Tau Pheno' Analysis

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LPNHE IJCLab

Virtual Workshop on Tau Decays

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Introduction

In the first part of the workshop (link), we have shown the tau measurements and their combination

In this part, we focus on the isospin breaking (IB) corrections:

$$\sigma^{I=1}_{e^+e^-\to\pi^+\pi^-} = \frac{4\pi\alpha^2}{s} v_{1,\pi^-\pi^0\nu_\tau}$$

$$v_{1,\pi^-\pi^0\nu_\tau} \propto \frac{B_{\pi\pi^0}}{B_e} \frac{dN_{\pi\pi^0}}{N_{\pi\pi^0} ds} \frac{m_\tau^2}{(1-s/m_\tau^2)^2 (1+2s/m_\tau^2)} \frac{R_{\rm IB}(s)}{S_{\rm EW}}$$

IB corrections

$$\frac{R_{\rm IB}(s)}{S_{\rm EW}} \quad \text{with} \quad R_{\rm IB}(s) = \frac{{\rm FSR}(s)}{G_{\rm EM}(s)} \frac{\beta_0^3(s)}{\beta_-^3(s)} \left|\frac{F_0(s)}{F_-(s)}\right|^2$$

Numerical Values of the IB Corrections

$$\frac{R_{\rm IB}(s)}{S_{\rm EW}} \quad \text{with} \quad R_{\rm IB}(s) = \frac{\mathrm{FSR}(s)}{G_{\rm EM}(s)} \frac{\beta_0^3(s)}{\beta_-^3(s)} \left| \frac{F_0(s)}{F_-(s)} \right|^2$$

Source	$\Delta a_\mu^{\rm Had,LO}[\pi\pi,\tau]$	$\Delta \mathcal{B}^{\rm CVC}_{\pi^-\pi^0}$
$S_{\rm EW}$	-12.21 ± 0.15	$+0.57 \pm 0.01$
$G_{\rm EM}$	-1.92 ± 0.90	-0.07 ± 0.17
FSR	$+4.67 \pm 0.47$	-0.19 ± 0.02
$m_{\pi^\pm} - m_{\pi^0}$ effect on σ	-7.88	$+0.19$
ρ–ω interference*	$+2.80 \pm 0.19$	-0.01 ± 0.01
$m_{\pi^\pm} - m_{\pi^0}$ effect on Γ_ρ	$+4.09$	-0.22
$m_{\rho^\pm} - m_{\rho^0_{\rm bare}}$	$0.20^{+0.27}_{-0.19}$	$+0.08 \pm 0.08$
$\pi\pi\gamma$, electrom. decays	-5.91 ± 0.59	$+0.34 \pm 0.03$
δ(GS – KS)**	-0.67	-0.03
Total	-16.07 ± 1.85	$+0.69 \pm 0.22$

* the ϱ–ω interference correction +2.80 was based on $|\delta_{\varrho\omega}|$=0.001997, arg($\delta_{\varrho\omega}$)=11.6° changed in DHLMZ23 to +3.99 using $|\delta_{\varrho\omega}|$=0.001990, arg($\delta_{\varrho\omega}$)=3.8° (Colangelo et al. 2023)

This is the only change over the last 15 years or so from our group

** Used GS (Gounaris-Sakurai) and KS (Kuhn-Santamaria) parameterisations

→ In the following, I shall introduce each correction term

→ Bogdan will show the energy dependence of the corrections and the corresponding uncertainties

→ Michel will comment on the context of the use of the tau data in the current confusing situation of the e+e- data

Short Distance Radiative Correction – S_{EW}

Leading EW correction: Marciano, Sirlin, 88

$$S_{\rm EW} = 1 + \frac{3\alpha}{4\pi}(1+2\overline{Q})\ln\frac{M_Z^2}{m_\tau^2} \simeq 1.0188 \quad \text{with} \quad \overline{Q} = \frac{1}{6} \quad \text{for semi-hadronic mode}$$

Improved by resuming all higher order logarithms using renormalisation group technique:

$$S_{\rm EW}^{\rm had} = \left[\frac{\alpha(m_b)}{\alpha(m_\tau)}\right]^{\frac{9}{19}} \left[\frac{\alpha(M_W)}{\alpha(m_b)}\right]^{\frac{9}{20}} \left[\frac{\alpha(M_Z)}{\alpha(M_W)}\right]^{\frac{36}{17}} \simeq 1.0194 \xrightarrow{\text{QCD corrections}} 1.0189$$

Braaten, Narison, Pich, 92
Sirlin, 82

Taking into account sub-leading non-logarithmic short distance correction (since the spectral function is normalised to the electron mode):

$$S_{\rm EW}^{\rm sub,lep} = 1 + \frac{\alpha(m_\tau)}{\pi}\left(\frac{25}{8} - \frac{\pi^2}{2}\right) \simeq 0.9957$$

$$v_{1,X^-}(s) = \frac{m_\tau^2}{6|V_{ud}|^2}\frac{\mathcal{B}_{X^-}}{\mathcal{B}_e}\frac{1}{N_X}\frac{dN_X}{ds} \times \left(1-\frac{s}{m_\tau^2}\right)^{-2}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\frac{R_{\rm IB}(s)}{S_{\rm EW}},$$

One has finally:

$$S_{\rm EW} = \frac{S_{\rm EW}^{\rm had}}{S_{\rm EW}^{\rm sub,lep}} \simeq 1.0233 \pm 0.0006$$

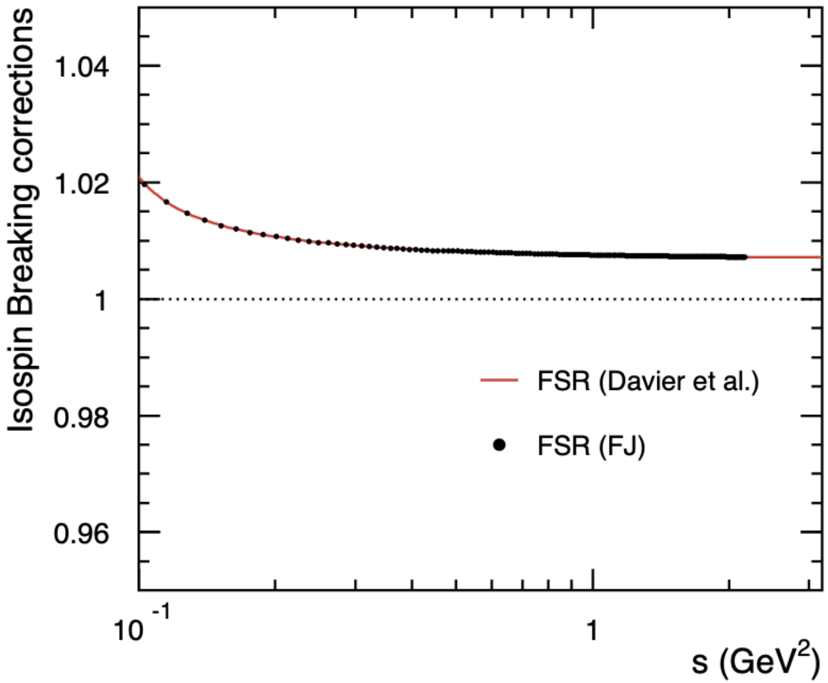
Uncertainty corresponds conservatively to the difference between the leading and resumed corrections

Final State Radiation (FSR) Correction

$$R_{\rm IB}(s) = \frac{\mathrm{FSR}(s)}{G_{\rm EM}(s)} \frac{\beta_0^3(s)}{\beta_-^3(s)} \left| \frac{F_0(s)}{F_-(s)} \right|^2$$

FSR corrections based on Schwinger 1989 (scalar QED with point-like pion) and are the same as F. Jegerlehner (FJ)

Figure from WP20



Quoting ~10% uncertainty

Real FSR production checked in e+e- data by BaBar 2023 (good agreement with scalar QED)

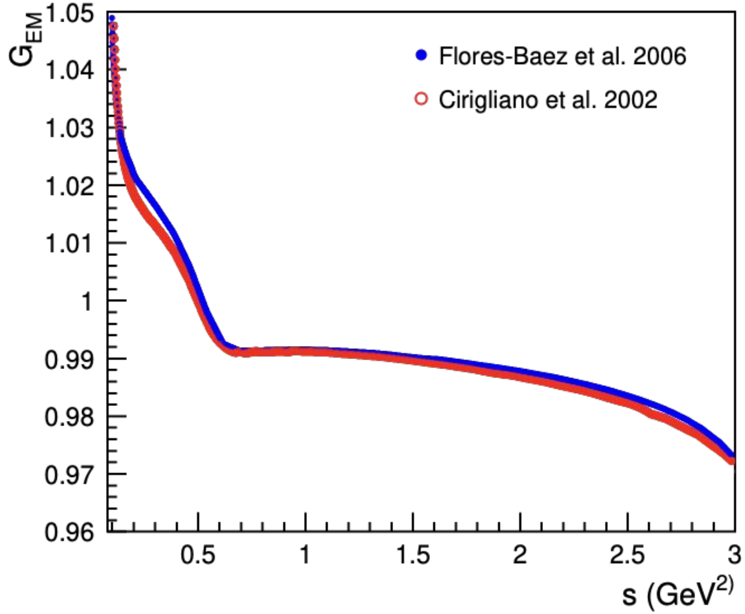
Structure dependence effect: aware of ongoing work @Bern (PhD Monnard 2021)

Long Distance EM Corrections – G_{EM}

$$R_{IB}(s) = \frac{\mathrm{FSR}(s)}{G_{EM}(s)} \frac{\beta_0^3(s)}{\beta_-^3(s)} \left| \frac{F_0(s)}{F_-(s)} \right|^2$$

Our G_{EM} corrections are based on vector meson dominance (VMD) model [1] since 2009

We quote the difference with corrections based on chiral perturbation theory (ChPT) [2] as uncertainty



[1] Flores-Baez et al. 2006
[2] Cirigliano, Ecker, Neufeld, 2001, 2002

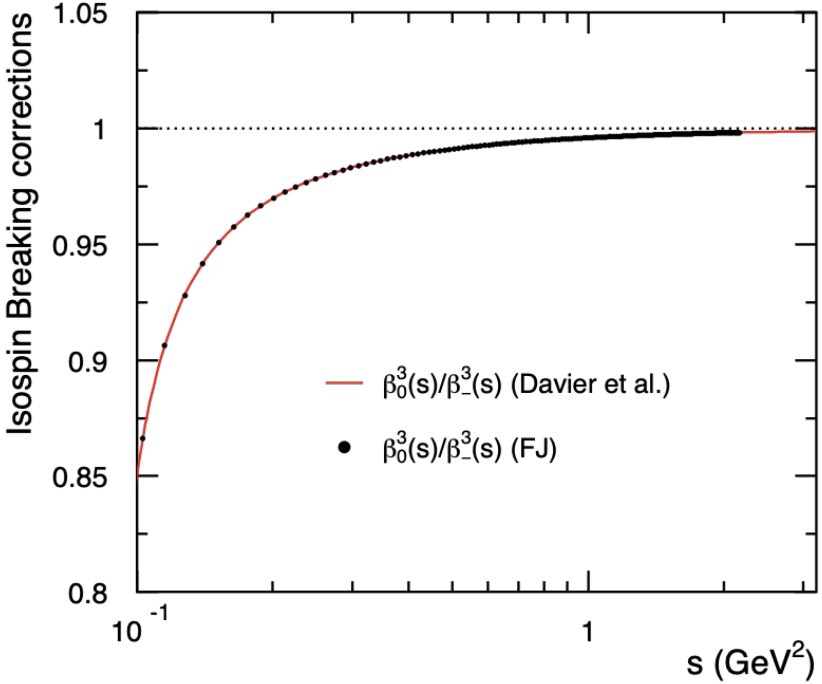
Phase Space Difference – Beta Ratio Term

$$R_{\mathrm{IB}}(s) = \frac{\mathrm{FSR}(s)}{G_{\mathrm{EM}}(s)} \frac{\beta_0^3(s)}{\beta_-^3(s)} \left| \frac{F_0(s)}{F_-(s)} \right|^2$$

$$\beta_{0,-} = \beta(s, m_{\pi^-}, m_{\pi^{0,-}})$$

$$\beta(s, m_1, m_2) = \left[\left(1 - \frac{(m_1 + m_2)^2}{s}\right) \left(1 - \frac{(m_1 - m_2)^2}{s}\right) \right]^{1/2}$$

Figure from WP20



Again no difference with Jegerlehner

Since the pion mass values are well known no uncertainty needed

Form Factor Ratio Term

$$F_0(s) = f_{\rho^0}(s)\left[1+\delta_{\rho\omega}\frac{s}{m_\omega^2 - s - im_\omega\Gamma_\omega(s)}\right]$$
$$F_-(s) = f_{\rho^-}(s)\,,$$

Using GS and KS parameterisations

$$R_{\rm IB}(s) = \frac{{\rm FSR}(s)}{G_{\rm EM}(s)}\frac{\beta_0^3(s)}{\beta_-^3(s)}\left|\frac{F_0(s)}{F_-(s)}\right|^2$$

$\rho-\omega$ mixing, only present in F_0, is one of the corrections

- previously (Davier et al. 2009) we have used $|\delta_{\rho\omega}|$=0.001997, arg($\delta_{\rho\omega}$)=11.6°
- in DHLMZ23 we changed to $|\delta_{\rho\omega}|$=0.001990, arg($\delta_{\rho\omega}$)=3.8° (Colangelo et al. 2023)

The ρ^0 and ρ^- rho width and mass difference is another correction

- for the width difference, we have

$$\delta\Gamma_\rho(s) = \Gamma_{\rho^0} - \Gamma_{\rho^-} = \frac{g^2_{\rho\pi\pi}\sqrt{s}}{48\pi}\left[\beta_0^3(s)(1+\delta_0) - \beta_-^3(s)(1+\delta_-)\right]$$

Flores-Baez et al. 2007

which is partly due to (a) $\pi^- - \pi^0$ mass splitting and partly due to (b) EM decays ($\delta_{0,-}$ terms, corresponding to radiative corrections to include $\pi\pi\gamma$ final state)
@775 MeV, only (a) gives $\delta\boldsymbol{\Gamma}_\rho \sim -1.06$ MeV, to be compared with (-0.42 ± 0.58) MeV [1] and (-0.61 ± 0.45) MeV [2]; including (b) gives $\delta\boldsymbol{\Gamma}_\rho \sim 0.76$ MeV

