χPT & dispersion relations

A personal view on open challenges

Gilberto Colangelo

^{b}
UNIVERSITÄT
BERN

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

Zürich Phenomenology Workshop January 7.2025

Outline

Outline

Introduction

SU(2) χPT
- Quark-mass dependence
- Momentum dependence
- Dispersion relations and matching to χPT

SU(3) χPT

Summary

Chiral Perturbation Theory

- Chiral symmetry of QCD spontaneously broken
  $\Rightarrow$ Goldstone Bosons $= \pi$'s (and K's and η)
- all other QCD particles: $M = \mathcal{O}(\Lambda_{\rm QCD})$
  $\Rightarrow$ for $E \ll \Lambda_{\rm QCD}$ QCD $\Leftrightarrow$ GB dynamics
- GB interact weakly for $p \to 0$
  $\Rightarrow \mathcal{L}_{\rm eff} = \sum \mathcal{L}_{2n}$ $2n =$n. of derivatives
- m_q break chiral symmetry explicitly
  $\Rightarrow m_q \ll \Lambda_{\rm QCD}$ expansion param. in $\mathcal{L}_{\rm eff}$, $m_q = \mathcal{O}(p^2)$

$$\mathcal{L}_{\chi\rm PT} = \sum_{n=1}^{\infty} \mathcal{L}_{2n}$$

with $\mathcal{L}_{2n}$ the most general, chiral invariant Lagrangian of $\mathcal{O}(p^{2n})$

χPT: technical implementation

Gasser and Leutwyler (84)

Aim: provide a faithful low-energy representation of QCD Green's functions $\Leftrightarrow$ Generating Functional

$$e^{iZ[j_k]} := \int [dG][dq][d\bar{q}]e^{iS_{\mathrm{QCD}}+i\int d^4x\,(\bar{q}\Gamma^k q)j_k}$$

$Z[j'_k] = Z[j_k]$ with j'_k chirally rotated sources $j_k = v_\mu, a_\mu, s, p$, expanded around

$$v_\mu = a_\mu = p = 0, \quad s = \mathrm{diag}(m_u, m_d, m_s)$$

Effective Lagrangian approach:

$$e^{iZ[j_k]} = \int [dU]e^{i\int d^4x\,\mathcal{L}_{\chi\mathrm{PT}}(U,j_k)} \qquad \text{with } U = \text{GB fields}$$

provides $\quad Z[j_k] = Z_2[j_k] + Z_4[j_k] + Z_6[j_k] + \ldots$

satisfying: $\quad Z_{2n}[j'_k] = Z_{2n}[j_k]$

χPT: technical implementation

Gasser and Leutwyler (84)

Remark:

While

$$S_{2n}[U, j_k] = \int d^4x\, \mathcal{L}_{2n}(U, j_k)$$

only contain powers of p and m_q

$$Z_{2n}[j_k]$$

also show non-analytic dependence on p and m_q.

This remains true for the effective vertices

Effectiveness of χPT

How well does this approach work?

Since $m_{u,d} =: m_\ell \sim \mathcal{O}(\mathrm{MeV})$ and $m_\ell \ll m_s$
$\Rightarrow$ split the question in two:

1. How well does the expansion in $(m_\ell, p)/\Lambda_{\mathrm{QCD}}$ work?
2. How well does the expansion in $(m_s, p)/\Lambda_{\mathrm{QCD}}$ work?

Outline

Introduction

SU(2) χPT
- Quark-mass dependence
- Momentum dependence
- Dispersion relations and matching to χPT

SU(3) χPT

Summary

Effectiveness of SU(2) χPT

Address this question by splitting it again in two

1a How well does the expansion in $m_\ell/\Lambda_{\rm QCD}$ work?

Need to consider “static” observables, e.g. M_π and F_π

1b How well does the expansion in $p/\Lambda_{\rm QCD}$ work?

Can we define a $p_{\rm max}$ such that for $p < p_{\rm max}$ we are sure to reach a certain precision?

Quark-mass expansion of M_π and F_π

Gasser and Leutwyler (84)

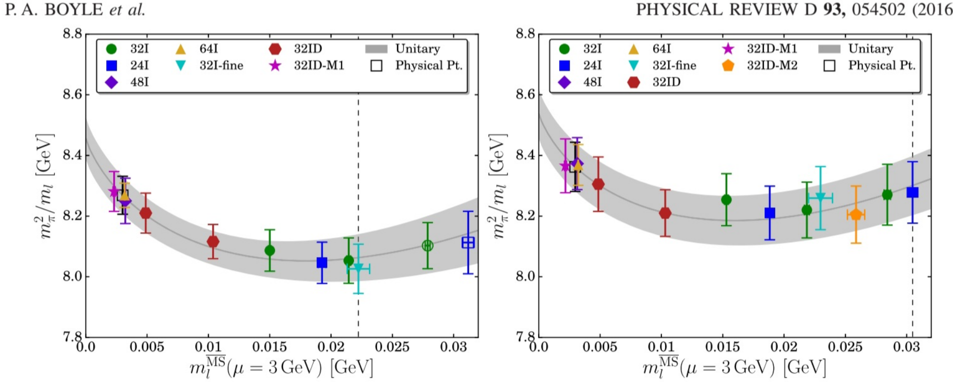
$$M_\pi^2 = M^2 \left[1 - \frac{M^2}{32\pi^2 F_\pi^2}\bar{\ell}_3 + \mathcal{O}(M^4)\right]$$
$$F_\pi = F \left[1 + \frac{M^2}{16\pi^2 F_\pi^2}\bar{\ell}_4 + \mathcal{O}(M^4)\right]$$

with

$$M^2 = B(m_u + m_d) \quad \text{and} \quad B = -\langle 0|\bar{q}q|0\rangle / F^2$$

NNLO expressions are also available   Bijnens, GC, Ecker, Gasser, Sainio (96)

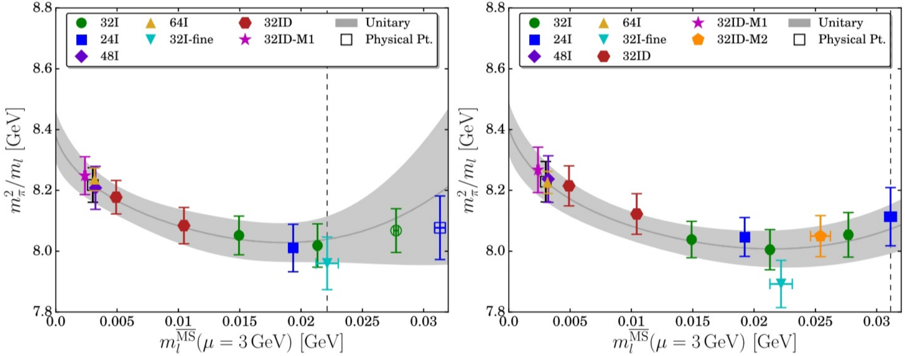
Quark-mass expansion of M_π and F_π



(a) NLO, $m_\pi^{\rm cut} = 370\,{\rm MeV}$ (b) NLO, $m_\pi^{\rm cut} = 450\,{\rm MeV}$

RBC/UKQCD (2016)

Quark-mass expansion of M_π and F_π

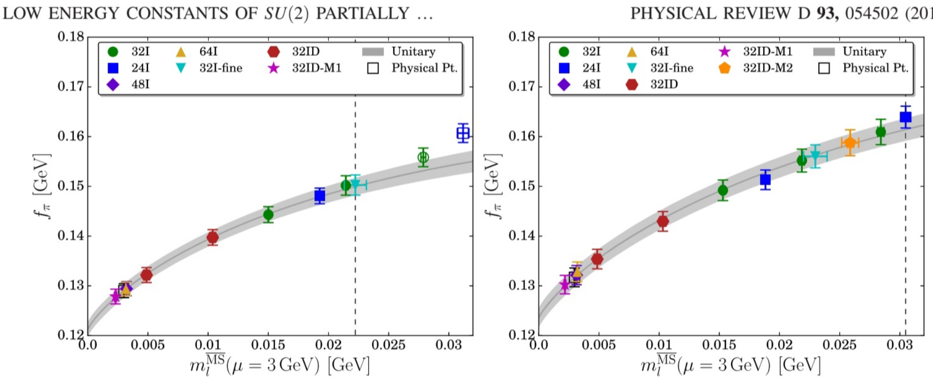


(c) NNLO, $m_\pi^{\rm cut} = 370\,{\rm MeV}$

(d) NNLO, $m_\pi^{\rm cut} = 450\,{\rm MeV}$

RBC/UKQCD (2016)

