Probing the early Universe with BSMPT v3: CP in the Dark, a test case

LHC Higgs Working Group WG3 (BSM) -
Extended Higgs Sector subgroup meeting
November 19th, 2024

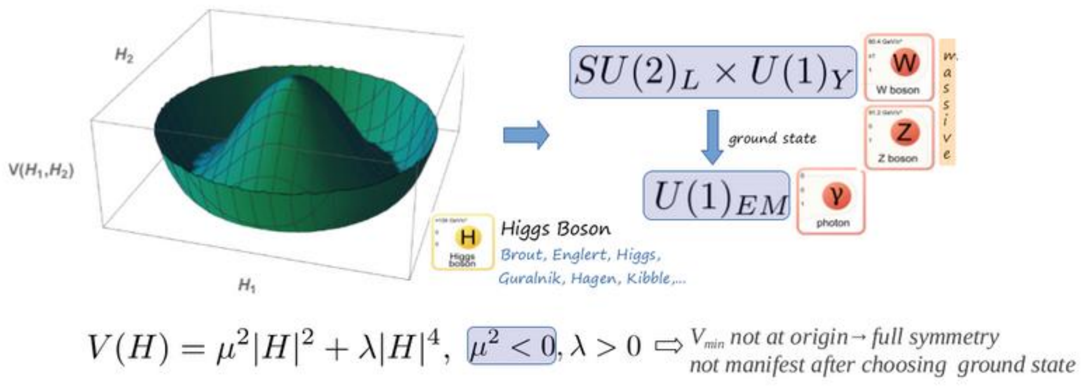
Philipp Basler, Lisa Biermann, Margarete Mühlleitner,
Jonas Müller, Rui Santos, Johann Plotnikov, **João Viana**

Fundação para a Ciência e a Tecnologia

Funded by FCT - in the framework of the projects no. CFTC: UIDB/00618/2020, UIDP/00618/2020, PRT/BD/154191/2022

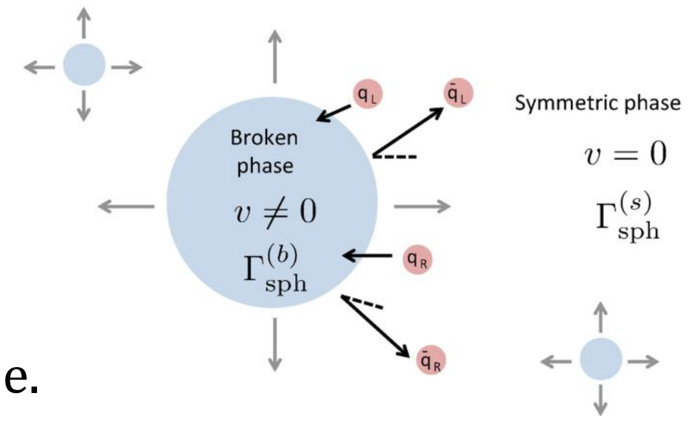
Electroweak phase transition

Vacuum expectation values (VEV) generated in the early Universe broke the electroweak gauge group. We call this **electroweak phase transition (EWPT).** ⟵ BSMPT v1/v2



https://www.mpi-hd.mpg.de/mpi/de/forschung/abteilungen-und-gruppen/unabhaengige-forschungsgruppen/newfo/forschung/elektroschwache-symmetriebrechung-und-das-higgs-potential

If **first order**, it can produce detectable **gravitational waves (GW)** in upcoming experiments, e.g. **LISA.** ⟵ BSMPT v3



DOI:10.1007/978-981-13-1008-9_2

Interactions out of thermal equilibrium in early Universe. If the model has **CP-violation** then all **Sakharov conditions** for baryogenesis are fulfilled. ⟵ Next version of BSMPT

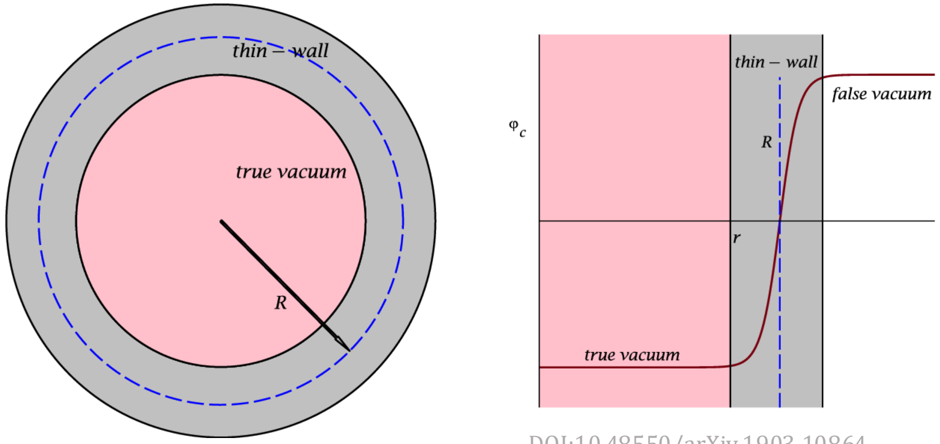
Primordial gravitational waves

In a first order EWPT, bubbles of **true vacuum** appear in a sea of **false vacuum.**

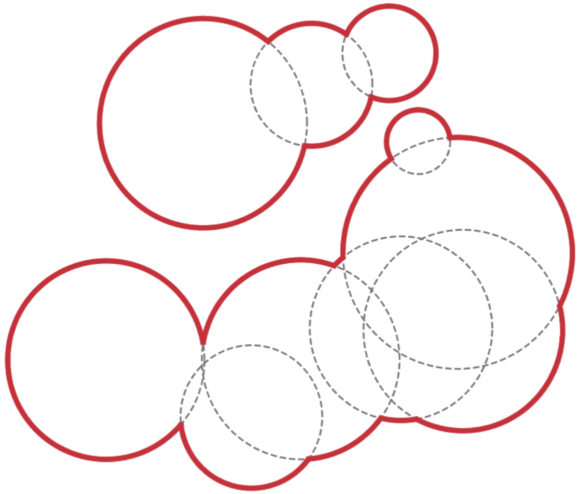
The bubbles expand **rapidly**, exerting pressure on the cosmic fluid creating **sound waves (SW),** and subsequently **GWs.**

After **collision time**, **turbulence** in the cosmic fluid will also generate **GWs.**

The **GWs** could live up to this day as a **stochastic gravitational wave background**.



DOI:10.48550/arXiv.1903.10864



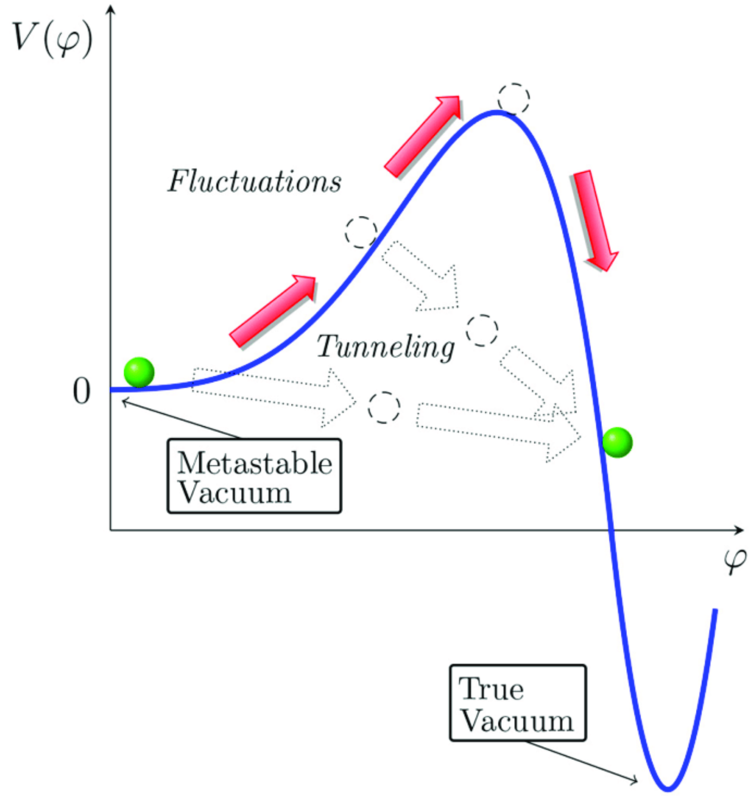
[10.1016/j.ppnp.2023.104094]

BSMPT v3

Recently released, **BSMPT v3** (e-Print: [2404.19037])

1. Calculates and tracks the **minima** of a model.
2. Solves the **bounce equation** - tunneling rate.
3. Calculates the **vacuum history** of the Universe
4. Calculates the **gravitational wave spectrum** and **signal-to-noise ratio** in LISA.

Available at: https://github.com/phbasler/BSMPT



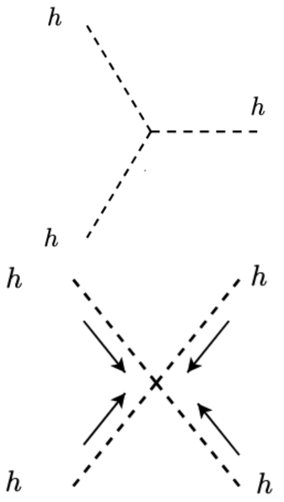
DOI:10.3389/fspas.2018.00040

Effective potential

We use the Coleman-Weinberg (CW) effective potential.
It is given by

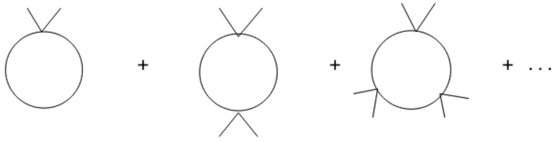
$$V_{\text{eff}}(\vec{\omega}, T) = V_o(\vec{\omega}) + V_{\text{CW}}(\vec{\omega}) + V_{\text{CT}}(\vec{\omega}) + \Delta V(\vec{\omega}, T)$$

Tree level potential



DOI:10.1142/S0217751X12300256

Coleman-Weinberg 1-loop effective potential at $T = 0$



arXiv:hep-th/0507214

$$V_{\text{CW}} = \sum_i (-1)^{F_i} n_i \frac{m_i^4(\phi_\alpha)}{64\pi^2} \left(\log\left[\frac{m_i^2(\phi_\alpha)}{\Lambda^2}\right] - c_i \right)$$

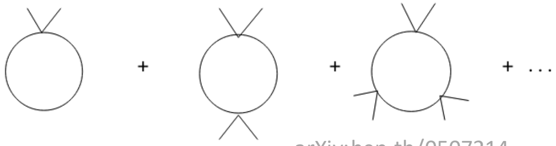
Counter term potential

Tree level VEV and mass matrix are kept at LO values.