- for the mass difference, we use $\delta m_\rho = m_{\rho^\pm} - m_{\rho^0_{\rm bare}} = (1.0 \pm 0.9)$ MeV, based on $m_{\rho^0} - m_{\rho^0_{\rm bare}} \approx 3\Gamma(\rho^0 \to e^+e^-)/(2\alpha) = 1.45$ MeV (Flores-Baez et al. 2007) and $m_{\rho^\pm} - m_{\rho^0} = (-0.4 \pm 0.9)$ MeV [3]

[1] ADH 1997, [2] Cirigliano, Ecker, Neufeld, 2001, 2002, [3] KLOE 2003

Two applications:

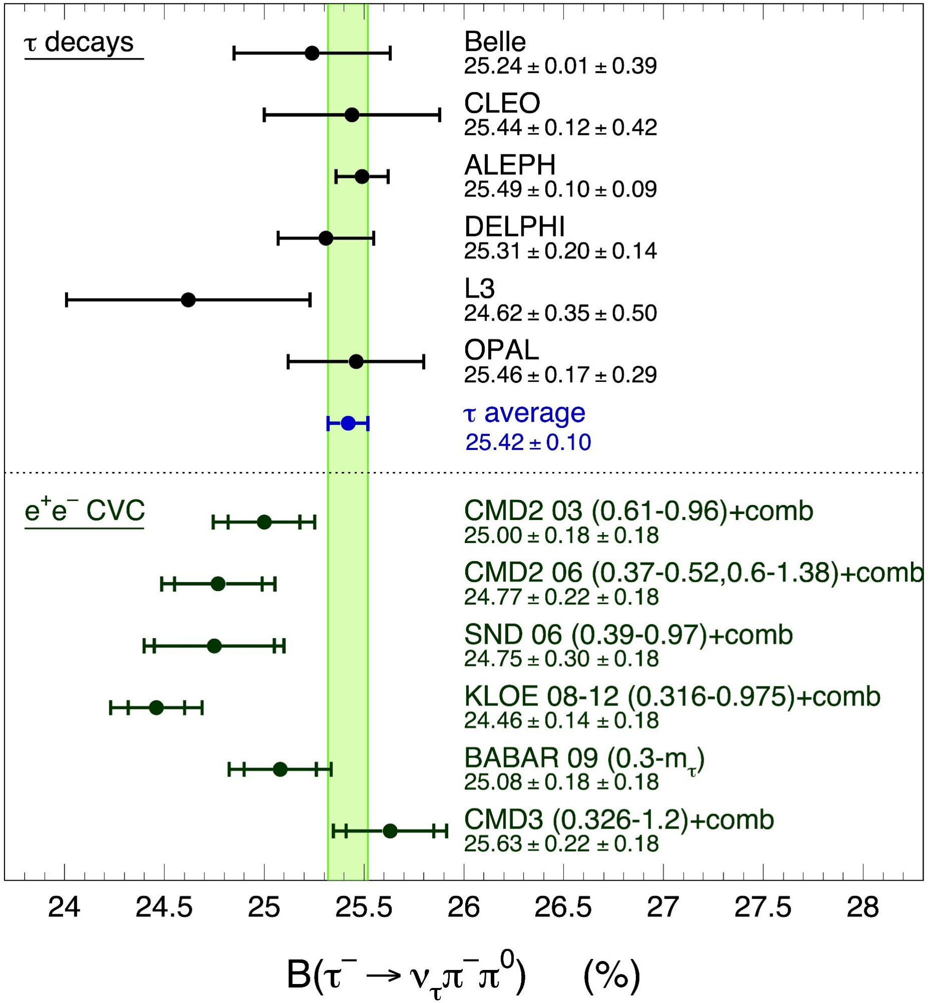
- Apply the IB corrections to tau data to get an equivalent e+e- spectrum for evaluating a_μ

$$a_\mu^{\mathrm{LO,had}}[\pi\pi,\tau] = \frac{\alpha^2 m_\tau^2}{6|V_{ud}|^2\pi^2}\frac{B_{\pi\pi^0}}{B_e}\int_{s_{4m_\pi^2}}^{m_\tau^2} ds\frac{K(s)}{s}\frac{dN_x}{N_x ds}\left(1-\frac{s}{m_\tau^2}\right)^{-2}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\frac{R_{\mathrm{IB}}}{S_{\mathrm{EW}}}$$

- Apply the IB corrections to e+e- data to get an equivalent tau spectrum for deriving a branching fraction B^{CVC}

$$B_{\pi^-\pi^0}^{\mathrm{CVC}} = \frac{3}{2}\frac{B_e|V_{ud}|^2}{\pi\alpha^2 m_\tau^2}\int_{s_{\min}}^{m_\tau^2} ds\, s\,\sigma_{\pi^-\pi^0}^0(s)\left(1-\frac{s}{m_\tau^2}\right)^2\left(1+\frac{2s}{m_\tau^2}\right)\frac{S_{\mathrm{EW}}}{R_{\mathrm{IB}}}$$

B_{CVC}

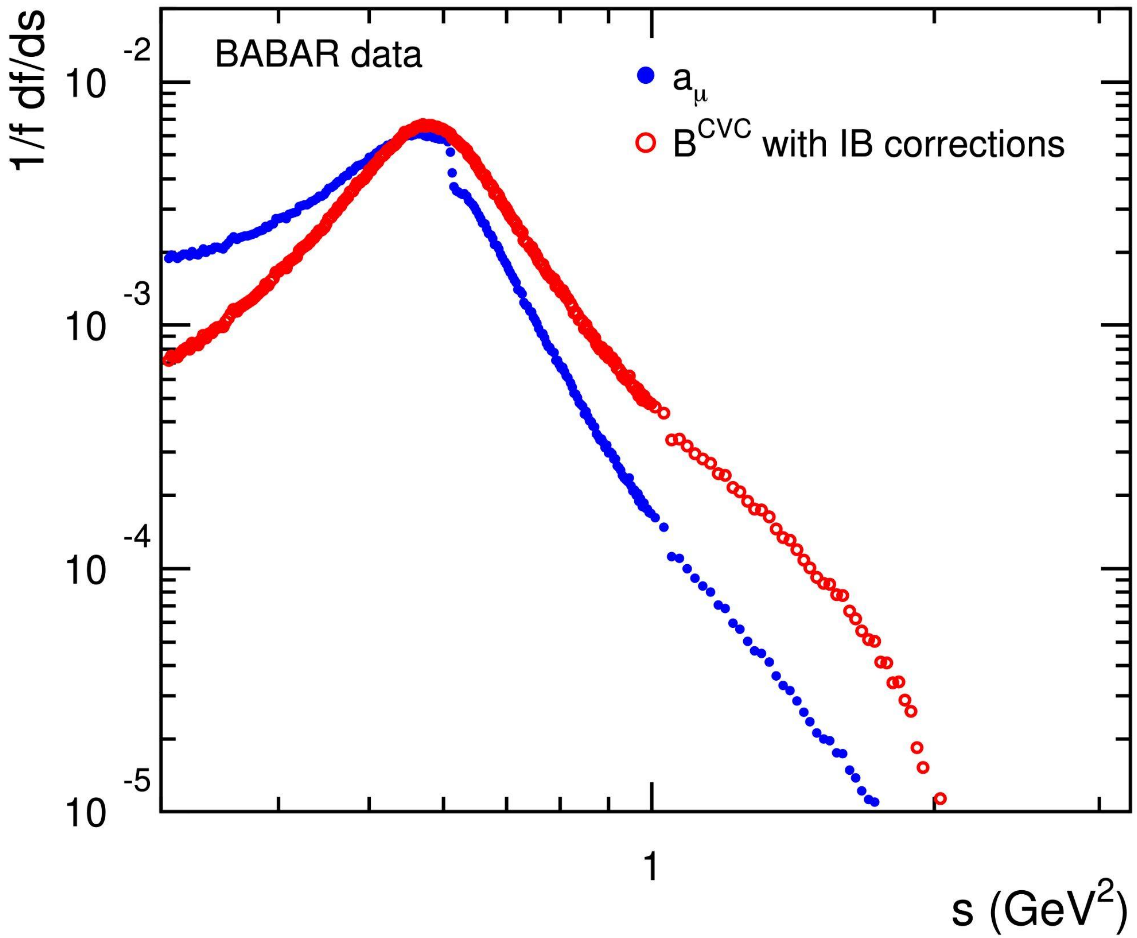


B^{CVC} values for some of the earlier e+e- measurements are lower than tau branching fraction measurements

BABAR and CMD-3 are however in fair agreement with the tau measurements

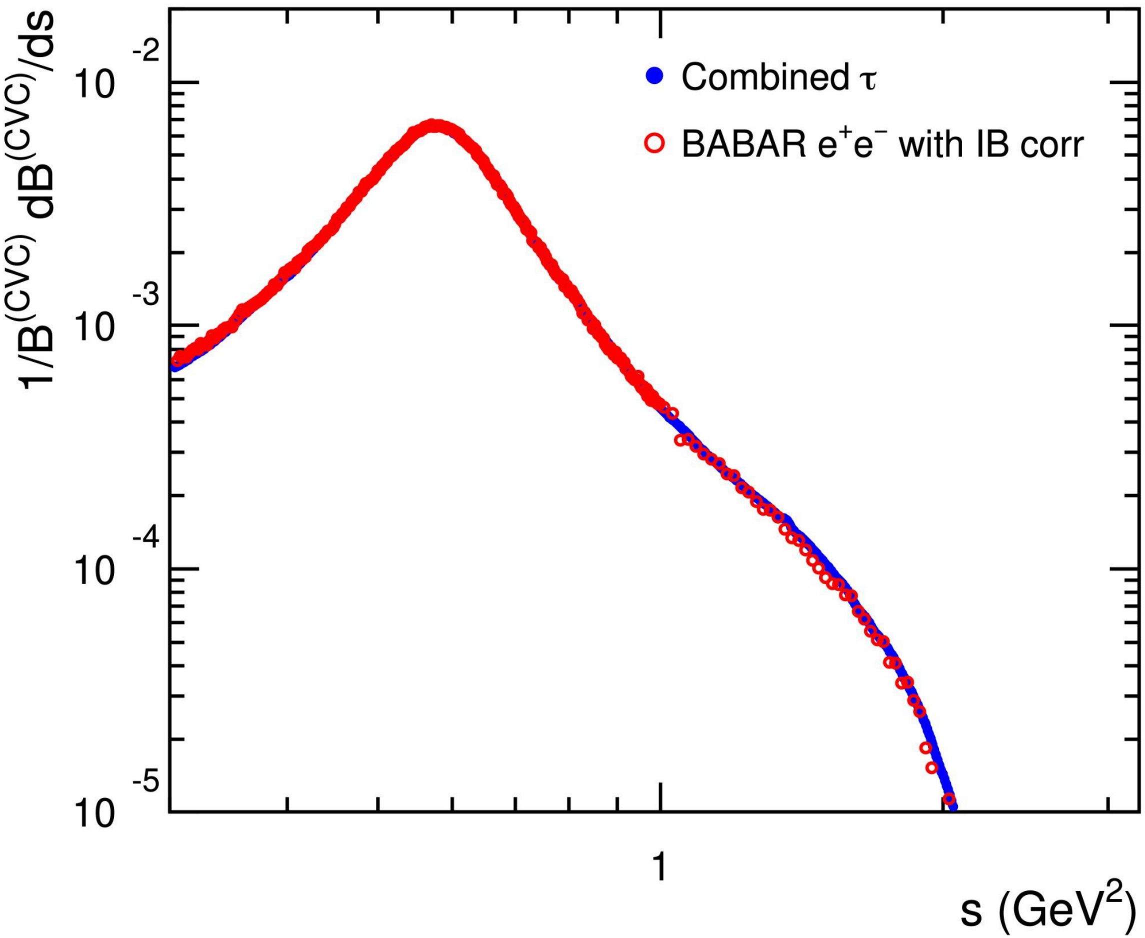
Tau BRs are in agreement and have competitive precision while e+e- data show large dispersion

Complementary on the IB Corrections between amu and B^{CVC}



The two applications are complementary as the weighted energy spectra for a_μ and B^{CVC} have very different shape though the rho peak region dominates in both cases

Compare Combined Tau Spectrum with e+e- BABAR Data*



Apply the IB corrections to e+e- BABAR $\pi\pi$ mass spectrum and compare with combined tau mass spectrum

⇒ Full agreement over a range of three orders of magnitude

see page 21 for zoomed comparison

* BABAR is chosen since it is the only measurement covering the full mass region

Moment integrals from τ data (2π channel) with IB corrections

a_μ [0.36 , 1.775 GeV] = $(507.51 \pm 1.86) \times 10^{-10}$

uncertainties from *combined spectrum*

$\pm 2.12 \times 10^{-10}$

uncertainties from *normalisation (Be & Bππ⁰)*

$\pm 1.9 \times 10^{-10}$

uncertainties from *IB uncertainties*

a_μ [0.36 , 1.775 GeV] = $(507.51 \pm 3.41) \times 10^{-10}$

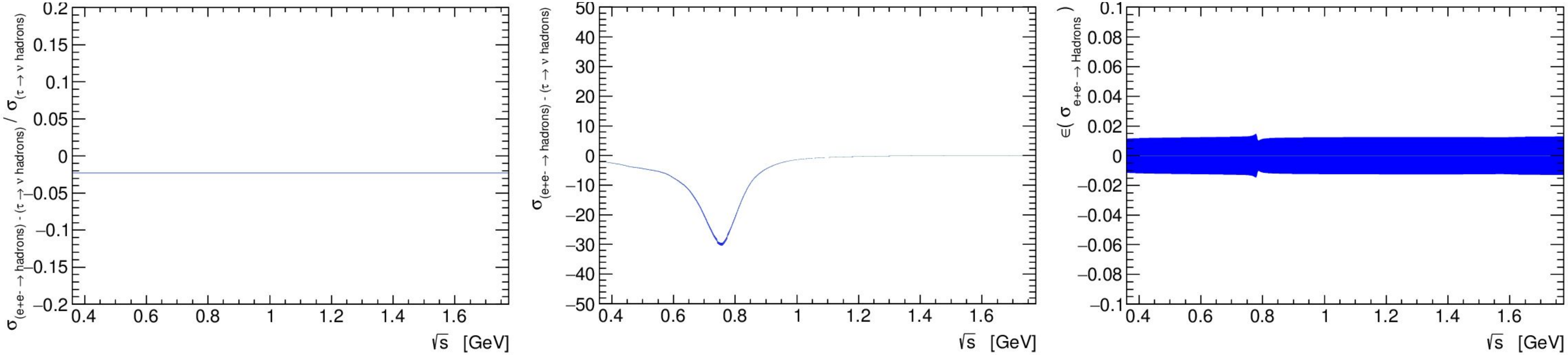
uncertainties from *combined spectrum, normalisation (Be & Bππ⁰)* and *IB uncertainties*

