

Model Building

1) In the type II seesaw model the $SU(2)_L$ triplet Higgs should get a small induced vev. Write down an extension of the SM (or MSSM) and calculate this vev. Under which conditions is it small? What changes in left-right extensions of the SM (discussed in the 2nd lecture)?

2) As will be discussed in lectures on model building, one can derive RGEs for the mixing angles from the RGEs for the neutrino mass operator. Are there any fixed points in the RG running of the mixing angles, i.e. any combinations of angles and phases which are stable? What does this imply? What can you conclude when you apply these considerations to the case of tri-bimaximal mixing?

Open-Ended

3) Construct a simple toy model for the lepton sector of the SM (or MSSM) with massive neutrinos where the flavour structure emerges from the breaking of a non-Abelian family symmetry (take $SO(3)$ or A_4 as an example with flavons in triplet representations and use additional discrete symmetries, e.g. Z_n symmetries). How do the flavon vevs have to look like for your toy model to be consistent with data? How many flavons do you need?

Design a Neutrino Factory

Open-Ended

4) Parts A-F use knowledge of how to keep particles in a storage ring (Gilardoni Lecture #1). Parts G-I use knowledge of how to accelerate beams and cool in a neutrino factory design.

- A) From a given set of oscillation parameters, choose distance and energy of the neutrino beams you desire. How you would decide the neutrino flux needed at the detector? What would be the dimension of the neutrino beam at the detectors and then at the exit of the storage ring? Evaluate a safety margin for radiation issues where neutrinos exit the rings near the site in your design.
- B) Compute the number of muon decays per year you need to get the neutrino flux, including the effect of beam divergence.
- C) Determine how many muons are needed in the storage ring at any one time.
- D) Why use a storage ring as the neutrino source but not a storage ring in the early stages of muon acceleration?
- E) Choose the shape of the storage ring. On which basis you choose?
- F) Design the ring: given the energy you need, compute the B field needed by the dipoles. Choose a technology, superconducting or normal conducting? Choose the quadrupoles, compute the gradient.
- G) Decide how many accelerator stages you want to feed the storage ring
- H) How do you accelerate? Linear accelerators? Circular accelerators?
- I) Determine where to put the cooling and parameters of the cooling design

Direct Neutrino Mass Experiments

Open-Ended

5) π^+ decays to μ^+ and ν_μ . Since this is a two body decay, one can in principle determine the neutrino mass by measuring quantities related to the π^+ and μ^+ in the decay. Design such an experiment. (Choose pions at rest or in flight in a beam.) What are the experimental limitations of this measurement? How precise do your measurements need to be in order to reach 1 eV uncertainty in the mass? What is the interpretation of the mass you measure in terms of the masses of the neutrino mass eigenstates? Is there a fundamental limitation to how precisely or accurately you can ever measure the neutrino mass with this type of experiment?

CP Violation

6) Prove that CP violation cannot be measured in a disappearance oscillation experiment

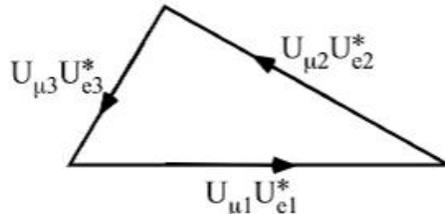
7) CP violation inferred from $|U_{\alpha i}|$ sizes

Suppose there are just 3 charged-lepton mass eigenstates (e , μ , τ), and just 3 neutrino mass eigenstates (ν_1 , ν_2 , ν_3). Assume that the 3×3 leptonic mixing matrix U is unitary.