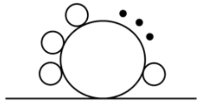
$$\left\langle \frac{\partial V_{\text{ct}}}{\partial h_i} \right\rangle = \left\langle -\frac{\partial V_{\text{CW}}^{(1)}}{\partial h_i} \right\rangle$$

$$\left\langle \frac{\partial^2 V_{\text{ct}}}{\partial h_i \partial h_j} \right\rangle = \left\langle -\frac{\partial^2 V_{\text{CW}}^{(1)}}{\partial h_i \partial h_j} \right\rangle$$

Coleman-Weinberg 1-loop effective potential thermal corrections + thermal masses



arXiv:hep-th/0507214



\+

arXiv:hep-ph/9901312

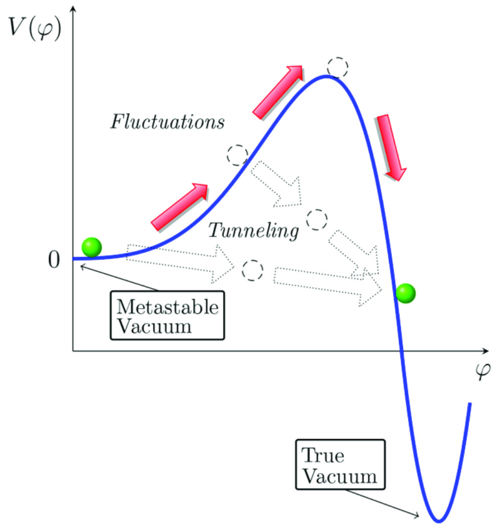
$$\Delta V(T) = \frac{T^4}{2\pi^2} \left\{ \sum_b n_b J_B \left[\frac{m_i^2(\phi_\alpha)}{T^2}\right] - \sum_f n_f J_F \left[\frac{m_i^2(\phi_\alpha)}{T^2}\right] \right\}$$

$$V_{\text{daisy}}(\omega, T) =$$
$$= -\frac{T}{12\pi} \left[\sum_{i=1}^{n_{\text{Higgs}}} \left((\overline{m}_i^2)^{3/2} - (m_i^2)^{3/2} \right) + \sum_{a=1}^{n_{\text{gauge}}} \left((\overline{m}_a^2)^{3/2} - (m_a^2)^{3/2} \right) \right]$$

Tunneling rate

The tunneling (transition) rate is calculated à la **Coleman** [doi: 10 . 1103 / PhysRevD . 15 . 2929], it is given by

$$\Gamma(\vec{\phi}_f \to \vec{\phi}_t) \equiv \Gamma = A(T)\, e^{-S_E}$$



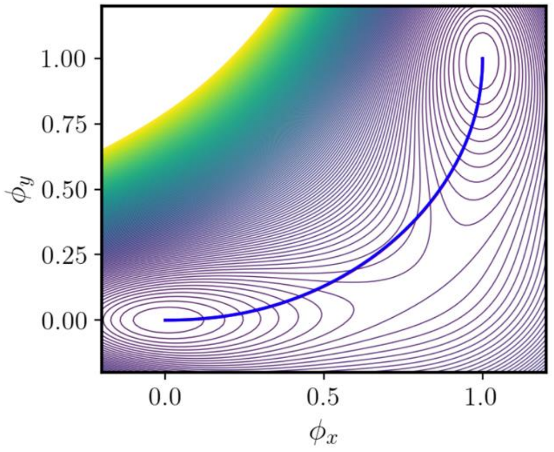
[DOI:10.3389/fspas.2018.00040]

where the Euclidian action $S_E = S_3$ is given by

$$S_3(T) = 4\pi \int_0^\infty d\rho\, \rho^2 \left[\frac{1}{2}\left(\frac{d\vec{\phi}}{d\rho}\right)^2 + V(\vec{\phi})\right]$$

by the path that fulfills the following ODE

$$\frac{d^2\vec{\phi}}{d\rho^2} + \frac{D-1}{\rho}\frac{d\vec{\phi}}{d\rho} = \nabla V(\vec{\phi}) \qquad \vec{\phi}(\rho)\big|_{\rho\to\infty} = \vec{\phi}_f, \quad \frac{d\vec{\phi}}{d\rho}\bigg|_{\rho=0} = 0$$



[https://arxiv.org/abs/2404.19037]

Characteristic temperatures

Critical temperature $\boldsymbol{T_c}$

$$V(\phi_f, T_c) = V(\phi_t, T_c)$$

Nucleation temperature $\boldsymbol{T_n}$

Decay rate matches Hubble rate. $\frac{\Gamma}{H^4} = 1$

$\sim$1 true vacuum bubble per Hubble volume.

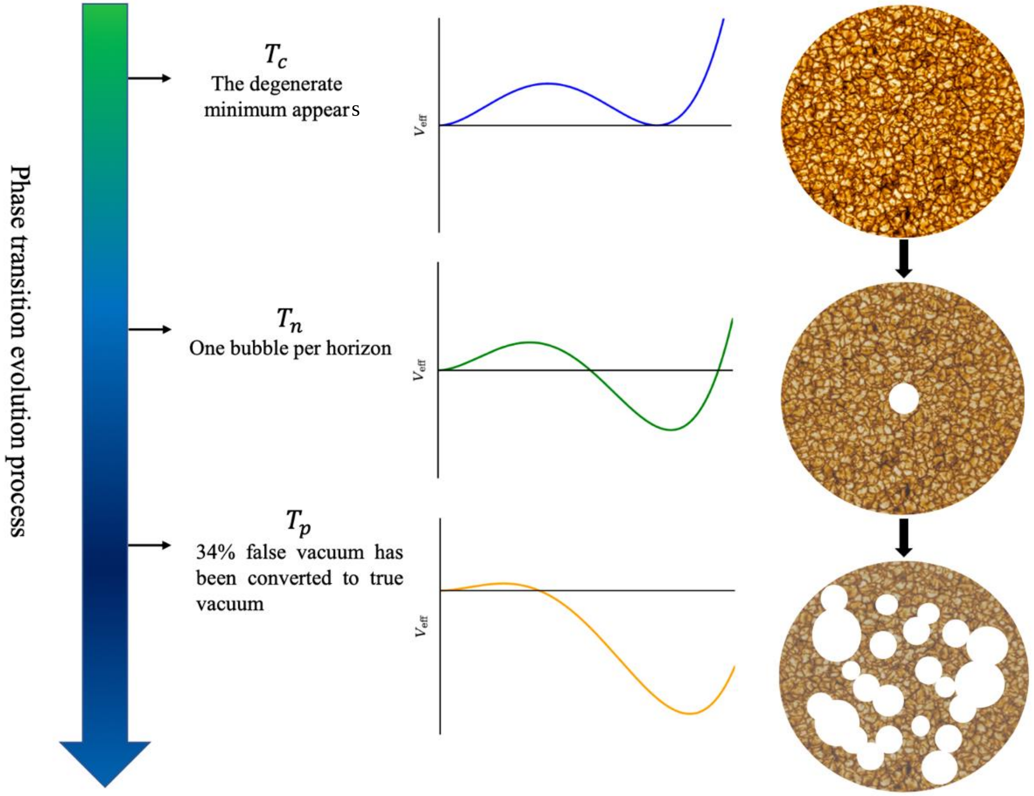
BSMPTv3 default transition temperature

Percolation temperature $\boldsymbol{T_p(P(T_p) = 0.71)}$

Temperature at which 29 % of the false vacuum decayed. (critical density for percolation of spheres in 3D)

Completion Temperature $\boldsymbol{T_f(P(T_f) = 0.01)}$

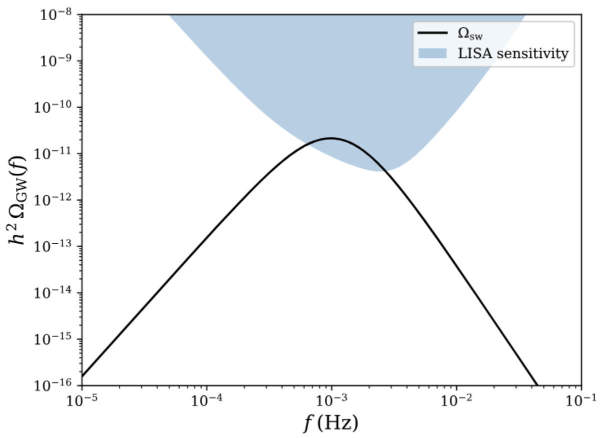
Temperature at which 99 % of the false vacuum decayed.