Quark-mass expansion of M_π and F_π

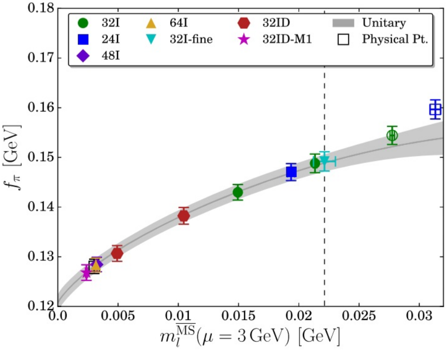


(a) NLO, $m_\pi^{\rm cut} = 370\,{\rm MeV}$

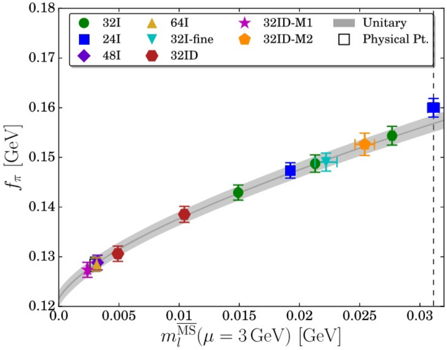
(b) NLO, $m_\pi^{\rm cut} = 450\,{\rm MeV}$

RBC/UKQCD (2016)

Quark-mass expansion of M_π and F_π



(c) NNLO, $m_\pi^{\text{cut}} = 370\,\text{MeV}$

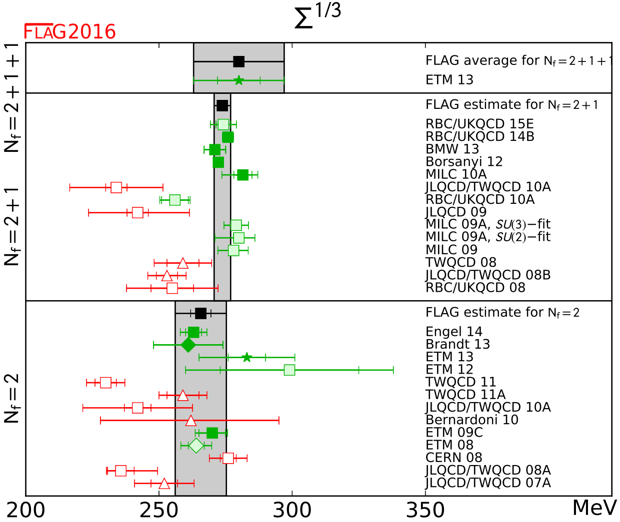


(d) NNLO, $m_\pi^{\text{cut}} = 450\,\text{MeV}$

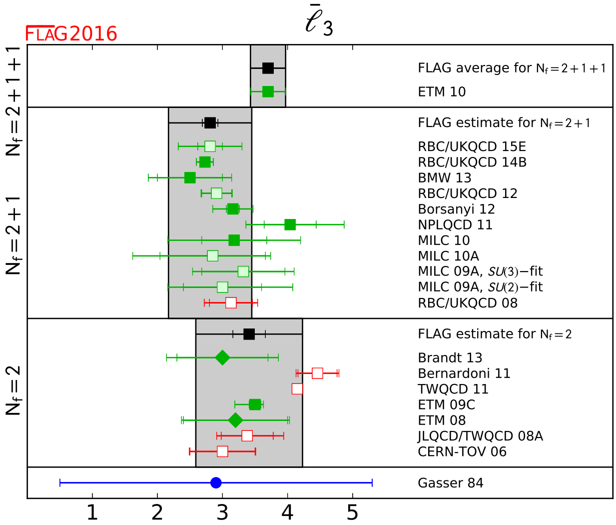
RBC/UKQCD (2016)

SU(2) LECs according to FLAG (2016)

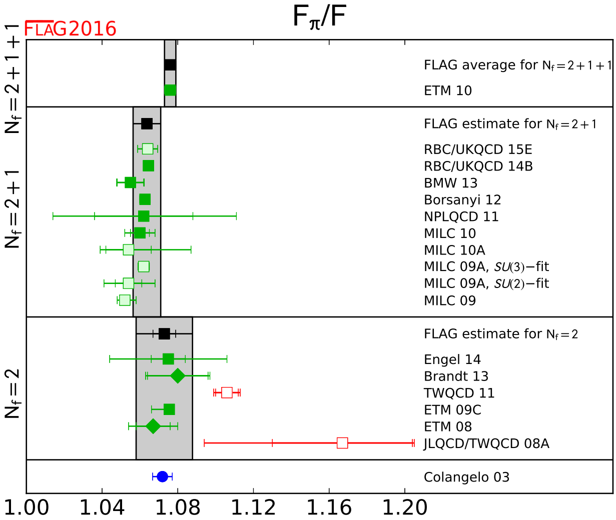
$$\Sigma = -\langle 0|\bar{q}q|0\rangle$$



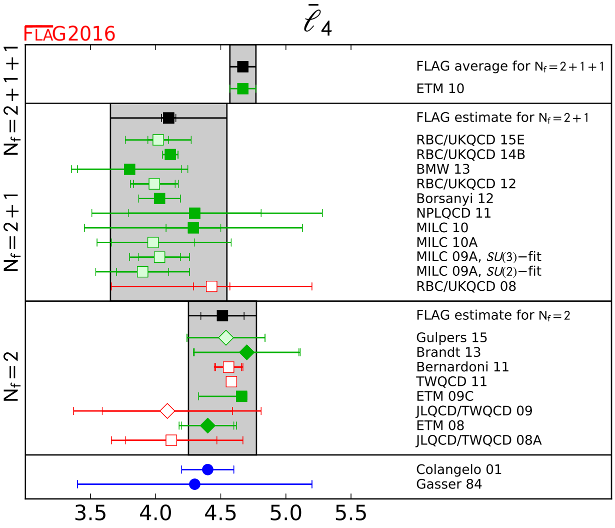
SU(2) LECs according to FLAG (2016)



SU(2) LECs according to FLAG (2016)



SU(2) LECs according to FLAG (2016)



Momentum dependence: Pion form factors

Vector and scalar form factors:

$$\langle \pi^i(p_2)|\bar{q}q|\pi^j(p_1)\rangle = \delta^{ij} F_S(s) \qquad s = (p_1+p_2)^2$$

$$\langle \pi^i(p_2)|\frac{1}{2}\bar{q}\tau^3\gamma_\mu q|\pi^j(p_1)\rangle = i\varepsilon^{i3j}(p_1+p_2)_\mu F_V(s)$$

and their NLO expressions: $N = 16\pi^2$

$$F_S(s) = F_S(0)\left[1 + \frac{s}{NF_\pi^2}(\bar{\ell}_4 - 1) + \frac{2s - M_\pi^2}{F_\pi^2}\bar{J}(s) + \mathcal{O}(p^4)\right]$$

$$F_V(s) = 1 + \frac{s}{6NF_\pi^2}\left(\bar{\ell}_6 - \frac{1}{3}\right) + \frac{s - 4M_\pi^2}{6F_\pi^2}\bar{J}(s) + \mathcal{O}(p^4)$$

[$\bar{J}(s)$ = loop function] Gasser, Leutwyler (84)

1a. How well can we predict the value at $s = 0$?

1b. How well do we understand the s dependence?

