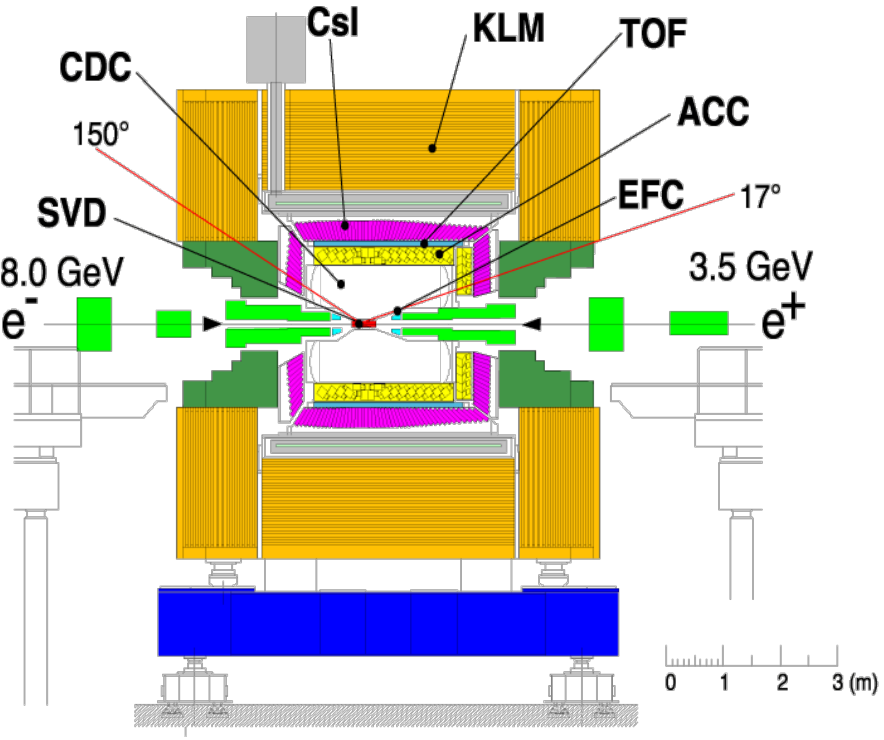
Belle 2π spectral function in τ lepton decay



Theory Initiative g-2 tau mini-workshop

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2π spectral function $v_{-}(s)$ in τ decay

2π spectral function $v_{-}(s)$ is defined by the differential width of τ lepton 2π decay $d\Gamma/ds$,

$$\frac{d\Gamma(\tau^{-}\to\pi^{-}\pi^{0}\nu_{\tau})}{ds} = \Gamma_e^0 \cdot \frac{6\pi|V_{ud}|^2 S_{EW}^{\pi\pi}}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right) v_{-}(s),$$

where $s = m_{\pi^-\pi^0}^2$.

Experimentally, we can obtain a $v_{-}(s)$, or a pion form factor $|F_\pi^-(s)|^2$, using the measured mass spectrum $dN_{\pi\pi}/ds$ and the decay branching fractions $B_e, B_{\pi\pi}$. The values of other parameters, m_τ, V_{ud}, S_{EW}, are well known.

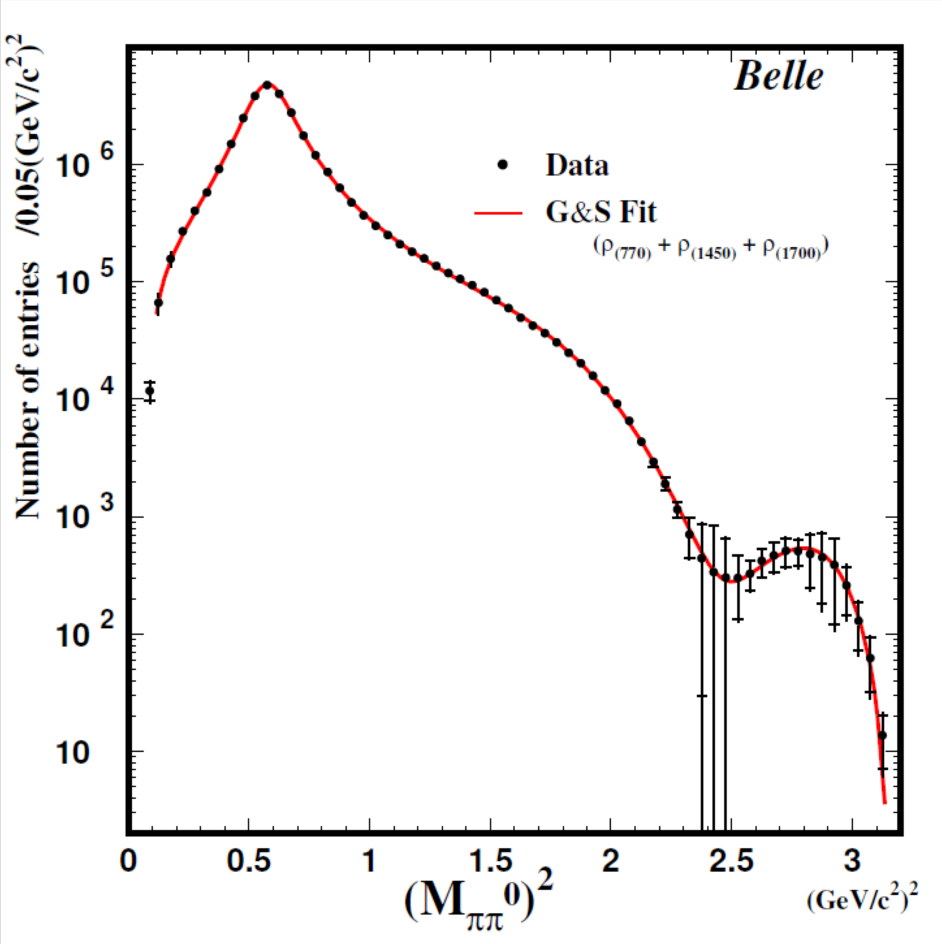
$$|F_\pi^-(s)|^2 = \frac{2m_\tau^2}{|V_{ud}|^2\left(1-\frac{s}{{m_\tau}^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)S_{EW}}\frac{1}{\beta_-^3}\left(\frac{B_{\pi\pi}}{B_e}\right)\left(\frac{1}{N_{\pi\pi}}\frac{dN_{\pi\pi}}{ds}\right)$$

