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# Complete Next-to-Leading-Order QCD corrections to ZZ production through gluon fusion

*LHC Higgs Working Group off-shell meeting 2024*

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*Based on https://arxiv.org/abs/2011.15113 and https://arxiv.org/abs/2404.05684*

# Motivation

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## Precision measurements:

Background to Higgs production through gluon fusion [CMS 2018] [ATLAS 2020]

## Higgs Width:

Indirect constraints on Higgs width through off-shell Higgs production [ATLAS 2018] [CMS 2019] [Caola, Melnikov 2013] [Campbell, Ellis, Williams 2013]

## BSM searches:

Searches for heavy diboson resonances decaying to 4 lepton final states [ATLAS 2020] [CMS 2023]

## Anomalous couplings:

Constrain anomalous $t\bar{t}Z$, triple gauge couplings [ATLAS 2023]

# Motivation

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## $gg \to ZZ$ at the LHC:

Loop induced; formally NNLO for $pp \to ZZ$ (starting at $O(\alpha_S^2)$)

Large contribution due to high gluon luminosity; $\sim 60\,\%$ of the total NNLO correction [Cascioli, Gehrmann, Grazzini, Kallweit, Maierhöfer, von Manteuffel, Pozzorini, Rathlev, Tancredi, Weihs (2014)]

$gg \to ZZ$ at NLO (massless quarks in the loop) increases total $pp \to ZZ$ by ~ 5%
[Grazzini, Kallweit, Wiesemann, Yook (2018)]

Top quark effects expected to be significant, especially for longitudinal modes due to Goldstone boson equivalence theorem

$\Longrightarrow$ Need a full NLO calculation

# NLO Calculation

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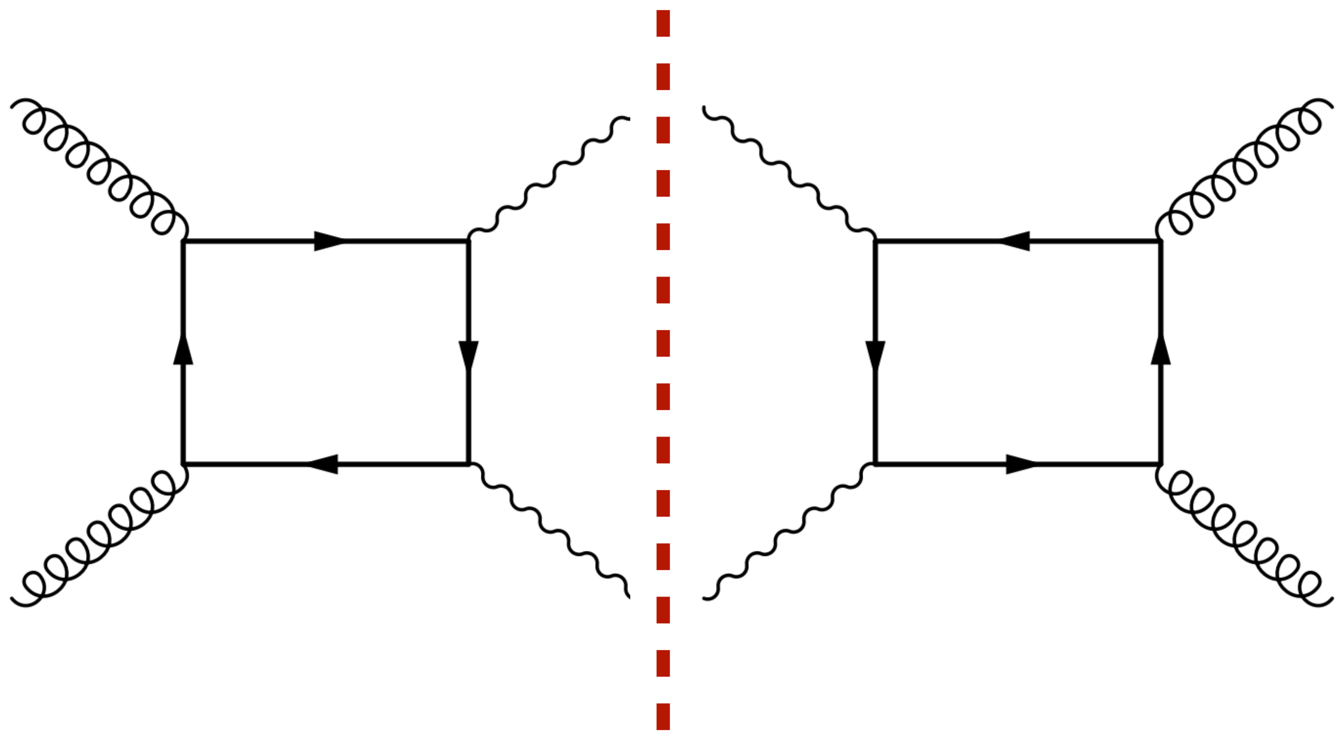
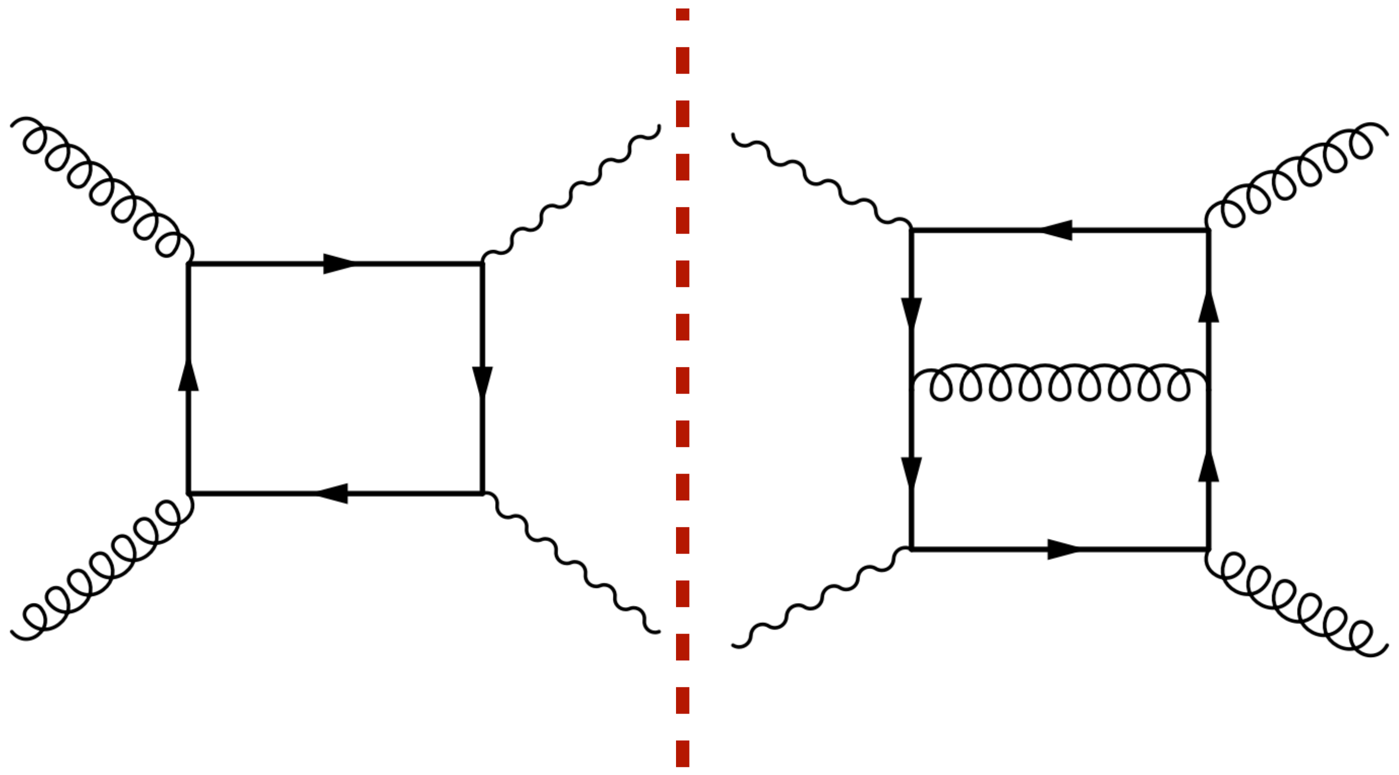
## Next-to-Leading Order cross-section:

$$\mathrm{d}\sigma_{NLO} = \mathrm{d}\sigma_B + \mathrm{d}\sigma_V + \mathrm{d}\sigma_R$$