Momentum dependence: Pion form factors

Vector and scalar form factors:

$$\langle \pi^i(p_2)|\bar{q}q|\pi^j(p_1)\rangle = \delta^{ij} F_S(s) \qquad s = (p_1 + p_2)^2$$

$$\langle \pi^i(p_2)|\frac{1}{2}\bar{q}\tau^3\gamma_\mu q|\pi^j(p_1)\rangle = i\varepsilon^{i3j}(p_1 + p_2)_\mu F_V(s)$$

1a. Value at $s = 0$

$\hat{m} = (m_u + m_d)/2$

$$F_S(0) = 2B\left[1 + \frac{M^2}{NF_\pi^2}\left(\bar{\ell}_3 - \frac{1}{2}\right) + \mathcal{O}(M^4)\right] = \frac{\partial M_\pi^2}{\partial \hat{m}}$$

$\Rightarrow$ understood as well as M_π^2

$F_V(0) = 1 \qquad \Leftrightarrow \qquad \bar{q}\vec{\tau}\gamma_\mu q =$ conserved current

$\Rightarrow$ protected from m_ℓ − effects

Momentum dependence: Pion form factors

Vector and scalar form factors:

$$\langle \pi^i(p_2)|\bar{q}q|\pi^j(p_1)\rangle = \delta^{ij} F_S(s) \qquad s = (p_1 + p_2)^2$$

$$\langle \pi^i(p_2)|\frac{1}{2}\bar{q}\tau^3\gamma_\mu q|\pi^j(p_1)\rangle = i\varepsilon^{i3j}(p_1 + p_2)_\mu F_V(s)$$

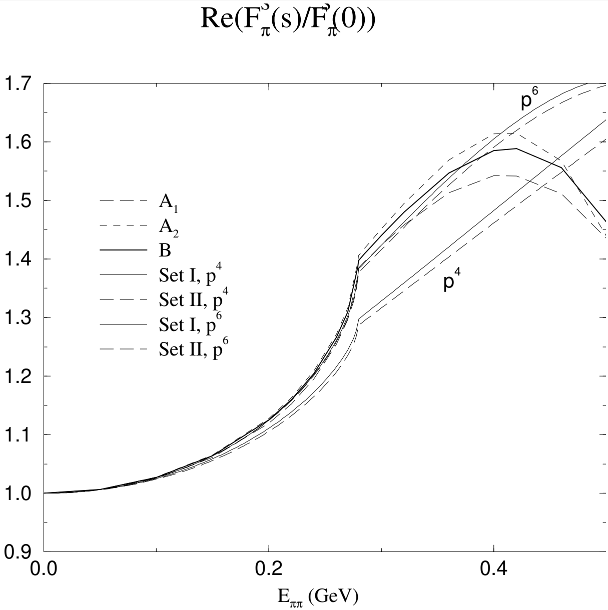
1b. *s*-dependence $\qquad \bar{F}_S(s) = F_S(s)/F_S(0)$

$$\bar{F}_S(s) = 1 + \frac{s}{NF_\pi^2}(\bar{\ell}_4 - 1) + \frac{2s - M_\pi^2}{F_\pi^2}\bar{J}(s) + \mathcal{O}(p^4)$$

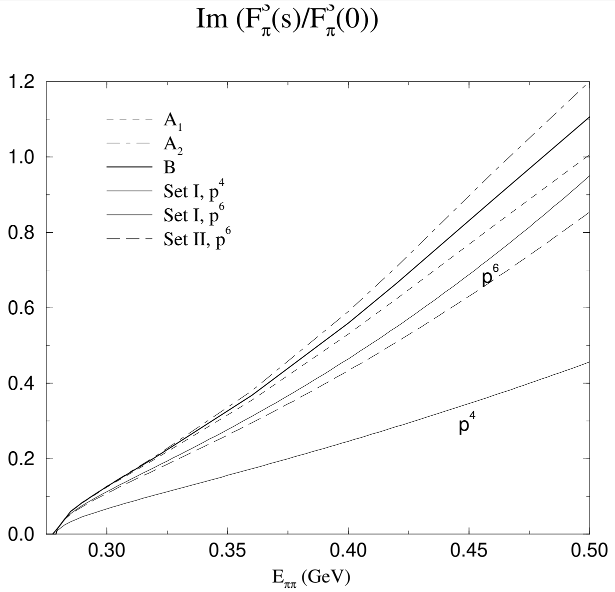
$$F_V(s) = 1 + \frac{s}{6NF_\pi^2}\left(\bar{\ell}_6 - \frac{1}{3}\right) + \frac{s - 4M_\pi^2}{6F_\pi^2}\bar{J}(s) + \mathcal{O}(p^4)$$

LEC FLAG values: $\bar{\ell}_4 = 4.02(45)$ $\qquad \bar{\ell}_6 = 15.1(1.2)$

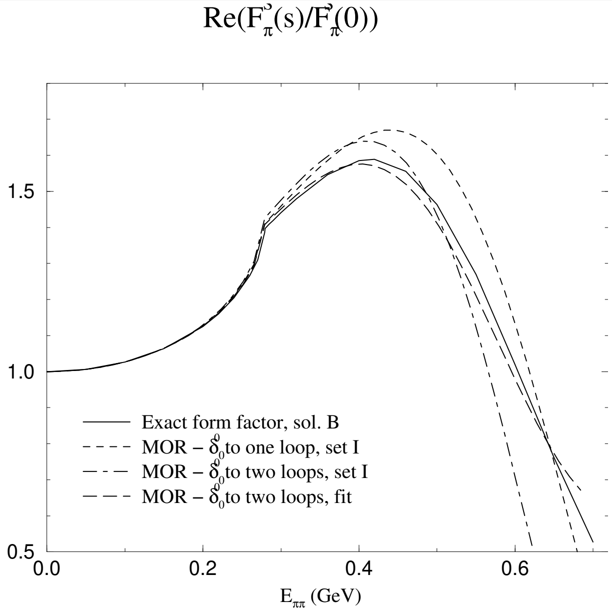
Momentum dependence: Pion form factors



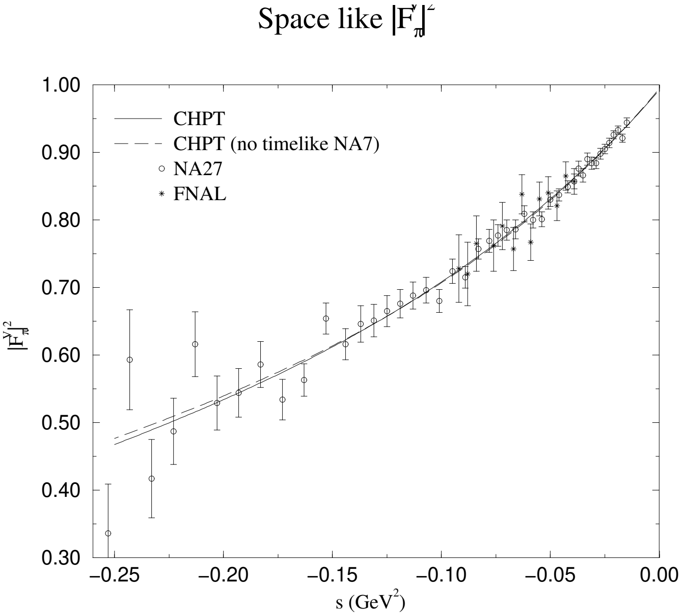
Momentum dependence: Pion form factors



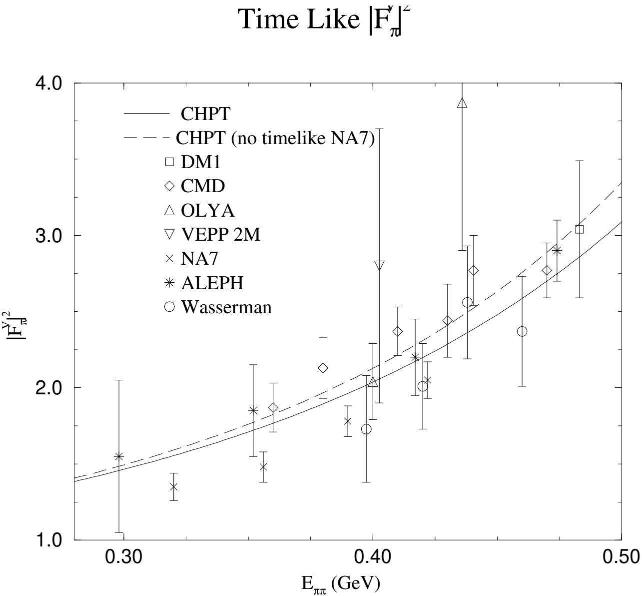
Momentum dependence: Pion form factors



Momentum dependence: Pion form factors



Momentum dependence: Pion form factors



Analytic properties of pion form factors

Mathematical problem:

1. $F(s)$ is an analytic function of s in the whole complex plane, with the exception of a cut for $4M_\pi^2 \leq s < \infty$;
2. approaching the real axis from above $e^{-i\delta(s)}F(s)$ is real on the real axis, where $\delta(s)$ is a known function.