Spectrum and SNR

The GW spectrum's **spectral shape** is given by

$$h^2\Omega_{\rm GW}(f) \simeq h^2\Omega_{\rm GW}^{\rm SW,peak}\left(\frac{4}{7}\right)^{-\frac{7}{2}}\left(\frac{f}{f_{\rm SW,peak}}\right)^3\left[1+\frac{3}{4}\left(\frac{f}{f_{\rm SW,peak}}\right)^2\right]^{-\frac{7}{2}}$$
$$+\, h^2\Omega_{\rm GW}^{\rm Turb,peak}\left(\frac{(f/f_{\rm Turb,peak})^3}{(1+f/f_{\rm Turb,peak})^{11/3}(1+8\pi f/H_*)}\right),$$



[DOI 10.1088/1475-7516/2020/03/024]

The **peak frequencies** $f_{\rm i,peak}$ and **peak amplitudes** $\Omega_{\rm i,peak}$ are calculated from the following parameters

- Strength of the phase transition (latent heat)

$$\alpha = \frac{1}{\rho_\gamma}\left[V(\vec{\phi}_f) - V(\vec{\phi}_t) - \frac{T}{4}\left(\frac{\partial V(\vec{\phi}_f)}{\partial T} - \frac{\partial V(\vec{\phi}_t)}{\partial T}\right)\right]_{T=T_*}$$

- Inverse time scale

$$\frac{\beta}{H_*} = T_*\,\frac{d}{dT}\left(\frac{S_3(T)}{T}\right)\bigg|_{T_*}$$

- Effective d.o.f. energy density

  g_*

- Transition temperature

  T_*

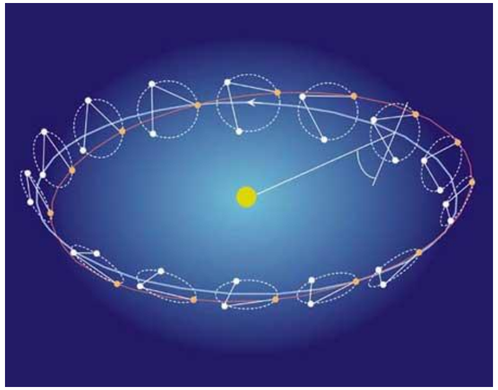
- Wall velocity (user defined)

  v_w

Signal-to-noise ratio in LISA

The **signal-to-noise (SNR)** ratio in **LISA** is given by

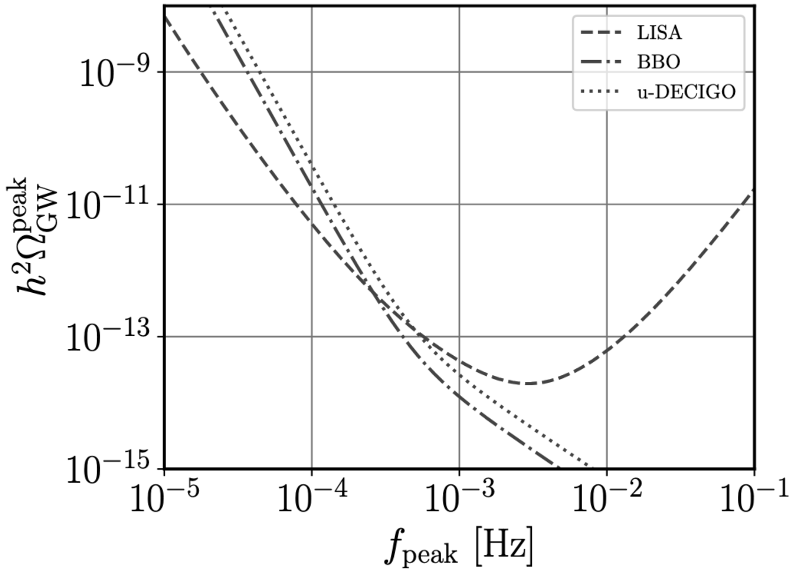
$$\mathrm{SNR} = \sqrt{\mathcal{T} \int_{f_{\min}}^{f_{\max}} \mathrm{d}f \left[\frac{h^2\Omega_{\mathrm{GW}}(f)}{h^2\Omega_{\mathrm{Sens}}(f)}\right]^2}$$



[https://web.archive.org/web/20070708045217/http://lisa.jpl.nasa.gov/gallery2.html]

LISA's orbit.

One of the three LISA satellites.



Peak integrated sensitivity curves (PISC) for various experiments. Points **above** a curve are detectable by that experiment.

BSMPT v3

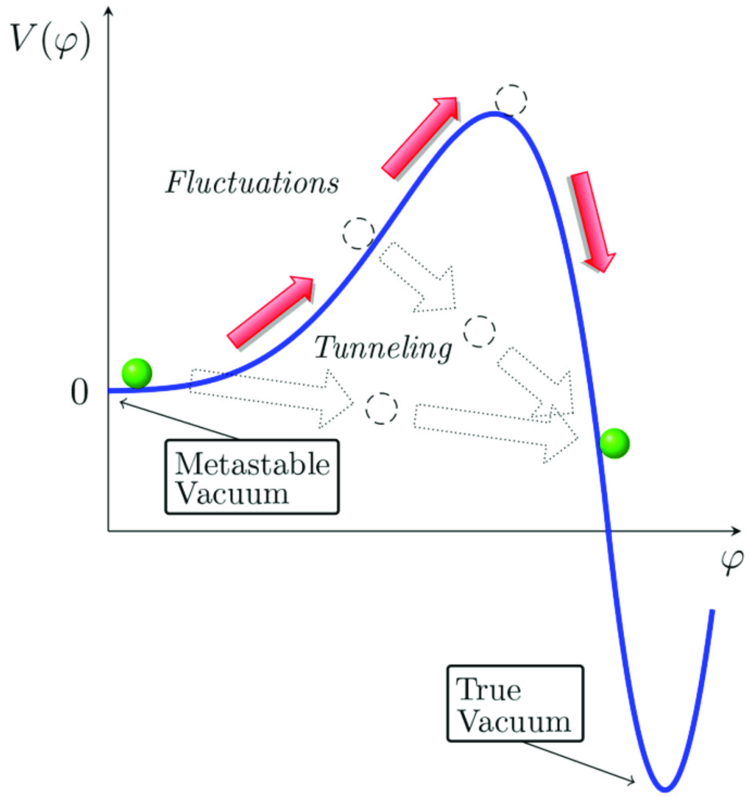
BSMPTv3 is shipped with the following models

- SM
- CxSM
- R2HDM / C2HDM
- N2HDM / CP in the Dark

You can also implement **your model**!

We have tools in *python* and *maple* to help.
(a *Mathematica* implementation tool will be released soon).

Any issue contact us in
https://github.com/phbasler/BSMPT/issues,
or by email at bsmpt@lists.kit.edu.



DOI:10.3389/fspas.2018.00040

Test case: CP in the Dark [1807.10322]

Scalar fields

$\mathbb{Z}_2$symmetry $\Phi_1 \to \Phi_1 \quad , \quad \Phi_2 \to -\Phi_2 \quad , \quad \Phi_S \to -\Phi_S$

$$\Phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_1 + i\eta_1 \\ \zeta_1 + \overline{\omega}_1 + i\Psi_1 \end{pmatrix}, \Phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} \rho_2 + \overline{\omega}_{\mathrm{CB}} + i\eta_2 \\ \zeta_2 + \overline{\omega}_2 + i(\Psi_2 + \overline{\omega}_{\mathrm{CP}}) \end{pmatrix}, \Phi_S = \zeta_S + \overline{\omega}_S$$

Spontaneous CP-violation if $\overline{\omega_{CP}} \neq 0$

Scalar potential

Explicit CP-violation if Im $A \neq 0$

$$\begin{aligned} V = &\ m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 + \frac{1}{2}m_S^2\Phi_S^2 + \left(A\Phi_1^\dagger\Phi_2\Phi_S + h.c.\right) \\ &+ \frac{1}{2}\lambda_1|\Phi_1|^4 + \frac{1}{2}\lambda_2|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{1}{2}\lambda_5\left[\left(\Phi_1^\dagger\Phi_2\right)^2 + h.c.\right] \\ &+ \frac{1}{4}\lambda_6\Phi_S^4 + \frac{1}{2}\lambda_7|\Phi_1|^2\Phi_S^2 + \frac{1}{2}\lambda_8|\Phi_2|^2\Phi_S^2 , \end{aligned}$$

Fermions transform trivially under $\mathbb{Z}_2$, so they only couple with Φ_1

$$-\mathcal{L}_Y = \lambda_t\bar{Q}_L\tilde{\Phi}_1 t_R + \lambda_b\bar{Q}_L\Phi_1 b_R + \lambda_\tau\bar{L}_L\Phi_1\tau_R + \ldots$$

Dark matter candidate + Dark CP violation

Test case: CP in the Dark

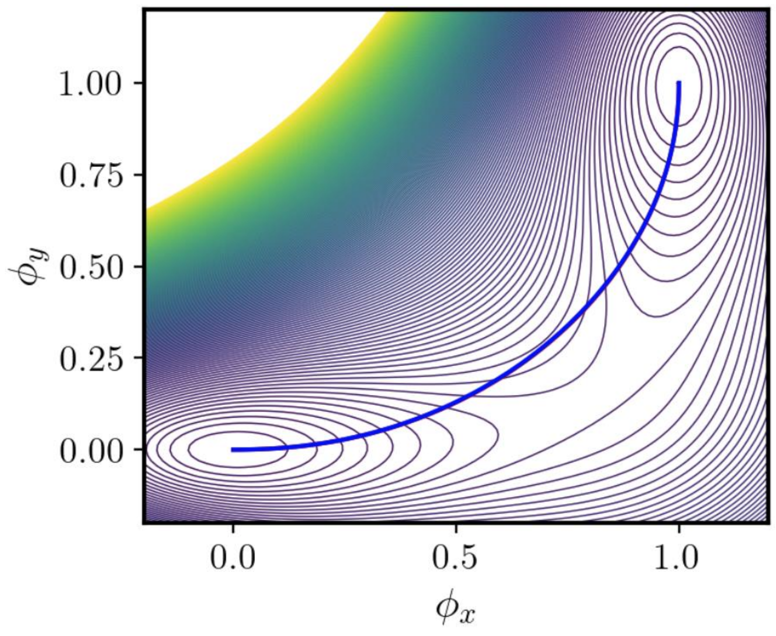
Parameter scan [Lisa Biermann's PhD thesis : DOI:10.5445/IR/1000174880]

m_h	$m_{h_{i,j}}$	$m_{H^\pm}$	α_i	λ_2	λ_6	λ_8	m_{22}^2	m_S^2
125.09	$[1, 10^3]$	$[65, 10^3]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$[0, 9]$	$[0, 17]$	$[-26, 26]$	$[0, 10^6]$	$[0, 10^6]$

Using **ScannerS** [2007.02985] and **micrOMEGAs** [1005.4133], it is imposed

- **Theoretical constraints -** Perturbative unitarity, boundedness from below and vacuum stability.
- **Experimental constraints –** EW precision constraints, flavor constraints, Higgs searches and measurements, EDM, relic density.

