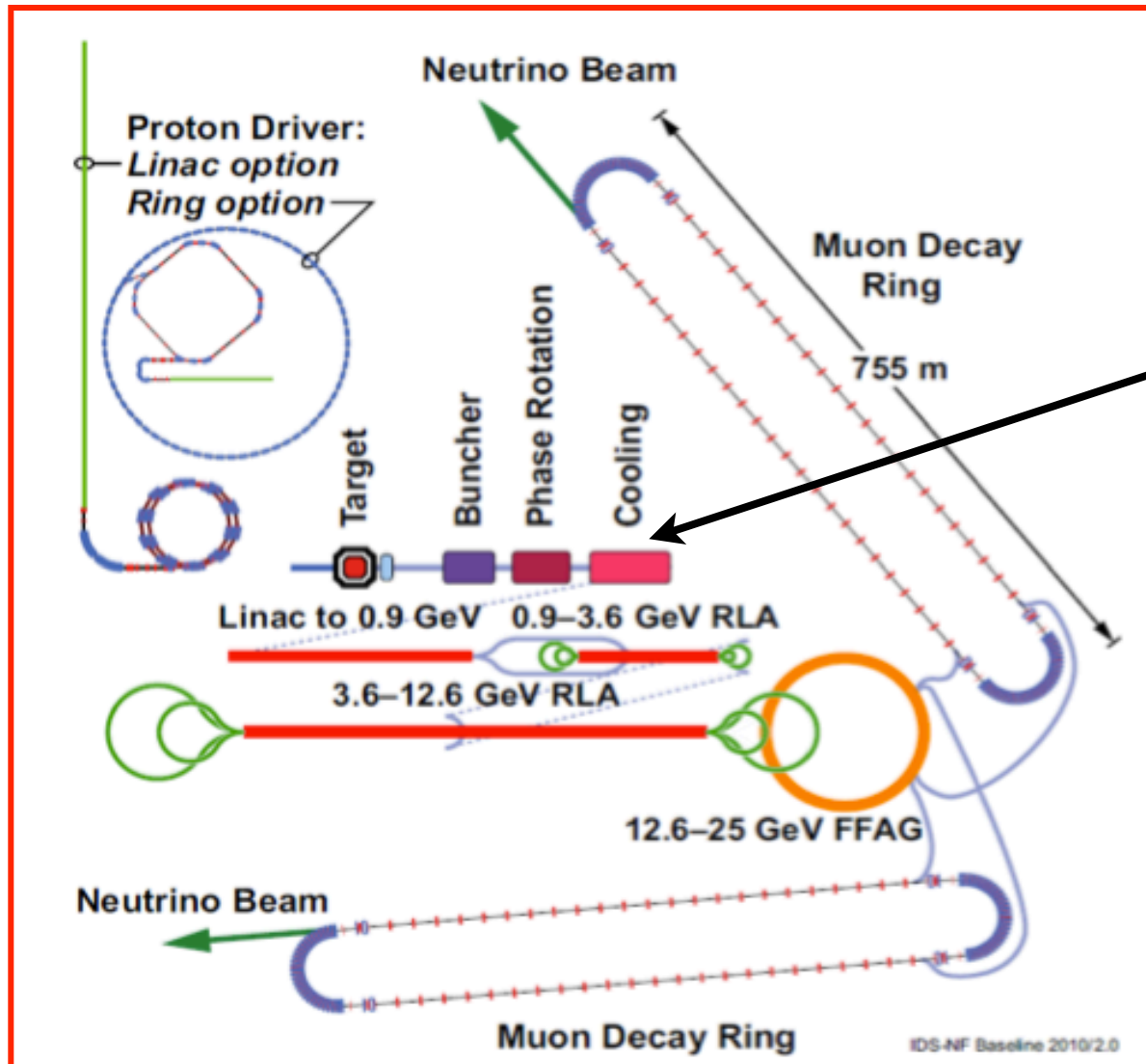
An low intensity ion beam is accumulated in the storage ring. To reduce the emittance, a very intense mono-energetic electron beam is sent parallel to the ion beam.

The large emittance ion beam is like an hot fluid which is cooled by the very ordinate electron beam. Same as putting one near another two fluids at different temperatures



# Why emittance is so important?

See in a moment, determine the beam dimension from the optics/lattice design



Here we want to reduce beam emittance to be able to feed the rest of the accelerator complex

# Classical mechanics.... spring with a mass

$$F = ma = m \frac{d^2 x}{dt^2} = -kx$$

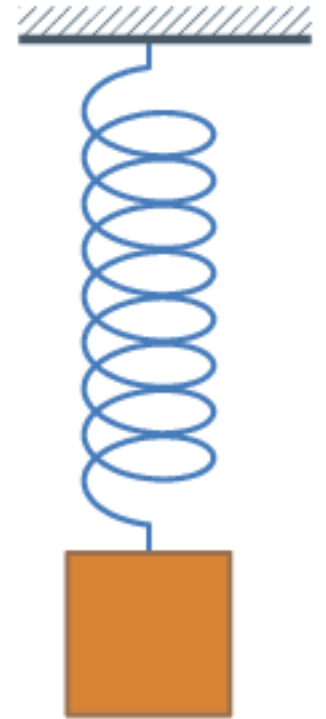
with  $k$  the spring constant and  $m$  the mass

**Solution of the equation of motion is a periodic function:**

$$x(t) = A \cos(2\pi f t + \phi)$$

with 1/period equals to

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$





# Equation of motion, not too in details

Equation of motion of a particle in an accelerator composed by a sequence of elements, each one eventually with a  $k$  at a position  $s$  of the ring, repeated at every  $C$

\*Hill's equation: pendulum-like with non-constant spring force wrt to  $s$ .

$$\frac{d^2 x}{ds^2} + K(s)x = 0 \xrightarrow{\text{beer} = \text{solution}} x(s) = a\sqrt{\beta(s)} \cos(\phi(s) + \phi_0)$$

Local force at a position  $s$  in the *ring*

$$K(s) = 1/\rho^2 + k(s)$$

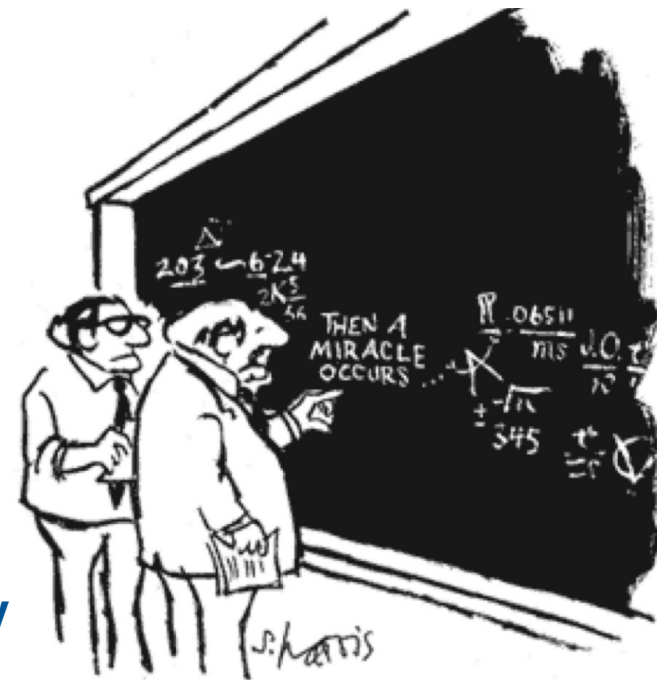
Dipoles

Quadrupoles

forget them for a moment

$$K(s) = K(s + C)$$

This imply periodicity of the solution



"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

me too... in a moment...

\*there was a Mr. Hill, an astronomer

# Solution of Hill's equation

$$x(s) = a \sqrt{\beta(s)} \cos(\phi(s) + \phi_0)$$

this "probably" contains k

Spring solution

$$x(t) = A \cos(2\pi f t + \phi)$$

This actually... look alike should not be there...

The **beta function** is a product of the locally changing force in the accelerator, i.e., of the **quadrupoles**.

Every section of an accelerator has a constant k, so alone would be similar to an harmonic oscillator

this contains k and m

By definition (*ipse dixit...*): 
$$\phi(s) = \int \frac{1}{\beta(s)} ds$$

is called the **phase advance**

# Relationship between beam ellipse and beta

*Nearly no beer ... full proof ...*

if the emittance is a surface this can be an amplitude (I am cheating... I know)

1) Let's suppose  $x''(s) + K(s) \cdot x(s) = 0$   $\longrightarrow$   $x(s) = \sqrt{\varepsilon} \cdot u(s) \cdot \cos(\phi(s) + \varphi_0)$

2) Let's apply  $\longleftarrow$  **What is this???**

$$\left[ u'' - u \cdot \phi'^2 + K \cdot u \right] \cdot \cos(\phi + \varphi_0) - \left[ 2 \cdot u' \cdot \phi' + u \cdot \phi'' \right] \sin(\phi + \varphi_0) = 0$$

*beer + trick. Coeffs in front of sin et cos should be zero and*  $\phi(s) = \int_0^s \frac{d\tilde{s}}{u^2(\tilde{s})}$

$$u'' - \frac{1}{u^3} + K \cdot u = 0 \xrightarrow{\text{def.}} \beta(s) := u^2(s) \longrightarrow x(s) = \sqrt{\varepsilon} \cdot \sqrt{\beta(s)} \cdot \cos(\phi(s) + \varphi_0)$$

$$\alpha(s) := -\frac{\beta'(s)}{2} \xrightarrow{\text{def.}} x'(s) = -\frac{\sqrt{\varepsilon}}{\sqrt{\beta(s)}} \left\{ \alpha(s) \cdot \cos(\phi(s) + \varphi_0) + \sin(\phi(s) + \varphi_0) \right\}$$

*beer*

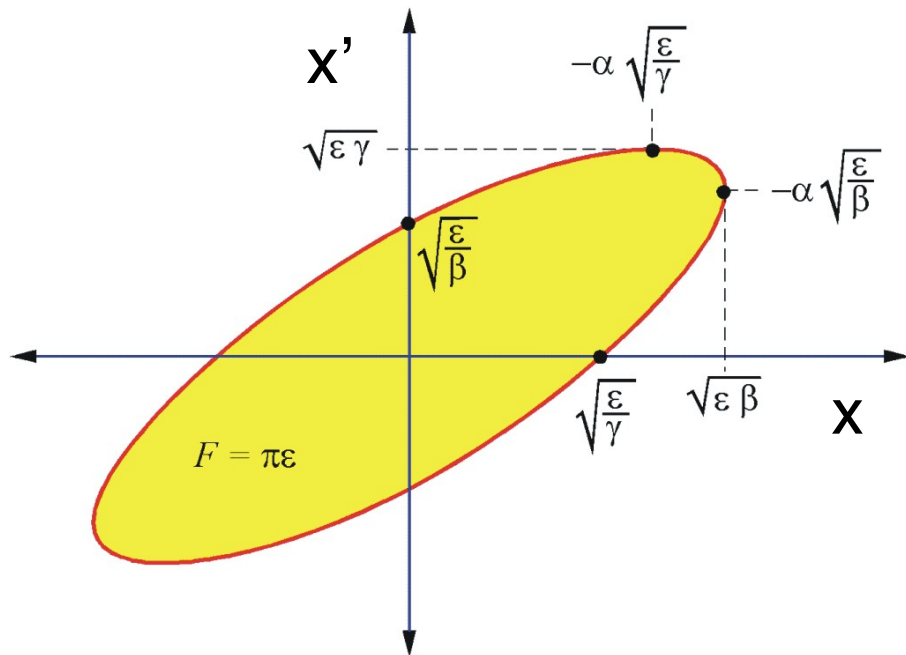
$$\sin^2(\phi + \varphi_0) = \left( \sqrt{\frac{\beta}{\varepsilon}} \cdot x' + \frac{\alpha}{\sqrt{\varepsilon \beta}} \cdot x \right)^2 \xrightarrow{\text{def.}} \gamma(s) := \frac{1 + \alpha^2(s)}{\beta(s)} \xrightarrow{\text{We brilliantly find...}}$$

..... what we wanted...

**oh surprise...**  $\longrightarrow \gamma x^2 + 2\alpha x x' + \beta x'^2 = \epsilon$

**Learned:**

- a) definition of Twiss parameters comes from the equation of motion and beta function
- b) The dynamics is really on/within an ellipse



Twiss parameters:

$$\alpha(s) := -\frac{\beta'(s)}{2}$$

$$\gamma(s) := \frac{1 + \alpha^2(s)}{\beta(s)}$$

$$\beta(s)$$



**Those are not  
the relativistic  
homonyms**



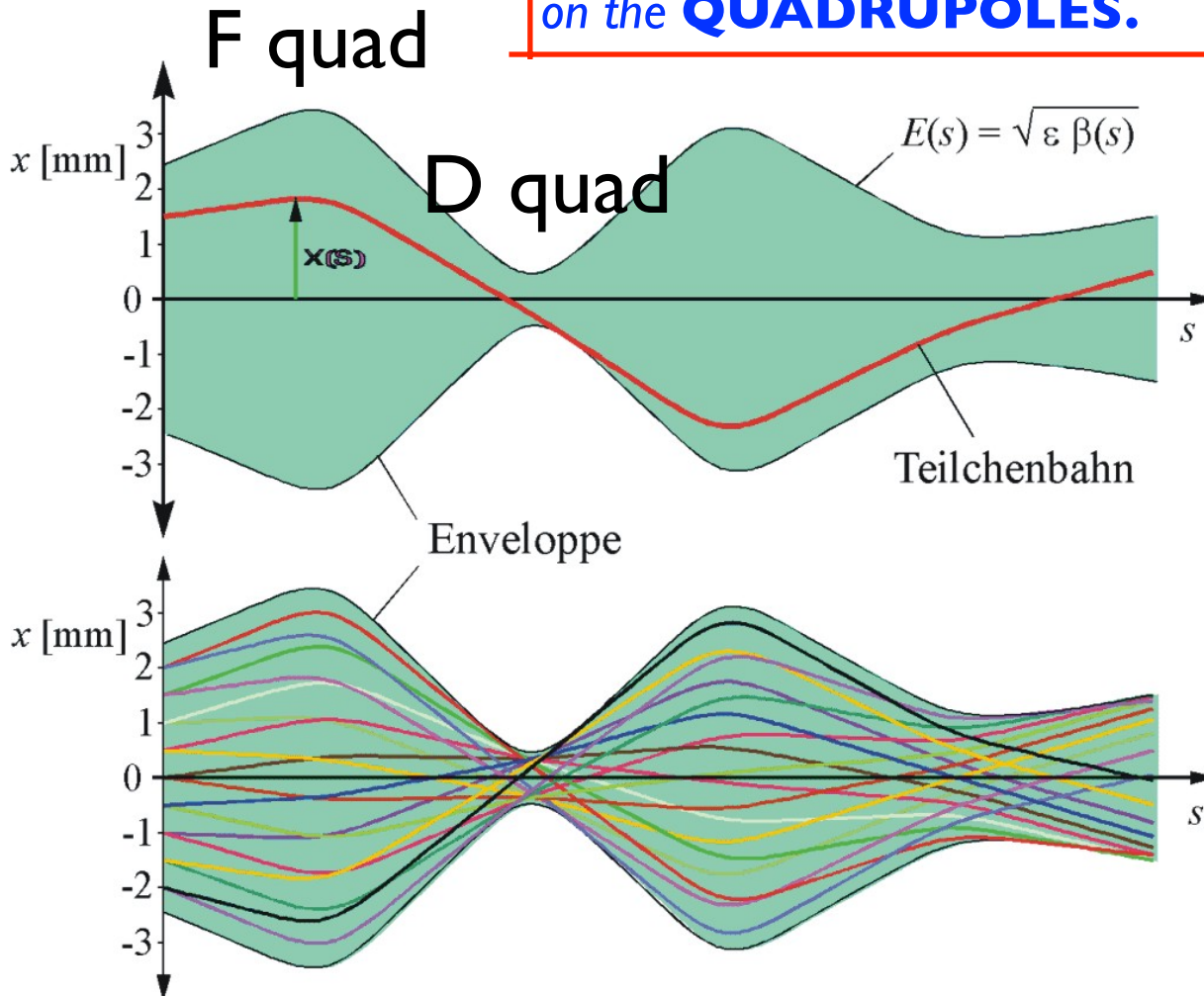
# Definition of envelope

$$\sigma_{x,y}^* = \sqrt{\beta_{x,y}^* \cdot \epsilon_{x,y}}$$

**Emittance:** Parameter which describes the **spread** of the particles in the phase space ( $xx'$ ) or ( $yy'$ ).

Optical machine parameter that depends on the lattice of the machine, in particular on the **QUADRUPOLES**.

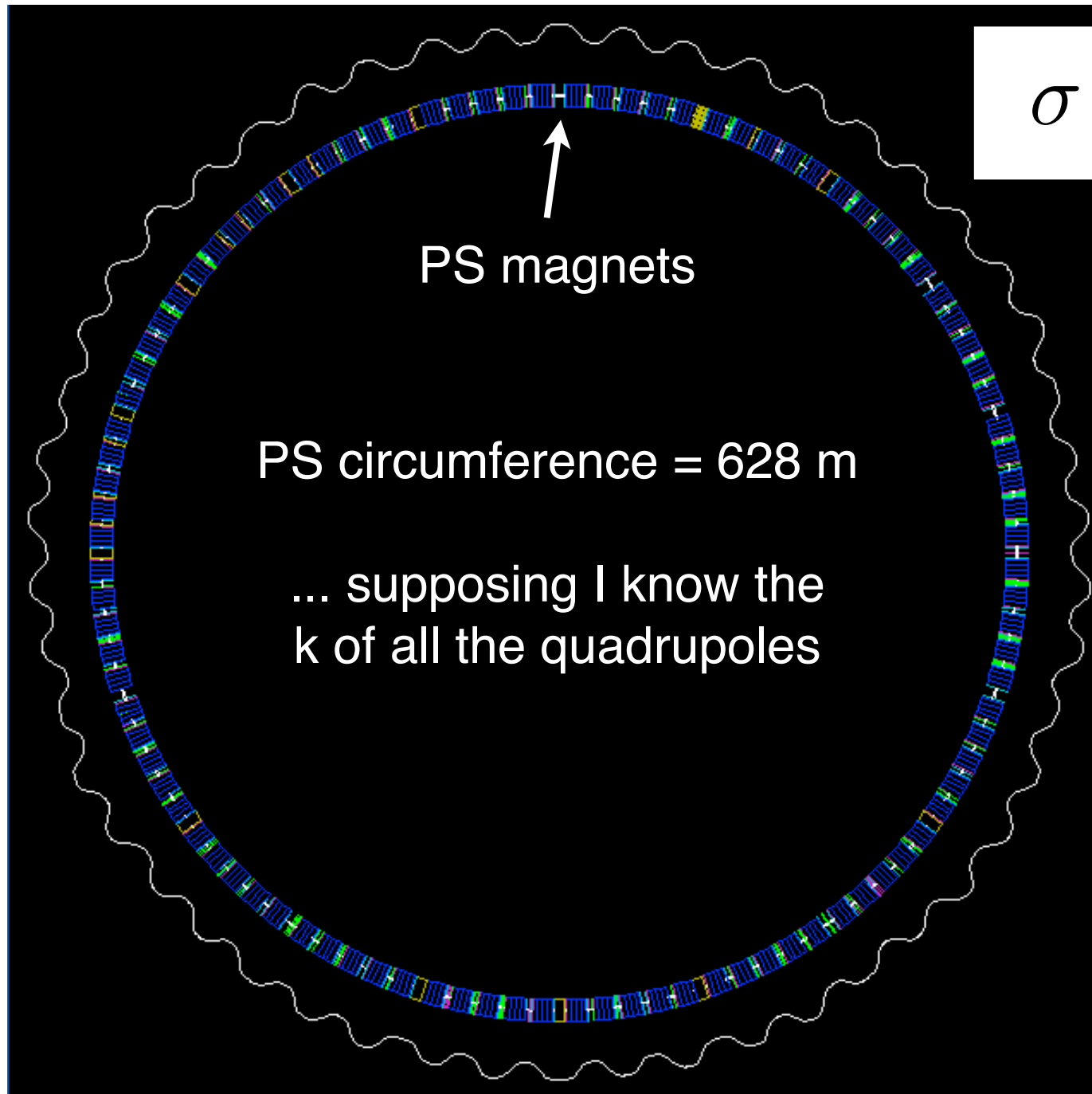
Beam physical dimension



The **envelope** is defined as the maximum amplitude for which the particle remains in the machine vacuum chamber at a given position  $s$ .

