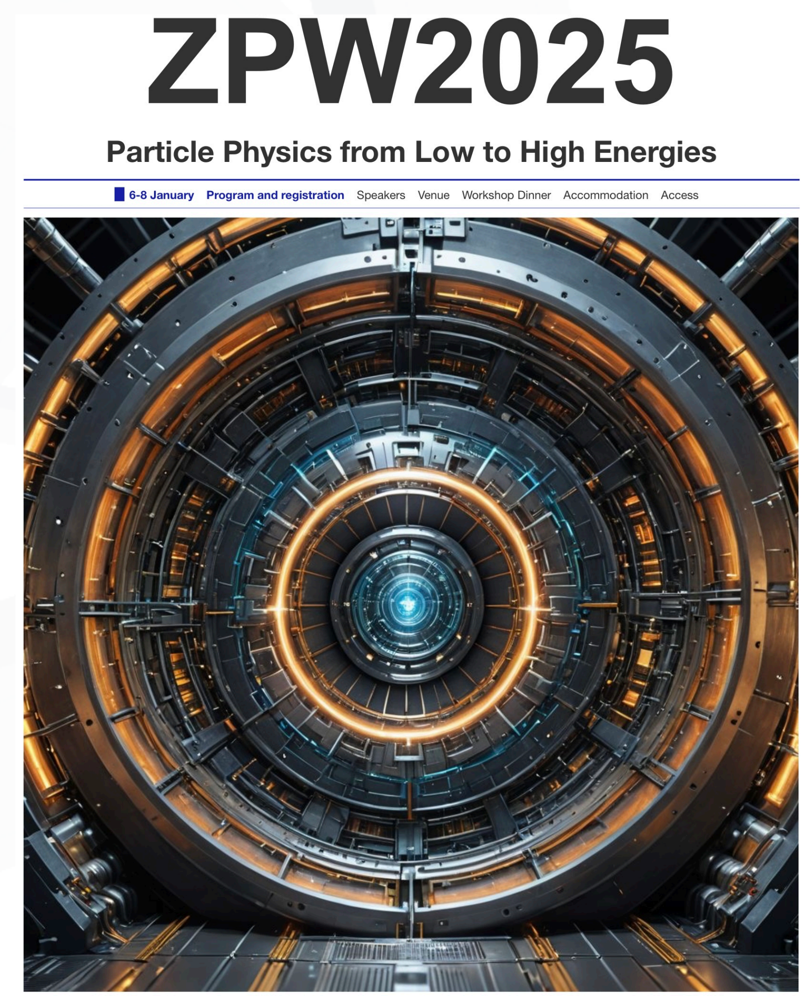
Progress in matching higher-order calculations to parton showers

Simone Alioli

MILANO-BICOCCA UNIVERSITY & INFN



ZURICH 6/1/2025

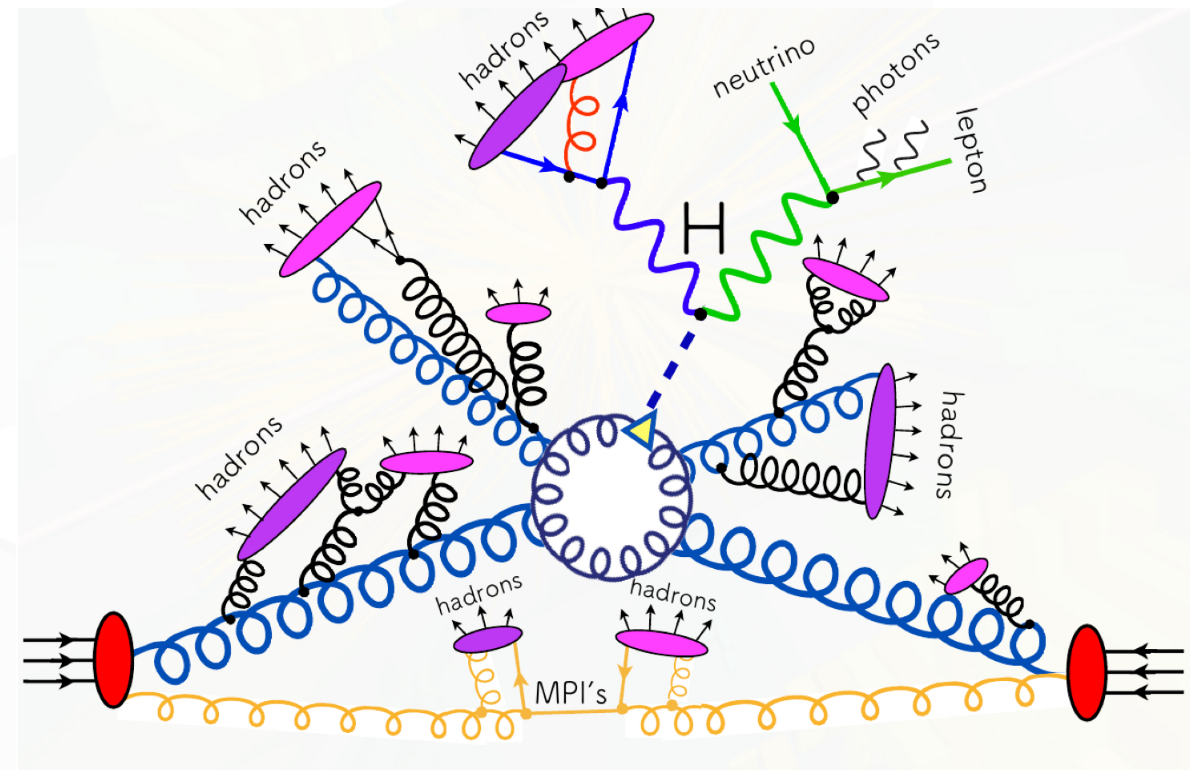
Outline:

- Overview of high-accuracy Monte Carlo event generators (NNLO+PS)
- MiNNLOps and GENEVA: similarities and differences
- Shower interface: status and challenges for new NLL showers
- Extension to processes with jets: roadmap and preliminary results

Apologies for the omissions and personal bias in the selection of topics

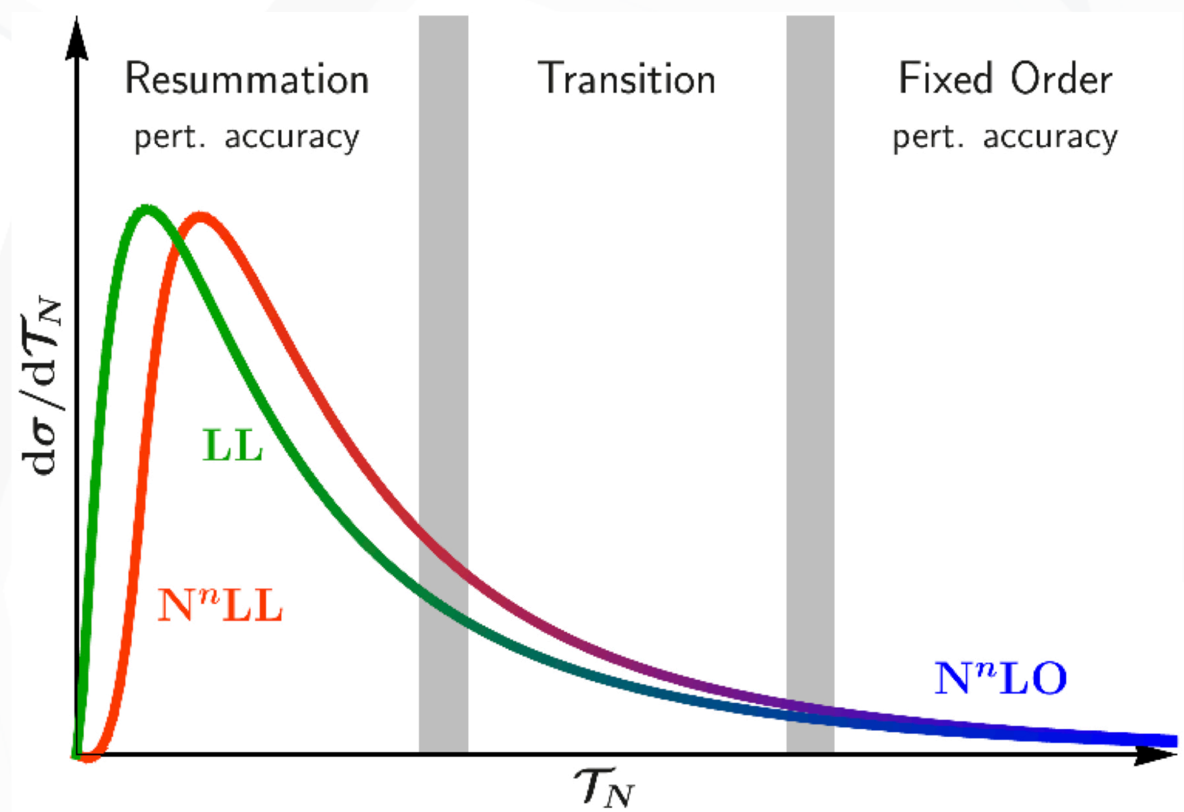
What is a precision MC ?

- Fully differential event generator producing hadronic final states, at high accuracy



- Precision enters in multiple ways:

- Perturbative accuracy of integrated total xs ($\mathrm{N}^n\mathrm{LO}$)
- Perturbative description of radiation pattern (resumm./shower)
- Description of hard tails (multi-jet)



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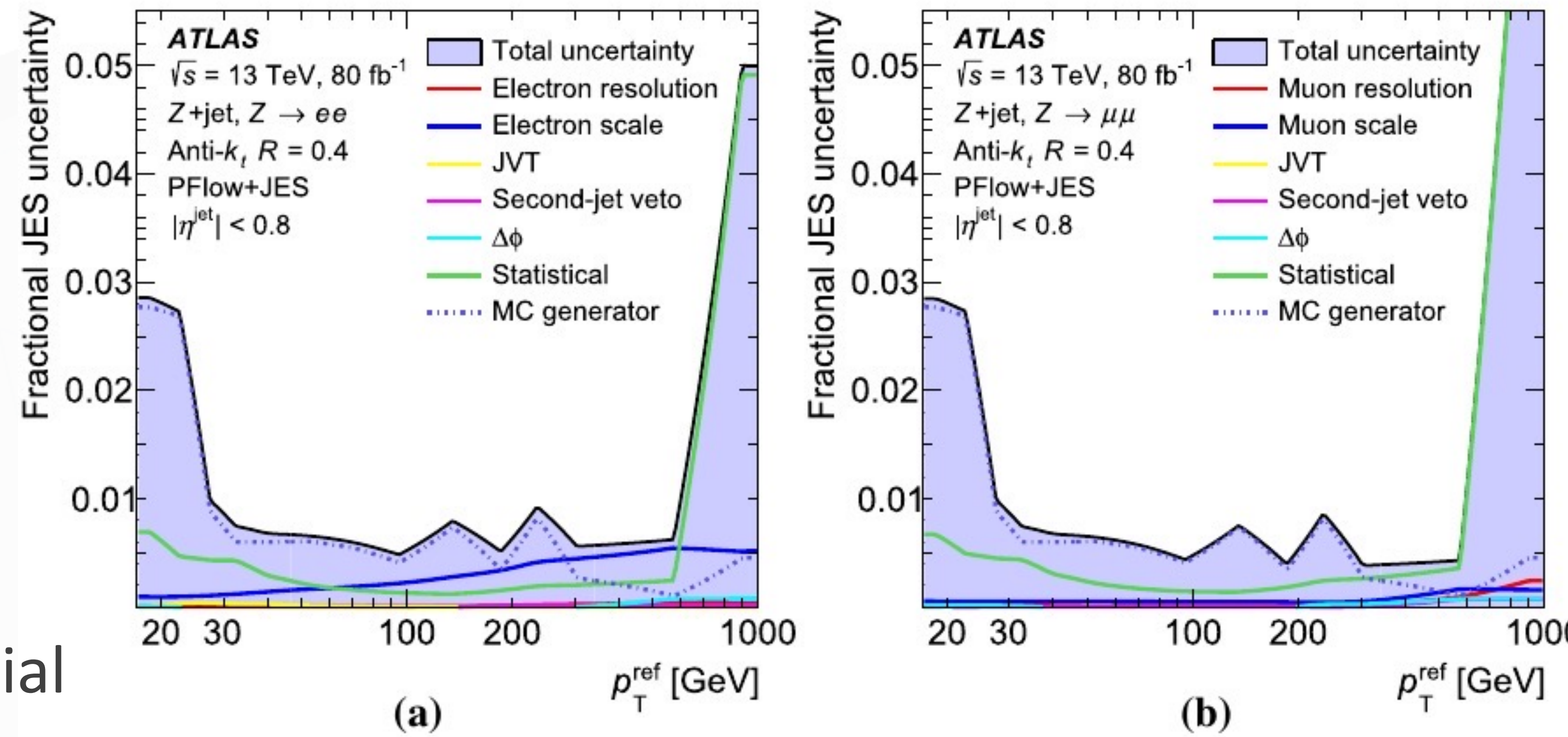
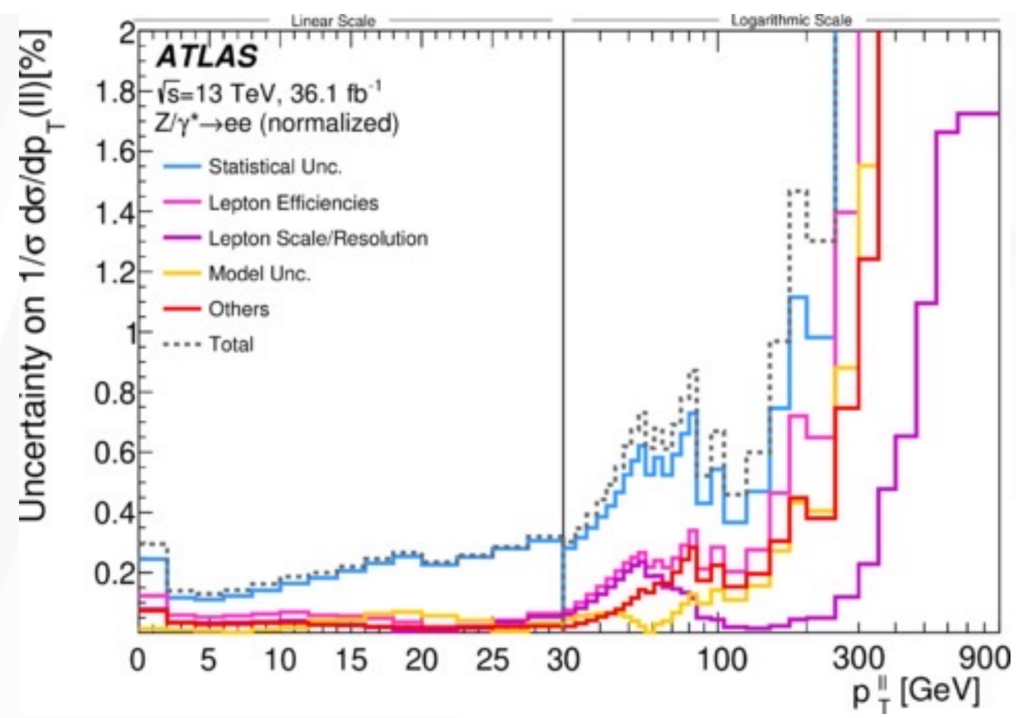
Why accuracy is important for SMC ?

Already the dominant source of uncertainty for Jet Energy Scale.
Everything involving jets is affected !



SMC are used to extrapolate theoretical predictions to fiducial regions.

LHC affected by 1-2% luminosity unc. but ratios cancels luminosity & other common systematics



At this level of precision need to start questioning every ingredient: perturbative calculations, nonperturbative PDFs, hadronization corrections …

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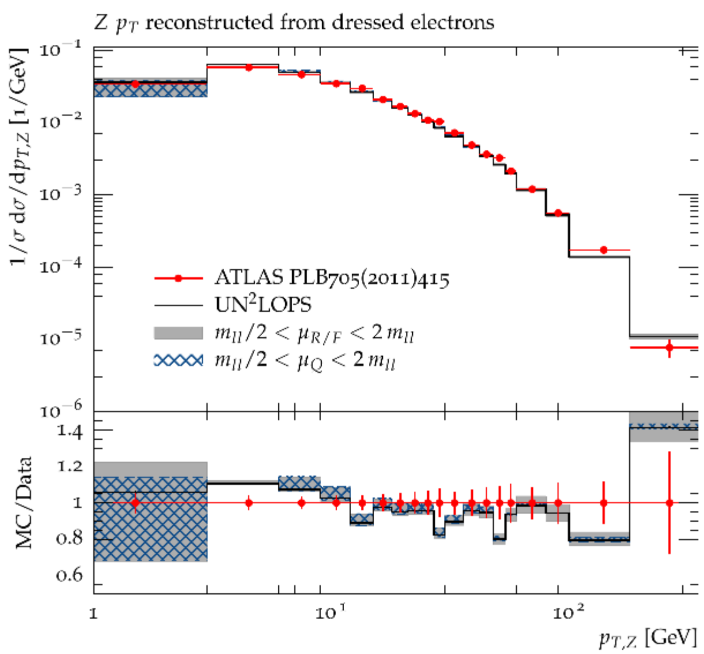
NNLO matched to parton showers

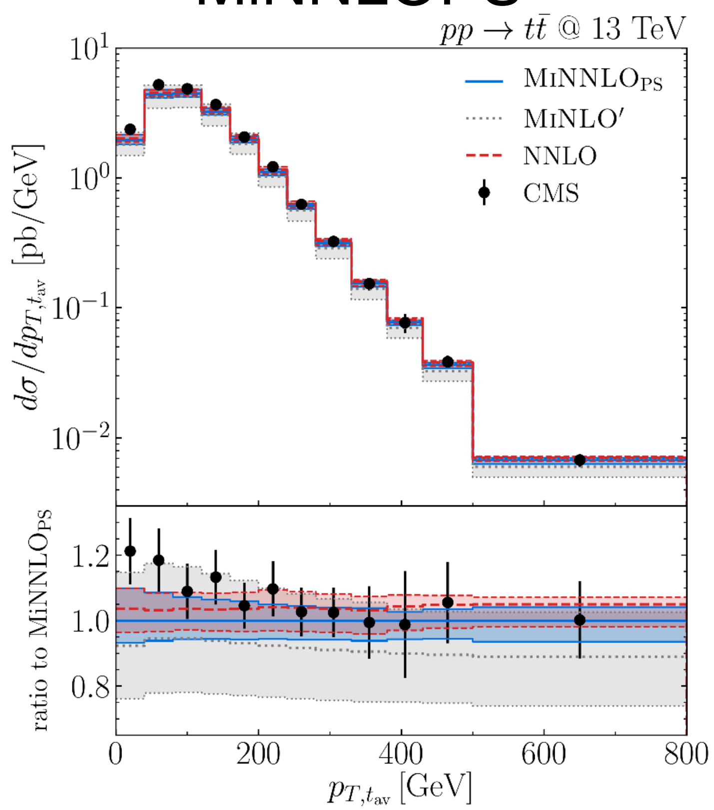
- The increasing experimental precision of LHC measurements challenges existing generators, pushing the request for higher accuracy
- The state-of-the-art is the inclusion of NNLO corrections into parton-shower Monte Carlo
- Three main approach to the problem:

UNNLOPS



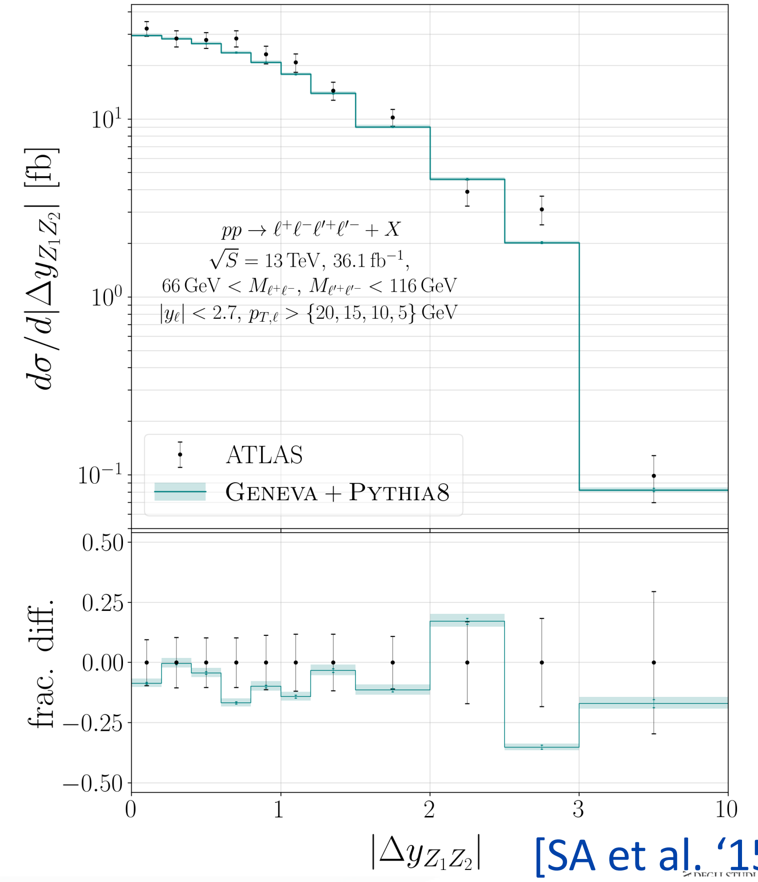
[Hoeche et al. '15]

MiNNLOPS



[Nason et al. '12 - '24]

GENEVA



[SA et al. '15 - '24]

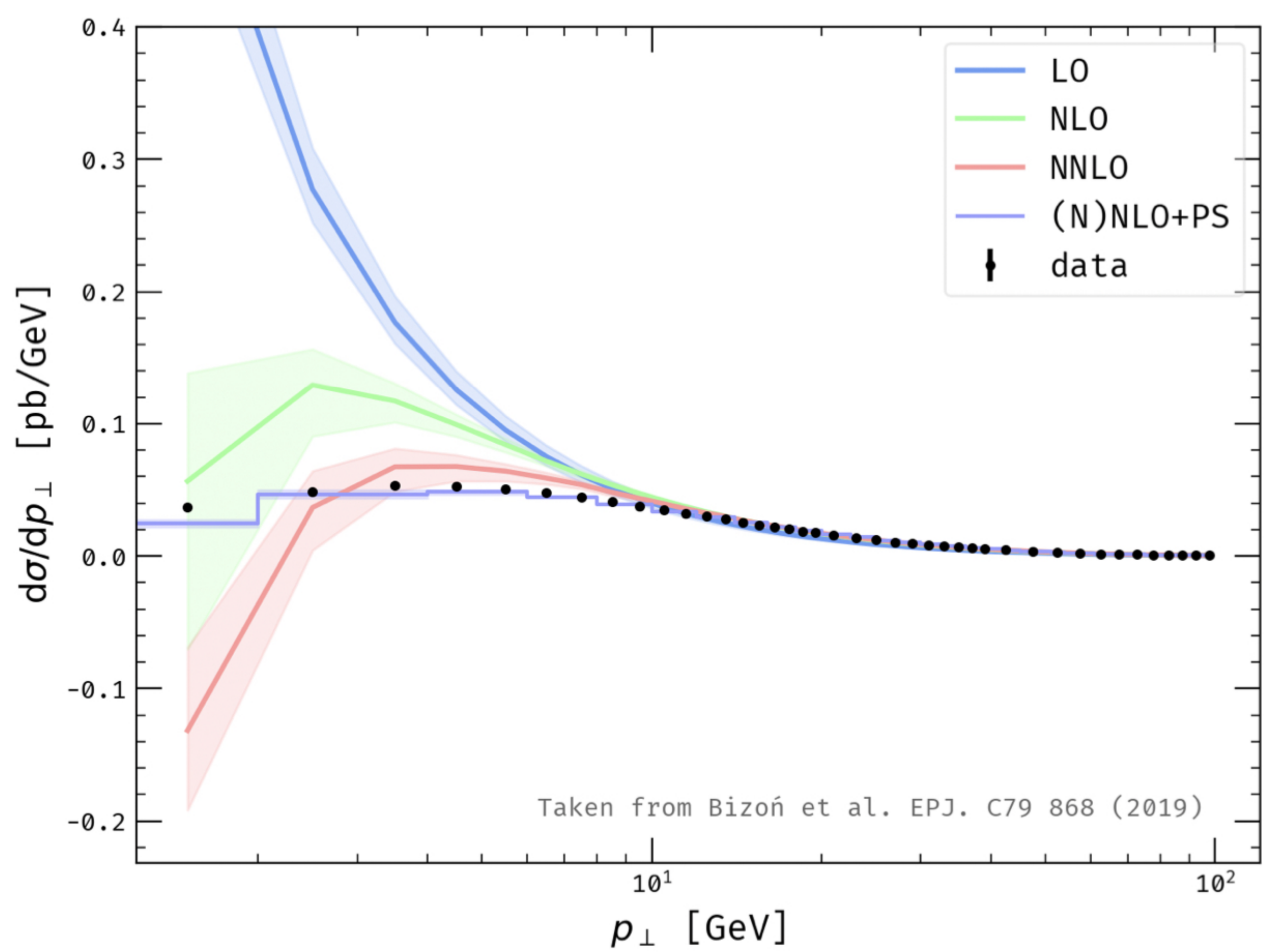
Also NNLO+PS with sector showers available for e^+e^- and $H \to b\bar{b}$

[Campbell et al. 2108.07133]

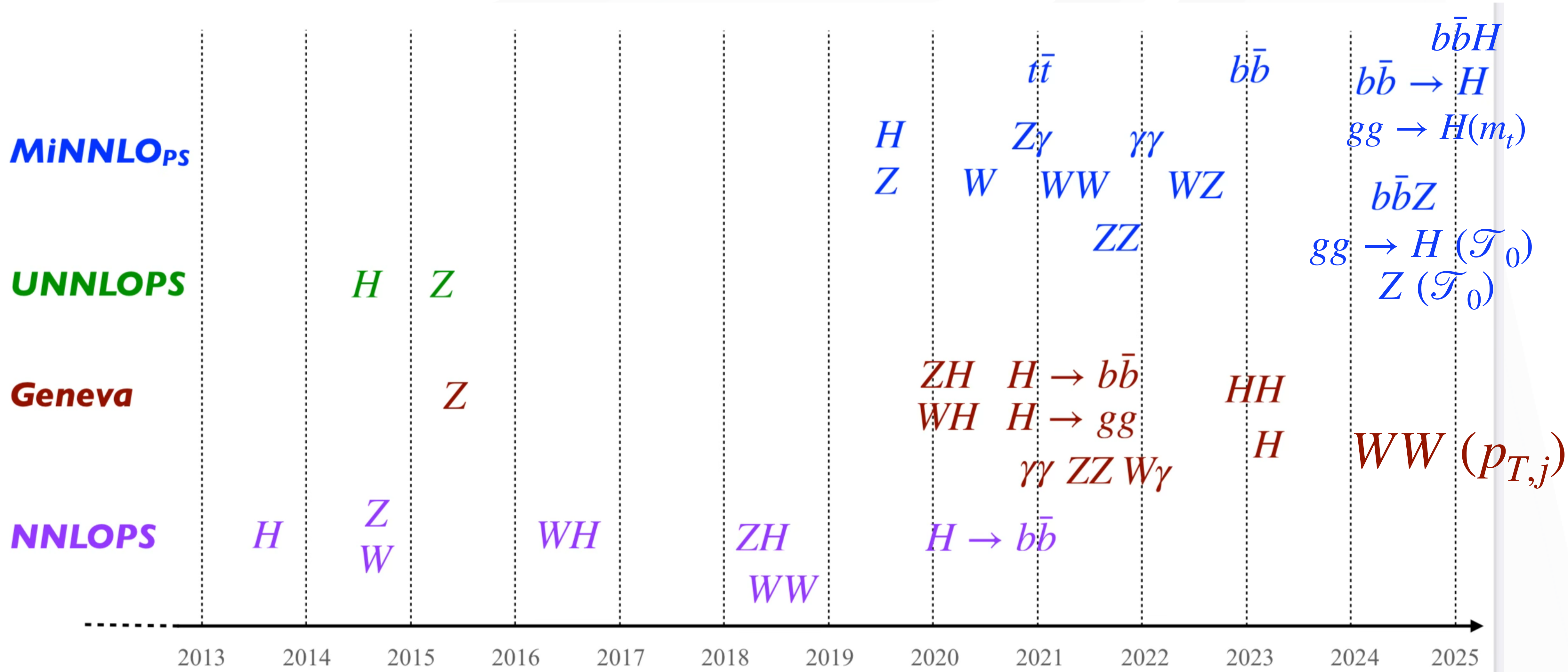
Accuracy goals - Example for $gg \to H$

- NNLO accuracy for observables inclusive over extra radiation, e.g. $d\sigma/dy_H$
- NLO accuracy for H+1 jet observables $d\sigma/dp_T^{j_1}$
- LO accuracy for H+2 jet observables $d\sigma/dp_T^{j_2}$ or $d\sigma/dm_{j_1,j_2}$

- Resumm. accuracy (or Shower Sudakov) for small p_T^H
- Further emissions only in shower soft/coll approximation.