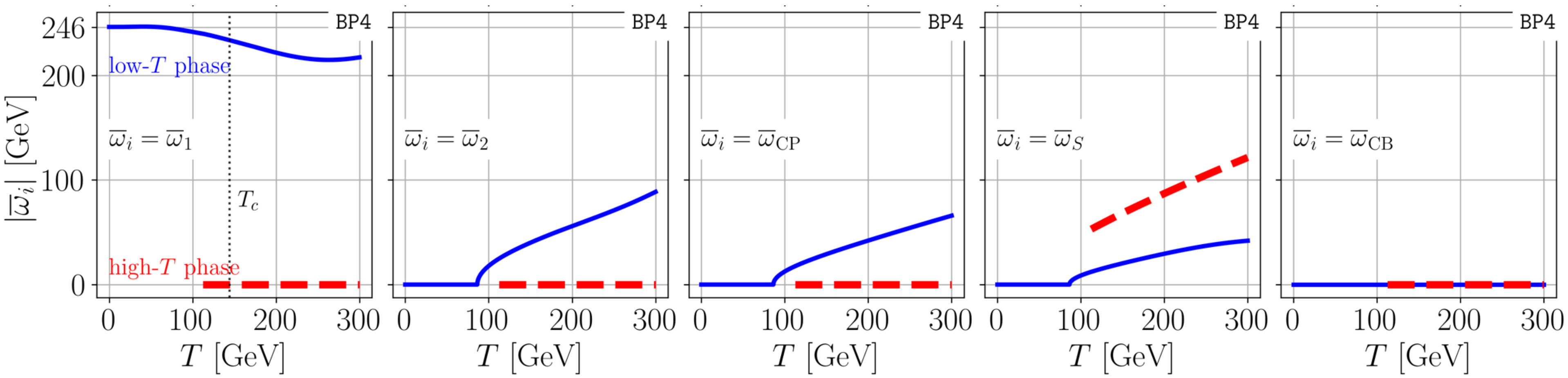
Then all points were checked for **strong first order electroweak phase transitions** in **BSMPTv3**, in a temperature range T $\in$ [0, 300] GeV.



Test case: CP in the Dark

Minimum tracking across the temperature range [DOI:10.5445/IR/1000174880]

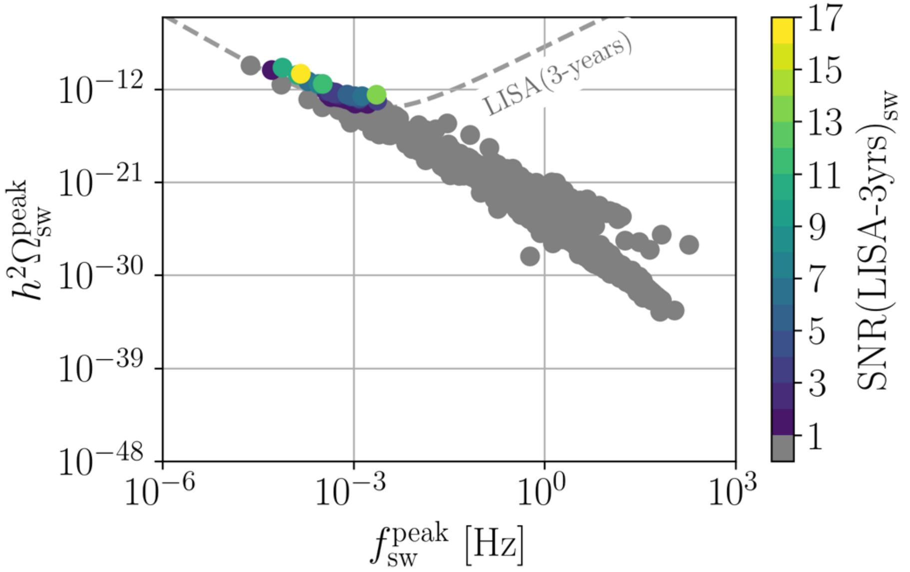
$m_{H^\pm}$	m_{h_1}	m_{h_2}	m_{h_3}	T_c	$\lvert\omega_{EW}^{true}(T_c)\rvert$	ξ_c
200.940 GeV	62.680 GeV	218.700 GeV	560.206 GeV	191.38 GeV	233.12 GeV	1.22

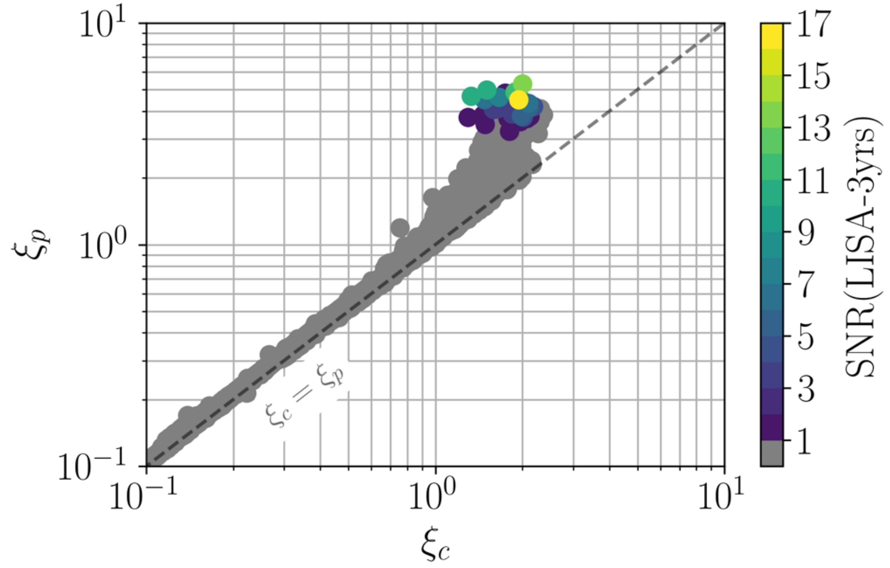


At high temperature, the Universe is in the false vacuum and the electroweak symmetry is unbroken.
For $T < T_c$ the true vacuum becomes the global minimum, but the transition only occurs at a lower temperature.

Test case: CP in the Dark

Gravitational wave strength and SNR [DOI:10.5445/IR/1000174880]





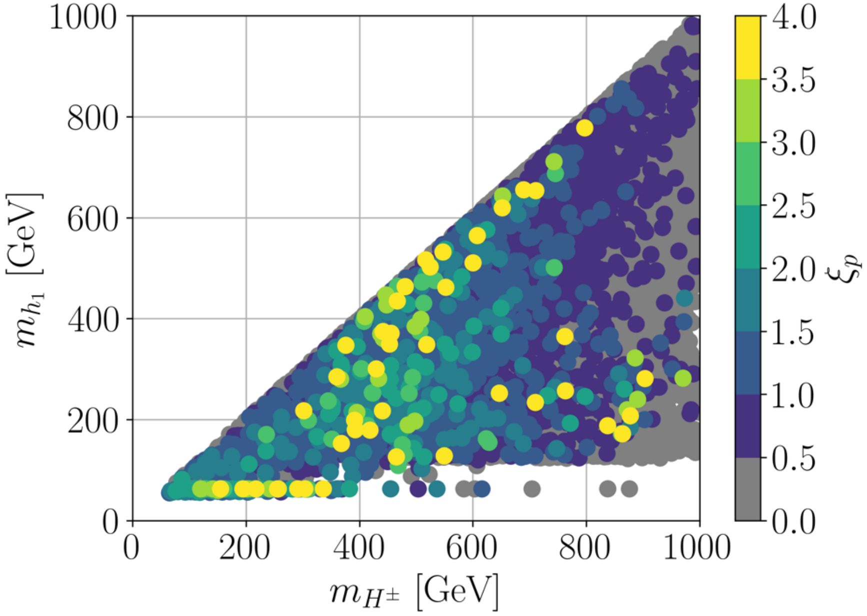
Detectable points (SNR > 1) in LISA are colored.