Omnès ('58) found an exact solution to this problem:

$$F(s) = P(s)\Omega(s) = P(s)\exp\left\{\frac{s}{\pi}\int_{4M_\pi^2}^{\infty}\frac{ds'}{s'}\frac{\delta(s')}{s'-s}\right\} \quad ,$$

where $P(s)$ is a polynomial which can only be constrained by the behaviour of $F(s)$ for $s\to\infty$, or by the presence of zeros.
$\Omega(s)$ is called the Omnès function

Scalar form factor of the pion

Omnès representation: (assuming no zeros)

$$F_S(s) = F_S(0)\Omega_S(s) \qquad \ln\Omega_S(s) = \frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\delta_S(s')}{s'(s'-s)}$$

Unitarity $\Rightarrow$ Watson's theorem:

$\delta_S(s) = \delta_0^0(s)$ for $s < 4M_K^2$ negligible inelasticity due to 4π

Scalar form factor of the pion

Omnès representation:

(assuming no zeros)

$$F_S(s) = F_S(0)\Omega_S(s) \qquad \ln \Omega_S(s) = \frac{s}{\pi}\int_{4M_\pi^2}^{\infty} ds' \frac{\delta_S(s')}{s'(s'-s)}$$

Unitarity $\Rightarrow$ Watson's theorem:

$$\Rightarrow \Omega_S(s) = \Omega_0^0(s)\cdot\exp\left[\frac{s}{\pi}\int_{4M_K^2}^{\infty} ds' \frac{\delta_S(s') - \delta_0^0(s')}{s'(s'-s)}\right] \simeq \Omega_0^0(s)\left(1 + c_1 \frac{s}{4M_K^2} + \dots\right)$$

Chiral vs. dispersive representation

Replace $\delta_0^0(s)$ with its chiral expansion; expand the exponential
$\Rightarrow$ chiral expansion of $F_S(s)$

Matching the chiral and the dispersive representation:
$\Rightarrow$ sum rules for the LECs

Scalar form factor of the pion

Conclusions:

- the low-energy behaviour of $F_S(s)$ is determined to a large extent by the $\pi\pi$ phase shift $\delta_0^0(s)$
- $F_S(0)$ ($\Leftrightarrow$ the σ-term of the pion) has a fast converging chiral expansion
- inelastic effects ($\bar{K}K$ channel) may be sizeable, but are well described by a polynomial at low energy (LECs)

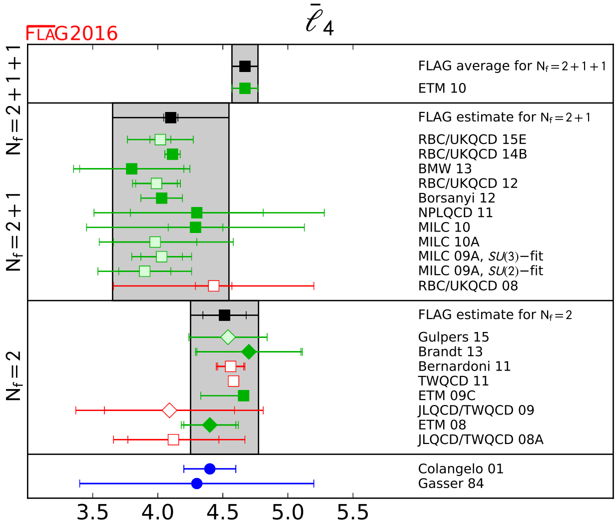
Scalar form factor of the pion

Conclusions:

- the low-energy behaviour of $F_S(s)$ is determined to a large extent by the $\pi\pi$ phase shift $\delta_0^0(s)$
- $F_S(0)$ ($\Leftrightarrow$ the σ-term of the pion) has a fast converging chiral expansion
- inelastic effects ($\bar{K}K$ channel) may be sizeable, but are well described by a polynomial at low energy (LECs)
- to have the latter under control a coupled-channel analysis is necessary Donoghue, Gasser, Leutwyler, 1990
- this leads to an accurate prediction for the scalar radius of the pion GC, Gasser, Leutwyler, 2001

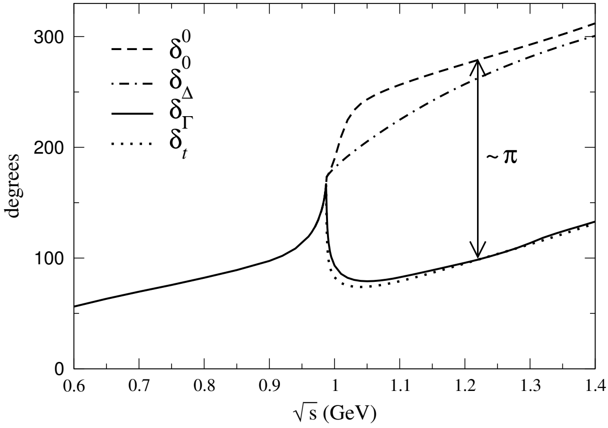
$$\langle r^2\rangle_s^\pi = \frac{6}{\pi}\int_{4M_\pi^2}^\infty \frac{ds\,\delta_S(s)}{s^2} = 0.61 \pm 0.04\mathrm{fm}^2$$

Scalar form factor of the pion



Scalar form factor: dispersive representation

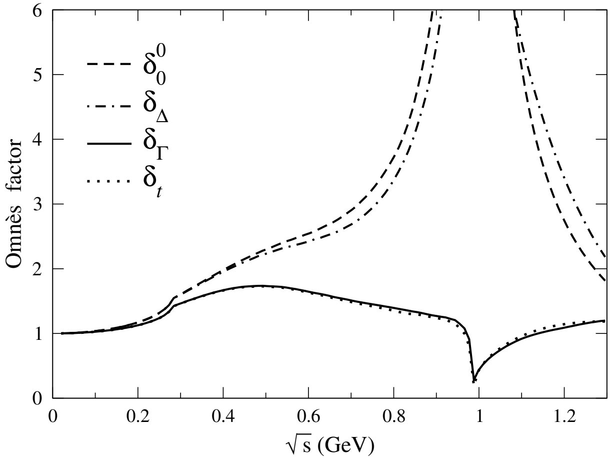
$\delta_\Gamma = \delta_S$



Ananthanarayan, Caprini, GC, Gasser, Leutwyler, 2004

Scalar form factor: dispersive representation

$\delta_\Gamma = \delta_S$



Ananthanarayan, Caprini, GC, Gasser, Leutwyler, 2004

Vector form factor of the pion

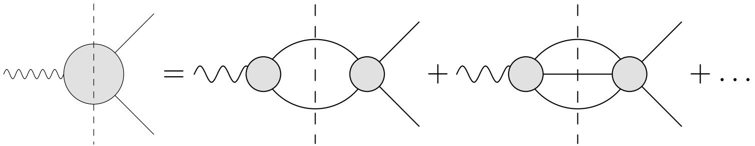
A similar discussion can be made for the vector form factor

- the normalization (subtraction constant) is fixed by gauge invariance:

$$F_V^\pi(0) = 1$$

- for this form factor there are data coming from $e^+e^- \to \pi^+\pi^-$ which allow one to pin down the free parameters in the Omnès representation