Suppose that incredibly precise measurements have determined that the ν_μ and ν_τ fractions of ν_3 are both exactly 49%, and the ν_μ and ν_τ fractions of ν_2 are both exactly 33%.

a) Find the magnitudes $|U_{\alpha i}|$ of all 9 elements of U .

b) The orthogonality constraint $\sum_{i=1}^3 U_{\mu i} U_{e i}^* = 0$ following from unitarity may be pictured as a closed triangle in the complex plane, shown below. This is known as a “unitarity triangle”. From



the magnitudes $|U_{\alpha i}|$ found in part (a), show that the area $|A_{\mu e}|$ of this triangle is not zero, and find its numerical magnitude.

c) We have shown in the lectures that when there are just 3 neutrino mass eigenstates,

$$P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) = 4 [|U_{\alpha 3} U_{\beta 3}|^2 \sin^2 \Delta_{31} + |U_{\alpha 2} U_{\beta 2}|^2 \sin^2 \Delta_{21} + 2 |U_{\alpha 3} U_{\beta 3} U_{\alpha 2} U_{\beta 2}| \sin \Delta_{31} \sin \Delta_{21} \cos(\Delta_{32} \pm \delta_{32})] , \quad (1)$$

where $\Delta_{ij} \equiv \Delta m_{ij}^2 \frac{L}{4E}$ and $\delta_{32} \equiv \arg(U_{\alpha 3} U_{\beta 3}^* U_{\alpha 2}^* U_{\beta 2})$.

From this expression, show that the CP-violating difference between the $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation probabilities is

$$P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = \pm 32 |A_{\mu e}| \sin \Delta_{31} \sin \Delta_{21} \sin \Delta_{32} . \quad (2)$$

Note that since the flavor fractions given at the beginning of the problem imply that $|A_{\mu e}| \neq 0$, these flavor fractions imply that CP is violated.

8)

When there are only 3 flavors, there is only one independent CP-violating effect.

Assume there are only 3 flavors (e, μ, τ). Assume CPT invariance, which implies that

$$P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\beta \rightarrow \bar{\nu}_\alpha) . \quad (3)$$

Let $\Delta_{\alpha\beta} \equiv P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$ be the CP-violating difference between corresponding neutrino and antineutrino oscillations. Note that CPT invariance implies that $\Delta_{\alpha\alpha} = 0$. That is, there can be no CP violating difference between the probabilities that a neutrino and its antineutrino survive with their original flavors. CP violation is possible only in $\Delta_{\alpha\beta}$ with $\beta \neq \alpha$; that is, in the appearance of a new flavor of neutrino ν_β in a beam of neutrinos born as ν_α .

Show that the 6 possible nonzero $\Delta_{\alpha\beta}$ with $\beta \neq \alpha$ are all equal apart from a sign:

$$\Delta_{e\mu} = \Delta_{\mu\tau} = \Delta_{\tau e} = -\Delta_{\mu e} = -\Delta_{\tau\mu} = -\Delta_{e\tau} \equiv \Delta . \quad (4)$$

Oscillation Phenomenology and Experimental Design

9) Suppose that an experiment with a detector only 15 m from a reactor reports that at 3 MeV, 6% of the electron antineutrino flux from the reactor has already disappeared before it reaches the detector. Meanwhile, an accelerator neutrino experiment is studying muon neutrino disappearance by comparing the observed muon neutrino fluxes at 0.6 GeV in a near detector at 3 km from the neutrino production point, and a far detector at 1000 km from the neutrino production point. Does the reactor result have any implications for this accelerator experiment? If so, what are they?

10) (a) Assuming that the neutrino mass spectrum is inverted, and neglecting θ_{13} and the mass splitting between $\nu(1)$ and $\nu(2)$, find the expression for the effective Majorana mass for neutrinoless double beta decay as a function of the common mass of $\nu(1)$ and $\nu(2)$, the "solar" mixing angle θ_{12} , and the CP-violating Majorana phases that appear in the leptonic mixing matrix. From what we already know experimentally, what is the smallest possible value of this effective Majorana mass?

(b) What would it take, experimentally and theoretically, to establish the presence of a nonvanishing Majorana phase? Is it realistic to imagine we could actually do what is necessary?