Timeline of NNLO + PS results



[Adapted from table by M.Wiesemann LoopFest'24]

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Geneva and MiNNLOps : comparison

GENEVA

Originally formulated for $\mathcal{T}_0$, now extended to use q_T and hardest-jet p_T as primary resolution variable

Employs higher-order resummation (≥NNLL’) of primary resolution variable

Used $\mathcal{T}_1$ as secondary resolution variable. Now also extended to $p^j_{T,2nd}$ Resummation done analytically at NLL

Mostly implemented with additive matching between resummation and fixed-order

MiNNLOps

Originally formulated with q_T as primary resolution variable, now extended to $\mathcal{T}_0$

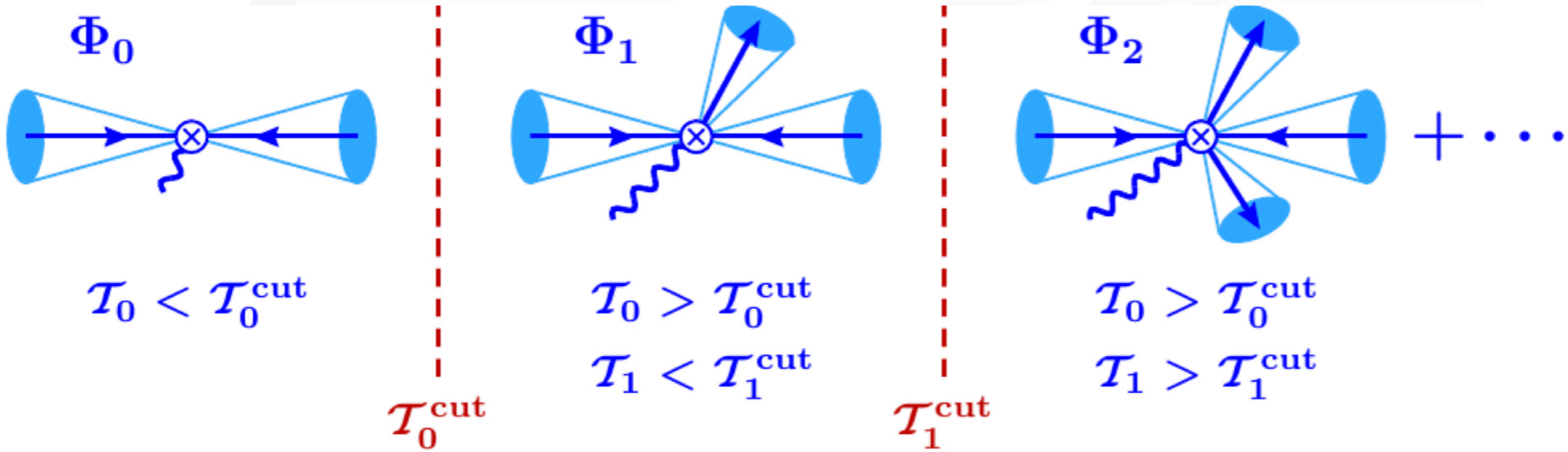
POWHEG-inspired Sudakov factor perform the resummation of primary resolution variable

Actual POWHEG Sudakov factor perform the resummation of secondary resolution variable

Multiplicative matching between resummation and fixed-order to achieve NNLO accuracy

How to build a NNLO + PS : GENEVA example

- Need a set of resolution parameters to measure hardness of first emission $p_T^H, p_T^{j_1}, \mathscr{T}_0$. For second emission $p_T^{j_2}, \mathscr{T}_1 \ldots$



- Partonic fixed-order weights have a log dependence on the resolution parameters
- Dependence is resummed either explicitly or by the shower Sudakov

- Results at partonic level can be further evolved by different shower matching and hadronization models

From resummation to event generation

Final GENEVA partonic formulae combine resummation and matching to fixed-order

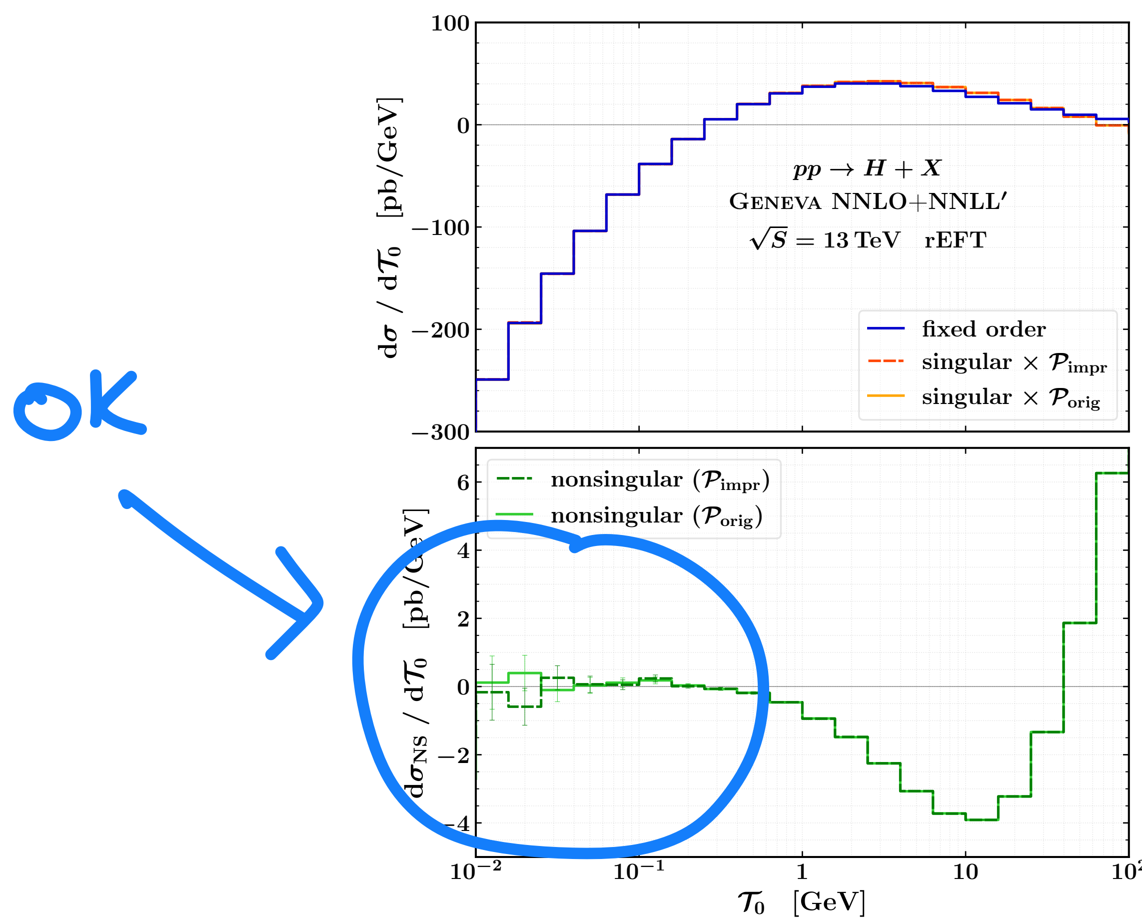
$$\frac{d\sigma^{\mathrm{MC}_0}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) = \frac{d\sigma^{\mathrm{NNLL}'}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) + \frac{d\sigma_0^{\mathrm{nons}}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}})$$

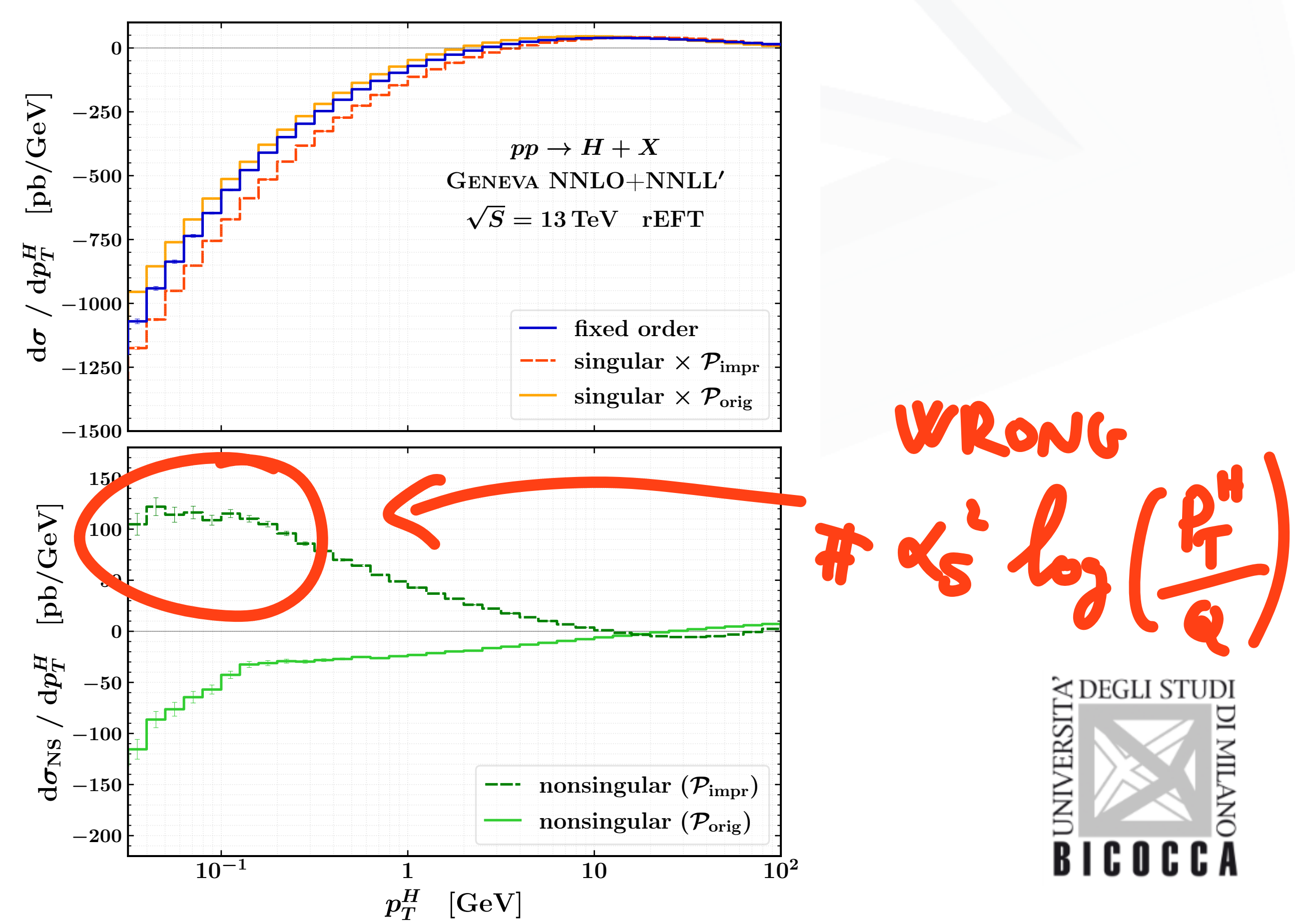
$$\frac{d\sigma_0^{\mathrm{nons}}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) = \frac{d\sigma_0^{\mathrm{NNLO}_0}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}}) - \left[\frac{d\sigma^{\mathrm{NNLL}'}}{d\Phi_0}(\mathcal{T}_0^{\mathrm{cut}})\right]_{\mathrm{NNLO}_0}$$

Lacking multi-differential resummation at this order, resummed results in $\mathcal{T}_0$ need to be made more differential via splitting functions, capturing the singular behaviour of different resolution variables as best as they can.

$$\frac{d\sigma^{\mathrm{MC}_1}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}; \mathcal{T}_1^{\mathrm{cut}}) = \frac{d\sigma^{C}_{\geq 1}}{d\Phi_1} U_1(\Phi_1, \mathcal{T}_1^{\mathrm{cut}})\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}) + \frac{d\sigma_1^{\mathrm{match}}}{d\Phi_1}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}; \mathcal{T}_1^{\mathrm{cut}})$$

$$\frac{d\sigma^{\mathrm{MC}_{\geq 2}}}{d\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}, \mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}}) = \frac{d\sigma^{C}_{\geq 1}}{d\Phi_1} U_1'(\Phi_1, \mathcal{T}_1)\, \theta(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}})\Big|_{\Phi_1 = \Phi_1^{\mathcal{T}}(\Phi_2)} \times \mathcal{P}(\Phi_2)\, \theta(\mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}}) + \frac{d\sigma^{\mathrm{match}}_{\geq 2}}{d\Phi_2}(\mathcal{T}_0 > \mathcal{T}_0^{\mathrm{cut}}, \mathcal{T}_1 > \mathcal{T}_1^{\mathrm{cut}})$$





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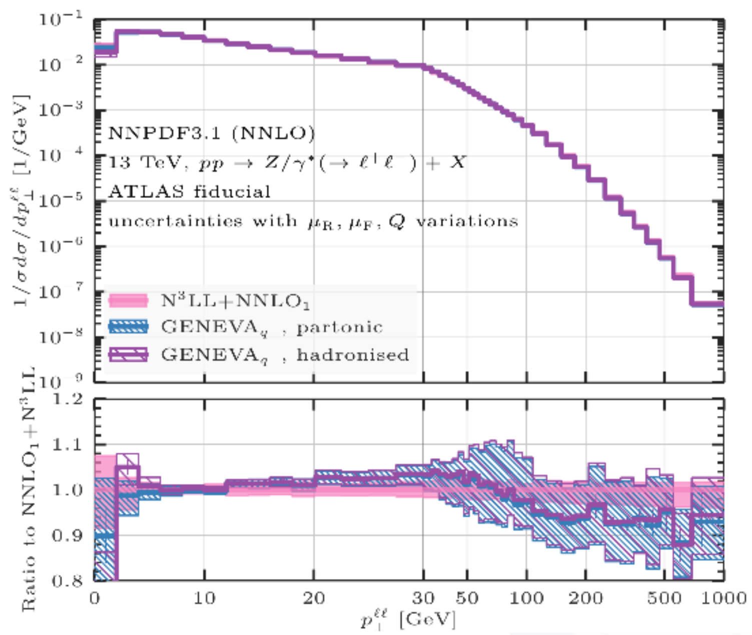
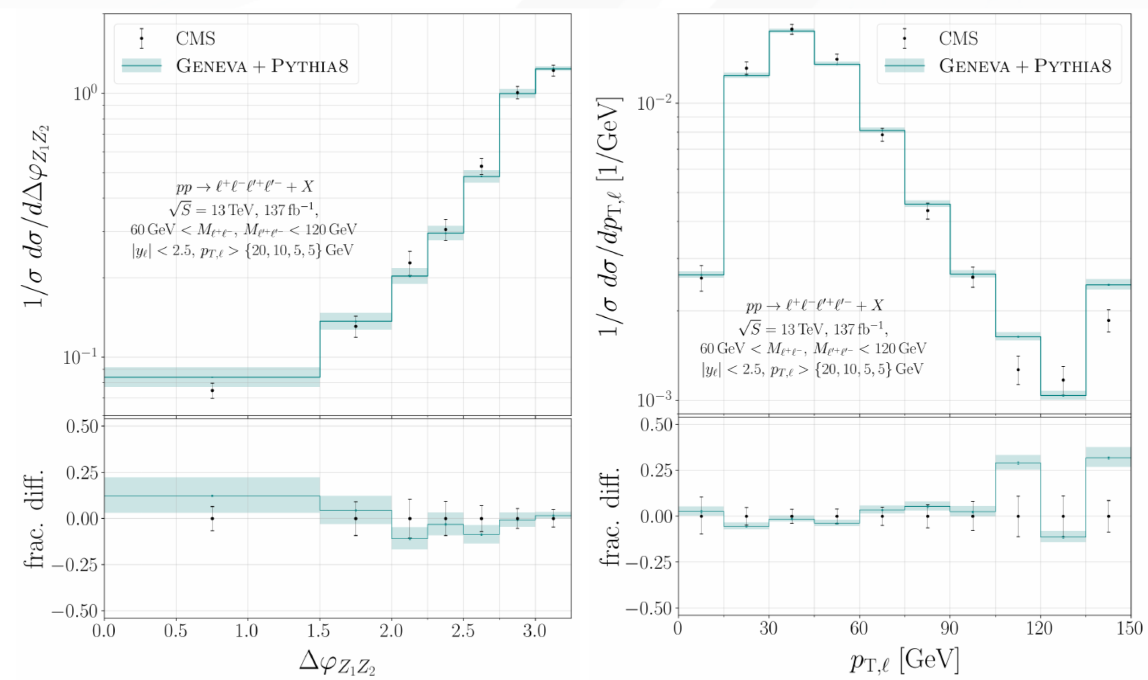
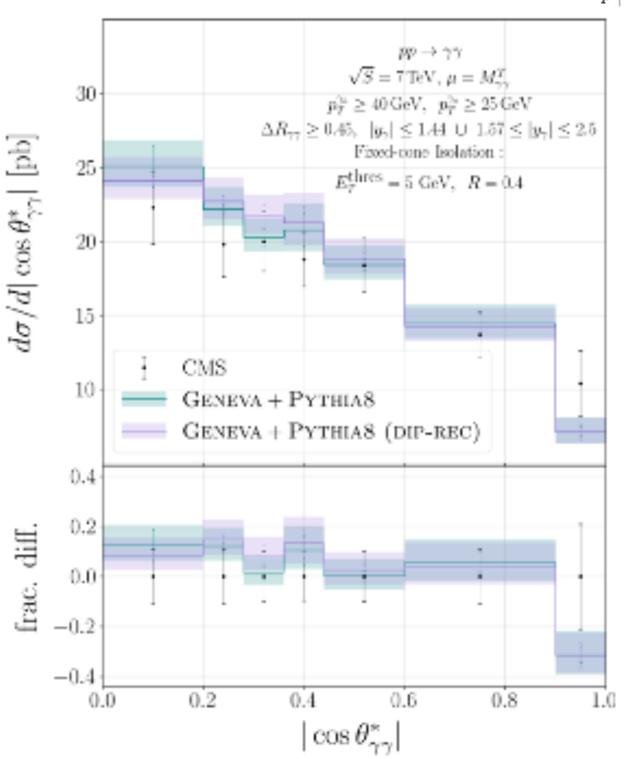
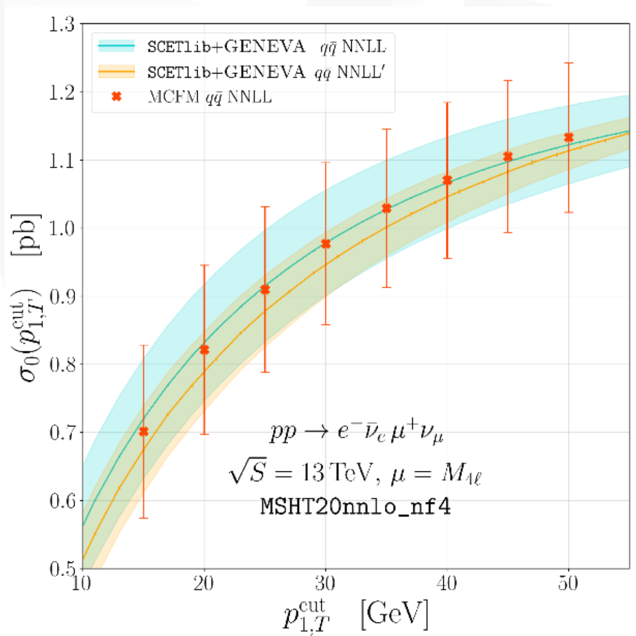
Implemented color-singlet processes

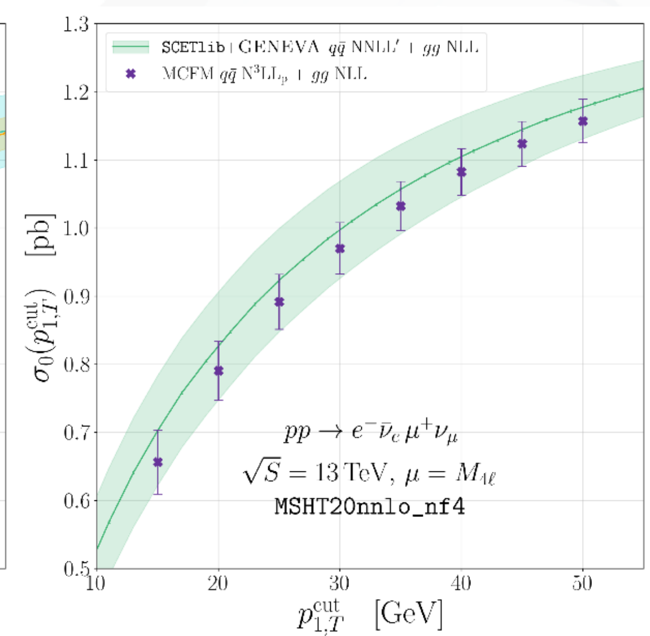
Method has been tested and validated with several color singlet production processes:
DY, ZZ, $W\gamma$, VH, $\gamma\gamma$, ggH, ggHH, WW using both $\mathcal{T}_0$, q_T and $p_T^{\rm jet}$











Method also extended to top-quark pair production with zero-jettiness resummation

MiNNLOps

To draw the parallel with GENEVA, let's start from an additive approach

$$d\sigma^F = d\sigma_F^{\text{res}} + [d\sigma_{FJ}]_{\text{f.o.}} - [d\sigma_F^{\text{res}}]_{\text{f.o.}}$$

The resummed component is recasted as a total derivative (exact up to the 2nd order)

$$\frac{d\sigma_F^{\text{res}}}{dp_T d\Phi_B} = \frac{d}{dp_T}\{e^{-S}\mathscr{L}\} = e^{-S}\underbrace{\{S'\mathscr{L} + \mathscr{L}'\}}_{\equiv D}$$

in terms of a luminosity function and a Sudakov form factor

$$\mathscr{L} \sim H(C\otimes f)(C\otimes f)$$

$$S(p_T) = \int_{p_T}^{Q} \frac{dq}{q}\left(A(\alpha_s(q))\ln\frac{Q^2}{q^2} + B(\alpha_s(q))\right)$$

i.e. the ingredients of a q_T resummation (one needs them up to NNLL' accuracy)

MiNNLOps

Turned into a multiplicative approach by factoring out the Sudakov

$$d\sigma^F = d\sigma_F^{\mathrm{res}} + [d\sigma_{FJ}]_{\mathrm{f.o.}} - [d\sigma_F^{\mathrm{res}}]_{\mathrm{f.o.}} = e^{-S}\left\{ D + \frac{[d\sigma_{FJ}]_{\mathrm{f.o.}}}{[e^{-S}]_{\mathrm{f.o}}} - \frac{[d\sigma_F^{\mathrm{res}}]_{\mathrm{f.o.}}}{[e^{-S}]_{\mathrm{f.o}}} \right\}$$

Re-expanding up to $\mathcal{O}(\alpha_s^3)$ gives the MiNNLOps formula

$$\frac{d\sigma_F^{\mathrm{MiNNLO}}}{dp_T d\Phi_B} = e^{-S(p_T)}\Bigg\{ \underbrace{\frac{\alpha_s(p_T)}{2\pi}\frac{d\sigma_{FJ}^{(1)}}{dp_T d\Phi_B}}_{\mathcal{O}(\alpha_s(p_T))}\underbrace{\left(1 + \frac{\alpha_s}{2\pi}S^{(1)}(p_T)\right) + \left(\frac{\alpha_s(p_T)}{2\pi}\right)^2 \frac{d\sigma_{FJ}^{(2)}}{dp_T d\Phi_B}}_{\mathcal{O}(\alpha_s(p_T^2))}$$

$$+ \underbrace{\left[D(p_T) - \frac{\alpha_s}{2\pi}D^{(1)}(p_T) - \left(\frac{\alpha_s(p_T)}{2\pi}\right)^2 D^{(2)}(p_T) \right]}_{\mathcal{O}(\alpha_s(p_T)^3)} + \text{regular terms } \mathcal{O}(\alpha_s^3) \Bigg\}$$

MiNNLOps

MiNLO' formula = NLO F+j generator which is also NLO accurate for F

$$\frac{d\sigma_F^{\rm MiNNLO}}{dp_T d\Phi_B} = e^{-S(p_T)}\Bigg\{ \underbrace{\frac{\alpha_s(p_T)}{2\pi}\frac{d\sigma_{FJ}^{(1)}}{dp_T d\Phi_B}}_{\mathcal{O}(\alpha_s(p_T))}\underbrace{\left(1+\frac{\alpha_s}{2\pi}S^{(1)}(p_T)\right)+\left(\frac{\alpha_s(p_T)}{2\pi}\right)^2\frac{d\sigma_{FJ}^{(2)}}{dp_T d\Phi_B}}_{\mathcal{O}(\alpha_s(p_T^2))}$$

$$+\underbrace{\left[D(p_T)-\frac{\alpha_s}{2\pi}D^{(1)}(p_T)-\left(\frac{\alpha_s(p_T)}{2\pi}\right)^2 D^{(2)}(p_T)\right]}_{\mathcal{O}(\alpha_s(p_T)^3)} + \text{regular terms } \mathcal{O}(\alpha_s^3)\Bigg\}$$

Sudakov factor exp. suppression when $q_T \to 0$

Additional terms that give NNLO accuracy upon integration over q_T

Terms beyond NNLO accuracy

MiNNLOps

The actual implementation starts from POWHEG implementation of F+j production

$$\frac{d\sigma}{d\Phi_{FJ}} = \tilde{B}^{\mathrm{MiNNLO_{PS}}}_{FJ} \times \left\{ \Delta_{\mathrm{pwg}}(\Lambda_{\mathrm{pwg}}) + \int d\Phi_{\mathrm{rad}} \Delta(p_{T,\mathrm{rad}}) \frac{R(\Phi_{FJ}, \Phi_{\mathrm{rad}})}{B(\Phi_{FJ})} \right\}$$

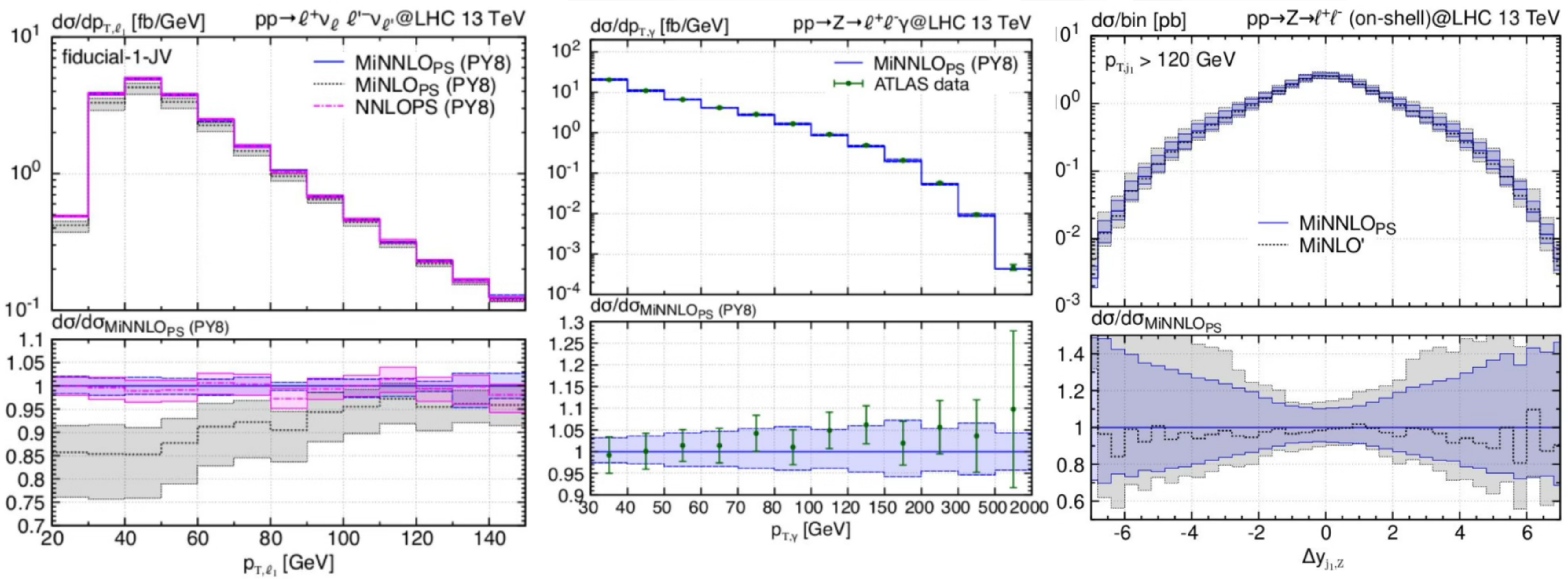
Where the MiNNLOps normalization is obtained spreading the integrated formula over F+J ps

$$\tilde{B}^{\mathrm{MiNNLOPS}}(\Phi_{FJ}) \simeq e^{-S(p_T)} \left\{ \frac{\alpha_s}{2\pi} \left[\frac{d\sigma}{d\Phi_{FJ}}\right]^{(1)} \left(1 + \frac{\alpha_s}{2\pi}[S(p_T)]^{(1)}\right) + \left(\frac{\alpha_s}{2\pi}\right)^2 \left[\frac{d\sigma}{d\Phi_{FJ}}\right]^{(2)} + \left(D(p_T) - D^{(1)}(p_T) - D^{(2)}(p_T)\right) \times \mathscr{P}(\Phi_{\mathrm{FJ}}) \right\}$$

The second emission is generated with the standard POWHEG mechanism

$$\frac{d\sigma}{d\Phi_{FJ}} = \tilde{B}^{FJ} \times \left\{ \Delta_{\mathrm{pwg}}(\Lambda_{\mathrm{pwg}}) + \int d\Phi_{\mathrm{rad}} \Delta(p_{T,\mathrm{rad}}) \frac{R(\Phi_{FJ}, \Phi_{\mathrm{rad}})}{B(\Phi_{FJ})} \right\}$$