The envelope can be determined by the beta function and by the beam emittance

# Envelope of PS beam



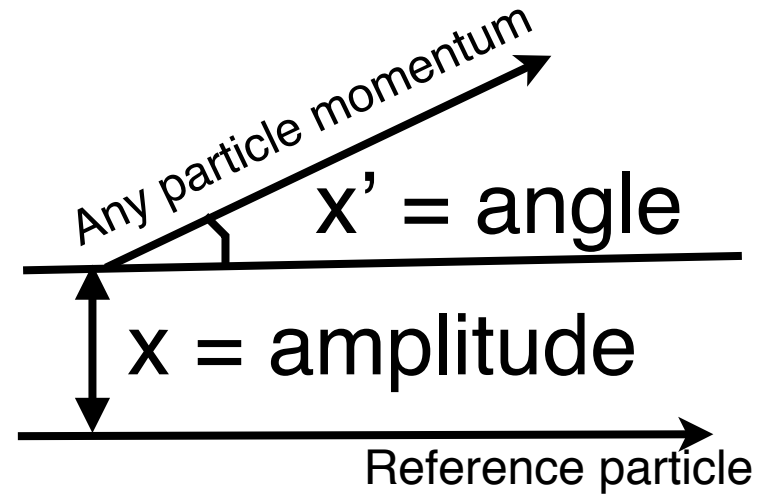
$$\sigma \propto \sqrt{\beta}$$

PS magnets

PS circumference = 628 m

... supposing I know the  
k of all the quadrupoles

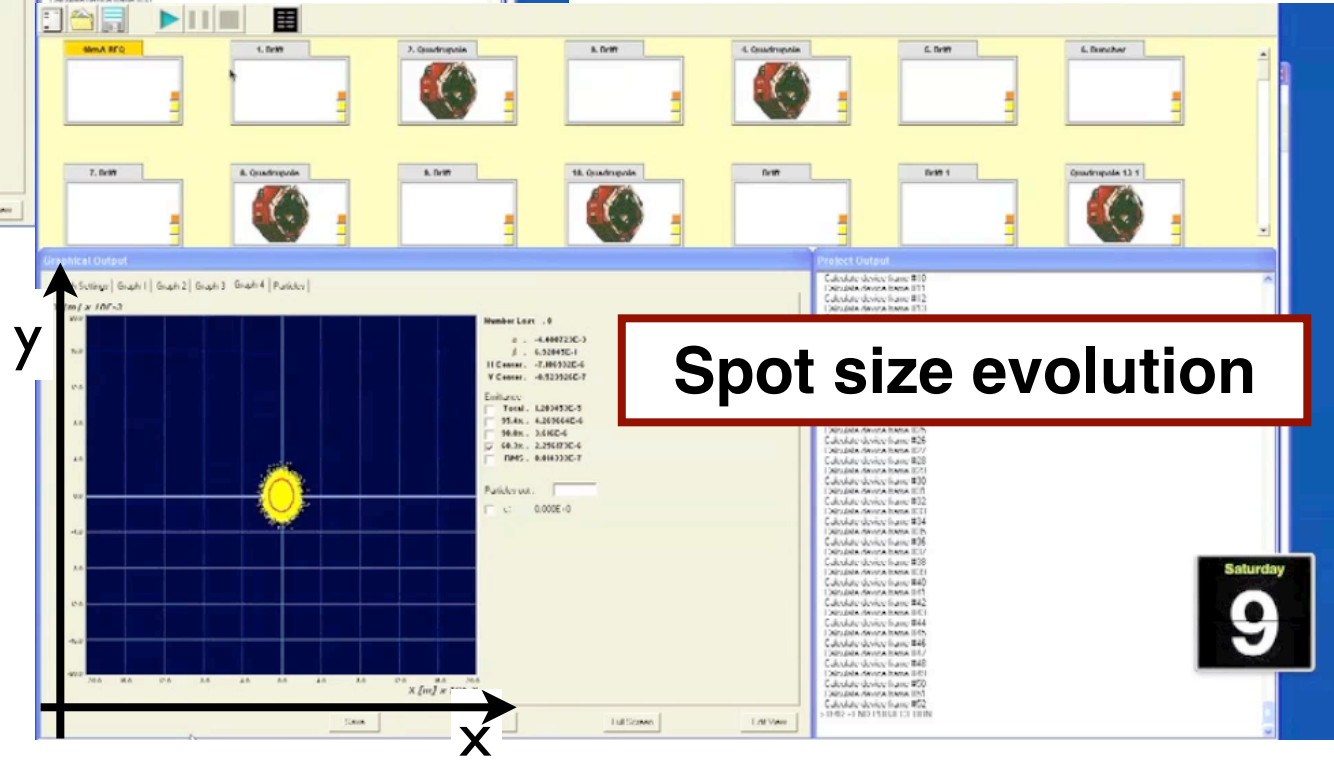
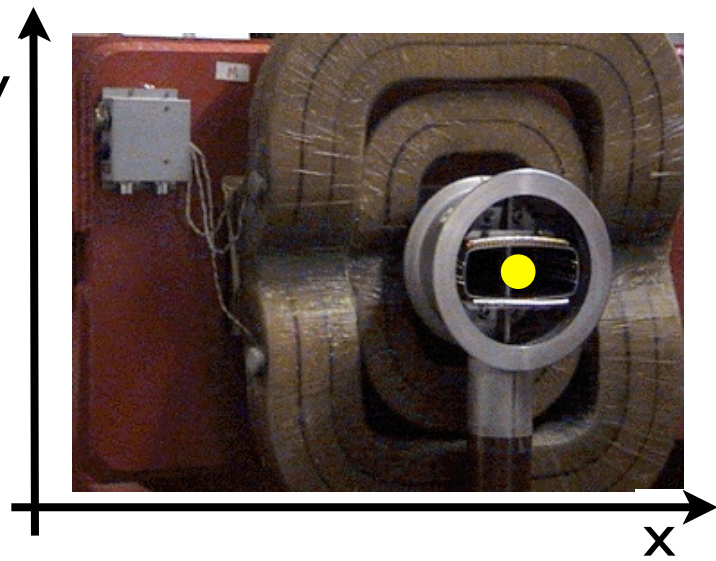
# Particle transport in a lattice



**Phase space evolution**



**Spot size evolution**

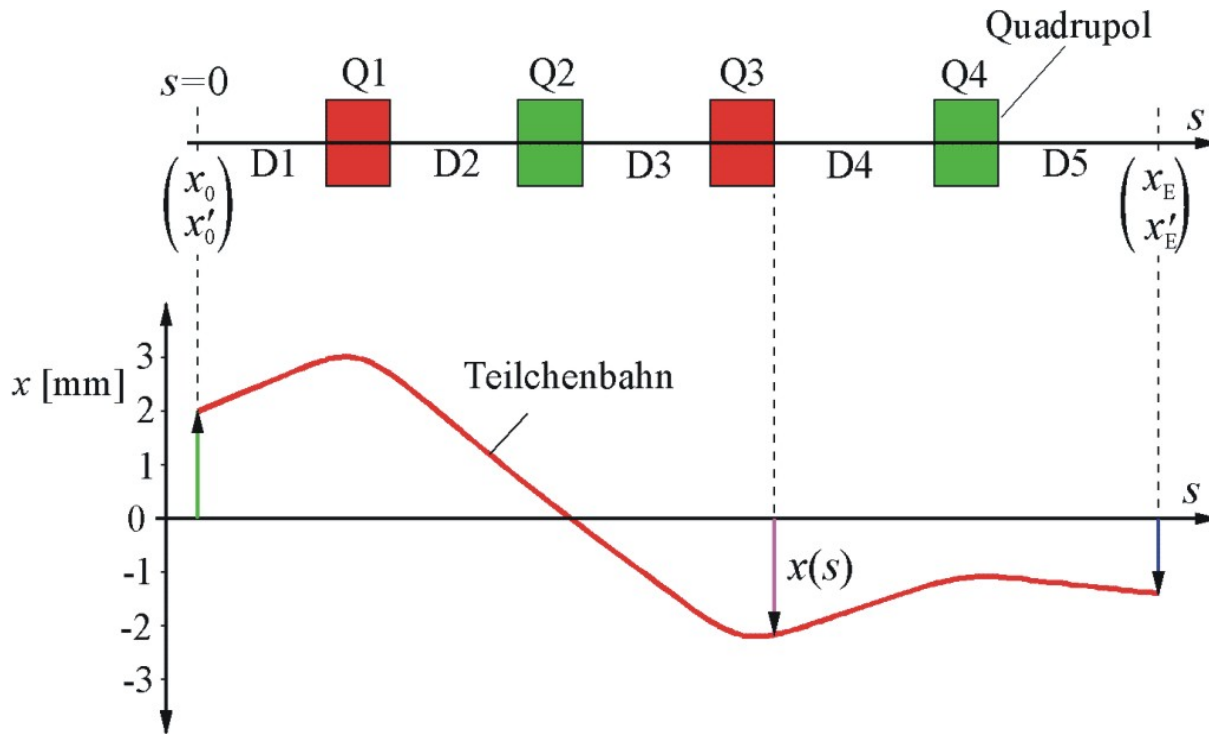


# How the transport is computed

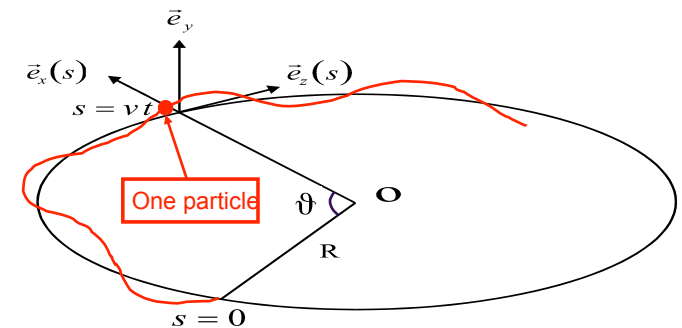
Every element of a machine, drift and quadrupoles, can be described AT FIRST ORDER by a **TRANSFER MATRIX (2x2 or 4x4)**, equivalent and deduced from the equation of motion.

$$\begin{pmatrix} x \\ x' \end{pmatrix}_f = M \begin{pmatrix} x \\ x' \end{pmatrix}_i$$

Phase space	Position	Drift	Dipole	Quadrupole
horizontal	$\vec{X}(s) = \begin{pmatrix} x \\ x' \end{pmatrix}$	$M_d = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$	$M_D = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$	$M_Q = \begin{pmatrix} 1 & 0 \\ -1/f & 1 \end{pmatrix}$



**Obs:** A dipoles is like a drift since the reference frame is turning with the central trajectory, i.e., the reference frame is following the bent trajectory

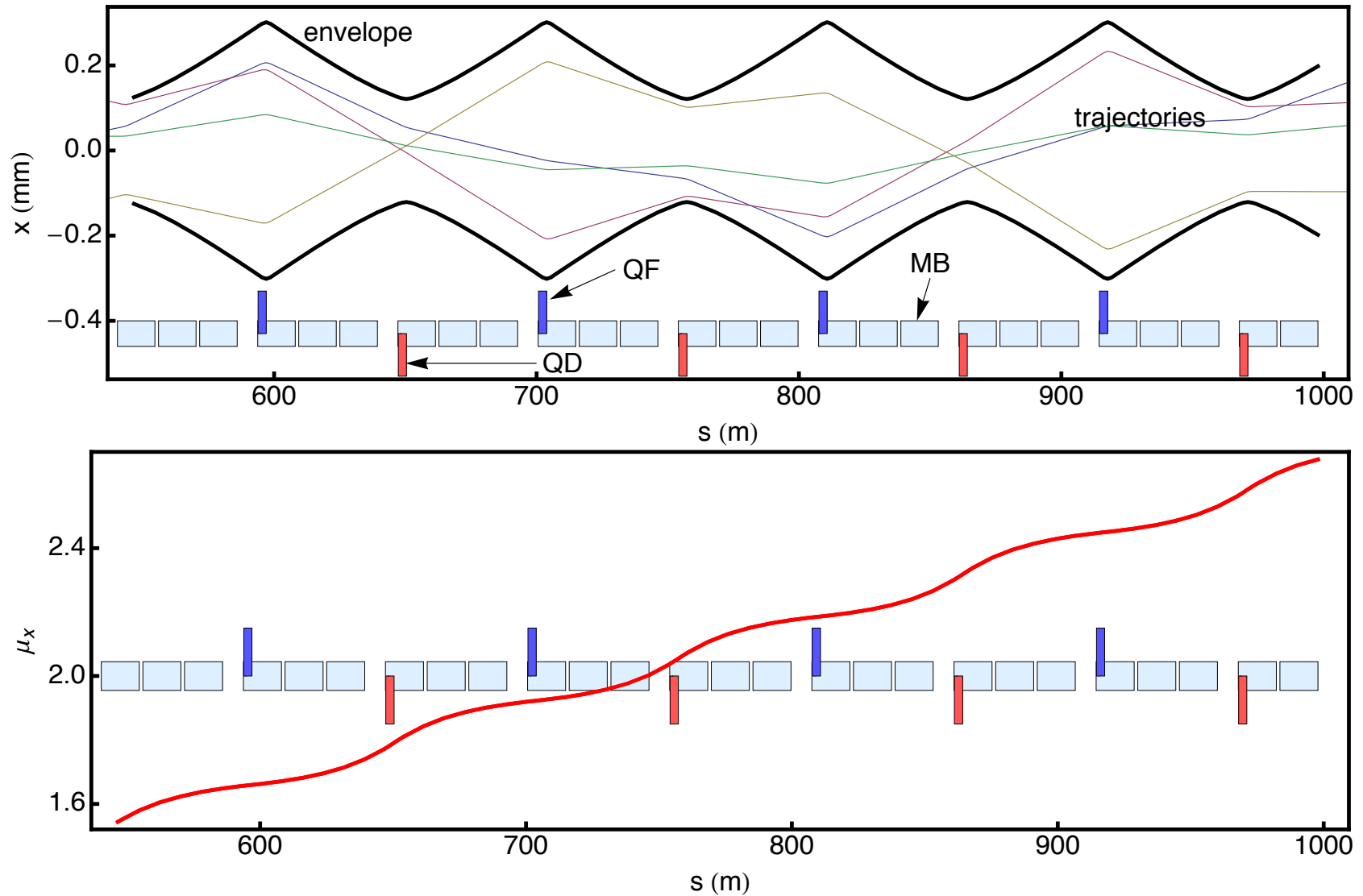


$$\vec{X}_E = M_{D5} \cdot M_{Q4} \cdot M_{D4} \cdot M_{Q3} \cdot M_{D3} \cdot M_{Q2} \cdot M_{D2} \cdot M_{Q1} \cdot M_{D1} \cdot \vec{X}_0$$

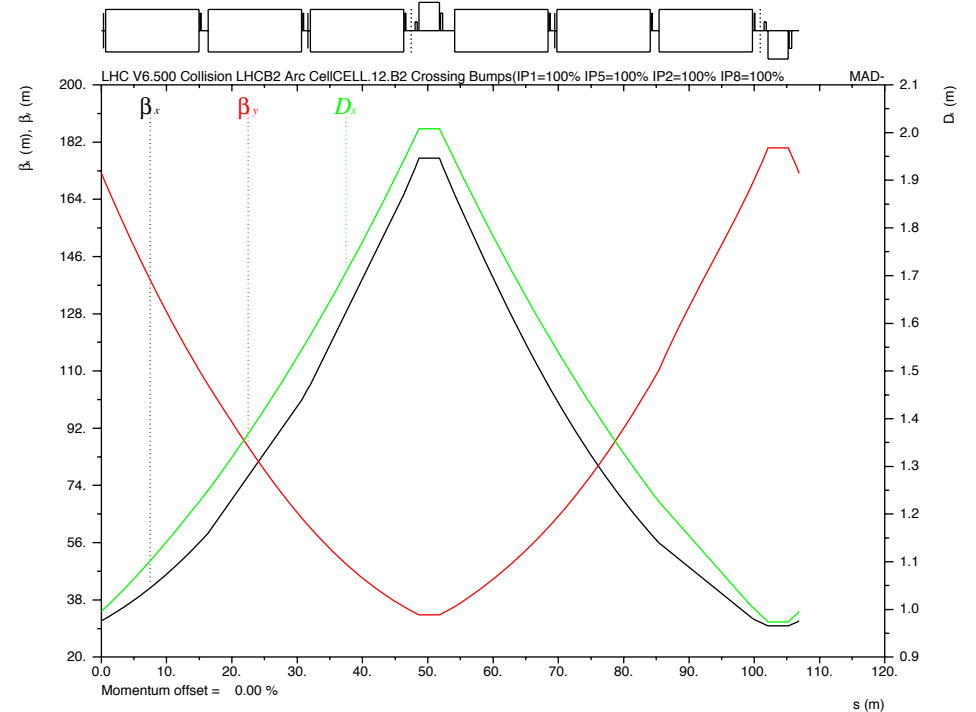
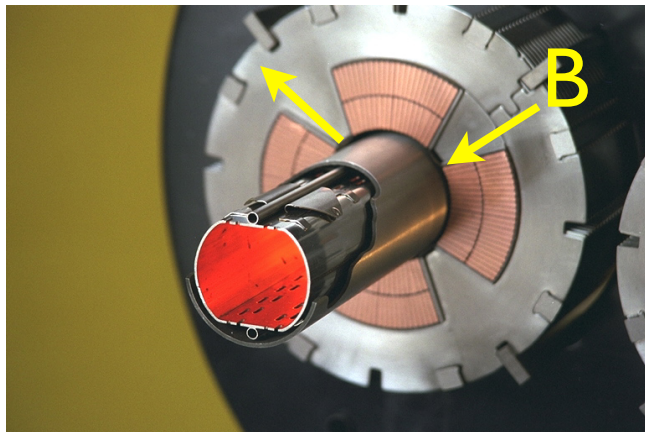
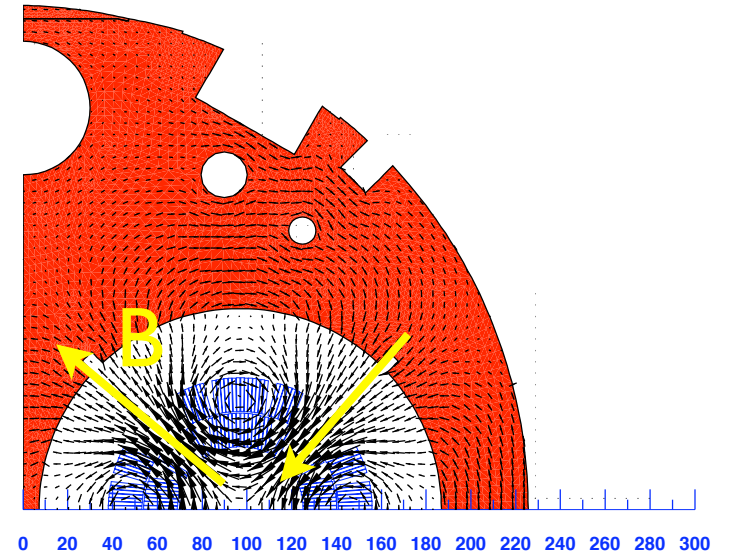
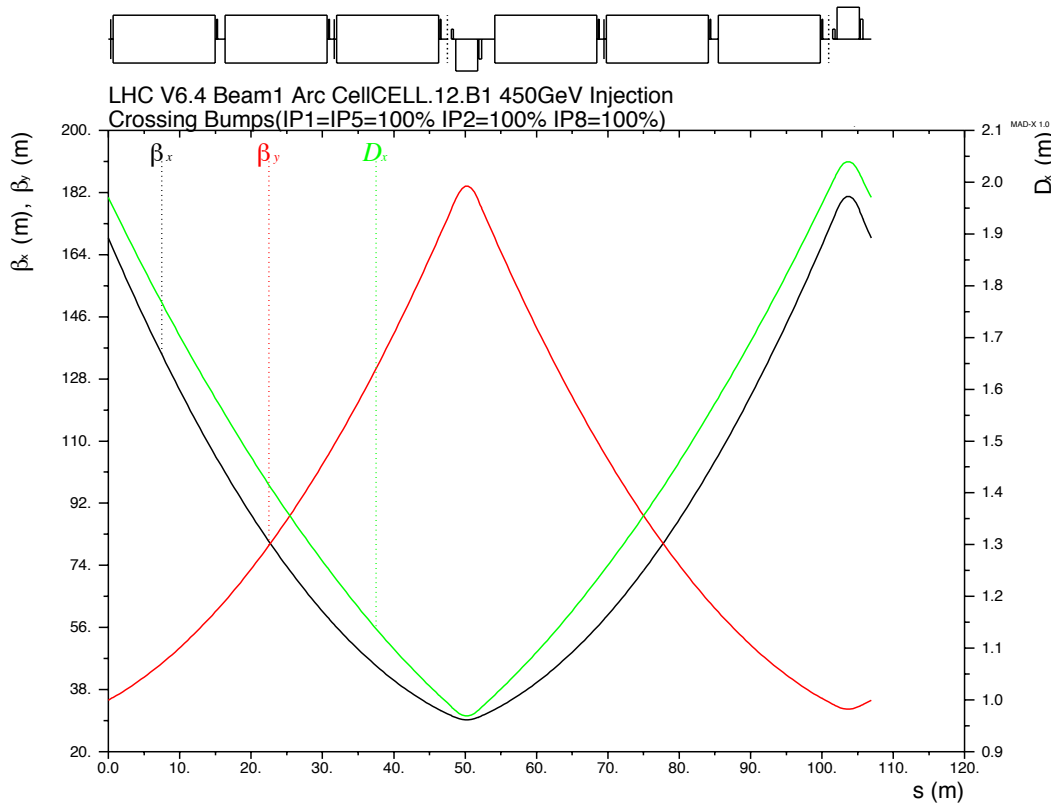


# Ray tracing and envelope in the LHC

The same apply for a machine like the LHC



# Arc cell at injection for beam 1 and beam 2



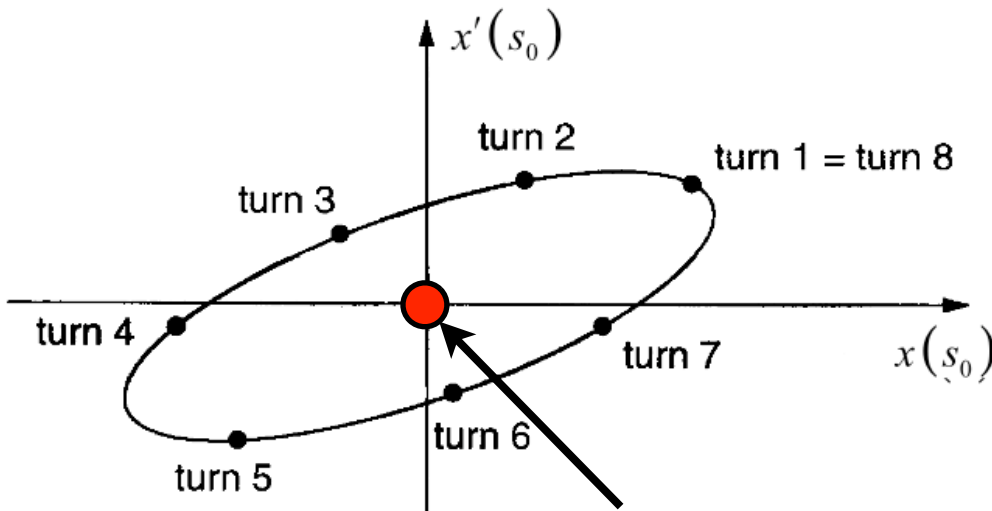
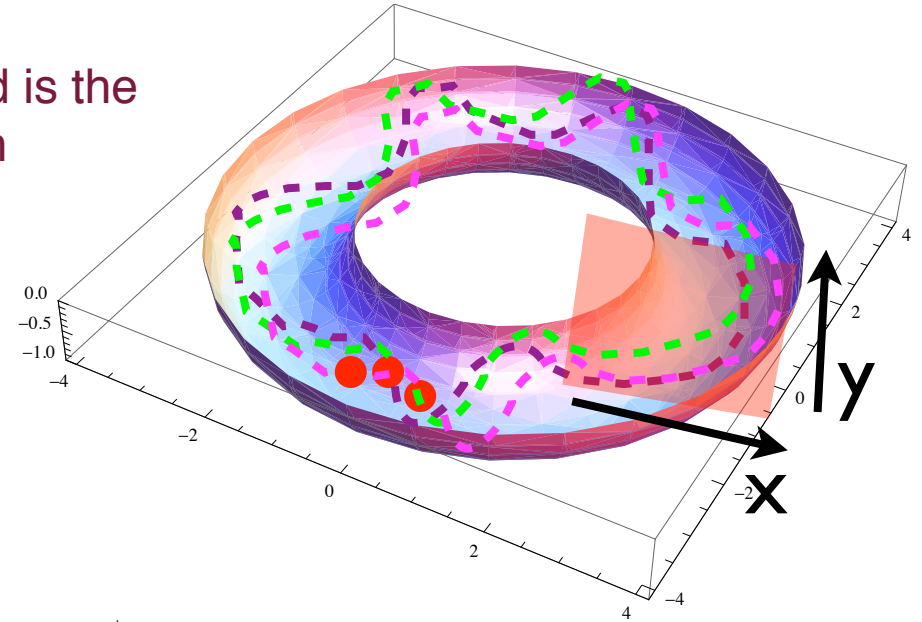
# Tune

## Tune:

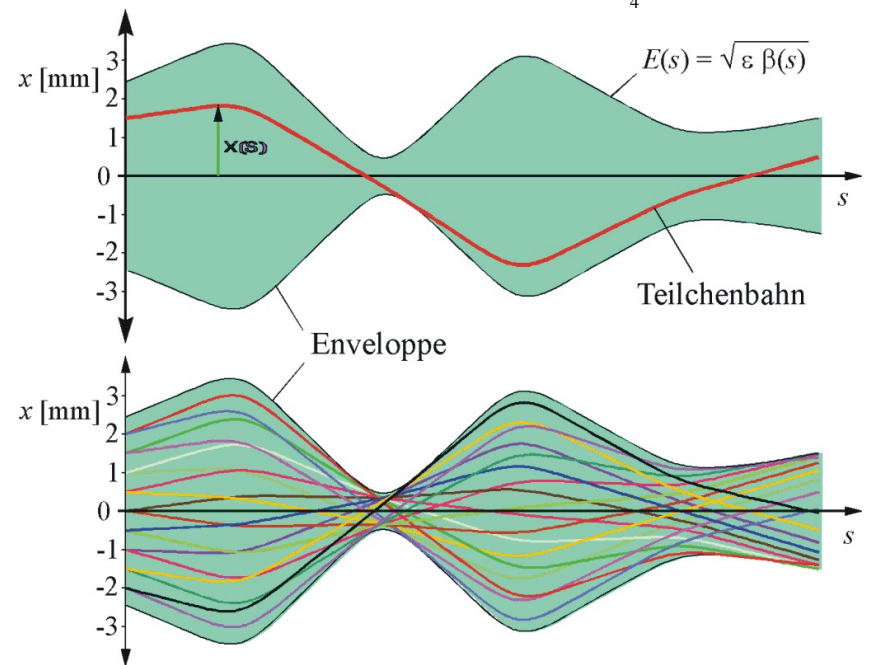
number of oscillations (called betatronic) in the  $xx'$  plane a particle does in one machine turn.