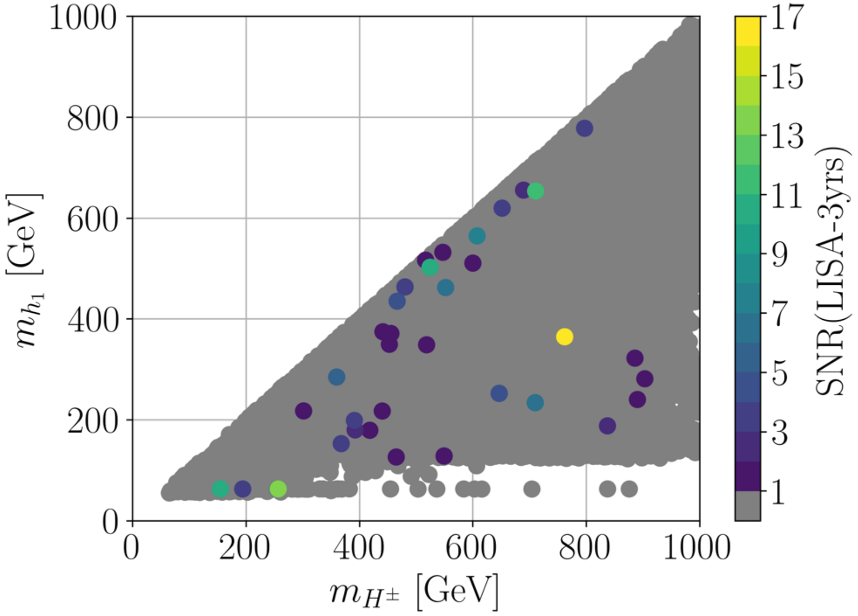
$\xi_p \approx \xi_c$ for non-detectable points. $\xi_p \equiv \xi(T_p) = \dfrac{\overline{\omega}_{\rm EW}^{\rm true}(T_p)}{T_p}$

$$\overline{\omega}_{\rm EW} \equiv \sqrt{\overline{\omega}_1^2 + \overline{\omega_2}^2 + \overline{\omega}_{\rm CP}^2 + \overline{\omega}_{\rm CB}^2}$$

Test case: CP in the Dark

Gravitational wave strength and SNR [DOI:10.5445/IR/1000174880]



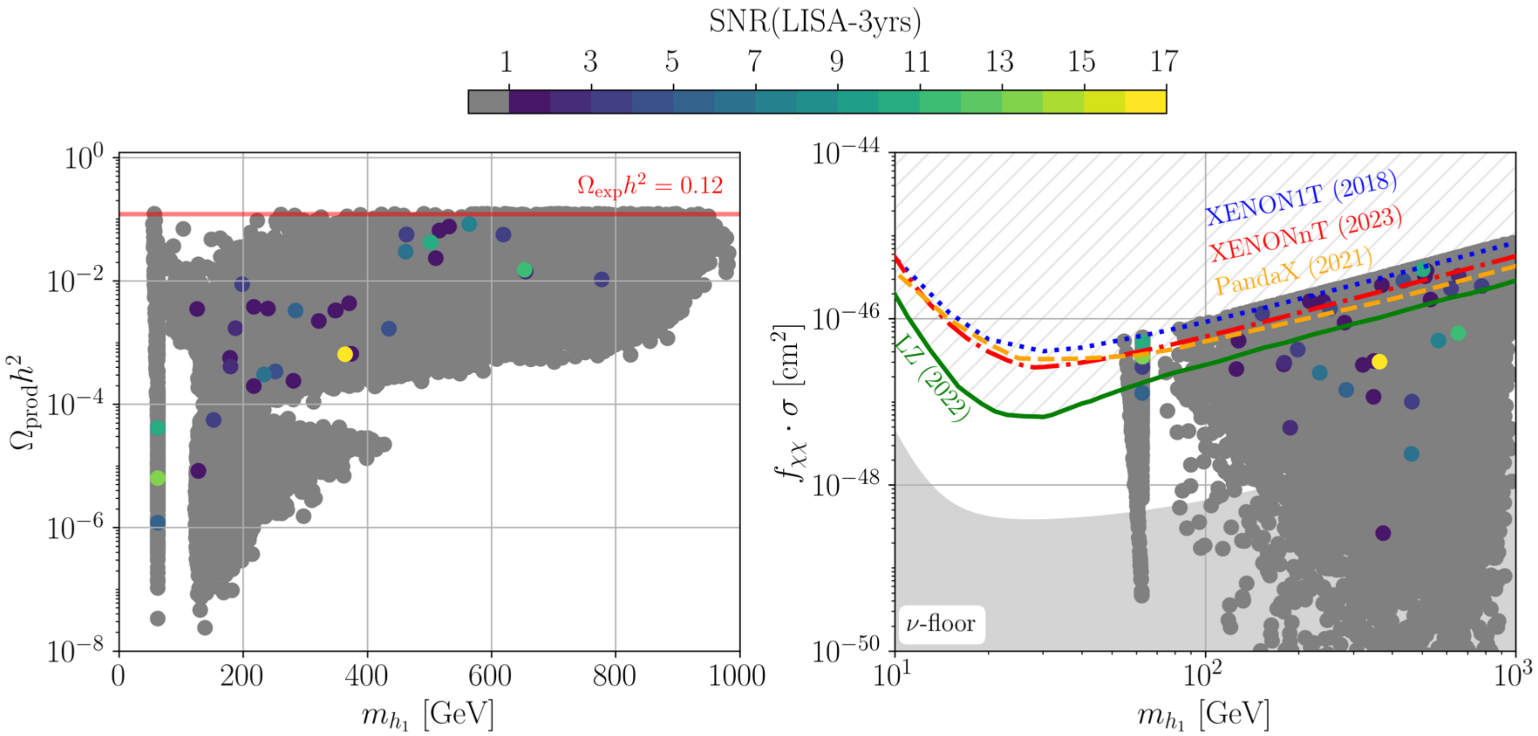


Abundance of points with $\xi_p > 1$ across the parameter space.
Good for BAU!

Points with SNR > 1 across the parameter space. No favored region.

Test case: CP in the Dark

Relic density and direct detection [DOI:10.5445/IR/1000174880]



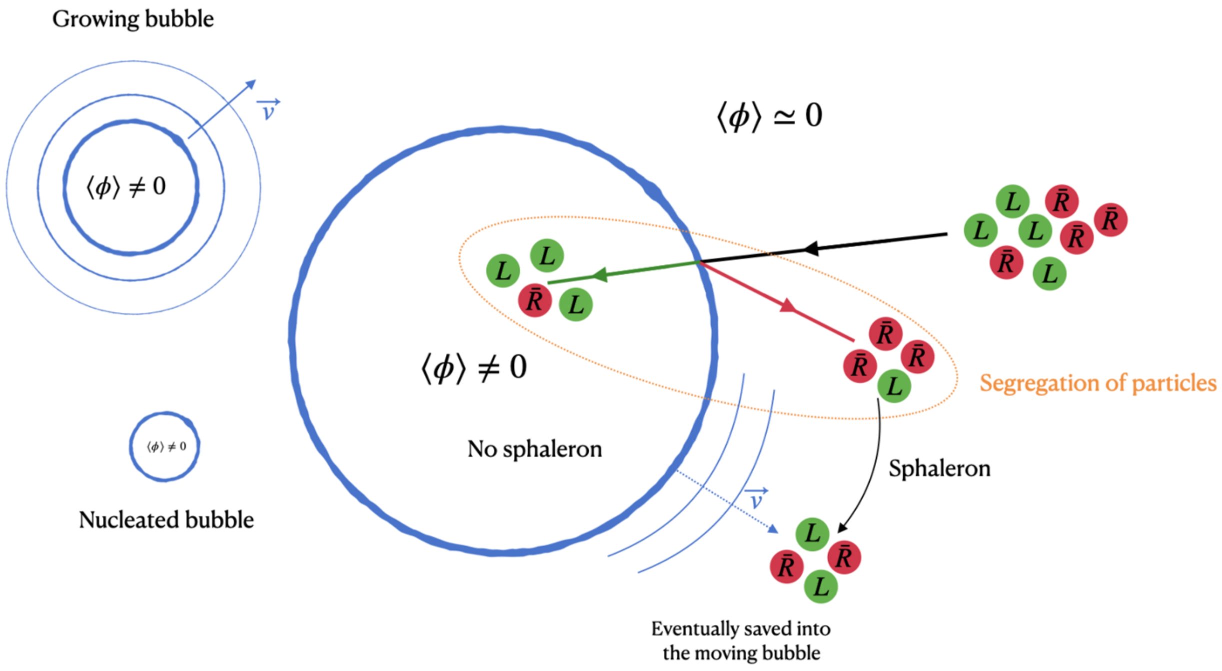
Detectable parameter points provide a **dark matter candidate** that does not exceed the relic density.

Some detectable parameter points **escape the direct detection constraints** (even for the most recent LZ results[2410.17036]).

Future of BSMPT

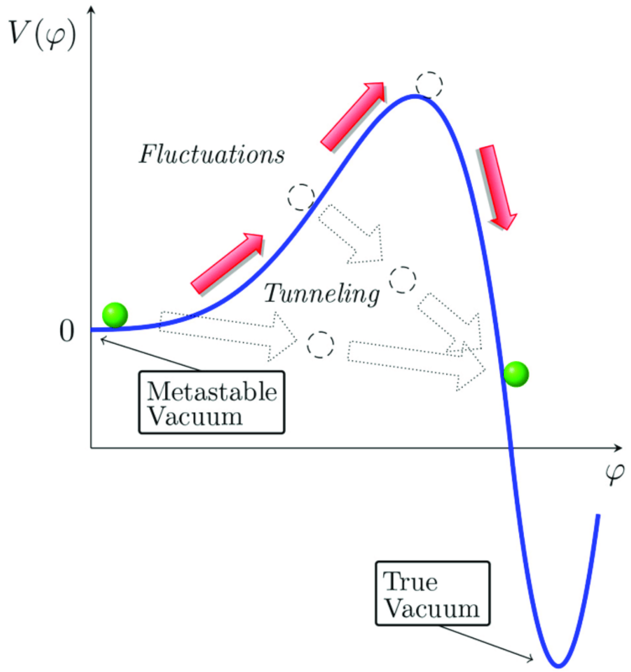
The next steps of BSMPT are

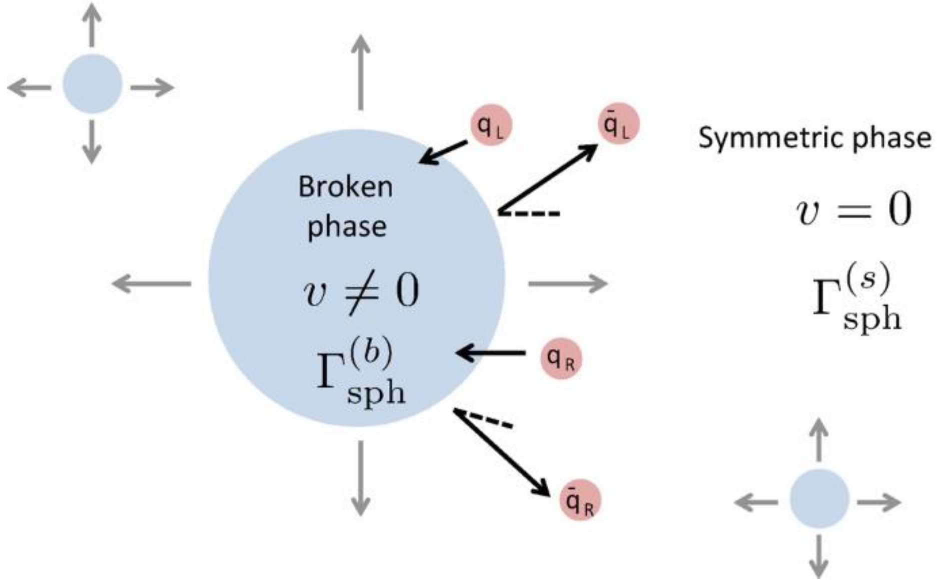
- Beside *python* and *maple*, models can soon be implemented using ***Mathematica***.
- Implement the **transport equations** (baryon asymmetry of the Universe).
- More stable and faster **bounce solver**.
- Calculate the **bubble wall velocity** using WallGo[2411.04970]
- Improve the **effective potential** perturbative expansion.