$$v_{-}(s) = \frac{\beta_-^3}{12\pi}|F_\pi^-(s)|^2$$

High Statistic Study of the $\tau^- \to \pi^-\pi^0\nu_\tau$ Decay.

- In this talk, how 2π mass spectrum, $\frac{dN_{\pi\pi}}{ds}$, is obtained by Belle exp.

$N_{\pi\pi}/ds$



Phys. Rev. D78, 072006 (2008).

arxiv: 0805.3773.

Outline

- Simulation programs for ISR/FSR estimation
- Dara and Event Selection
- Background/Efficiency
- Mass resolution and Unfolding
- Systematic uncertainties
- Fit results

Simulation programs

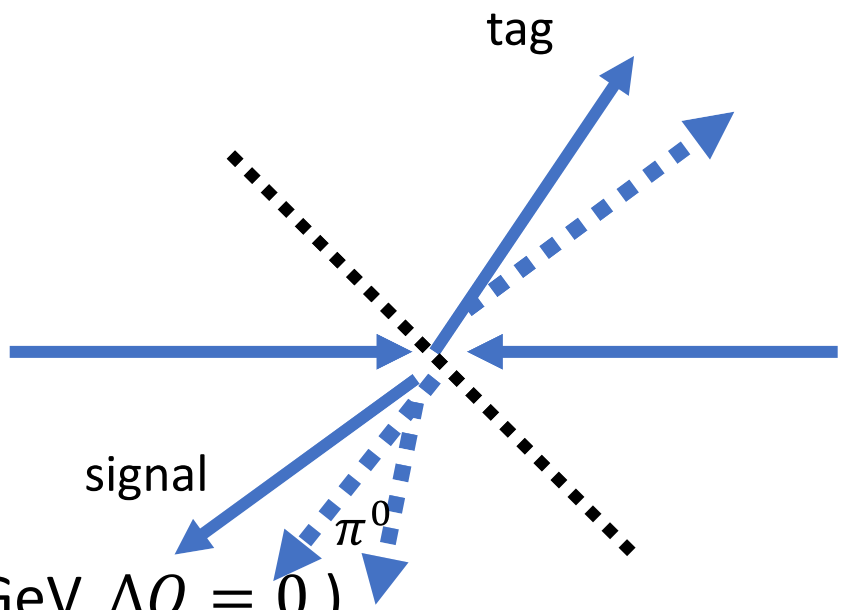
- Signal simulation
  - KKMC + TAUOLA+ PHOTOS
- $e^+ + e^- \rightarrow \tau^+ + \tau^-(\gamma) \rightarrow (\pi^\pm\pi^0\nu_\tau(\gamma))$+ (1 or 3 charged particles + nγ)

  KKMC: simulate ISR and FSRs of $e^+ + e^- \rightarrow \tau^+ + \tau^-(\gamma)$.

  TAUOLA: simulate τ $lepton$ decays.

  PHOTOS: simulate photon radiations from charged hadrons ($\pi^\pm, K^\pm$).
- Background estimation
  - TAUOLA: Feed down from other tau decays.
  - PYTHIA: Continuum $e^+e^- \rightarrow q\bar{q}(\gamma)$ productions.
  - Others: KKMC($\mu^+\mu^-\gamma$), BHLUMI(Bhabha), AAFH(two photons)

Data and Event Selection



- Data 72 fb^{-1}, accumulated by Belle.

1)Signal selection

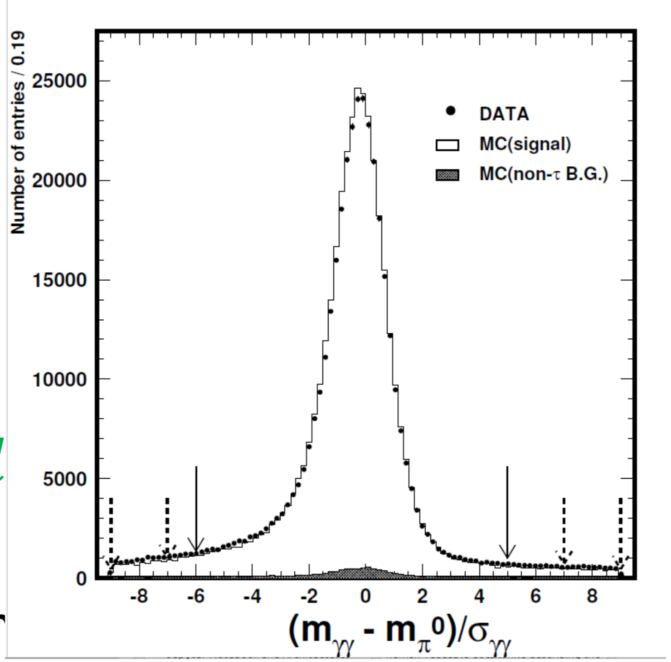
- Low multiplicity events.
  - 2-4 charged tracks (dr<1cm, |dz|<5cm, $p_T > 0.1$GeV, $\Delta Q = 0$)
  - any number of photons with $E_\gamma > 0.05 - 0.1$ GeV.
  - Fiducial region $35 < \theta^* < 145$ in CMS.
- Divide events in two-hemispheres in CMS, by a plane perpendicular to the highest mom. particle.
- Tag side: one or three charged tracks+ any photons.
- Signal side: one charged track + one π^0 ($\tau^- \rightarrow \pi^-\pi^0\nu_\tau$).