The tune depends on the quadrupoles settings and is the integral of the phase advance on one machine turn

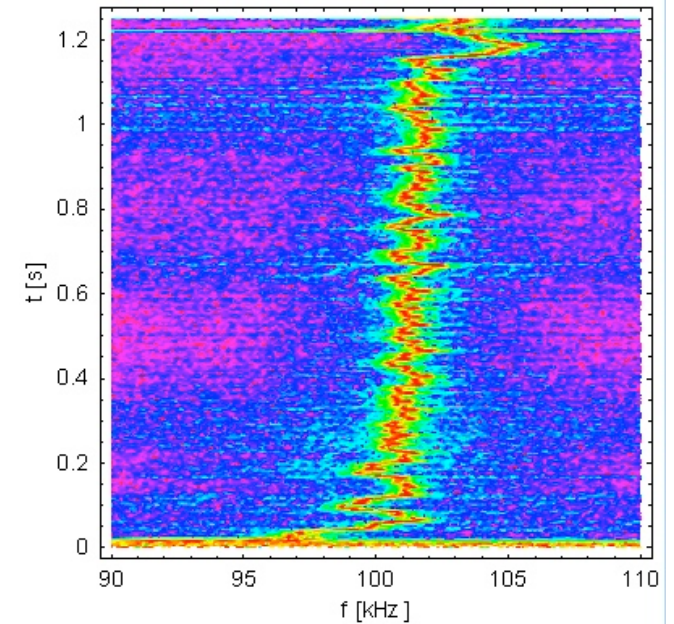
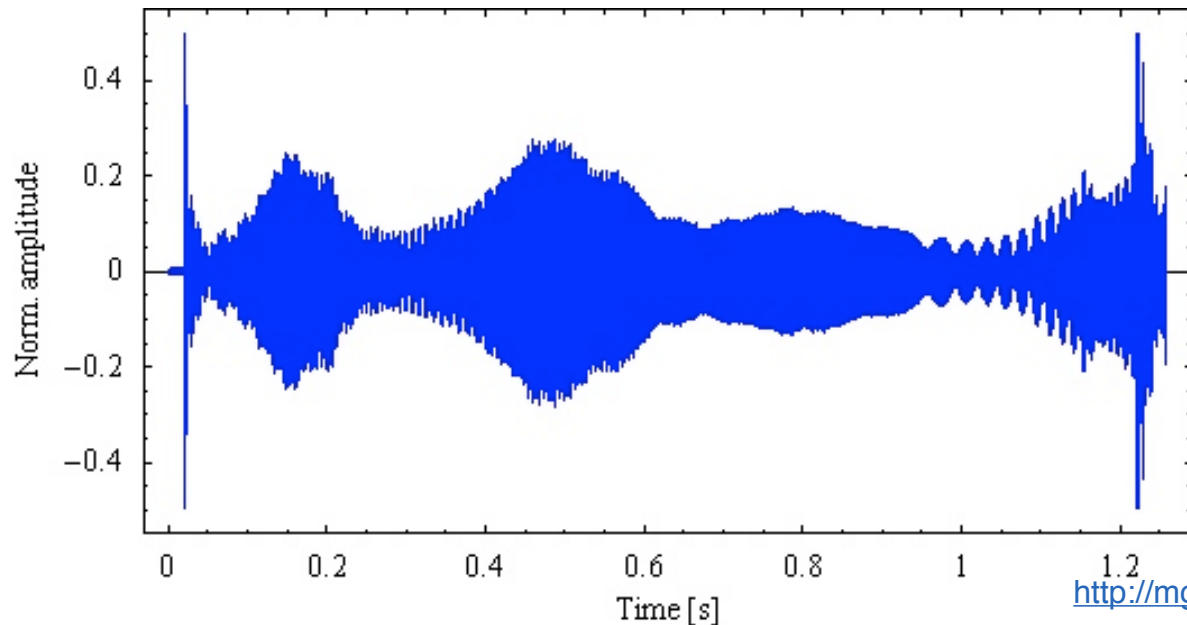
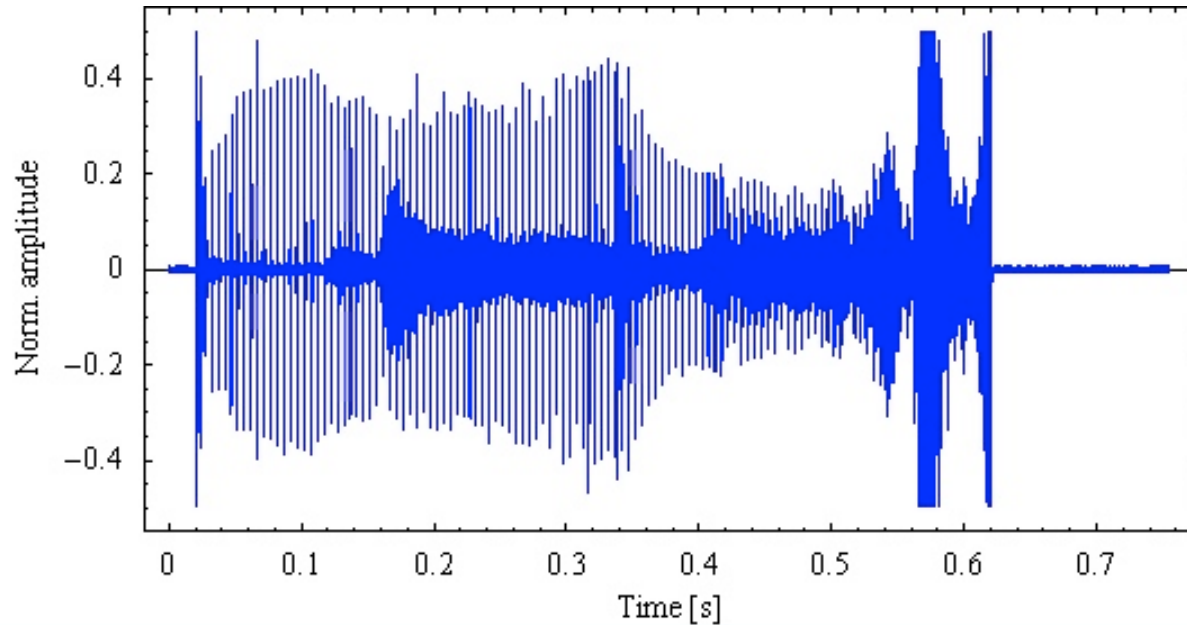
$$Q_x = \frac{1}{2\pi} \oint \frac{ds}{\beta_x(s)}$$



Reference particle



# Tune: number of betatron oscillation in the transverse plane





# Machine imperfections ....

## Real LHC orbit - correction of dipolar error

LHC average momentum at injection =  $450.5 \pm 0.2$  GeV

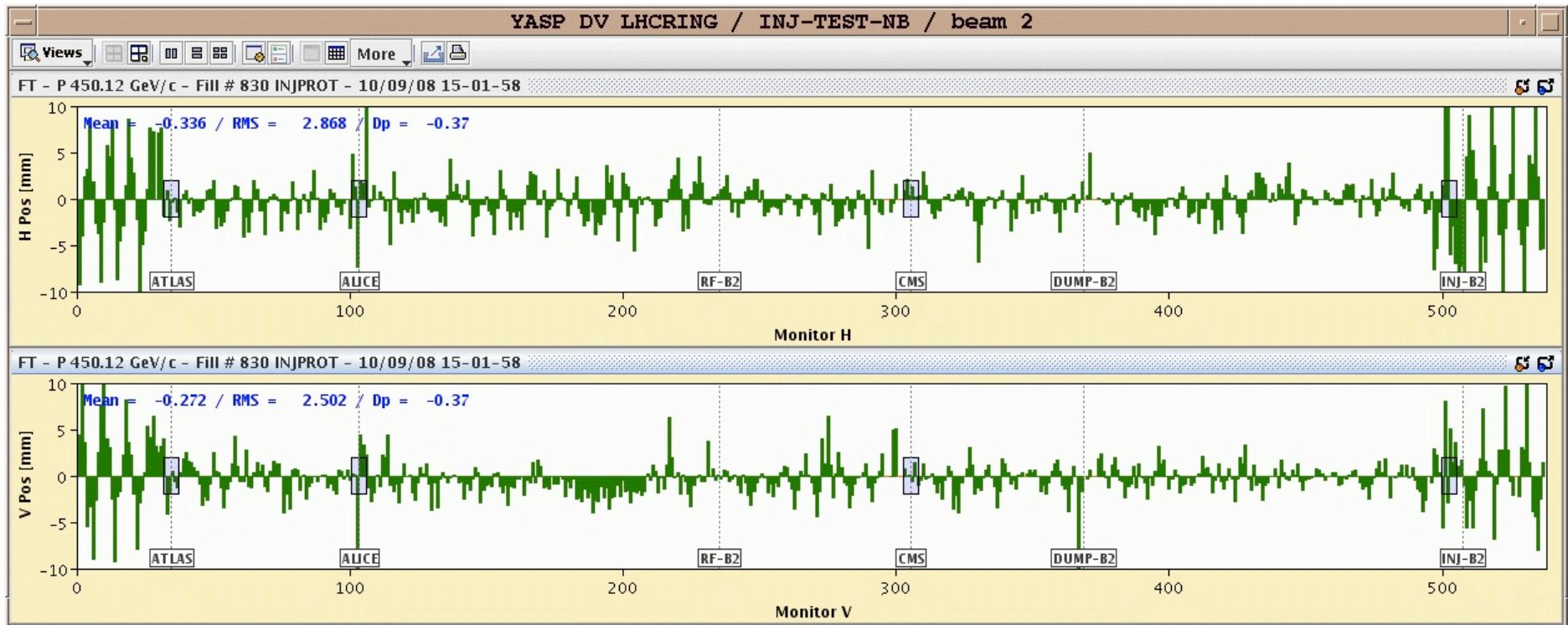
$\Rightarrow$  Dipole error  $\Delta b_l$  between rings  $\approx 1.5 \times 10^{-4}$

$\Rightarrow \Delta b_l$  among 8 sectors  $\approx 3 \times 10^{-4}$

Courtesy of J. Wenninger

# Machine imperfections ...

## Real LHC orbit - correction of dipolar error



LHC average momentum at injection =  $450.5 \pm 0.2$  GeV

$\Rightarrow$  Dipole error  $\Delta b_l$  between rings  $\approx 1.5 \times 10^{-4}$

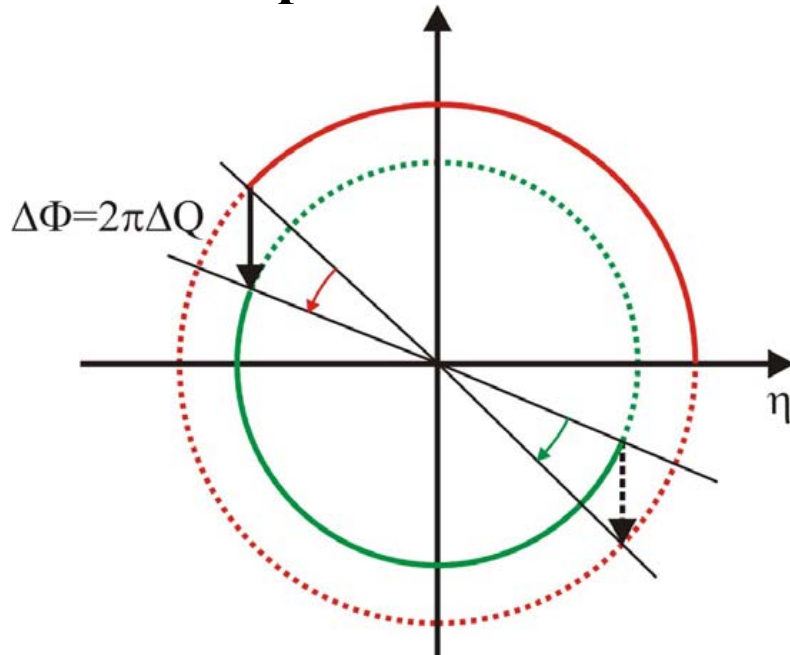
$\Rightarrow \Delta b_l$  among 8 sectors  $\approx 3 \times 10^{-4}$

Courtesy of J. Wenninger

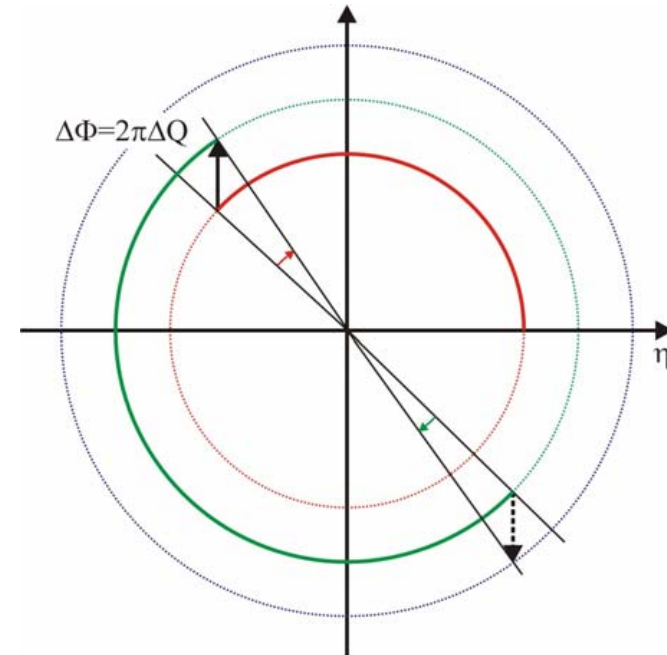
# Machine imperfections

Magnetic errors can cause beam losses/instability if appears as certain multipoles of the tune

## Dipole Errors:



## Gradient Errors:



### resonance type

integer resonance:  $Q = n$

half-integer resonance  $2 \cdot Q = n$

third-integer resonance  $3 \cdot Q = n$

### driving multipole

dipole errors

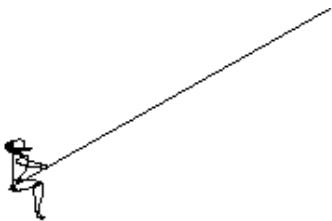
quadrupole errors

sextupole errors

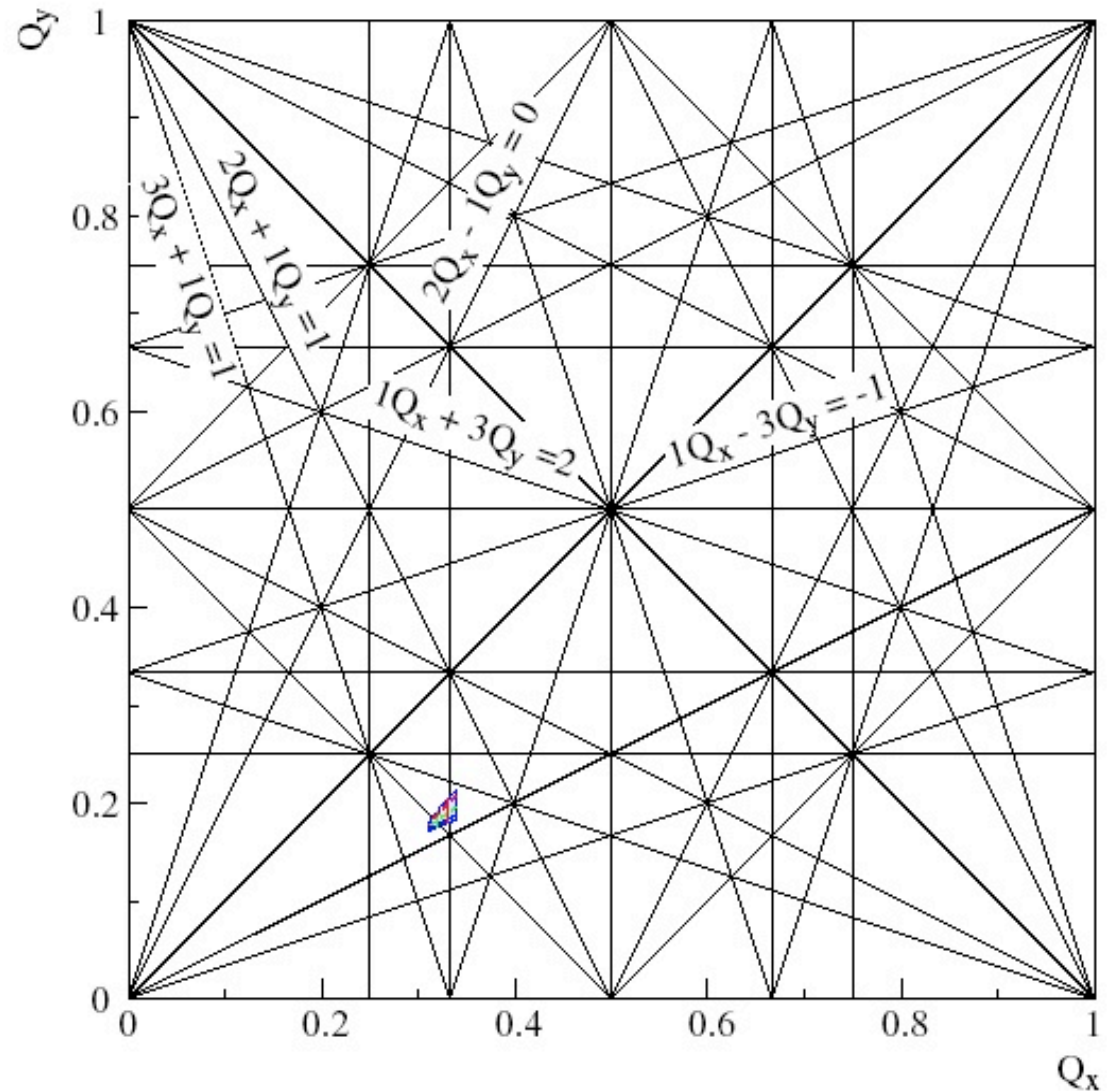
# Tune and resonances

Like on a “**swing**”, to keep the oscillations bounded in amplitude, one has to avoid to excite the beam in a resonant way, in particular due to magnets imperfections.

The tune has to be far away from some values, like exciting the beam with the same force at each turn (dipolar errors)



To be avoided  $M Q_x + N Q_y = P$

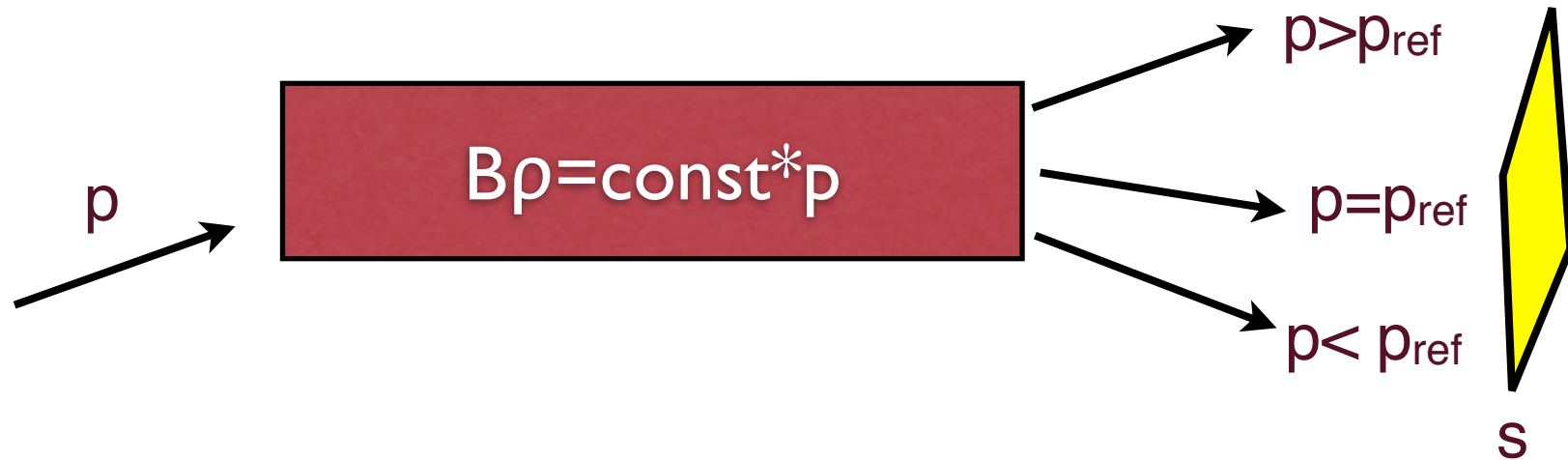




# Dispersion

So far, beam considered monoenergetic, in fact, it is not ...