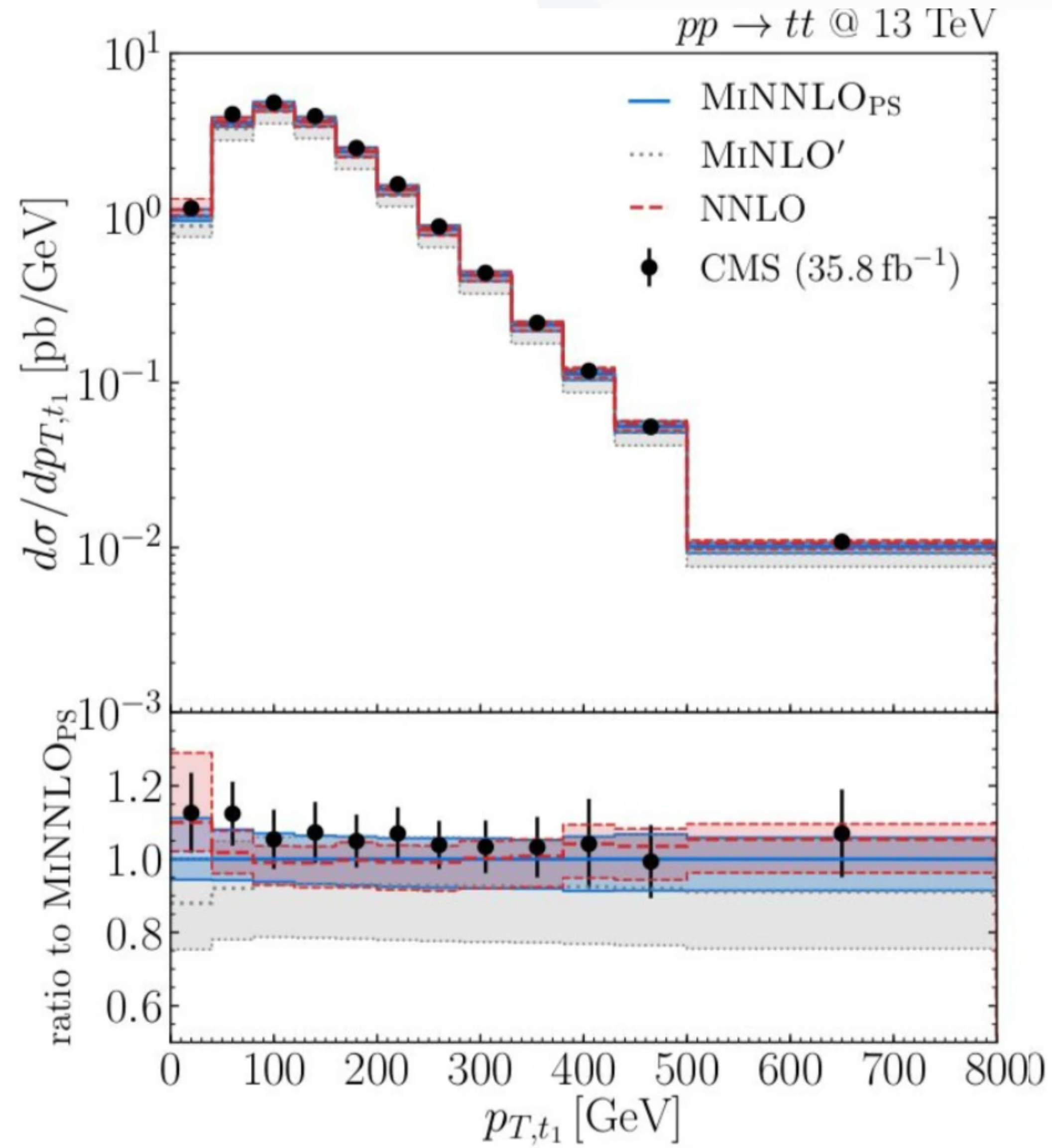
MiNNLOps results - color singlets

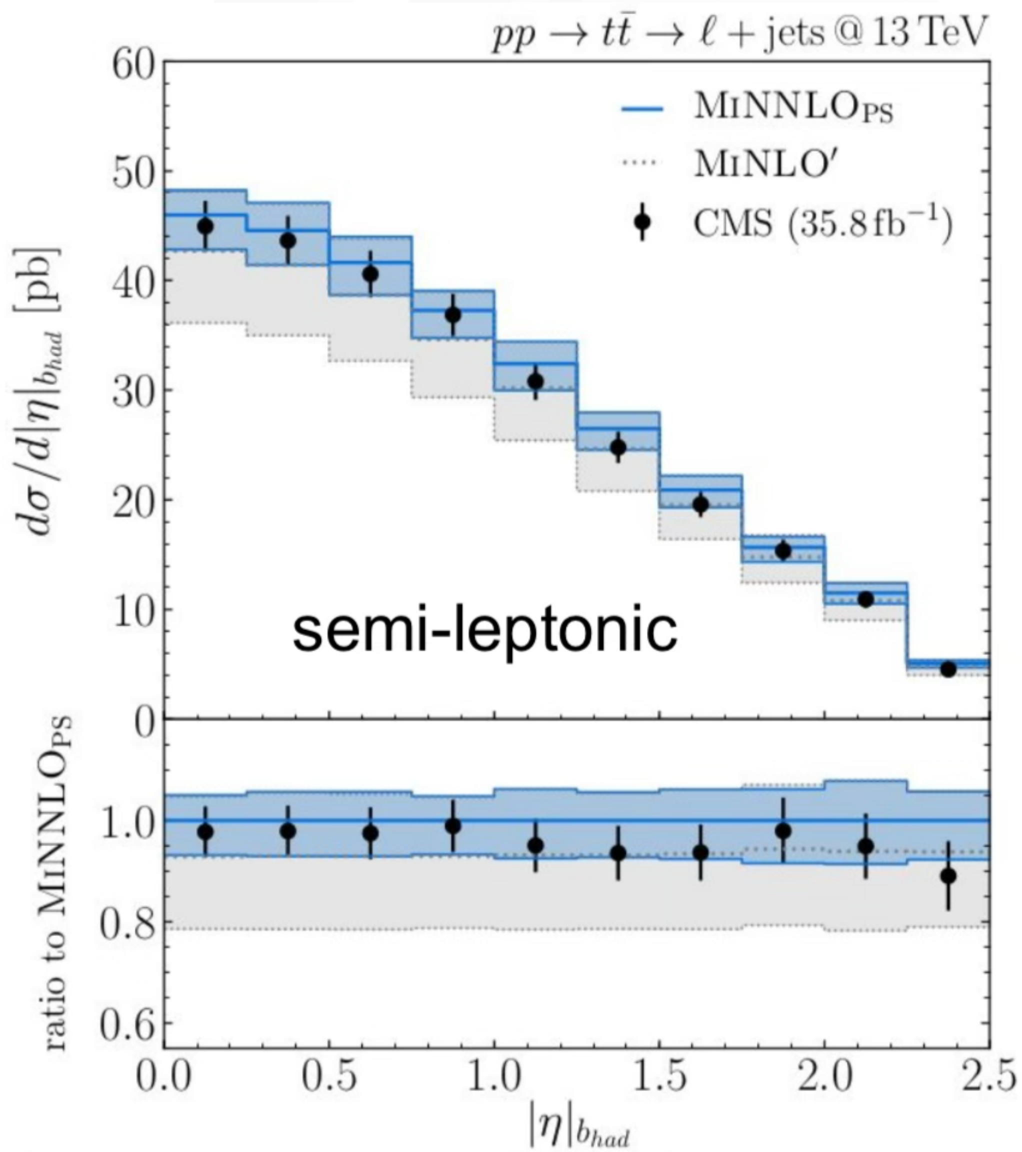


Many color singlet production processes implemented: diboson, Higgs, Drell-Yan etc.

MiNNLOps results - heavy quarks pair

Method extended to top-quark (and bottom-quark) pairs relying on transverse momentum resummation expansion up to NNLO
(no need to resum color matrices in Sudakov factor)





[Mazzitelli et al. '20-'21, '23]

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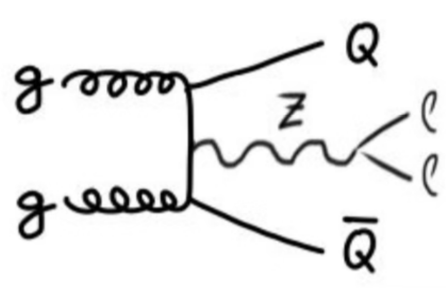
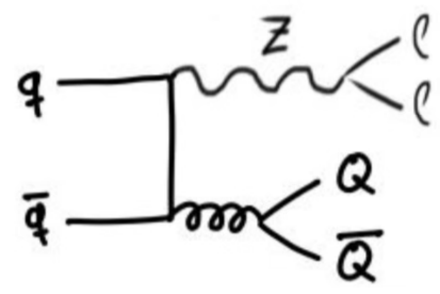
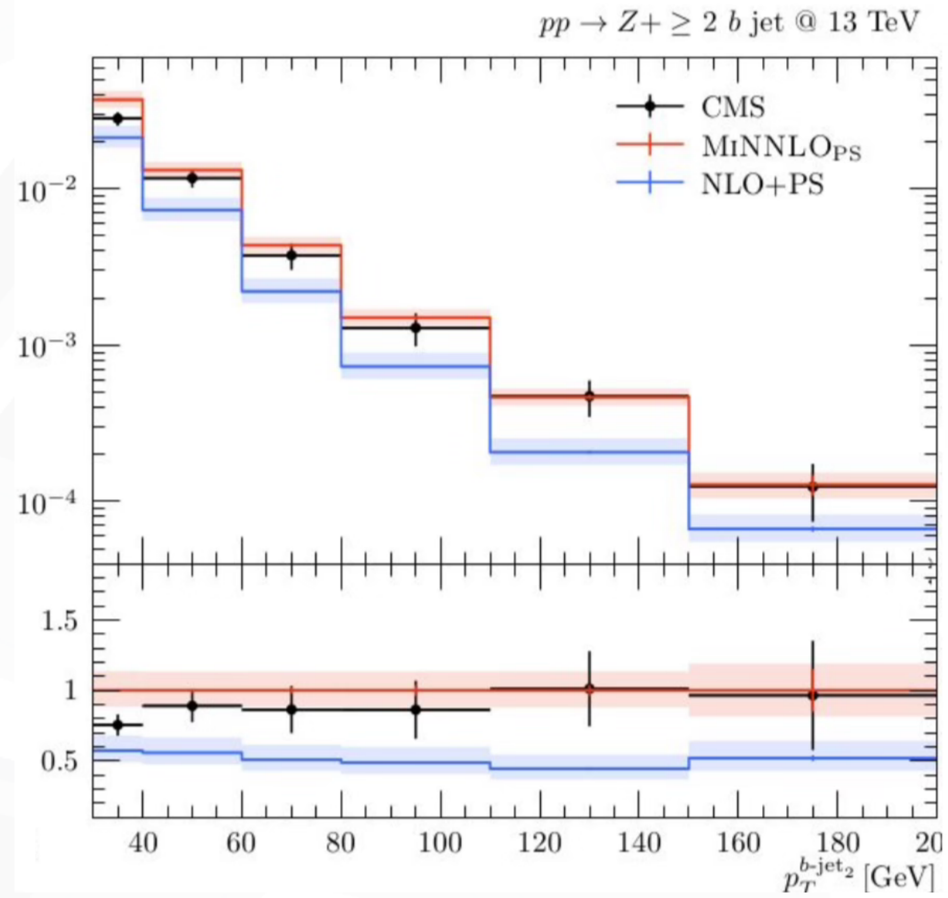
MiNNLOps results - heavy quarks plus boson

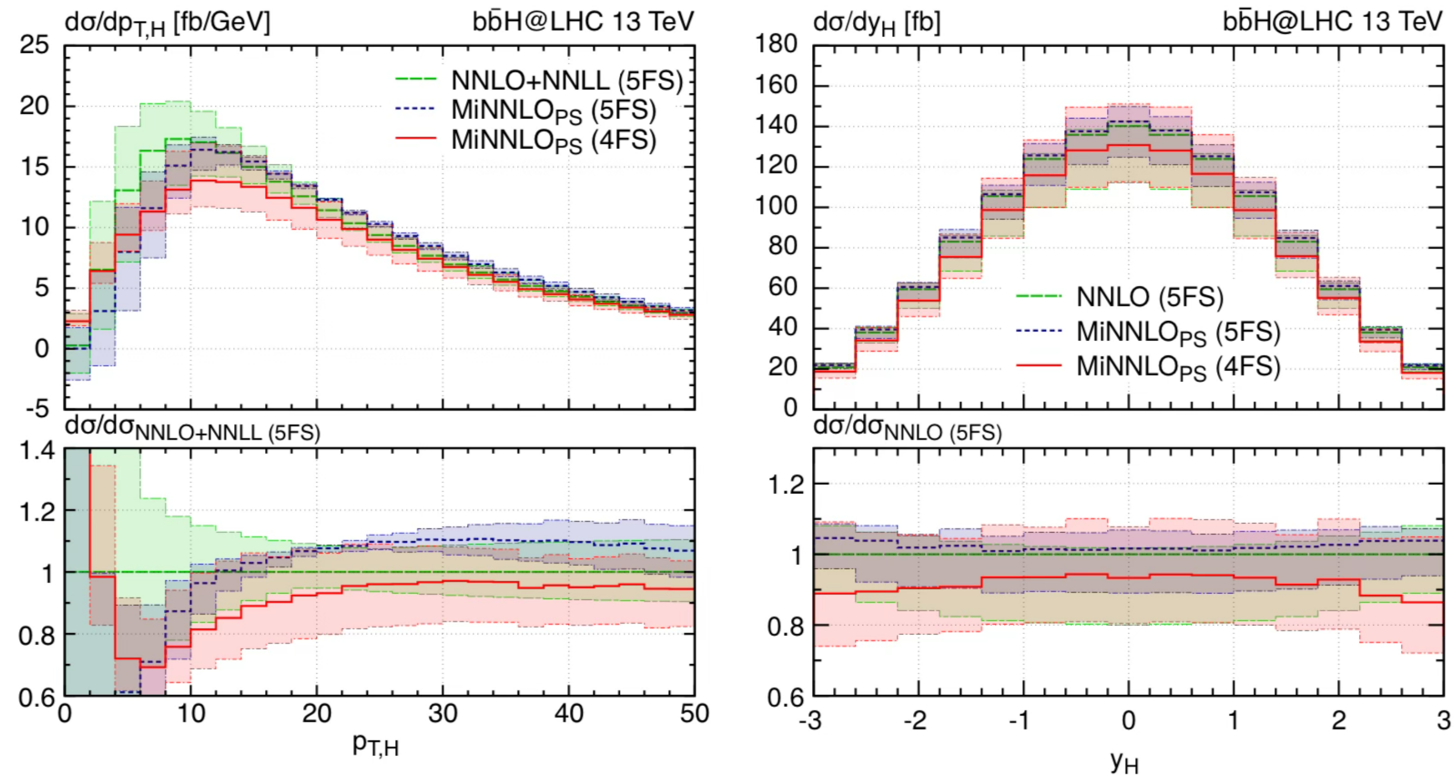
Recently extended to 2->3 processes with massive colored partons ($Z\bar{b}b$ and $H\bar{b}b$)







[Mazzitelli et al. '24]



[Biello et al. '24]

Massification procedure for double virtual amplitudes.

State-of-the-art NNLO calculations immediately available as NNLO+PS

MiNNLOps: extension to N-jettiness

Using a different resolution variable allows to study the robustness of the method and assess the uncertainties associated to its choice

The choice has important consequences for the interface with the parton shower and for the extension to more complicated processes

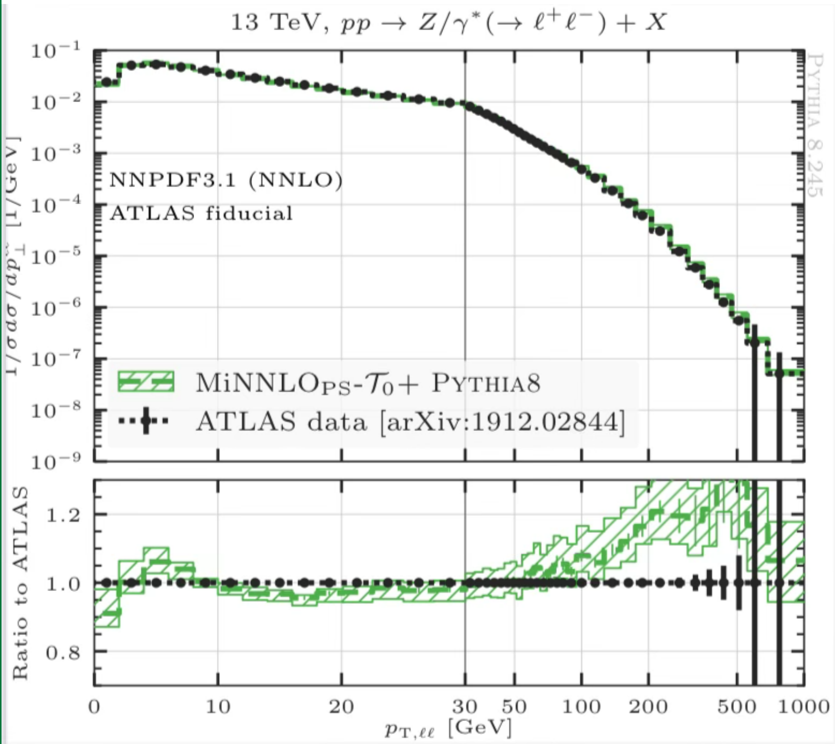
MiNNLOps recently has been extended to use $\mathcal{T}_0$ as primary resolution

$$\tilde{B}^{\mathrm{MiNNLOPS}}(\Phi_{FJ}) \simeq e^{-S(\mathcal{T}_0)} \left\{ \frac{\alpha_s}{2\pi} \left[\frac{d\sigma}{d\Phi_{FJ}} \right]^{(1)} \left(1 + \frac{\alpha_s}{2\pi} [S(\mathcal{T}_0)]^{(1)} \right) + \left(\frac{\alpha_s}{2\pi} \right)^2 \left[\frac{d\sigma}{d\Phi_{FJ}} \right]^{(2)} + \left(D(\mathcal{T}_0) - D^{(1)}(\mathcal{T}_0) - D^{(2)}(\mathcal{T}_0) \right) \times \mathscr{P}(\Phi_{\mathrm{FJ}}) \right\}$$

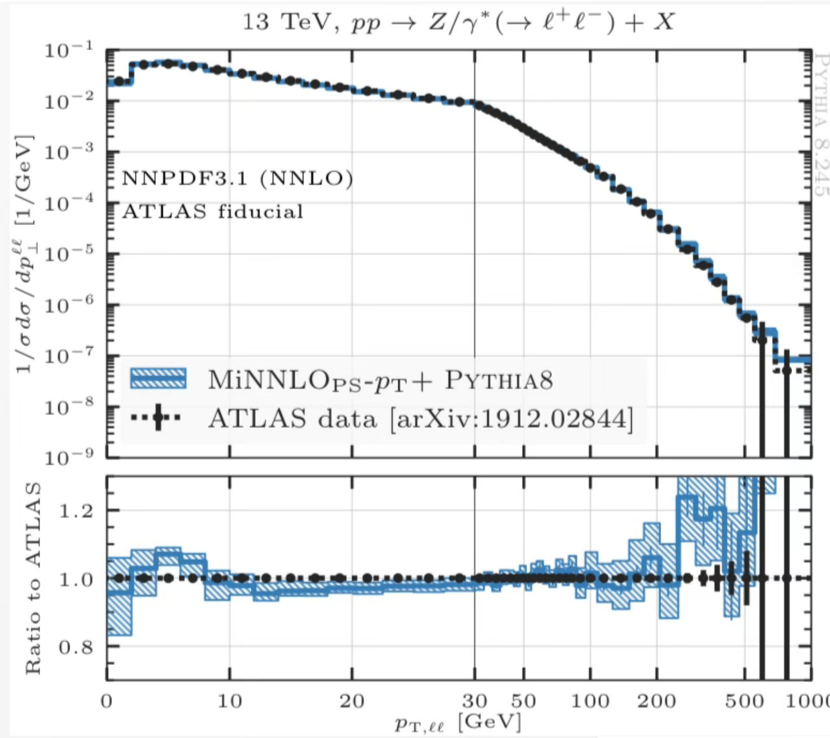
[Ebert et al. '24]

MiNNLOps for V+j using $\mathcal{T}_1$ as primary resolution formulae presented.

MiNNLOps: extension to zero-jettiness

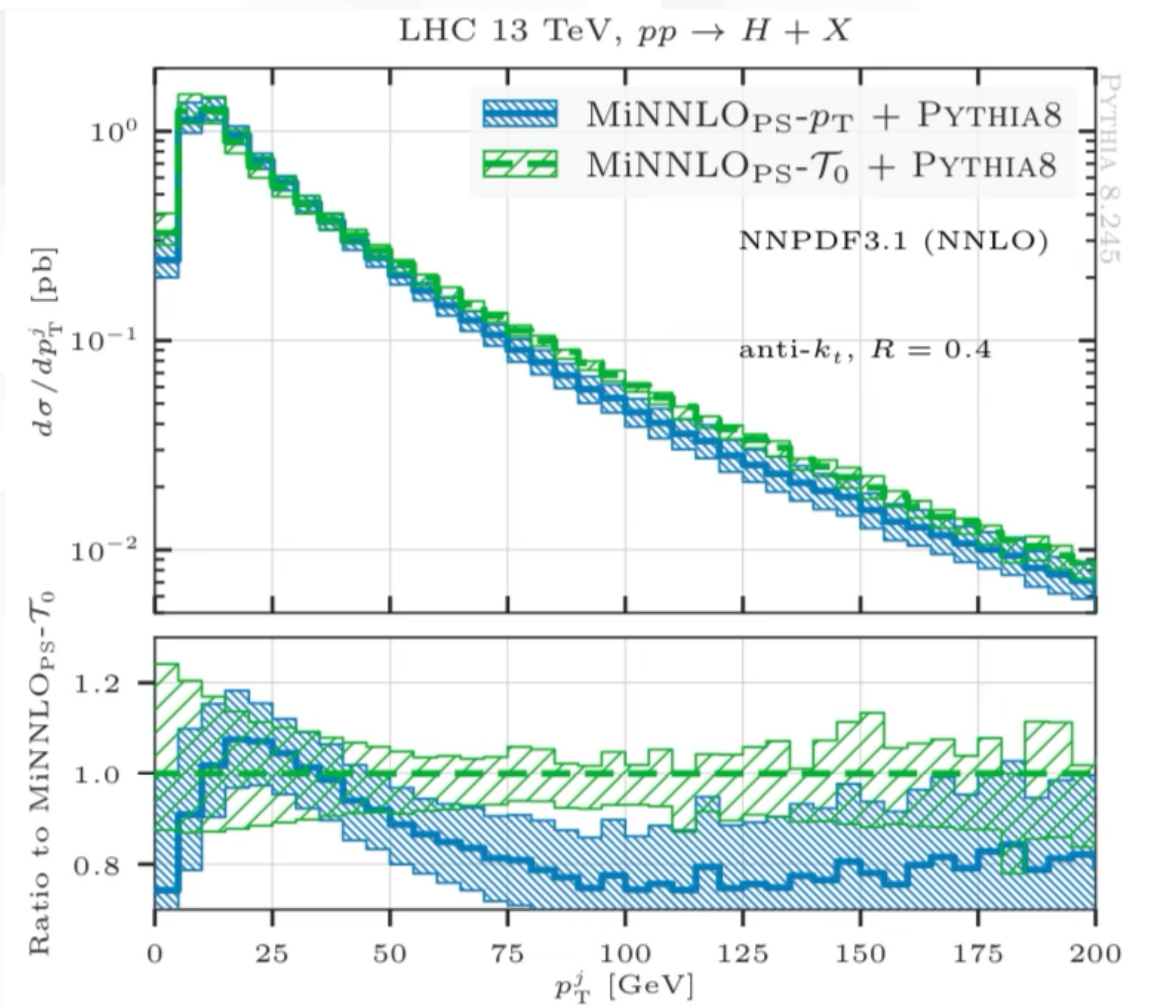


$\mathcal{T}_0$



q_T

Good agreement for transverse momentum and integrated NNLO distributions



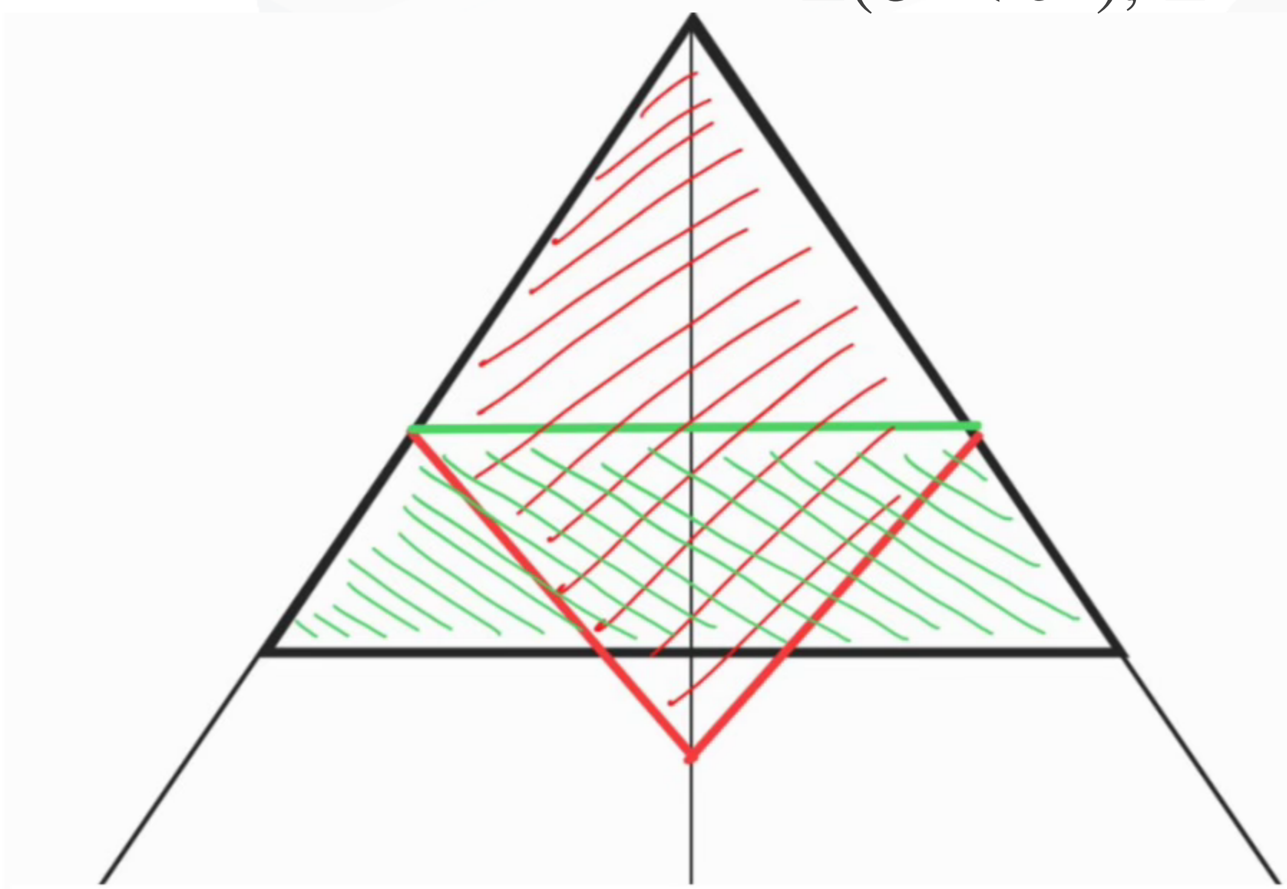
NLO F+j quantities seem more problematic and dependent on the details of the implementation (e.g. on how the resummation and the higher-order terms are spread out to the full F+j phase space)

Shower interface

Interfacing to the shower is one of the most dedicated points of NNLO+PS, even for simple LL showers. The problem starts already at NLO+PS.

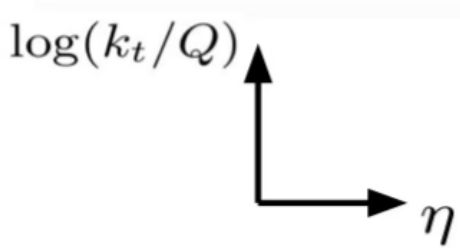
Calculating the cumulant

$$\Sigma(O < e^L),\ L = \log\left(\frac{k_t}{Q}\right)$$



$$O_{PS} \sim \frac{k_T}{Q} e^{-\beta_{PS}|\eta|}$$

$$O_{EG} \sim \frac{k_T}{Q} e^{-\beta_{EG}|\eta|}$$

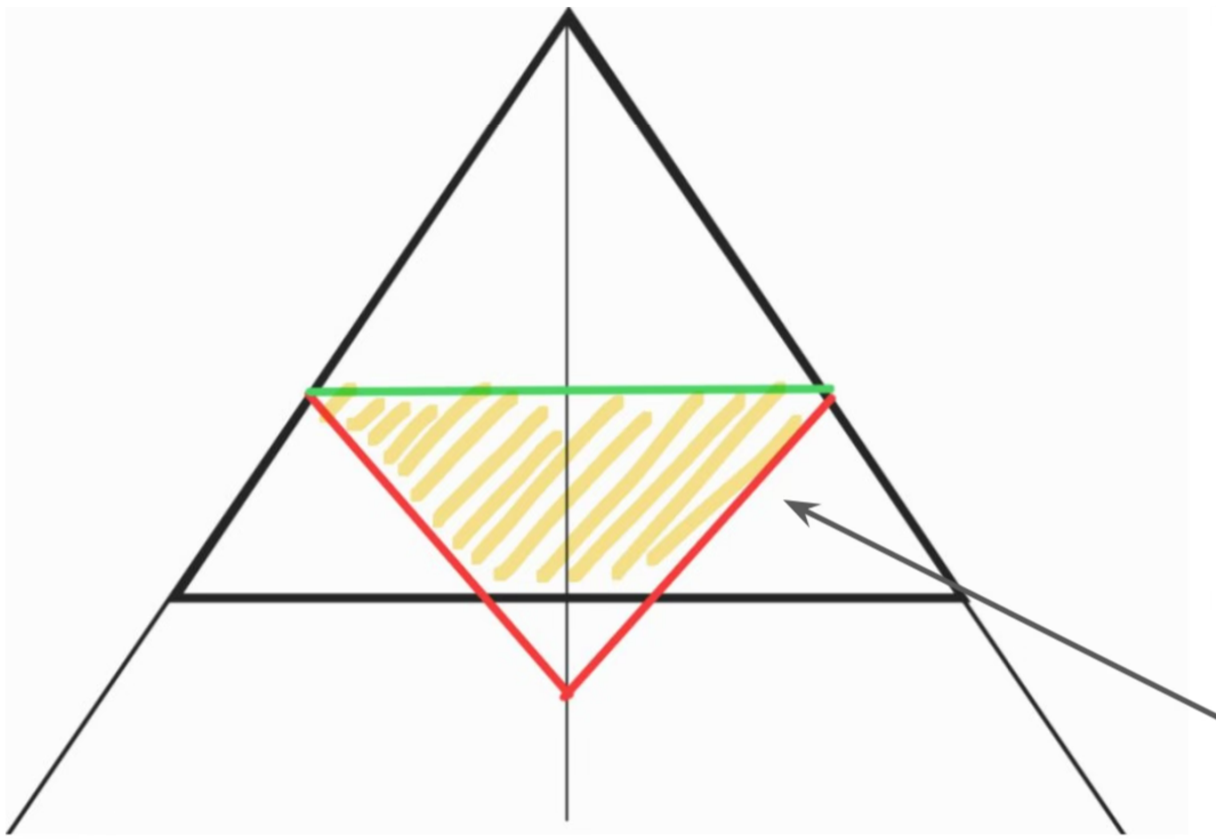


Correct LL given by area in black triangle

Event generator ordering vetoes red area ($\beta_{EG} = 1$)

Parton shower vetoes green area ($\beta_{PS} = 0$)

Shower interface



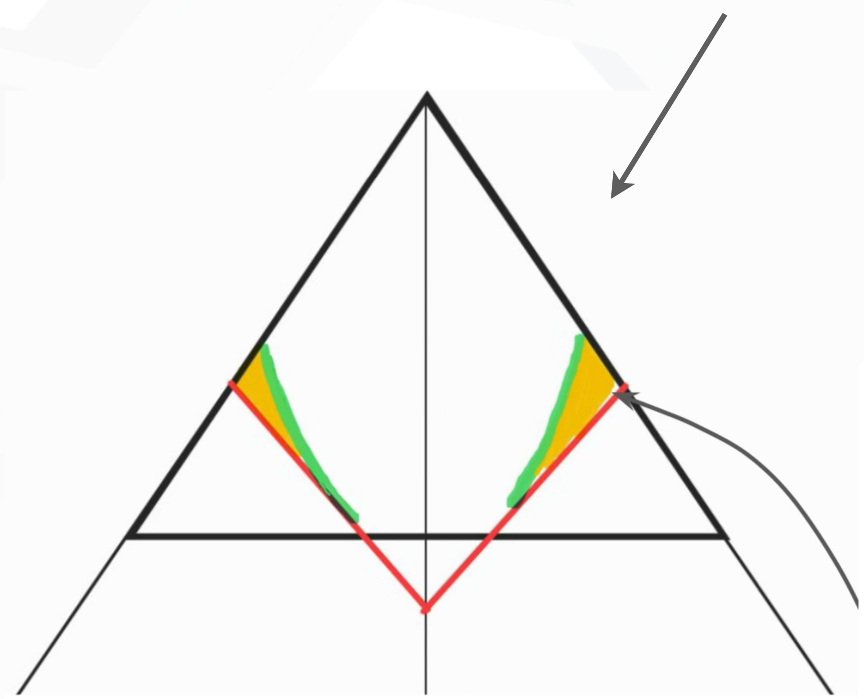
Mismatch of ordering could spoil LL accuracy.