Open-Ended

11) Evaluate T2K's sensitivity to CP violation. You may obtain fluxes from:

Neutrino: <http://www.mpi-hd.mpg.de/personalhomes/globes/glb/JHFplus.dat>

Anti-neutrino: <http://www.mpi-hd.mpg.de/personalhomes/globes/glb/JHFminus.dat>

(a) Compute the signal expected for $\sin^2(2\theta_{13})=0.1$, 1.5×10^{20} pot assuming 22.5 kton fiducial volume at 295 km and an electron efficiency of 70% (flat in energy).

(b) using the antineutrino flux compute the sensitivity to leptonic CP violation, assuming the previous value of θ_{13} , by running 5 years with neutrinos and 5 years with antineutrinos (6.6×10^{21} POT/year in both cases), and using a fiducial volume of 500 kton at the same distance. To start on this sensitivity evaluation, it might be good to first evaluate whether or not the experiment will be sensitive to $\delta = \pi/4$ or not in the normal hierarchy.

(c) discuss the effect of backgrounds and systematic errors.

Open-Ended

12) In their next result, T2K finds that $\sin^2 2\theta_{13}$ is >0.07 at 99% confidence level. Just as they announce their results, the entire near detector plan for a next generation project to measure the mass hierarchy and CP violation in muon neutrino and antineutrino oscillations to electron neutrinos and antineutrinos is lost due to a previously unknown bug in Microsoft Project. Bill Gates is so grateful we found this bug that he offers to pay any amount you want in order to design and build a new near detector. You may assume any far detector technology and design of your choosing, subject to the constraint that the optimized oscillation analysis will have 45% electron neutrino signal, 45% neutral current background and 10% charged current background. What does this new near detector complex look like?

Neutrino Decay

13) Since we know that neutrinos are massive, the heavier ones can decay radiatively to the lighter ones (via a loop diagram). First estimate the lifetime of the neutrinos (don't actually do the loop calculation). Next, analyze whether the radiation from neutrino decay could be used to detect cosmic neutrinos. This requires some consideration of the background radiation from other astrophysical and cosmological sources, which can be found in the book by Kolb and Turner.

Reactor (and related) Neutrino Experiments

Open-Ended

14) What is the influence of the recent +3% shift of the reactor neutrino flux from nuclear reactors on the forthcoming reactor neutrino experiments (Daya Bay, Double Chooz, Reno) constraining θ_{13} ? What is the influence on the limit of Chooz? If you were doing a global fit to oscillation parameters, give arguments for and against re-evaluating the experimental limit of Chooz in your fits given this new information about reactor fluxes.

Open-Ended

15) An underground anti-science terrorist organization is nevertheless developing nuclear weapons in secret. Work out parameters for a neutrino monitor that would be able to pick up nuclear tests or production of fuel at a nuclear reactor.

Open-Ended

16) A consortium of governments with nuclear weapons hires you to make a 1% precision measurement of the flux of neutrino released from the explosion of a 200 kTon TNT-equivalent Uranium fission bomb. Because the nuclear test ban treaty must be waived to enable this experiment, you get only one attempt at the measurement. Design such an experiment using a detector of the smallest size achievable. (Of course, having the detector survive long enough to make the measurement is a prerequisite!)

Neutrino Beams

Open-Ended

17) How to get a neutrino beam containing a large fraction of ν_τ (>10%) in order to study charged current ν_τ interactions? Propose a possible detector capable of this measurement. (You should propose an experiment that can do much better than the detector that originally performed this measurement!)

18) Compare and critically discuss the following options for a very high energy (~500 GeV) neutrino beam at LHC (assume LHC design values):

- conventional neutrino beam by extracting the LHC proton beam
- neutrino beam from interactions of the proton beam on an internal gas-jet target
- neutrino beam from extracted proton beam in a beam dump
- neutrino beam from pp interactions at the interaction points

What would roughly be the $\nu_e/\nu_\mu/\nu_\tau$ composition of such beams? What physics goal(s) could you think of for studying these very high energy neutrinos?