Born

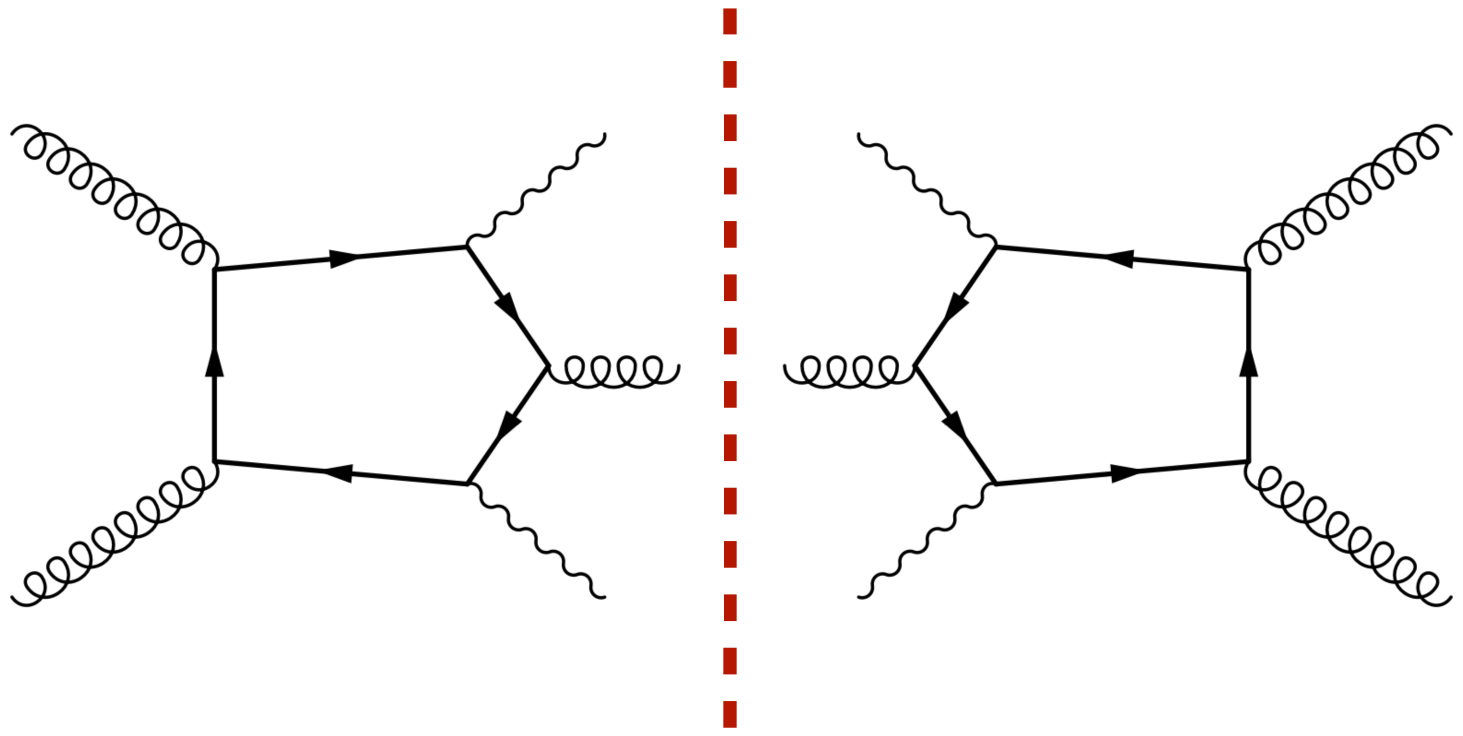
$2 \to 2$ amplitude
at 1-loop



Virtuals

$2 \to 2$ amplitude
at 2-loops

Conceptually
challenging



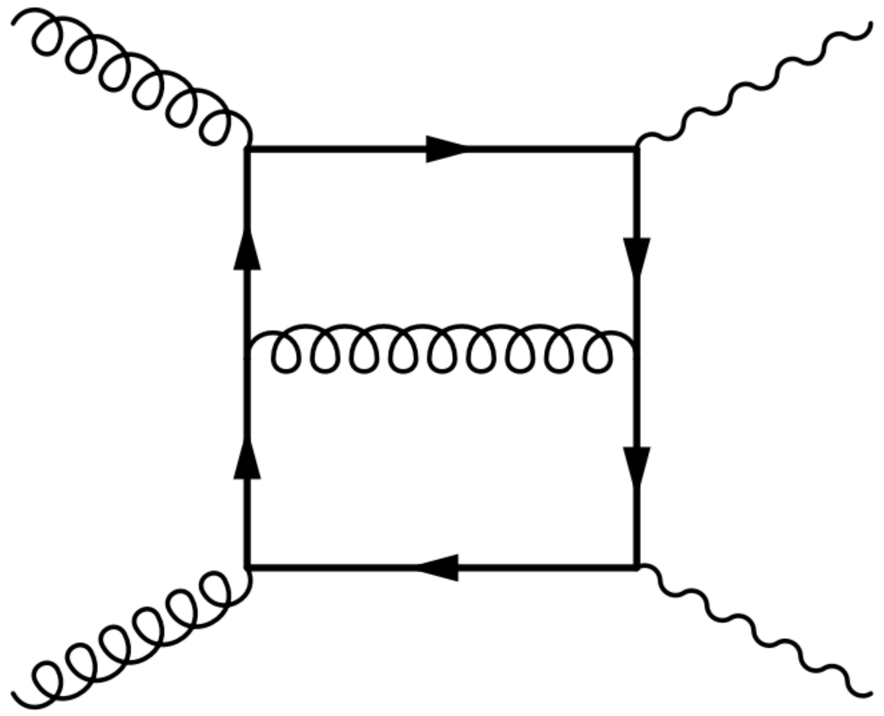
Reals

$2 \to 3$ amplitude
at 1-loop

Numerically
challenging

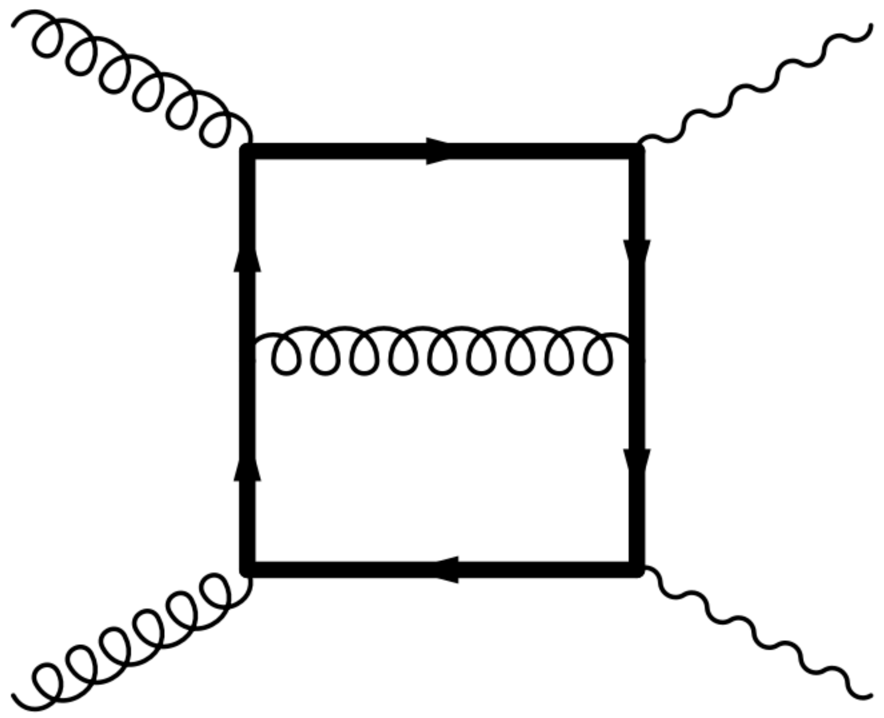
# Two-loop Amplitude





## Massless quarks $(\mathbf{A_l})$

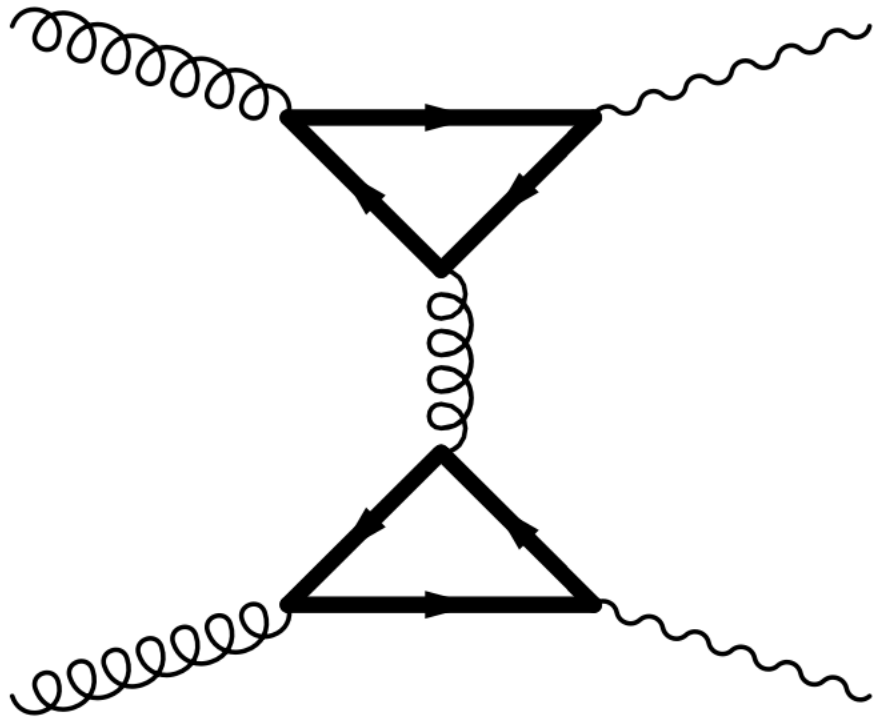
[von Manteuffel, Tancredi (2015)]
[Caola, Henn, Melnikov, Smirnov, Smirnov (2015)]



## Massive $(\mathbf{A_h})$

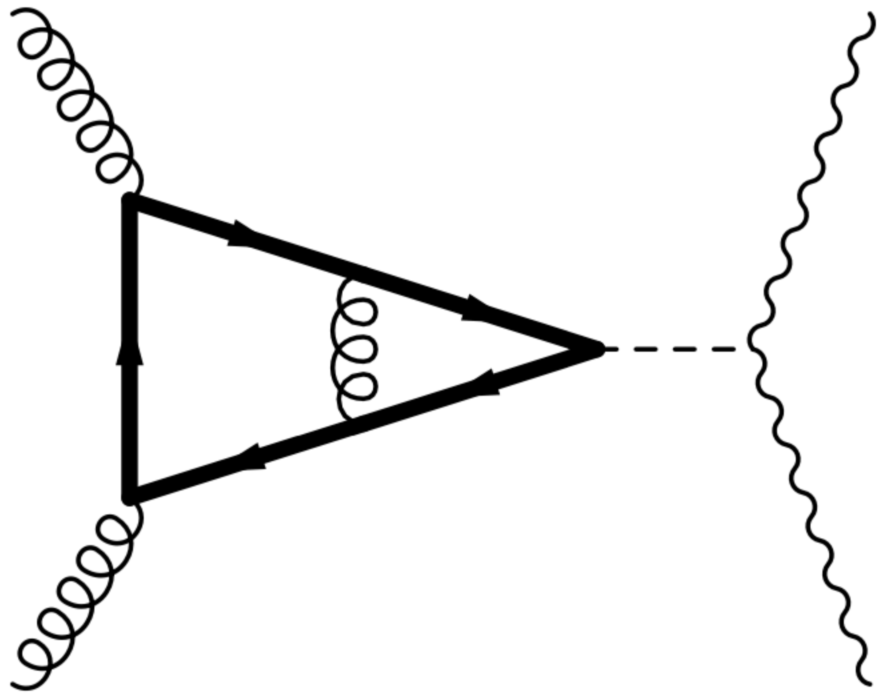
**[BA, Jones, von Manteuffel (2020)]**
[Brønnum-Hansen, Wang (2021)]

And for various expansions: [Melnikov, Dowling (2015)] [Caola et al (2016)] [Cambell, Ellis, Czakon, Kirchner (2016)] [Gröber, Maier, Rauh (2019)] [Davies, Mishima, Steinhauser, Wellmann (2020)] [DeGrassi, Gröber, Vitti (2024)]



## Anomaly type $(\mathbf{B})$

[Kniehl, Kühn (1990)] [Cambell, Ellis, Zanderighi (2007)] [Cambell, Ellis, Czakon, Kirchner (2016)]



## Higgs mediated $(\mathbf{C})$

[Spira et al (1995)] [Harlander & Kant (2005)] [Anastasiou et al (2006)] [Bonciani et al (2006)]

# Results

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Write the UV and IR finite amplitudes (after UV renormalisation and IR subtraction respectively) as:

$$\mathcal{M}^{fin}_{\lambda_1\lambda_2\lambda_3\lambda_4} = \left(\frac{\alpha_S}{2\pi}\right)\mathcal{M}^{(1)}_{\lambda_1\lambda_2\lambda_3\lambda_4} + \left(\frac{\alpha_S}{2\pi}\right)^2\mathcal{M}^{(2)}_{\lambda_1\lambda_2\lambda_3\lambda_4} + O\left(\alpha_S\right)^3$$