In MiNNLOps-qT (POWHEG) or GENEVA with pT jet no such mismatch because ordering variables are similar

For NLL showers things get more complicated ! Boundaries in the hard-collinear regions have to match as well

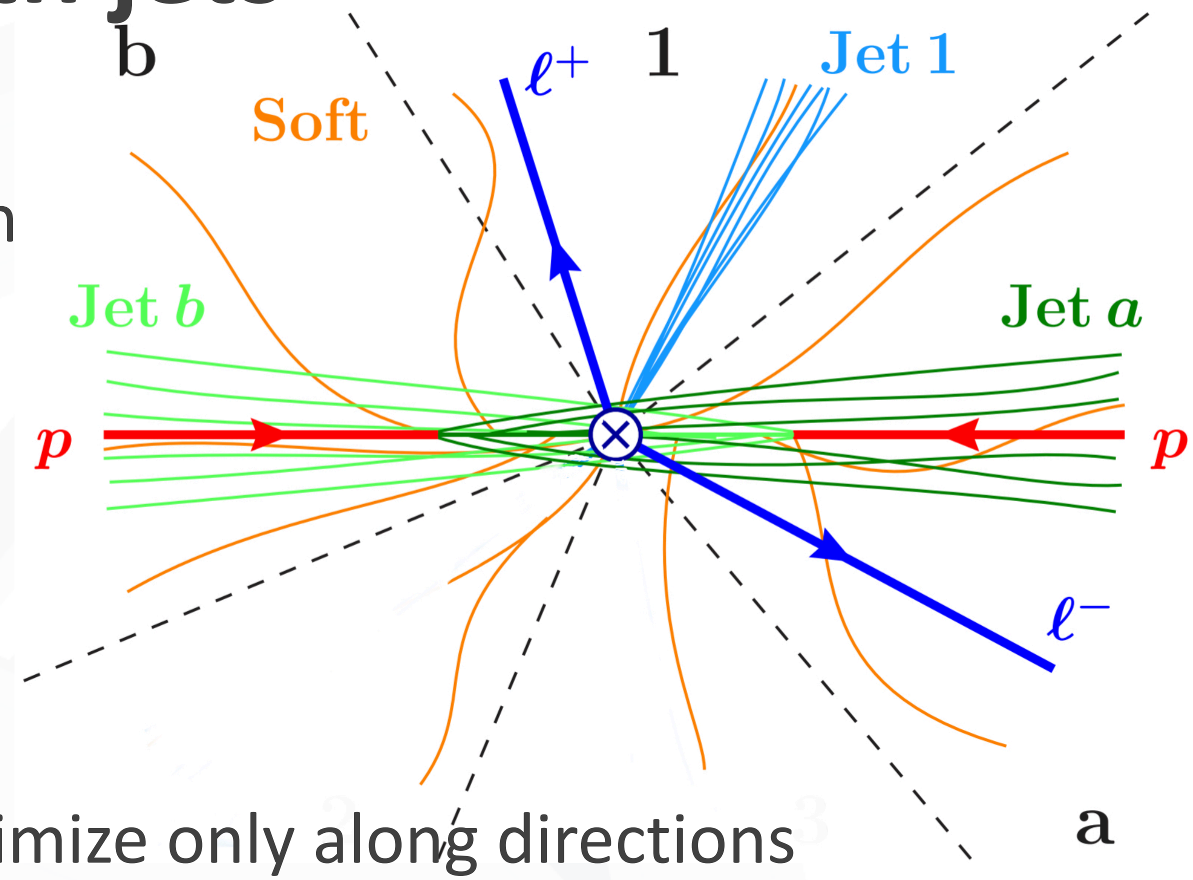
In Geneva $\mathcal{T}_0$ explicit veto is required after shower to avoid double counting (simulating a truncated-vetoed shower)

MiNNLOPS-$\mathcal{T}_0$ formally breaks LL because a change in the POWHEG mapping is required to handle 2nd emission



Boundaries mismatch spoils NLL

Extension to processes with jets



- Focus of color-singlet plus jet production

$$\mathcal{T}_1 = \sum_k \min\{\frac{2q_a \cdot p_k}{Q_a}, \frac{2q_b \cdot p_k}{Q_b}, \frac{2q_J \cdot p_k}{Q_J}\}$$

- To remove energy-dependence and minimize only along directions $Q_i = 2E_i$'s must be frame-dependent

$$\hat{\mathcal{T}}_1 = \sum_k \min\{\frac{\hat{n}_a \cdot \hat{p}_k}{\rho_a}, \frac{\hat{n}_b \cdot \hat{p}_k}{\rho_b}, \frac{\hat{n}_J \cdot \hat{p}_k}{\rho_J}\}$$

- The choice of the ρ_i's determines the frame in which the one-jettiness resummation is performed. Possible choices:
  LAB or CS-frame $Y_V = 0$

$$\rho_a = e^{\hat{Y}_V}\,,$$
$$\rho_b = e^{-\hat{Y}_V}\,,$$
$$\rho_J = \frac{e^{-\hat{Y}_V}(\hat{p}_J)_+ + e^{\hat{Y}_V}(\hat{p}_J)_-}{2\hat{E}_J}$$

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GENEVA to-do list for color-singlet plus jet:

- ✔ Derive factorization theorem and perform the resummation of the main resolution variable (at least at NNLL')
- ✔ Implement GENEVA formula and validate NNLO accuracy of results for fully differential distributions (NNLO integrator)
- Construct the maps that preserve the main resolution variable ($\mathcal{T}_1$), building a true NNLO event generator with events whose weights are IR-finite and properly resummed.
- Add (N)LL resummation of secondary resolution variable and interface with the shower.

Resummation of one-jettiness for Z+jet

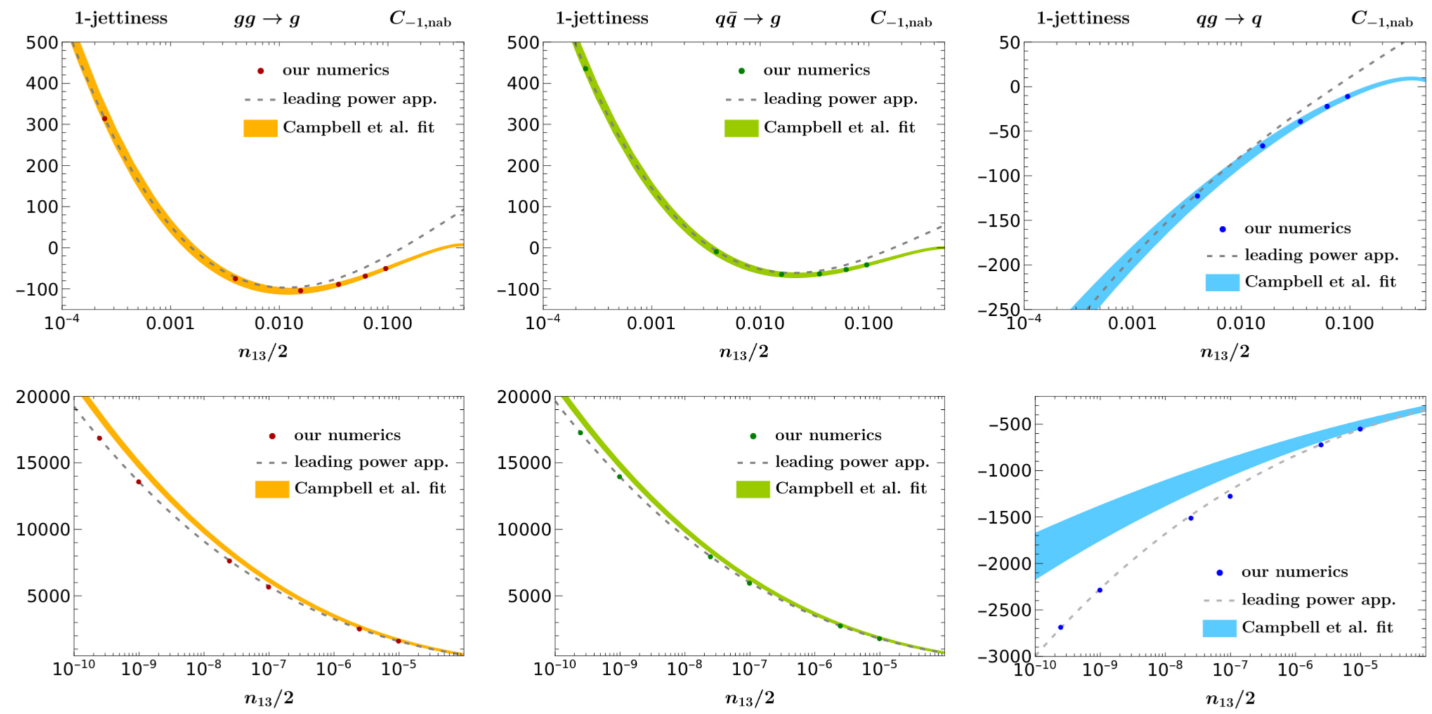
Factorization formula in the region $\mathcal{T}_1 \ll Q$ hard scale: $\sqrt{s}, M_{\ell^+\ell^-}, M_{T,\ell^+\ell^-}, \mathcal{T}_0$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_1 \mathrm{d}\mathcal{T}_1} = \sum_{\kappa=\{q\bar{q}g, qgq, ggg\}} H_\kappa(\Phi_1) \int \mathrm{d}t_a \mathrm{d}t_b \mathrm{d}s_J \, B_{\kappa_a}(t_a) B_{\kappa_b}(t_b) J_{\kappa_J}(s_J) \times S_\kappa\left(n_{a,b} \cdot n_J, \mathcal{T}_1 - \frac{t_a}{Q_a} - \frac{t_b}{Q_b} - \frac{s_J}{Q_J}\right)$$

Hard, beam and jet functions all known

The 2-loop Soft provided by SoftSERVE collaboration in the form of an interpolation grid

Reproduces leading power behavior at extreme angles, important for N3LL and for N3LO singular contribution



Hard evolution

For every channel ($q\bar{q}g$, qgq,ggg,...), ***hard anomalous dimension*** has the form [T. Becher and M. Neubert 1908.11379]

$$\mathbf{\Gamma}_C^\kappa(\mu) = \Gamma_C^\kappa(\mu)\ \mathbf{1} \ = \Bigg\{ \frac{\Gamma_{\text{cusp}}(\alpha_s)}{2}\Bigg[(C_c - C_a - C_b) \ln\frac{\mu^2}{(-s_{ab} - \text{i}0)} + \text{cyclic permutations} \Bigg]$$

4-loops

$$+ \gamma_C^a(\alpha_s) + \gamma_C^b(\alpha_s) + \gamma_C^c(\alpha_s) + \frac{C_A^2}{8} f(\alpha_s)(C_a + C_b + C_c) \Bigg\} \mathbf{1}$$

3-loops

$$+ \sum_{(i,j)} \Bigg[- f(\alpha_s)\boldsymbol{\mathcal{T}}_{iijj} + \sum_{R=F,A} g^R(\alpha_s)(3\boldsymbol{\mathcal{D}}_{iijj}^R + 4\boldsymbol{\mathcal{D}}_{iiij}^R) \ln\frac{\mu^2}{(-s_{ij} - \text{i}0)} \Bigg] + \mathcal{O}(\alpha_s^5)$$

$f(\alpha_s)$ and $g^R(\alpha_s)$ start at $\mathcal{O}(\alpha_s^3)$ and $\mathcal{O}(\alpha_s^4)$ computed in [Henn, Korchemsky, Mistlberger 1911.10174], [Von Manteuffel, Panzer, Schabinger 2002.04617]. Evaluated these contributions as functions of N_c using the *colour space formalism*

$$\boldsymbol{\mathcal{D}}_{ijkl}^R = d_R^{abcd} \boldsymbol{T}_i^a \boldsymbol{T}_j^b \boldsymbol{T}_k^c \boldsymbol{T}_l^d \qquad \boldsymbol{\mathcal{T}}_{ijkl} = f^{ade} f^{bce} (\boldsymbol{T}_i^a \boldsymbol{T}_j^b \boldsymbol{T}_k^c \boldsymbol{T}_l^d)_+$$

$$d_R^{a_1 \dots a_n} = \text{Tr}_R(\boldsymbol{T}^{a_1} \dots \boldsymbol{T}^{a_n})_+ \equiv \frac{1}{n!} \sum_\pi \text{Tr}(\boldsymbol{T}_R^{a_{\pi(1)}} \dots \boldsymbol{T}_R^{a_{\pi(n)}})$$

Using color conservation and symmetry properties of d_R^{abcd}, we found the following relations

$$3(\boldsymbol{\mathcal{D}}_{iijj}^R + \boldsymbol{\mathcal{D}}_{jjii}^R) + 4(\boldsymbol{\mathcal{D}}_{iiij}^R + \boldsymbol{\mathcal{D}}_{jjji}^R) = (D_{kR} - D_{iR} - D_{jR})\ \mathbf{1} \qquad i \neq j \neq k$$

Quartic Casimirs

Similarity to the quadratic case $\quad \boldsymbol{T}_a \cdot \boldsymbol{T}_b = [\boldsymbol{T}_c^2 - \boldsymbol{T}_a^2 - \boldsymbol{T}_b^2]/2$

$$C_4(R_i, R) = \frac{d_{R_i}^{abcd} d_R^{abcd}}{N_{R_i}} \equiv D_{iR}$$

N3LL resummed formula

Combine the solutions to the RG equations for the hard, soft, beam and jet functions to obtain

$$\frac{d\sigma^{N^3LL}}{d\Phi_1 d\mathcal{T}_1} = \sum_\kappa \exp\Big\{ 4(C_a+C_b)K_{\Gamma_{\rm cusp}}(\mu_B,\mu_H) + 4C_c K_{\Gamma_{\rm cusp}}(\mu_J,\mu_H) - 2(C_a+C_b+C_c)K_{\Gamma_{\rm cusp}}(\mu_S,\mu_H)$$
$$- 2C_c L_J\, \eta_{\Gamma_{\rm cusp}}(\mu_J,\mu_H) - 2(C_a L_B + C_b L'_B)\eta_{\Gamma_{\rm cusp}}(\mu_B,\mu_H) + K_{\gamma_{\rm tot}}$$
$$+ \left[C_a \ln\left(\frac{Q_a^2 u}{st}\right) + C_b \ln\left(\frac{Q_b^2 t}{su}\right) + C_{\kappa_j}\ln\left(\frac{Q_J^2 s}{tu}\right) + (C_a+C_b+C_c)L_S\right]\eta_{\Gamma_{\rm cusp}}(\mu_S,\mu_H)$$
$$+ \sum_{R=F,A}\Big[8\,(D_{aR}+D_{bR})K_{g^R}(\mu_B,\mu_H) + 8\,D_{cR}K_{g^R}(\mu_J,\mu_H)$$
$$- 4\,(D_{aR}+D_{bR}+D_{cR})K_{g^R}(\mu_S,\mu_H) - 4D_{cR}L_J\eta_{g^R}(\mu_J,\mu_H) - 4\,(D_{aR}L_B + D_{bR}L'_B)\eta_{g^R}(\mu_B,\mu_H)$$
$$+ 2\left[D_{aR}\ln\left(\frac{Q_a^2 u}{st}\right) + D_{bR}\ln\left(\frac{Q_b^2 t}{su}\right) + D_{cR}\ln\left(\frac{Q_J^2 s}{tu}\right) + (D_{aR}+D_{bR}+D_{cR})L_S\right]\eta_{g^R}(\mu_S,\mu_H)\Big]\Big\}$$
$$\times H_\kappa(\Phi_1,\mu_H)\tilde{S}^\kappa(\partial_{\eta_S}+L_S,\mu_S)\tilde{B}_{\kappa_a}(\partial_{\eta_B}+L_B,x_a,\mu_B)\tilde{B}_{\kappa_b}(\partial_{\eta'_B}+L'_B,x_b,\mu_B)\,\tilde{J}_{\kappa_J}(\partial_{\eta_J}+L_J,\mu_J)$$
$$\times \frac{Q^{-\eta_{\rm tot}}}{\mathcal{T}_1^{\,1-\eta_{\rm tot}}}\frac{\eta_{\rm tot}\, e^{-\gamma_E \eta_{\rm tot}}}{\Gamma(1+\eta_{\rm tot})}$$

Up to NNLL′

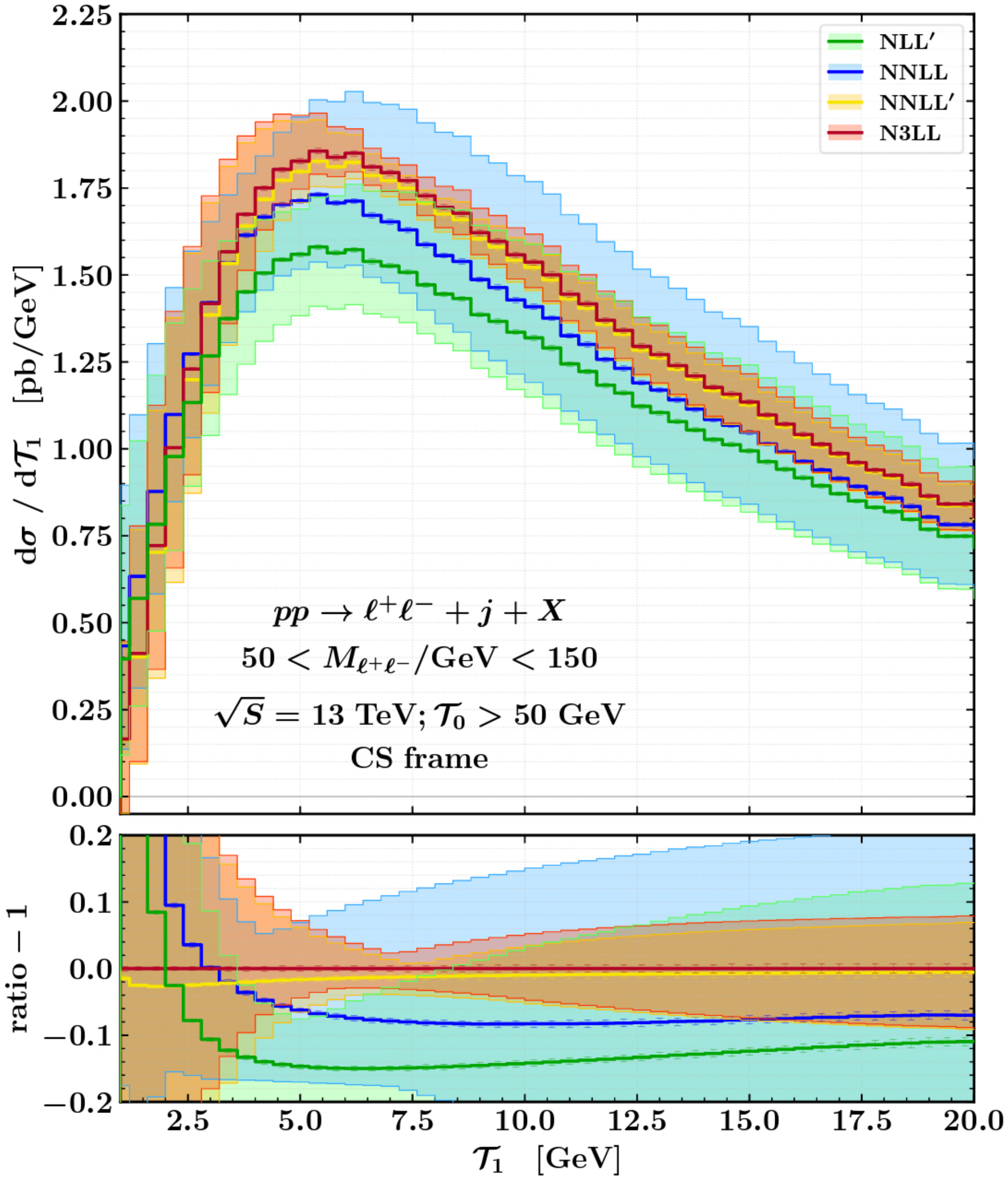
where we defined $\eta_{\rm tot} = \eta_B + \eta'_B + \eta_J + 2\eta_S$

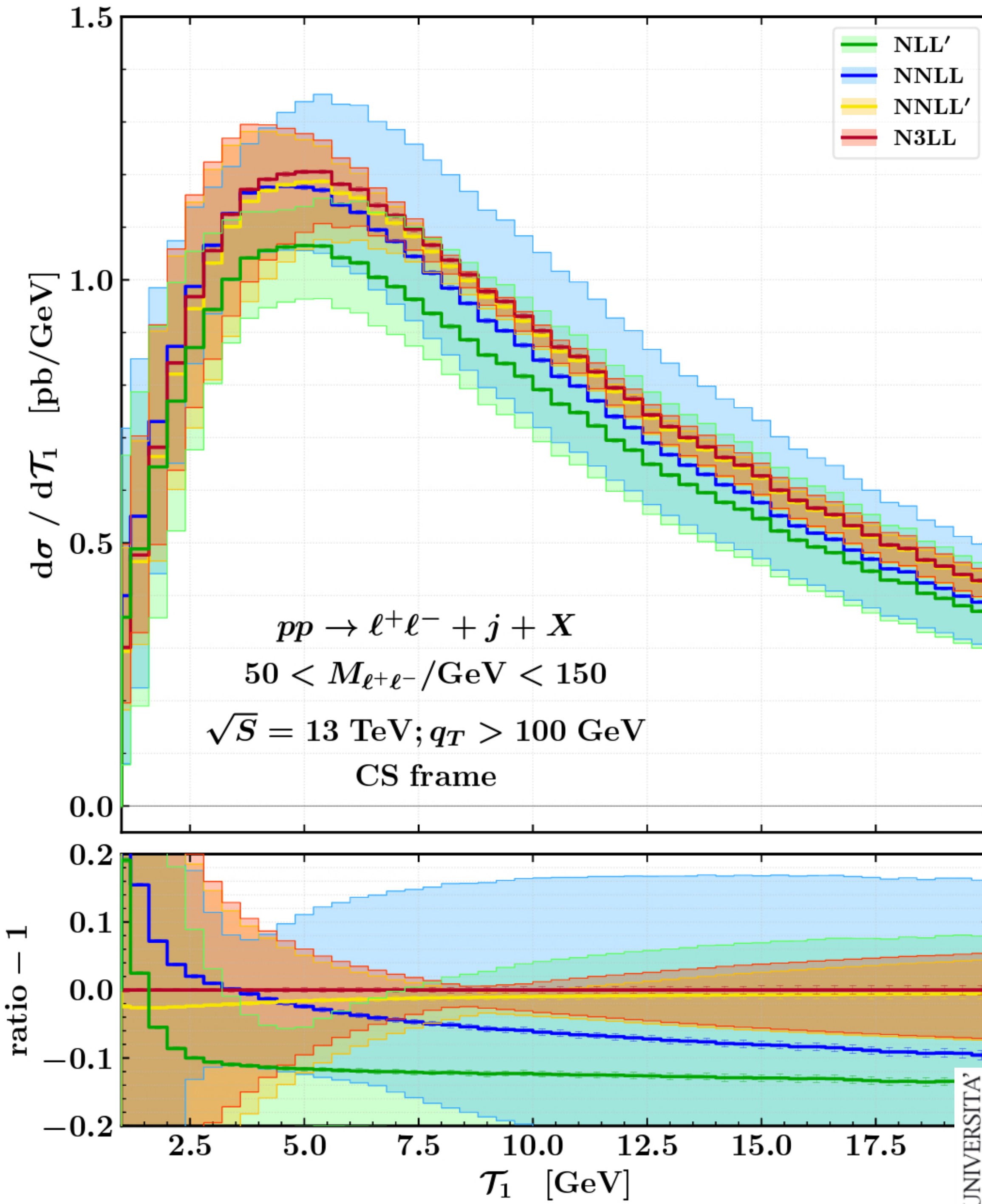
$$L_H = \ln\left(\frac{Q^2}{\mu_H^2}\right),\ L_B = \ln\left(\frac{Q_a Q}{\mu_B^2}\right),\ L'_B = \ln\left(\frac{Q_b Q}{\mu_B^2}\right)$$
$$L_J = \ln\left(\frac{Q_J Q}{\mu_J^2}\right),\ L_S = \ln\left(\frac{Q^2}{\mu_S^2}\right)$$

$$K_{g^R}(\mu_H,\mu) \equiv \int_{\alpha_s(\mu_H)}^{\alpha_s(\mu)} \frac{d\alpha_s}{\beta(\alpha_s)} g^R(\alpha_s) \int_{\alpha_s(\mu_H)}^{\alpha_s} \frac{d\alpha'_s}{\beta[\alpha'_s]}$$
$$\eta_{g^R}(\mu_H,\mu) \equiv \int_{\alpha_s(\mu_H)}^{\alpha_s(\mu)} \frac{d\alpha_s}{\beta(\alpha_s)} g^R(\alpha_s)$$
$$K_f(\mu_H,\mu) \equiv \int_{\alpha_s(\mu_H)}^{\alpha_s(\mu)} \frac{d\alpha_s}{\beta(\alpha_s)} f(\alpha_s)$$

Resummed results

- Summing in quadrature profile scales variations and fixed-order ones
- Nice convergence and reduction of theoretical uncertainties





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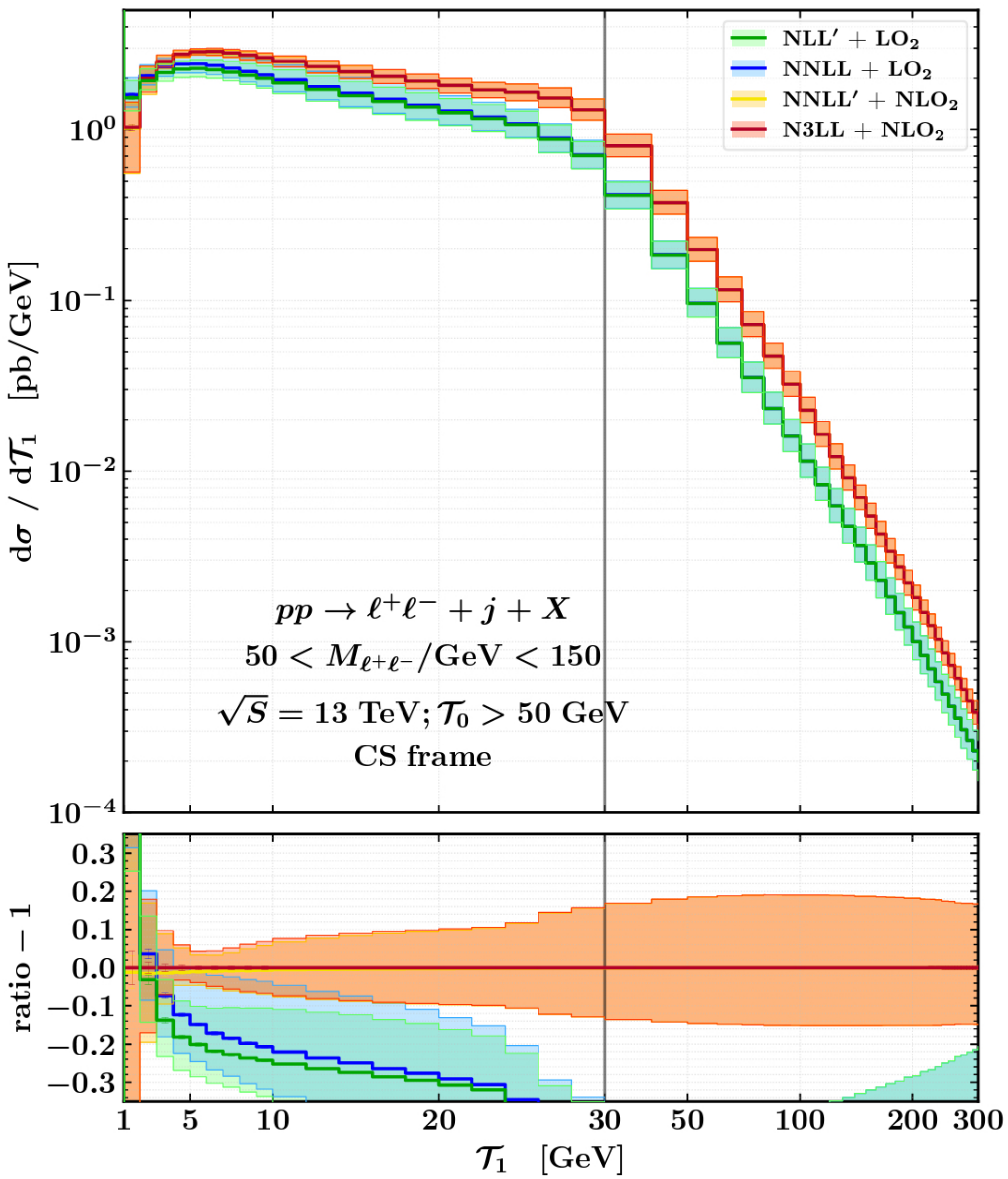
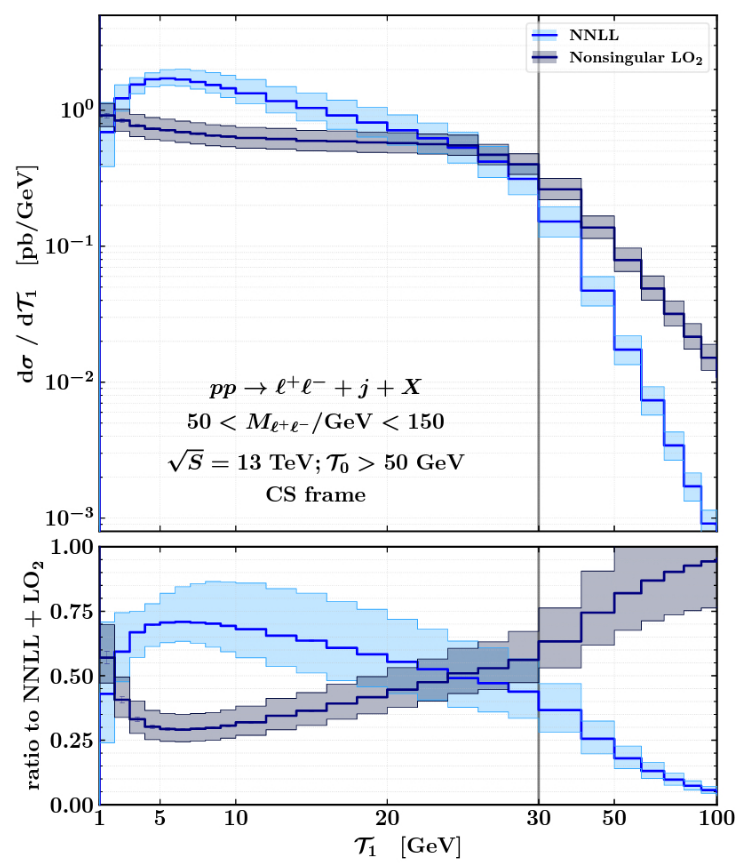
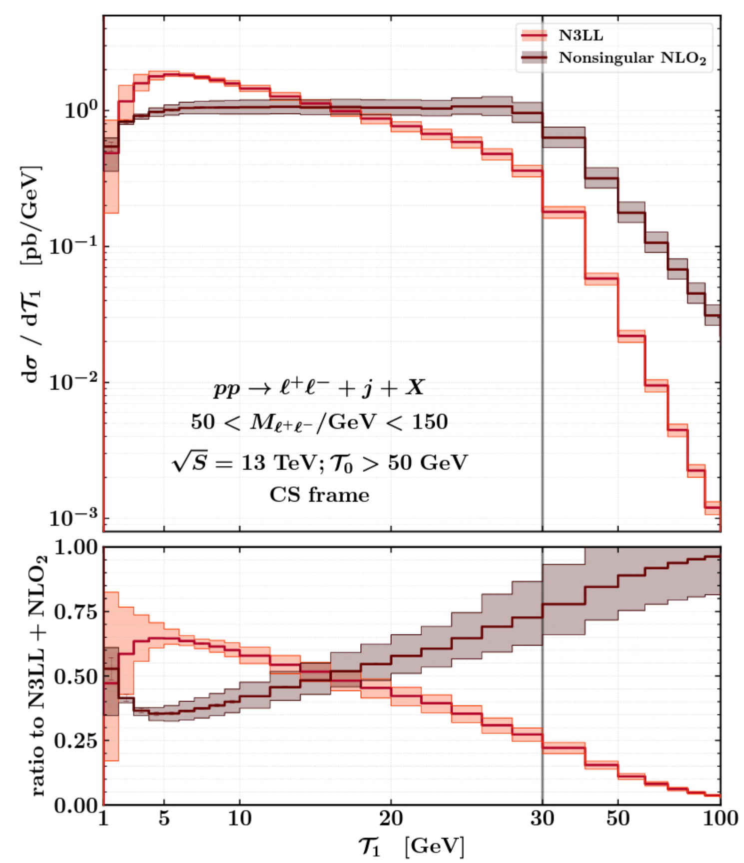
Matched results

$$\frac{d\sigma^{\text{match.}}}{d\Phi_1 d\mathcal{T}_1} = \frac{d\sigma^{\text{res.}}}{d\Phi_1 d\mathcal{T}_1} + \frac{d\sigma^{\text{f.o.}}}{d\Phi_1 d\mathcal{T}_1} - \frac{d\sigma^{\text{res.exp.}}}{d\Phi_1 d\mathcal{T}_1}$$