→ New (next slides):
Display of energy dependence for IB corrections and uncertainties

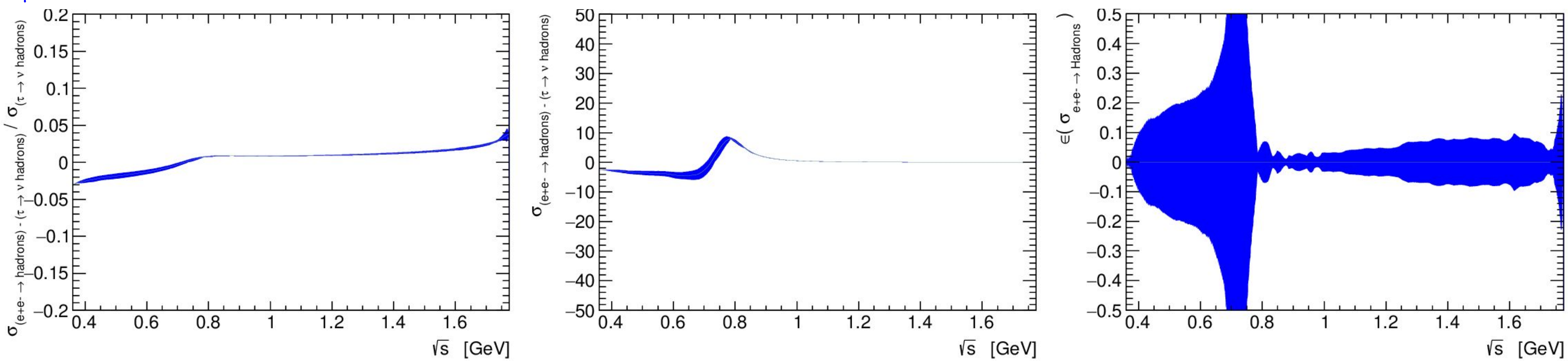
$$R_{\mathrm{IB}}(s) = \frac{\mathrm{FSR}(s)}{G_{\mathrm{EM}}(s)} \frac{\beta_0^3(s)}{\beta_-^3(s)} \left| \frac{F_0(s)}{F_-(s)} \right|^2$$

Moment integrals from IB corrections for τ data (2π channel)

a_μ [0.36 , 1.775 GeV] = (-11.94 ± 0.15) × 10^{-10} → Corr. & unc. from *IB Sew (Short-distance EW radiative effects)*

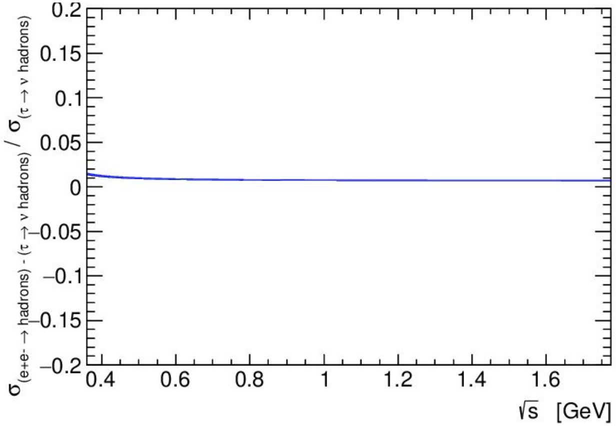


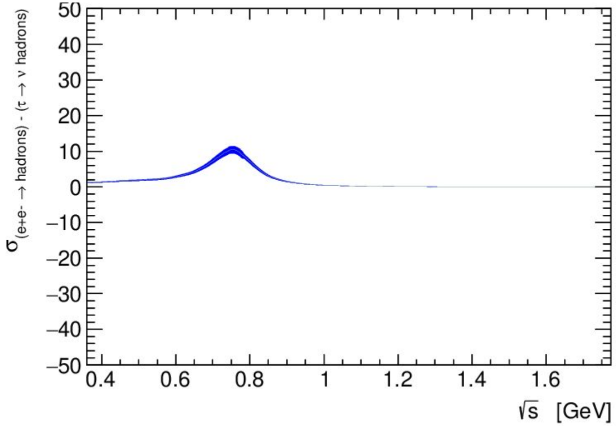
a_μ [0.36 , 1.775 GeV] = (-1.31 ± 0.94) × 10^{-10} → Corr. & unc. from *IB Gem (long-distance radiative corrections)*

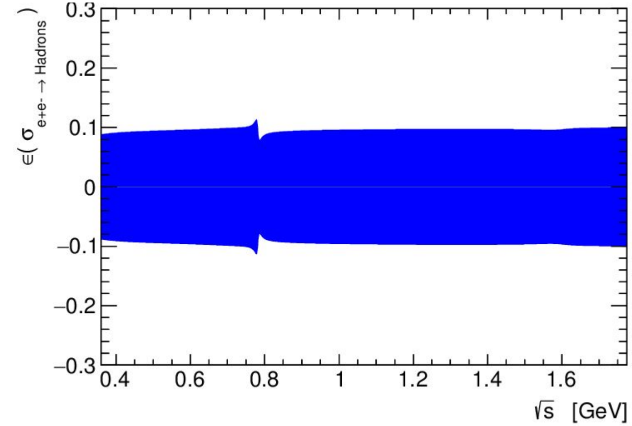


Moment integrals from IB corrections for τ data (2π channel)

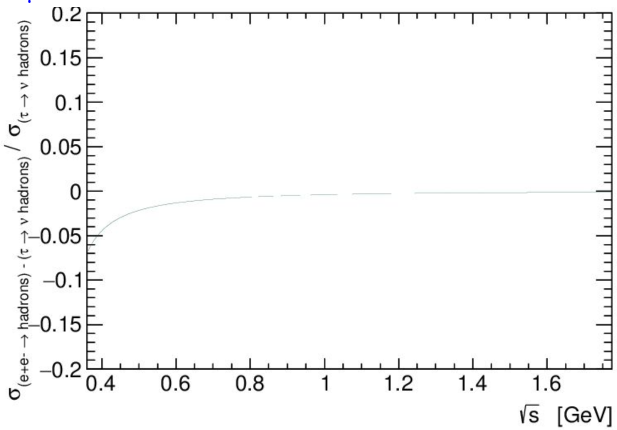
a_μ [0.36 , 1.775 GeV] = (4.41 ± 0.43) × 10^{-10} → Corrections and uncertainties from *IB FSR*

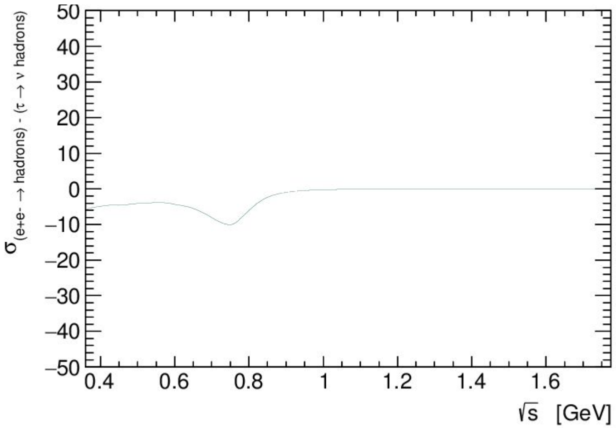






a_μ [0.36 , 1.775 GeV] = (-6.05 ± 0) × 10^{-10} → Corrections and uncertainties from *IB beta ($\pi^{\pm} - \pi^0$ mass splitting)*

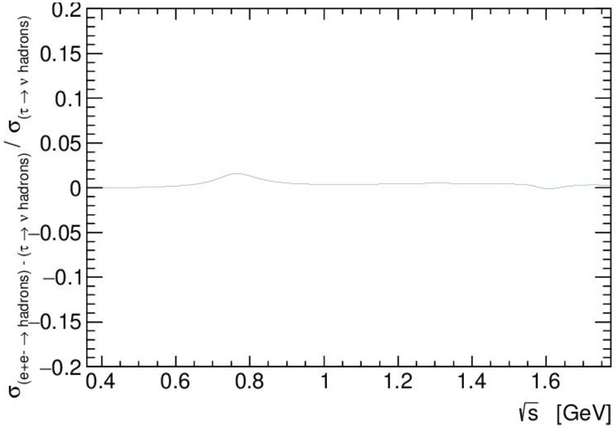


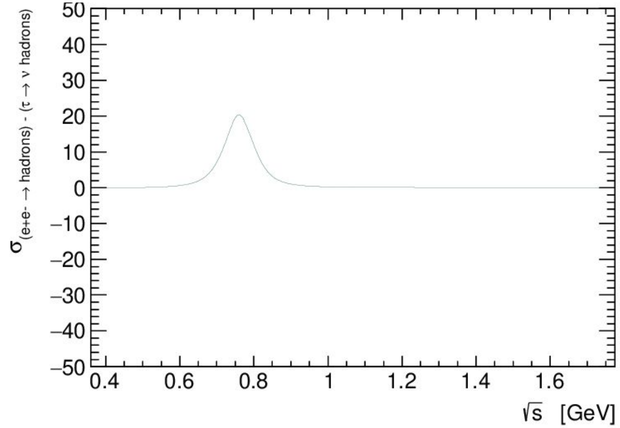


$$\frac{\beta_0^3(s)}{\beta_-^3(s)}$$

Moment integrals from IB corrections for τ data (2π channel)

a_μ [0.36 , 1.775 GeV] = $(4.11 \pm 0) \times 10^{-10}$ → Corrections and uncertainties from *IB mPi (impact on ρ width)*

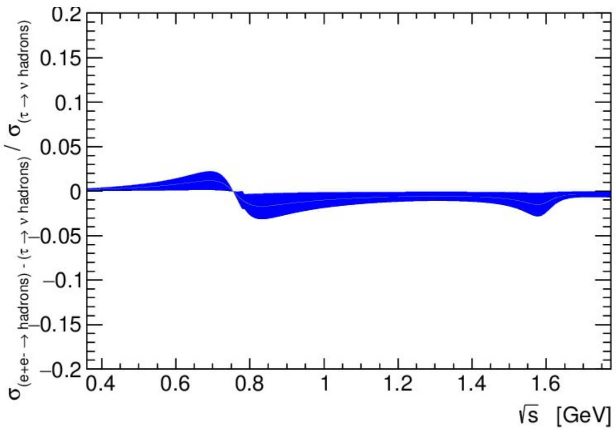


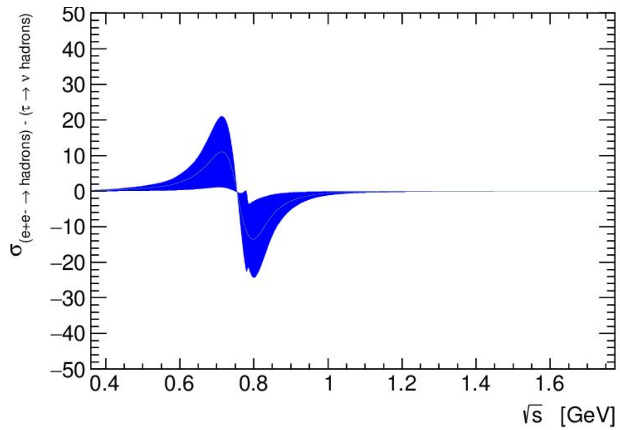


$$\left|\frac{F_0(s)}{F_-(s)}\right|^2$$

a_μ [0.36 , 1.775 GeV] = $(0.20 \pm 0.27) \times 10^{-10}$ → Corrections and uncertainties from *IB mRho*

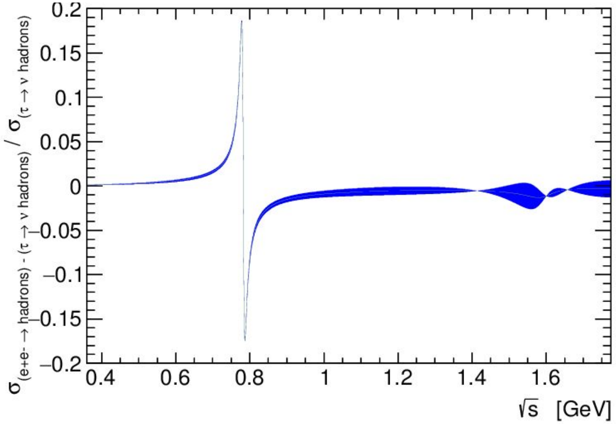
$$\left|\frac{F_0(s)}{F_-(s)}\right|^2$$

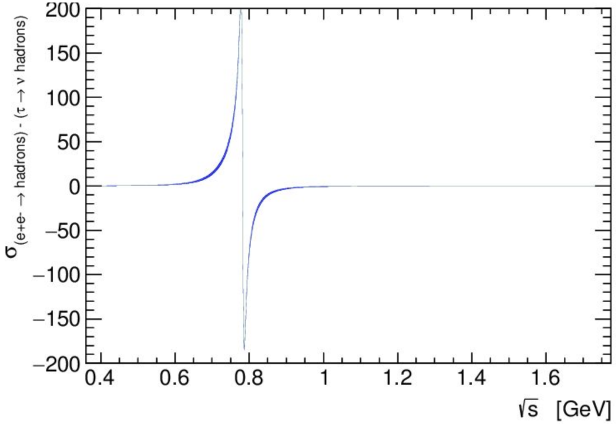


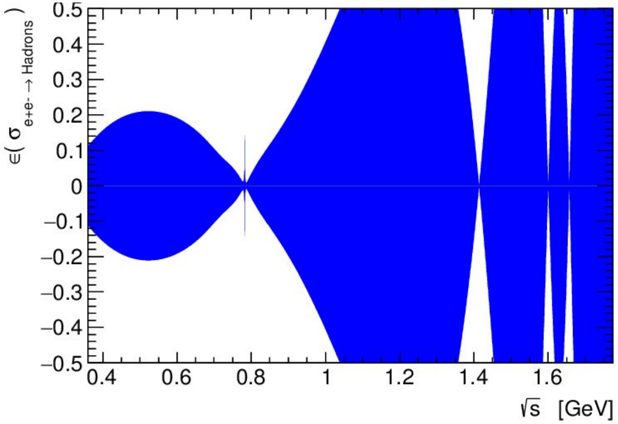


Moment integrals from IB corrections for τ data (2π channel)

a_μ [0.36 , 1.775 GeV] = (3.99 ± 0.98) × 10^{-10} → Corrections from *IB interference* and uncertainties from *IB KS-GS*