Define 1-loop squared and interference between 1-loop and 2-loop amplitudes:

$$\mathcal{V}^{(1)}_{\lambda_1\lambda_2\lambda_3\lambda_4} = |\mathcal{M}^{(1)}_{\lambda_1\lambda_2\lambda_3\lambda_4}|^2$$

$$\mathcal{V}^{(2)}_{\lambda_1\lambda_2\lambda_3\lambda_4} = 2\,\mathrm{Re}\left(\mathcal{M}^{*(1)}_{\lambda_1\lambda_2\lambda_3\lambda_4}\mathcal{M}^{(2)}_{\lambda_1\lambda_2\lambda_3\lambda_4}\right)$$

Note that in the following results, only the pure top-quark contributions are included (i.e. no Higgs mediated diagrams or massless internal quarks)

# Numerical Evaluation

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## Integration strategy

Helicity amplitudes $\mathscr{M}^{(2)}_{\lambda_1\lambda_2\lambda_3\lambda_4}$ written as a linear combination of $\sim O(10^4)$ integrals after sector decomposition i.e. each sector of a master integral is considered and evaluated separately

Number of evaluations for each integral set dynamically to minimise the evaluation time for $\mathscr{M}^{(2)}_{\lambda_1\lambda_2\lambda_3\lambda_4}$ instead of each integral [Borowka et al (2016)]

$$T = \Sigma\, t_i \;+\; \lambda\,(\sigma^2 - \Sigma_i\, \sigma_i^2)$$

$T$ : Total integration time
$t_j$ : Integration time for integral $j$
$\sigma$ : Required precision
$\sigma_i$ : Estimated precision for integral $i$
$\lambda$ : Lagrange Multiplier

Quasi-Monte Carlo algorithm for quadrature [Li, Wang, Zhao (2015)] [Borowka et al (2017)]

Request per-cent precision on each helicity amplitude (and ~10% on form factors $A_i$); much better precision obtained usually

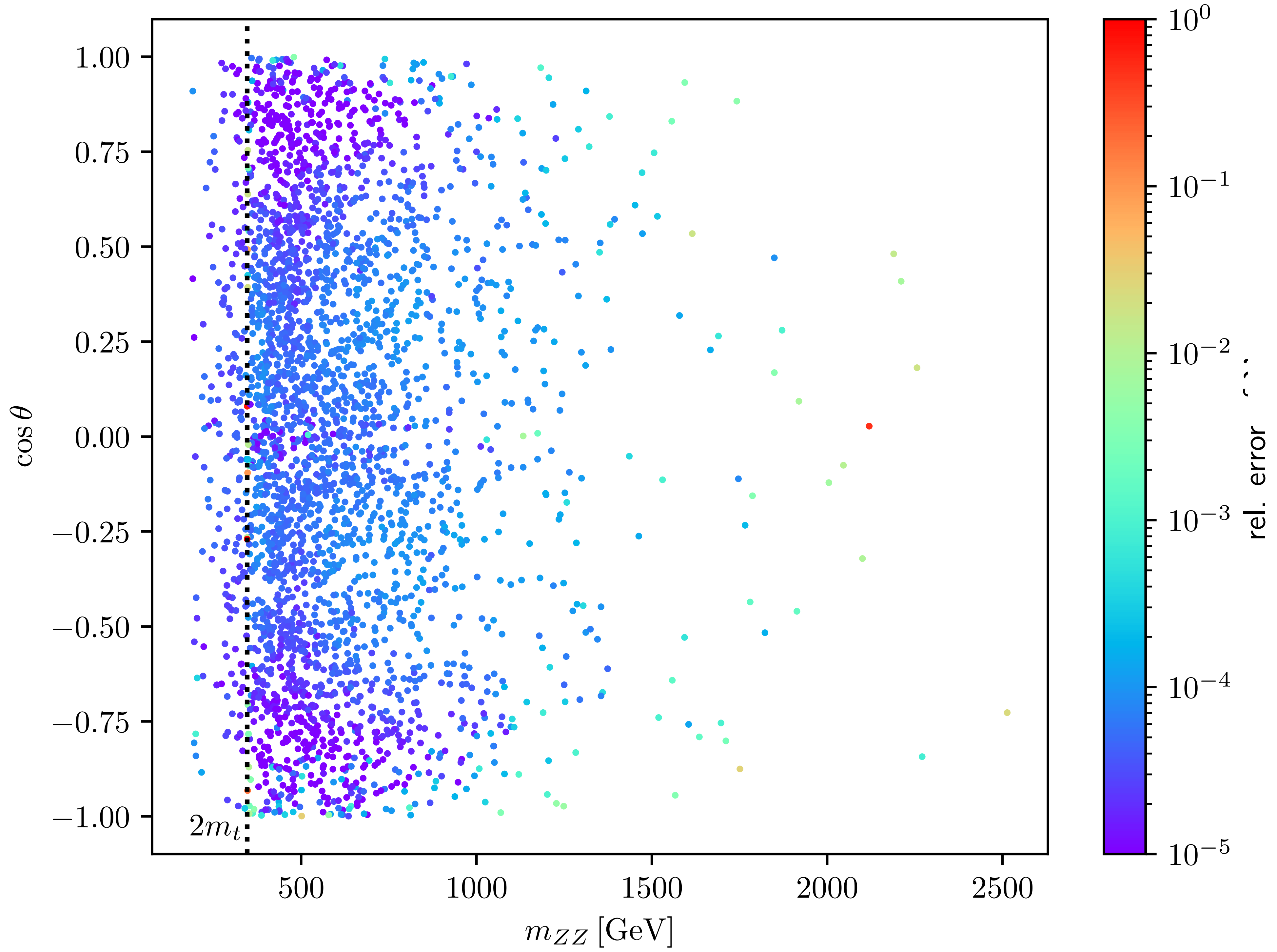
# Numerical Evaluation

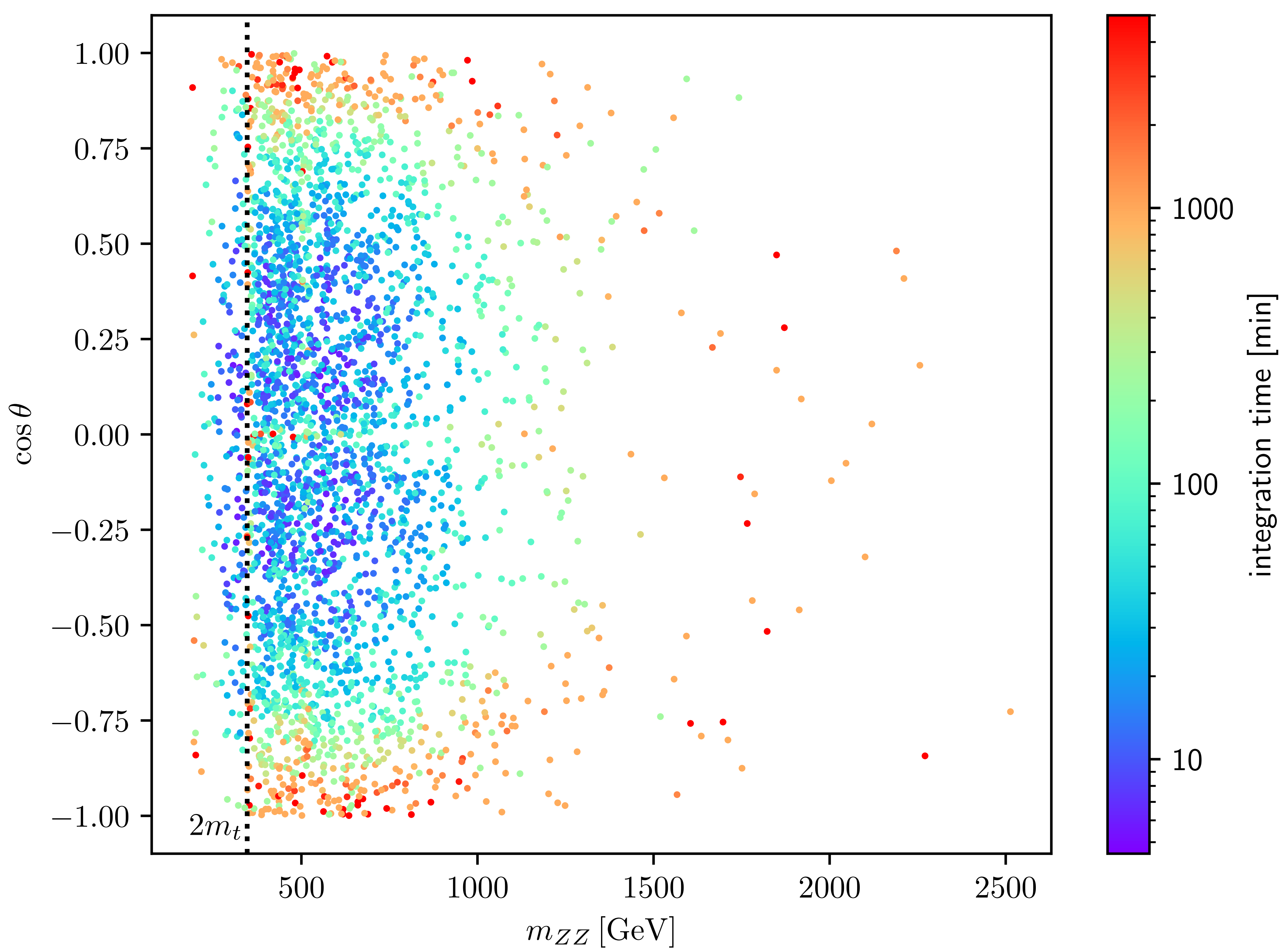
Use the born calculation (with only top quarks) to generate unweighted events to sample the virtual corrections (~3000 points)

Good numerical stability in most regions of phase space, in particular around the top-quark threshold

Runtimes in $O(10)$ min for large part of the phase space with expected difficulties for $|cos\theta| \sim 1$ (very small $p_T$)