2) π^0 selection

- Normalized invariant mass

$$S_{\gamma\gamma} = \frac{m_{\gamma\gamma} - m(\pi^0)}{\sigma_{\gamma\gamma}} \qquad \sigma_{\gamma\gamma}: mass\ resolution\ (5-8\ MeV)$$

- $-6 < S_{\gamma\gamma} < 5$: signal region
- $7 < |S_{\gamma\gamma}| < 9$: side –band region



- Side-band region is used to estimate combinatorial $\gamma\gamma$ $background$
- Rejection of the background $\tau^- \to \pi^-(n\pi^0)\nu_\tau (n \geq 2)$.

Require no. additional photons with $E_\gamma \geq 0.2$ GeV in the signal hem

3) Other requirements for background rejection.

- Max p_T>0.5 GeV to reduce 2-photon processes.
- Max$|p|$<9.0 GeV/c to reduce Bhabha and $\mu^+\mu^-\gamma$.
- Select tau-pair events in M_{miss} vs θ^*_{miss}plane.

$$M^2_{miss} = \left(p_{ini} - \sum p_{track} + \sum p_{photon}\right)^2$$

0.5 GeV $< M_{miss} <$ 7 GeV and $20^\circ < \theta^*_{miss} < 160^\circ$

(to reduce 2-photon, $\mu^+\mu^-\gamma, Bhabha$)

➡ Final sample contain $5.4 \times 10^6\ \tau^- \to \pi^-\pi^0\nu_\tau$ candidates.

Background remained

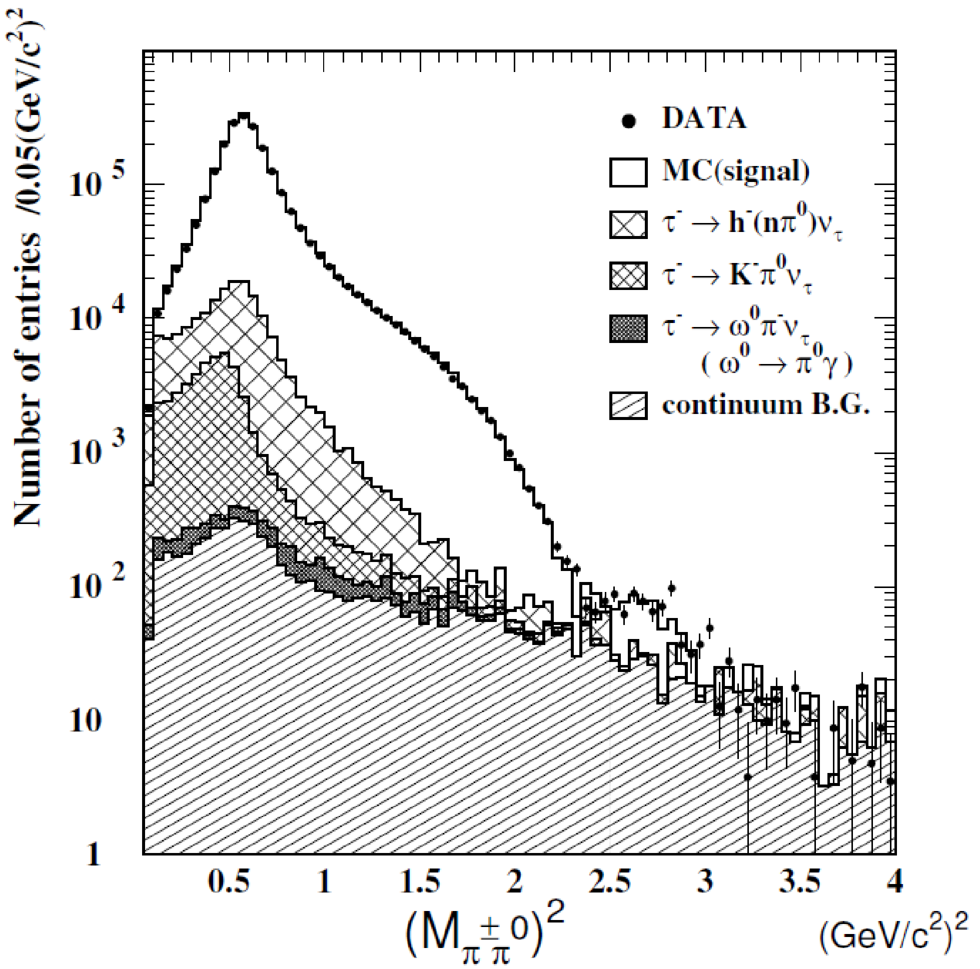
- **Feed down background**
  - $\tau^- \to \pi^-(n\pi^0)\nu_\tau (n \geq 2)$

    Largest background in ρ peak region (7%).
  - $\tau^- \to K^-\pi^0\nu_\tau$
    - Second largest background in ρ peak region (1.5%).

○ *Rely on the* TAUOLA *simulation for these modes.*

○ Uncertainty of Br of these modes are taken into account the systematic errors.