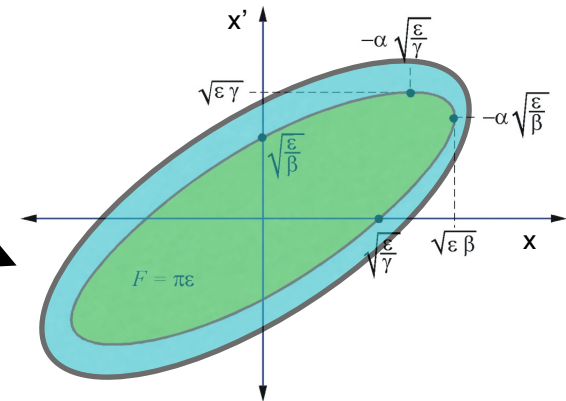
Bending strength of the dipoles depends on the energy



Dispersion **D** at position **s** parametrize the different amplitude **x** due to the difference in energy

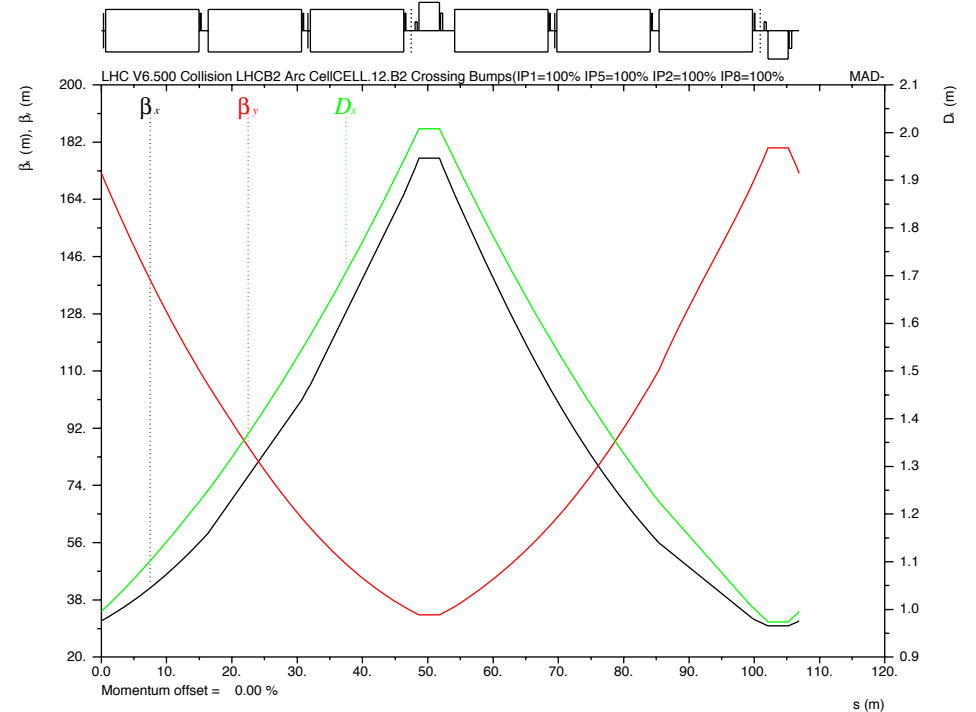
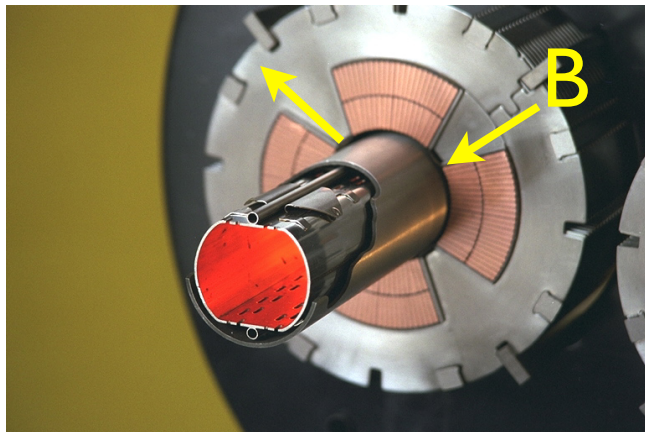
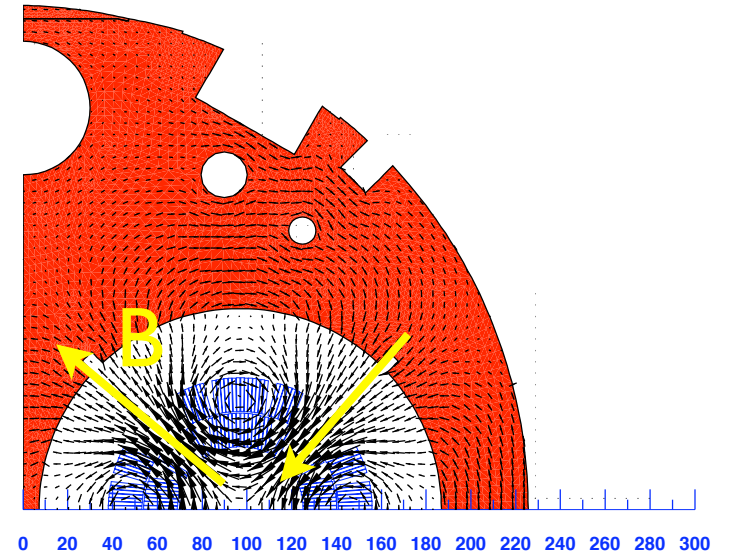
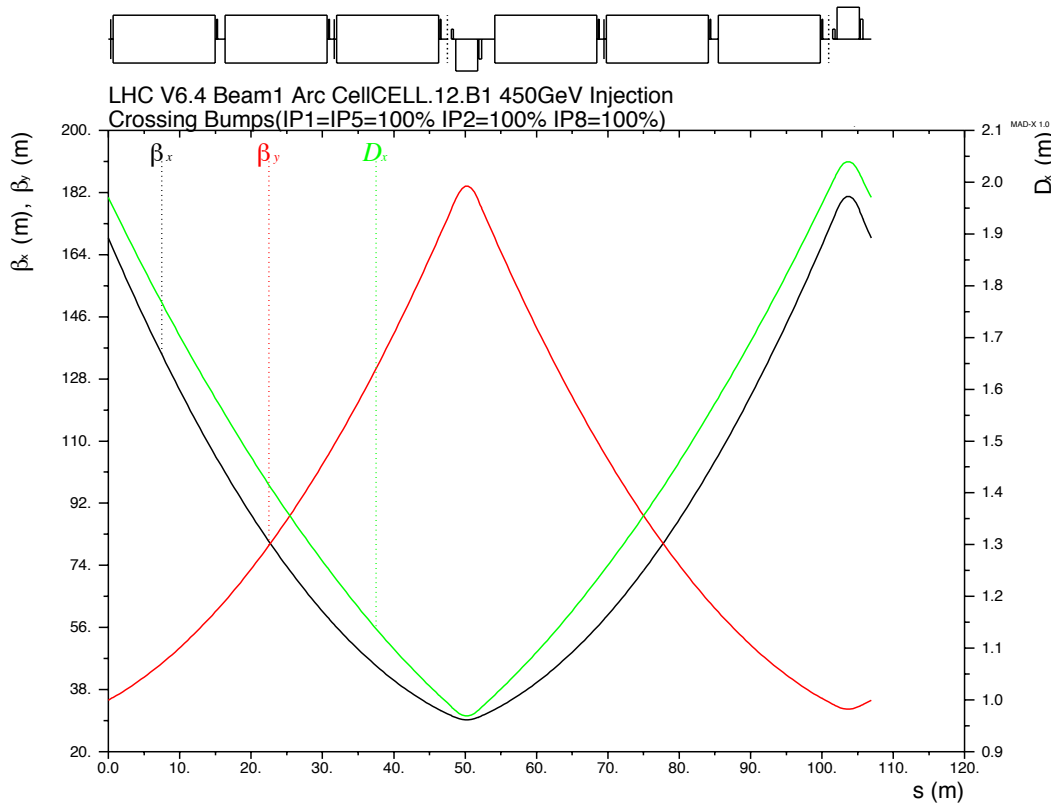
$$\frac{\Delta x}{x} = D \delta \quad \delta = \frac{\Delta p}{p}$$

The dispersion changes also the beam ellipse



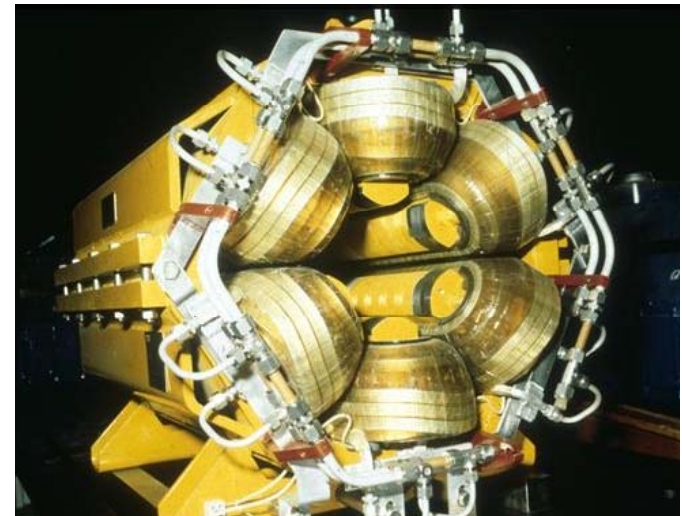
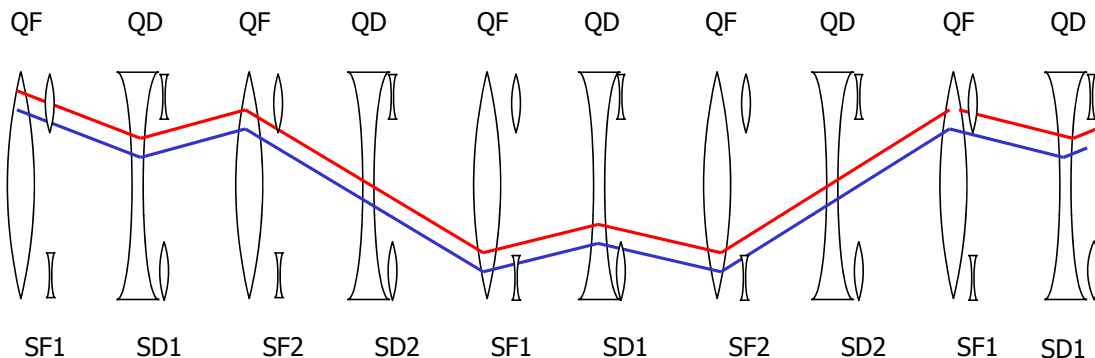
$$\sigma_x(s) = \sqrt{\epsilon_x \beta_x(s) + \langle \delta^2 \rangle D_x^2(s)}.$$

# Arc cell at injection for beam 1 and beam 2



# Chromaticity

- If the energy of a particle is different from the energy of the reference particle, the quadrupoles will focus less or more, so the tune will change according to the energy, as if the accelerator suffer from **ASTIGMATISM** (or **MIOPHY**).
- This is defined as **CHROMATICITY**  $\xi$  
$$\frac{\Delta Q}{Q} = \xi \frac{\Delta p}{p}$$
- Since one want to avoid crossing resonances, the **CHROMATICITY** has to be kept small and corrected.
- This can be done by using **SEXTUPOLES**, thanks to the lattice design, can focus differently different energies



# Actually ... that's chromaticity...



$$k = \frac{g}{p/e}$$

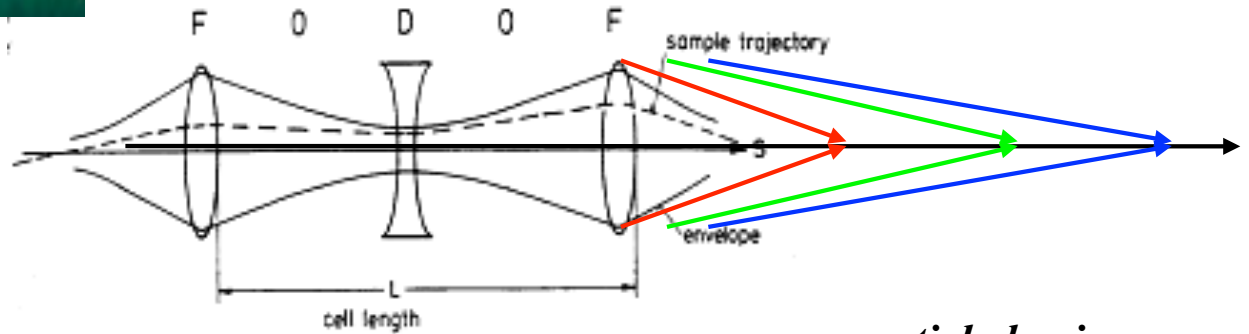


Figure 29: FODO cell

*particle having ...  
to high energy  
to low energy  
ideal energy*

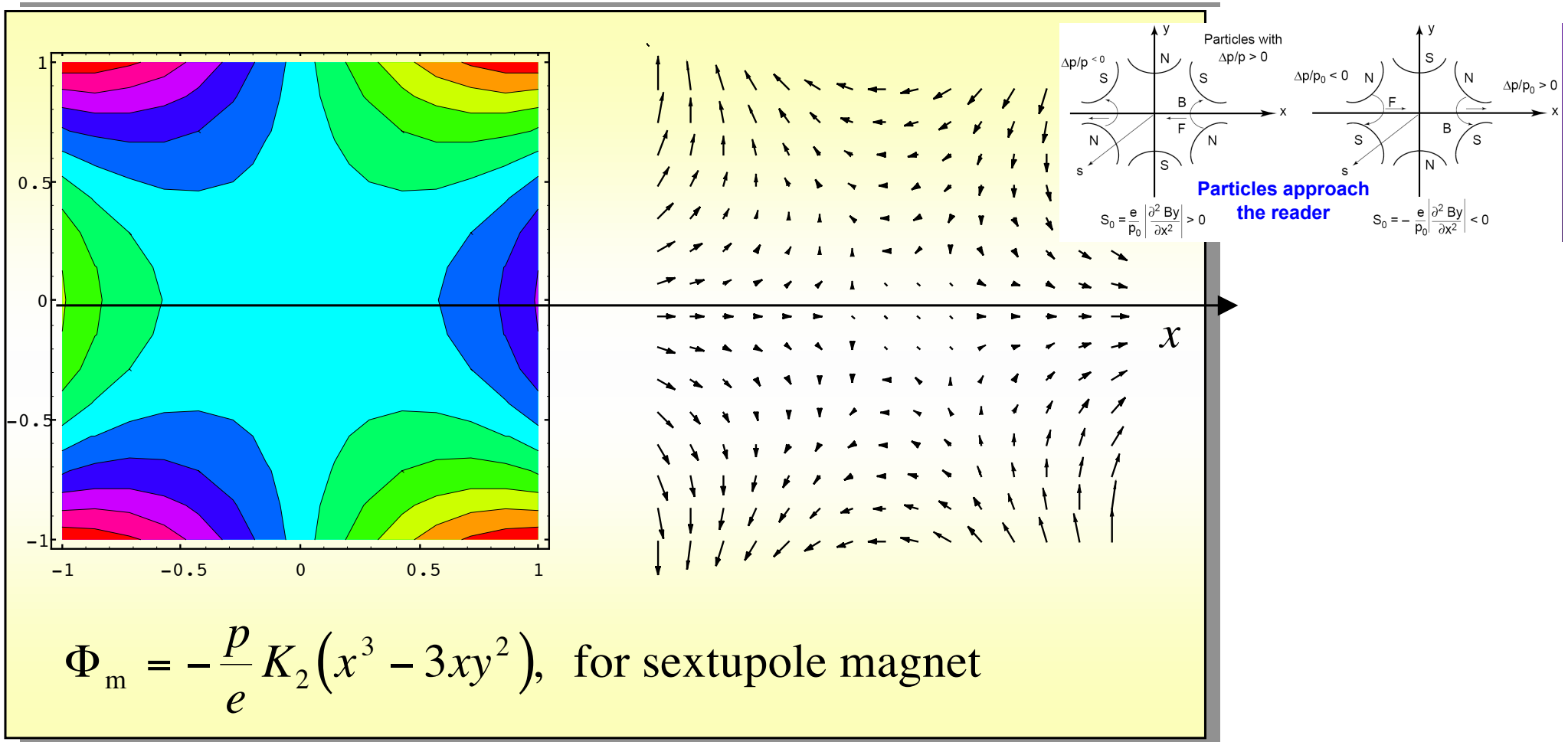


# Sextupoles

$$B = -\nabla \Phi_m$$

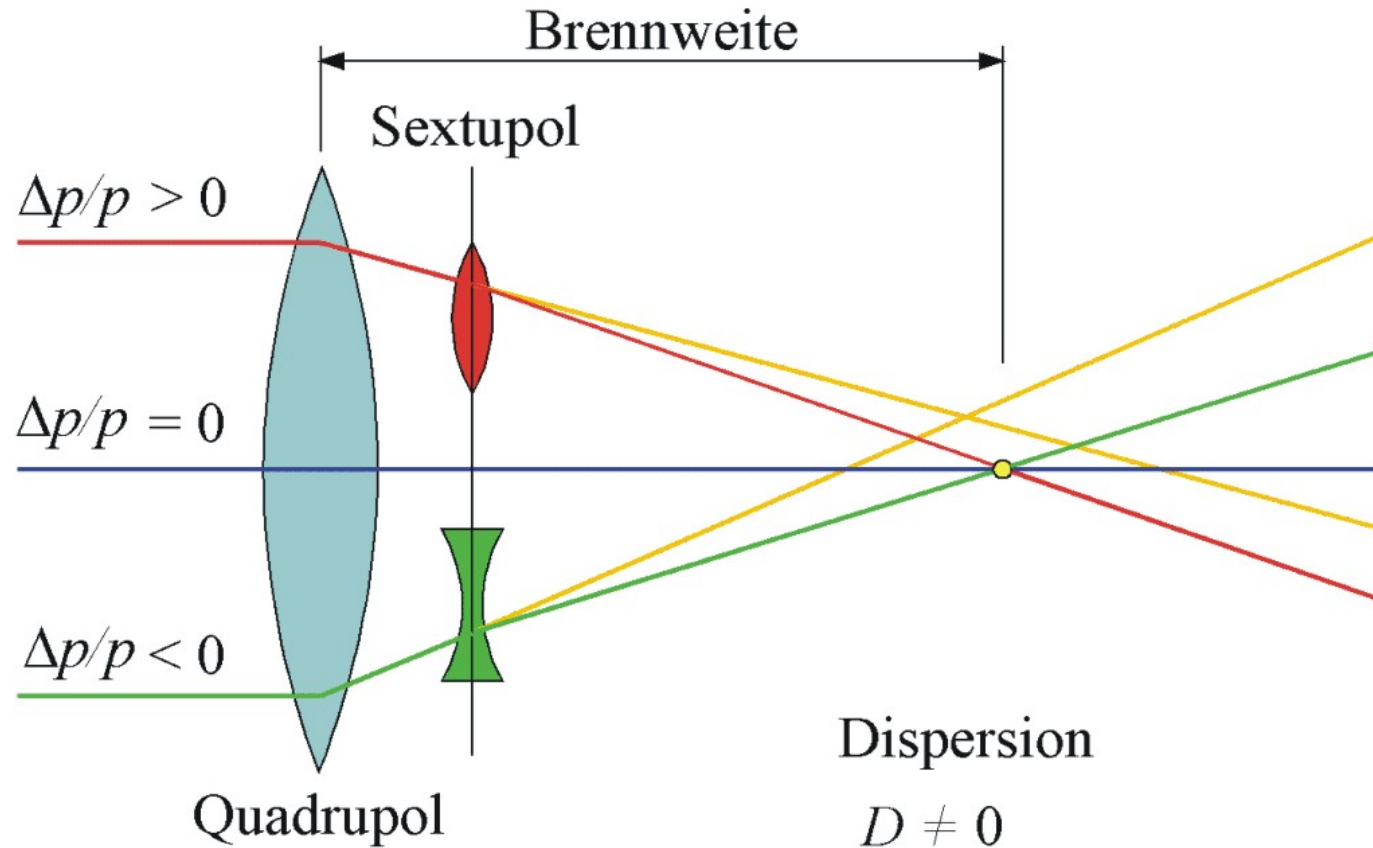
$$\Phi_m = \text{Vector potential}$$

The force changes with the distance, and couples vertical and horizontal plane



# Chromaticity correction scheme

In a region where Dispersion is not zero, the sextupoles uses the correlation between energy and displacement to correct the different focal point for the different energies

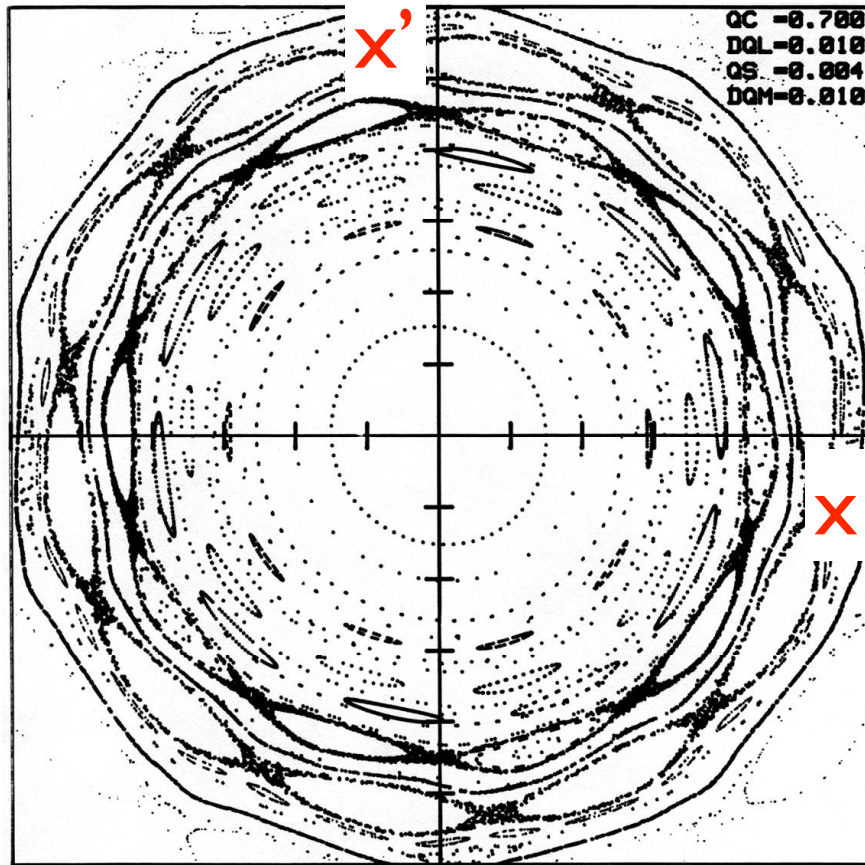


**Why chromaticity is bad?** In fact not all the time, but due to  $\frac{\Delta Q}{Q} = \xi \frac{\Delta p}{p}$  particles with large energy errors could intercept some resonances

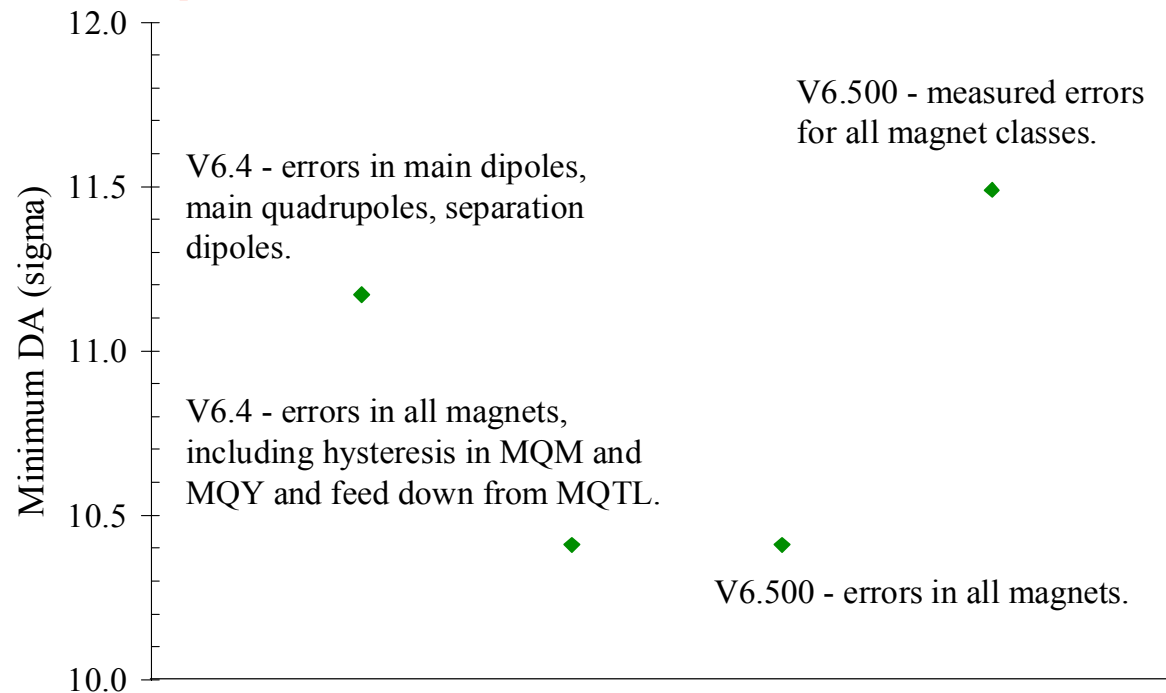
# Dynamical Aperture

Dynamic aperture (DA) is largest region of phase space where stable motion occurs.  
Computed by particle tracking simulating different sets of machine imperfection due to:

- (A) machine alignment/mechanical aperture
- (B) **multipoles generated by the magnets (see sextupoles...)**



## Dynamic aperture for different LHC "realisation"

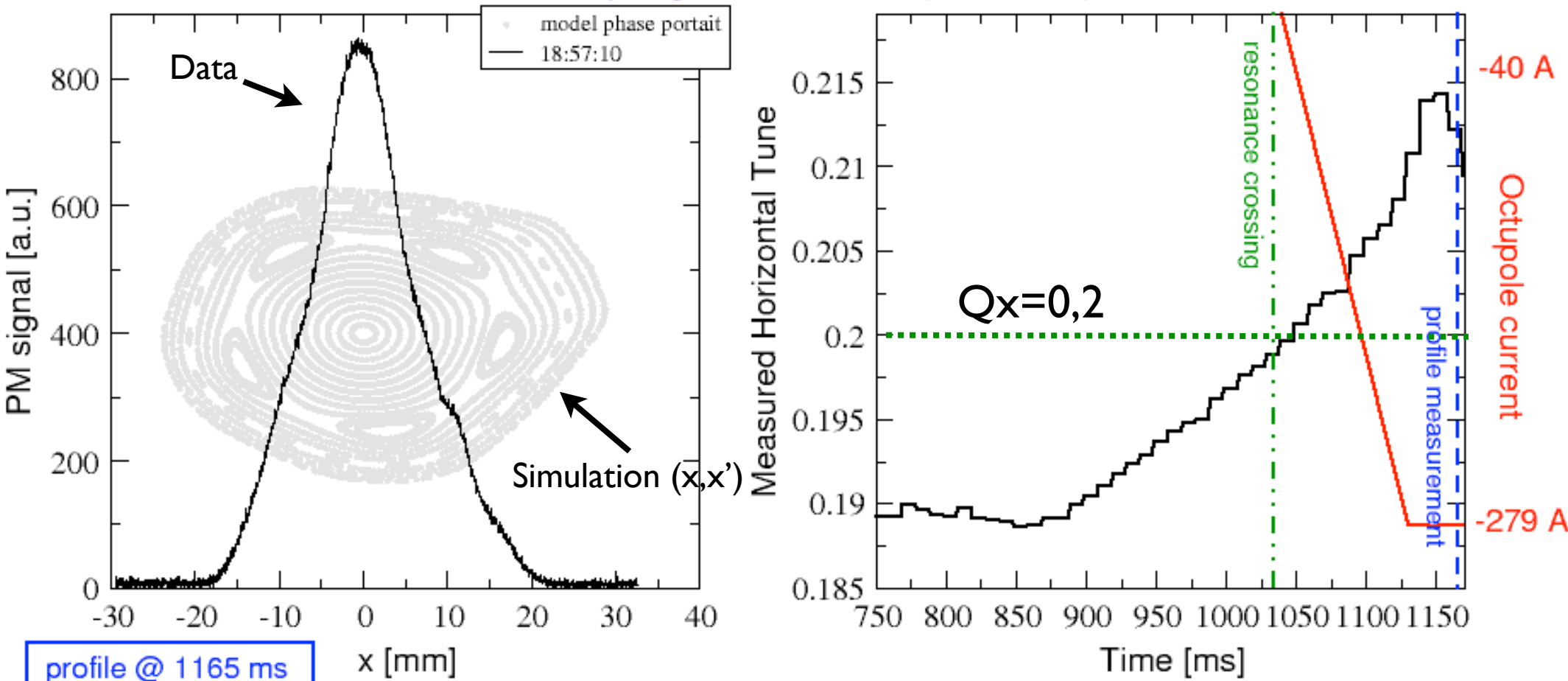


In the transverse phase space, one cannot really see an ellipse which describes the normal betatronic phase space in a FODO lattice. Islands and chaotic (both in mathematical sense as in how it looks like) motion appear due to fields that vary with amplitudes ( $x$ )  $\Rightarrow$  Multipoles

In fact, if can control properly the non-linearities you can even split up the beam

## PS Multi-Turn Extraction experiment, 27 July 2007

horizontal beam splitting in five stable islands (1/5 resonance)



Experiment done in the PS at 14 GeV/c (not @ the LHC) to show the splitting in 5 islands of the beam by using sextupoles and octupoles and crossing the 1/5 resonance, i.e.  $Q_x = 0,2$

*If this not done not under controlled conditions, losses appear*

*If one can produce multibeams, one can for sure produce tails in a nice gaussian beam profile...*



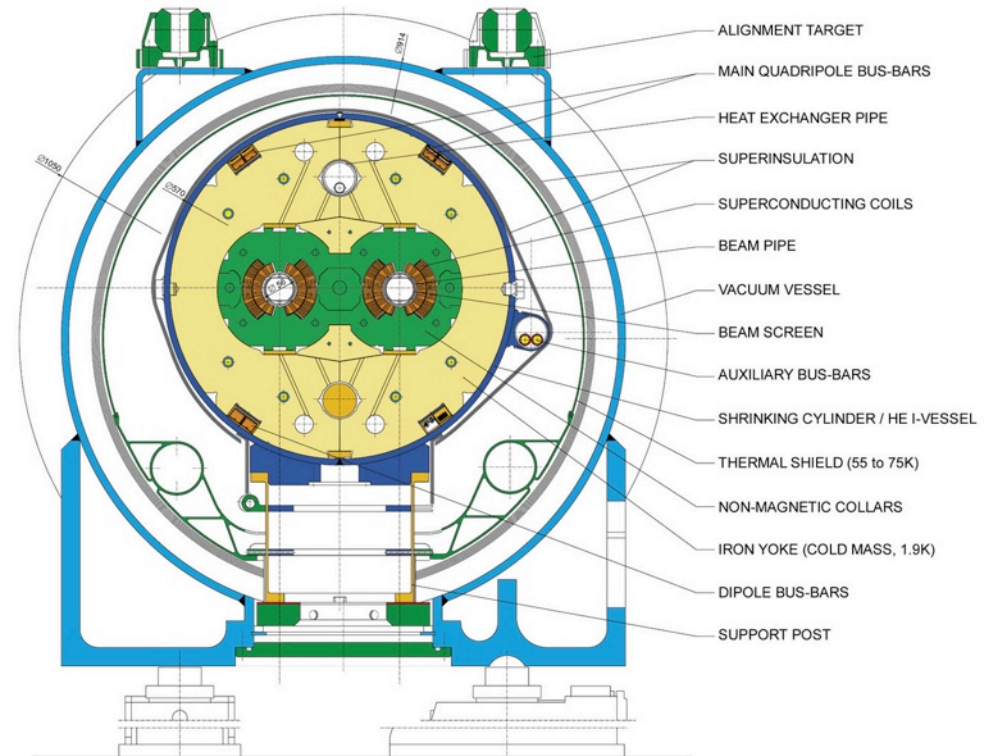
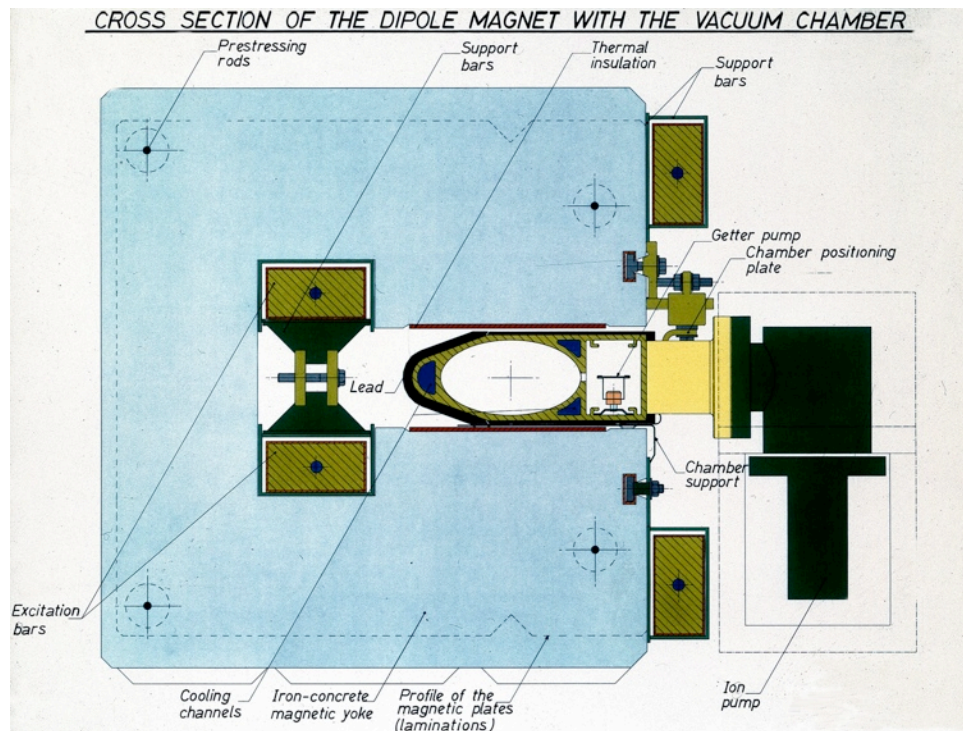
# Magnets, a change in technology when energy increases

**Bending Field**  $\rightarrow$   **$p(\text{TeV}) = 0.3 B(\text{T}) R(\text{Km})$**   
(earth magnetic field is between 24 mT and 66 mT)

Tunnel  $R \approx 4.3 \text{ Km}$  LHC  $7 \text{ TeV} \rightarrow B \approx 8.3 \text{ T} \rightarrow$  **Superconducting coils**  
LEP  $0.1 \text{ TeV} \rightarrow B \approx 0.1 \text{ T} \rightarrow$  **Room temperature coils**

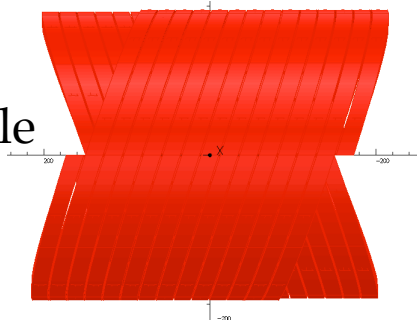
## LHC DIPOLE : STANDARD CROSS-SECTION

CERN AC/DI/MM - HE107 - 30 04 1999

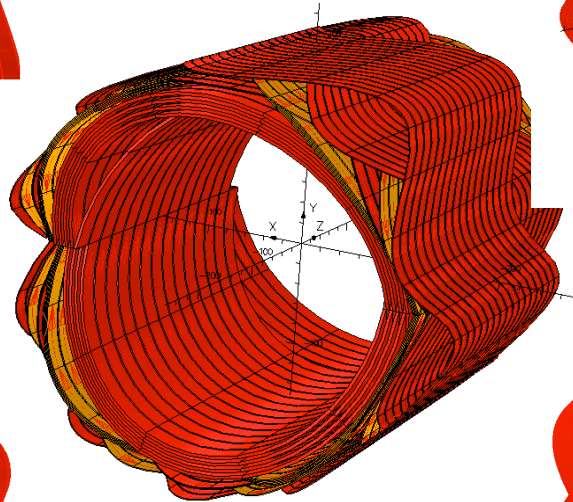


# Remember for combined function magnets...

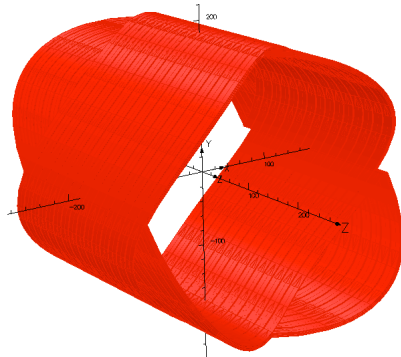
dipole



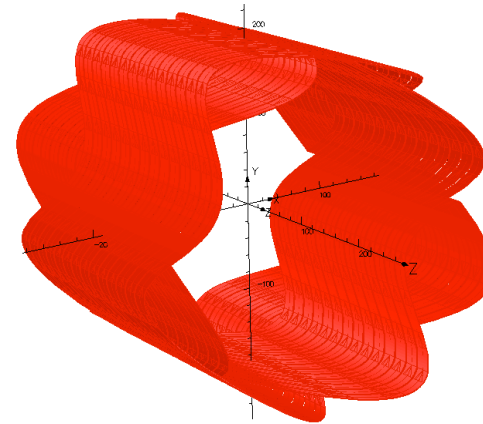
combination



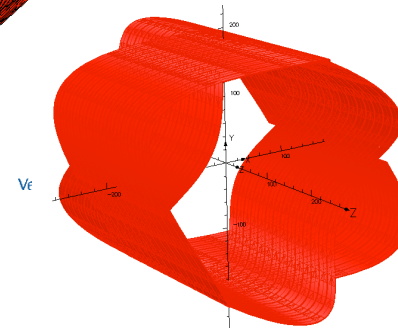
quadrupole



octupole



sextupole



Why this shape ?

# Very, very short introduction to Superconductivity for accelerators

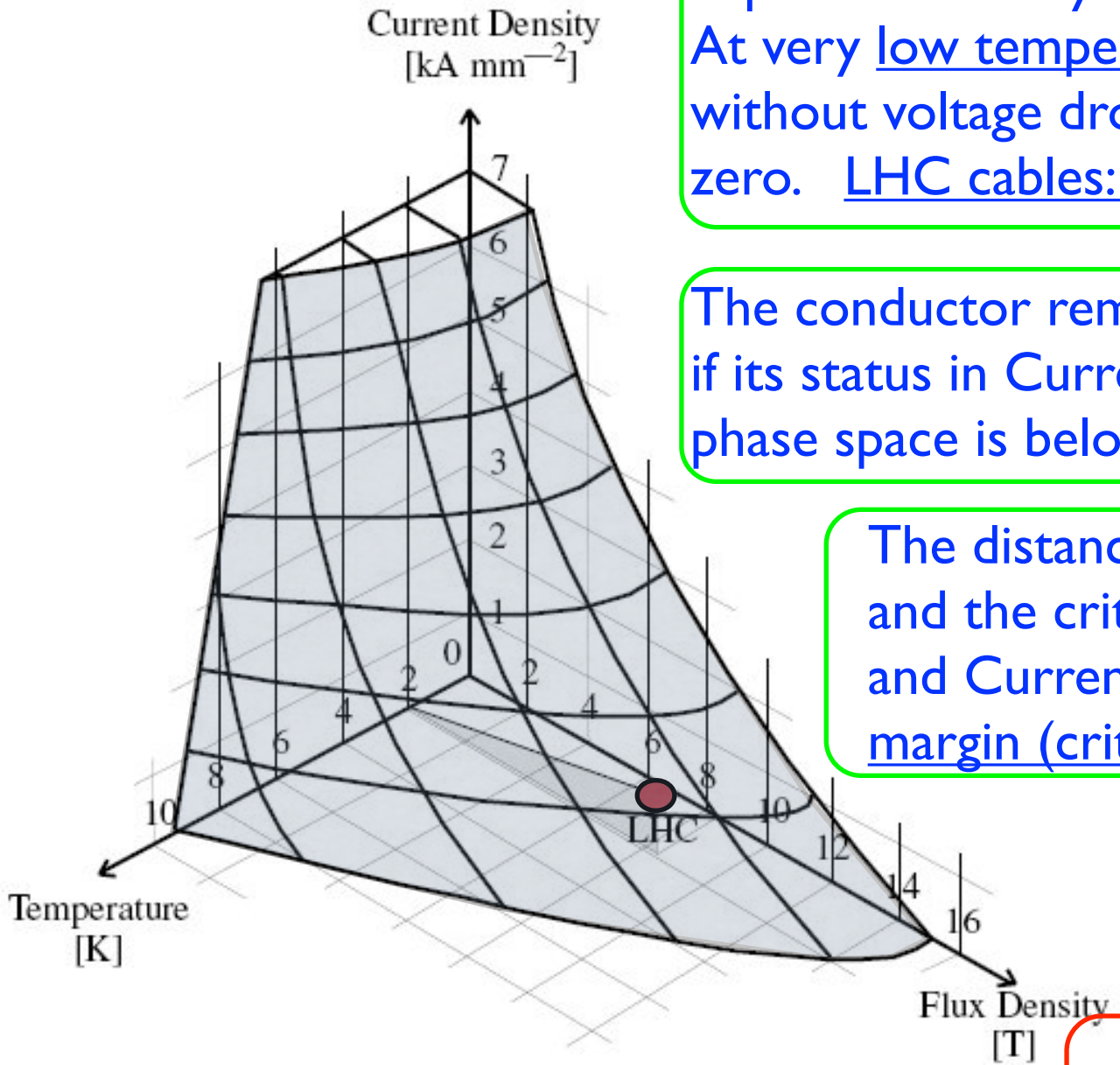
Superconductivity is a property of some materials. At very low temperature they can carry currents without voltage drop, i.e. their resistivity goes to zero. LHC cables: Nb-Ti working at 1.9 K

The conductor remains Superconductor if its status in Current Density, Temperature, B field phase space is below the Critical Surface

The distance between the working point and the critical surface for a fixed B field and Current Density is the temperature margin (critical temperature)

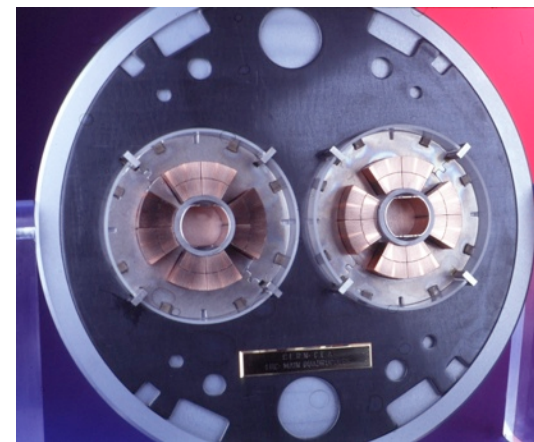
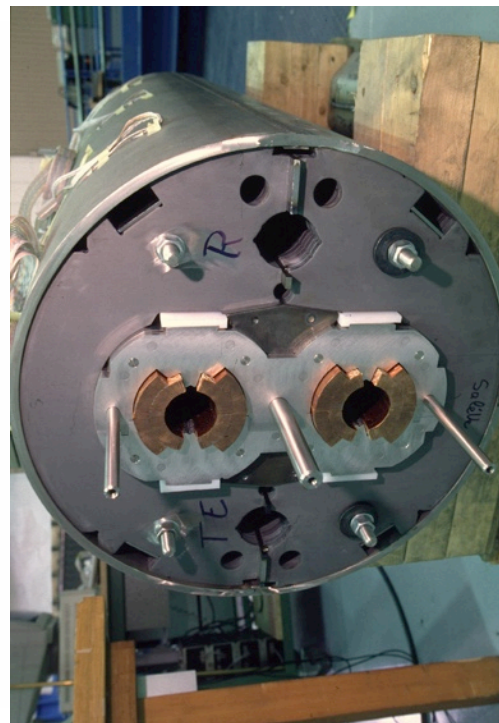
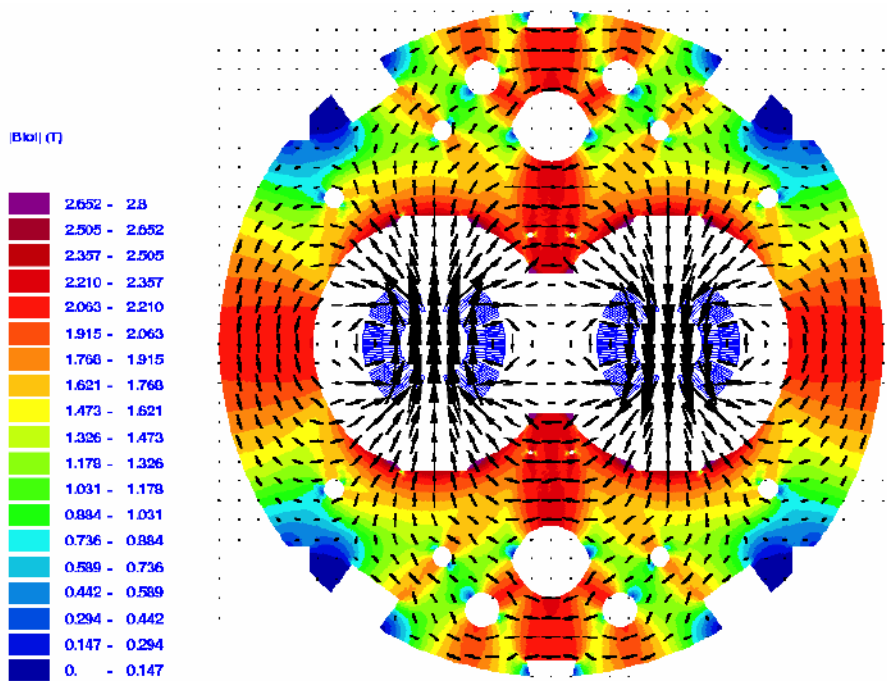
Transition to a normal conducting state is called magnet quench

What can increase the temperature in a magnet ?