Better than per-mille precision for most of the phase-space



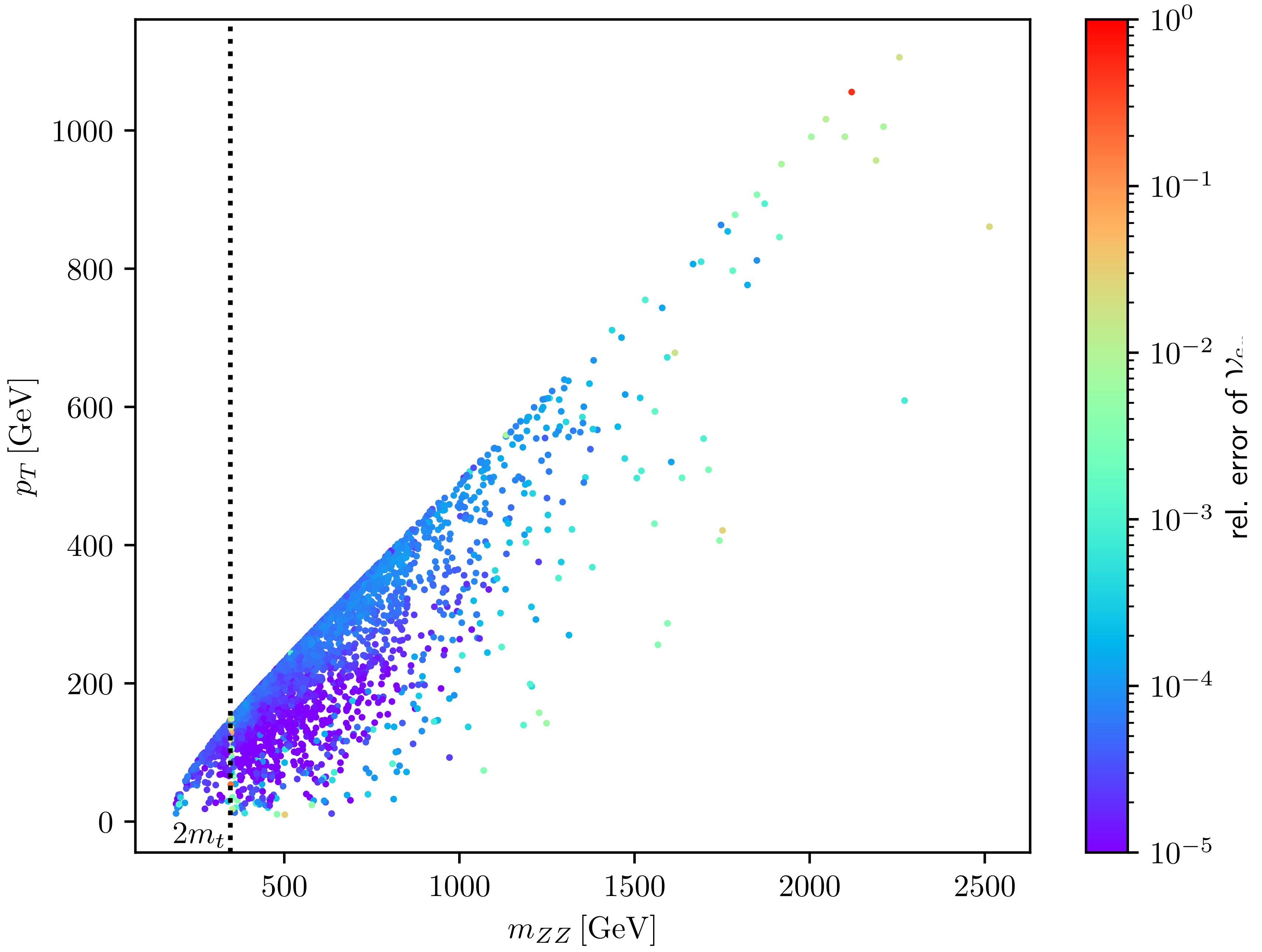
# Numerical Evaluation

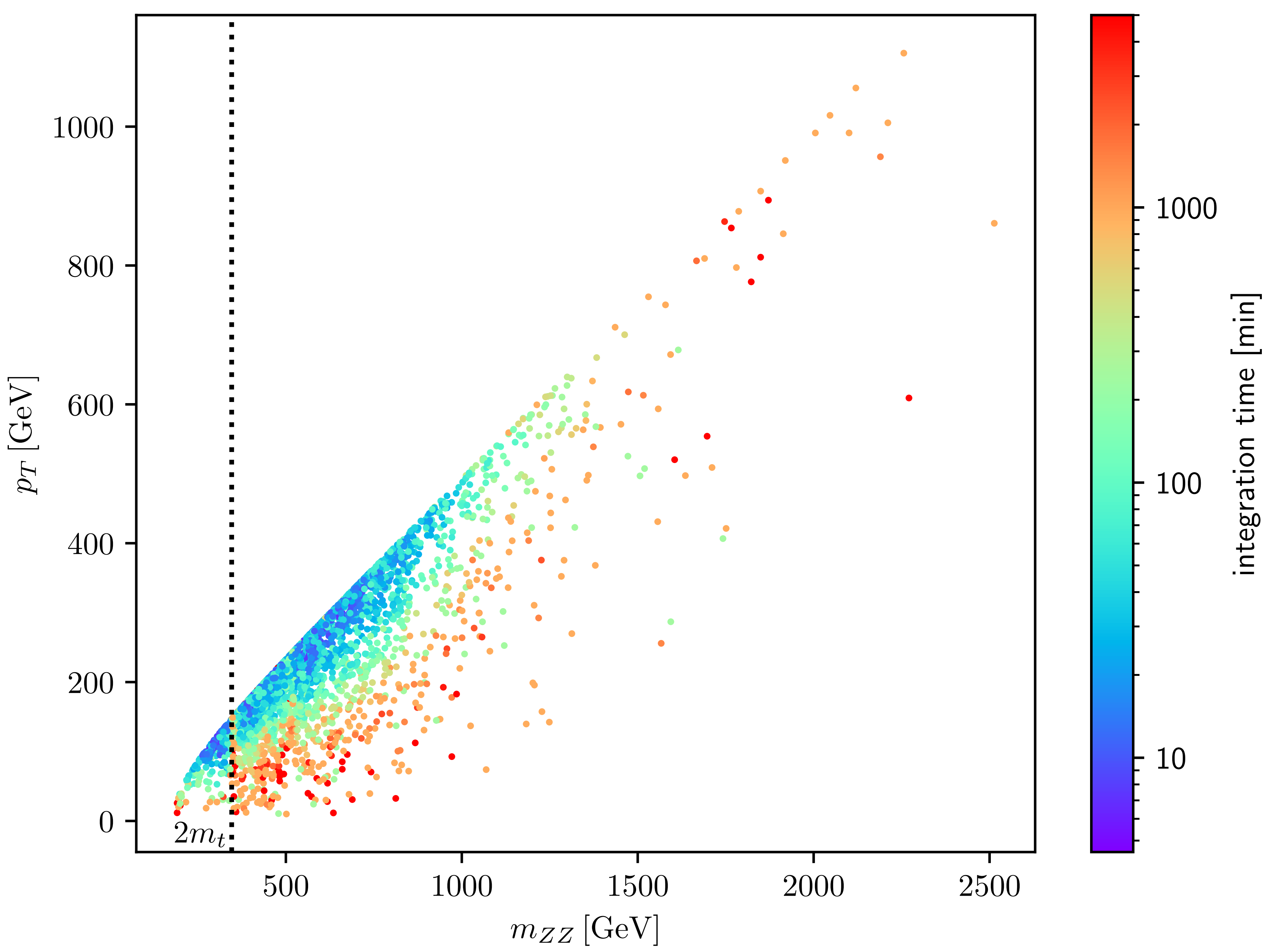
Good numerical stability in most regions of phase space, in particular around the top-quark threshold (except for small $p_T$)

Runtimes in $O(10)$ min for large part of the phase space with expected difficulties for very small $p_T$

Can access high energy and high $p_T$ region without much difficulty, but very high energy $\left(\sqrt{s} > 2\,TeV\right)$ challenging

Better than per-mille precision for most of the phase-space



# Comparison to expansions

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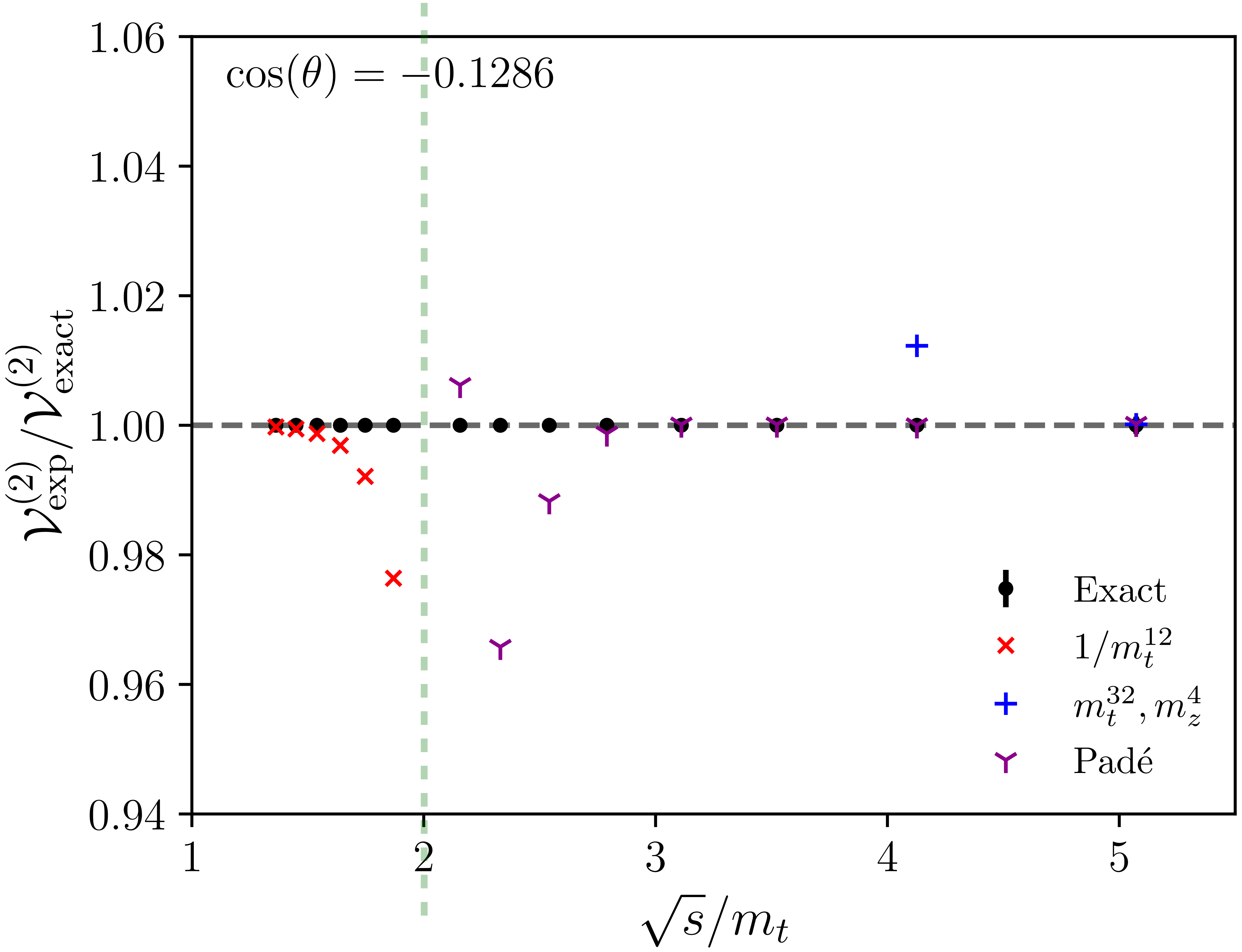


Comparison of $\sqrt{s}$ dependence of the unpolarised interference with expansion results at fixed $\cos\theta = -0.1286$. Exact results from **[BA, Jones, von Manteuffel (2020)]**. Expansion and Padé results from [Davies, Mishima, Steinhauser, Wellmann (2020)] (see also [Davies, Mishima, Schönwald, Steinhauser (2023)]). Error bars for the exact result are plotted but they are too small to be visible.