Omnès representation including isospin breaking



Omnès representation including isospin breaking

- Omnès representation

$$F_V^\pi(s) = \exp\left[\frac{s}{\pi}\int_{4M_\pi^2}^\infty ds' \frac{\delta(s')}{s'(s'-s)}\right] \equiv \Omega(s)$$

- Split elastic ($\leftrightarrow \pi\pi$ phase shift, δ_1^1) from inelastic phase

$$\delta = \delta_1^1 + \delta_{\rm in} \quad \Rightarrow \quad F_V^\pi(s) = \Omega_1^1(s)\Omega_{\rm in}(s)$$

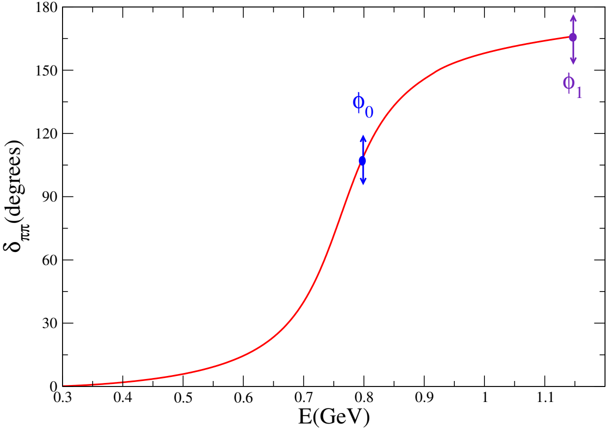
Eidelman-Lukaszuk: unitarity bound on $\delta_{\rm in}$

$$\sin^2\delta_{\rm in} \le \frac{1}{2}\left(1-\sqrt{1-r^2}\right), \; r = \frac{\sigma^{I=1}_{e^+e^-\to\neq 2\pi}}{\sigma_{e^+e^-\to 2\pi}} \Rightarrow s_{\rm in} = (M_\pi + M_\omega)^2$$

- $\rho-\omega$–mixing $\qquad F_V(s) = \Omega_{\pi\pi}(s)\cdot\Omega_{\rm in}(s)\cdot G_\omega(s)$

$$G_\omega(s) = 1 + \epsilon\frac{s}{s_\omega - s} \qquad \text{where} \qquad s_\omega = (M_\omega - i\Gamma_\omega/2)^2$$

Essential free parameters



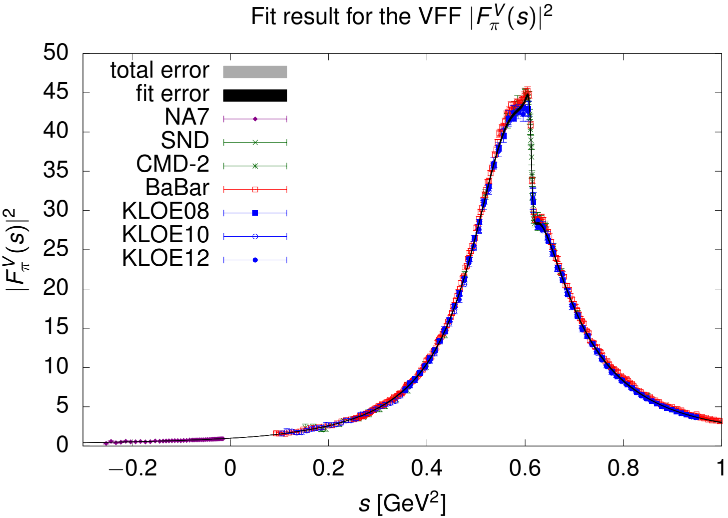
Estimated range ($\pi N \to \pi\pi N$):

Caprini, GC, Leutwyler (12)

$$\phi_0 = 108.9(2.0)^\circ \qquad \phi_1 = 166.5(2.0)^\circ$$

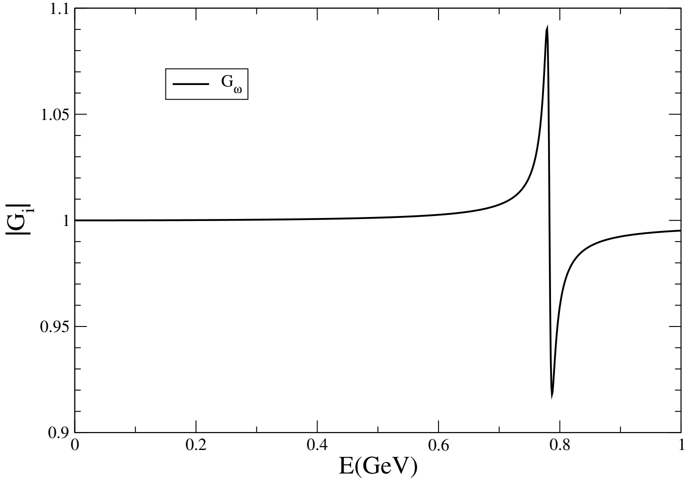
Fit results

GC, Hoferichter, Stoffer (18)



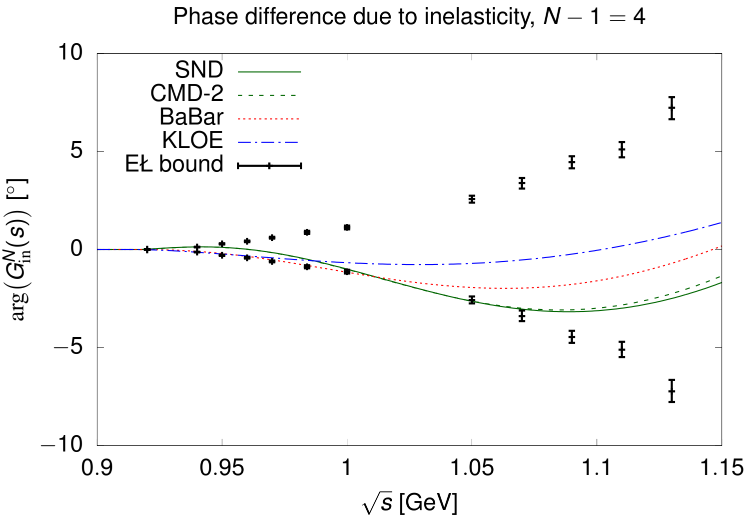
Fit results

GC, Hoferichter, Stoffer (18)



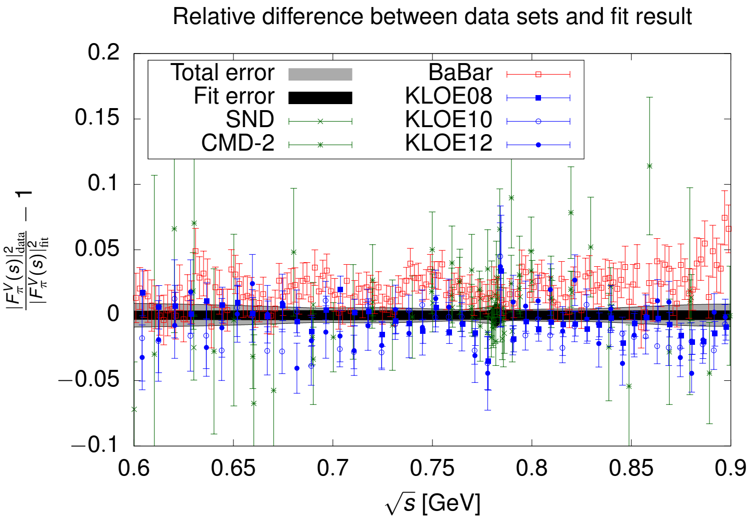
Fit results

GC, Hoferichter, Stoffer (18)