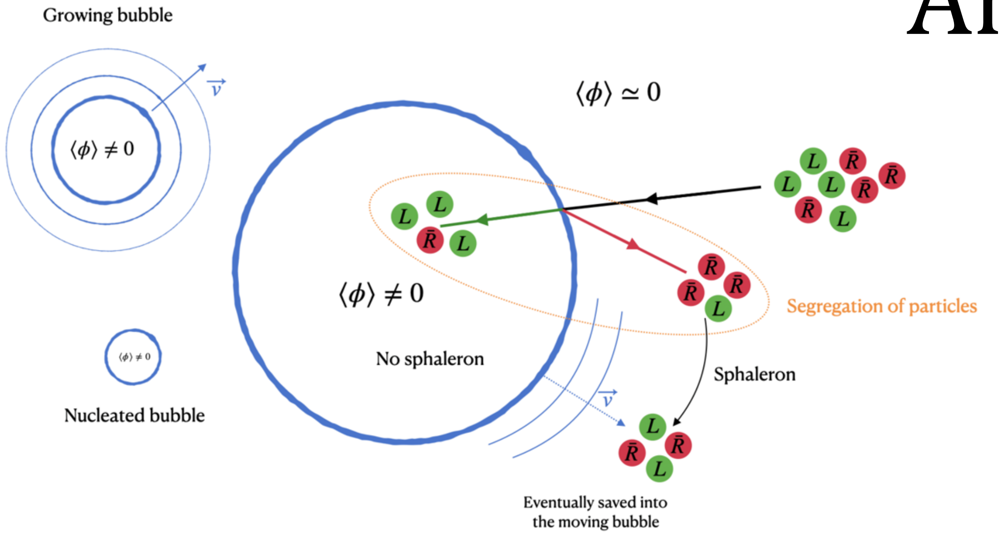
[DOI:10.3390/galaxies10060116]

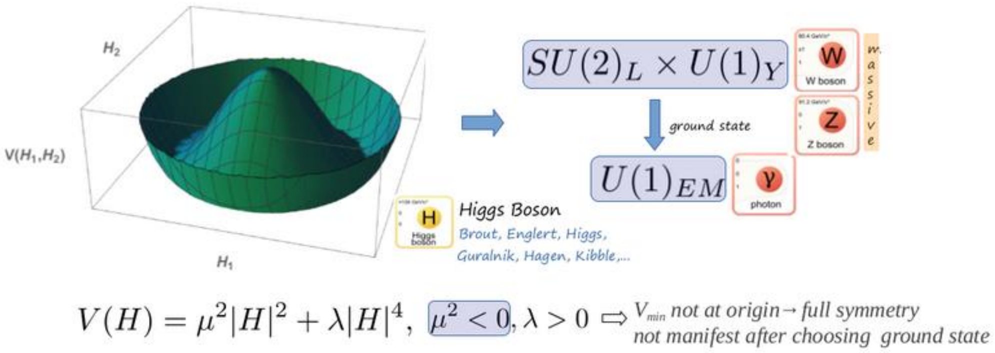
Your ideas and suggestion are welcome! Contact us at https://github.com/phbasler/BSMPT/discussions





Thanks for listening!
Any questions?



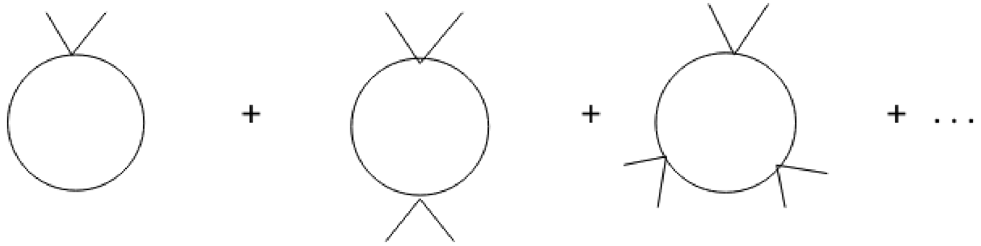


Additional slides

Effective potential

We use the Coleman-Weinberg (CW) effective potential.
It is given by

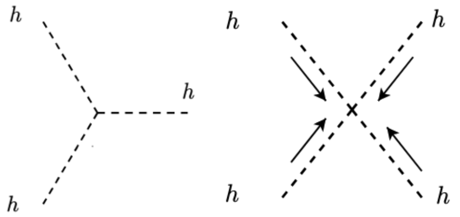
$$V_{\text{eff}}(T) = V_0 + V_{\text{CW}}^{(1)} + \Delta V(T) + V_{\text{ct}}$$



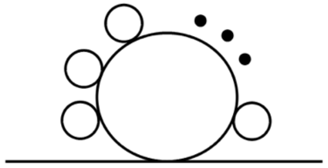
arXiv:hep-th/0507214

- Tree level potential

$$V_0$$



DOI:10.1142/S0217751X12300256



arXiv:hep-ph/9901312

- CW 1-loop potential

$$V_{\text{CW}} = \sum_i (-1)^{F_i} n_i \frac{m_i^4(\phi_\alpha)}{64\pi^2} \left(\log\left[\frac{m_i^2(\phi_\alpha)}{\Lambda^2}\right] - c_i \right)$$

- Thermal corrections

$$\Delta V(T) = \frac{T^4}{2\pi^2} \left\{ \sum_b n_b J_B \left[\frac{m_i^2(\phi_\alpha)}{T^2}\right] - \sum_f n_f J_F \left[\frac{m_i^2(\phi_\alpha)}{T^2}\right] \right\}$$

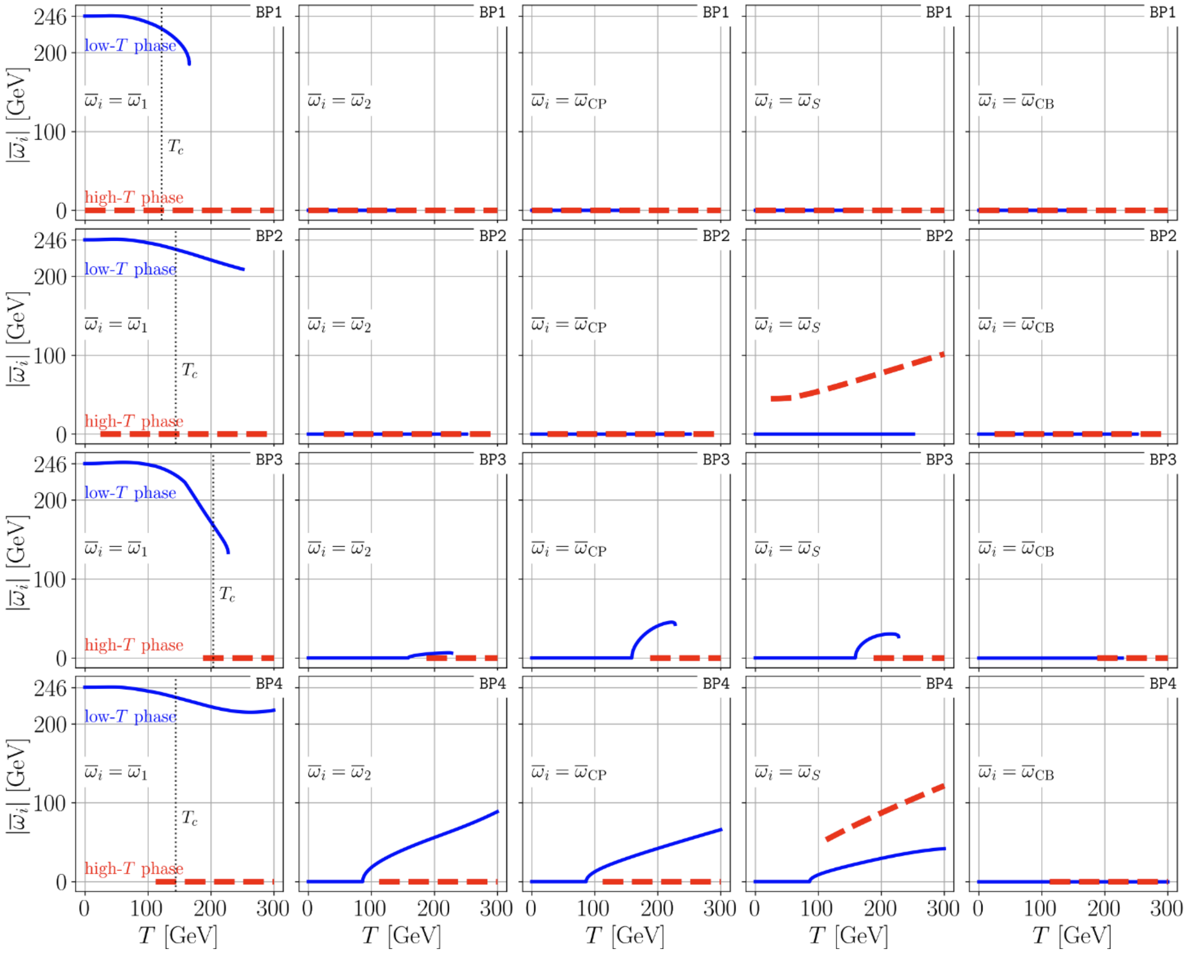
$$V_{\text{daisy}}(\omega, T) = -\frac{T}{12\pi} \left[\sum_{i=1}^{n_{\text{Higgs}}} \left((\overline{m}_i^2)^{3/2} - (m_i^2)^{3/2} \right) + \sum_{a=1}^{n_{\text{gauge}}} \left((\overline{m}_a^2)^{3/2} - (m_a^2)^{3/2} \right) \right]$$