# Comparison to expansions

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For previous results, "$q_T$" subtraction scheme

Transformation between Catani's original scheme and $q_T$ scheme

$$A_i^{(2),fin,Catani} = A_i^{(2),fin,q_T} + \Delta I_1 A_i^{(1),fin}$$

$$\Delta I_1 = -\frac{1}{2}\pi^2 C_A + i\pi\beta_0 \quad \sim 15$$

For interference terms, 1-loop result multiplied by $\sim 30$ => Leads to a very different qualitative behaviour

**Relative comparisons highly dependent on IR scheme**

# Comparison to expansions

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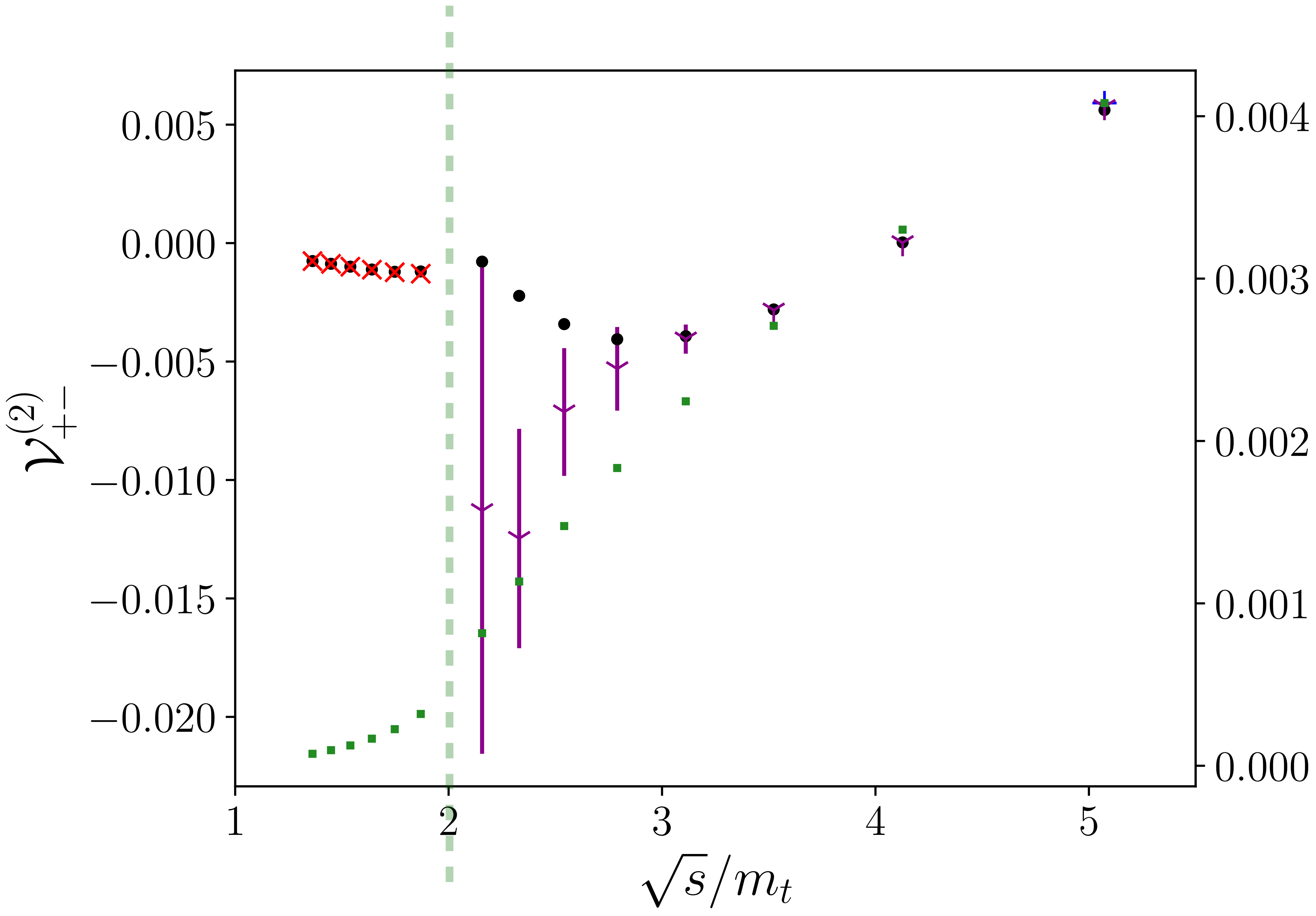
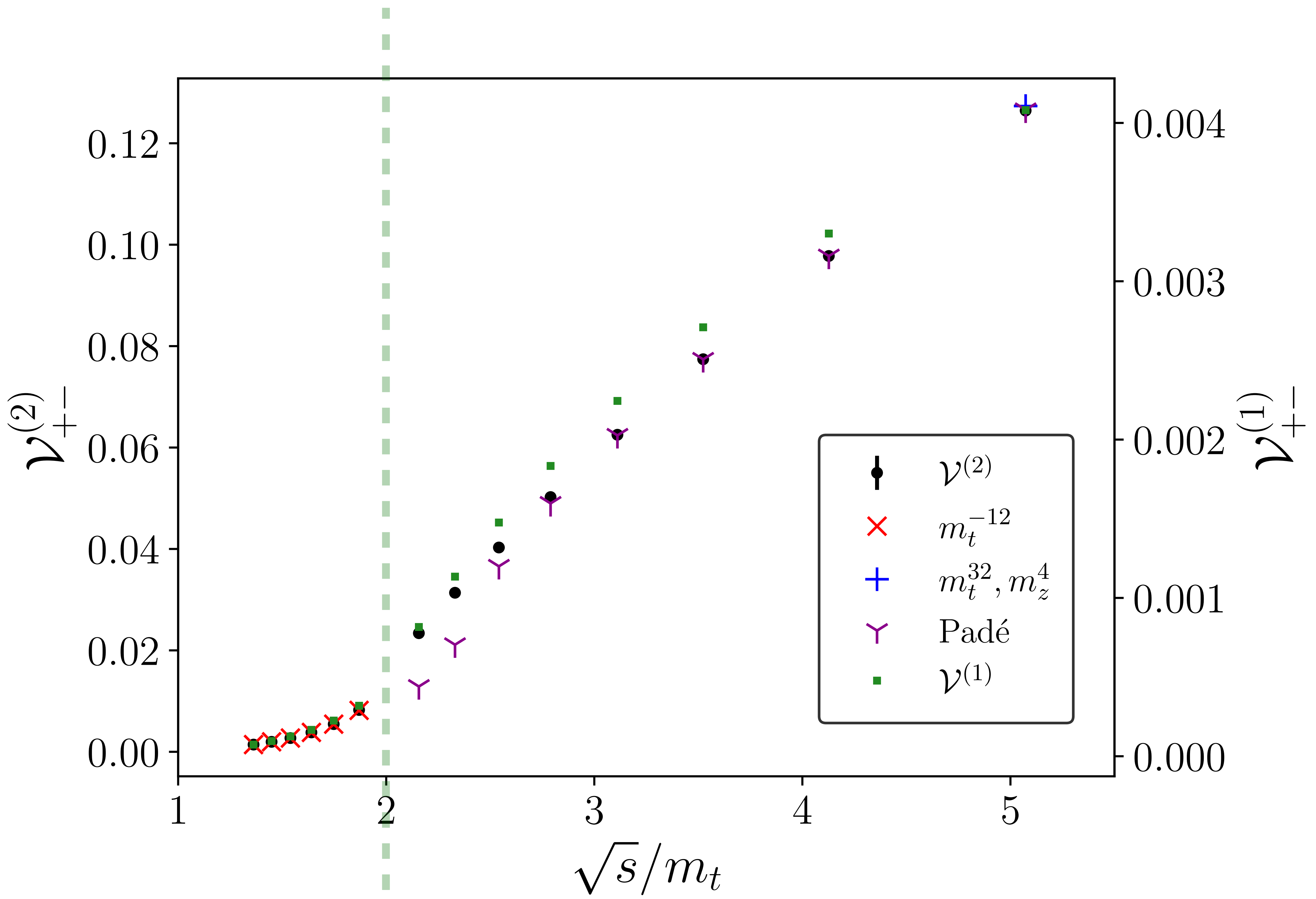