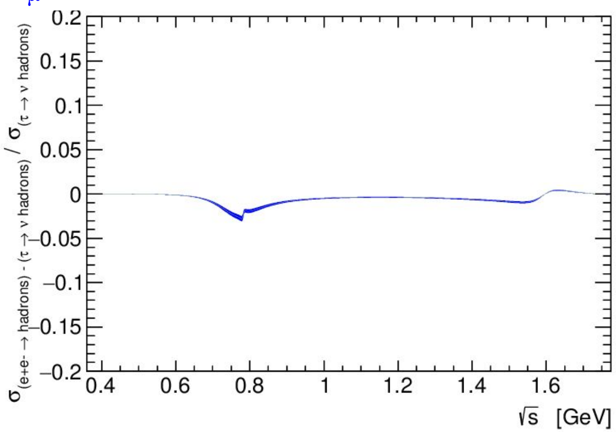


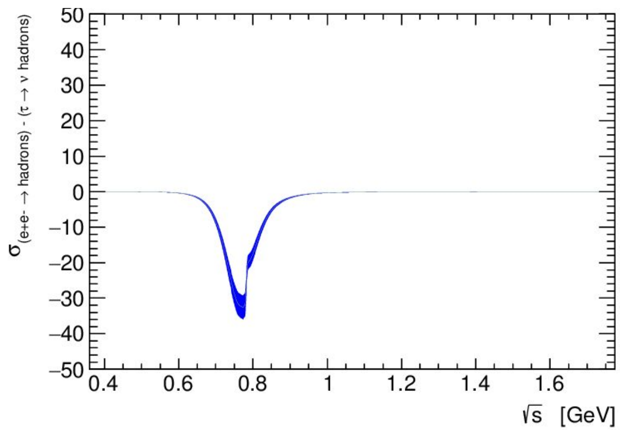


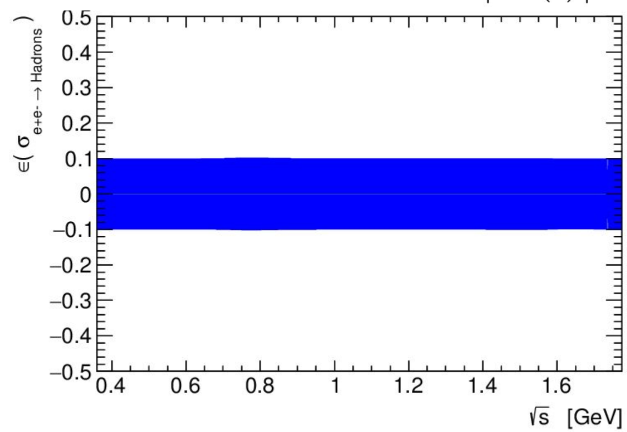


a_μ [0.36 , 1.775 GeV] = (-5.82 ± 0.59) × 10^{-10} → *IB EM decay* corrections and uncertainties from *IB EM decay*

$$\left|\frac{F_0(s)}{F_-(s)}\right|^2$$



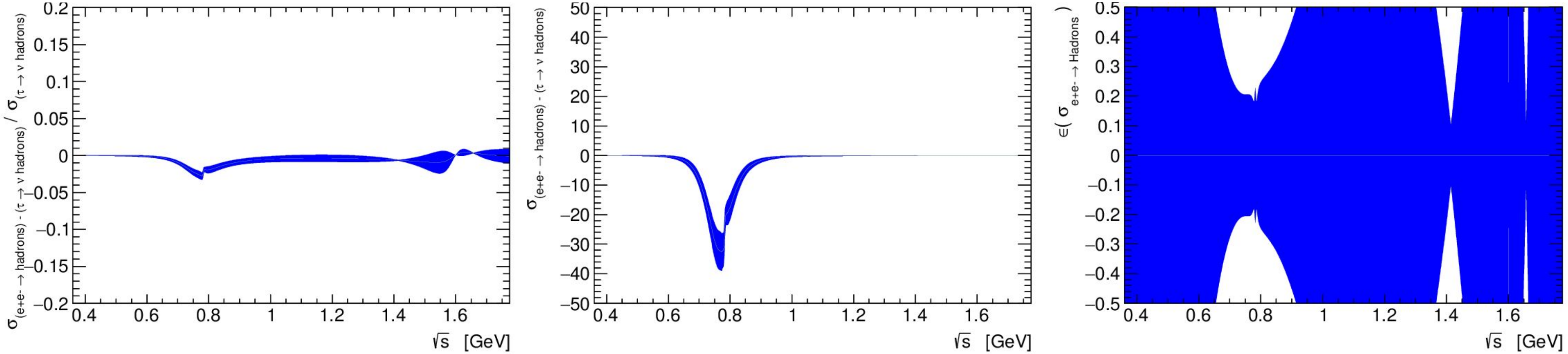




Moment integrals from IB corrections for τ data (2π channel)

a_μ [0.36 , 1.775 GeV] = (-5.82 ± 1.57) × 10^{-10}

→ *IB EM decay* corrections and uncertainties from *IB EM decay + KS-GS (conservative sum of uncertainties)*



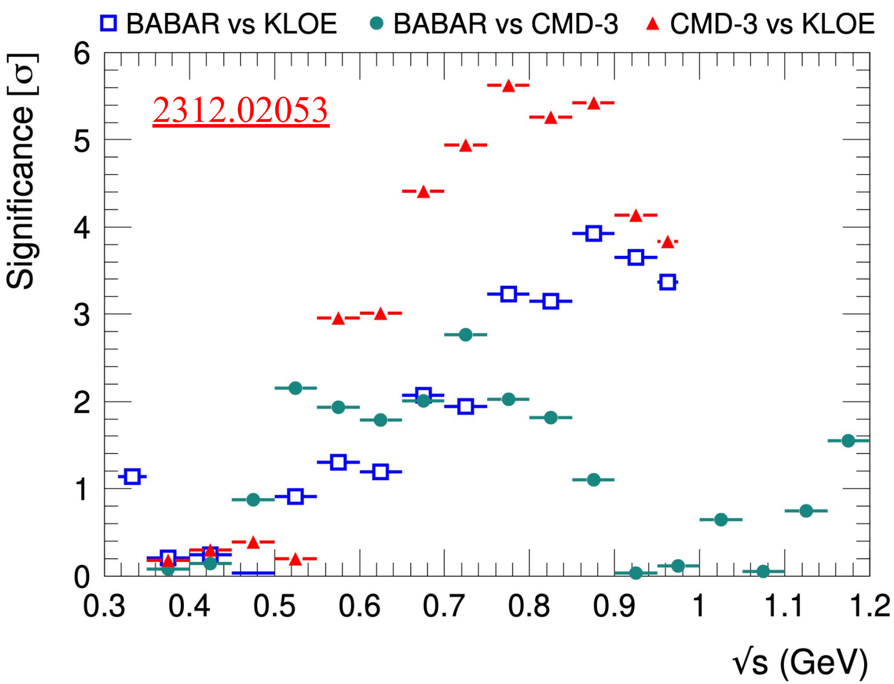
→ Note:

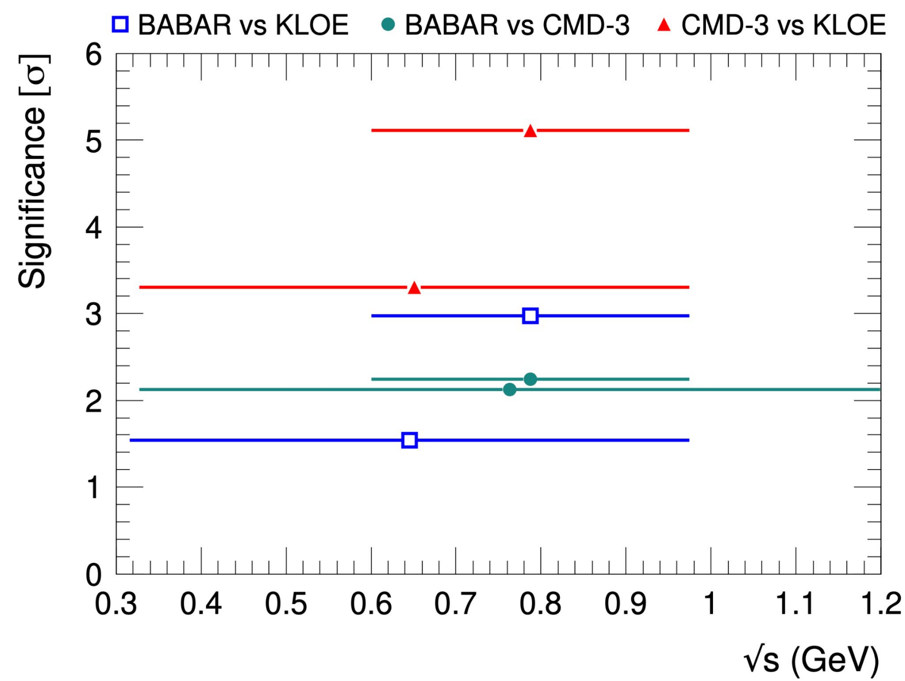
Various models for description of ρ-ω interference in IB corrections adjusted to the same e+e- data

KS-GS uncertainty, using external parameters, conservatively covers this effect

Quantitative comparisons for a_μ^{HVP}

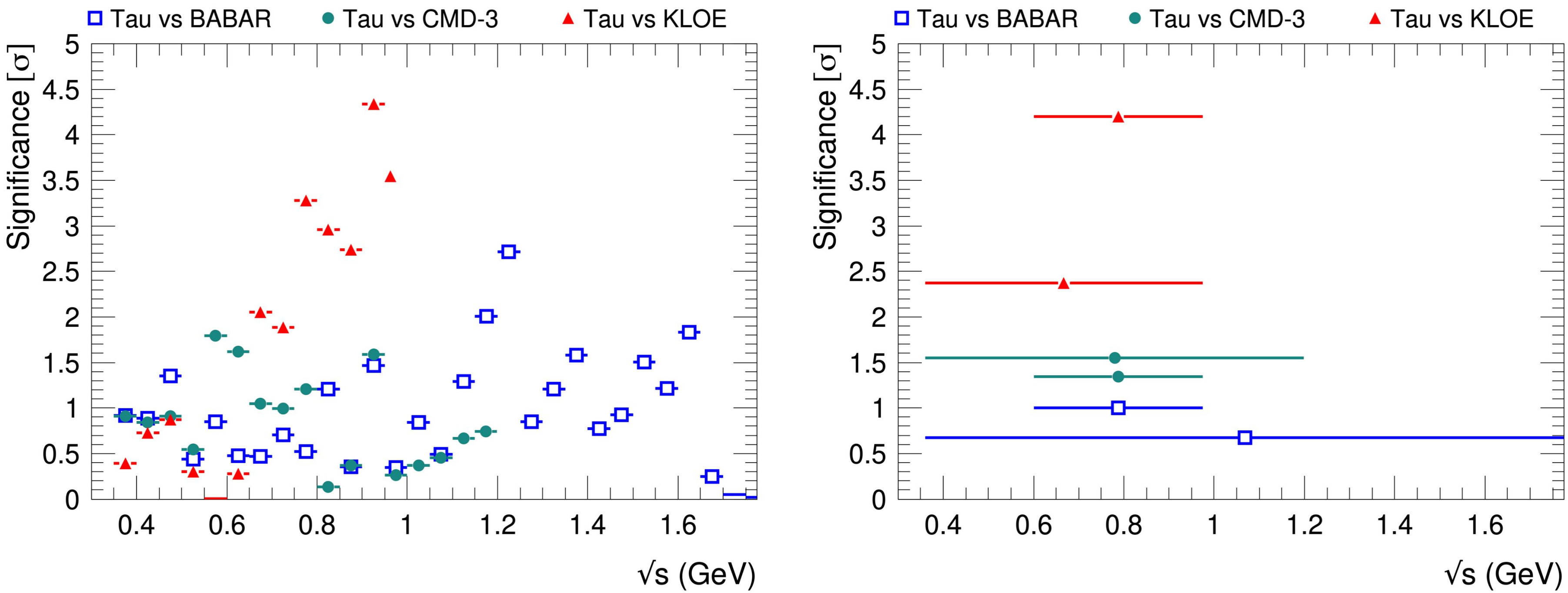
→ Comparison of integrals computed in various restricted energy ranges, for individual e^+e^- experiments: significance of the difference between different experiments, taking into account correlations





→ Largest tensions between CMD3 and KLOE

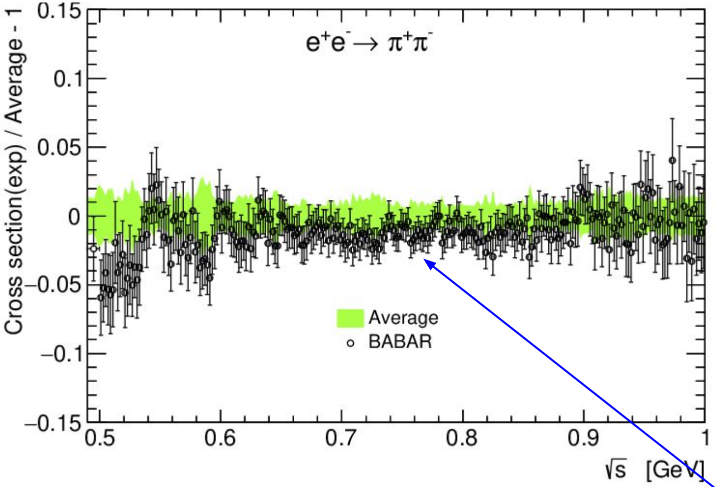
→ Comparison of integrals computed in various restricted energy ranges, for τ / individual e^+e^- experiments: significance of the difference between different experiments, taking into account correlations