- Counter terms

$$\left\langle \frac{\partial V_{\text{ct}}}{\partial h_i} \right\rangle = \left\langle -\frac{\partial V_{\text{CW}}^{(1)}}{\partial h_i} \right\rangle \qquad \left\langle \frac{\partial^2 V_{\text{ct}}}{\partial h_i \partial h_j} \right\rangle = \left\langle -\frac{\partial^2 V_{\text{CW}}^{(1)}}{\partial h_i \partial h_j} \right\rangle$$

Test case: CP in the Dark

Minimum tracking across the temperature range [DOI:10.5445/IR/1000174880]



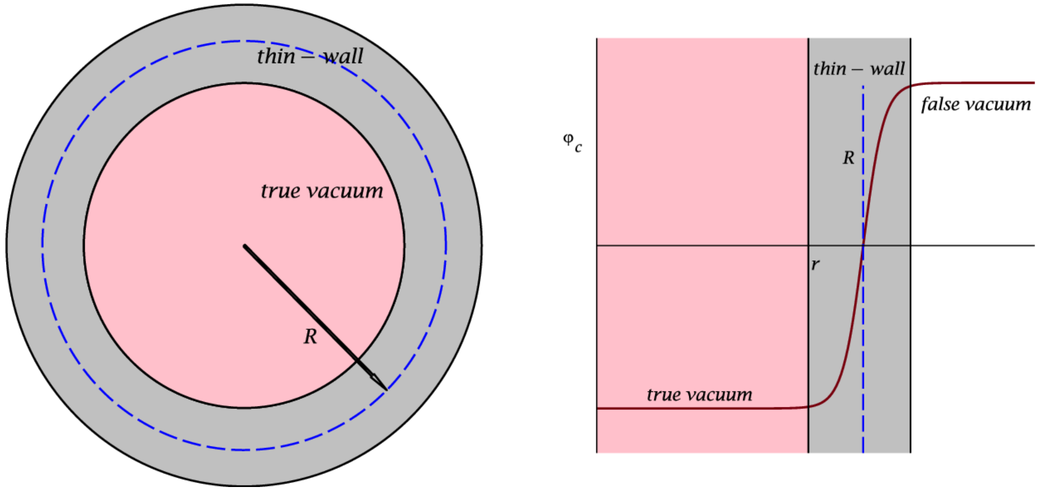
point	m_{22}^2 [GeV^2]	m_S^2 [GeV^2]	Re (A) [GeV]	Im (A) [GeV]	λ_2	
BP1	529 186.148	356 345.493	476.542	−678.778	4.299	
BP2	96 703.414	32 442.949	159.627	−325.391	3.532	
BP3	34 330.331	206 553.473	142.797	814.968	4.679	
BP4	65 258.809	36 279.847	279.502	−326.645	3.660	
point	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8
BP1	−0.782	0.572	0.071	1.053	16.810	−2.093
BP2	−0.796	0.787	−0.055	10.446	7.596	4.683
BP3	2.779	3.093	0.840	10.188	1.243	−4.563
BP4	−0.821	0.220	−0.371	4.715	7.760	14.781

point	$m_{H^\pm}$	m_{h_1}	m_{h_2}	m_{h_3}	T_c	$\overline{\omega}_{\mathrm{EW}}^{\mathrm{true}}(T_c)$
BP1	710.979	653.473	723.595	980.264	121.96	230.06
BP2	269.386	241.718	308.943	549.265	144.45	234.08
BP3	344.358	102.422	486.105	649.695	204.04	175.41
BP4	200.940	62.680	218.700	560.206	191.38	233.12
point	ξ_c	$\lvert\overline{\omega}_{\mathrm{CB}}^{\mathrm{true}}(T_c)\rvert$	$\lvert\overline{\omega}_1^{\mathrm{true}}(T_c)\rvert$	$\lvert\overline{\omega}_2^{\mathrm{true}}(T_c)\rvert$	$\lvert\overline{\omega}_{\mathrm{CP}}^{\mathrm{true}}(T_c)\rvert$	$\lvert\overline{\omega}_S^{\mathrm{true}}(T_c)\rvert$
BP1	1.89	0	230.06	0	0	0
BP2	1.62	0	234.08	0	0	0
BP3	0.85	0	167.65	5.89	41.83	29.64
BP4	1.22	0	223.42	53.25	39.91	27.97

Baryogenesis on BSMPT v2

BSMPT v2 already has the baryonic asymmetry of the Universe (BAU) calculation implemented, but is uses a few approximations

- Kink solution, an interpolated tunneling path between false and true vacuum.
- Assumes a low wall velocity.
- Only works for the C2HDM (complex 2HDM).



DOI:10.48550/arXiv.1903.10864

BSMPT v2 has the Fromme-Huber (FH) method [10.1088/1126-6708/2007/03/049] and VEV insertion approximation (VIA) method [10.1103/PhysRevD.53.5834] implemented.

Recent results [10.1007/JHEP12(2022)121] showed that the source term in the VIA method vanishes. For this reason, we will not consider it in BSMPT v3 until the source term is properly calculated.

We plan to improve the implementation as well as generalize it for any model.

FHCK / Semiclassical force method

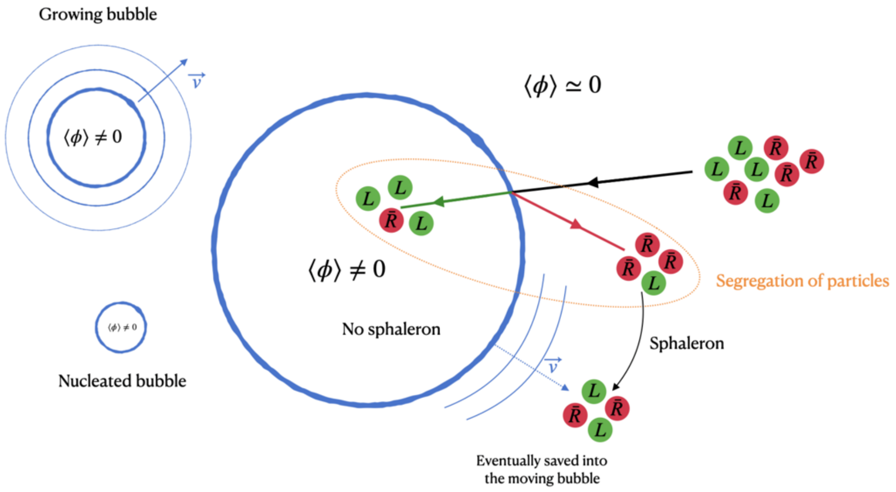
Our work is based on J. Cline and K. Kainulainen's method [10.1103/PhysRevD.101.063525], the generalization of the FH method for any bubble wall velocity.

Model with a CP-violating complex fermionic mass

$$\mathcal{M} = m(z)e^{i\theta(z)}$$

Using the Wentzel–Kramers–Brillouin (WKB) ansatz on the Dirac equation

$$\Psi \sim e^{-i\omega t + i\int^z p_{cz}(z')dz'}$$



DOI:10.3389/0.3390/galaxies10060116

We get the semiclassical group velocity $\boldsymbol{v_g}$ and force F given by

$$v_g = \frac{p_z}{E} + s_h s_{k_0}\frac{m^2\theta'}{2E^2E_z} \quad \Bigg| \quad F = -\frac{(m^2)'}{2E} + s_h s_{k_0}\left(\frac{(m^2\theta')'}{2EE_z} - \frac{m^2(m^2)'\theta'}{4E^3E_z}\right) \quad \Bigg| \quad s_h = h\gamma_{||}\frac{p_z}{|\mathbf{p}|} \equiv h s_{\mathrm{p}}$$

with $s_{k_0} = 1(-1)$ for particles (anti-particles) and $s = \pm 1$ for spin eigenstates in the z −direction (bubble wall). Particles and anti-particles "feel" a different force.

Liouville and collision operator

The distribution function is parameterized as

$$f = \frac{1}{e^{\beta[\gamma_w(E_w + v_w p_z) - \mu]} \pm 1} + \delta f$$

, where μ is the chemical potential and

$$\int \mathrm{d}^3 p \delta f = 0.$$

δf does not change local particle density.