Fit results

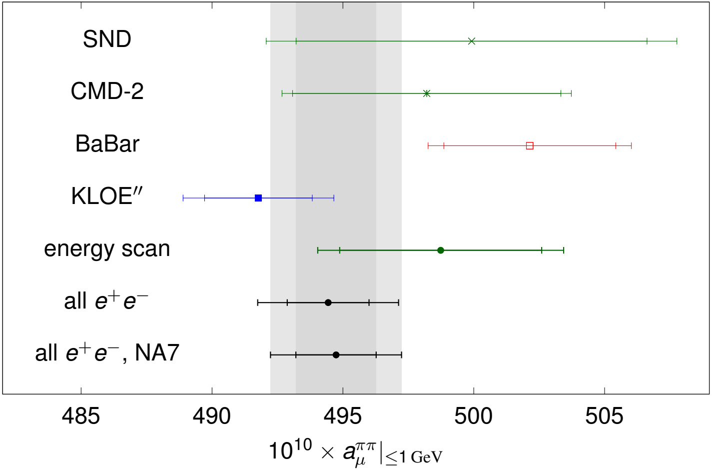
GC, Hoferichter, Stoffer (18)



Fit results

GC, Hoferichter, Stoffer (18)

Result for $a_\mu^{\pi\pi}|_{\leq 1\,\mathrm{GeV}}$ from the VFF fits to single experiments and combinations



$\pi\pi$ scattering

Partial waves:

$$t^I_\ell(s) \text{ with } I = \text{isospin}, \ \ell = \text{angular momentum}$$

χPT: $t^I_\ell(s)$ known up to $\mathcal{O}(p^6)$ (NNLO)

$$t^0_0(s) = \frac{2s - M_\pi^2}{32\pi F_\pi^2} + \mathcal{O}(p^4)$$

LEC values?

Dispersive representation: Roy eqs. (1971)

$$t^I_\ell(s) = k^I_\ell(s) + \sum_{I'=0}^{2}\sum_{\ell'=0}^{\infty}\int_{4M_\pi^2}^{\infty} ds' K^{II'}_{\ell\ell'}(s,s')\,\mathrm{Im}\,t^{I'}_{\ell'}(s')$$

with $K^{II'}_{\ell\ell'}(s,s')$ analytically known kernels and

$$k^I_\ell(s) = a^I_0\delta^0_\ell + \frac{s - 4M_\pi^2}{72M_\pi^2}(2a^0_0 - 5a^2_0)\left(6\delta^I_0\delta^0_\ell + \delta^I_1\delta^1_\ell - 3\delta^I_2\delta^0_\ell\right)$$

Roy equations

In the elastic region:

$$t_\ell^I(s) = \frac{\sin \delta_\ell^I(s) e^{i\delta_\ell^I(s)}}{\sqrt{1 - 4M_\pi^2/s}}$$

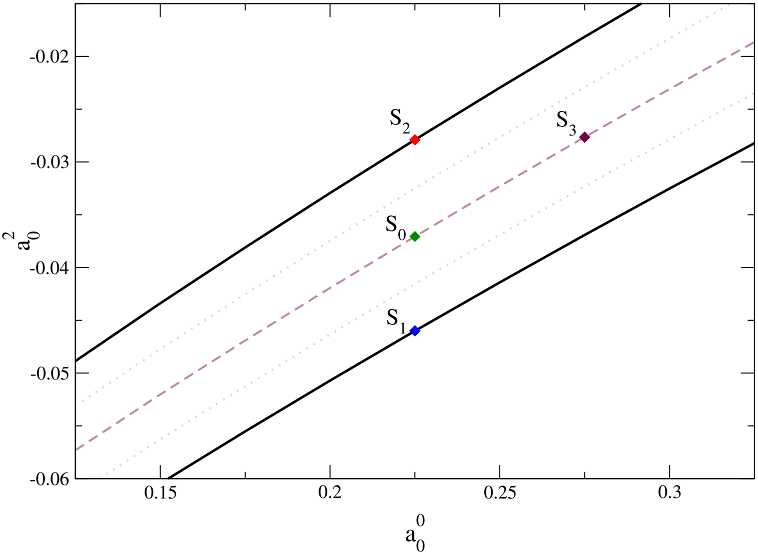
$\Rightarrow$ Roy eqs. become coupled, nonlinear, integral eqs. for $\delta_\ell^I(s)$

For:

- a given input $\mathrm{Im} t_\ell^I(s)$, for $\sqrt{s} \geq \mathcal{O}(1\mathrm{GeV})$
- a fixed value for a_0^0 and a_0^2 (inside the universal band)

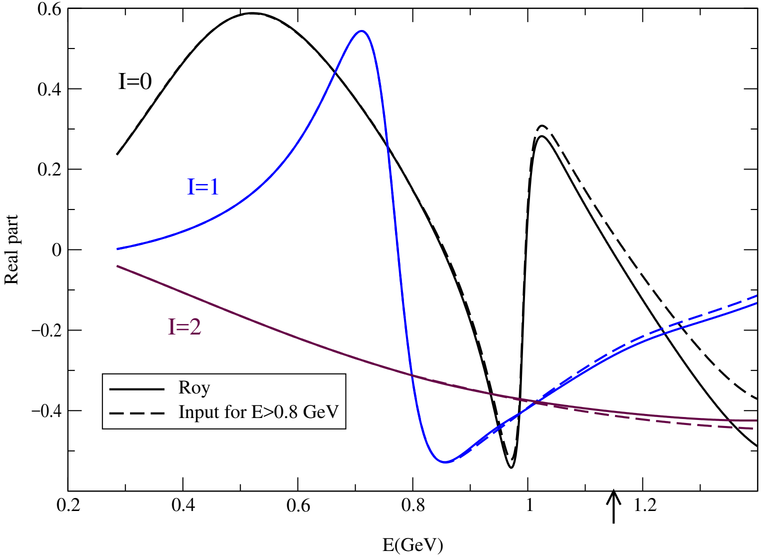
they can be solved numerically

Roy equations



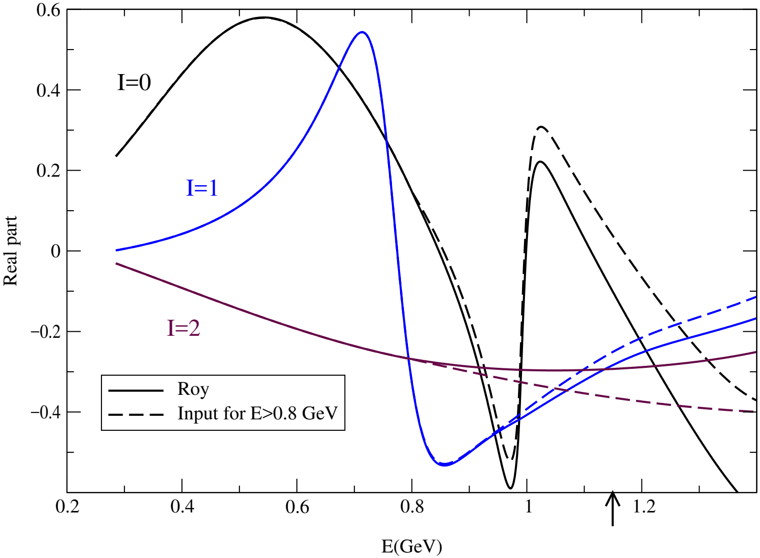
Ananthanarayan, GC, Gasser, Leutwyler (00)

Roy equations



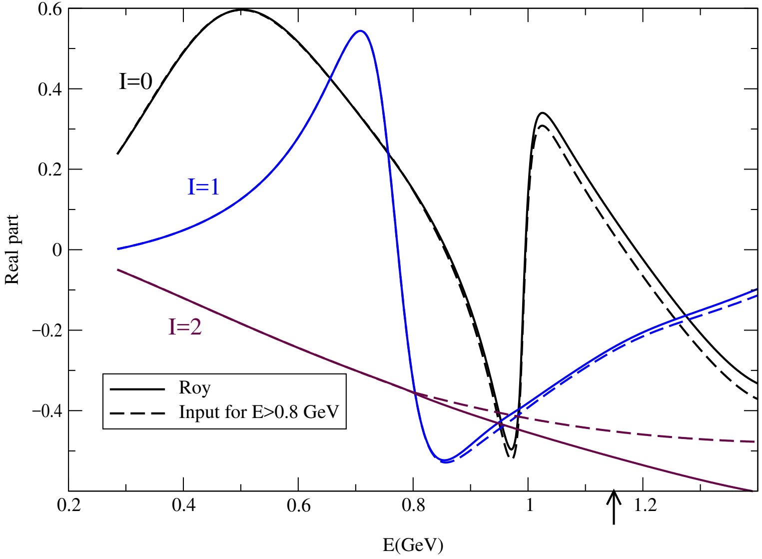
Ananthanarayan, GC, Gasser, Leutwyler (00)