Traditional Catani Scheme

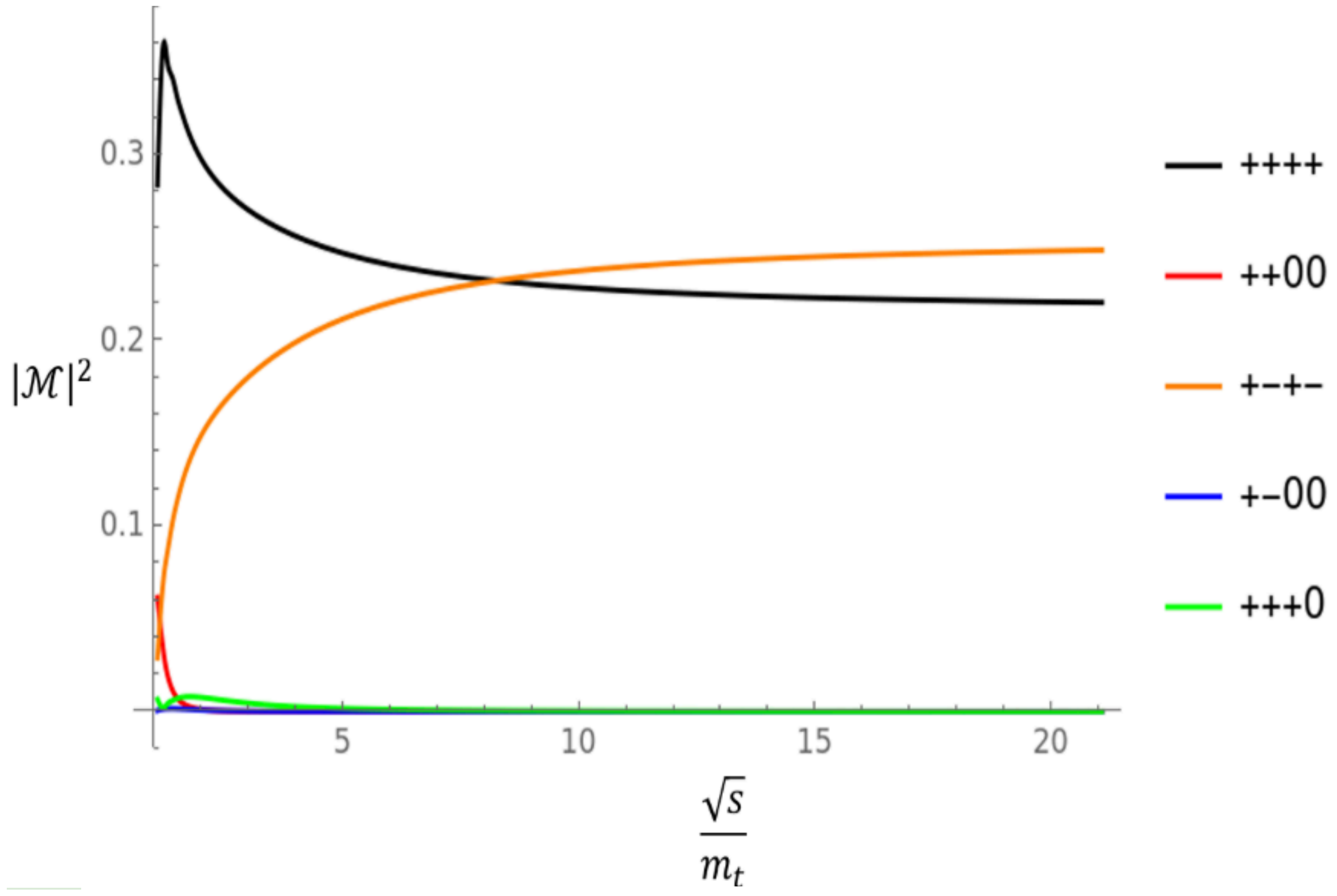



"$q_T$" scheme

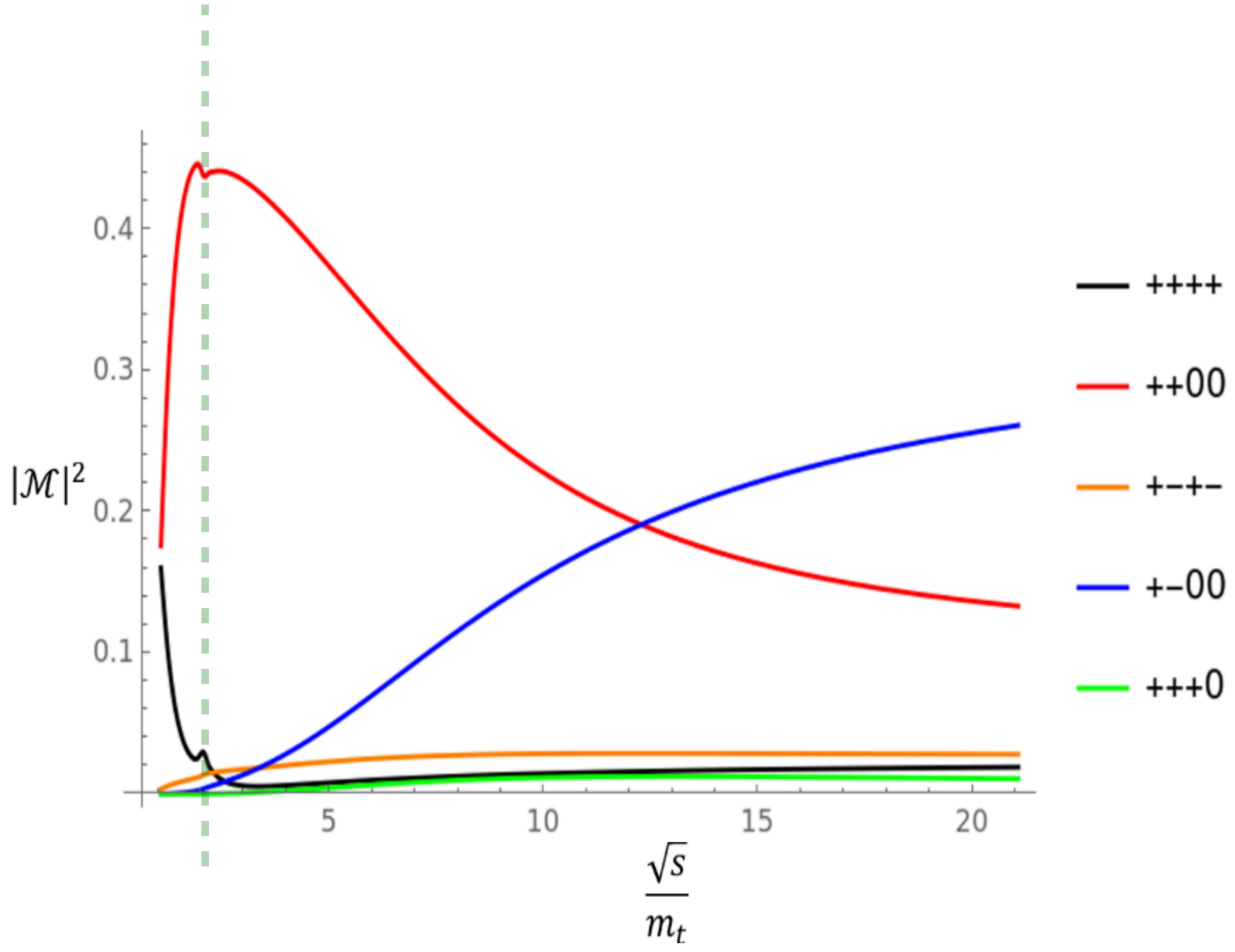
Comparison of $\sqrt{s}$ dependence of the polarised interference with expansion results at fixed $\cos\theta = -0.1286$. Exact results from **[BA, Jones, von Manteuffel (2020)]**. Expansion and Padé results from [Davies, Mishima, Steinhauser, Wellmann (2020)] (see also [Davies, Mishima, Schönwald, Steinhauser (2023)]).

# Higgs and Top quark






Comparison of Born $|\mathcal{M}|^2$ against $\sqrt{s}$ for different helicity contributions for massless quarks




Comparison of Born $|\mathcal{M}|^2$ against $\sqrt{s}$ for different helicity contributions for massive quarks and Higgs mediated diagrams