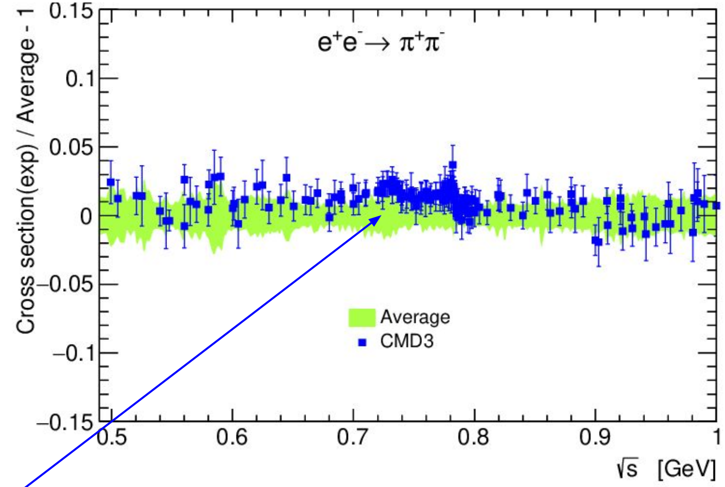


→ Largest tensions between Tau and KLOE

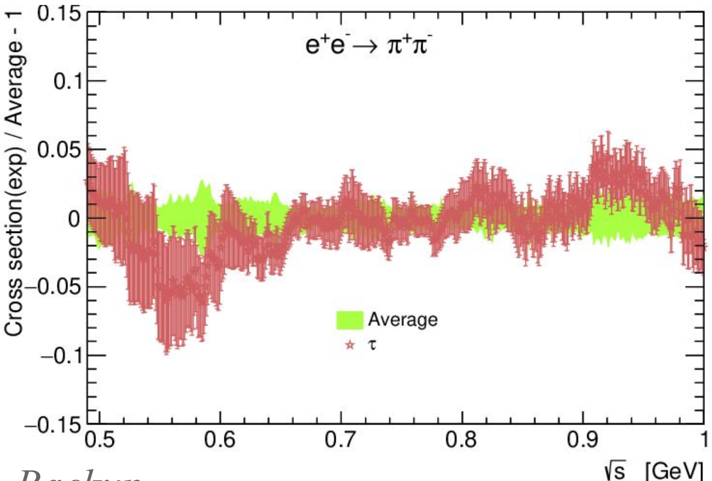
→ Good agreement among the Tau measurements (see talk @ previous mini-workshop & Backup)

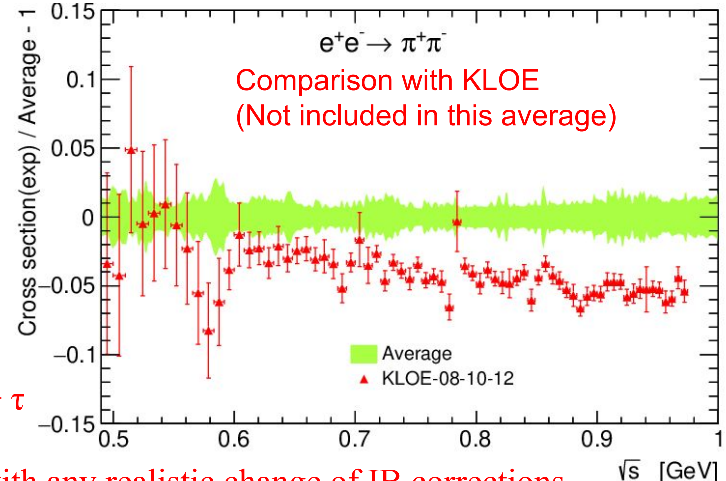
Combining the $e^+e^- \to \pi^+\pi^-$ data, BaBar & CMD3 & Tau(+IB)





Some (reduced) systematic tensions





Much larger tension (slope and shift) for KLOE vs. BABAR + CMD-3 + τ combination

$\to$ Not compatible with any realistic change of IB corrections

Backup

Discussion of Tau data inputs in White Paper 2020

- Using τ data in the dispersive method discontinued by DHMZ after 2016 (before TI)
- Special section in WP (Zhiqing) on input from hadronic τ BR and spectral functions (2.2.6)
- No other mention elsewhere

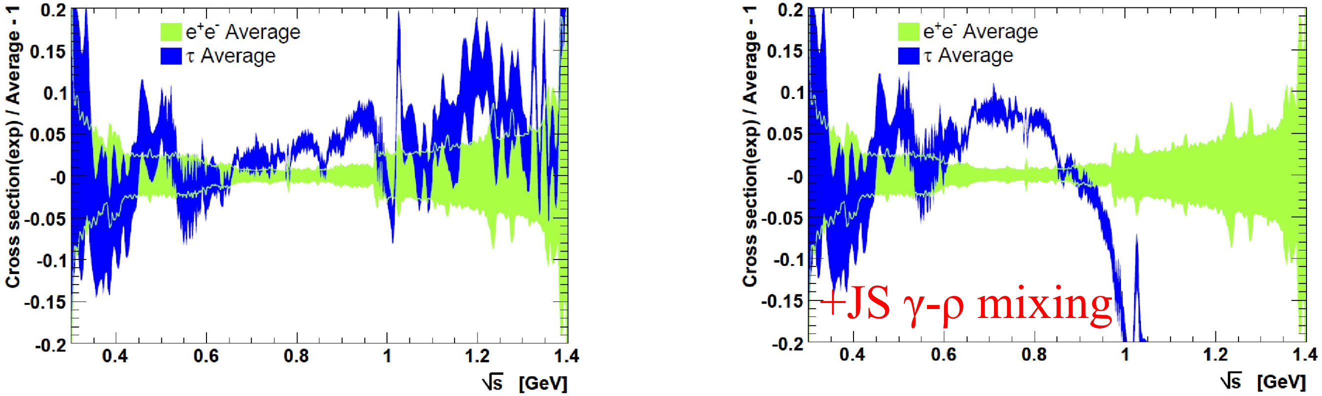
Despite the improved IB corrections, there is still a sizable difference between the e^+e^- based prediction of $692.3(4.2)\times 10^{-10}$ and the τ based one of $703.0(4.4)\times 10^{-10}$ [191]. The difference amounts to $10.7(4.9)\times 10^{-10}$, corresponding to a deviation of 2.2σ. The shape of the combined τ spectral function after the IB corrections in the two-pion channel is also different from the one from e^+e^- data (Fig. 20). The discrepancy is further reflected in the τ branching fractions (Fig. 21).

A model-dependent ρ–γ mixing, occurring only in the e^+e^- data, was proposed in Ref. [178] to explain the e^+e^-–τ discrepancy. The proposed correction corresponds to the difference between the open blue points and the solid black points in Fig. 19 (bottom right), showing an increasing effect above the ρ peak that appears uncomfortably large. Unlike γ–Z mixing on the Z resonance, well established theoretically and experimentally, the description of photon mixing with a strongly interacting ρ may be affected by significant difficult-to-assess uncertainties. The correction [178], shown in Fig. 22, seems to overestimate the observed difference.

Concluding this part, it appears that, at the required precision to match the e^+e^- data, the present understanding of the IB corrections to τ data is unfortunately not yet at a level allowing their use for the HVP dispersion integrals. It remains a possibility, however, that the alternate lattice approach, discussed in Section 3.4.2, may provide a solution to this problem.

ee-τ discrepancy driven by KLOE, not by BABAR and nor CMD-3

Jegerlehner-Szafron(JS) γ-ρ mixing



The new context for dispersive HVP since White Paper 2020 and Tau data

- At the time of WP 2020 $\Delta a_\mu^{\text{HVP LO}}$ (10^{-10})

    $\text{KLOE}_{\text{peak}}$ (0.6-0.9+comb) 2.3

    BABAR 3.8

    BABAR − KLOE difference 9.8 (5.6 found with all-KLOE/all-BABAR)

- now

→ γ-ρ mixing not justified from theoretical point of view (discussions with several TI theorists)

    CMD-3 4.2 *result changing e+e- data landscape*

    CMD-3 − KLOE difference 21.6

    BABAR LO/NLO/NNLO study: *points to a necessary revisiting of KLOE analysis*

- Focusing on τ for 2π (competitive with best e^+e^- 2π) + e^+e^- for the rest (non-2π + I=0)

    data $1.9_{\text{spectrum}} \oplus 2.2_{\text{BR}} = 2.9$

    IB correction -14.9 ± 1.9 uncertainty x11 smaller than CMD3-KLOE ≠

Questions from Vincenzo

- Vincenzo Cirigliano, cirigv@uw.edu : Note that the following questions apply to both the 'Present status of phenomenological analysis' and 'Long distance EM corrections' talks.
    - Q1: Please discuss the uncertainty in G_EM(s) and F_0(s)/F_- (s) arising from using different model parameterizations of the form factors. Are there strategies to mitigate this intrinsic model dependence?
    - Q2: Please discuss the uncertainty in F_0(s)/F_−(s) due to rho resonance parameters (difference in masses and widths, …). How robust are the current determinations of these parameters?
    - Q3: Please discuss the uncertainties in G_EM(s) induced by the structure-dependent effects, both in loops involving virtual photons and in real photon emission.
    - Q4: Please discuss uncertainties in the short-distance correction S_EW associated with the renormalization group running and the matching to the long-distance corrections G_EM(s). To a given order, the product S_EW*G_EM(s) should be independent on the renormalization scale and scheme. Do we control the scheme (in)dependence to O(alpha/pi)?

Backup

Present IB corrections and uncertainties

- Summary of IB corrections applied in arxiv:2312.02053 ($\times 10^{-10}$)
- Short-distance radiative EW (SEW): -12.21 ± 0.15
- Long-distance radiative (GEM): -1.92 ± 0.90
- FSR: $+4.67 \pm 0.47$
- $\pi^-\pi^0$ mass difference (β^3) in cross section: -7.88
- $\pi^-\pi^0$ mass difference (β^3) in ρ width: $+4.09$
- $\rho^-\rho^0$ mass difference: $+0.20 \pm^{0.27}_{0.19}$
- EM decays, mostly $\pi\pi\gamma$ in ρ width: -5.91 ± 0.59
- $\rho-\omega$ interference: $+4.0 \pm 0.4$
- Sum: -14.9 ± 1.9

Combine cross section data: goal and requirements

→ Goal: combine experimental spectra with arbitrary binning (/point spacing)

→ Requirements:

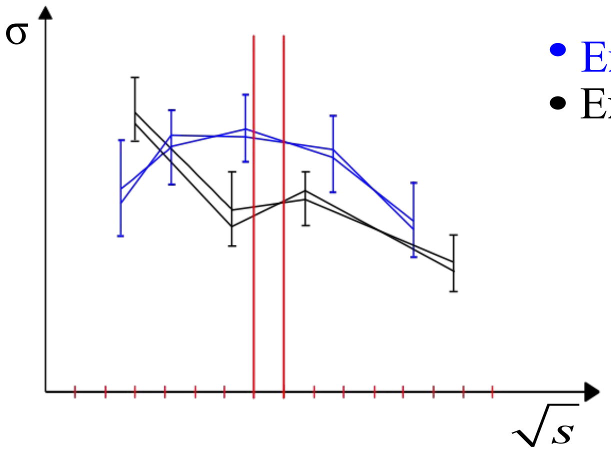
- Properly propagate uncertainties and correlations
  - *Between measurements (data points/bins) of a given experiment* (covariance matrices and/or detailed split of uncertainties in sub-components)
  - *Between experiments* (common systematic uncertainties) based on detailed information provided in publications
  - *Between different channels* – motivated by understanding of the meaning of systematic uncertainties and identifying the common ones
- Minimize biases
- Optimize g-2 integral uncertainty (*without overestimating the precision with which the uncertainties of the measurements are known*)

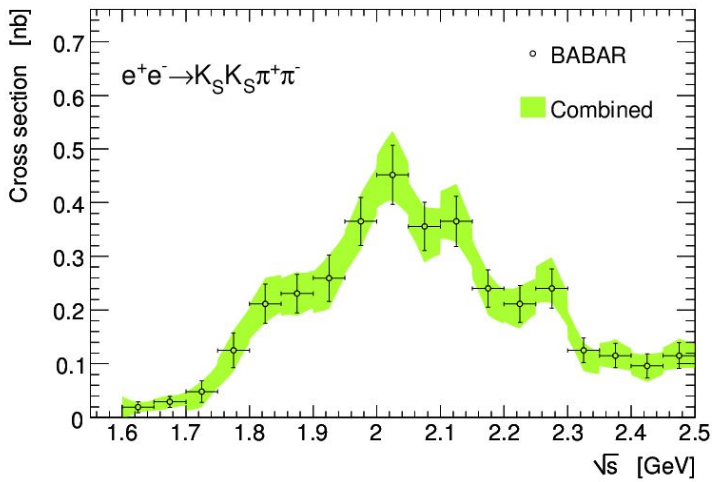
Procedure and software (*HVPTools - Since 2009*) for combining cross section data with arbitrary binning

→ Validated through closure test

→ Featuring full & realistic (i.e. not too optimistic) treatment of uncertainties and correlations, fully accounting for possible systematic tensions between experiments.

Combination procedure implemented in HVPTools software





→ Define a (fine) final binning (to be filled and used for integrals etc.)