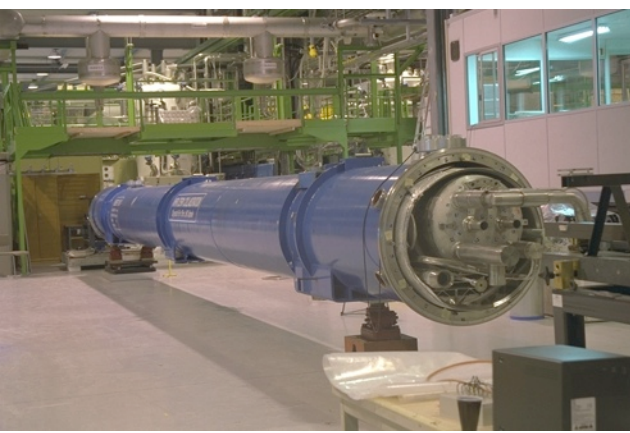
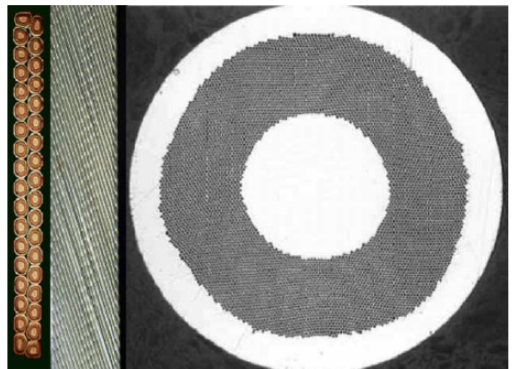
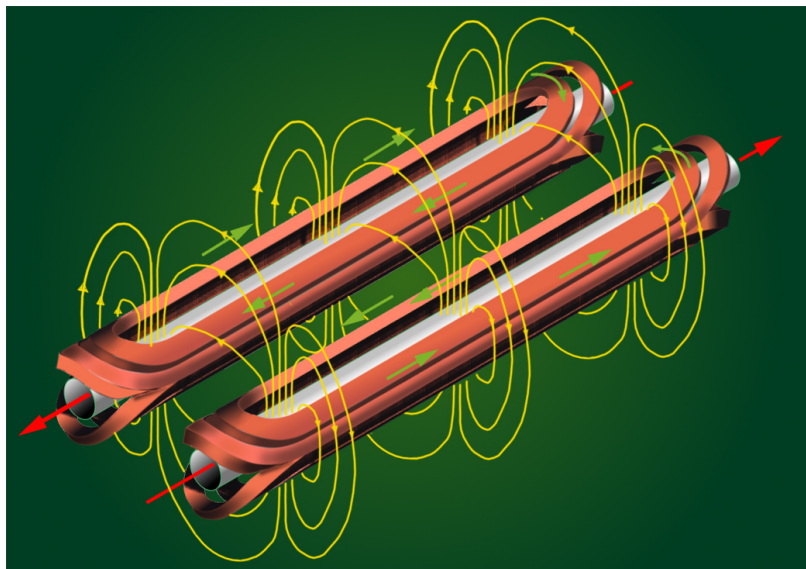


# Two-in-one magnet design

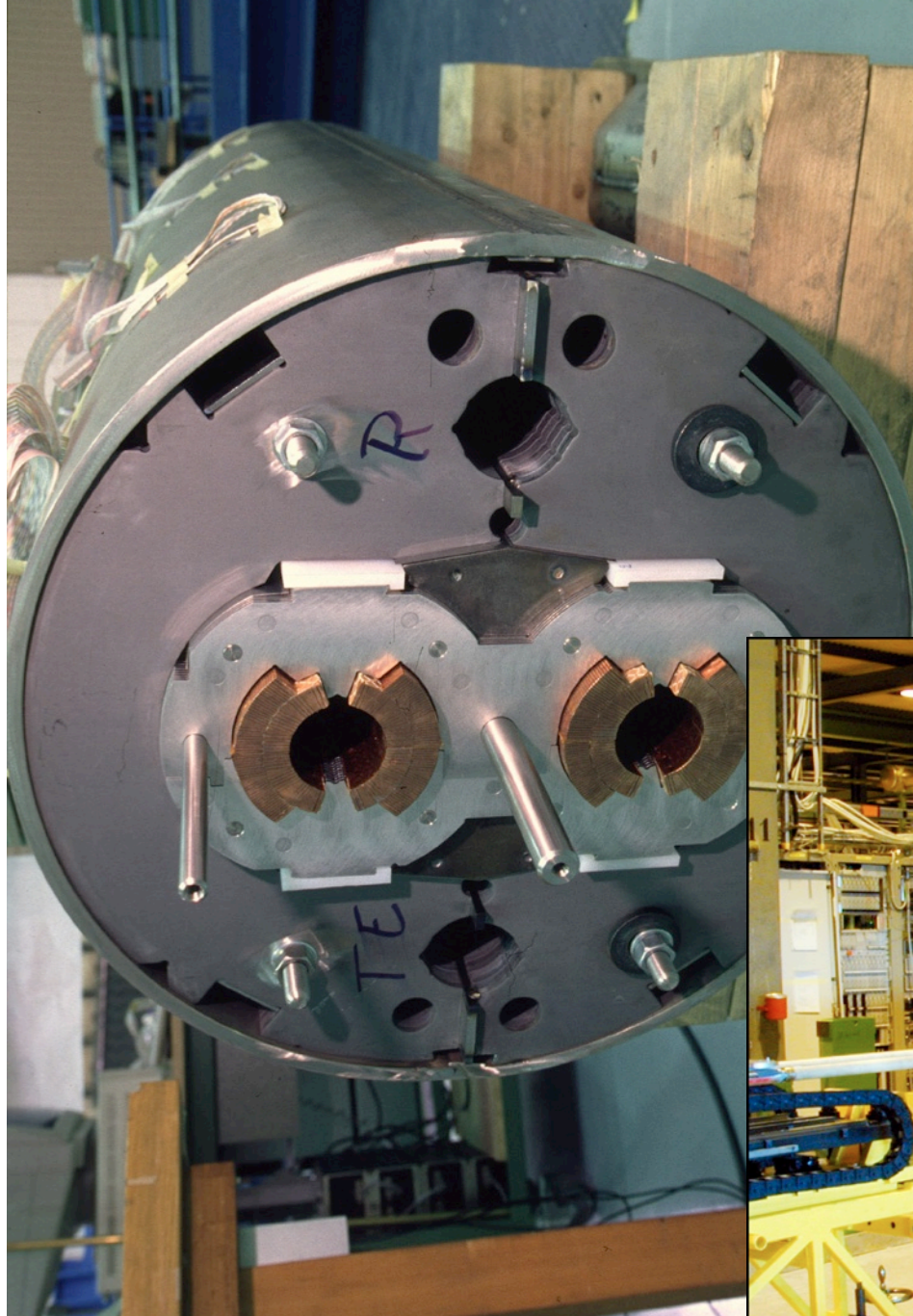


The LHC is one ring where two accelerators are coupled by the magnetic elements.

Nb -Ti  
superconducting cable  
in a Cu matrix







At 7 TeV:

$I_{\max} = 11850 \text{ A}$  Field=8.33 T

Stored energy= 6.93 MJ

Weight = 27.5 Tons

Length = 15.18 m at room temp.

Length (1.9 K)=15 m - ~10 cm

Test bench for magnetic measurements at 1.9 K



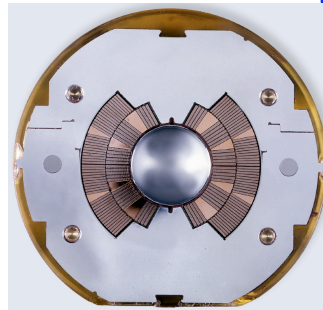
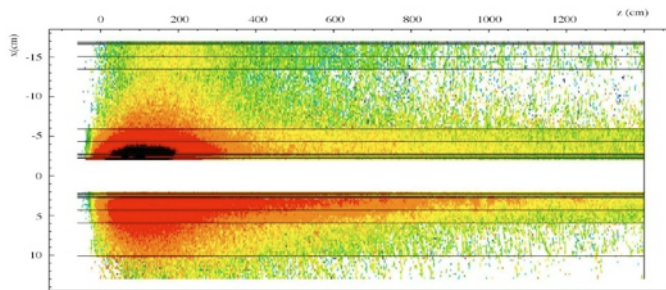
PS: they are not straight,  
small bending of 5.1 mrad



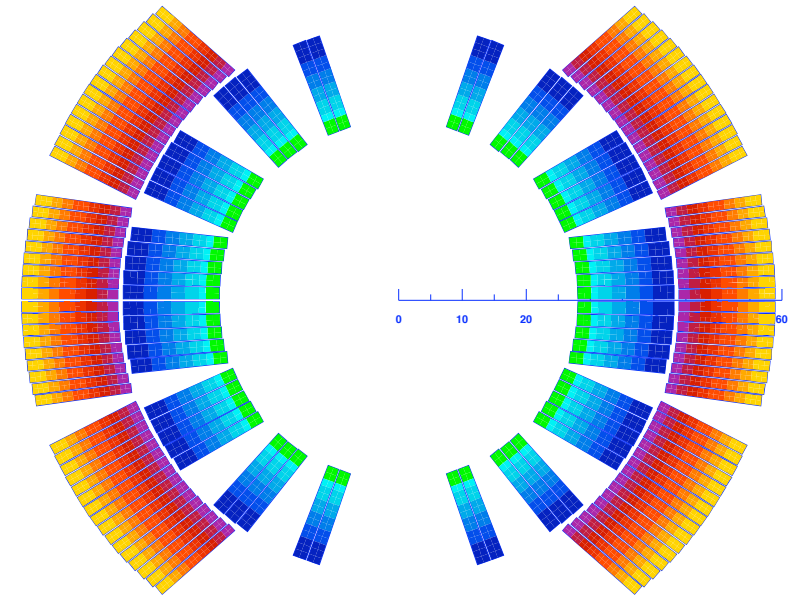
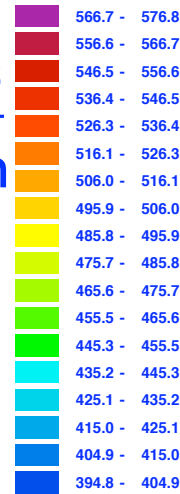
# V. V. S. Introduction to Superconductivity II

Beam losses can eat the temperature margin because of energy deposition

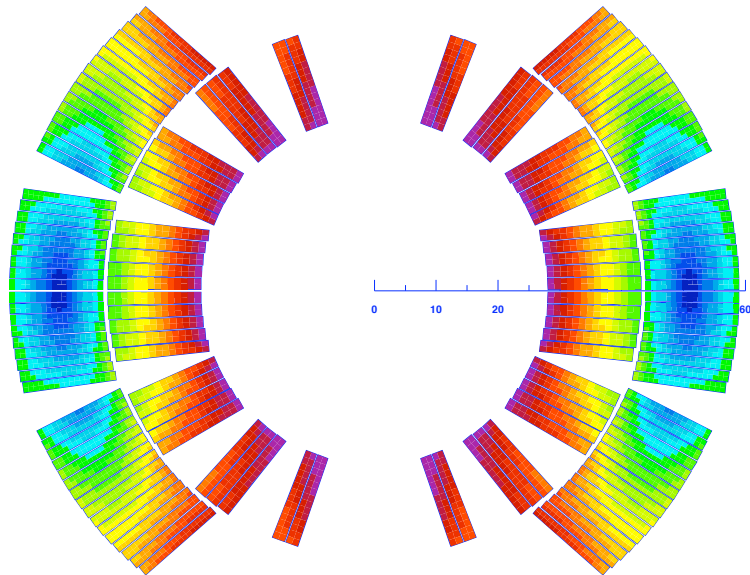
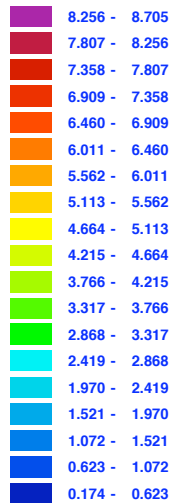
Limit of accepted losses:  $\sim 10 \text{ mW/cm}^3$  to avoid  $\Delta T > 2 \text{ K}$ , the temperature margin



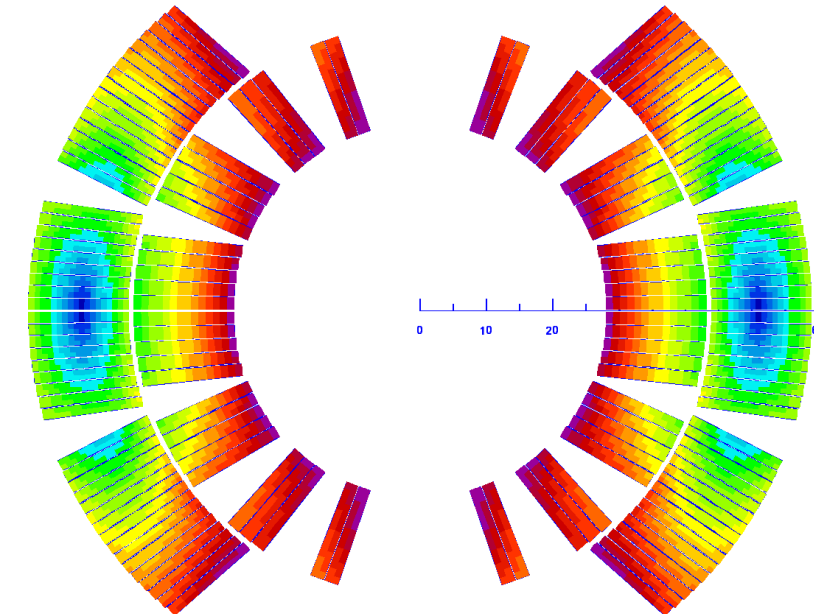
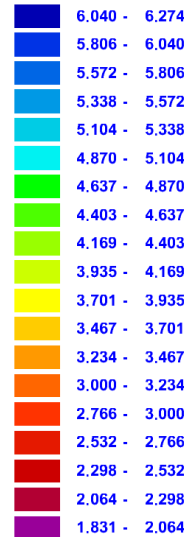
ljl (A/mm<sup>2</sup>)



lBI (T)



Temperature margin (K)

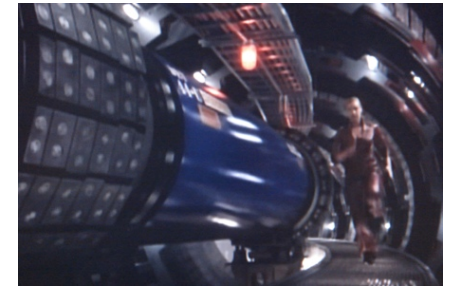
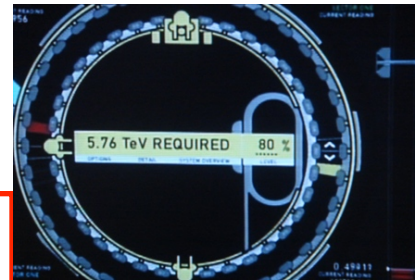


# INTERLUDE: THE TERMINATOR-3 ACCELERATOR

We apply some concepts to the accelerator shown in Terminator-3 [Columbia Pictures, 2003]

- Estimation of the magnetic field

- Energy = 5760 GeV
- Radius ~30 m
- Field =  $5760/0.3/30 \sim 700$  T (a lot !)



Energy of the machine (left) and size of the accelerator (right)

- Why the magnet is not shielded with iron ?
  - Assuming a bore of 25 mm radius, inner field of 700 T, iron saturation at 2 T, one needs  $700 \cdot 25 / 2 = 9000$  mm = 9 m of iron ... no space in their tunnel !
  - In the LHC, one has a bore of 28 mm radius, inner field of 8 T, one needs  $8 \cdot 25 / 2 = 100$  mm of iron
- Is it possible to have 700 T magnets ??



A magnet whose fringe field is not shielded

# How much is $10 \text{ mW/cm}^3$ ?



A fluorescent (known as neon) tube can be typically 1.2 m long with a diameter of 26 mm, with an input power of 36 W.

This makes a power density of about  $56 \text{ mW/cm}^3$ .

**The power of a neon tube can quench about 5 LHC dipoles at collision energy... because one does not need  $10 \text{ mW/cm}^3$  for the entire volume of a magnet, but for about  $1 \text{ cm}^3$ .**

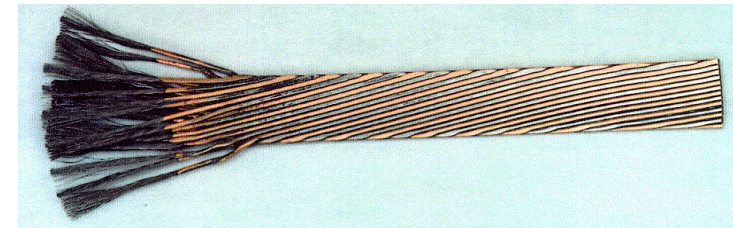
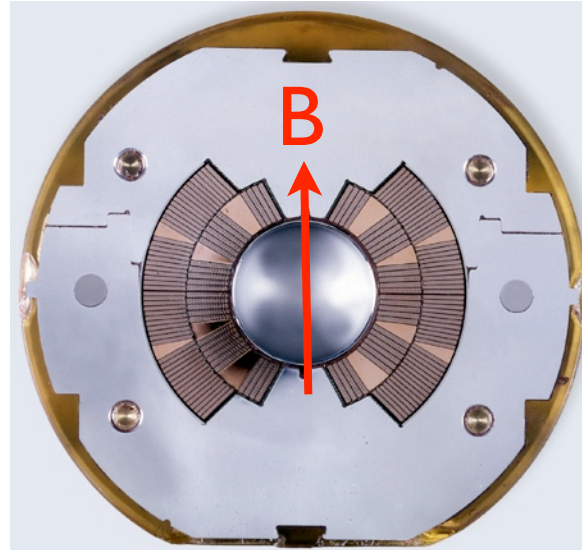
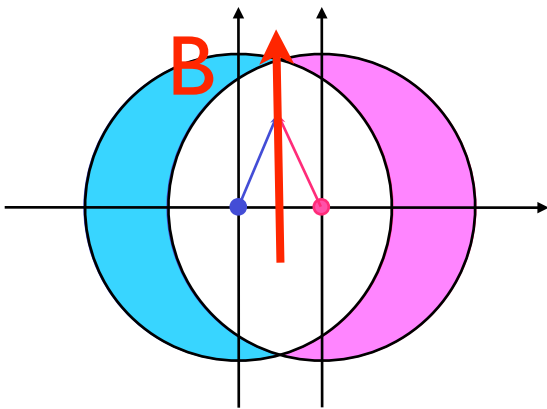


**If you do the same basic computation with a normal 100 W resistive bulbs is even worst**



# Cosθ coil of main dipoles

A 2D cosθ current distribution generates a quasi-perfect vertical field in the aperture between the two conductors.



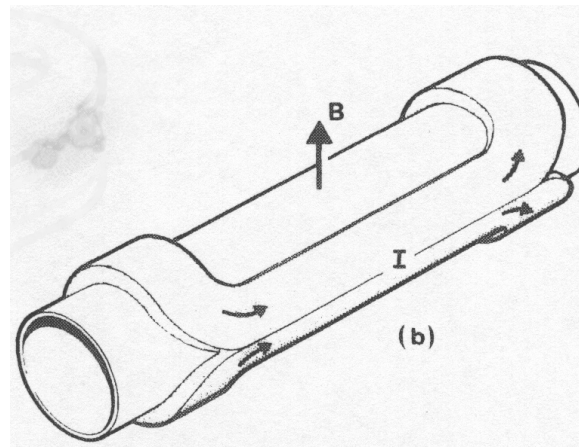
$$I = I_0 \cos \vartheta$$

$$B_{\vartheta} = \frac{\mu_0 I_0}{2 r_0} \cos \vartheta$$

$$B_{\vartheta} = \frac{\mu_0 I_0}{2 r_0} \sin \vartheta$$

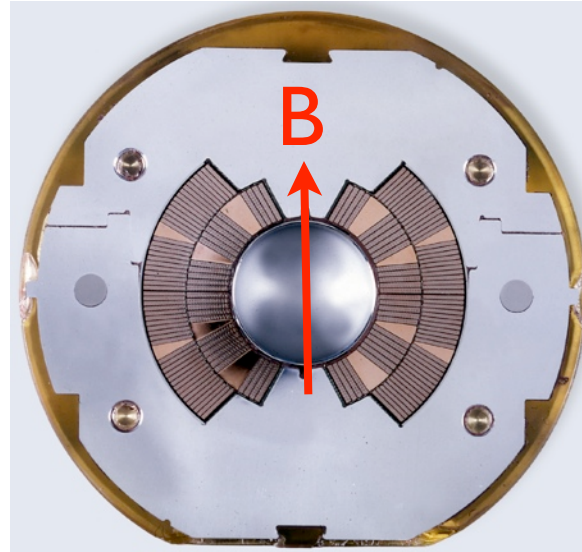
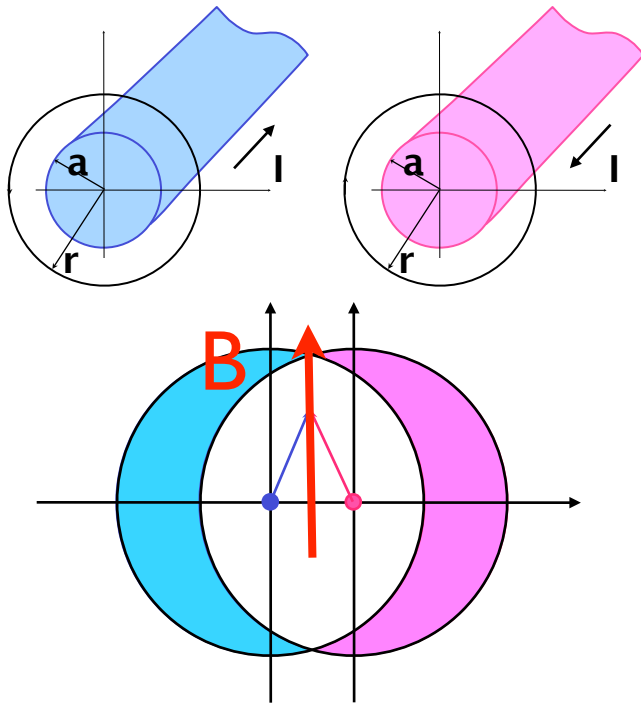
$$B_x = 0$$

$$B_y = \frac{\mu_0 I_0}{2 r_0}$$

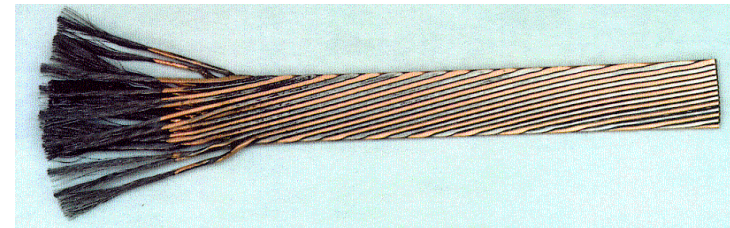


Dipolar Vertical field

# Cosθ coil of main dipoles



A 2D cosθ current distribution generates a quasi-perfect vertical field in the aperture between the two conductors.