# Higgs and Top quark

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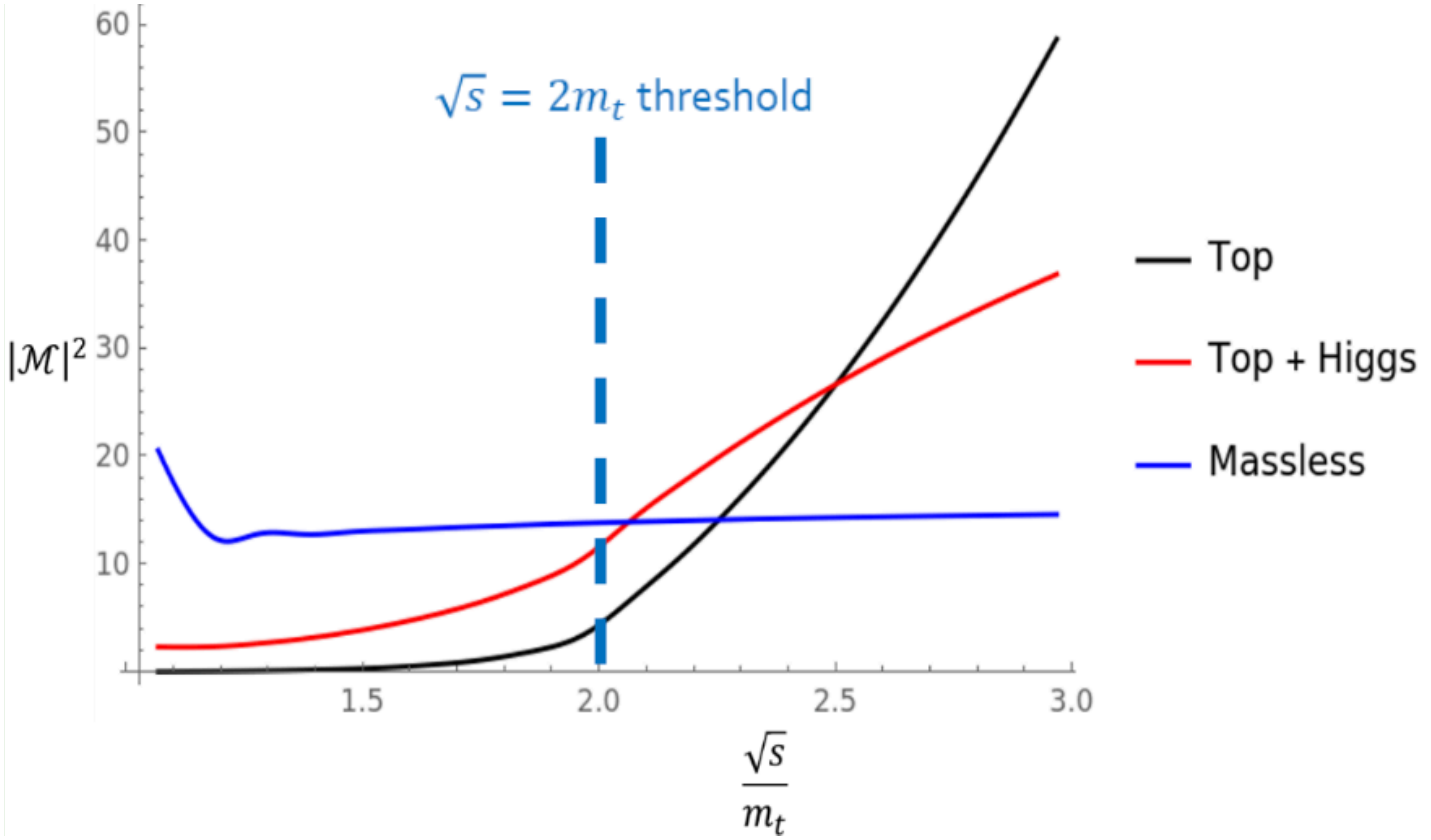
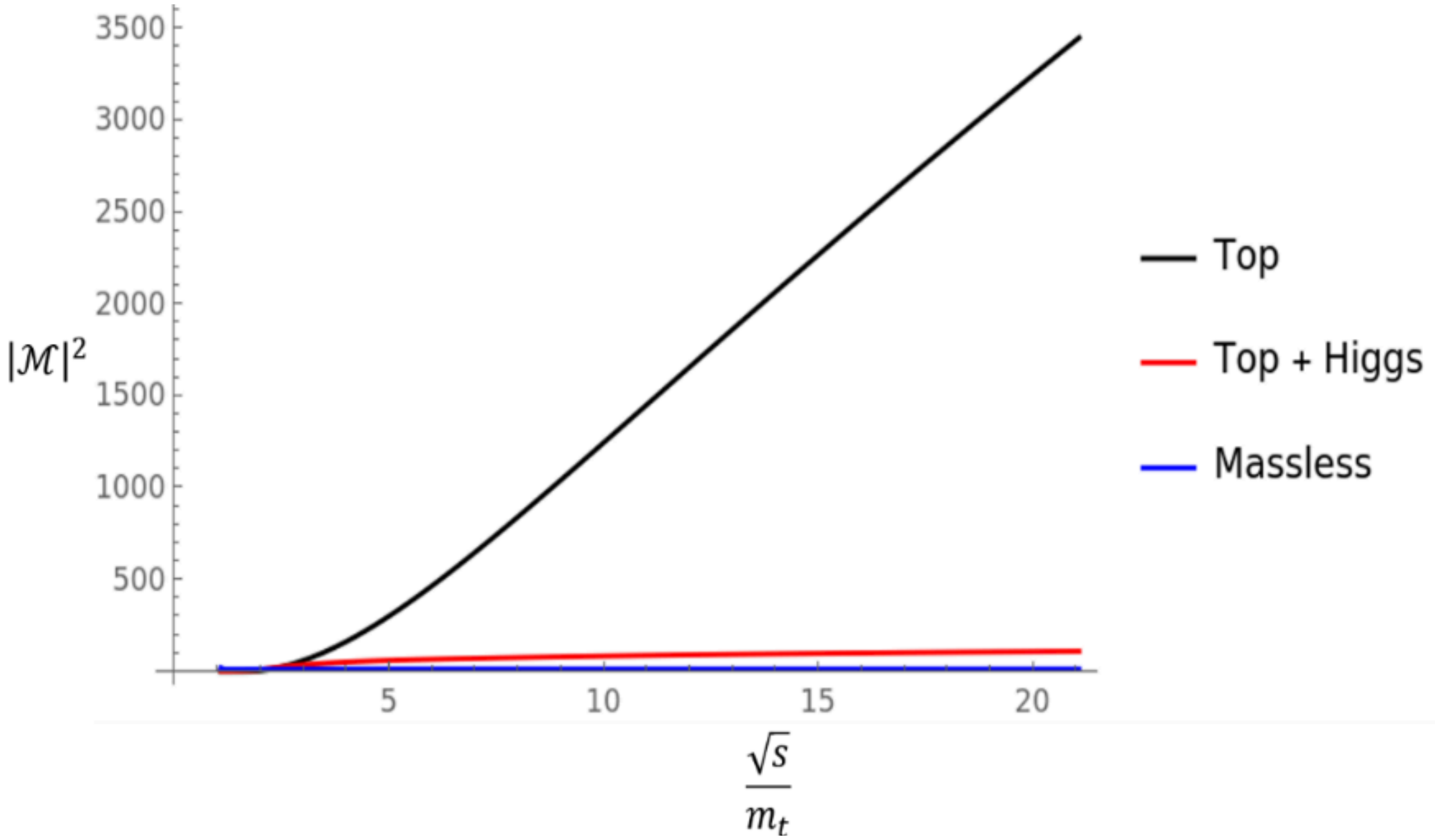
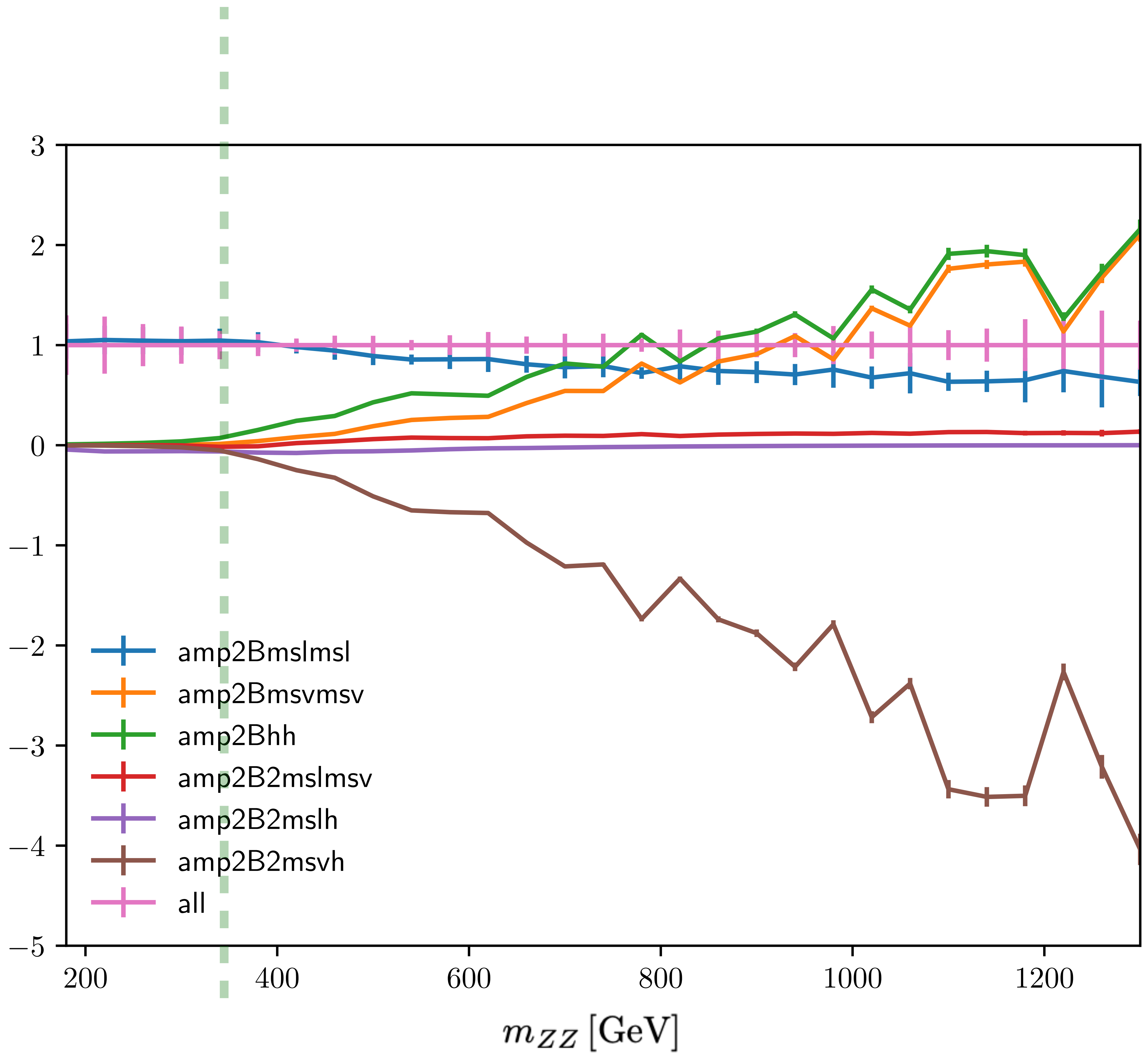



Comparison of Born $|\mathcal{M}|^2$ against $\sqrt{s}$ for different contributions

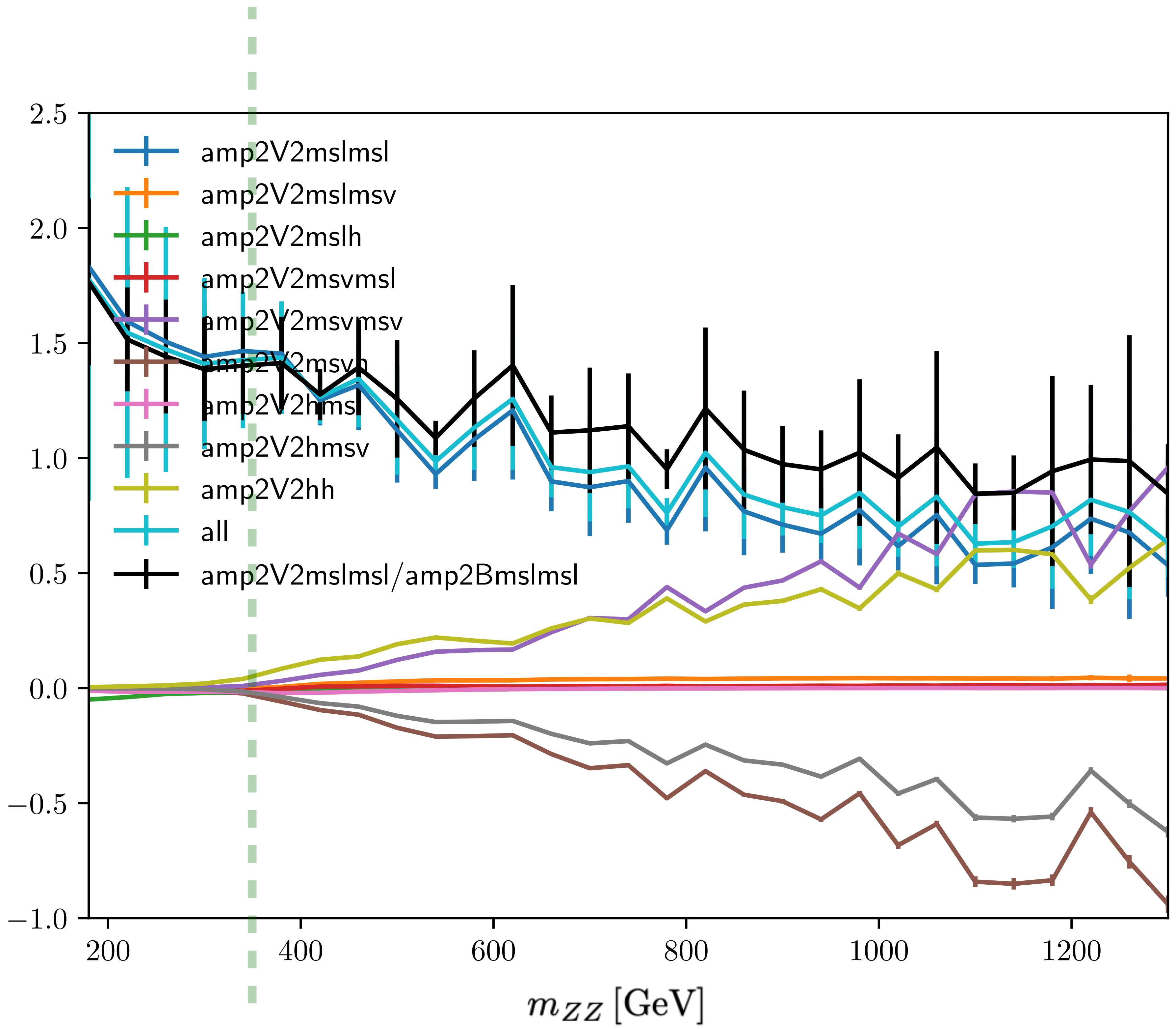



Comparison of Born $|\mathcal{M}|^2$ against $\sqrt{s}$ for different contributions at very high energies

# Higgs and Top quark




Comparison of ratios of different interferences (normalised to full) at 1-loop level against $m_{ZZ}$




Comparison of ratios of different interferences (normalised to full) at 2-loop level against $m_{ZZ}$

**Delicate cancellations between top-only and Higgs mediated contributions**

# Results: Complete NLO Corrections

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Top-only contributions:

$$\sigma_{\text{LO}}^{\text{A}_h} = 19.00^{+29.4\%}_{-21.4\%}\ \text{fb}$$

$$\sigma_{\text{NLO}}^{\text{A}_h} = 34.46(6)^{+16.4\%}_{-14.4\%}\ \text{fb}$$

Including all contributions:

$$\sigma_{\text{LO}} = 1316^{+23.0\%}_{-18.0\%}\ \text{fb}$$

$$\sigma_{\text{NLO}} = 2275(12)^{+14.0\%}_{-12.0\%}\ \text{fb}$$

(Number in parentheses indicates the Monte-carlo error)