→ Linear/quadratic splines to interpolate between the points/bins of each experiment
- for binned measurements: preserve integral inside each bin
- closure test: replace nominal values of data points by Gounaris-Sakurai model and re-do the combination
  → *(non-)negligible bias for (linear)quadratic interpolation*

→ Fluctuate data points taking into account correlations & re-do the splines for each (pseudo-)experiment
- each uncertainty fluctuated coherently for all the points/bins that it impacts
- eigenvector decomposition for (statistical) covariance matrices

Combination procedure implemented in HVPTools software

For each final bin:

→ Compute an average value for each measurement and its uncertainty

→ Compute correlation matrix between experiments

→ Minimize χ^2 and get average coefficients (weights)

→ Compute average between experiments and its uncertainty

Evaluation of integrals and propagation of uncertainties:

→ Integral(s) evaluated for nominal result and for each set of toy pseudo-experiments; uncertainty of integrals from RMS of results for all toys

→ The pseudo-experiments also used to derive (statistical & systematic) covariance matrices of combined cross sections → Integral evaluation

→ Uncertainties also propagated through $\pm 1\sigma$ shifts of each uncertainty: also allows to account for correlations between different channels (for integrals and spectra)

→ *Checked consistency between the different approaches*

Combining the τ data in the ππ channel

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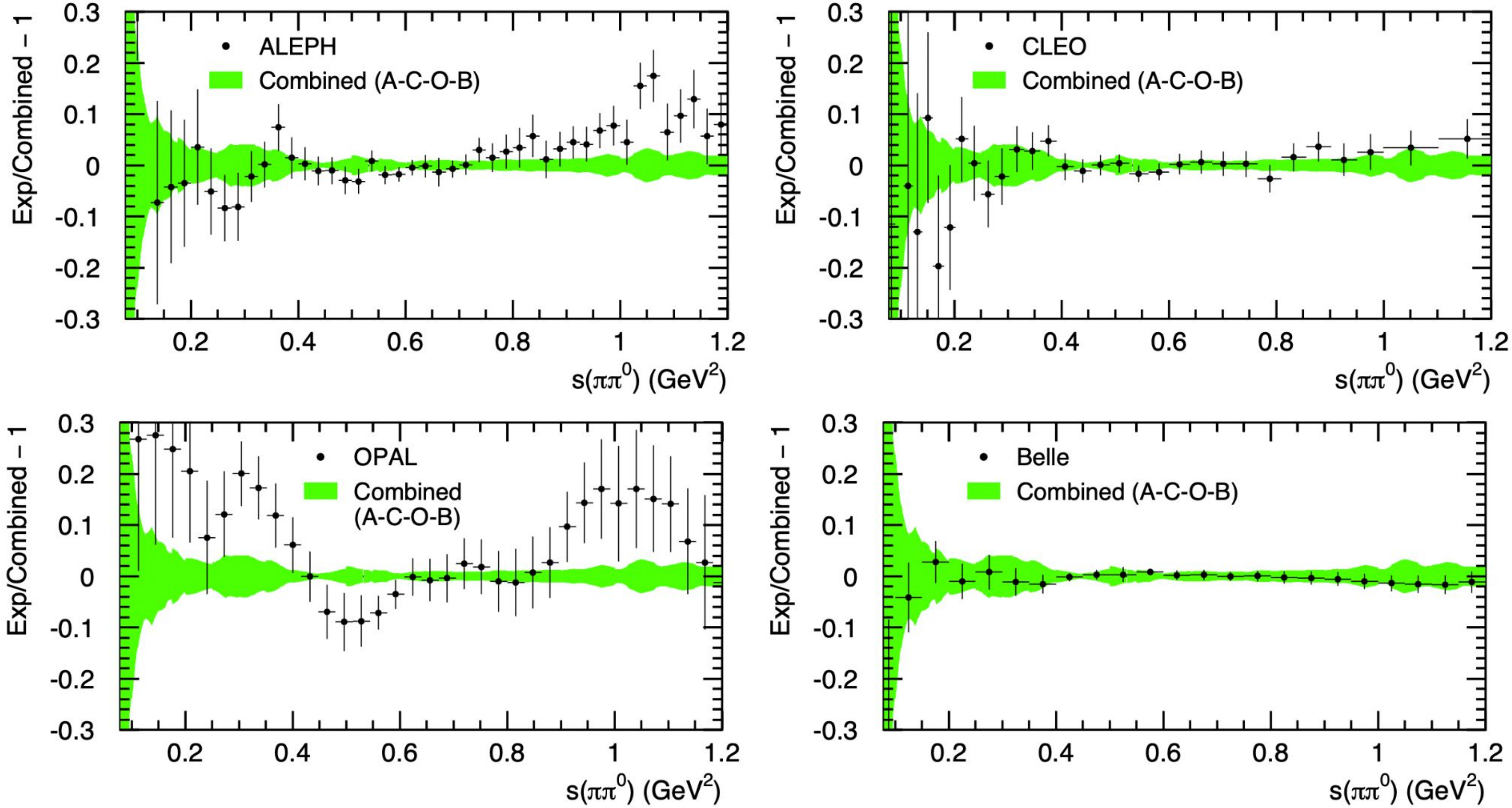
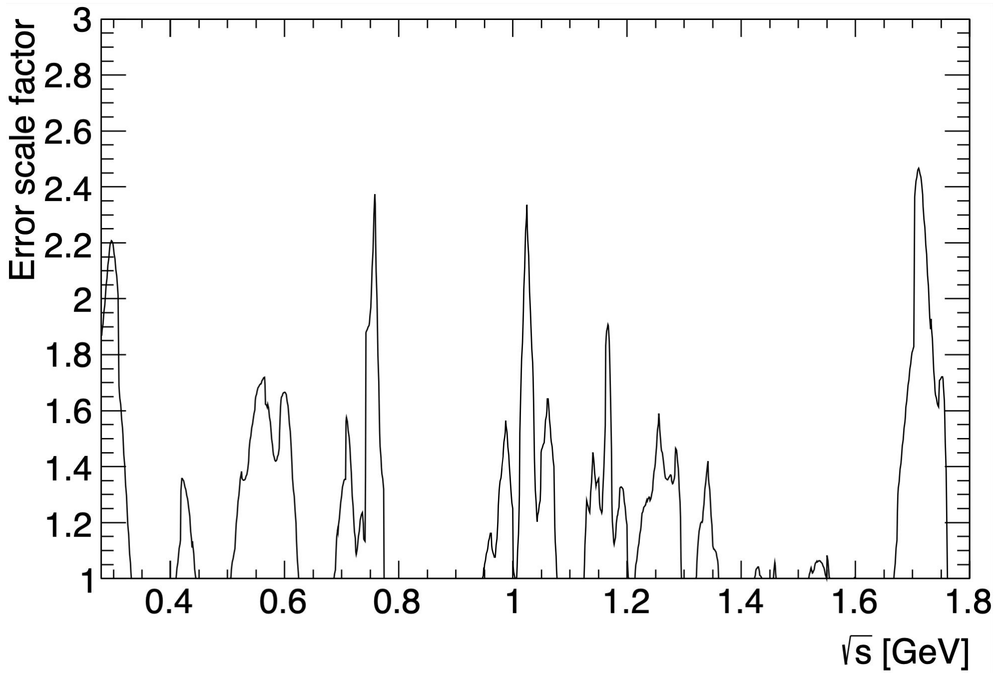


Combination: compatibility between measurements

For each final bin:

→ χ^2/ndof: test locally the level of agreement between input measurements, *taking into account correlations*

→ Scale uncertainties in bins with χ^2/ndof > 1 (PDG)



→ Level of agreement significantly better than the one observed for $e^+e^- \to \pi^+\pi^-$ data

Combination: weights of various measurements

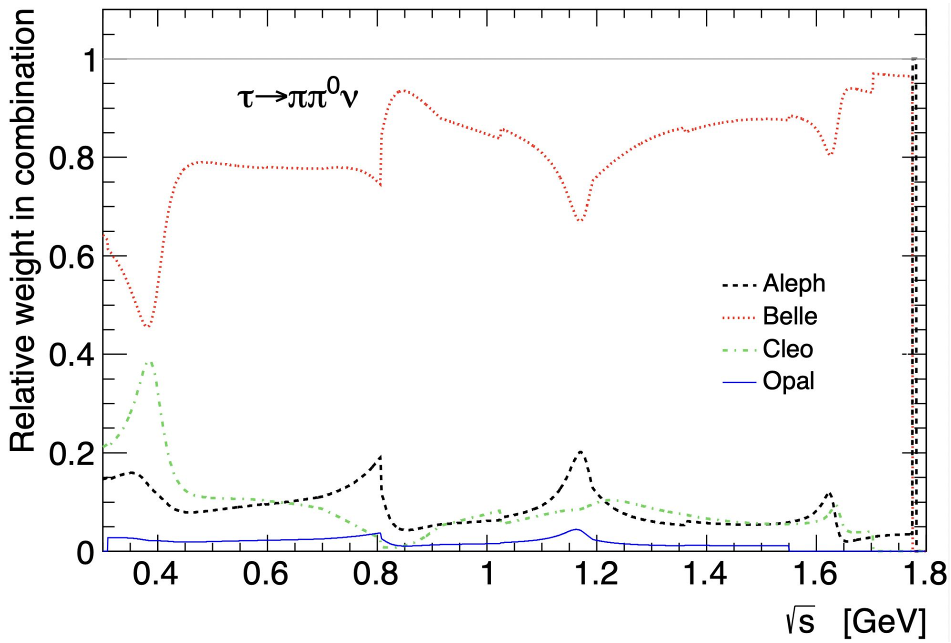
For each final bin:

→ Minimize χ^2 and get average coefficients

Note: average weights must account for bin sizes / point spacing of measurements
(Compare the precisions on the same footing: do not over-estimate the weight of experiments with large bins)

→ Weights in fine bins evaluated using a common (large) binning for measurements + interpolation

→ Their determination also integrates bin-to-bin statistical & systematic correlations on moderate energy ranges



→ Shape information provided mainly by Belle (reflected by the weights from the combination of spectra)

Combining the τ data in the $\pi\pi$ channel

→ Normalisation dominated by ALEPH (directly impacting and very relevant for the integrals)

Experiment	$a_\mu^{\rm had,LO}[\pi\pi,\tau]$ (10^{-10}) $2m_{\pi^\pm} - 0.36$ GeV	$0.36 - 1.8$ GeV
ALEPH	$9.80 \pm 0.40 \pm 0.05 \pm 0.07$	$501.2 \pm 4.5 \pm 2.7 \pm 1.9$
CLEO	$9.65 \pm 0.42 \pm 0.17 \pm 0.07$	$504.5 \pm 5.4 \pm 8.8 \pm 1.9$
OPAL	$11.31 \pm 0.76 \pm 0.15 \pm 0.07$	$515.6 \pm 9.9 \pm 6.9 \pm 1.9$
Belle	$9.74 \pm 0.28 \pm 0.15 \pm 0.07$	$503.9 \pm 1.9 \pm 7.8 \pm 1.9$
Combined	$9.82 \pm 0.13 \pm 0.04 \pm 0.07$	$506.4 \pm 1.9 \pm 2.2 \pm 1.9$

Table 6. The isospin-breaking-corrected $a_\mu^{\rm had,LO}[\pi\pi,\tau]$ (in units of 10^{-10}) from the measured mass spectrum by ALEPH, CLEO, OPAL and Belle, and the combined spectrum using the corresponding branching fraction values. The results are shown separately in two different energy ranges. The first errors are due to the shapes of the mass spectra, which also include very small contributions from the τ-mass and $|V_{ud}|$ uncertainties. The second errors originate from $B_{\pi\pi^0}$ and B_e, and the third errors are due to the isospin-breaking corrections, which are partially anti-correlated between the two energy ranges. The last row gives the evaluations using the combined spectra.

Individual measurements with the corresponding uncertainties:

ALEPH: 511.0 ± 5.3 (±1.9 common, from IB)

CLEO: 514.2 ± 10.1

OPAL: 526.9 ± 12.3

Belle: 513.7 ± 8.0

→ Most precise determination from ALEPH, due to most precise Br

→ Uncertainty from combined spectra (±2.9) smaller than uncertainty from weighted average of integrals (±3.8):
Due to better use of the available information on the precision of the measurements (Br and mass-dependent uncertainties)

χ^2 : 1.45/3 dof, when averaging the 4 individual integrals

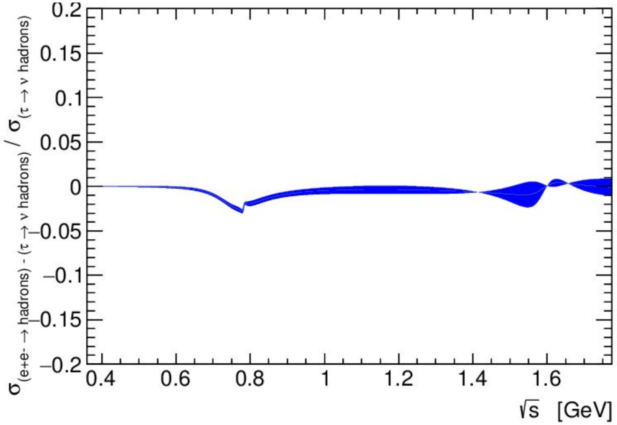
χ^2 : 1.88/3-4 dof, when comparing the 4 individual integrals with the integral of the combined spectrum

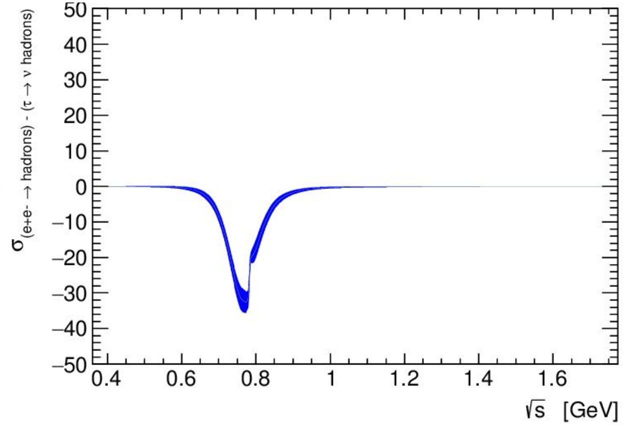
→ Excellent agreement among the 4 measurements

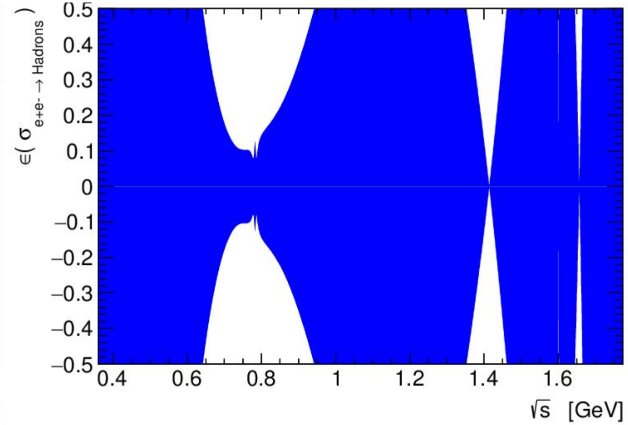
Moment integrals from IB corrections for τ data (2π channel)

a_μ [0.36 , 1.775 GeV] = (-5.82 ± 0.98) × 10^{-10}

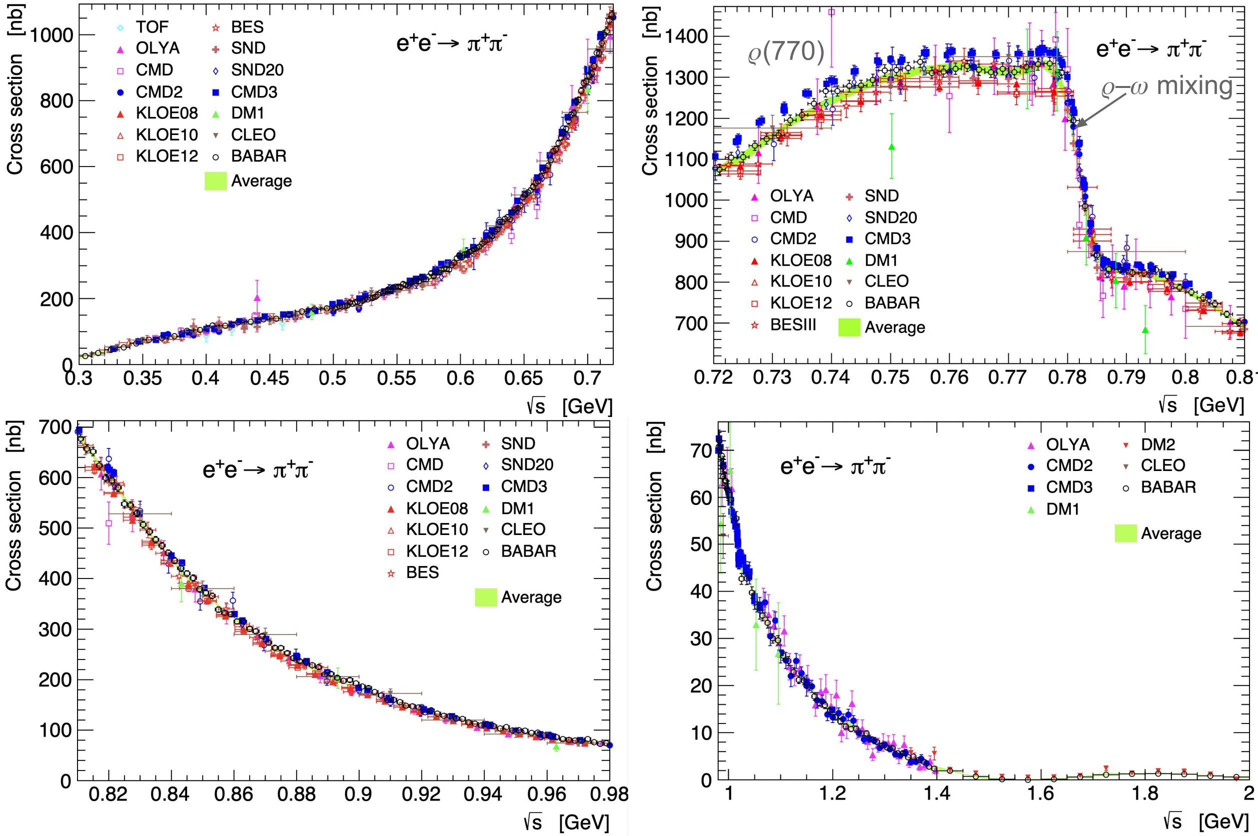
→ *IB EM decay* corrections and uncertainties from *IB KS-GS*