The Boltzmann equation acting on the distribution function, reads

$$L[\mu_h, \delta f_h] = \mathcal{S}_h + \delta\mathcal{C}_h$$

CP-conserving interactions with the bubble wall

CP-violating interactions with the bubble wall

Interaction between particles

where the Liouville operator $\boldsymbol{L}$ and source term $\boldsymbol{\mathcal{S}_h}$ are defined as

$$L[\mu, \delta f] \equiv -\frac{p_z}{E} f'_{0w} \partial_z \mu + v_w \gamma_w \frac{(m^2)'}{2E} f''_{0w} \mu + \frac{p_z}{E} \partial_z \delta f - \frac{(m^2)'}{2E} \partial_{p_z} \delta f,$$

$$\mathcal{S}_h = -v_w \gamma_w h s_\mathrm{p} \frac{\left(m^2 \theta'\right)'}{2EE_z} f'_{0w} + v_w \gamma_w h s_p \frac{m^2 \left(m^2\right)' \theta'}{4E^2 E_z} \left(\frac{f'_{0w}}{E} - \gamma_w f''_{0w} \right)$$

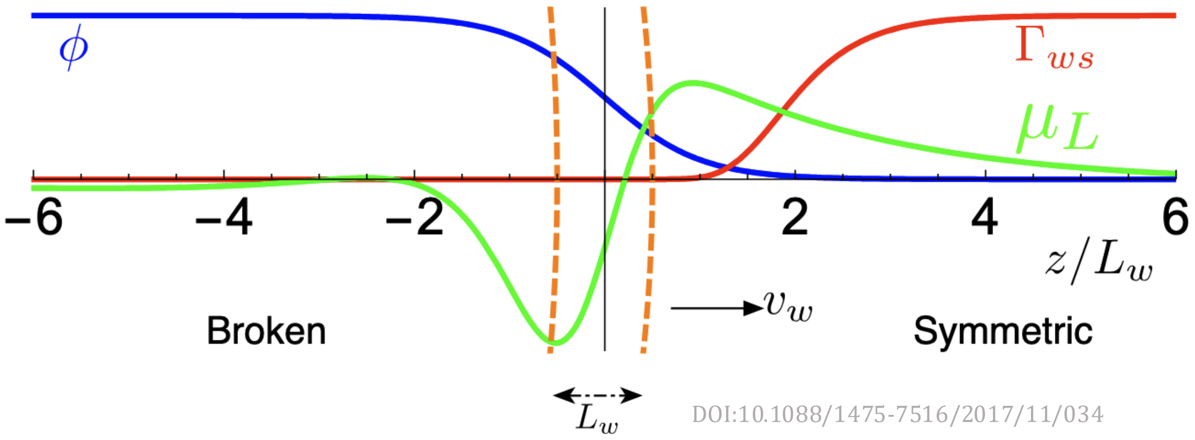
$$s \to s_h = h \gamma_{||} \frac{p_z}{|\mathbf{p}|} \equiv h s_\mathrm{p}$$

h is helicity
v_w is the wall velocity
γ_w is the Lorentz factor
f_{0w} is f with $\delta f = \mu = 0$

The collision operator $\delta\mathcal{C}_h$ is model dependent.

BAU

We solve for the chemical potentials $\boldsymbol{\mu_L}$



DOI:10.1088/1475-7516/2017/11/034

- ϕ – EWPT order parameter
- L_w – Bubble wall thickness
- μ_L – Chemical potentials
- $\Gamma_{ws} = \Gamma_{sph}$ – EW sphaleron rate

To calculate baryonic asymmetry in the Universe we integrate the chemical potential

$$\eta_B = \frac{405\Gamma_{\rm sph}}{4\pi^2 v_w \gamma_w g_* T} \int_{-\infty}^{\infty} dz \mu_{B_{\rm L}} f_{\rm sph} e^{-45\Gamma_{\rm sph}|z|/4v_w\gamma_w}$$

$$f_{\rm sph}(z) = \min\left(1, 2.4\frac{\Gamma_{\rm sph}}{T} e^{-40h(z)/T}\right)$$

f_{sph}(z) describes the sphaleron rate as a function of the distance to the bubble wall

The experimental value is given by

$$\eta_B \equiv \frac{n_B}{n_\gamma} = (6.2 \pm 0.4) \cdot 10^{-10}$$

Computational problems

The semi-classical force method is straightforward to use but we face some numerical challenges

- The different scales of the involved sources might produce stiff systems (numerically unstable). We must find an appropriate integration method.
- The thermal transport coefficients, i.e. $D_l, Q_l, etc.$, are computationally expensive to calculate. We must find fast methods/approximations for them.
- CP-violation can be present without an imaginary mass, e.g. CKM phase. How can we have a general method for all cases.

The project is still in its early days. Ideas/suggestions are welcome!

Moment expansion

We define the momentum averages as

$$\langle X \rangle \equiv \frac{1}{N_1} \int \mathrm{d}^3 p X \qquad N_1 \equiv \int \mathrm{d}^3 p f'_{0w,\mathrm{FD}} = -\gamma_w \frac{2\pi^3}{3} T^2$$

$$[\mathcal{X}] \equiv \frac{1}{N_0} \int d^3 p \mathcal{X} f_{0w} \qquad N_0 = \int d^3 p f_{0w} = \gamma_w \int d^3 p f_0 \equiv \gamma_w \hat{N}_0$$

The velocity pertubations are defined as

$$u_\ell \equiv \left\langle \left(\frac{p_z}{E} \right)^\ell \delta f \right\rangle$$

The ***l***-th moment is given by

$$\left\langle \left(\frac{p_z}{E} \right)^\ell L \right\rangle = \left\langle \left(\frac{p_z}{E} \right)^\ell (\mathcal{S} + \delta\mathcal{C}) \right\rangle$$

Moment expansion

The Liouville term is given by

$$\langle L \rangle = -D_1\mu' + u_1' + v_w\gamma_w(m^2)'Q_1\mu,$$

$$\left\langle \frac{p_z}{E} L \right\rangle = -D_2\mu' + u_2' + v_w\gamma_w(m^2)'Q_2\mu$$

$$+ (m^2)'\left\langle \frac{1}{2E^2}\delta f \right\rangle,$$

The source term is given by

$$S^o_{h\ell} = -v_w\gamma_w h[(m^2\theta')'Q^{8o}_\ell - (m^2)'m^2\theta' Q^{9o}_\ell]$$

The collision term is given by

$$\delta C_1 \equiv \langle \delta\mathcal{C} \rangle \qquad \delta C_1 = K_0 \sum_i \Gamma_i \sum_j s_{ij} \frac{\mu_j}{T},$$

$$\delta C_2 \equiv \langle (p_z/E)\delta\mathcal{C} \rangle \qquad \delta C_2 = -\Gamma_{\text{tot}} u - v_w \delta C_1.$$

BAU

Neglecting electroweak sphalerons we have

$$B = \sum_q (n_q - \bar{n}_q) = 0$$

so that the baryonic chemical potential is given by

$$\mu_{B_L} = \frac{1}{2}(1 + 4D_0^t)\mu_{t_L} + \frac{1}{2}(1 + 4D_0^b)\mu_{b_L} + 2D_0^t \mu_{t_R}$$

To calculate baryonic asymmetry in the Universe we integrate the chemical potential

$$\eta_B = \frac{405\Gamma_{\text{sph}}}{4\pi^2 v_w \gamma_w g_* T} \int dz \mu_{B_L} f_{\text{sph}} e^{-45\Gamma_{\text{sph}}|z|/4v_w\gamma_w} \qquad f_{\text{sph}}(z) = \min(1, 2.4\frac{\Gamma_{\text{sph}}}{T} e^{-40h(z)/T})$$

The experimental value is given by

$$\eta \equiv \frac{n_B}{n_\gamma} = (6.2 \pm 0.4) \cdot 10^{-10}$$

Collision terms

$$\delta\bar{C}_1^{t_L} = \Gamma_y(\mu_{t_L} - \mu_{t_R} + \mu_h) + \Gamma_m(\mu_{t_L} - \mu_{t_R}) + \Gamma_W(\mu_{t_L} - \mu_{b_L}) + \tilde{\Gamma}_{SS}[\mu_i],$$

$$\delta\bar{C}_1^{b_L} = \Gamma_y(\mu_{b_L} - \mu_{t_R} + \mu_h) + \Gamma_W(\mu_{b_L} - \mu_{t_L}) + \tilde{\Gamma}_{SS}[\mu_i],$$

$$\delta\bar{C}_1^{t_R} = -\Gamma_y(\mu_{t_L} + \mu_{b_L} - 2\mu_{t_R} + 2\mu_h) + \Gamma_m(\mu_{t_R} - \mu_{t_L}) - \tilde{\Gamma}_{SS}[\mu_i],$$

$$\delta\bar{C}_1^{h} = \tilde{\Gamma}_y(\mu_{t_L} + \mu_{b_L} - 2\mu_{t_R} + 2\mu_h) + \Gamma_h\mu_h$$

Strong sphaleron

$$\tilde{\Gamma}_{SS}[\mu_i] = \Gamma_{SS}((1 + 9D_0^t)\mu_{t_L} + (1 + 9D_0^b)\mu_{b_L} - (1 - 9D_0^t)\mu_{t_R}).$$

interaction	rate
$t_L \leftrightarrow t_R + h$ $b_L \leftrightarrow t_R + h$	$\Gamma_{y,t}$
$b_L \leftrightarrow b_R + h$ $c_L \leftrightarrow c_R + h$ $s_L \leftrightarrow s_R + h$ $t_L \leftrightarrow b_R + h$ $s_L \leftrightarrow c_R + h$ $c_L \leftrightarrow s_R + h$	$\Gamma_{y,b}$
$t_L \leftrightarrow t_R$	$2\Gamma_{m,t}$
$b_L \leftrightarrow b_R$ $c_L \leftrightarrow c_R$ $s_L \leftrightarrow s_R$	$2\Gamma_{m,b}$
$t_L \leftrightarrow b_L$ $c_L \leftrightarrow s_L$	Γ_W
$h \leftrightarrow 0$	Γ_h
all L $\leftrightarrow$ all R	Γ_{ss}

inelastic rates
$\Gamma_{y,t} = 4.2 \times 10^{-3}\, y_t^2\, T$ $\Gamma_{y,b} = 4.2 \times 10^{-3}\, y_b^2\, T$ $\Gamma_{m,t} = \frac{m_t^2}{63T}$ $\Gamma_{m,b} = \frac{m_b^2}{63T}$ $\Gamma_W = \frac{T}{60}$ $\Gamma_h = \frac{m_W^2}{50T}$ $\Gamma_{ss} = 4.9 \times 10^{-4} T$

elastic rates
$\Gamma_{tot,q} = \frac{T}{18}$ $\Gamma_{tot,h} = \frac{T}{60}$