Geneva School INSS2011

Question 1

Demonstrate that the threshold for delta production for a proton hitting a target of photons

$$E_p = \frac{m_\Delta^2 - m_p^2}{4E_\gamma}$$

Then calculate the energy threshold of protons on the cosmic microwave background.

Question 2

The Sun converts protons into He according to the reaction:

$$4p \rightarrow {}^4He + 2e^+ + 2\nu_e$$

The solar constant describing the power of the Sun at Earth is P = 1400 W/m^2. How many solar neutrinos arrive at Earth?

Question 3

This problem is on the radiation exposure due to solar neutrinos. Use the result of Question 2 to evaluate:

a) The number of interactions of solar neutrinos in a typical human body (assume its density is ρ = 1 g/cm^3 and its weight 80 kg; use N_A = 6.022 x 10^{23} g^{-1} = Avogadro number; and the value of the cross section $\sigma(\nu_e N) \approx 10^{-45} cm^2 / nucleon$ in the energy range for solar neutrinos for the dominant interaction neutrino-nucleon scattering $\nu_e + N \rightarrow e^- + N'$);

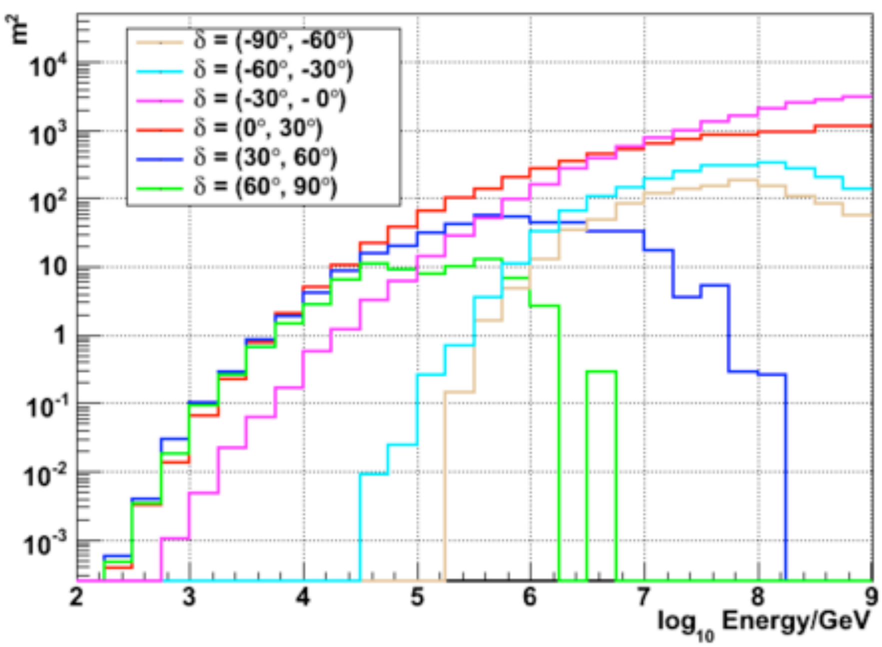
b) The radiation damage is caused by electrons. The equivalent dose is defined as:

$$H = \left(\frac{\Delta E}{m}\right) w_R$$

where m is the mass of the human body, w_R the radiation weighting factor (=1 for electrons), ΔE the energy deposit in the human body and the units for H is defined as 1 Sievert = 0.01 J to be absorbed per kg. Work out the annual equivalent dose due to solar neutrinos for a typical man of 80 kg and compare it with the normal natural dose H_0 = 2 mSv. Assume that on average 50% of the neutrino energy is transferred to the electron and take a typical energy for solar neutrinos of 200 keV.

Group Activity on Neutrino Fluxes from knowledge of gamma fluxes

1. Consider one of the TeV sources in http://tevcat.uchicago.edu/ that you think could be a good candidate neutrino emitter and that is in the field of view of IceCube. Describe the kind of source you picked up and its main features. Describe relevant observations and measurements by gamma-ray and other photon experiments that make the source an interesting neutrino emitter, for instance the spectral emission distribution and fits of this curve to hadronic or leptonic models, the flaring properties.
2. Investigate if there are existing models on hadronic emission. This can help you to answer the question if the dominant process for hadronic production can be proton-proton or proton-gamma or make your own motivated guess. Then, derive the neutrino flux from the measured gamma one. You can assume that the gammas are not absorbed and ignore the cascading process in the source and use their observed spectral index in the TeV range. For neutrinos use the spectral index typical of Fermi 1^{st} order acceleration, E^{-2}.
3. Using the given effective area, derive the muon-neutrino event rate per year for the estimated neutrino flux from that source. [Do not forget oscillations in your calculation.]
4. Collect all the information in slides to be discussed with your colleagues.



IceCube 40strings area in declination bins vs energy.

Solution

Problem 1

Equating the square of the 4-momentum in the CM and Lab.

$$s = E_{CM}^2 = E_\gamma^2 + E_p^2 - 2 \times (E_p E_\gamma - p_p E_\gamma \cos\theta) = m_p^2 - 2E_p E_\gamma (1 - \cos\theta)$$

For opposite beams cos(theta) = -1. The threshold is calculated for final states at rest:

$$m_\Delta^2 = m_p^2 - 4E_p E_\gamma$$

For the cosmic background radiation

$$E_\gamma = 3 \times 2.73 \times 8.62 \times 10^{-5} \text{ eV}$$

$$E_p = \frac{(1236\times10^6)^2 - (938\times10^6)^2}{4\times3\times2.73\times8.62\times10^{-5}} = 2.3 \times 10^{20} \text{ eV}$$

This is the energy the protons need to have when hitting the photons. To get the right value though one should integrate on the black body spectrum of photons.

Problem 2

The solar neutrino flux is given by the fusion process:

$$4p \rightarrow {}^4He + 2e^+ + 2\nu_e$$

We estimated its Q value of 26.7 MeV during lectures. Hence, since there are 2 neutrinos per reaction chain:

P/Q = 8.53 x 1011 MeV cm-2 s-1/26.7MeV x 2 = 6.5 x 10^{10} electron neutrinos

Problem 3

a) The interaction rate is

$$R = \sigma_N N_A \rho V \phi_\nu$$

where σ_N is the interaction cross section, $N_A = 6.022 \times 10^{23}$ g^{-1} is the Avogadro number, Vρ = mass of a typical man = 80 kg, and the total solar neutrino flux is about $\phi_\nu \sim 6.2 \times 10^{10}$ cm^{-2} s^{-1}. So R = 6.2 x 10^{10} x 80 x 10^3 x 6.022 10^{23} x 10^{-45} = 3.0 x 10^{-6} s^{-1} = 94.2 yr^{-1}.

b) A typical energy for solar neutrinos is 200 keV, i.e. 0.1 MeV are transferred to the electron. Consequently, the total annual energy transfer to the electrons is: ΔE = 94.2*0.1=9.4 MeV/yr = 1.5 10^{-12} J/yr.
Hence H = 1.5e-12/80 = 1.9 x 10-11 mSv << H_0. The contribution of neutrinos due to the normal dose is negligible.