Combination and comparison of Tau 2π measurements

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Combine cross section data: goal and requirements

→ Goal: combine experimental spectra with arbitrary binning (/point spacing)

→ Requirements:

- Properly propagate uncertainties and correlations
  - *Between measurements (data points/bins) of a given experiment* (covariance matrices and/or detailed split of uncertainties in sub-components)
  - *Between experiments* (common systematic uncertainties) based on detailed information provided in publications
  - *Between different channels* – motivated by understanding of the meaning of systematic uncertainties and identifying the common ones

- Minimize biases

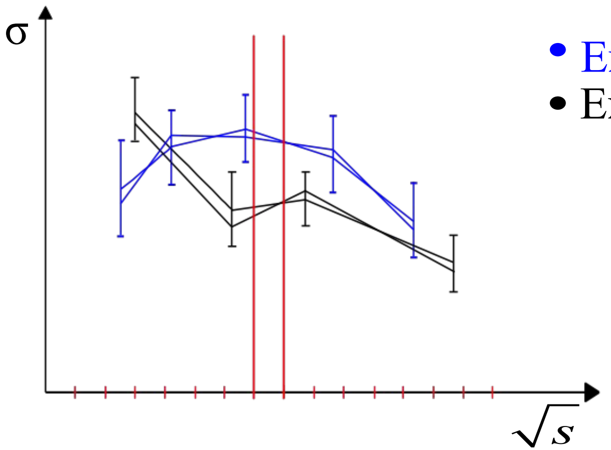
- Optimize g-2 integral uncertainty (*without overestimating the precision with which the uncertainties of the measurements are known*)

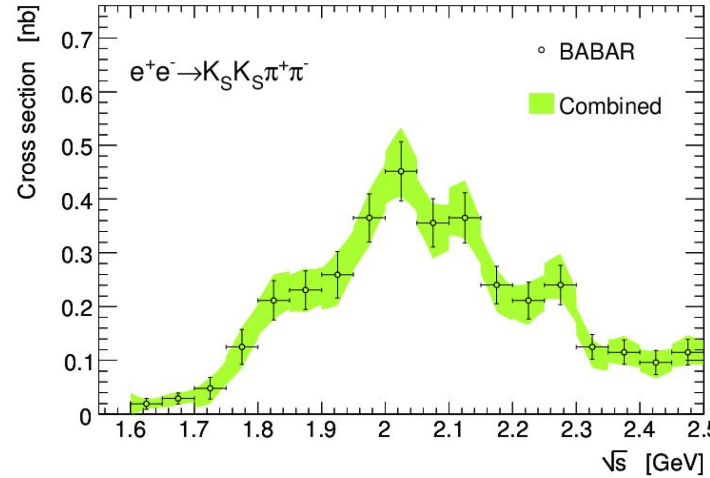
Procedure and software (*HVPTools - Since 2009*) for combining cross section data with arbitrary binning

→ Validated through closure test

→ Featuring full & realistic (i.e. not too optimistic) treatment of uncertainties and correlations, fully accounting for possible systematic tensions between experiments.

Combination procedure implemented in HVPTools software





→ Define a (fine) final binning (to be filled and used for integrals etc.)

→ Linear/quadratic splines to interpolate between the points/bins of each experiment
- for binned measurements: preserve integral inside each bin
- closure test: replace nominal values of data points by Gounaris-Sakurai model and re-do the combination

→ *(non-)negligible bias for (linear)quadratic interpolation*

→ Fluctuate data points taking into account correlations & re-do the splines for each (pseudo-)experiment
- each uncertainty fluctuated coherently for all the points/bins that it impacts
- eigenvector decomposition for (statistical) covariance matrices

Combination procedure implemented in HVPTools software

For each final bin:

→ Compute an average value for each measurement and its uncertainty

→ Compute correlation matrix between experiments

→ Minimize χ^2 and get average coefficients (weights)

→ Compute average between experiments and its uncertainty

Evaluation of integrals and propagation of uncertainties:

→ Integral(s) evaluated for nominal result and for each set of toy pseudo-experiments; uncertainty of integrals from RMS of results for all toys

→ The pseudo-experiments also used to derive (statistical & systematic) covariance matrices of combined cross sections → Integral evaluation

→ Uncertainties also propagated through $\pm 1\sigma$ shifts of each uncertainty: also allows to account for correlations between different channels (for integrals and spectra)

→ *Checked consistency between the different approaches*

Combining the τ data in the ππ channel

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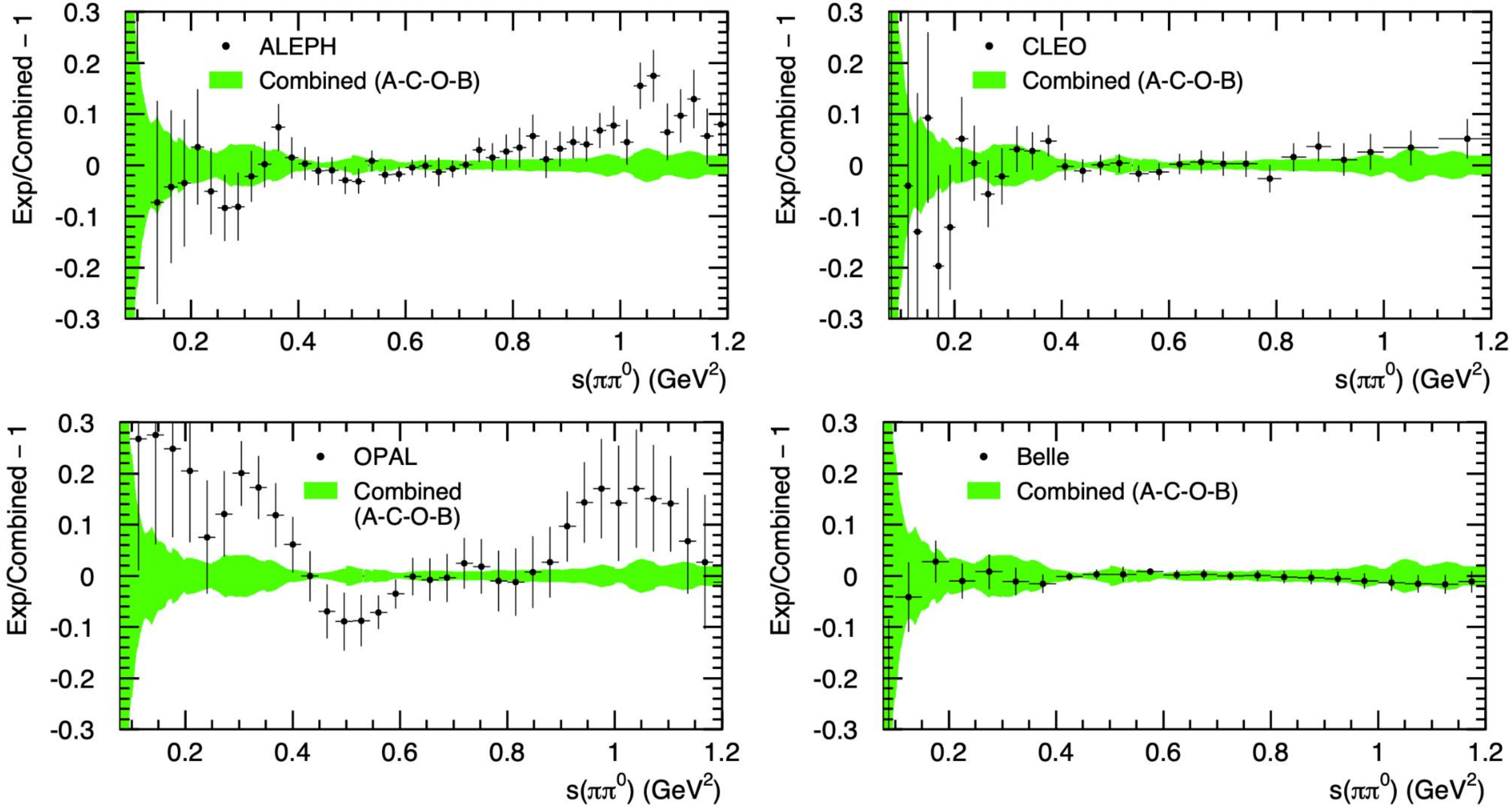
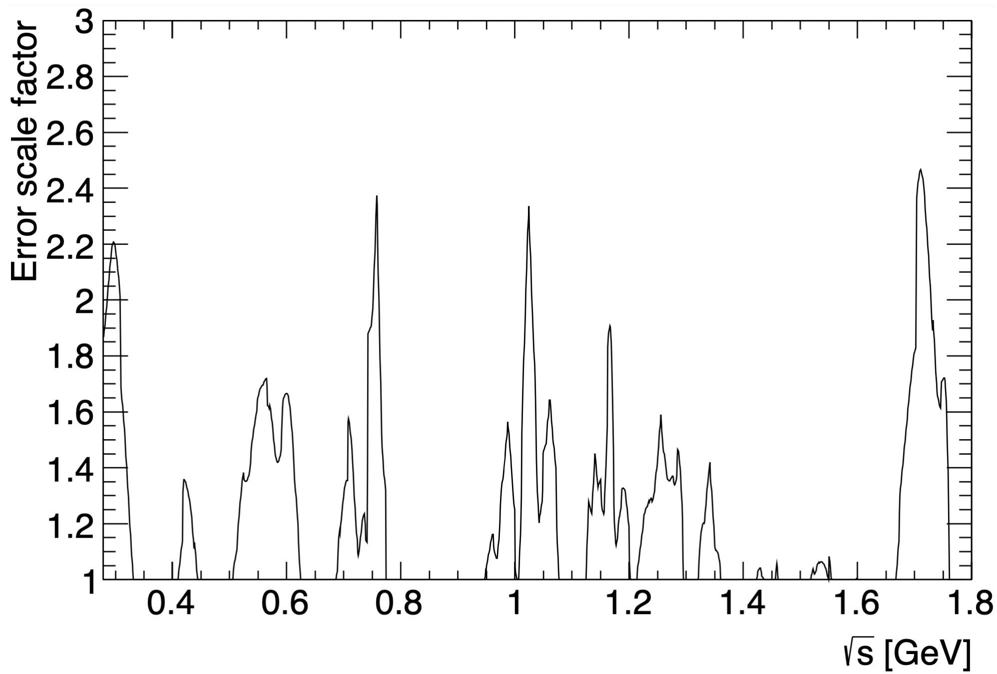