- **Continuum background**
  - $e^+e^- \to q\bar{q}$
    - Contribution is negligible in in ρ peak region (0.1%).
    - Dominated at high mass region $m_{\pi\pi}^2 \geq 2\ \mathrm{GeV}^2$



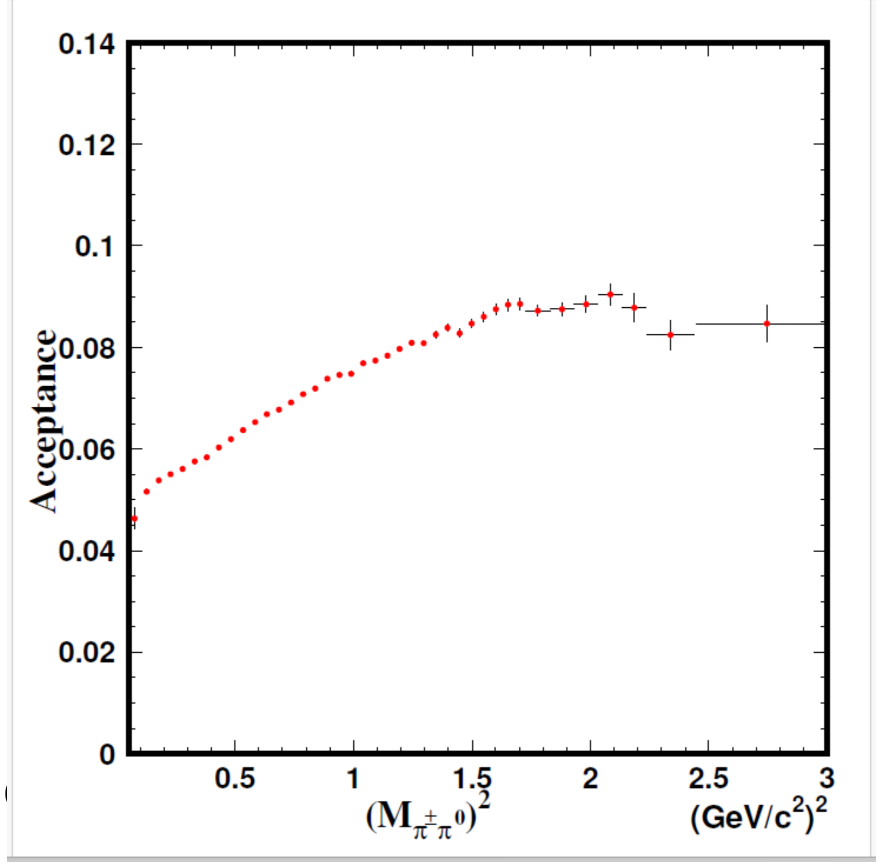
- Control sample, selected with looser condition and in 1.7 GeV< m_{hadron}< 2.0 GeV, is used to estimate the normalization and their uncertainties.

Signal efficiency

- The signal efficiency varies smoothly as a function of $m^2_{\pi\pi}$.
- The average value is *7 %.* This include the effects of the tag-side branching factions and detector fiducial region . These are not affect to the mass spectrum.
- *π^0 detection efficiency is ~ 50% .*
- *We calibrate the π^0 efficiency using η meson decays*

$$R_i = \frac{N(\eta\to\pi^0\pi^0\pi^0)}{N(\eta\to\gamma\gamma)}$$

- Further consistency check for the data and the simulati is done using $\gamma \to e^+e^-$conversion.



Mass resolution and correlation btw gen. and obs.

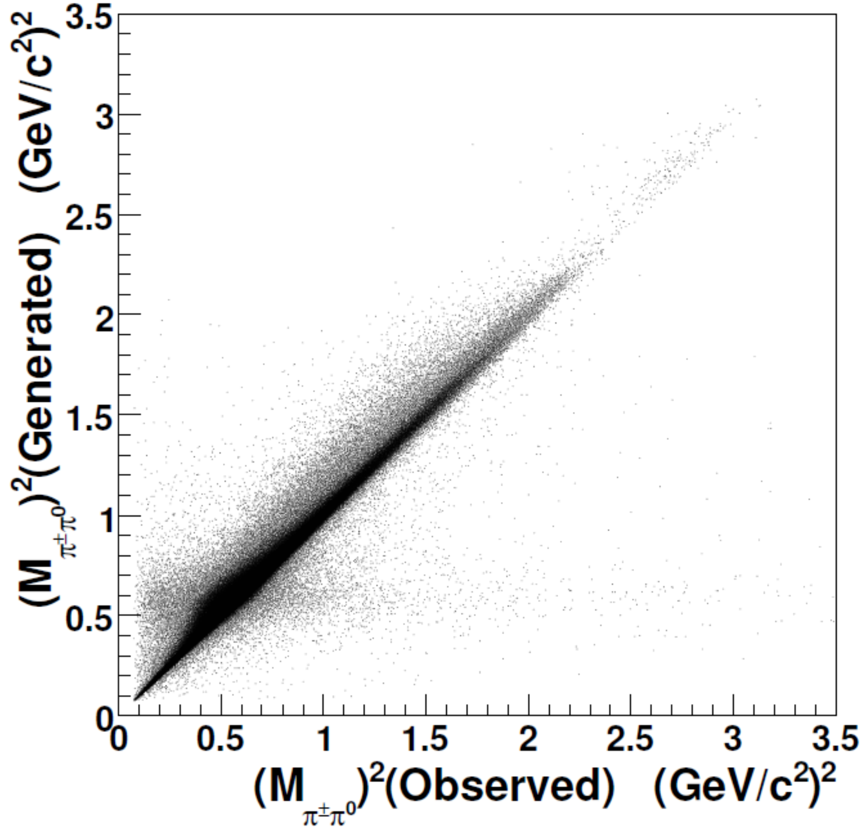
- $m^2_{\pi\pi}$ resolution
  - 0.005 GeV^2 for ρ peak region
  - 0.03 GeV^2 for high mass region.

  (4 MeV – 10 MeV in Δm)
- Correlation between the observeded mass

$m^2_{\pi\pi,obs}$ versus the generated mass $m^2_{\pi\pi,gen}$ (See right figure).