- $\mathcal{O}(\alpha_s^3)$ gives sizable contribution, important to include it for small values of $\mathcal{T}_0$





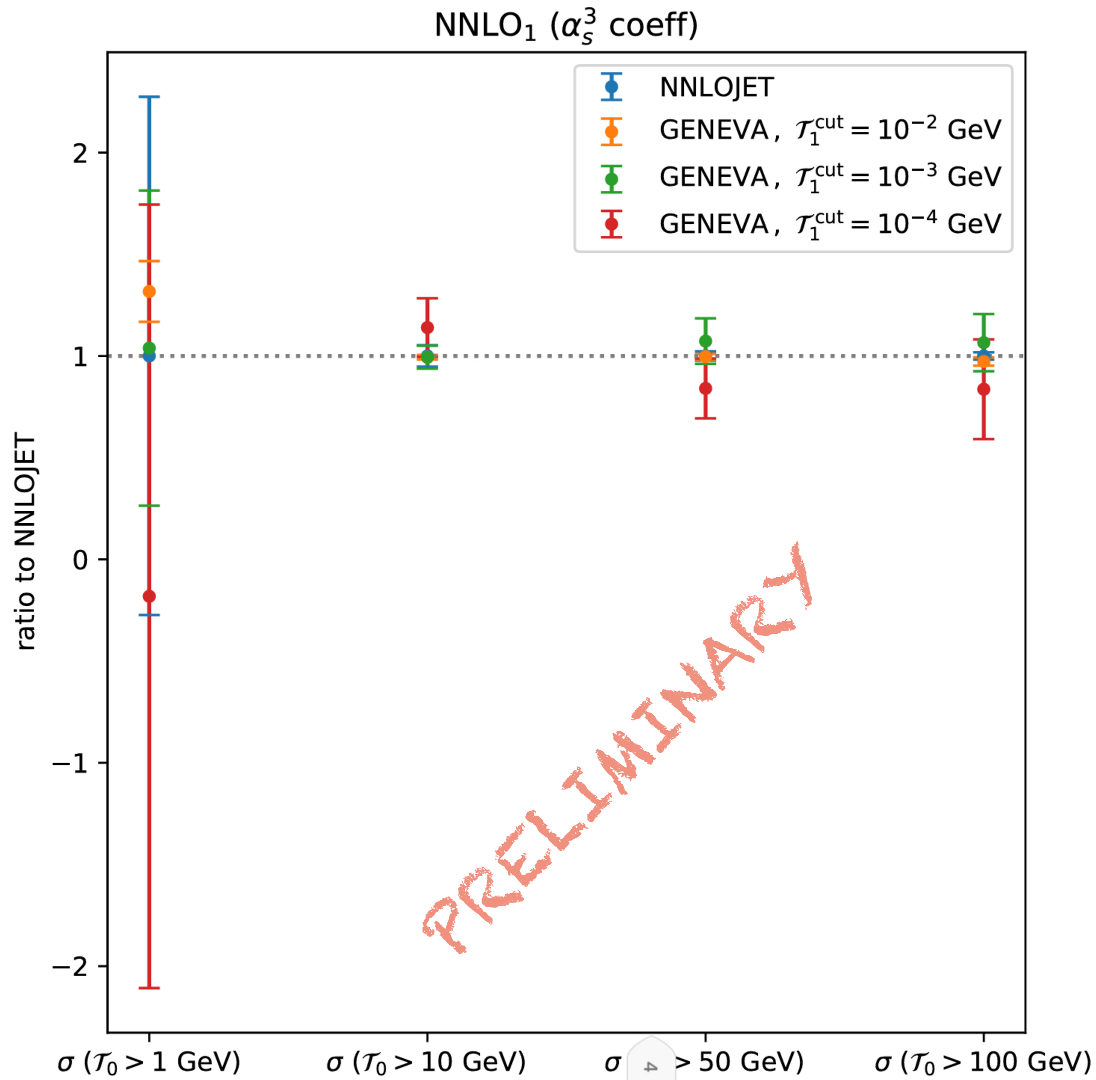
- Nonsingular divergent for $\mathcal{T}_0 \to 0$. Joint $(\mathcal{T}_0, \mathcal{T}_1)$ resummation required to handle both divergencies

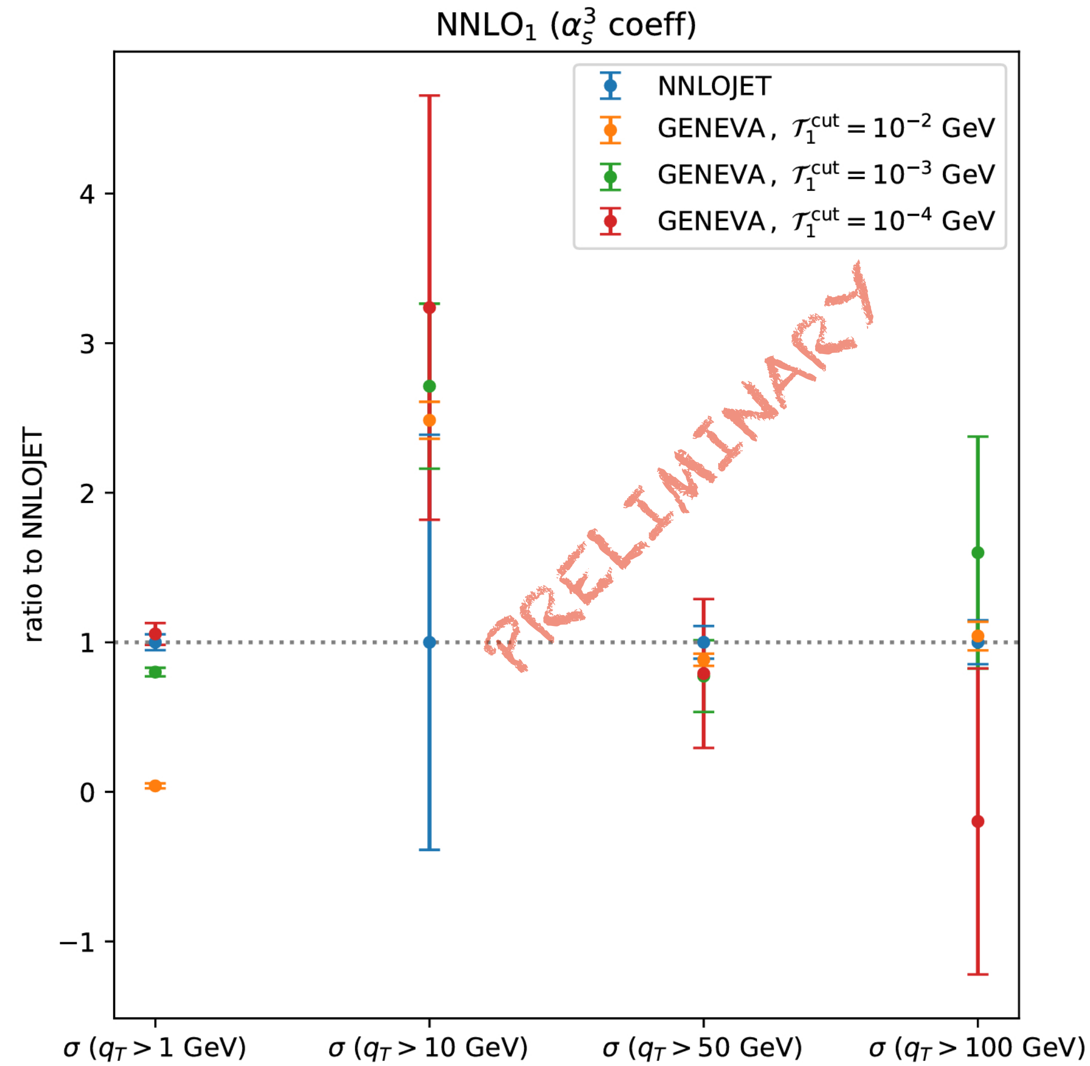
NNLO validation - 1-jettiness slicing

- Crucial to check the NNLO accuracy: expand matching formula to NNLO ($\mathcal{T}_1 -$ slicing) and compare with NNLOJET the pure $\mathcal{O}(\alpha_s^3)$ coeff.

$$O^{\delta \mathrm{NNLO}_1}(\Phi_1) = \frac{d\sigma^{\mathrm{N3LL}}}{d\Phi_1}(\mathcal{T}_1^{\mathrm{cut}}) \bigg|_{\mathcal{O}(\alpha_s^3)} O(\Phi_1) + \int_{\mathcal{T}_1^{\mathrm{cut}}}^{\mathcal{T}_1^{\mathrm{max}}} \frac{d\Phi_2}{d\Phi_1} \frac{d\sigma^{\delta \mathrm{NLO}_2}}{d\Phi_2} O(\Phi_{\{2,3\}})$$

Analytic cumulant expanded — NLO with local FKS subtraction





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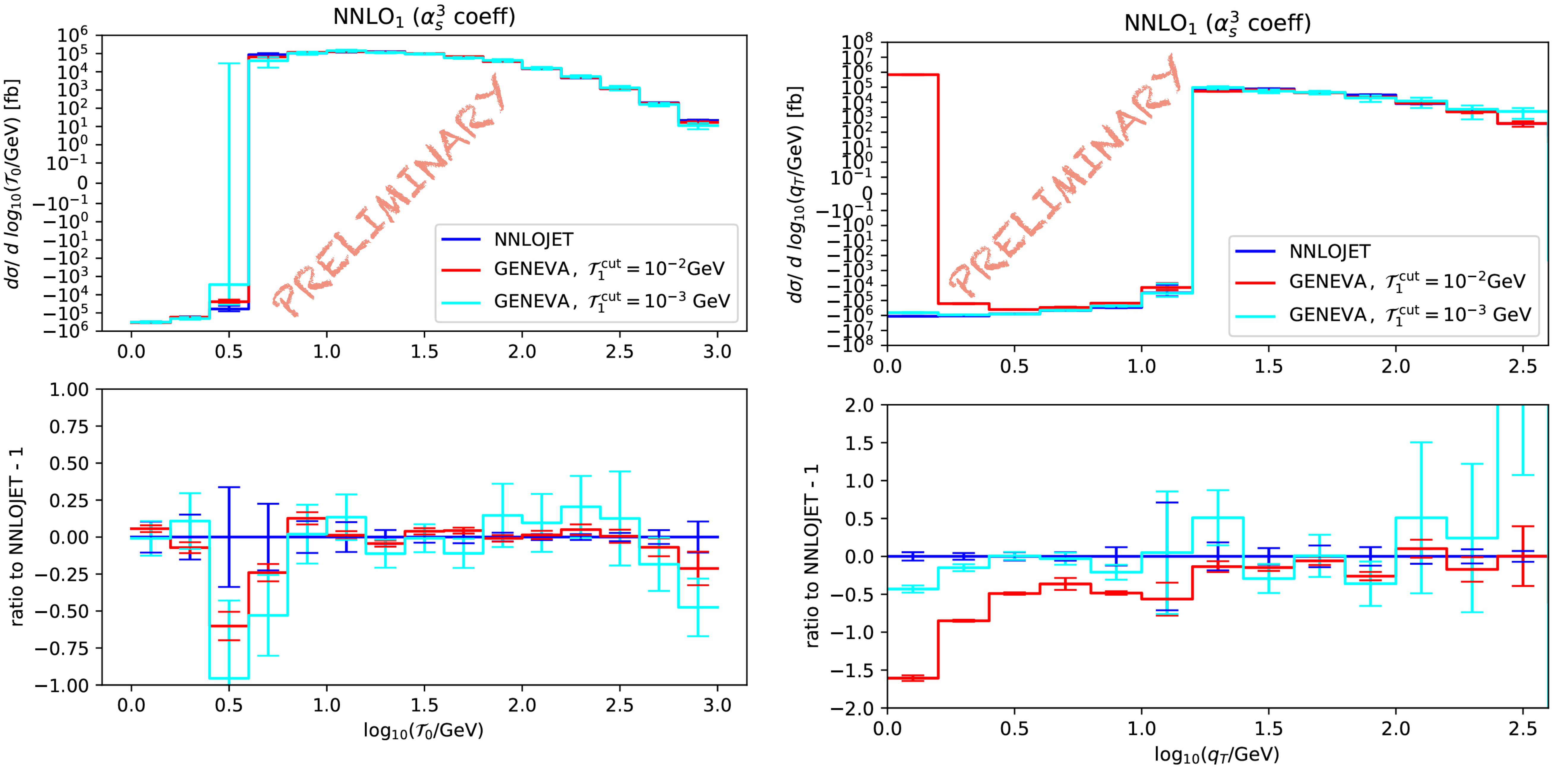
NNLO differential distributions $\mathscr{T}_1$−slicing



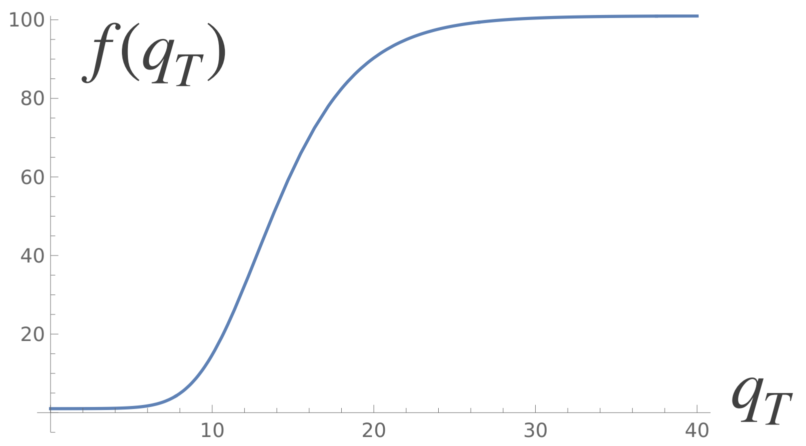
- Small $\mathscr{T}_1^{\rm cut}$ needed to correctly capture the low q_T behaviour, but increased stat errors at large q_T due to larger numerical cancellations

$\mathcal{T}_1$- slicing and subtraction with dynamic cuts

- Solution is to dynamically adapt the $\mathcal{T}_1^{\rm cut}$ value according to the kinematics (multi-scale problem).

One can use $m_T^Z, \mathcal{T}_0, q_T, \ldots$

$$\mathcal{T}_1^{\rm cut} = \min\{10^{-4} f(q_T), \mathcal{T}_0/2\}$$



- Additionally we can subtract the singular spectrum locally in $\mathcal{T}_1$

$$O^{\delta{\rm NNLO}_1}(\Phi_1) = \frac{d\sigma^{\rm N3LL}}{d\Phi_1}(\mathcal{T}_1^{\rm cut})\bigg|_{\mathcal{O}(\alpha_s^3)} O(\Phi_1) + \int_{\mathcal{T}_1^{\rm cut}}^{\mathcal{T}_1^{\rm max}} \frac{d\Phi_2}{d\Phi_1}\frac{d\sigma^{\delta{\rm NLO}_2}}{d\Phi_2}\, O(\Phi_{\{2,3\}})$$

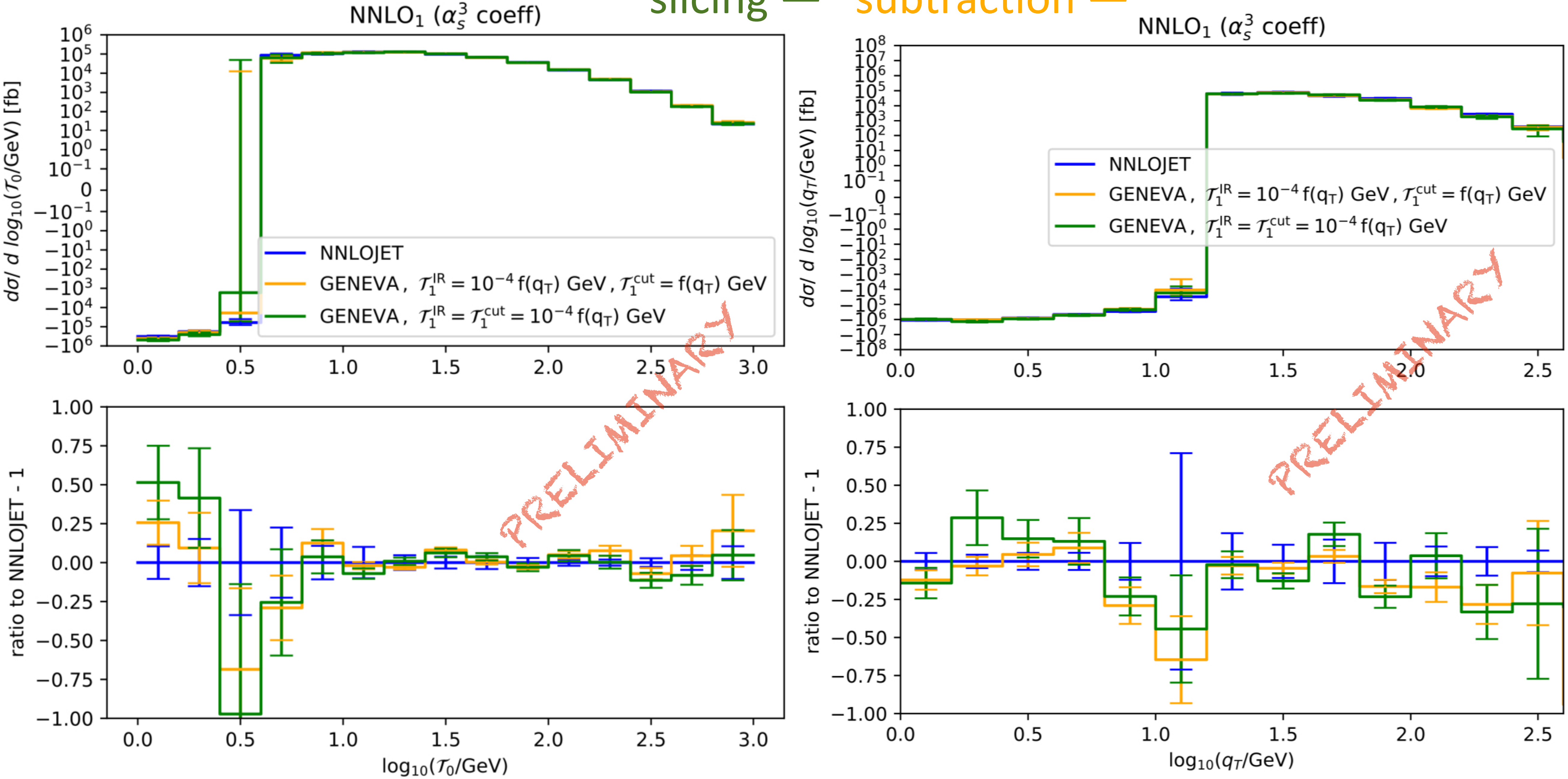
$$+\int_{\mathcal{T}_1^{\rm IR}}^{\mathcal{T}_1^{\rm cut}} \frac{d\Phi_2}{d\Phi_1}\left[\frac{d\sigma^{\delta{\rm NLO}_2}}{d\Phi_2}\, O(\Phi_{\{2,3\}}) - \frac{d\sigma^{\rm N3LL}}{d\Phi_1 d\mathcal{T}_1}\bigg|_{\mathcal{O}(\alpha_s^3)} \mathcal{P}(z,\varphi)\, O(\Phi_1)\right]$$

Allows for larger $\mathcal{T}_1^{\rm cut}$ while still providing complete inclusive power corrections down to $\mathcal{T}_1^{IR} \ll \mathcal{T}_1^{\rm cut}$

$\mathcal{P}(z,\varphi)$ normalized splitting functions $\int dz d\varphi \mathcal{P}(z,\varphi) \equiv 1$

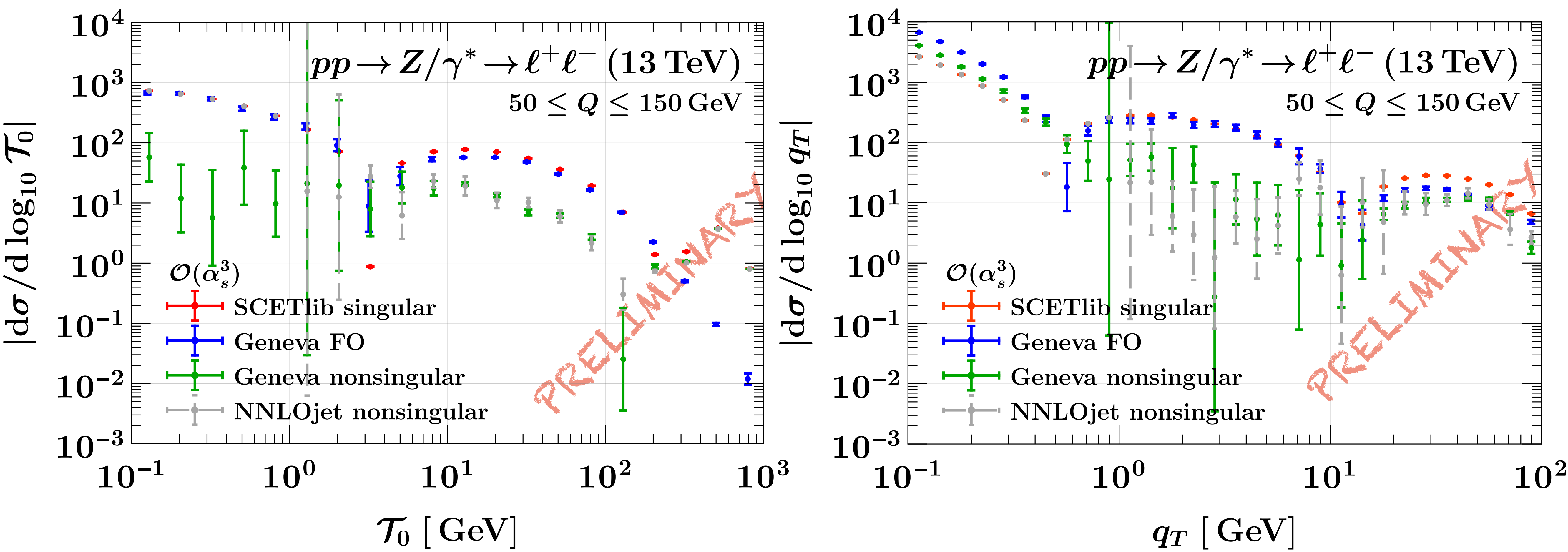
$\mathscr{T}_1$ slicing and subtraction with dynamic cuts

slicing — subtraction —



- Complete agreement with NNLOJET and statistical errors comparable with similar running times (~ 80k CPU hours)

Validation with $\mathscr{T}_0$ and q_T singular spectra



- Comparison with $\mathscr{O}(\alpha_s^3)$ singular spectra in $\mathscr{T}_0$ and q_T tells us how much we can push our approach before it breaks down, due to internal technical cuts or just by large numerical cancellations.
- GENEVA nonsingular well behaved for $\mathscr{T}_0$ down to 0.5 GeV, this NNLOJET run has generation cut at 1 GeV.
- Both approached more demanding for q_T, seems OK down to ~ 1 GeV but singular still decreasing….

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Inclusion of fiducial power corrections

- Using P2B it is possible to completely capture all fiducial power corrections of any observable $\mathscr{O}$ and only neglect the dynamical (inclusive) ones below the IR cutoff $\mathscr{T}_\delta$

$$\mathcal{O}_{\delta\mathrm{NNLO}}(\Phi_N) = \int \mathrm{d}\mathcal{T}_\mathrm{N} \frac{\mathrm{d}\sigma^{\delta\mathrm{NNLO}}_{\mathrm{N,sub.}}}{\mathrm{d}\Phi_\mathrm{N}} \, \mathcal{O}(\Phi_\mathrm{N}) \, \theta\Big(\mathcal{T}_\mathrm{N}(\Phi_{\mathrm{N+X}}) < \mathcal{T}^{\mathrm{cut}}_\mathrm{N}\Big)$$

Resummed-expanded cumulant

Neglected inclusive pwr corr.

$$+ \int \mathrm{d}\mathcal{T}_\mathrm{N} \left[\frac{\mathrm{d}\sigma^{\delta\mathrm{NNLO}}_{\mathrm{N}}}{\mathrm{d}\Phi_\mathrm{N}} - \frac{\mathrm{d}\sigma^{\delta\mathrm{NNLO}}_{\mathrm{N,sub.}}}{\mathrm{d}\Phi_\mathrm{N}}\right] \mathcal{O}(\Phi_\mathrm{N}) \, \theta\Big(\mathcal{T}_\mathrm{N}(\Phi_{\mathrm{N+X}}) < \mathcal{T}_\delta\Big)$$

FPC captured by P2B

$$+ \int \frac{\mathrm{d}\Phi_{\mathrm{N+1}}}{\mathrm{d}\Phi_\mathrm{N}} \frac{\mathrm{d}\sigma^{\delta\mathrm{NLO}}_{\mathrm{N+1}}}{\mathrm{d}\Phi_{\mathrm{N+1}}} \Big[\mathcal{O}(\Phi_{N+X}) - \mathcal{O}(\Phi_N)\Big] \, \theta\Big(\mathcal{T}_N(\Phi_{N+X}) < \mathcal{T}_\delta\Big)$$

N-jettiness subtraction between $\mathscr{T}_\delta$ and $\mathscr{T}_N^{\mathrm{cut}}$

$$+ \int \frac{\mathrm{d}\Phi_{\mathrm{N+1}}}{\mathrm{d}\Phi_\mathrm{N}} \left[\frac{\mathrm{d}\sigma^{\delta\mathrm{NLO}}_{\mathrm{N+1}}}{\mathrm{d}\Phi_{\mathrm{N+1}}} \, \mathcal{O}(\Phi_{N+X}) - \frac{\mathrm{d}\sigma^{\delta\mathrm{NNLO}}_{\mathrm{N,sub.}}}{d\Phi_N} \, P(\Phi_{N+1}) \, \mathcal{O}(\Phi_N)\right] \times \theta\Big(\mathcal{T}_\delta < \mathcal{T}_N(\Phi_{N+X}) < \mathcal{T}^{\mathrm{cut}}_N\Big)$$