Combination: compatibility between measurements

For each final bin:

→ χ^2/ndof: test locally the level of agreement between input measurements, *taking into account correlations*

→ Scale uncertainties in bins with χ^2/ndof > 1 (PDG)

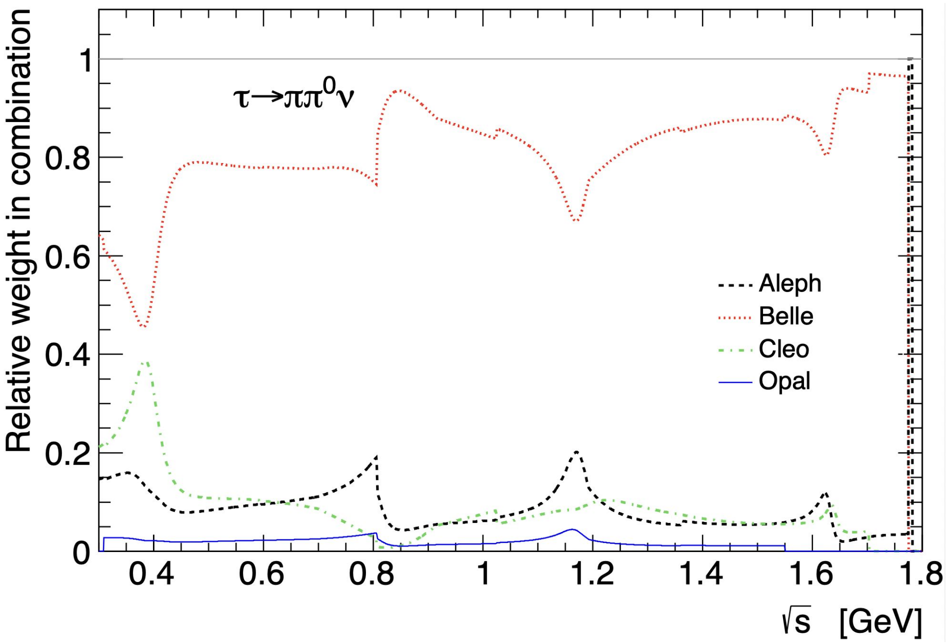


→ Level of agreement significantly better than the one observed for $e^+e^- \to \pi^+\pi^-$ data

Combination: weights of various measurements

For each final bin:
→ Minimize χ^2 and get average coefficients

Note: average weights must account for bin sizes / point spacing of measurements
(Compare the precisions on the same footing: do not over-estimate the weight of experiments with large bins)
→ Weights in fine bins evaluated using a common (large) binning for measurements + interpolation
→ Their determination also integrates bin-to-bin statistical & systematic correlations on moderate energy ranges



→ Shape information provided mainly by Belle (reflected by the weights from the combination of spectra)

Combining the τ data in the ππ channel

→ Normalisation dominated by ALEPH (directly impacting and very relevant for the integrals)

Experiment	$a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau]$ (10^{-10}) $2m_{\pi^\pm} - 0.36$ GeV	$0.36 - 1.8$ GeV
ALEPH	$9.80 \pm 0.40 \pm 0.05 \pm 0.07$	$501.2 \pm 4.5 \pm 2.7 \pm 1.9$
CLEO	$9.65 \pm 0.42 \pm 0.17 \pm 0.07$	$504.5 \pm 5.4 \pm 8.8 \pm 1.9$
OPAL	$11.31 \pm 0.76 \pm 0.15 \pm 0.07$	$515.6 \pm 9.9 \pm 6.9 \pm 1.9$
Belle	$9.74 \pm 0.28 \pm 0.15 \pm 0.07$	$503.9 \pm 1.9 \pm 7.8 \pm 1.9$
Combined	$9.82 \pm 0.13 \pm 0.04 \pm 0.07$	$506.4 \pm 1.9 \pm 2.2 \pm 1.9$

Table 6. The isospin-breaking-corrected $a_\mu^{\mathrm{had,LO}}[\pi\pi,\tau]$ (in units of 10^{-10}) from the measured mass spectrum by ALEPH, CLEO, OPAL and Belle, and the combined spectrum using the corresponding branching fraction values. The results are shown separately in two different energy ranges. The first errors are due to the shapes of the mass spectra, which also include very small contributions from the τ-mass and $|V_{ud}|$ uncertainties. The second errors originate from $B_{\pi\pi^0}$ and B_e, and the third errors are due to the isospin-breaking corrections, which are partially anti-correlated between the two energy ranges. The last row gives the evaluations using the combined spectra.

Individual measurements with the corresponding uncertainties:

ALEPH: 511.0 ± 5.3 (±1.9 common, from IB)

CLEO: 514.2 ± 10.1

OPAL: 526.9 ± 12.3

Belle: 513.7 ± 8.0

→ Most precise determination from ALEPH, due to most precise Br

→ Uncertainty from combined spectra (±2.9) smaller than uncertainty from weighted average of integrals (±3.8): Due to better use of the available information on the precision of the measurements (Br and mass-dependent uncertainties)

χ^2 : 1.45/3 dof, when averaging the 4 individual integrals

χ^2 : 1.88/3-4 dof, when comparing the 4 individual integrals with the integral of the combined spectrum

→ Excellent agreement among the 4 measurements