Experimental data combination (Example: $e^+e^- \to \pi^+\pi^-$ channel)



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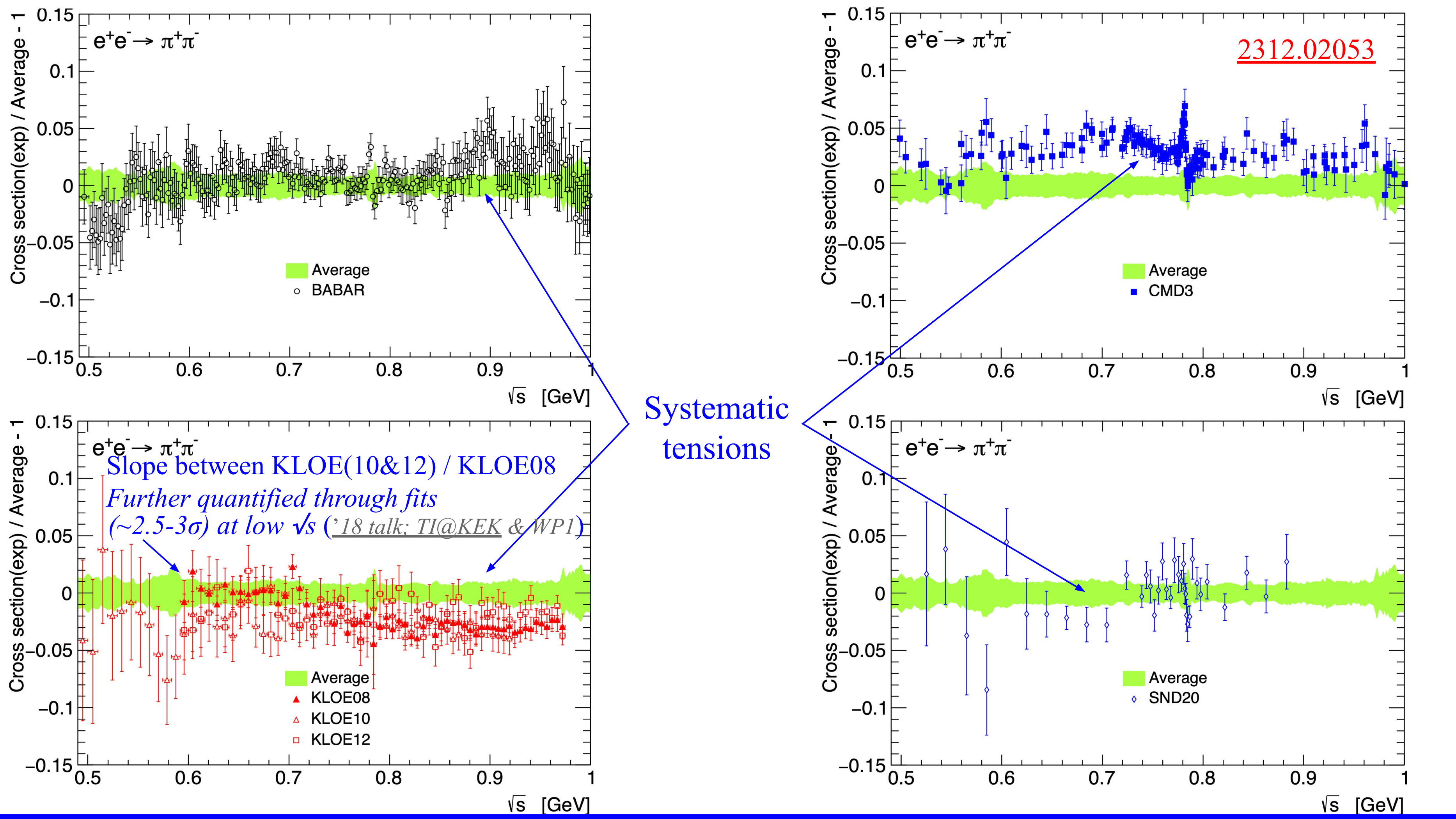
→ *New since g-2 Theory Initiative White Paper*: Large tensions, especially between KLOE & CMD3, which provide the smallest / largest cross-sections in the ρ region

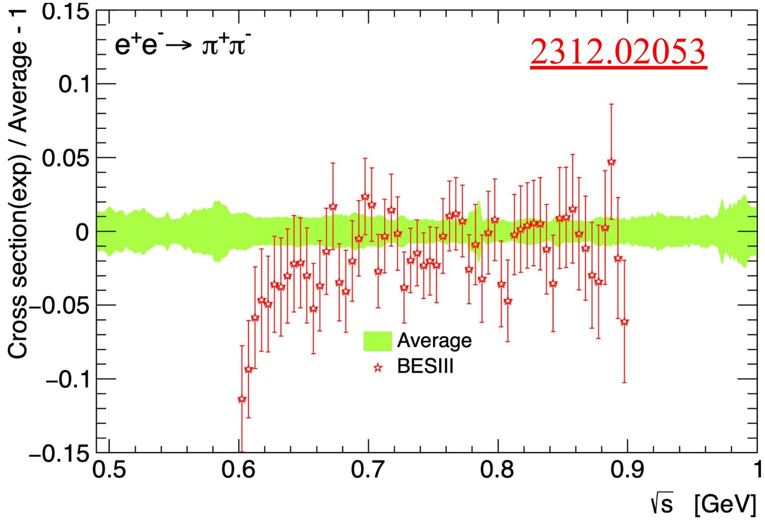
Backup

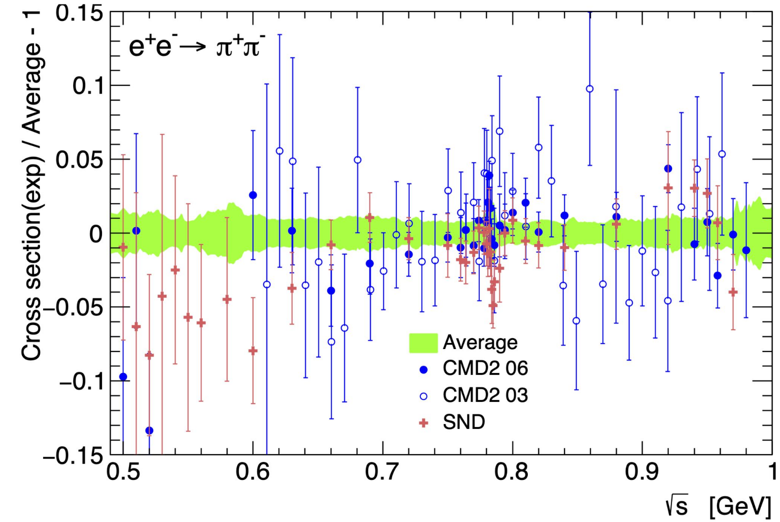
Procedure and software (*HVPTools - Since 2009*) for combining cross section data with arbitrary point spacing/binning → Validated through closure test. Featuring full & realistic (i.e. not too optimistic) treatment of uncertainties and correlations (between measurements (data points/bins) of a given experiment, b. experiments, b. different channels), fully accounting for systematic tensions between experiments.

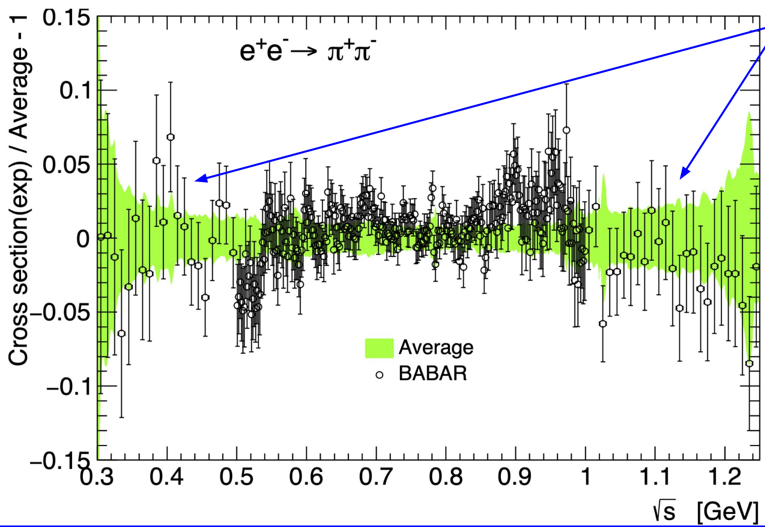
Combining the $e^+e^- \to \pi^+\pi^-$ data: relative differences



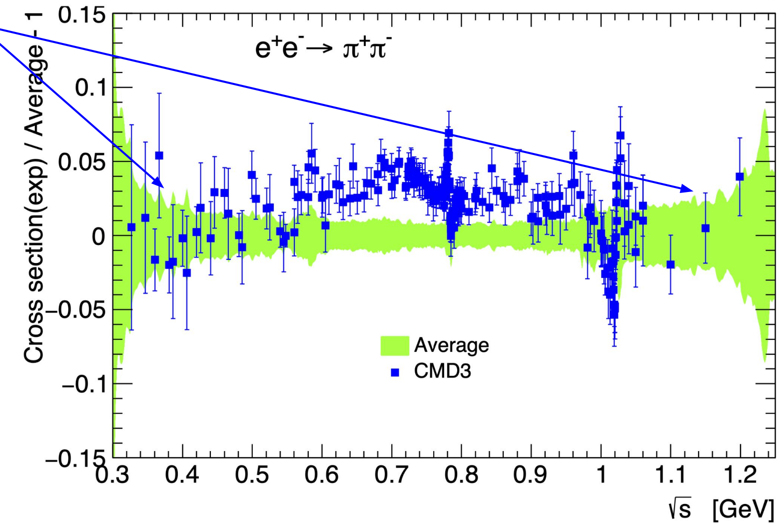
Combining the $e^+e^- \to \pi^+\pi^-$ data: relative differences





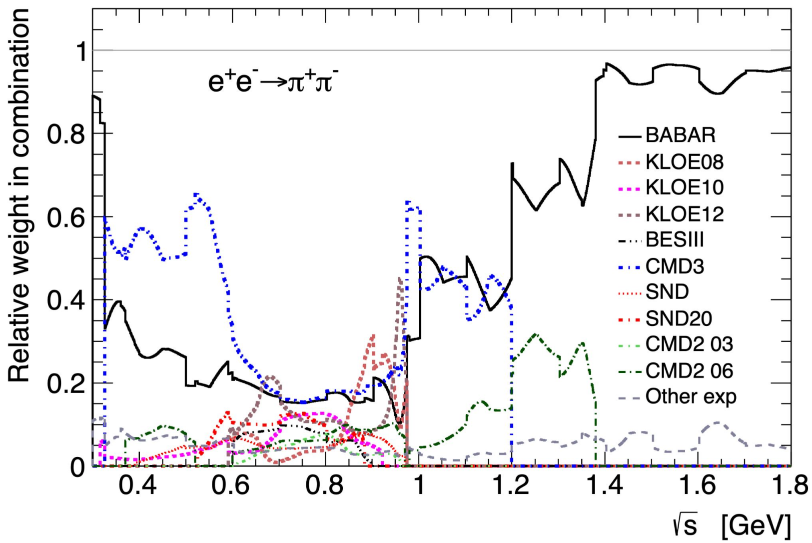


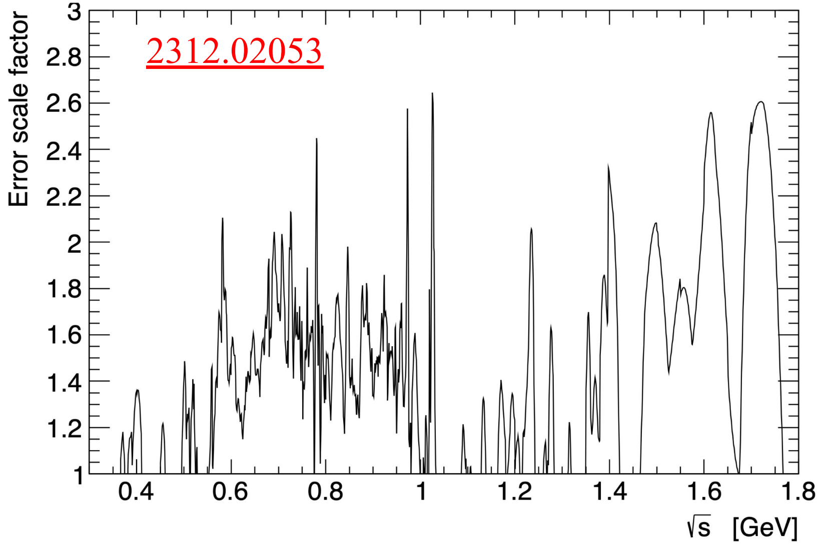
Reasonable BABAR/CMD3 agreement at low & high E