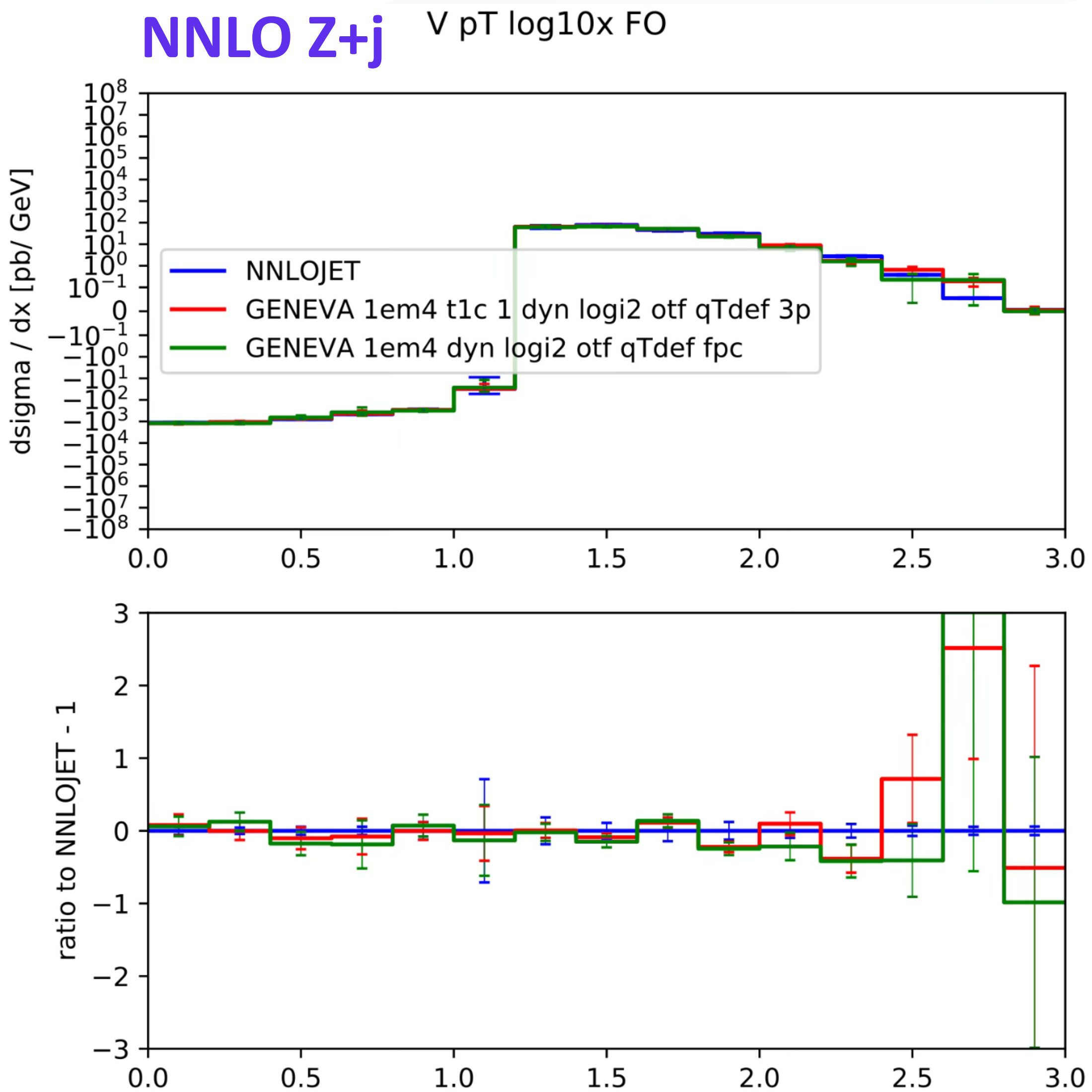
Standard NLO N+1 calculation with higher cutoff $\mathscr{T}_N^{\mathrm{cut}}$

$$+ \int \frac{\mathrm{d}\Phi_{\mathrm{N+1}}}{\mathrm{d}\Phi_\mathrm{N}} \frac{\mathrm{d}\sigma^{\delta\mathrm{NLO}}_{\mathrm{N+1}}}{\mathrm{d}\Phi_{\mathrm{N+1}}} \, \mathcal{O}(\Phi_{N+X}) \, \theta\Big(\mathcal{T}_N(\Phi_{N+X}) > \mathcal{T}^{\mathrm{cut}}_N\Big) \, .$$

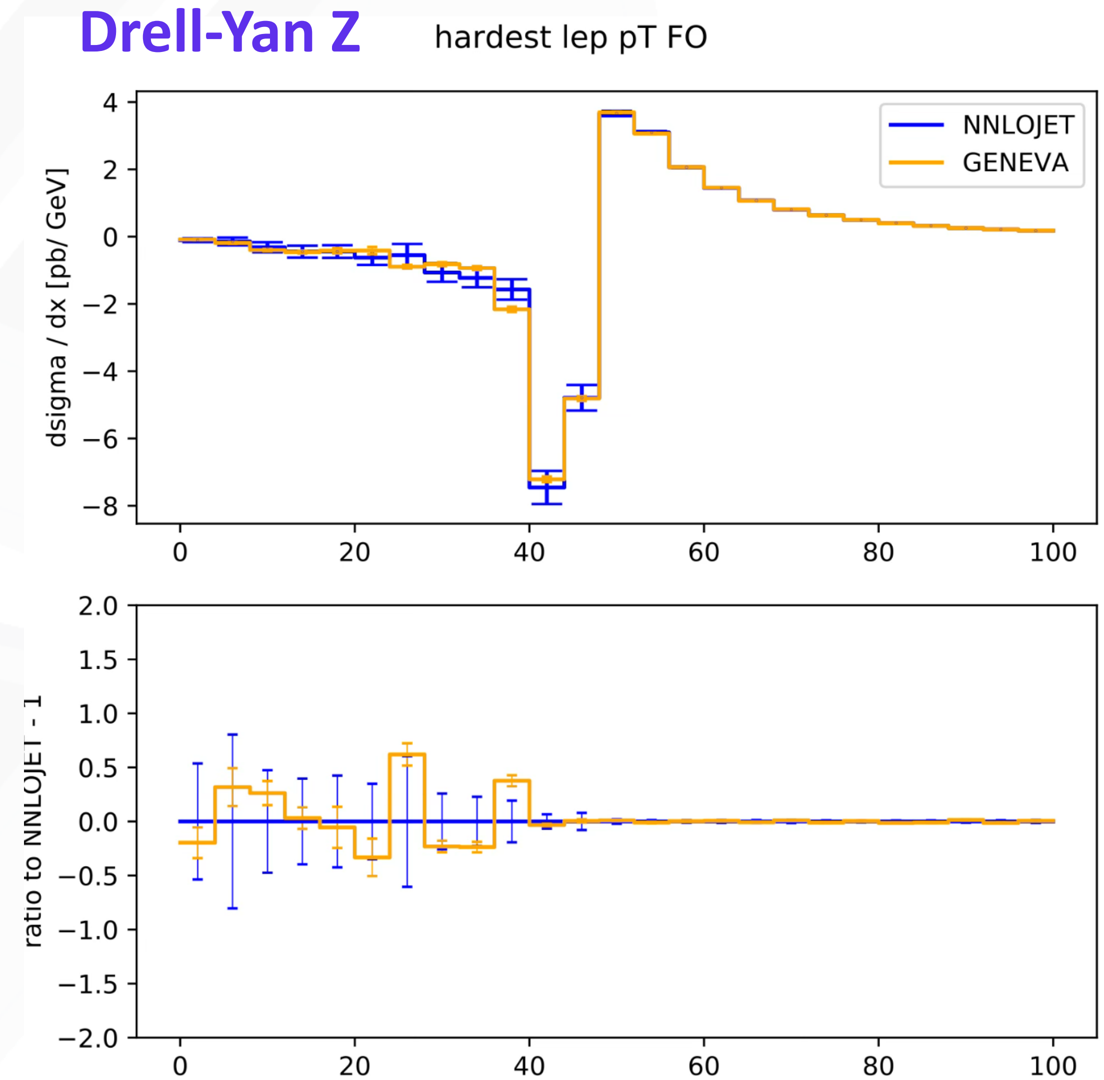
Inclusion of fiducial power corrections

- Using P2B it is possible to completely capture all fiducial power corrections and only remain with the dynamical (inclusive) ones

NNLO Z+j



Drell-Yan Z



- Drell-Yan can use FKS mapping for P2B, but for Z+j one needs a defining-cut-preserving mapping.

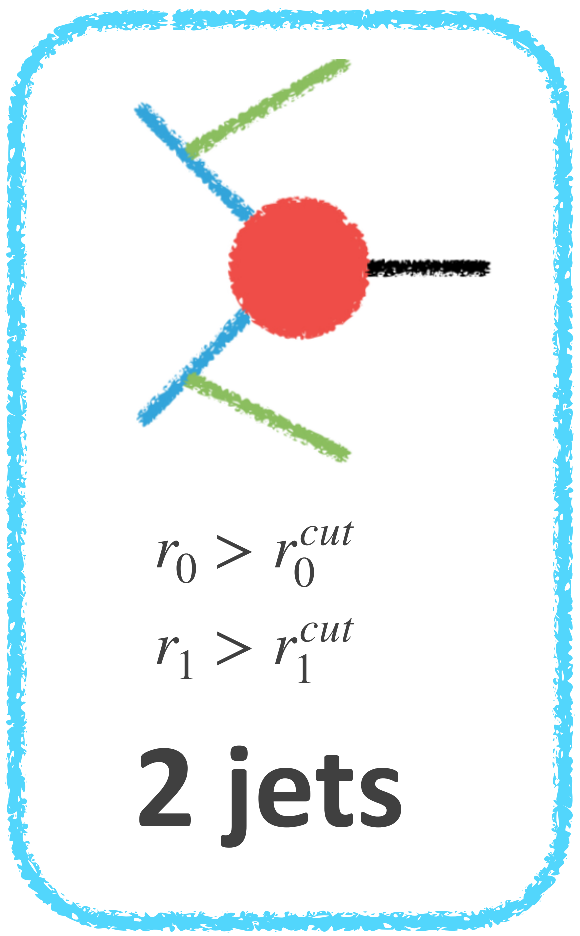
Conclusion and outlook

- The inclusion of state-of-the-art theoretical predictions in SMC generators is mandatory to match the experimental precision and fully exploit the discovery potential of LHC measurements
- MiNNLOps and GENEVA methods allow for event generation with NNLO accuracy matched to parton showers.
- Several color-singlet processes implemented, using different resolution variables: N-jettiness, qT, jet veto…
- The availability of multiple approaches is crucial to get an handle on theoretical uncertainties
- Next challenges are the interface to more accurate parton-showers (NLL and beyond) and the extension to more complicated processes with jets. Work in progress …

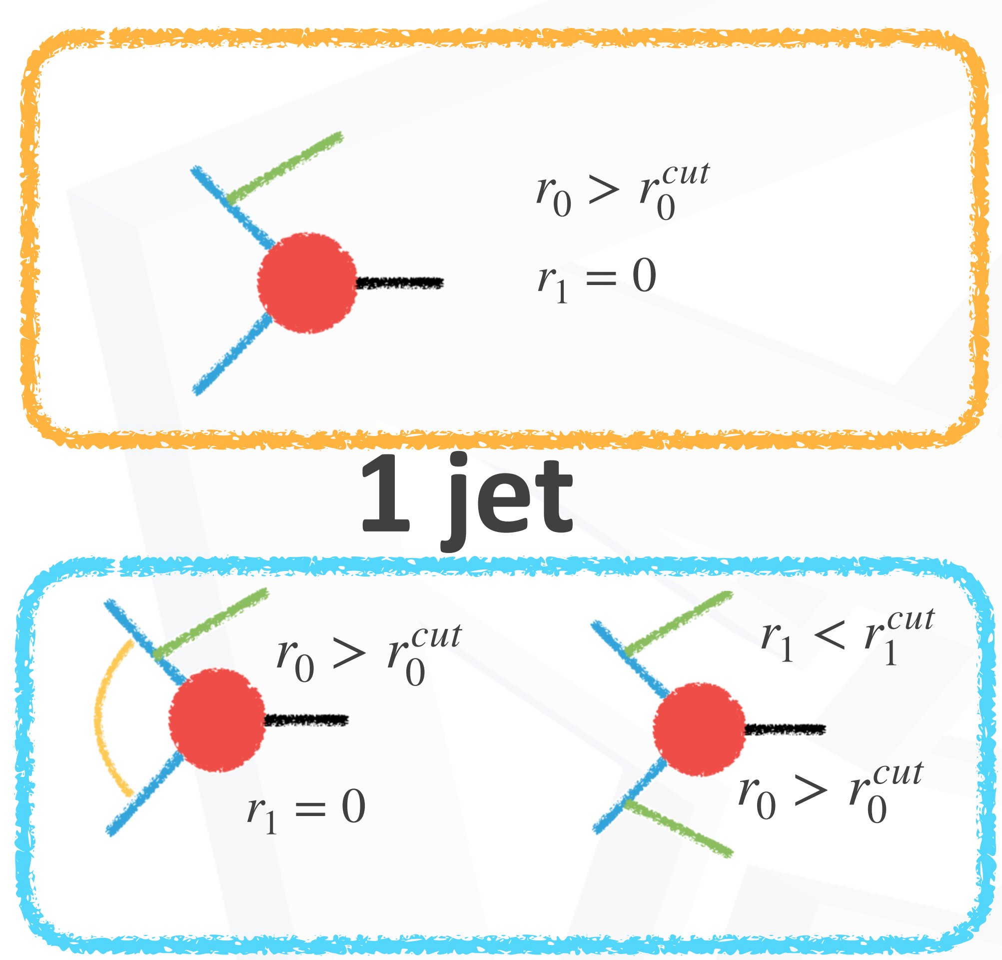
Thank you for your attention.

BACKUP

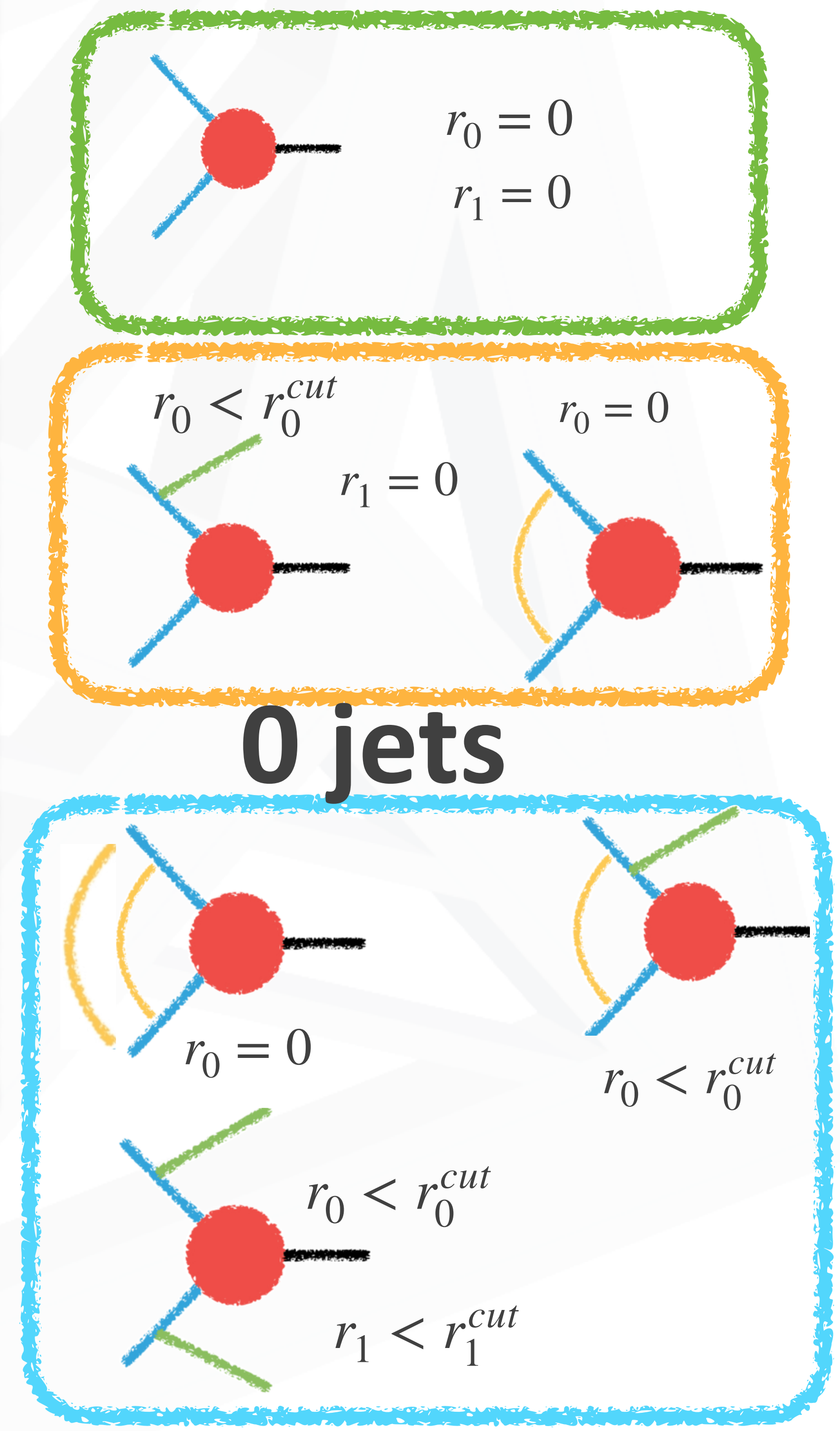
Partitioning phase space with resolution cuts



$$\frac{d\sigma}{d\Phi_2}(r_0 > r_0^{\text{cut}}, r_1 > r_1^{\text{cut}})$$



$$\frac{d\sigma}{d\Phi_1}(r_0 > r_0^{\text{cut}}, r_1^{\text{cut}})$$



$$\frac{d\sigma}{d\Phi_0}(r_0^{\text{cut}})$$

When emissions become unresolved, cuts must be resummed.
Differential information below cut is lost during projection.
No difference for preserved quantities, in general can be made a power correction.
Mapping that preserves singular behavior is required for correct event definition.

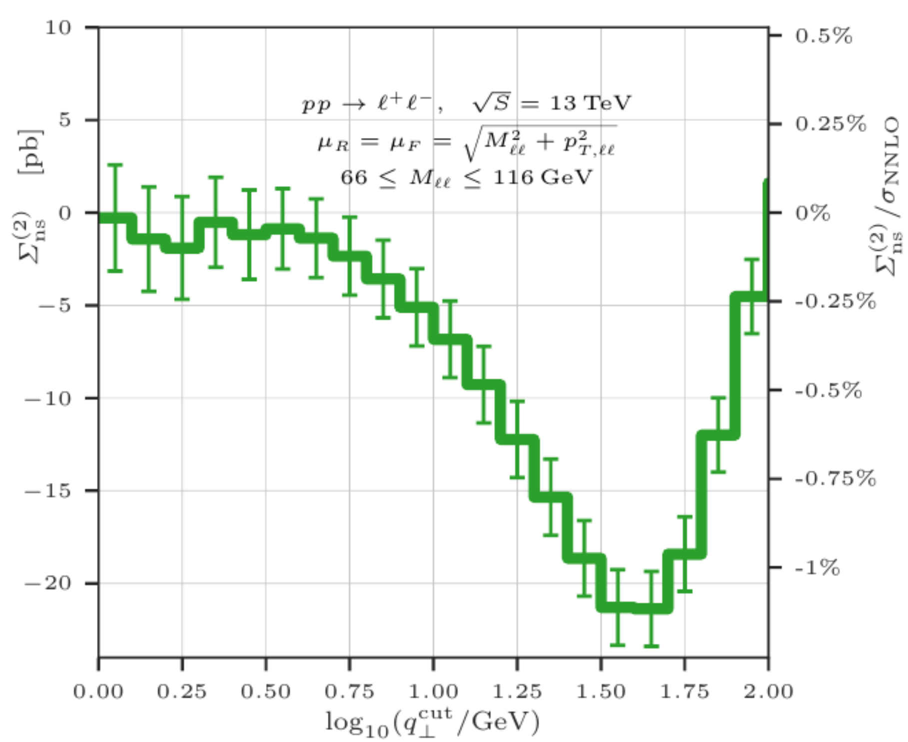
Resummation of resolution parameters

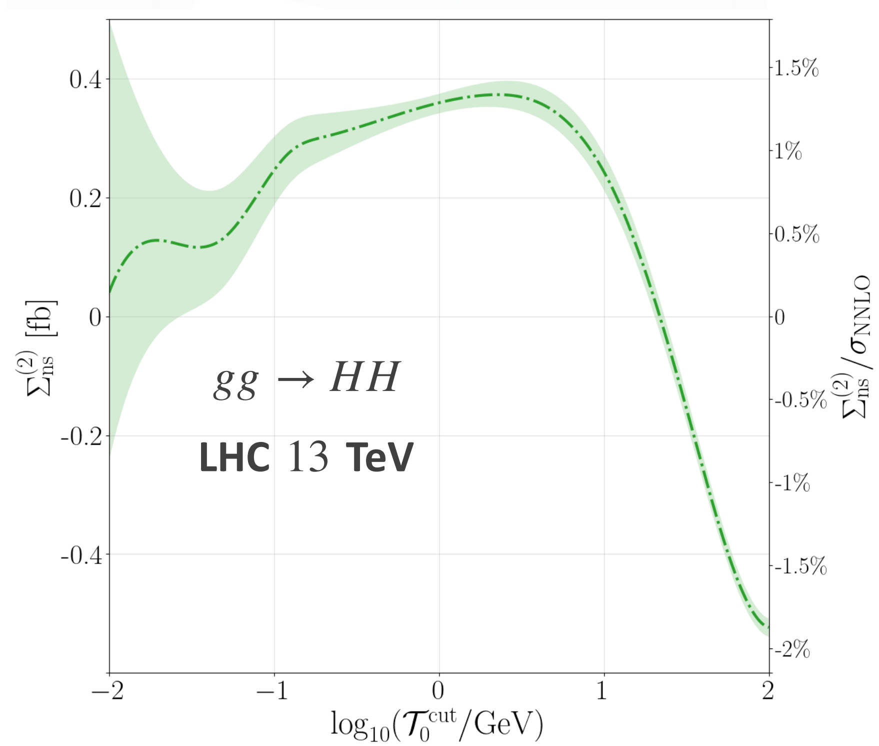
Resumming resolutions parameters not really a new idea, SMCs have been doing it since the ‘80s with Sudakov factors

Using resummation at higher orders has several benefits: systematically improvable (NLL,NNLL,N3LL,…), lowering theoretical uncertainty at each step.

Higher accuracy allows to lower the cuts without risking missing higher logarithms being numerically relevant.
The lower the cut the smaller the nonsingular power corrections due to phase-space projections will affect the results differentially.

For NNLO event generation one needs at least NNLL’$_{r_0}$ + NNLO accuracy to control the full α_s^2 singular contributions.





Using the jet pT as resolution variable

GENEVA recently extended to jet veto resummation in [Gavardi et al. 2308.11577].

Factorization most easily derived for cumulant of the cross-section. SCET II problem. Numerical derivative to get the spectrum. For hardest-jet we have

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_0}(p_T^{\mathrm{cut}},\mu,\nu)=\sum_{a,b} H_{ab}(\Phi_0,\mu)B_a(Q,p_T^{\mathrm{cut}},R,x_a,\mu,\nu)B_b(Q,p_T^{\mathrm{cut}},R,x_b,\mu,\nu)S_{ab}(p_T^{\mathrm{cut}},R,\mu,\nu)$$

Two loop Beam and Soft functions recently computed in [Abreu et al. 2207.07037, 2204.02987]

Focus on $W^+W^- \to \mu^+\nu_\mu e^-\bar{\nu}_e$ with jet veto, in 4-flavor scheme to avoid top contaminations. Massless two-loop hard function taken from `qqVVamp` [Gehrmann et al. 1503.04812]

Interface to SCETlib [Tackmann et al.] allows to perform also resummation also for pT of the second jet at the cumulant level. Refactorization of soft sector into global soft, soft-coll and nonglobal contributions [Cal et al.]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_1}(p_T^{\mathrm{cut}},\mu,\nu) = \sum_{\kappa} H_\kappa(\Phi_1,\mu)B_a(Q,p_T^{\mathrm{cut}},R,x_a,\mu,\nu)\, B_b(Q,p_T^{\mathrm{cut}},R,x_b,\mu,\nu) S_\kappa(p_T^{\mathrm{cut}},y_J,\mu,\nu)$$
$$\times \mathcal{S}_j^R(p_T^{\mathrm{cut}}R,\mu)\, J_j(p_T^J R,\mu)\mathcal{S}_j^{\mathrm{NG}}\left(\frac{p_T^{\mathrm{cut}}}{p_T^J}\right).$$

[Banfi et al. hep-ph/0206076]

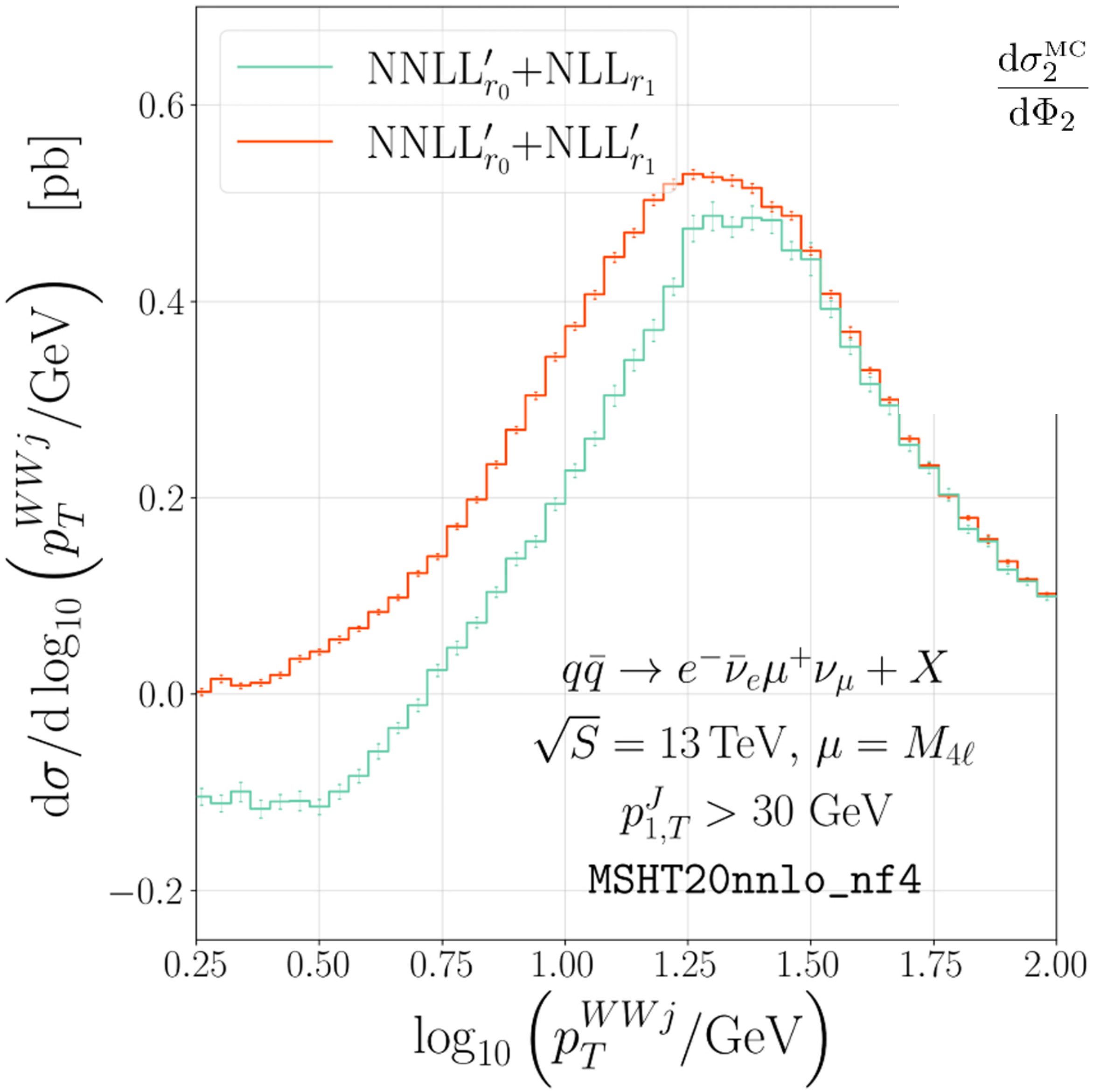
Resumming second jet resolution at NLL’ in GENEVA

Extension of the GENEVA approach to include resummation of $r_1^{\rm cut}$ to NLL’ accuracy

Now truly capturing the correct nonsingular behaviour when approaching the single-jet limit

$$\begin{aligned}\frac{d\sigma_1^{\rm MC}}{d\Phi_1}(r_1^{\rm cut}) = &\Bigg\{\Bigg[\frac{d\sigma^{{\rm NNLL'}r_0}}{d\Phi_0\, dr_0} - \frac{d\sigma^{{\rm NNLL'}r_0}}{d\Phi_0\, dr_0}\bigg|_{\rm NLO_1}\Bigg]\mathcal{P}_{0\to1}(\Phi_1)\, U_1(\Phi_1, r_1^{\rm cut}) \\ &+ \frac{d\sigma^{\rm NLO_1}}{d\Phi_1}(r_1^{\rm cut}) + \frac{d\sigma^{{\rm NLL'}r_1}}{d\Phi_1}(r_1^{\rm cut}) - \frac{d\sigma^{{\rm NLL'}r_1}}{d\Phi_1}(r_1^{\rm cut})\bigg|_{\rm NLO_1}\Bigg\}\theta(r_0 > r_0^{\rm cut}) \\ &+ \frac{d\sigma^{\rm LO_1}_{\rm nonproj}}{d\Phi_1}\theta(r_0 < r_0^{\rm cut})\end{aligned}$$

$$\begin{aligned}\frac{d\sigma_2^{\rm MC}}{d\Phi_2} = &\Bigg\{\Bigg[\frac{d\sigma^{{\rm NNLL'}r_0}}{d\Phi_0\, dr_0} - \frac{d\sigma^{{\rm NNLL'}r_0}}{d\Phi_0\, dr_0}\bigg|_{\rm NLO_1}\Bigg]\mathcal{P}_{0\to1}(\Phi_1)\, U_1'(\Phi_1, r_1)\, \mathcal{P}_{1\to2}(\Phi_2) \\ &+ \frac{d\sigma^{\rm LO_2}}{d\Phi_2} + \Bigg[\frac{d\sigma^{{\rm NLL'}r_1}}{d\Phi_1\, dr_1} - \frac{d\sigma^{{\rm NLL'}r_1}}{d\Phi_1\, dr_1}\bigg|_{\rm LO_2}\Bigg]\mathcal{P}_{1\to2}(\Phi_2)\Bigg\}\theta(r_1 > r_1^{\rm cut})\,\theta(r_0 > r_0^{\rm cut}) \\ &+ \frac{d\sigma^{\rm LO_2}_{\rm nonproj}}{d\Phi_2}\theta(r_1 < r_1^{\rm cut})\,\theta(r_0 > r_0^{\rm cut})\,.\end{aligned}$$



NLL’ accuracy of the second jet only maintained in presence of an hard first jet.

Resummation formula not able to handle the $r_0 \sim r_1 \ll \mu_H$ hierarchy, double resummation required there.