- Note:

In order to take into account FSR in hadronic τ decays, the invariance mass of the $\pi^+\pi^0\gamma$ system is taken as the $m^2_{\pi\pi,gen}$ value, where γ *is* the one generated by PHOTOS.



Unfolding

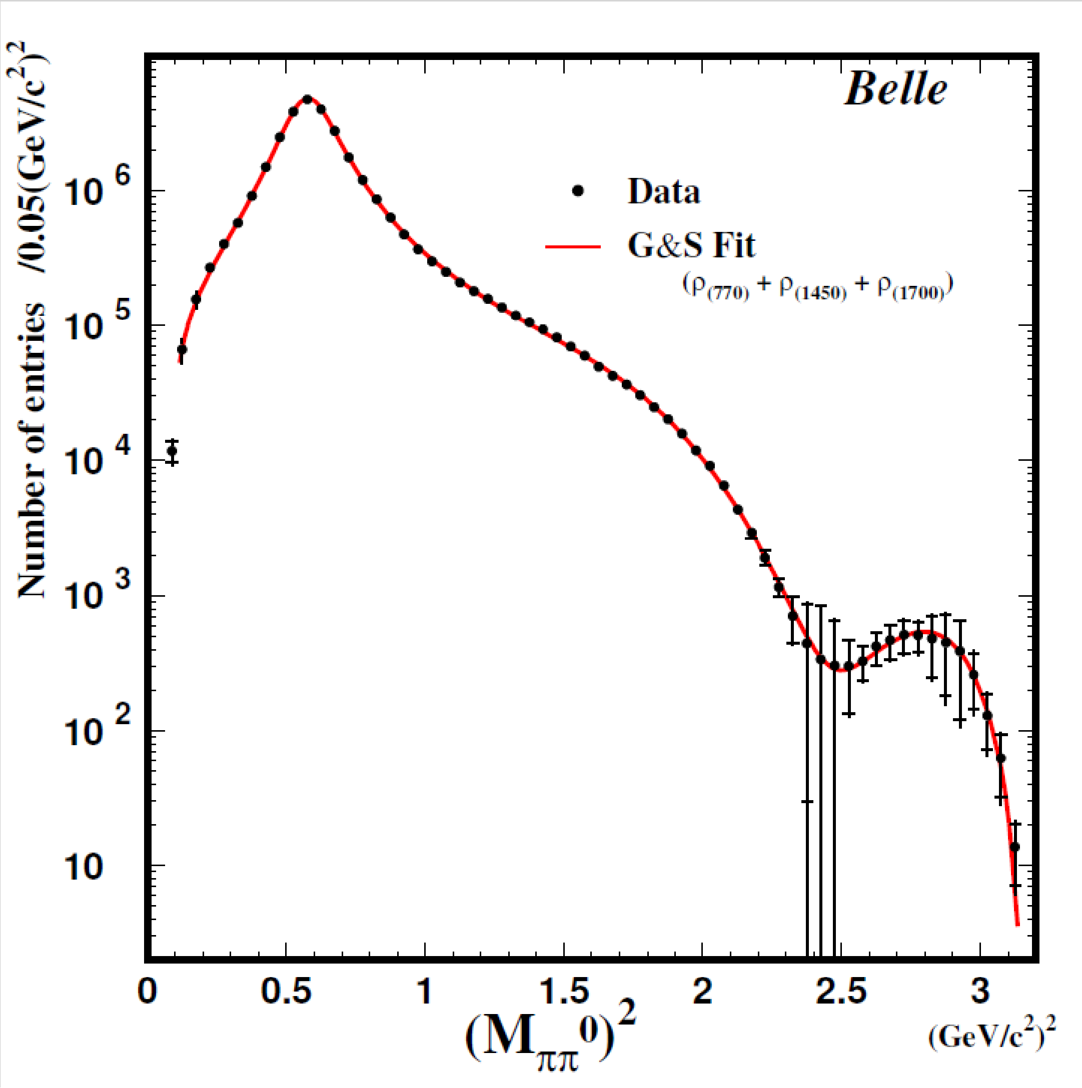
- In order to correct the detector effects (finite efficiency, mass resolution), and the ISR and FSR effects, the data are "unfolded".

  - $\boldsymbol{b} = A\ \boldsymbol{x}$
  - $\boldsymbol{b}$: Vector of observed distribution
  - $\mathbf{x}$: Vector of true distribution to be determined.
  - A : Response matrix which include detector efficiency and resolution and ISR, FSR effects.

- A Singular Value Decomposition (SVD) method is used for the unfolding.

(SVD: NIM A372, 469 (1996))

Unfolded results



- The error bars include both static and systematic errors.

- Red line: a fit with BW form including ρ, $\rho(1450)$, $\rho(1700)$ resonances.

Systematic uncertainties of the unfolded spectrum in (%)

$M^2_{\pi\pi^0}$ region ((GeV/c^2)^2)	First bin (0.08)	Threshold region (0.2-0.3)	ρ region (0.55-0.60)	(1.0-1.2)	ρ' region (1.9-2.0)	ρ'' region (2.5-2.7)
UNF1	2.50	0.79	0.31	0.85	1.50	1.50
UNF2	2.60	0.53	0.09	0.27	0.58	9.19
BKG1	1.13	0.09	0.01	0.04	0.52	5.76
BKG2	4.90	0.65	0.10	0.10	...	0.50
BKG3	25.21	4.80	...	...	...	...
ACC	5.36	1.44	0.03	0.15	0.15	0.40
PES	1.24	1.08	0.59	0.99	0.05	0.50
Total	26.5	5.3	0.7	1.5	1.8	11.4

Sources of Uncertainty

UNF1/2: Unfolding

BKG1: Continuum BKG

BKG2: Feed-down $\pi^{\pm}(n\pi^0)\nu_\tau$ $(n \geq 2)$

BKG3: π^0combinatorial BKG.