Neutrino Interactions

19) Neutrino-electron scattering: Consider the following processes:

- $\nu_l + e \rightarrow l + \nu_e$ ($l = \mu$ or τ)
- $\bar{\nu}_e + e \rightarrow l + \bar{\nu}_l$ ($l = \mu$ or τ)
- $\nu_l + e \rightarrow \nu_l + e$ ($l = \mu$ or τ)
- $\bar{\nu}_l + e \rightarrow \bar{\nu}_l + e$ ($l = \mu$ or τ)
- $\nu_e + e \rightarrow \nu_e + e$
- $\bar{\nu}_e + e \rightarrow \bar{\nu}_e + e$

What is the cross section dependence from the incoming neutrino energy for these processes?

Why are processes c) and e) important for the interpretation of the solar neutrino signal by Super-Kamiokande? Outline an experiment, with a suitable neutrino source, where one or a combination of the above processes could be used for a precise measurement of the weak mixing angle $\sin^2\theta_W$. Could these processes be used to search for 'New Physics'? Why is the process $\bar{\nu}_\mu + e^- \rightarrow \mu^- + \bar{\nu}_e$ forbidden in the Standard Model?

20) Neutrino helicity flip: Calculate the minimum pion momentum needed for a decay neutrino to be right handed in the lab frame. Note: you must assume a value for the neutrino's mass; state what value you assume and why. What must be true in order for such a neutrino to be detected? What would be detected?

21) Neutrino energy reconstruction

- Derive the expression for the neutrino energy in a charged-current quasi-elastic interaction, $\nu + n \rightarrow \mu^- + p$, in terms of the muon energy and angle (in the lab frame, with respect to the incoming neutrino) and the particle masses. Assume the neutron is free and at rest and the neutrino is massless.
- In a real experiment, the neutrino interactions are with nucleons bound in nuclei. What are the effects of this to the neutrino energy measurement, compared to the case above? Estimate the magnitude of these effects, for the case of measuring neutrinos in the T2K beam at Super-Kamiokande.

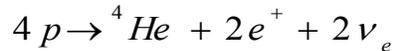
Neutrinos and Astrophysics

22) Demonstrate that the threshold for delta production for a proton hitting a target of photons

$$E_p = \frac{m_\Delta^2 - m_p^2}{4E_\gamma}$$

Then calculate the energy threshold of protons on the cosmic microwave background.

23) The Sun converts protons into He according to the reaction:



The solar constant describing the power of the Sun at Earth is $P = 1400 \text{ W/m}^2$. How many solar neutrinos arrive at Earth?

24) This problem is on the radiation exposure due to solar neutrinos. Use the result of Question 2 to evaluate:

a) The number of interactions of solar neutrinos in a typical human body (assume its density is $\rho = 1 \text{ g/cm}^3$ and its weight 80 kg; use $N_A = 6.022 \times 10^{23} \text{ g}^{-1}$ = Avogadro number; and the value of the cross section $\sigma(\nu_e N) \approx 10^{-45} \text{ cm}^2 / \text{nucleon}$ in the energy range for solar neutrinos for the dominant

interaction neutrino-nucleon scattering $\nu_e + N \rightarrow e^- + N'$);

b) The radiation damage is caused by electrons. The equivalent dose is defined as:

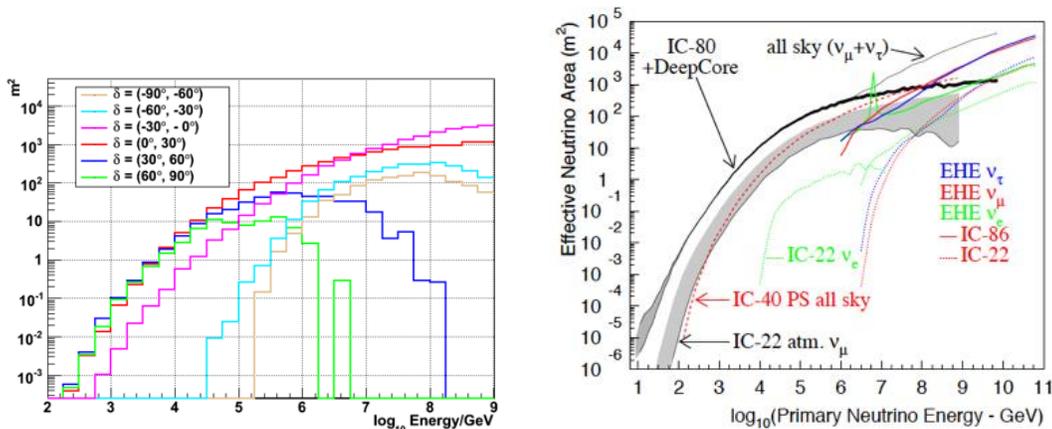
$$H = \left(\frac{\Delta E}{m} \right) w_R$$

where m is the mass of the human body, w_R the radiation weighting factor (=1 for electrons), ΔE the energy deposit in the human body and the units for H is defined as 1 Sievert = 0.01 J to be absorbed per kg. Work out the annual equivalent dose due to solar neutrinos for a typical man of 80 kg and compare it with the normal natural dose $H_0 = 2 \text{ mSv}$. Assume that on average 50% of the neutrino energy is transferred to the electron and take a typical energy for solar neutrinos of 200 keV.

Open-Ended 25)

- A. Consider one of the TeV sources in <http://tevcat.uchicago.edu/> that you think could be a good candidate neutrino emitter and that is in the field of view of IceCube (see plot below). Describe the kind of source you picked up and its main features. Describe relevant observations and measurements by gamma-ray and other photon experiments that make the source an interesting neutrino emitter, for instance the spectral emission distribution and fits of this curve to hadronic or leptonic models, the flaring properties.
- B. Investigate if there are existing models on hadronic emission. This can help you to answer the question if the dominant process for hadronic production can be proton-proton or proton-gamma or make your own motivated guess. Then, prediction the ratio of neutrino to photon fluxes at earth so that you may derive the neutrino flux from the measured gamma flux. (You can assume that the gammas are not absorbed and ignore the cascading process in the source and use their observed spectral index in the TeV range. For neutrinos use the spectral index typical of Fermi 1st order acceleration, E^{-2} . Don't worry about getting every detail right in your calculation; just make reasonable assumptions and state what they are.)
Once you've made your best estimate, you may find full calculations for the p+ γ and p+p cases in <http://prd.aps.org/abstract/PRD/v78/i3/e034013> and <http://arxiv.org/pdf/astro-ph/0606058>, respectively.
- C. Using the given effective area below, derive the muon-neutrino event rate per year for the estimated neutrino flux from that source. [Do not forget oscillations in your calculation.]

(Left) IC40 effective area vs. energy for different angles. (Right) Effective area vs. Energy for different neutrino sources



Neutrino Charge

Open-Ended

26) As we learned on the first day, one could write down a variant of the Standard Model where the neutrino has a small electric charge (a small fraction of the electron's charge). What would be some consequences of this charge? Can you derive any empirical limits on that electric charge from existing experimental data, astrophysical observations, or thought experiments?

Relic Neutrinos

27) Derive:

- A) The present number density of a massless species of neutrinos in cm^{-3} . How does this change if the neutrino has a mass?
- B) The contribution of the known neutrinos to the present density of the Universe. What if the neutrinos had a degenerate mass spectrum with each neutrino more massive than current limits would allow? Derive a limit on such a mass from the requirement that $\Omega_\nu < 1$. Derive a limit from $\Omega_\nu < \Omega_m$.
- C) The decoupling temperature of relic neutrinos. What would T_ν/T_V become if the coupling constant of $SU(2)_L$ were increased by a factor of two?
- D) The increase in the number density of relic neutrinos in a model with one extra light scalar which only couples to the Standard Model particles via a Z' of mass 1000 TeV with the same coupling constants as the Standard Model Z . Express this increase as an increase in the effective number of neutrinos.