~2% decrease in full NLO cross-section after including top quark and Higgs contributions

# Results: Complete NLO Corrections

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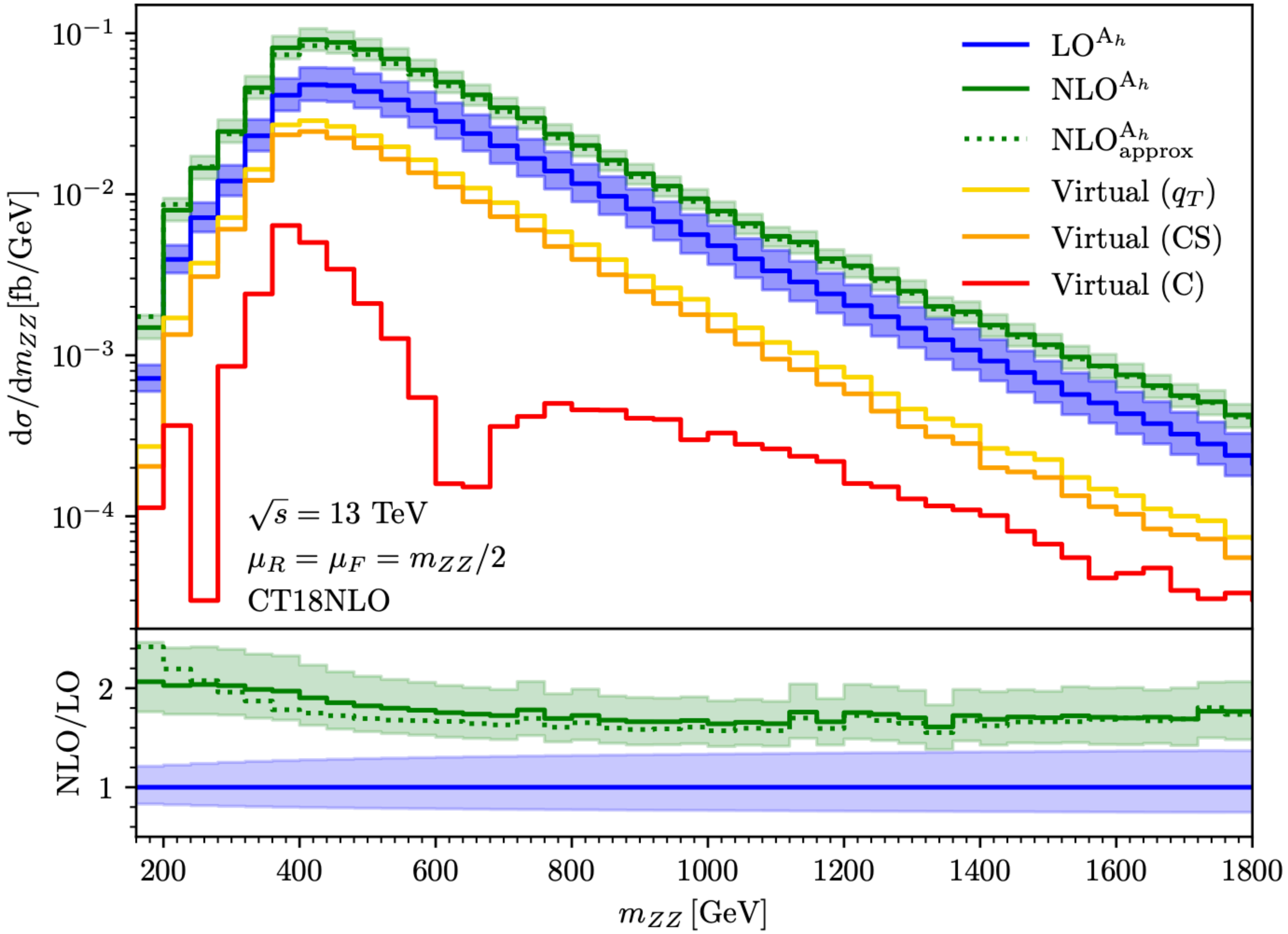


Top-quark-only contributions to the ZZ invariant mass distribution in pp collisions. The absolute value of the two-loop virtual correction is shown separately in the qT , Catani-Seymour (CS), and Catani (C) schemes. The dashed curve represents an approximate NLO result obtained by rescaling the massive Born amplitude with the massless K-factor.
Plot from **[BA, Jones, Kerner, von Manteuffel (2024)]**

# Results: Complete NLO Corrections

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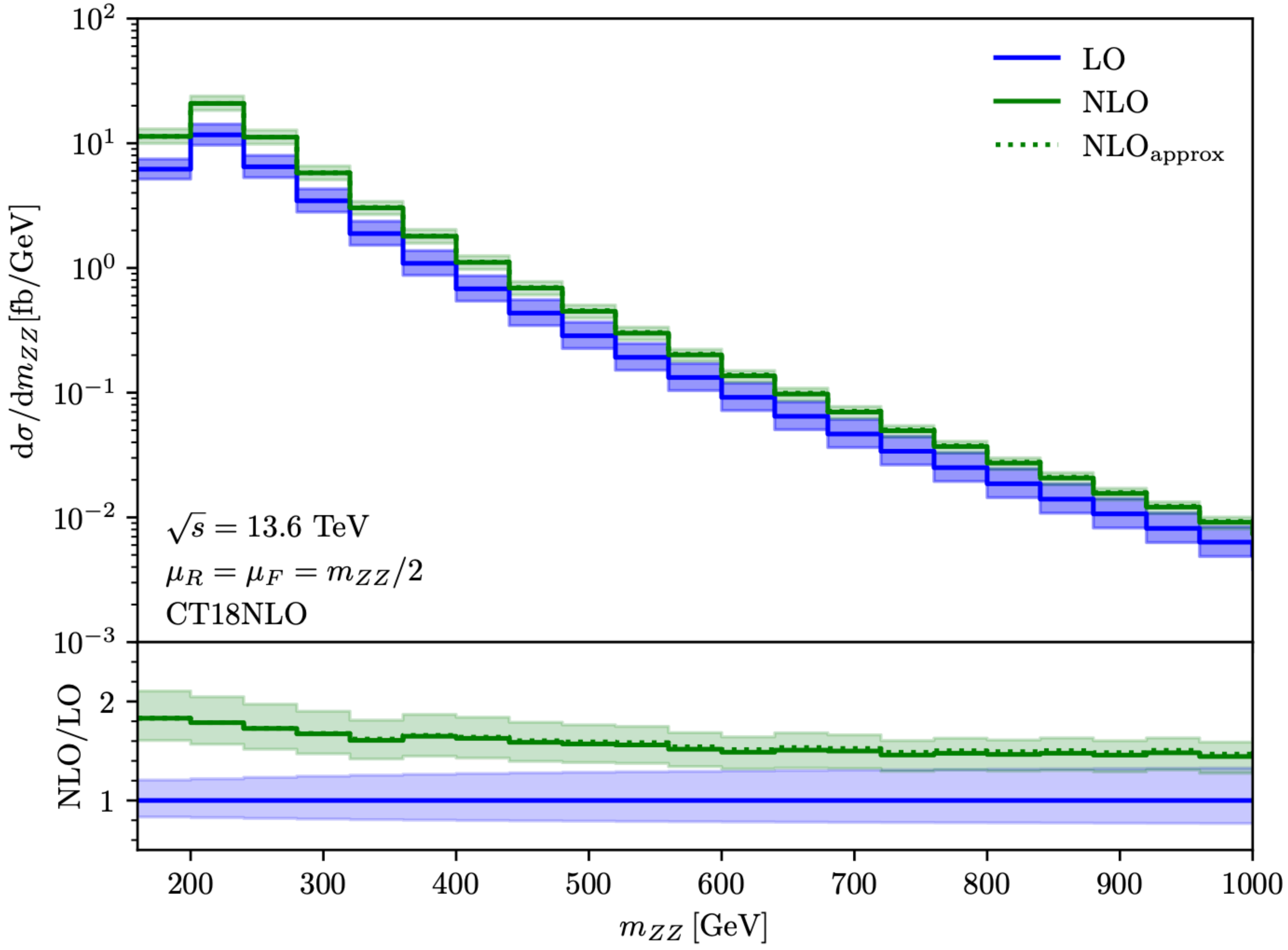


Diboson invariant mass distribution for gluon- initiated ZZ production at the LHC. The Solid curves represent the LO and NLO results with complete massless and massive contributions, including Higgs-mediated diagrams. The dashed curve represents an approximate NLO result obtained as described in the text. Plot from **[BA, Jones, Kerner, von Manteuffel (2024)]**

# Top mass scheme uncertainty

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We can estimate the mass uncertainty by comparing the numbers between on-shell and $\overline{\text{MS}}$ schemes. For $\overline{\text{MS}}$ scheme, we use $m_t(2m_t^{\text{OS}}) = 154.6 \text{ GeV}$

At Leading Order:

$$\sigma_{LO}^{OS} = 18.97 \text{ fb}$$

$$\sigma_{LO}^{\overline{MS}} = 20.62 \text{ fb} \qquad \Longrightarrow \sim 9\,\% \text{ increase}$$

At NLO, we can estimate the uncertainty by varying everything except the bare 2-loop amplitudes, which are not available with symbolic top mass dependence.

However, the impact of these finite 2-loop amplitudes can be reduced by working in Catani scheme to get a better estimate.

# Top mass scheme uncertainty

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Significant reduction in the mass scheme uncertainty at NLO (similar to the effect observed in $gg \to hh$)

| | OS | MS | % Change |
|---|---|---|---|
| **Born** | 18.97 | 20.62 | **8.7** |
| **Reals (Catani)** | 14.89 | 16.33 | |
| **Virtuals (Catani)** | 0.59 | -1.32 | |
| **Reals (qT)** | 5.80 | 6.22 | |
| **Virtuals (qT)** | 9.65 | 8.64 | |
| **NLO (Catani)** | 34.48 | 35.90 | **4.1** |
| **NLO (qT)** | 34.45 | 35.75 | **3.7** |

Difference between the two schemes gives an estimate of the “correctness” of our mass scheme uncertainty

# Conclusions

Efficient integration strategy using sector decomposition to minimise the total integration time; able to get good statistics for distributions

Numerically very stable in most regions of phase-space, even close to top-quark pair production threshold, at high invariant mass and forward scattering

Significant top-quark only corrections (~100%)

Great impact due to the choice of IR scheme on virtual (and reals)

Existing approximations based on rescaling the massive Born by massless k-factor quite good for unpolarised cross-section

Extreme cancellations between Higgs and Top-quark contributions; sensitive to exact SM couplings

Estimate of top-mass uncertainty; difference between Catani and qT schemes assesses the stability of the estimate