ACC : Uncertainty of efficiency

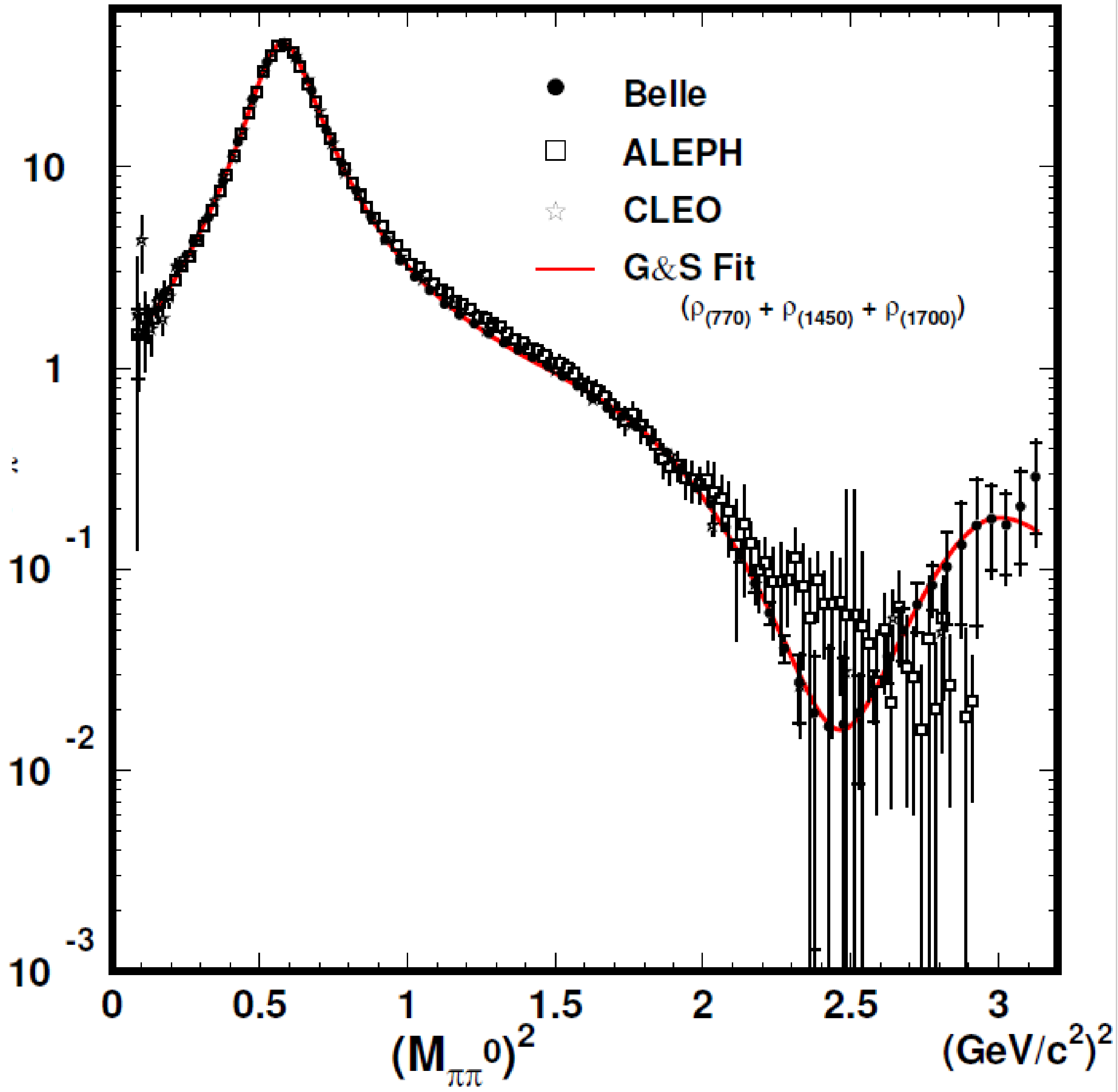
PES : Photon energy scale.

Total systematic uncertainty at ρ peak region is 0.7 %, which is dominated by the uncertainties of the unfolding and the photon energy scale. (stat. error is 0.3 %).