Roy equations



Ananthanarayan, GC, Gasser, Leutwyler (00)

Roy equations

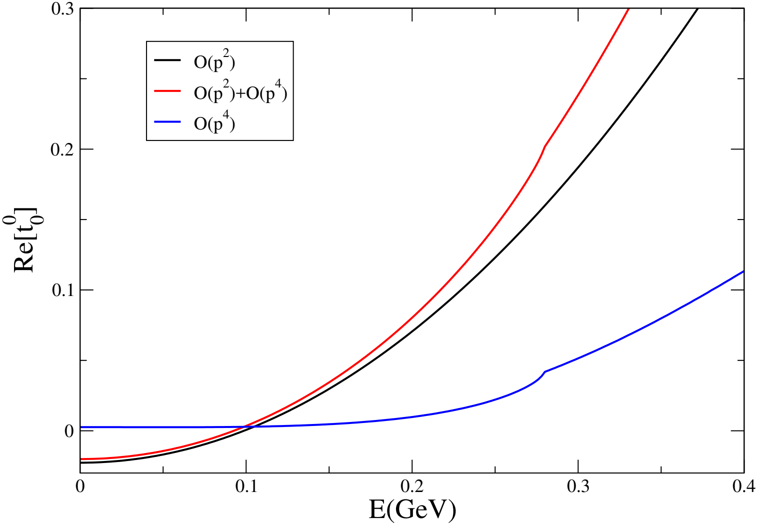


Ananthanarayan, GC, Gasser, Leutwyler (00)

Roy + χPT

- at fixed input above 1 GeV, the only free parameters in the Roy eqs. are a_0^0 and a_0^2
- chiral perturbation theory predicts these
- the most reliable χPT prediction is below threshold
- fixing the two subtraction constants in this way leads to a very precise prediction

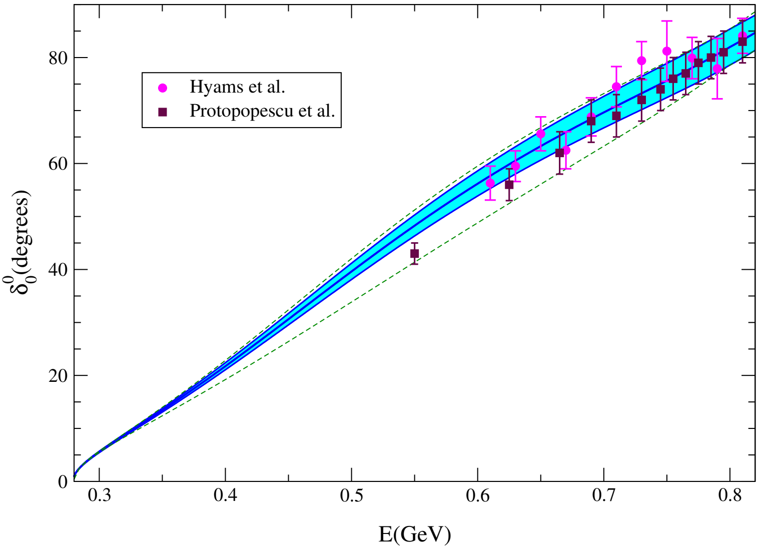
Roy + χPT



Roy+ChPT: final results

GC, Gasser and Leutwyler (01)

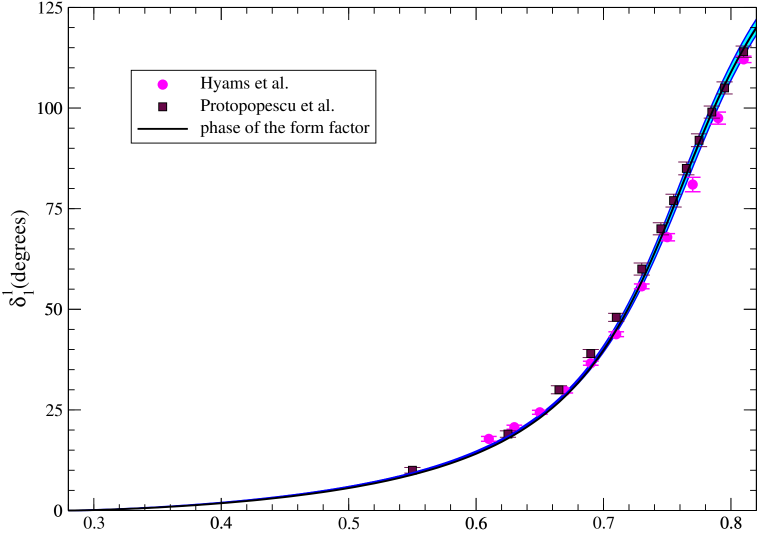
Phase shifts:



Roy+ChPT: final results

GC, Gasser and Leutwyler (01)

Phase shifts:



Roy+ChPT: final results

GC, Gasser and Leutwyler (01)

Scattering lengths:
convergence of the direct χPT calculation:

$$\begin{array}{rcrcrcr} a_0^0 & = & 0.159 & \to & 0.200 & \to & 0.216 \\ 10 \cdot a_0^2 & = & -0.454 & \to & -0.445 & \to & -0.445 \\ & & p^2 & & p^4 & & p^6 \end{array}$$

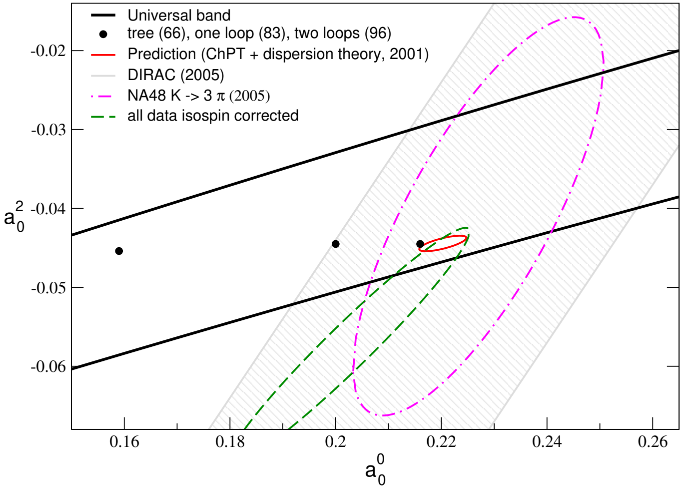
convergence with the matching below threshold

$$\begin{array}{rcrcrcr} a_0^0 & = & 0.197 & \to & 0.220 & \to & 0.220 \\ 10 \cdot a_0^2 & = & -0.402 & \to & -0.446 & \to & -0.444 \\ & & p^2 & & p^4 & & p^6 \end{array}$$

Final prediction

$$\begin{array}{rcr} a_0^0 & = & 0.220 \pm 0.005 \\ 10 \cdot a_0^2 & = & -0.444 \pm 0.01 \end{array}$$

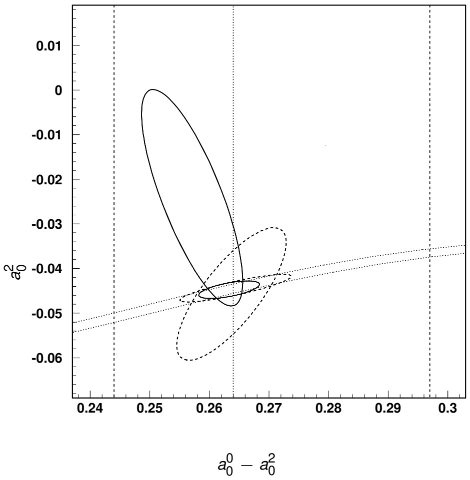
Experimental confirmation



“all data” refers to K_{e4} data, isospin correction from GC, Gasser, Rusetsky (09)

Experimental confirmation

Figure from NA48/2 Eur.Phys.J.C64:589,2009



Outline

Effectiveness of SU(3) χPT

Address this question by splitting it again in two

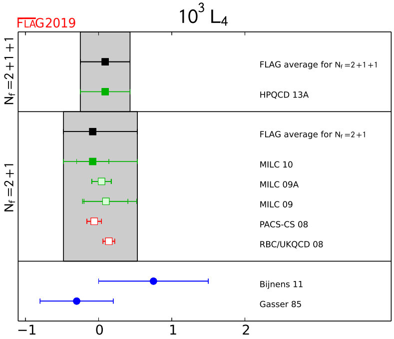
1a How well does the expansion in $m_s/\Lambda_{\rm QCD}$ work?