Spline-based combination procedure: weights and tension

→ For each narrow final bin minimize χ^2 to get average coefficients test locally the level of agreement
Average weights account for bin sizes/point-spacing of measurements (compare precisions on same footing), while their determination integrates bin-to-bin statistical & systematic correlations on moderate energy ranges





→ Average dominated by BaBar, CMD3, KLOE, SND20; *BaBar covers full energy range*

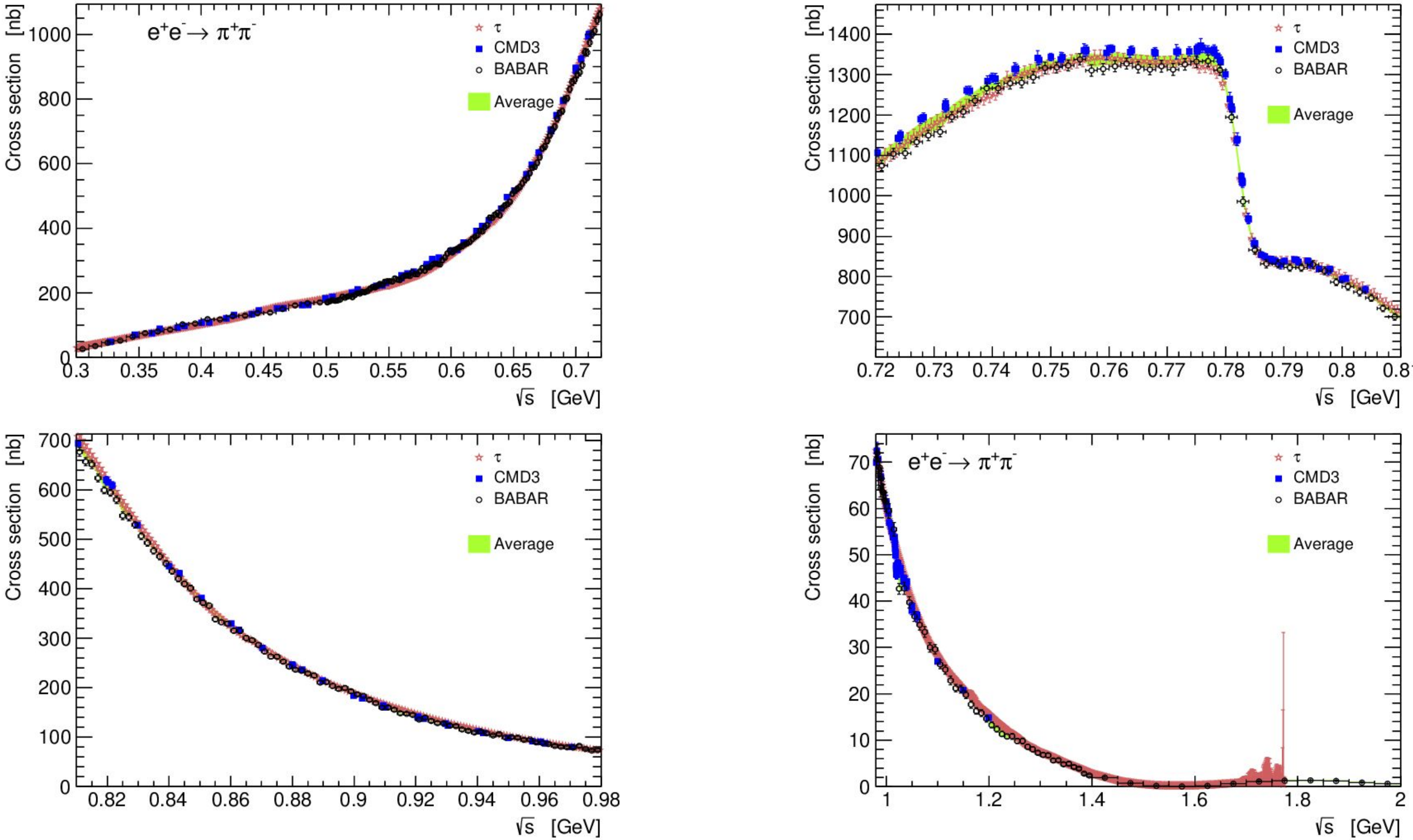
→ Enhanced tensions, especially between KLOE & CMD3, which provide the smallest / largest cross-sections in the ρ region: *clear indication of underestimated uncertainties* (*'18 talk; TI@Mainz & WP1*)

→ *Calls for conservative uncertainty treatment* in combination fit (fits / evaluation of weights)

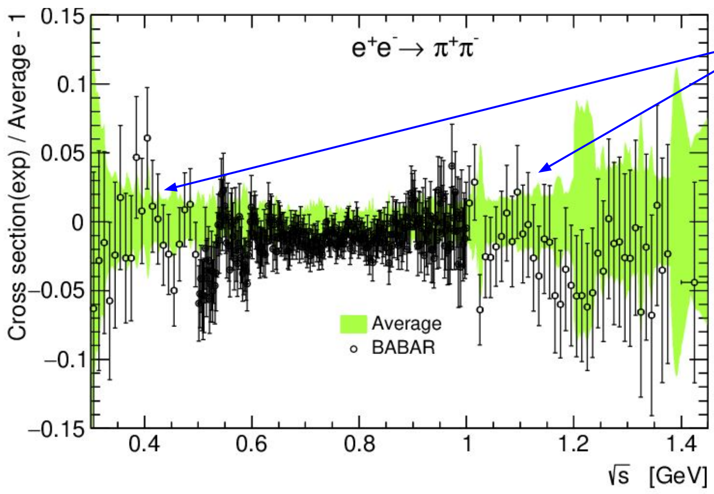
→ Systematic effects beyond the local χ^2 /ndof rescaling: had already motivated the inclusion of the dominant BABAR-KLOE systematic by DHMZ since 2019, but *tensions are larger now*

Combining the $e^+e^- \to \pi^+\pi^-$ data, BaBar & CMD3 & Tau(+IB)

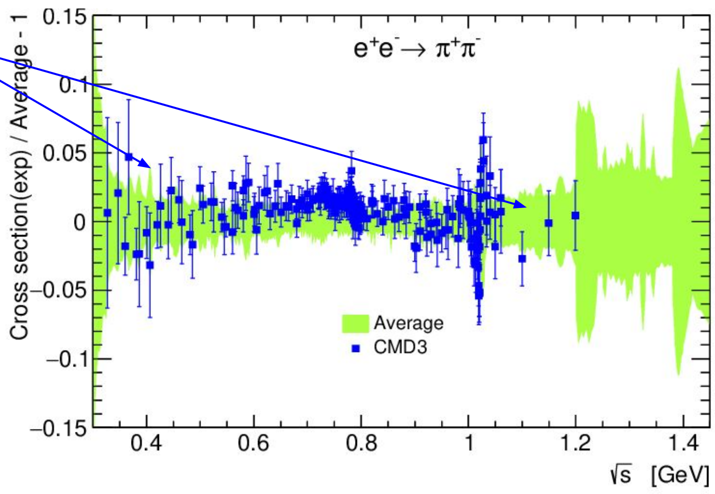
→ Motivated by the previous findings, combine τ, BABAR and CMD-3 spectra

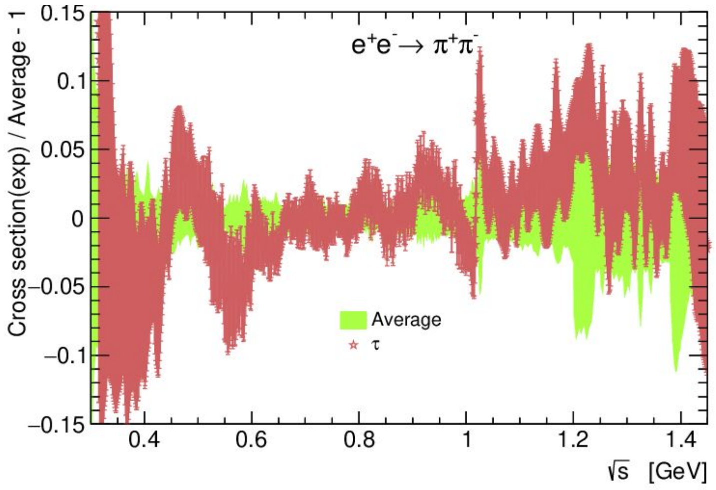


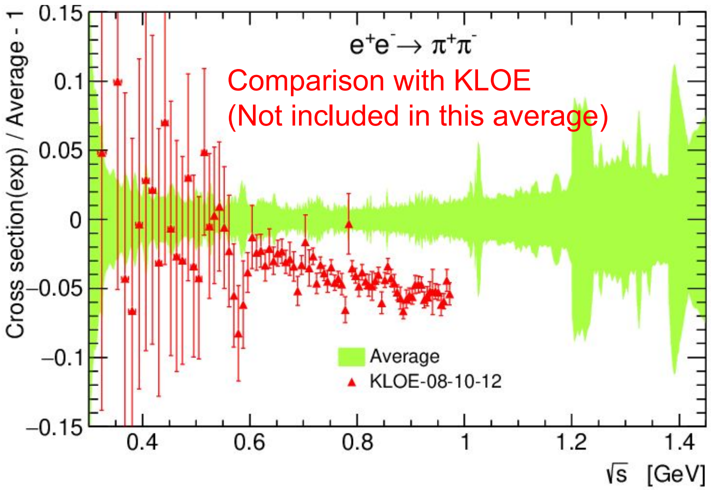
Combining the $e^+e^- \to \pi^+\pi^-$ data, BaBar & CMD3 & Tau(+IB)



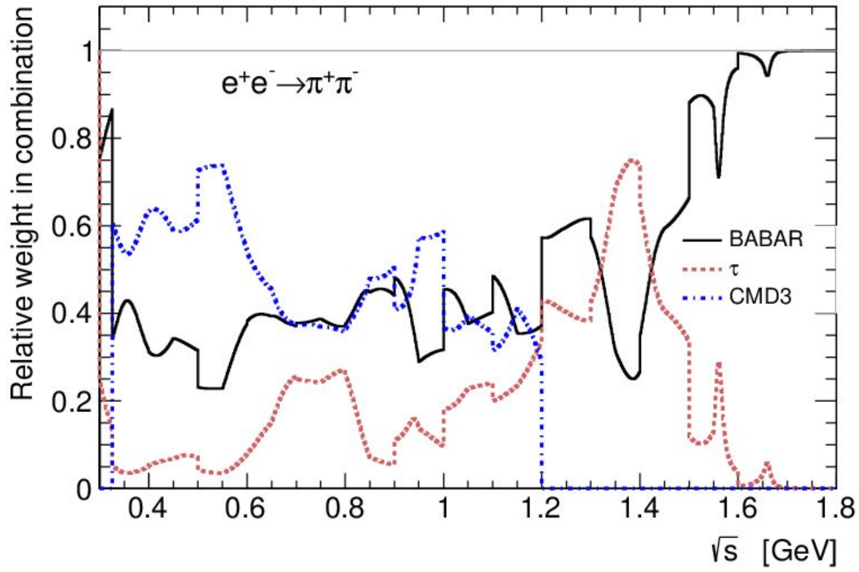
Reasonable BABAR/CMD3 agreement at low & high E

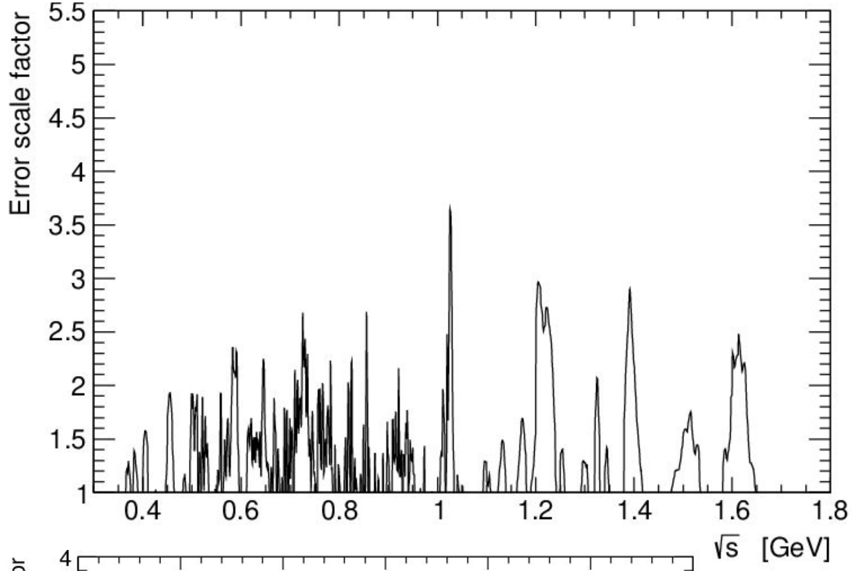


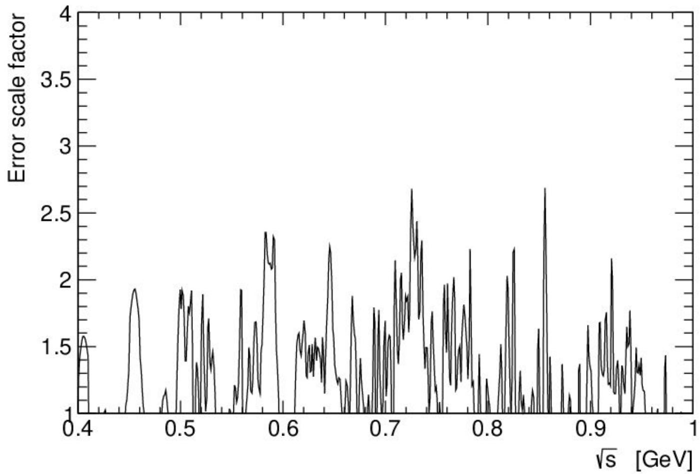




Combining the $e^+e^- \to \pi^+\pi^-$ data, BaBar & CMD3 & Tau(+IB)







→ Average dominated by BaBar and CMD3;
BaBar and τ cover full energy range

→ Some tension between BaBar & CMD3 in the ρ region

→ Much larger tension (slope and shift) when comparing KLOE with the BABAR + CMD-3 + τ combination