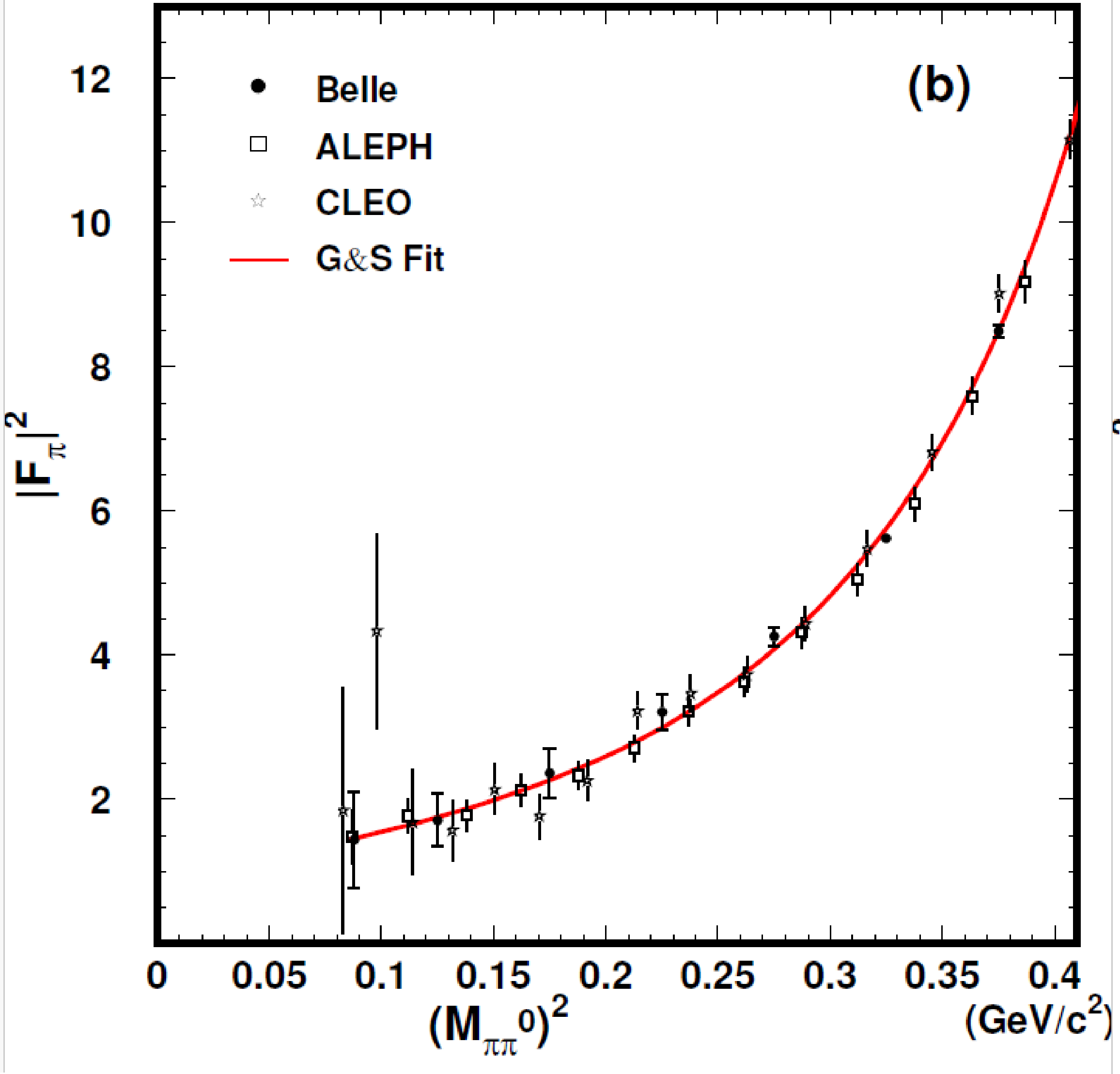
Pion form factors

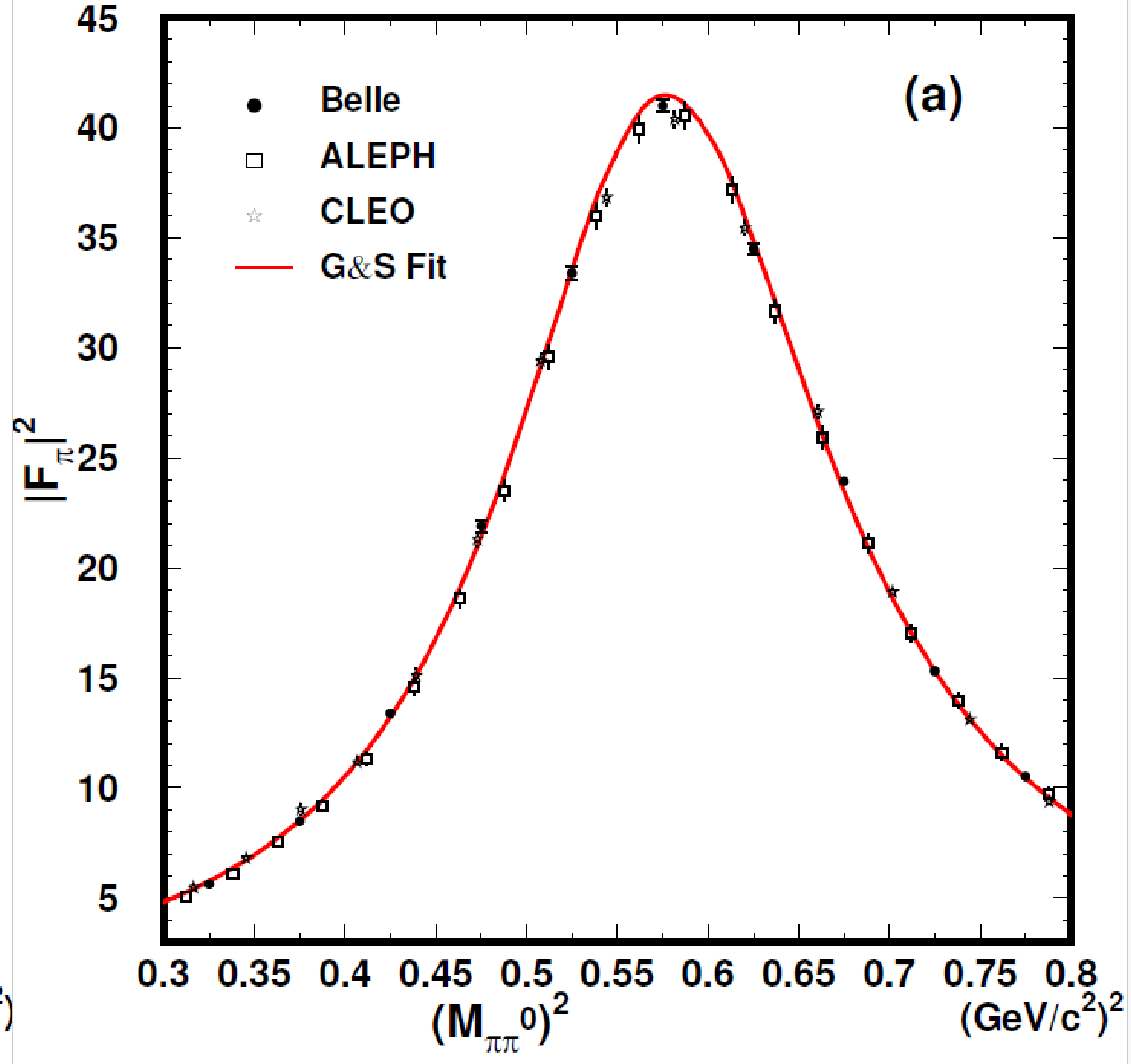
$$|F_\pi^-(s)|^2 = \frac{2m_\tau^2}{|V_{ud}|^2\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)S_{EW}}\frac{1}{\beta_-^3}\left(\frac{\mathcal{B}_{\pi\pi}}{\mathcal{B}_e}\right)\left(\frac{1}{N_{\pi\pi}}\frac{dN_{\pi\pi}}{ds}\right),$$