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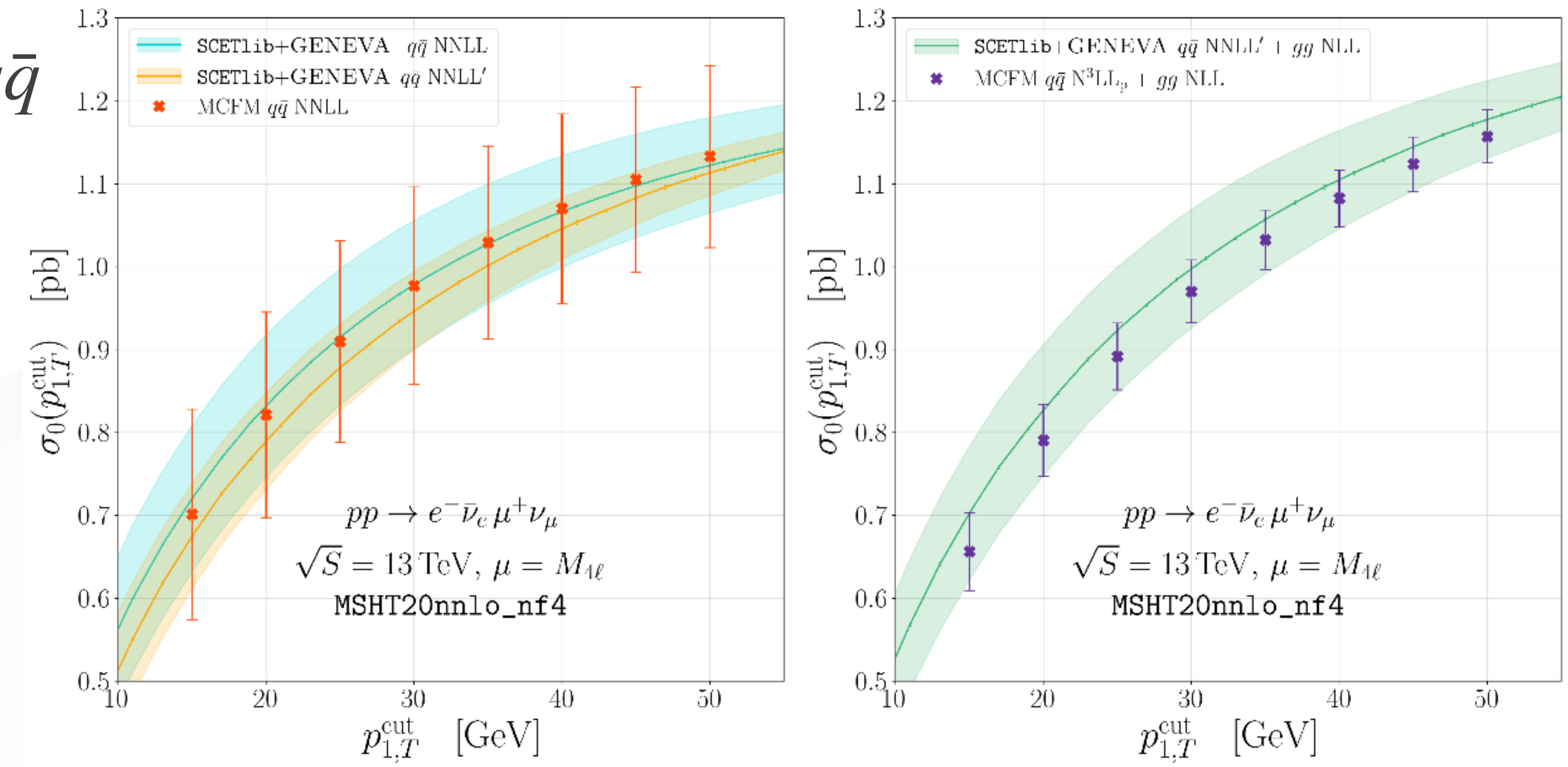
Validation of WW production

We include the resummation of the $q\bar{q}$ channel at NNLL' and the gg channel at NLL

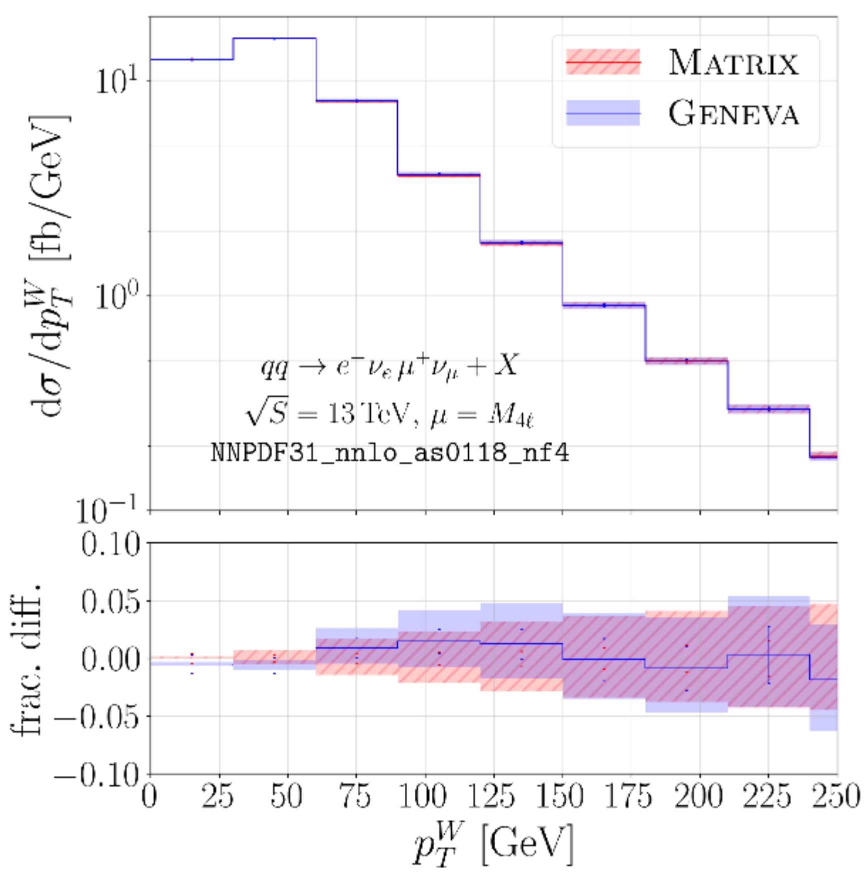
Jet veto resummation available in MCFM up to partial N3LL accuracy. Different treatment of uncertainties.

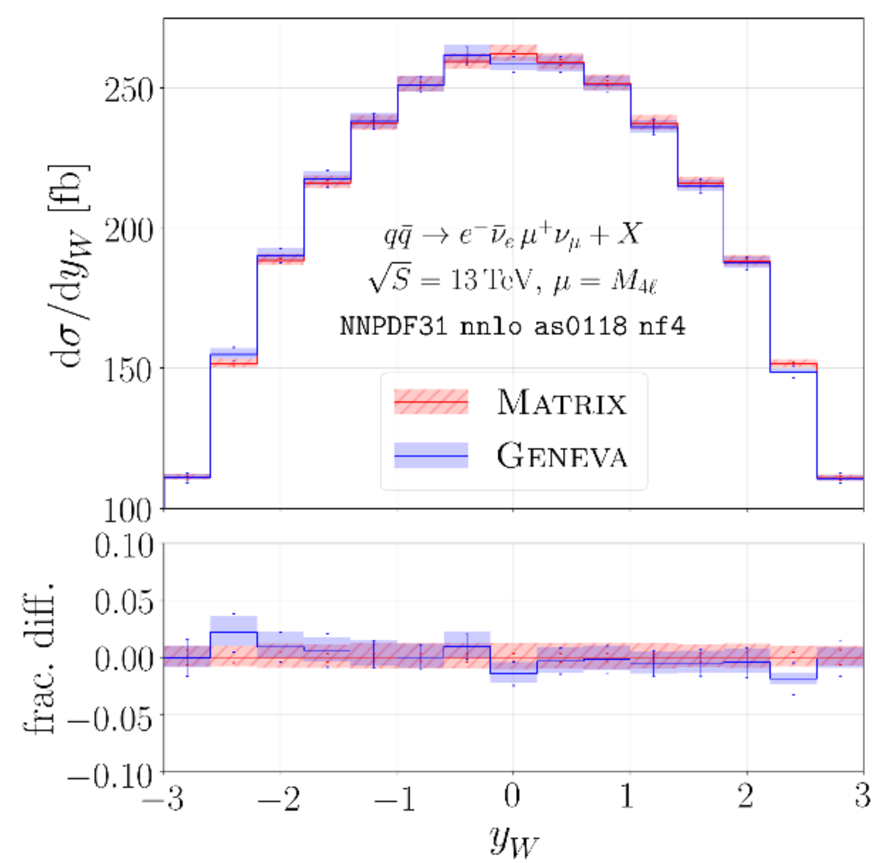
[Campbell et al. 2301.11768]



NNLO validation against MATRIX

[Grazzini et al. 1711.06631]





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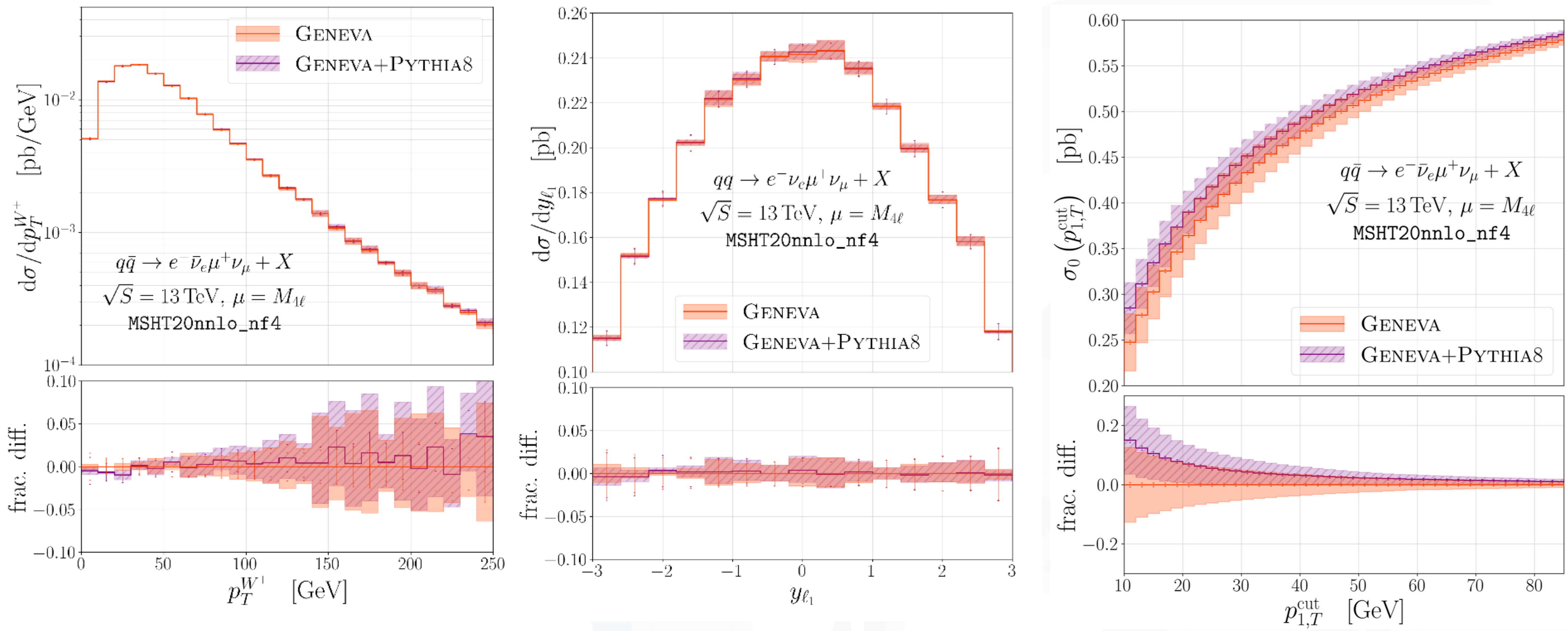
Showering



Inclusive quantities well-preserved by the shower, pT of the hardest jet is extremely sensitive to shower effects and gets mildly shifted. Few percent effect at 30 GeV.

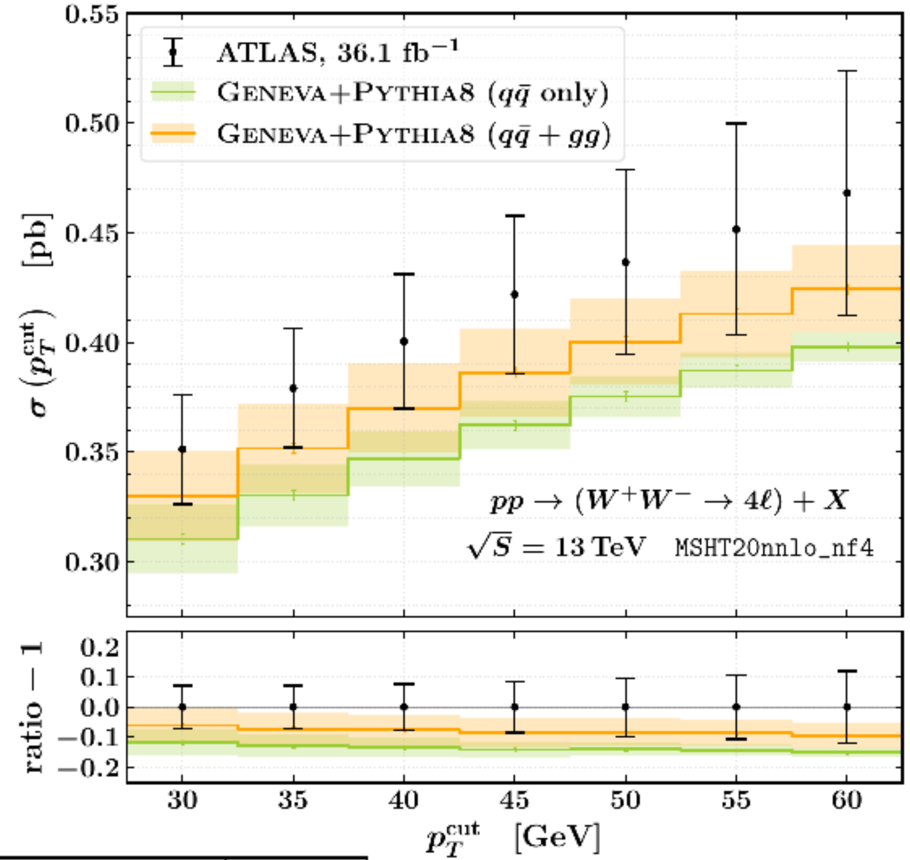
This is entirely due to FSR emissions (the shower splits the hardest jet above pT cut into 2 jets below pT cut). Placing constraints to avoid this preserves pT_{1st} but not physically motivated.

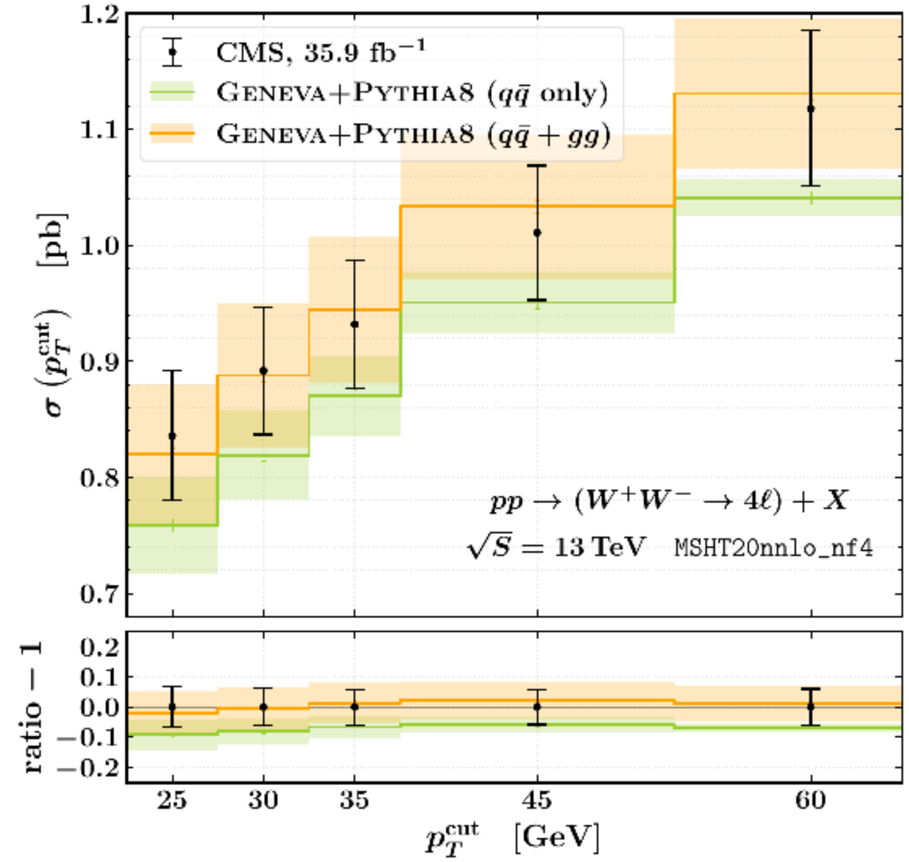
Investigating resummation of different 1-jet resolution variable $\mathcal{T}_1^{k_T}$ (SCET II fact.)

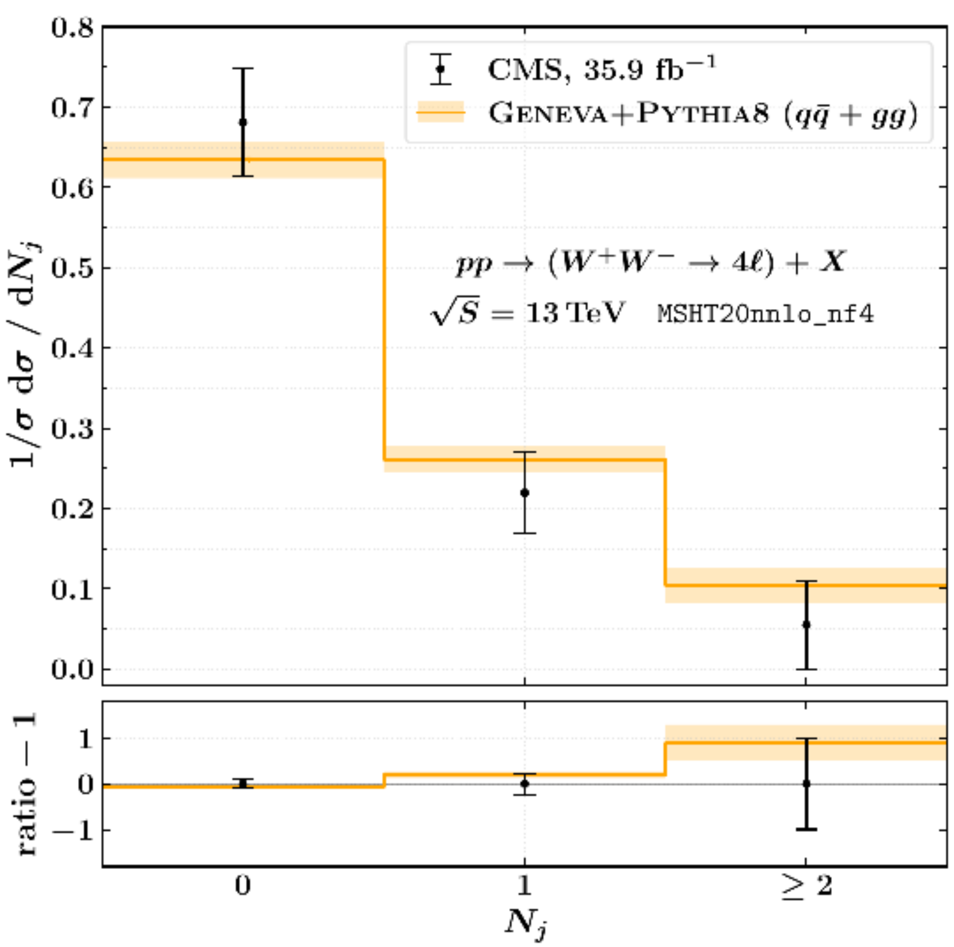
Data comparison

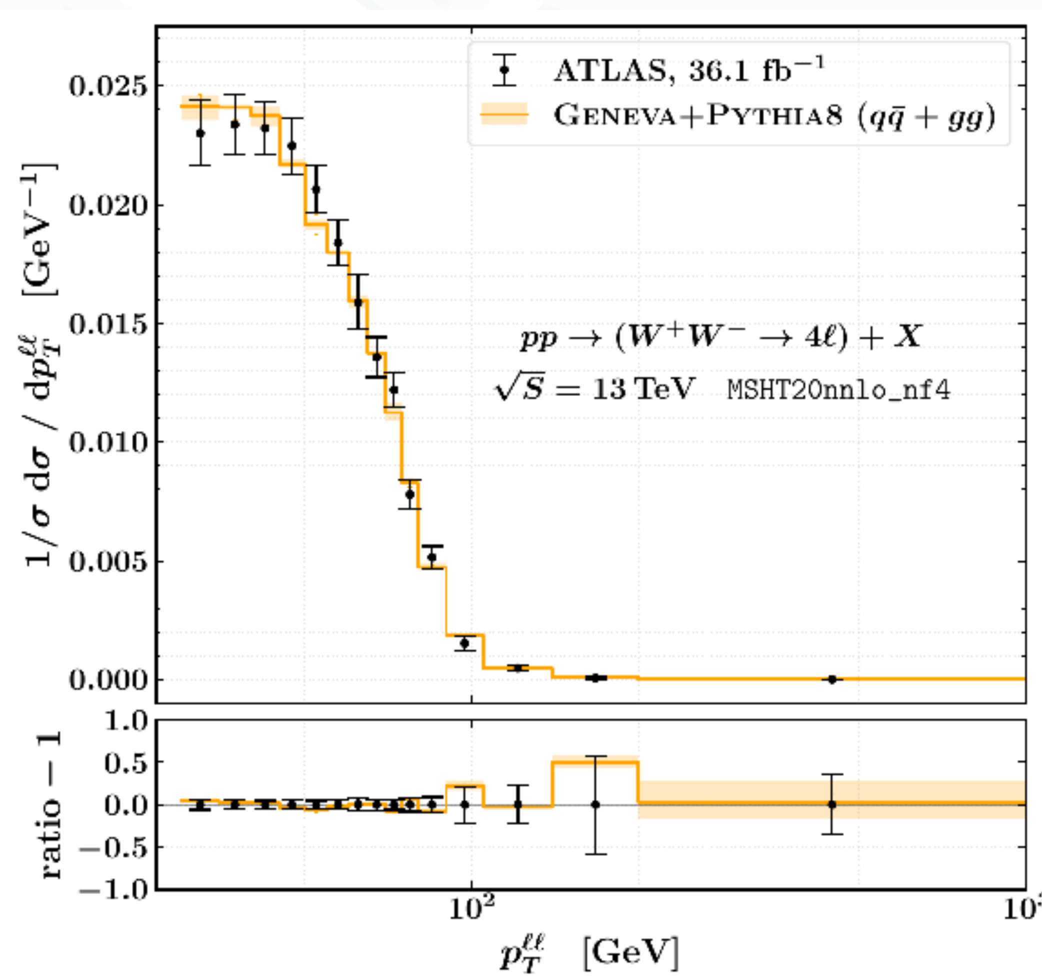
Inclusion of gg channel necessary for agreement with data.

Extension of gg channel to NLO+NLL' ongoing









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Zero-jettiness factorization for top-quark pairs

Factorization formula derived using SCET+HQET in the region where $M_{t\bar{t}} \sim m_t \sim \sqrt{\hat{s}}$ are all hard scales. [SA et al. 2111.03632]

In case of boosted regime $M_{t\bar{t}} \gg m_t$ one would instead need a modified two-jettiness [Fleming, Hoang,Mantry,Stewart `07][Bachu,Hoang,Mateu,Pathak,Stewart `21]

$$\frac{d\sigma}{d\Phi_0 d\tau_B} = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \int dt_a\, dt_b\, B_i(t_a, z_a, \mu)\, B_j(t_b, z_b, \mu)\, \mathrm{Tr}\left[\mathbf{H}_{ij}(\Phi_0, \mu)\, \mathbf{S}_{ij}\left(M\tau_B - \frac{t_a + t_b}{M}, \Phi_0, \mu\right)\right]$$

Beam functions [Stewart, Tackmann, Waalewijn, [1002.2213], known up to N^{3}LO

Hard functions (color matrices)

Soft functions (color matrices)

It is convenient to transform the soft and beam functions in Laplace space to solve the RG equations, the factorization formula is turn into a product of (matrix) functions

$$\mathscr{L}\left[\frac{d\sigma}{d\Phi_0 d\tau_B}\right] = M \sum_{ij=\{q\bar{q},\bar{q}q,gg\}} \tilde{B}_i\left(\ln\frac{M\kappa}{\mu^2}, z_a\right) \tilde{B}_j\left(\ln\frac{M\kappa}{\mu^2}, z_b\right) \mathrm{Tr}\left[\mathbf{H}_{ij}\left(\ln\frac{M^2}{\mu^2}, \Phi_0\right) \tilde{\mathbf{S}}_{ij}\left(\ln\frac{\mu^2}{\kappa^2}, \Phi_0\right)\right]$$

Zero-jettiness resummation for top pairs

Resummed formula valid up to NNLL' accuracy

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Phi_0\mathrm{d}\tau_B} = U(\mu_h,\mu_B,\mu_s,L_h,L_s)$$
$$\times \mathrm{Tr}\Big\{\mathbf{u}(\beta_t,\theta,\mu_h,\mu_s)\,\mathbf{H}(M,\beta_t,\theta,\mu_h)\,\mathbf{u}^\dagger(\beta_t,\theta,\mu_h,\mu_s)\,\tilde{\mathbf{S}}_B(\partial_{\eta_s}+L_s,\beta_t,\theta,\mu_s)\Big\}$$
$$\times \tilde{B}_a(\partial_{\eta_B}+L_B,z_a,\mu_B)\tilde{B}_b(\partial_{\eta'_B}+L_B,z_b,\mu_B)\frac{1}{\tau_B^{1-\eta_{\rm tot}}}\frac{e^{-\gamma_E\eta_{\rm tot}}}{\Gamma(\eta_{\rm tot})}\,.$$

where

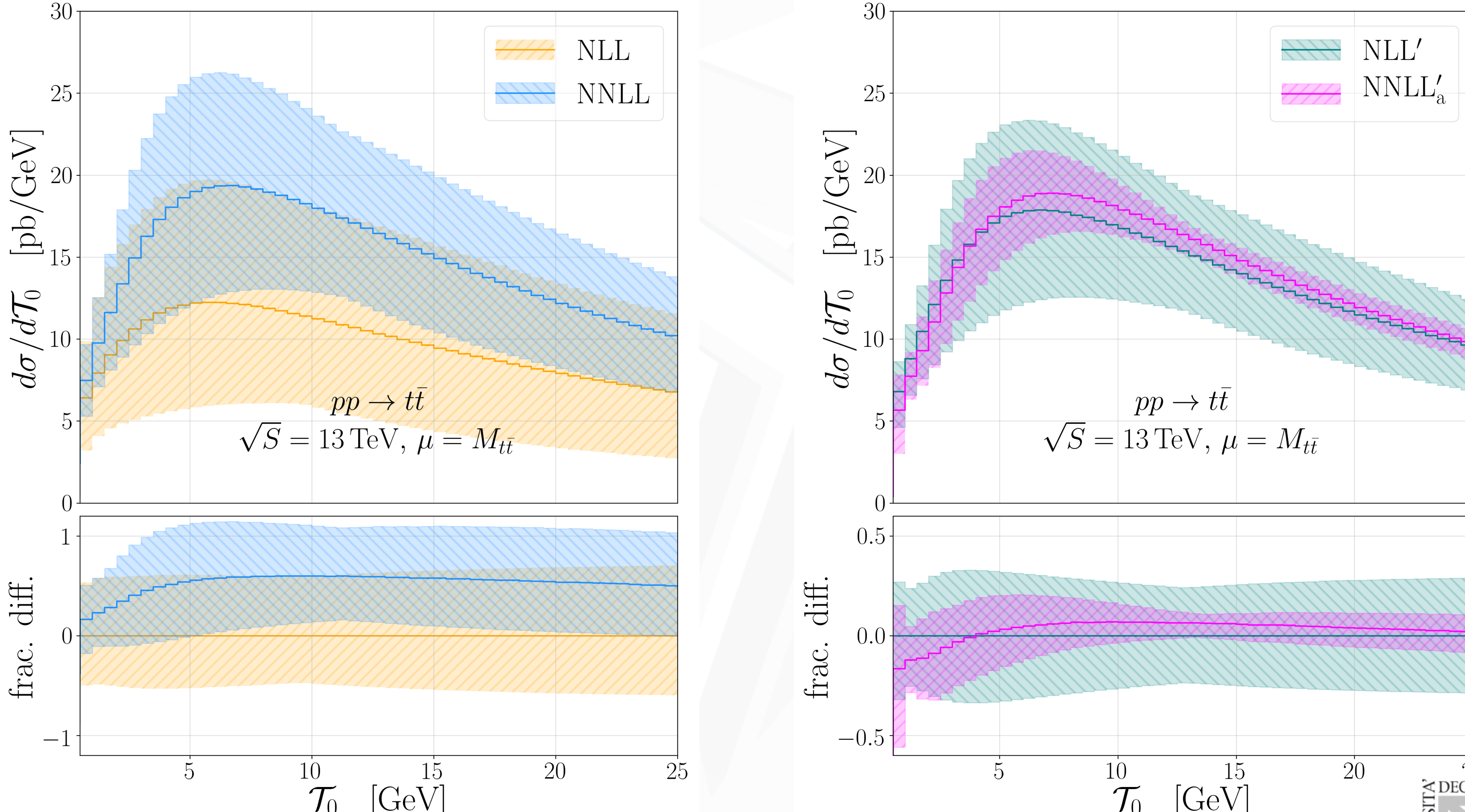
$$U(\mu_h,\mu_B,\mu_s,L_h,L_s) =$$
$$\exp\left[4S(\mu_h,\mu_B)+4S(\mu_s,\mu_B)+2a_{\gamma^B}(\mu_s,\mu_B)-2a_\Gamma(\mu_h,\mu_B)\,L_h-2a_\Gamma(\mu_s,\mu_B)\,L_s\right]$$

and $L_s=\ln(M^2/\mu_s^2)$, $L_h=\ln(M^2/\mu_h^2)$, $L_B=\ln(M^2/\mu_B^2)$ and $\eta_{\rm tot}=2\eta_S+\eta_B+\eta_{B'}$

The final accuracy depends on the availability of the perturbative ingredients

Resummed results

$NNLL'_a$ is our best prediction, it includes NNLO beam functions, all mixed NLO x NLO terms, NNLL evolution matrices, all NNLO soft logarithmic terms. Resummation is switched off via profile scales