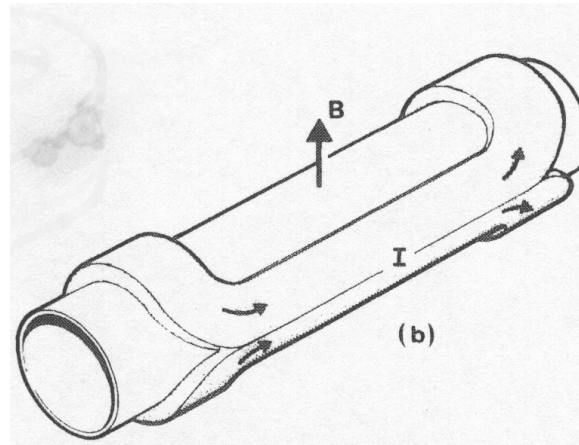
$$I = I_0 \cos \vartheta$$

$$B_{\vartheta} = \frac{\mu_0 I_0}{2 r_0} \cos \vartheta$$

$$B_{\vartheta} = \frac{\mu_0 I_0}{2 r_0} \sin \vartheta$$

$$B_x = 0$$

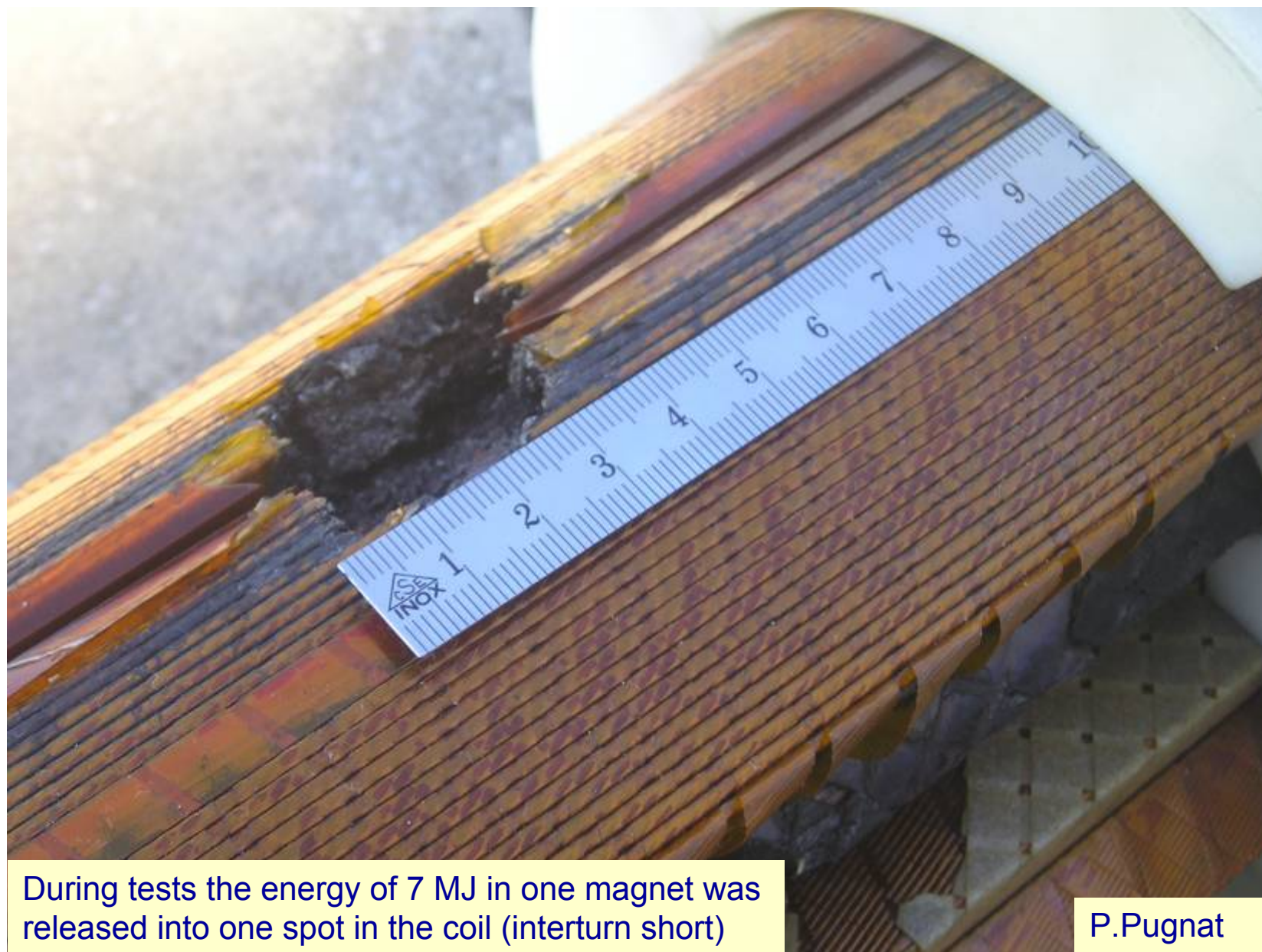
$$B_y = \frac{\mu_0 I_0}{2 r_0}$$



Dipolar Vertical field



# When something goes wrong.... bad quench...

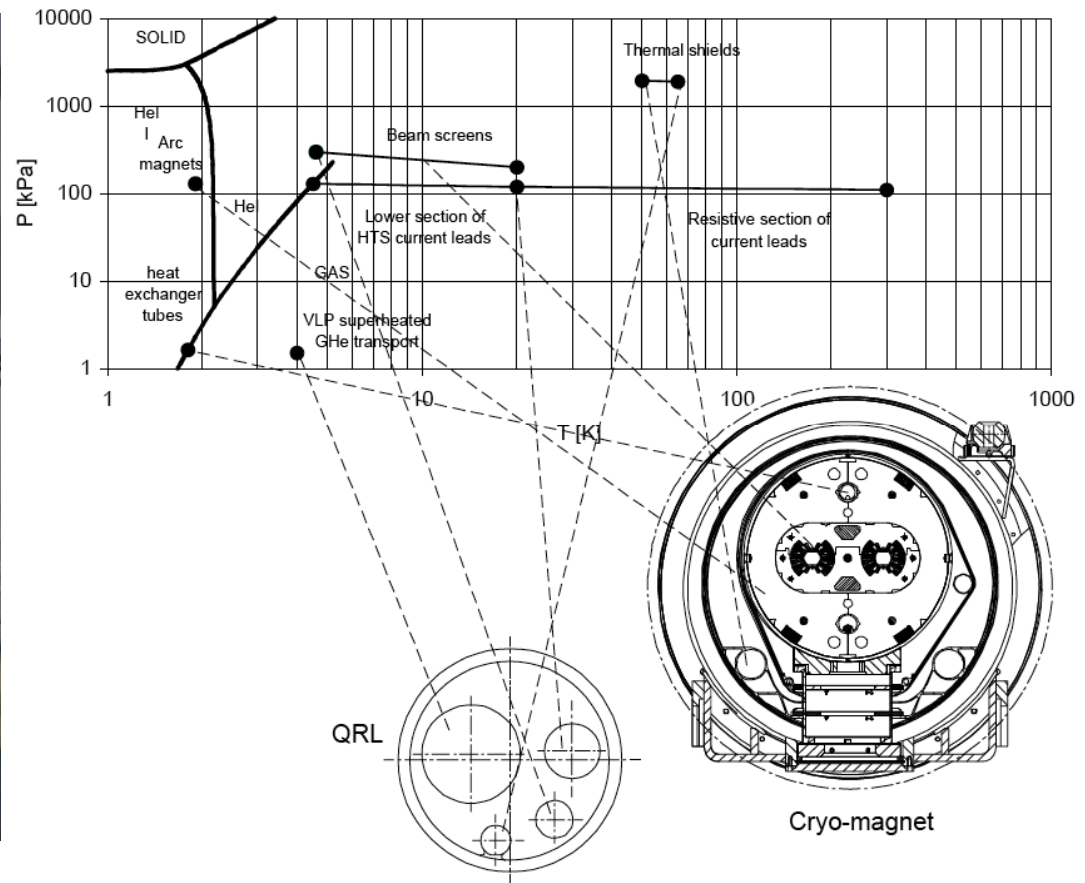
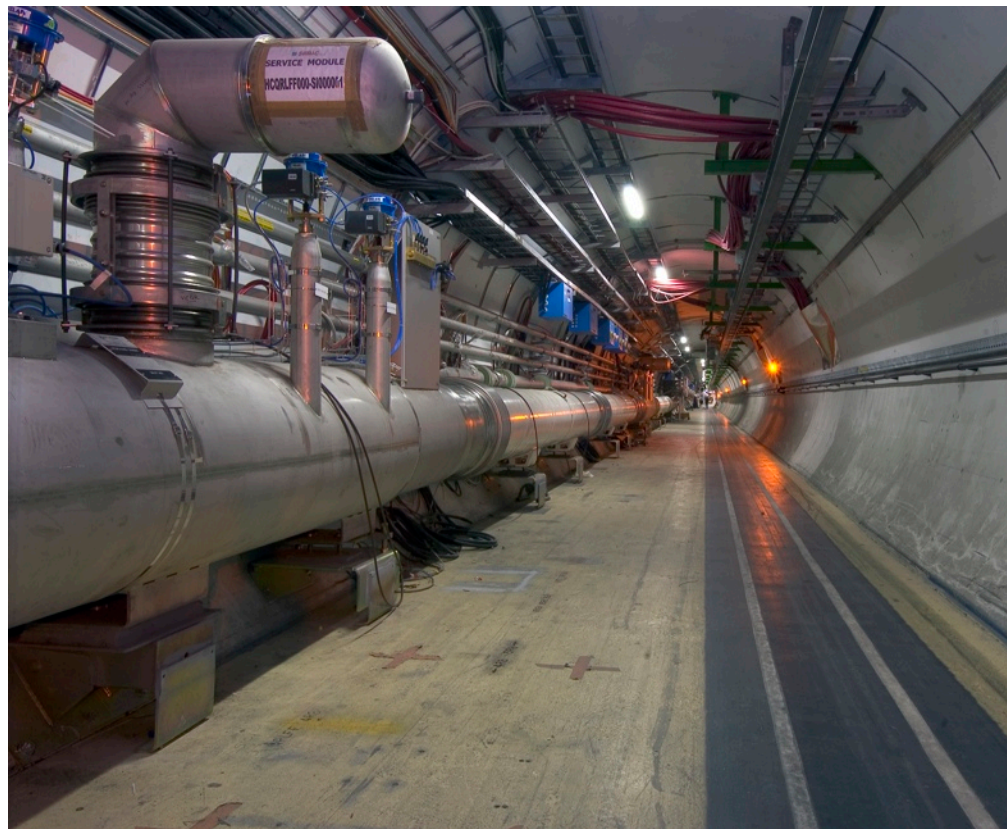


During tests the energy of 7 MJ in one magnet was released into one spot in the coil (interturn short)

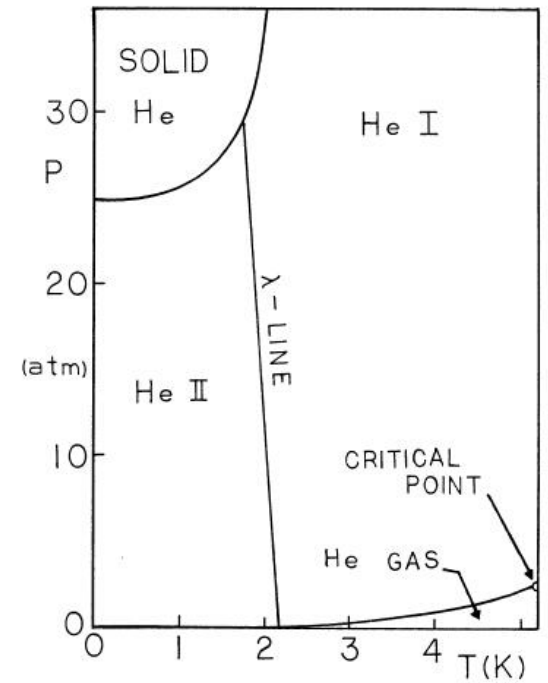
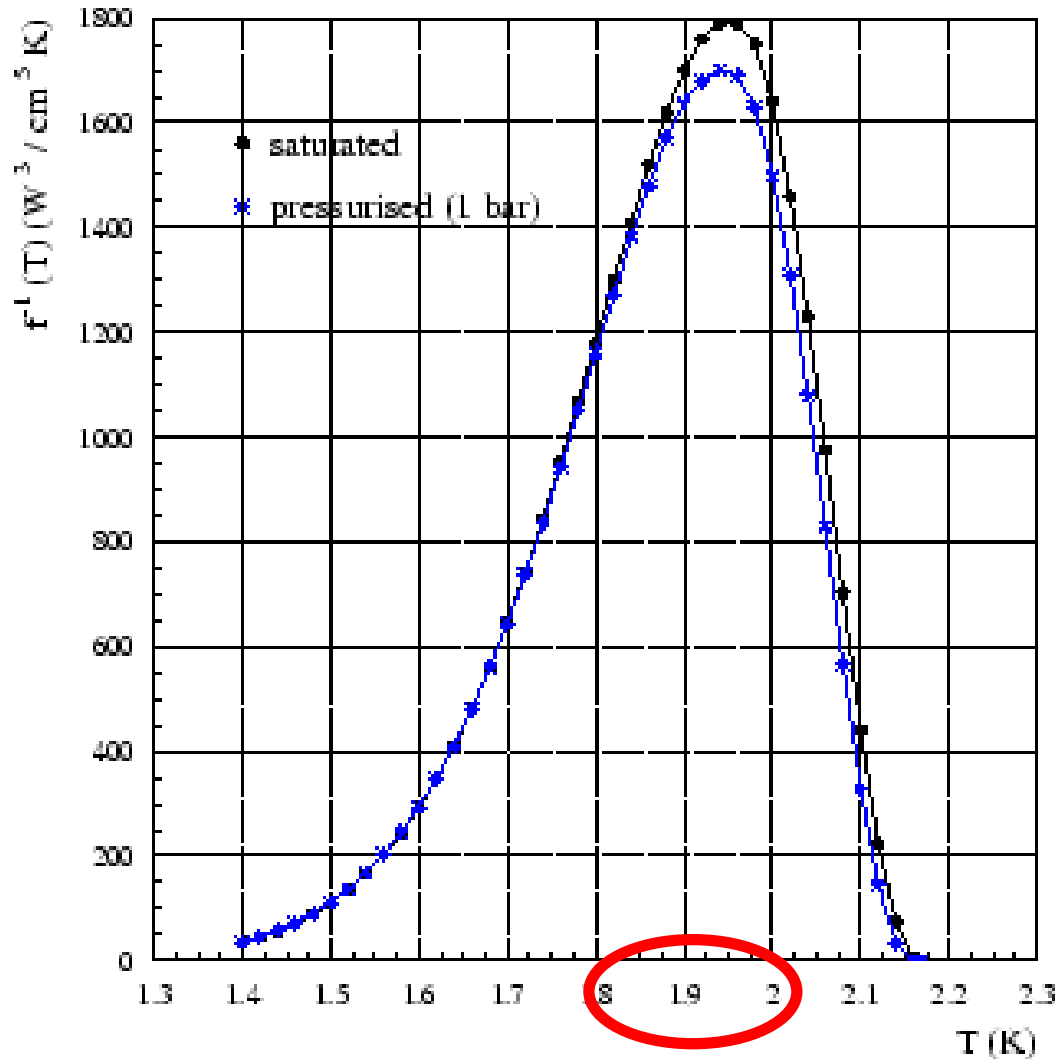
P.Pugnat

# Which coolant ? Liquid superfluid helium

LHC cryogenics will need 40,000 leak-tight pipe junctions.  
12 million litres of liquid nitrogen will be vaporised during the initial cooldown of 31,000 tons of material and the total inventory of liquid helium will be 700,000 l (about 100 tonnes).



# Why helium ?



He at 1.8-2 K has a very large thermal conductivity and very low viscosity



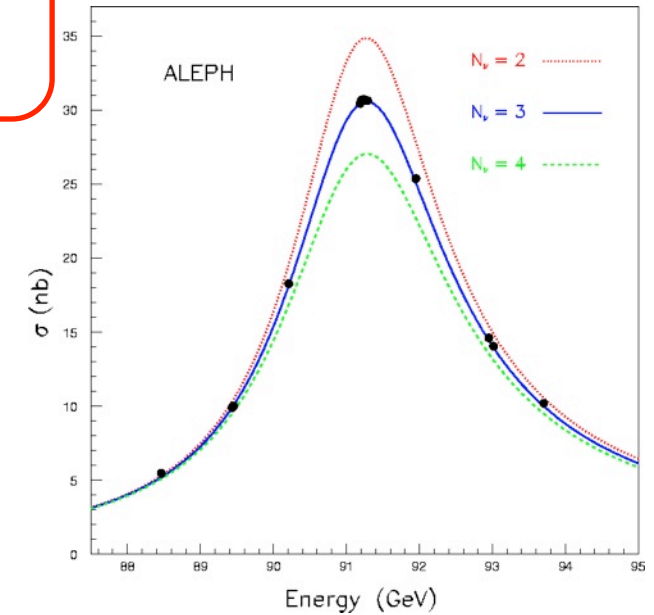
# What can influence an accelerator?

The physics case:

the Z mass at LEP has been measured with an error of 2 MeV.  
Energy of the accelerator has to be known better than 20 ppm.

Energy measurements obtained by  
during last years of LEP operation

Nominal (GeV)	$E_{CM}$ (LEP) (GeV)
181	$180.826 \pm 0.050$
182	$181.708 \pm 0.050$
183	$182.691 \pm 0.050$
184	$183.801 \pm 0.050$
Combined	$182.652 \pm 0.050$

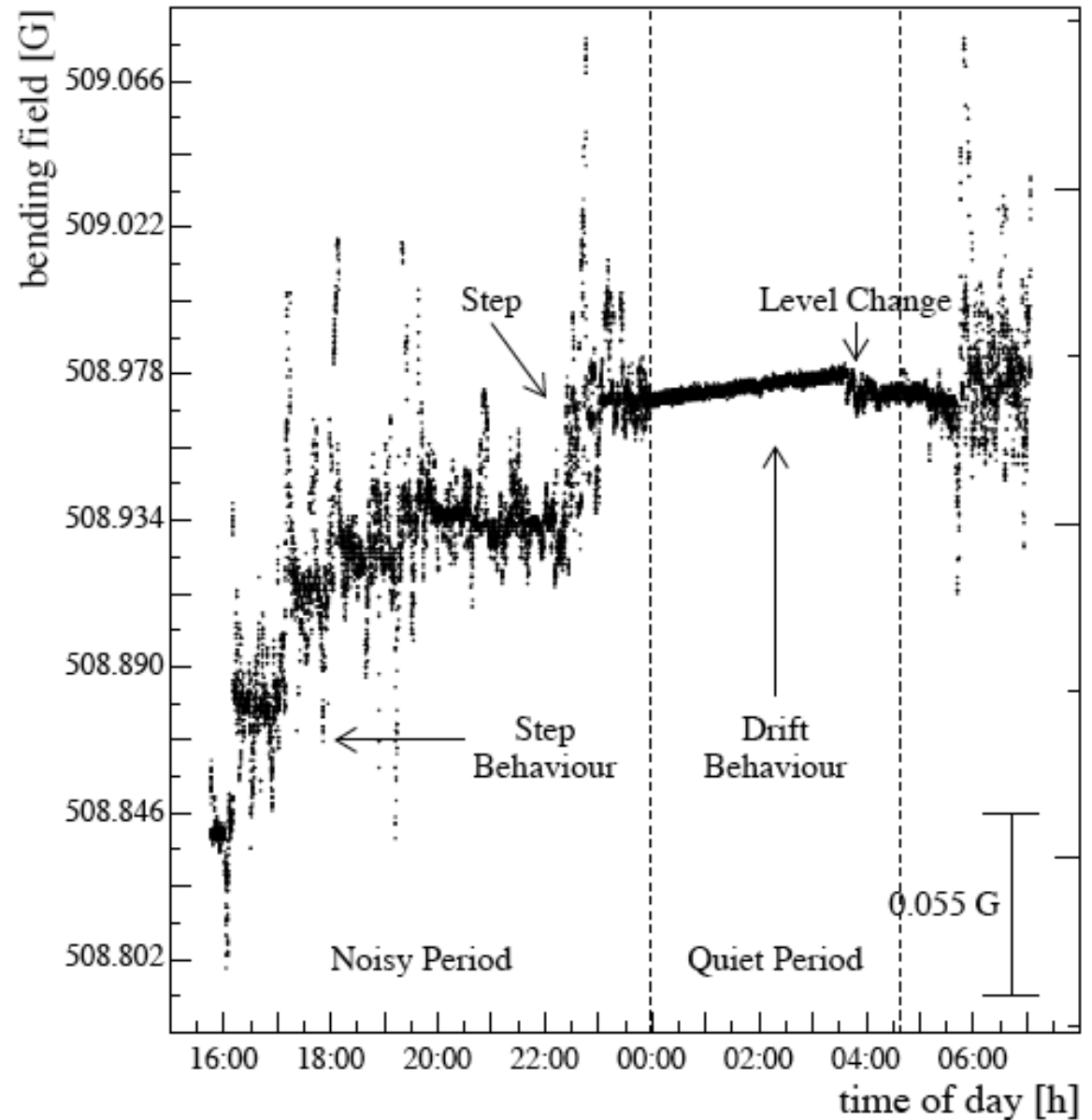
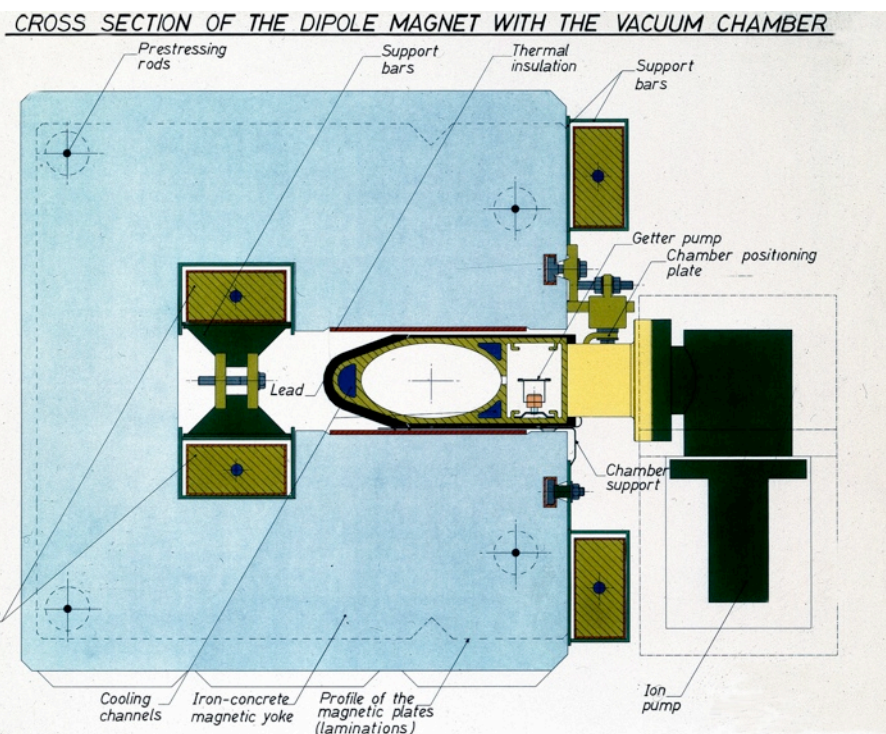


What can influence the energy of a collider?