$S_{EW} = 1.0235 \pm 0.0003$
$V_{ud} = 0.97377 \pm 0.00027$
$B_{\pi\pi} = (25.24 \pm 0.10)\%$



Pion form factor in threshold and ρ peak.





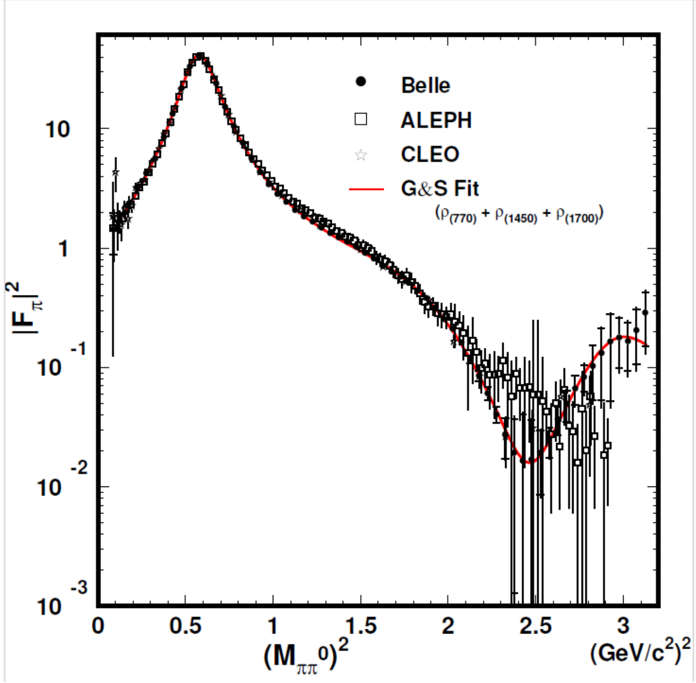
Fit results

BW fits with ρ, $\rho'(1450)$, $\rho''(1700)$ resonances

$$F_\pi(s) = \frac{1}{1+\beta+\gamma}\left(BW_\rho + BW_{\rho'} + BW_{\rho''}\right)$$

$$BW_\rho^{GS} = \frac{M_i^2 + d\cdot M_i\Gamma_i(s)}{(M_i^2 - s) + f(s) - i\sqrt{s}\Gamma_i(s)}$$

GS: Gounaris-Sakurai model



Parameter	Fit result (fixed $\lvert F(0)\rvert^2$)	Fit result (all free)
M_ρ, MeV/c^2	$774.6 \pm 0.2 \pm 0.5$	$774.9 \pm 0.3 \pm 0.5$
Γ_ρ, MeV	$148.1 \pm 0.4 \pm 1.7$	$148.6 \pm 0.5 \pm 1.7$
$M_{\rho'}$, MeV/c^2	$1446 \pm 7 \pm 28$	$1428 \pm 15 \pm 26$
$\Gamma_{\rho'}$, MeV	$434 \pm 16 \pm 60$	$413 \pm 12 \pm 57$
$\lvert\beta\rvert$	$0.15 \pm 0.05^{+0.15}_{-0.04}$	$0.13 \pm 0.01^{+0.16}_{-0.04}$
ϕ_β, degree	$202 \pm 4^{+41}_{-\ 8}$	$197 \pm 9^{+50}_{-\ 5}$
$M_{\rho''}$, MeV/c^2	$1728 \pm 17 \pm 89$	$1694 \pm 41 \pm 89$
$\Gamma_{\rho''}$, MeV	$164 \pm 21^{+89}_{-26}$	$135 \pm 36^{+50}_{-26}$
$\lvert\gamma\rvert$	$0.037 \pm 0.006^{+0.065}_{-0.009}$	$0.028 \pm 0.020^{+0.059}_{-0.009}$
ϕ_γ, degree	$24 \pm 9^{+118}_{-\ 28}$	$-3 \pm 13^{+136}_{-\ 29}$
$\lvert F(0)\rvert^2$	[1.0]	$1.02 \pm 0.01 \pm 0.04$
χ^2/NDF	80/52	65/51
$\rho''(1700)$ signif., σ	6.5	7.0

Summary

- Belle 2π spectral function is reviewed.
- See Phys. Rev. D 78, 072006 (2008), arxiv: 0805.3773 for a full description.
- Covariance matrix is available at

https://belle.kek.jp/bdocs/b_journal.html

See [Fig] folder in publication 264.

tableV.data: 2pi mass spectrum, 1/N dN/ds.

tableV-ematsys.dat : covariance matrix of 1/N dN/ds.

README.txt: an explanation of table format.

Back up

M_{miss} vs θ^*_{miss}