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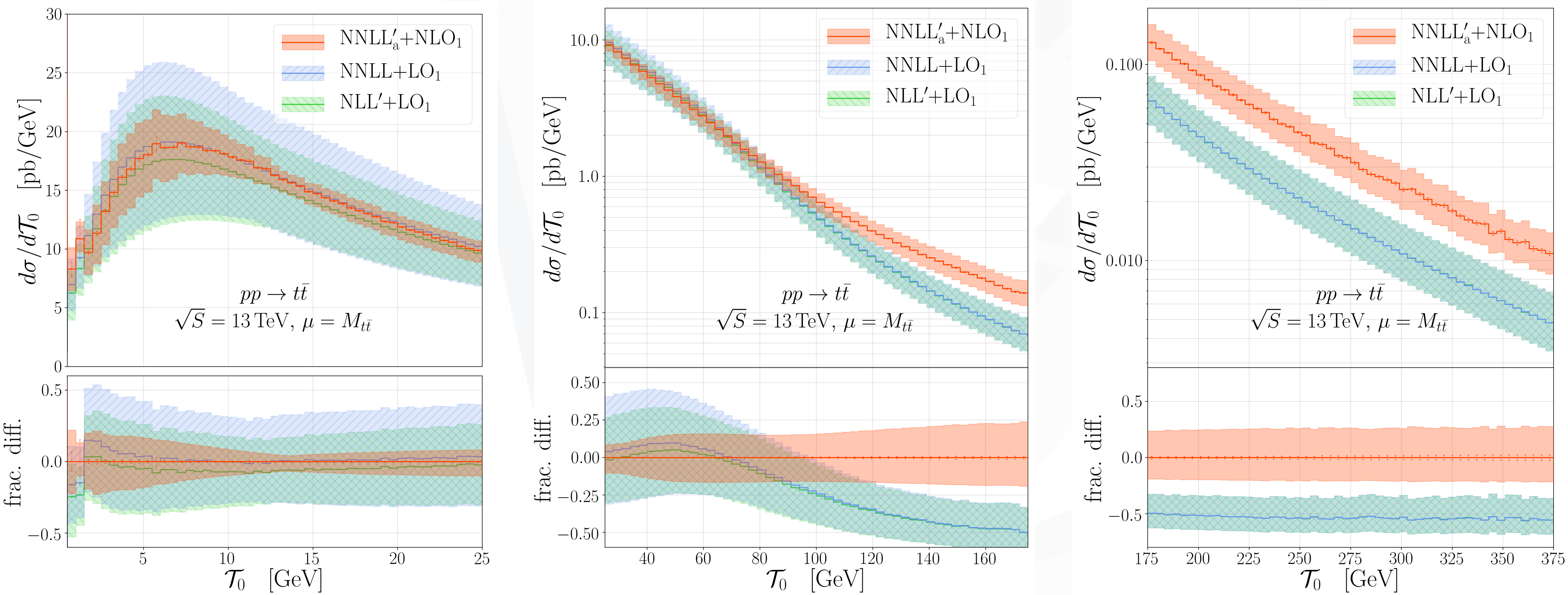
Matched results

Matching to $t\bar{t}+j$ @NLO improves the perturbative accuracy across the whole spectrum

$$\frac{\mathrm{d}\sigma^{\mathrm{match}}}{\mathrm{d}\mathcal{T}_0} = \frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_0} + \frac{\mathrm{d}\sigma^{\mathrm{FO}}}{\mathrm{d}\mathcal{T}_0} - \left[\frac{\mathrm{d}\sigma^{\mathrm{resum}}}{\mathrm{d}\mathcal{T}_0}\right]_{\mathrm{FO}}$$

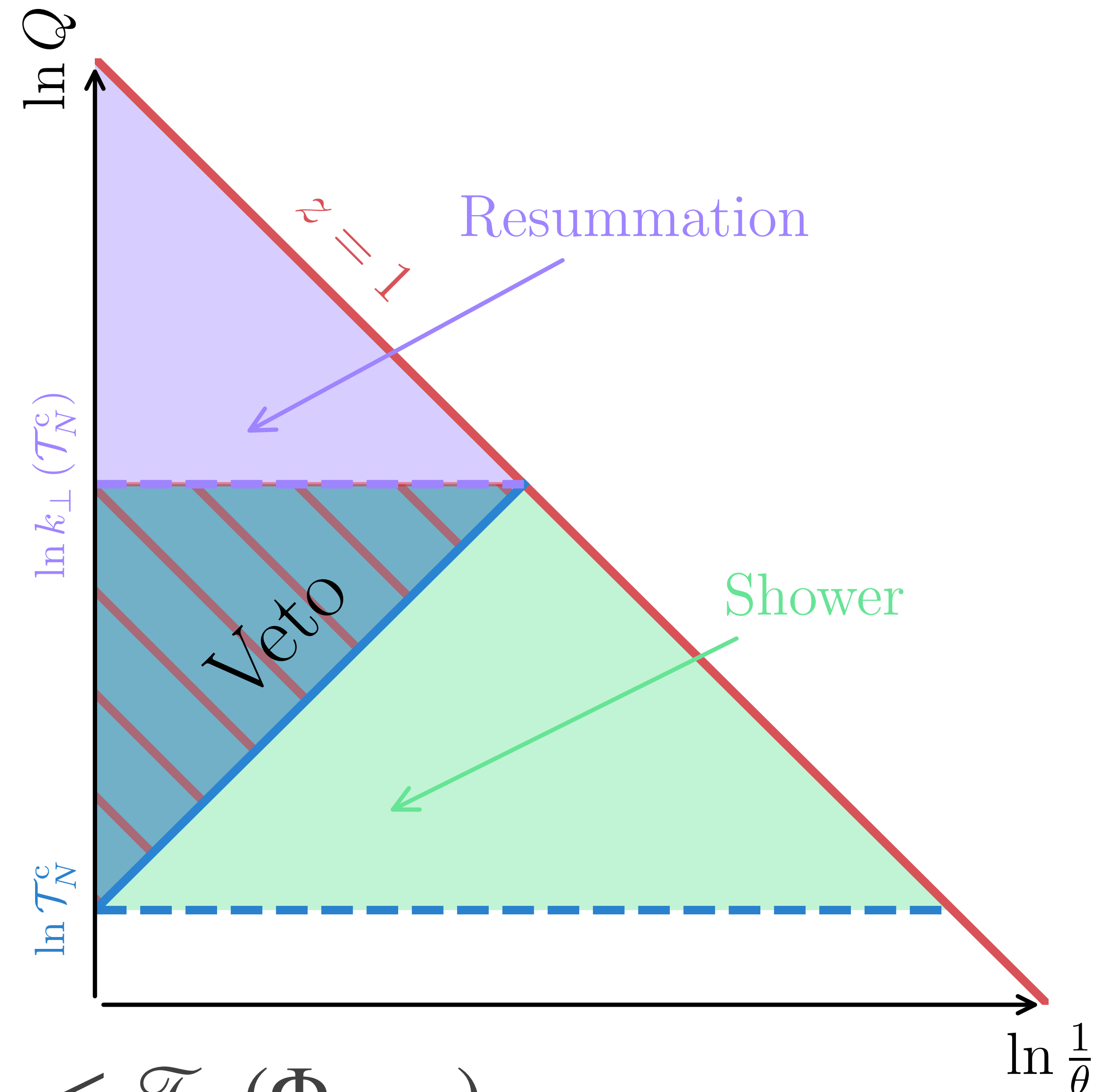


Extension to full NNLL' and to event generation is in progress.

Interface with the parton shower

$\mathcal{T}_N(\Phi_{N+1})$ measures the hardness of the N+1-th emission

- If shower ordered in k_T, start from largest value allowed by N-jettiness
- Let the shower evolve unconstrained.
- At the end veto an event if after $M \geq 1$ shower emissions $\mathcal{T}_N(\Phi_{N+M}) > \mathcal{T}_N(\Phi_N + 1)$ and **retry** the whole shower.



$$\mathcal{T}_{N+M-1}(\Phi_{N+M}) \leq \mathcal{T}_{N+M-2}(\Phi_{N+M}) \leq \ldots \leq \mathcal{T}_N(\Phi_{N+M})$$

Ensures the relevant phase space is correctly covered to avoid spoiling the resummation accuracy for $\mathcal{T}$. Shower accuracy for other observables is more delicate for dipole shower, effects numerically negligible .

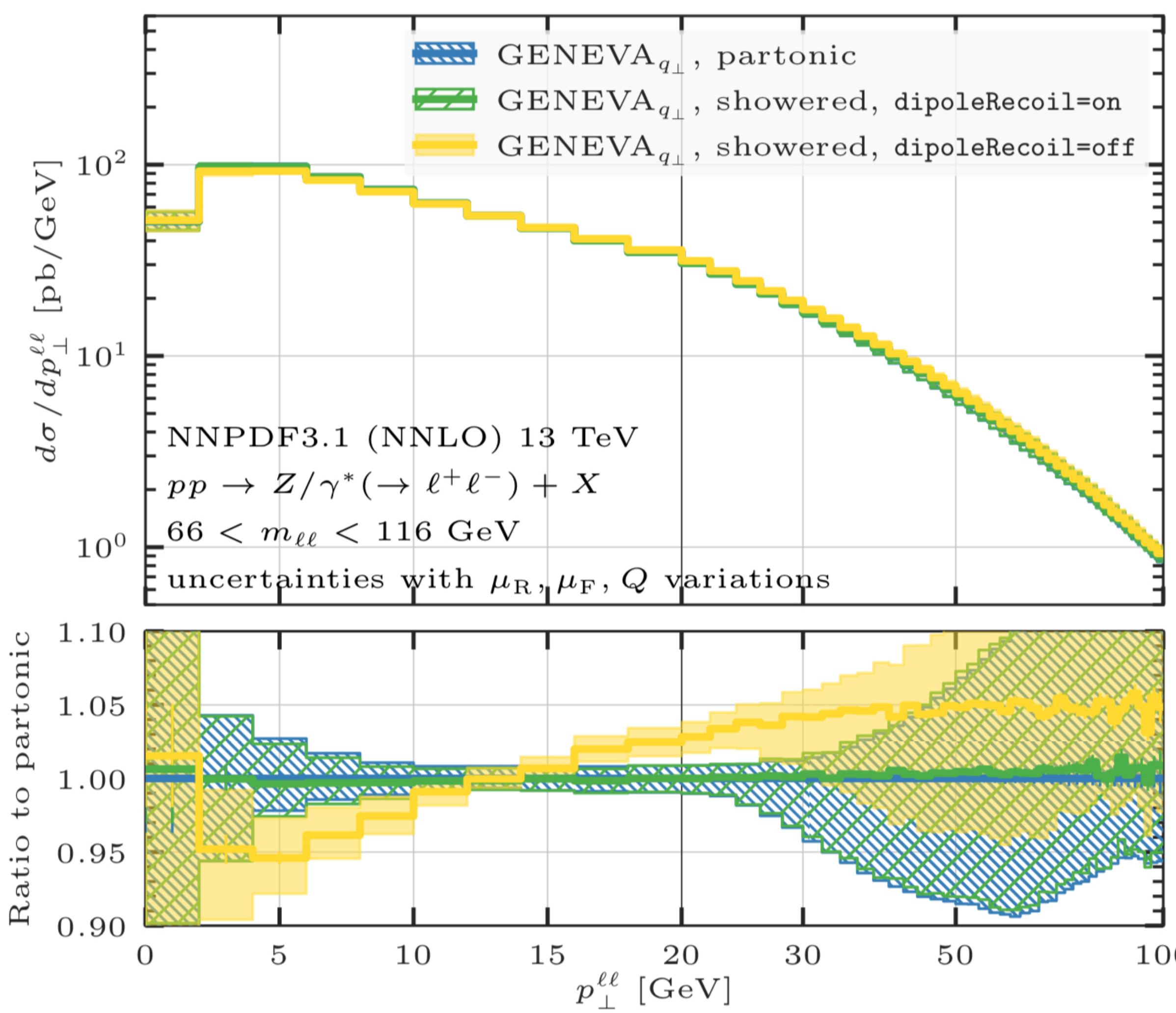
0-jet and 1-jet bins are treated differently: starting scale is resolution cutoff.

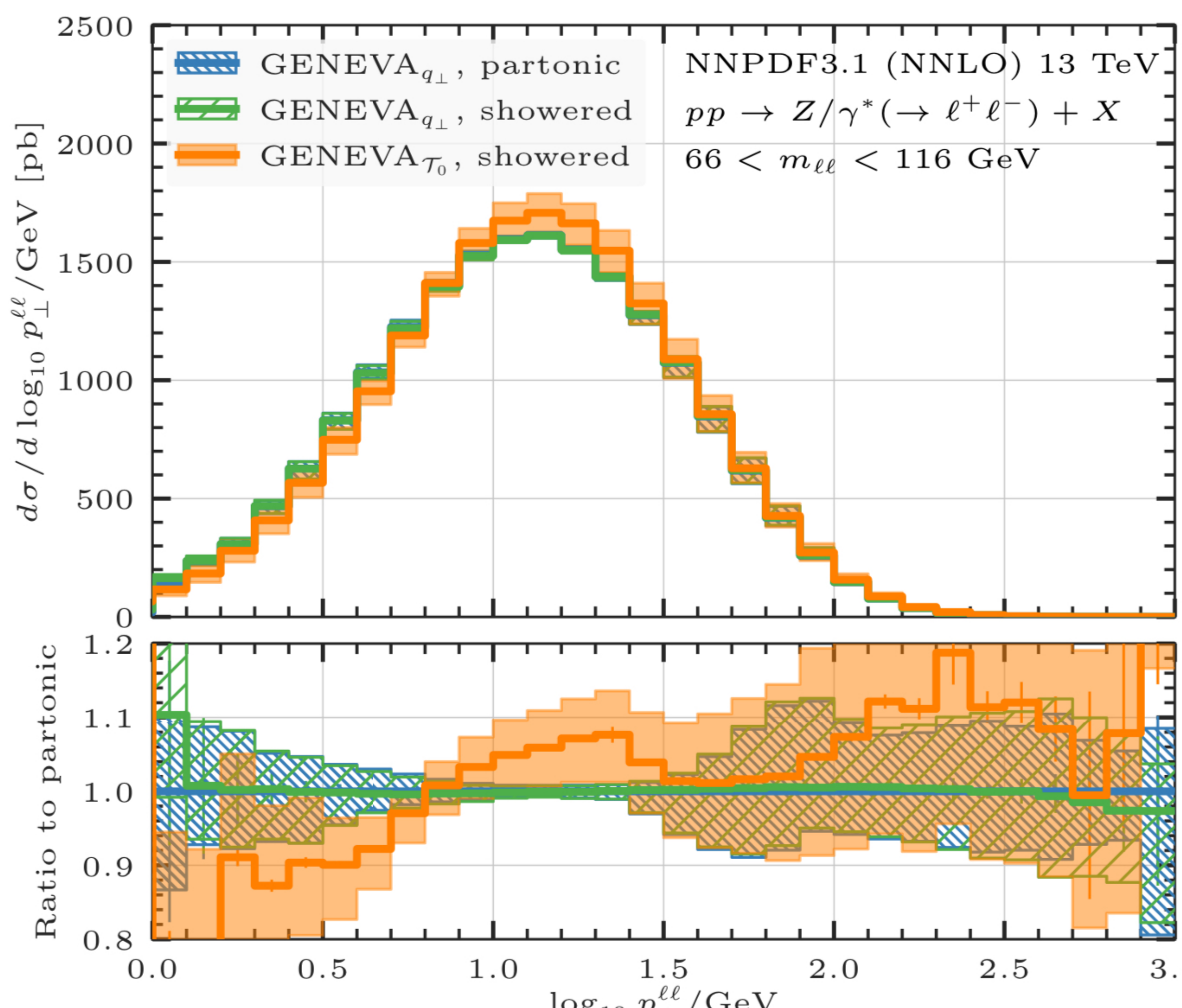
Method rather independent from shower used: PYTHIA8, DIRE & SHERPA.

Interface with the parton shower

Effect of shower on resolution variables different from what is resummed more marked, albeit shower accuracy is maintained.
GENEVA framework allows this comparison for DY when resumming q_T or $\mathcal{T}_0$

Best approach here would be joint $(\mathcal{T}_0, \vec{q}_T)$ resummation, avoids need of splitting func.



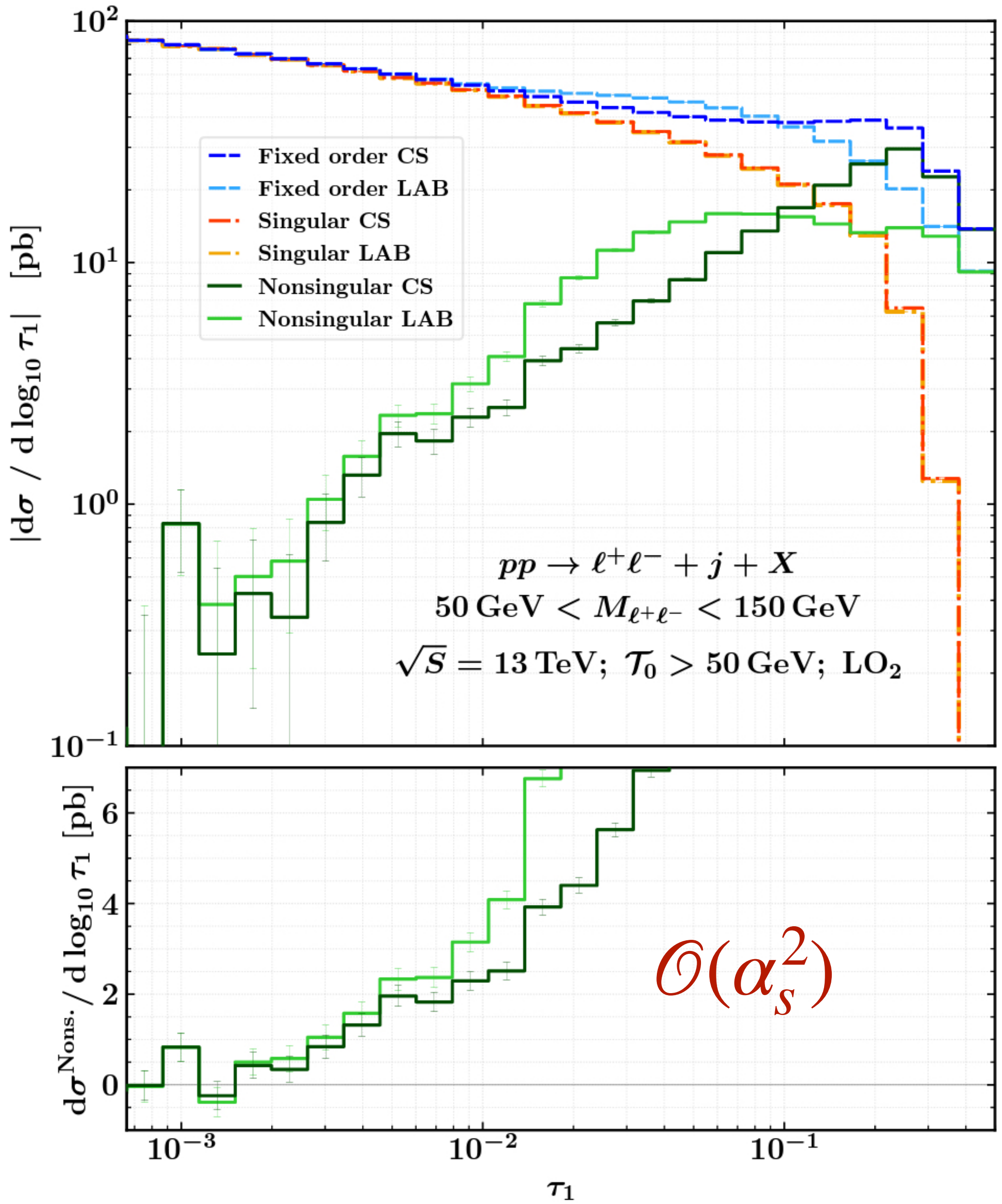


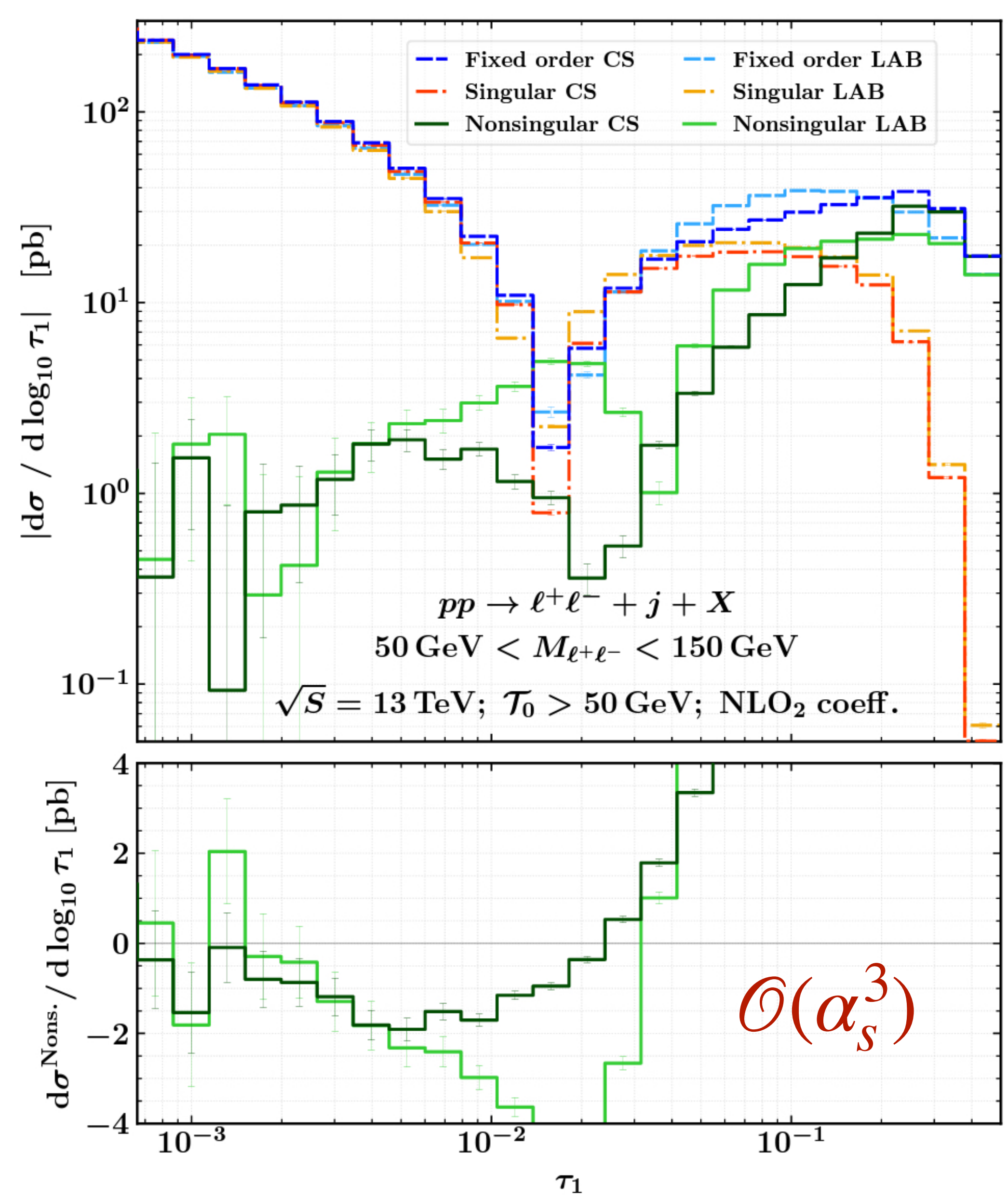
Nonsingular behavior

Dimensionless definition

$$\tau_1 = 2\mathcal{T}_1/\sqrt{M^2_{\ell^+\ell^-} + q_T^2}$$

- Different $\mathcal{T}_1$ choices have different subleading power corrections
- Investigated for one-jettiness subtraction at LL NLP [Boughezal, Isgro', Petriello `20]





- CS frame better than LAB across different cuts. UB frame delicate for IR safety.

Two dimensional profile scales

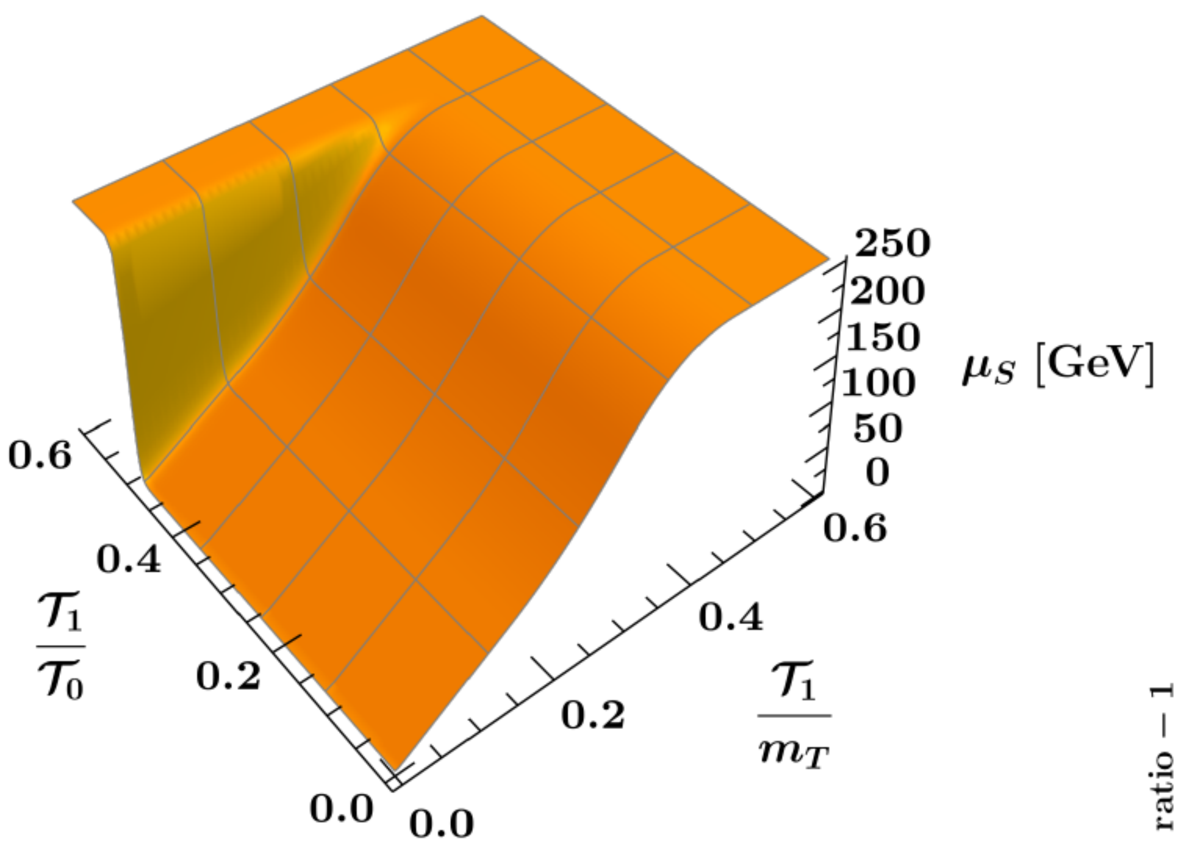
A final state with N particles is subject to the constraint

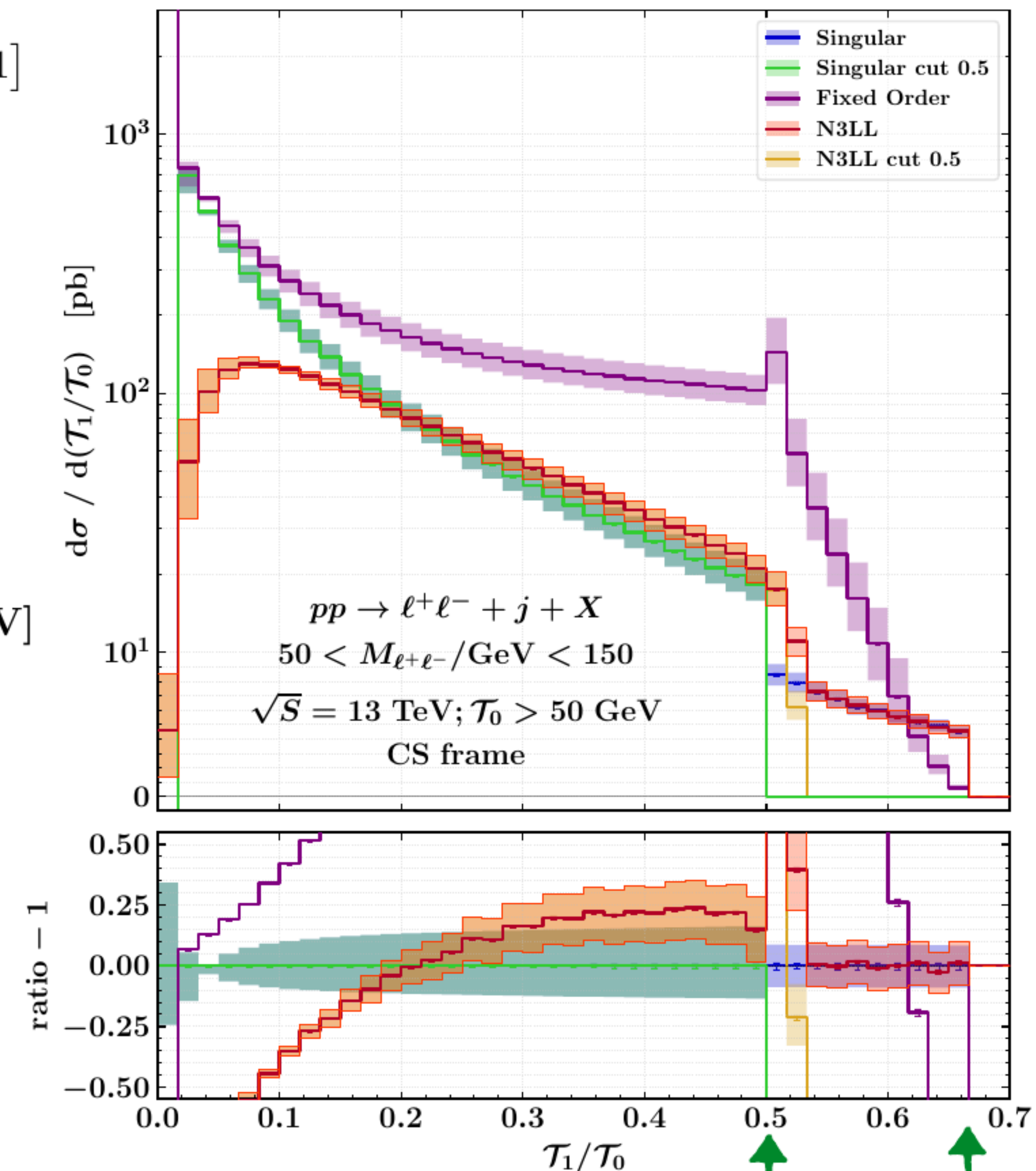
$$\frac{\mathcal{T}_1(\Phi_N)}{\mathcal{T}_0(\Phi_N)} \leq \frac{N-1}{N} = \begin{cases} 1/2\,, & N=2 \\ 2/3\,, & N=3 \end{cases}$$

$$\mu_S(\mathcal{T}_1/\mu_{\rm FO},\mathcal{T}_1/\mathcal{T}_0) = \mu_{\rm FO}\big[\big(f_{\rm run}(\mathcal{T}_1/\mu_{\rm FO})-1\big)s^{(p,k)}(\mathcal{T}_1/\mathcal{T}_0)+1\big]$$

Behaves as smooth Theta function

$$s^{(p,k)}(\mathcal{T}_1/\mathcal{T}_0) = \frac{1}{1+e^{pk(\mathcal{T}_1/\mathcal{T}_0-1/p)}}$$





Kinematical boundaries

We use $p = 2$ (determines the transition point) and $k = 100$ (slope of the transition)

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