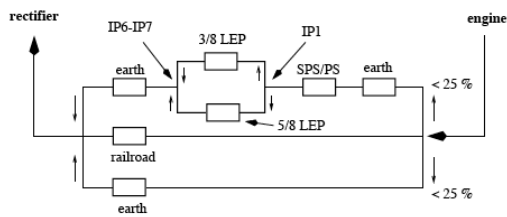
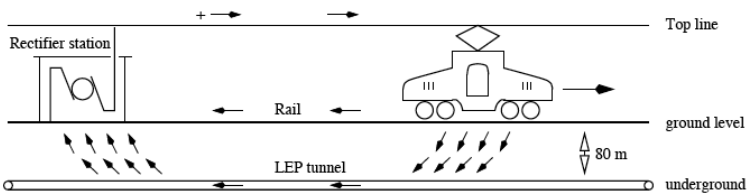
# The problem: an accelerator is not in the middle of nothing

Observed variation of the bending strength of the LEP dipoles during the day



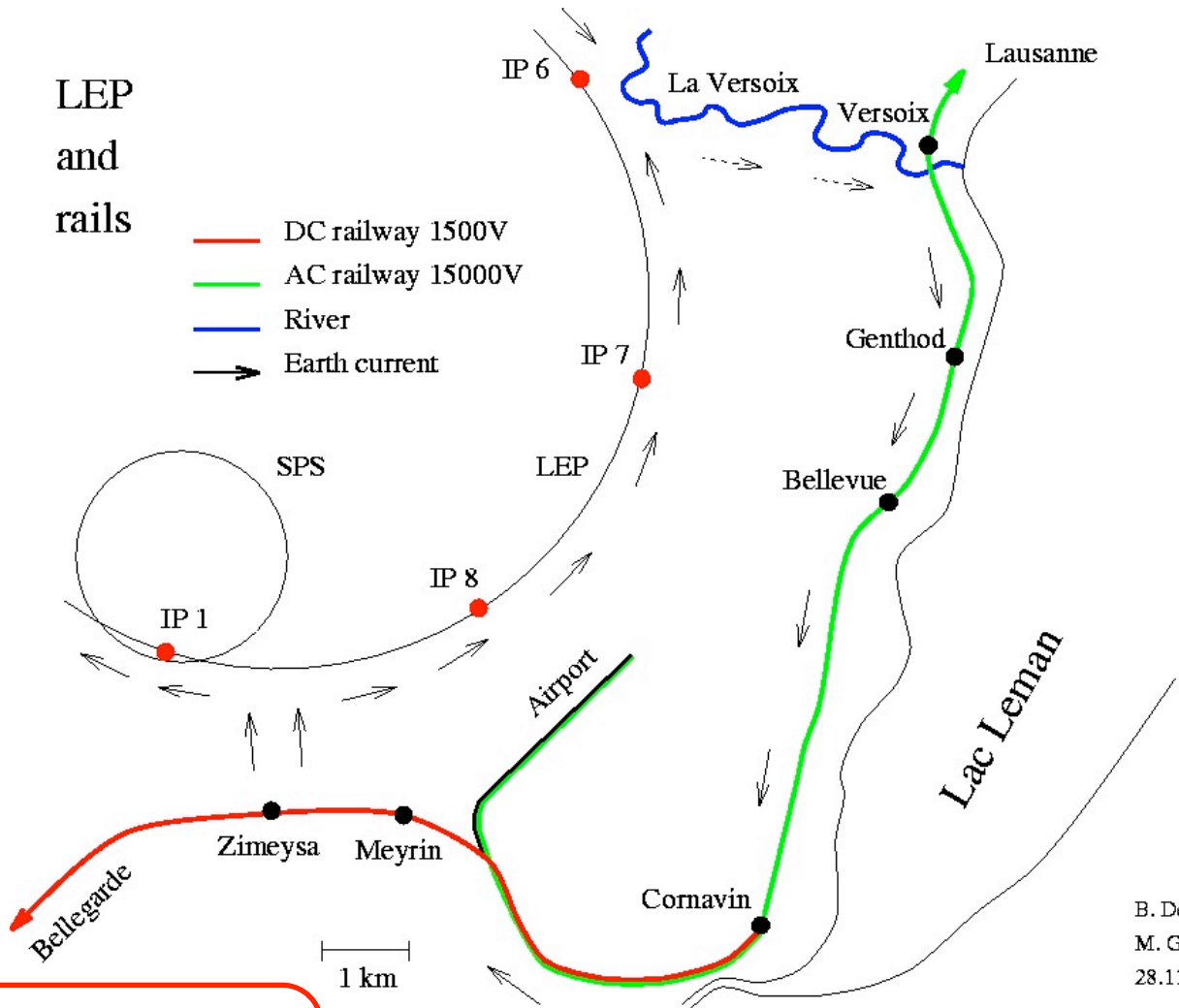


# Influence of train leakage current



LEP  
and  
rails

- DC railway 1500V
- AC railway 15000V
- River
- $\rightarrow$  Earth current

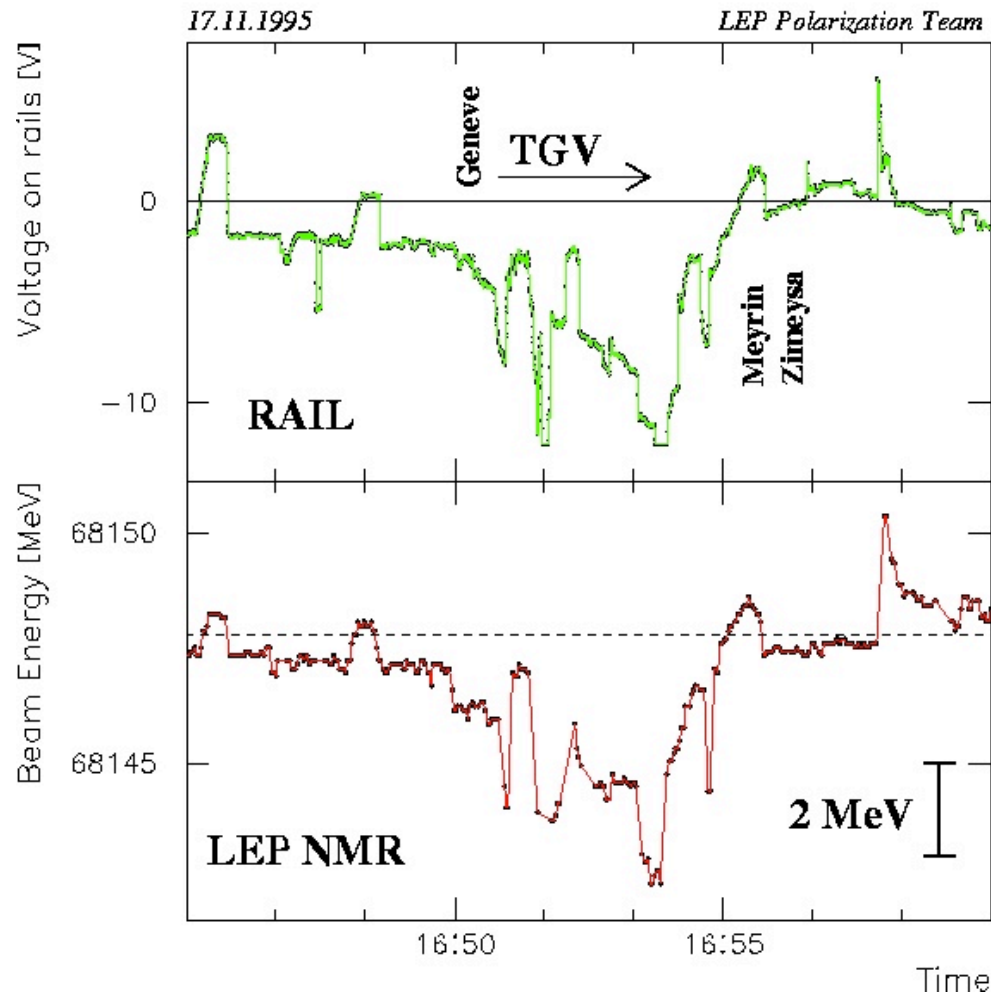


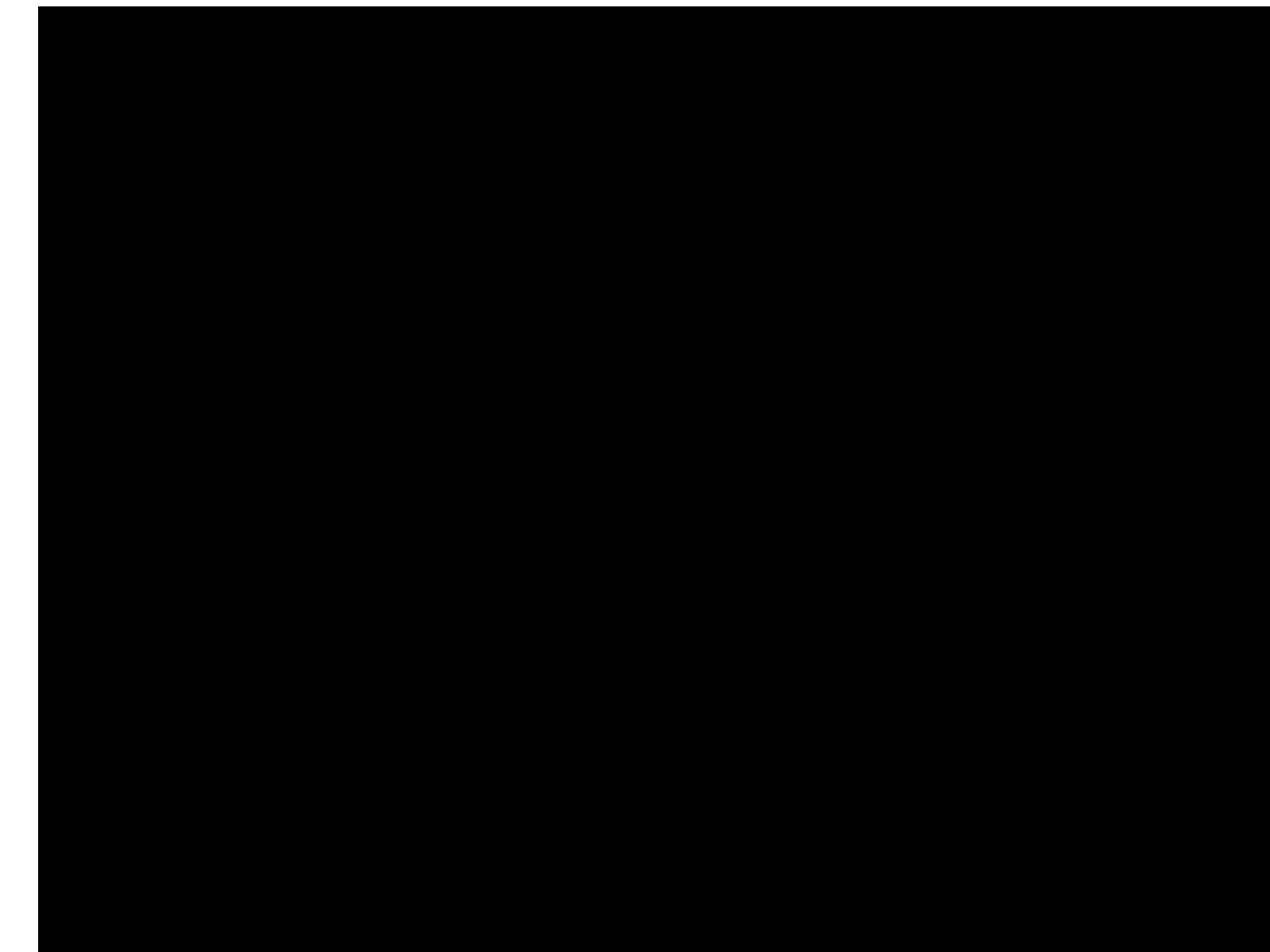
B. Dehning  
M. Geitz  
28.11.1995

LEP beam pipe as ground for leakage current.  
Variation of the dipole field due to the current .  
Change in energy following the SNCF train table

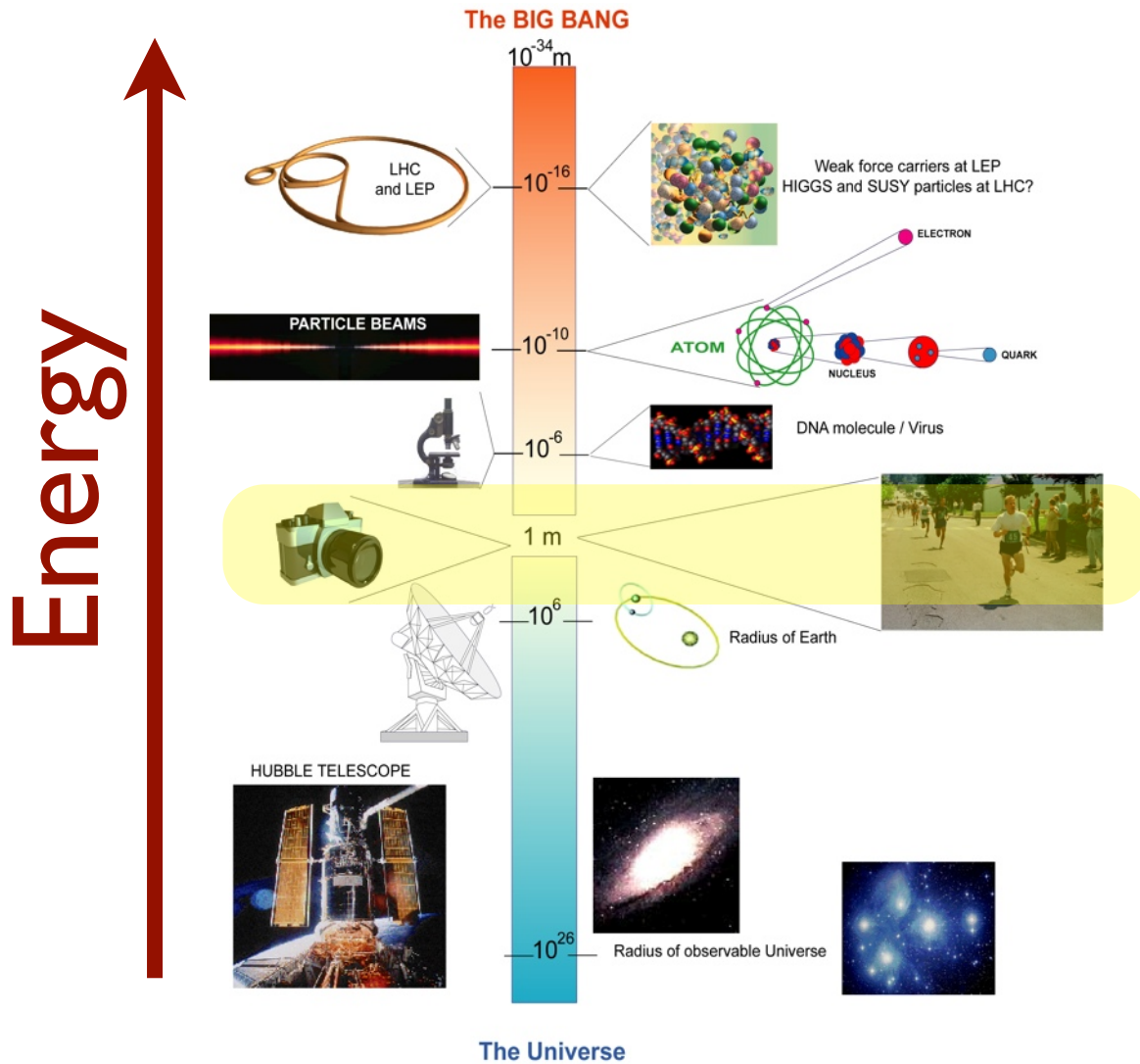
# The evidence, TGV to Paris at 16:50 ...

## Correlation between trains and LEP energy





# The right instrument for a given dimension



Wavelength of probe radiation should be smaller than the object to be resolved

$$\lambda \ll \frac{h}{p} = \frac{hc}{E}$$

Object	Size	Energy of Radiation
Atom	10 <sup>-10</sup> m	0.00001 GeV (electrons)
Nucleus	10 <sup>-14</sup> m	0.01 GeV (alphas)
Nucleon	10 <sup>-15</sup> m	0.1 GeV (electrons)
Quarks	?	> 1 GeV (electrons)

Radioactive sources give energies in the range of MeV

Need accelerators for higher energies.



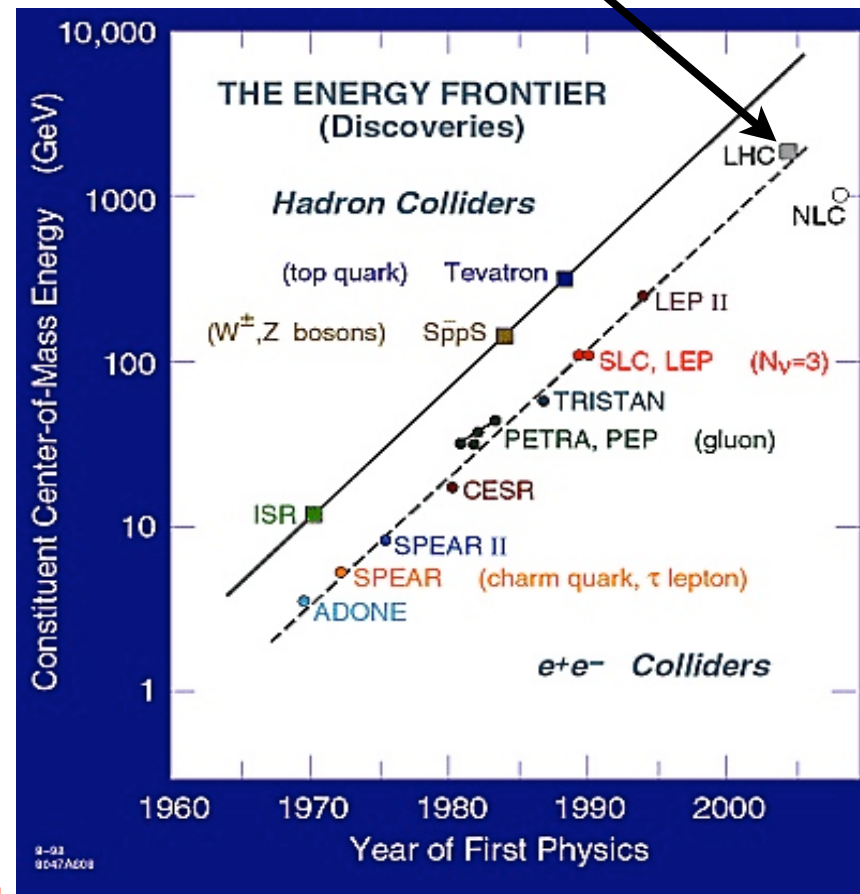
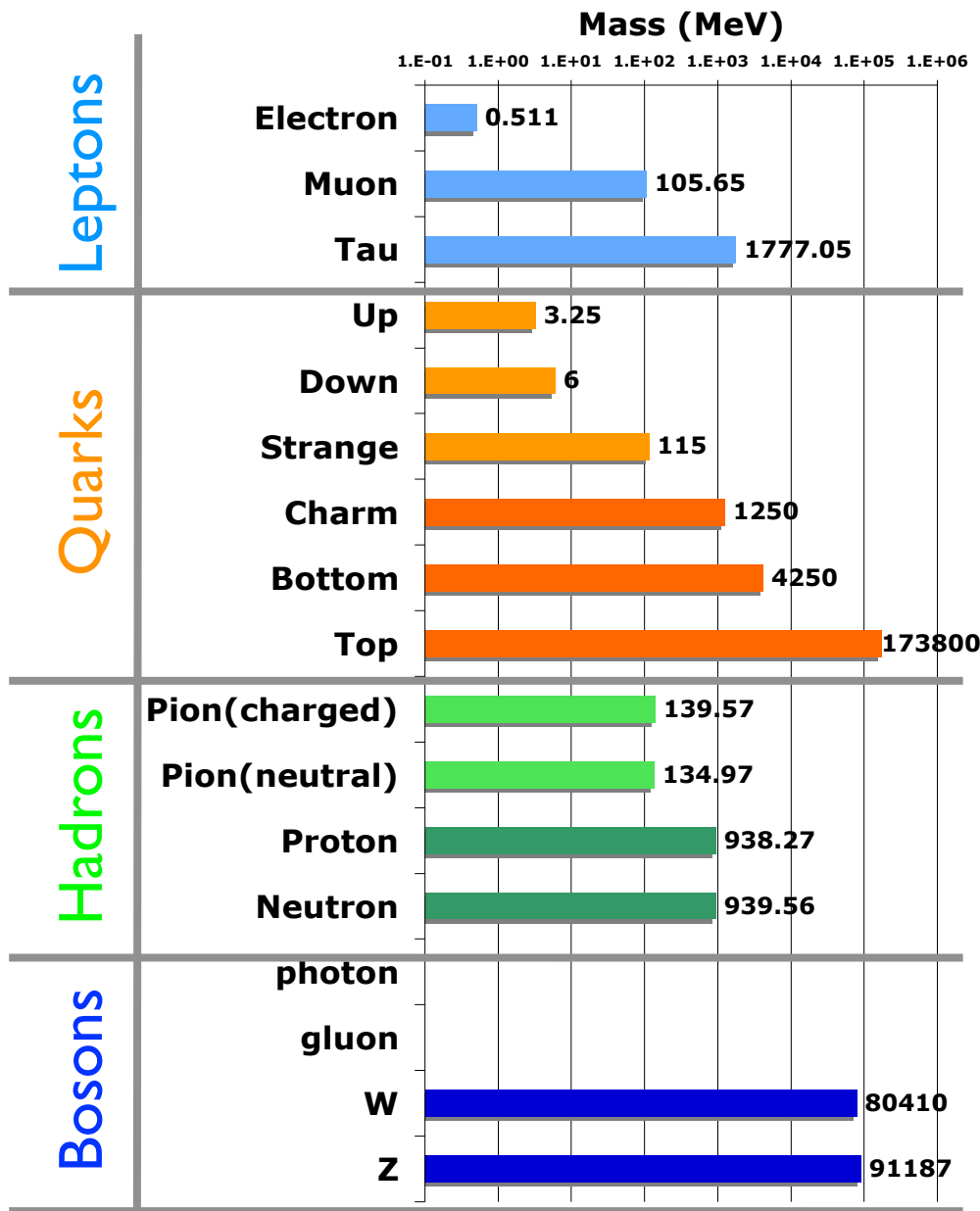
"electronic eyes"

The typical energy of our life is eV

So, how we can reach the energy/dimension of the big bang?

# History/Energy line vs discovery

Higgs and super-symmetry ?  
Or something else maybe



**Constant increase in energy to discover heavier and heavier particles or very rare processes**

Obs: you can notice different particle species used in the different colliders  
electron-positrons and hadron colliders (either  $p\bar{p}$  as Tevatron,  $p-p$  as LHC)

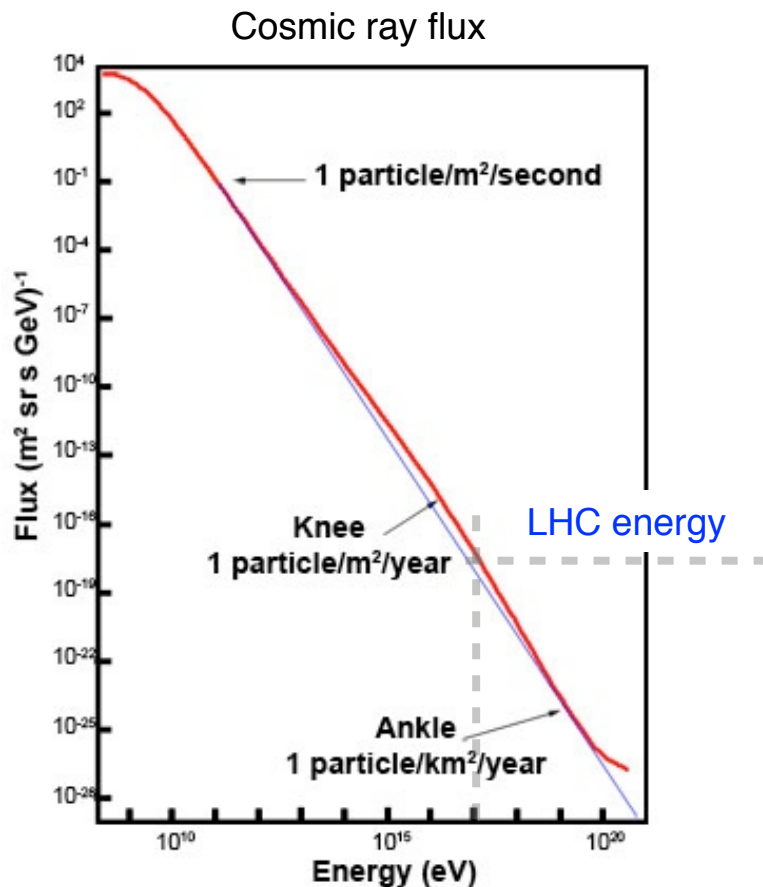


# Why particle accelerators ?

- *Why accelerators?:* need to produce under controlled conditions HIGH INTENSITY, at a CHOSEN ENERGY particle beams of GIVEN PARTICLE SPECIES to do an EXPERIMENT
- An experiment consists of studying the results of colliding particles either onto a fixed target or with another particle beam.



*The cosmos accelerates already particles more than the TeV*  
While I am speaking about  $66 \cdot 10^9$  particles/cm<sup>2</sup>/s are traversing your body, about  $10^5$  LHC-equivalent experiment done by cosmic rays  
**With a space distribution too dispersed for today's HEP physics!**



from: Swinburne University web site



Cloud chamber, from YOUTUBE

# Typical LHC Operational cycle