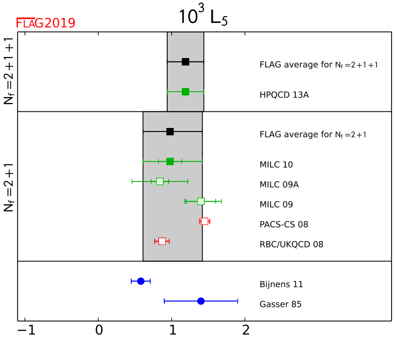
Many more "static" observables available!
$M_{\pi,K,\eta}$ and $F_{\pi,K,\eta}$

1b How well does the expansion in $p/\Lambda_{\rm QCD}$ work?

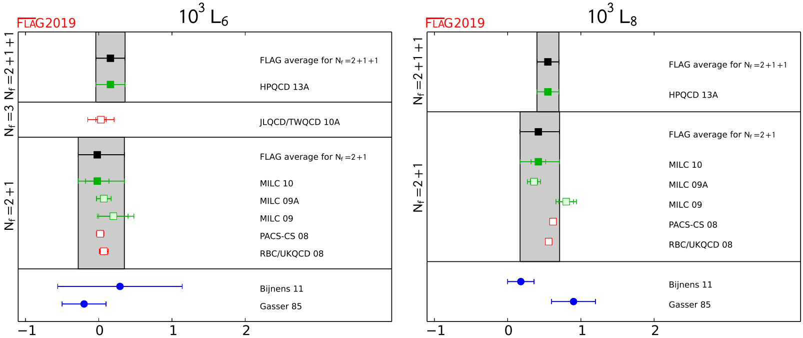
With $p \sim \mathcal{O}(M_K)$ in the $S = 1$ sector

Lattice determination of LECs relevant for M_P and F_P

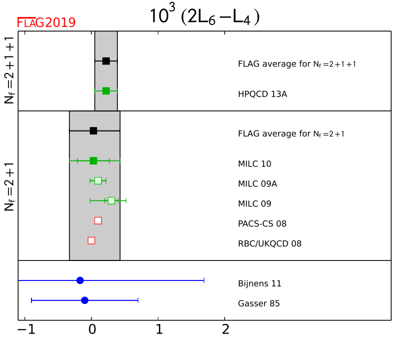


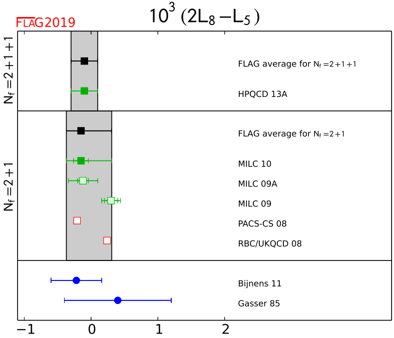


Lattice determination of LECs relevant for M_P and F_P



Lattice determination of LECs relevant for M_P and F_P





Bijnens-Ecker (14) fits

Table 3 Next-to-next-to-leading-order fits[a] for the low-energy constants L_i^r

Fit	BE14	Free fit
$10^3 L_A^r$	0.24(11)	0.68(11)
$10^3 L_1^r$	0.53(06)	0.64(06)
$10^3 L_2^r$	0.81(04)	0.59(04)
$10^3 L_3^r$	−3.07(20)	−2.80(20)
$10^3 L_4^r$	≡0.3	0.76(18)
$10^3 L_5^r$	1.01(06)	0.50(07)
$10^3 L_6^r$	0.14(05)	0.49(25)
$10^3 L_7^r$	−0.34(09)	−0.19(08)
$10^3 L_8^r$	0.47(10)	0.17(11)
χ^2	1.0	0.5
F_0 (MeV)	71	64

[a]The second column contains our preferred fit (BE14) with fixed $L_4^r = 0.3 \times 10^{-3}$; the third column contains the general free fit without any restrictions on the L_i^r. Numerical values are in units of 10^{-3}. No estimate of the error due to higher orders is included.

Bijnens-Ecker (14) fits

Analysis of the convergence of the SU(3) chiral series

$$\frac{F_K}{F_\pi} = 1 + 0.176(0.121) + 0.023(0.077),$$
$$\frac{F_\pi}{F_0} = 1 + 0.208(0.313) + 0.088(0.127),$$
$$\frac{M_\pi^2}{M_{\pi\,\mathrm{phys}}^2} = 1.055(0.937) - 0.005(+0.107) - 0.050(-0.044),$$
$$\frac{M_K^2}{M_{K\,\mathrm{phys}}^2} = 1.112(0.994) - 0.069(+0.022) - 0.043(-0.016),$$
$$\frac{M_\eta^2}{M_{\eta\,\mathrm{phys}}^2} = 1.197(0.938) - 0.214(-0.076) + 0.017(0.014).$$

Bijnens-Ecker (14) fits

The $\pi\pi$ scattering lengths show a very good convergence for both

$$a_0^0 = 0.160 + 0.044(0.046) + 0.012(0.012),$$
$$a_0^2 = -0.0456 + 0.0016(0.0017) - 0.0001(-0.0003).$$

The πK scattering lengths have a worse convergence:

$$a_0^{1/2} = 0.142 + 0.031(0.027) + 0.051(0.057),$$
$$a_0^{3/2} = -0.071 + 0.007(0.005) + 0.016(0.019).$$

Bad convergence of πK scattering lengths due to m_s or to p?

πK scattering

Calculated in χPT up to $\mathcal{O}(p^6)$ Bijnens, Dhonte, Talavera (04)

Roy-Steiner equation solved numerically

Büttiker, Descotes-Genon, Moussallam (04)

Matching below threshold to disentangle m_s from p dependence not yet done work in progress, GC, Ruiz de Elvira, Hermansson-Truedsson

K_{e4} decays

Analogous to πK scattering are K_{e4} decays:

Calculated in χPT up to $\mathcal{O}(p^6)$ — Amoros, Bijnens, Talavera (00)

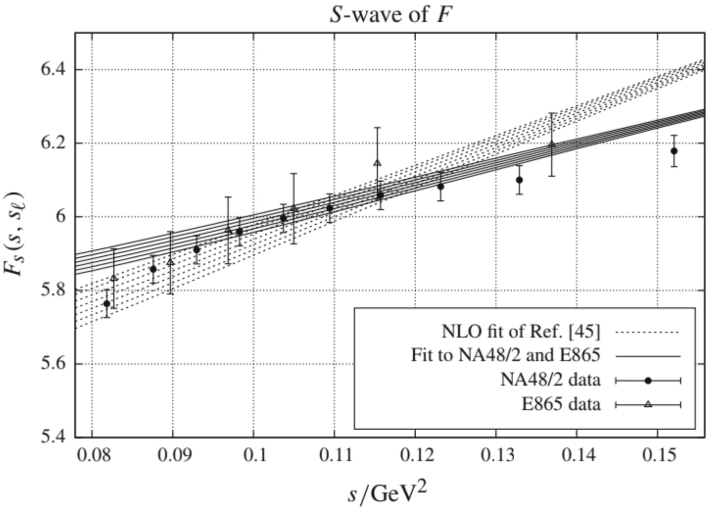
Dispersion relations solved numerically — GC, Passemar, Stoffer (15)

Radiative corrections calculated in χPT — Stoffer (14)

Matching below threshold to disentangle m_s from p dependence also done — GC, Passemar, Stoffer (15)

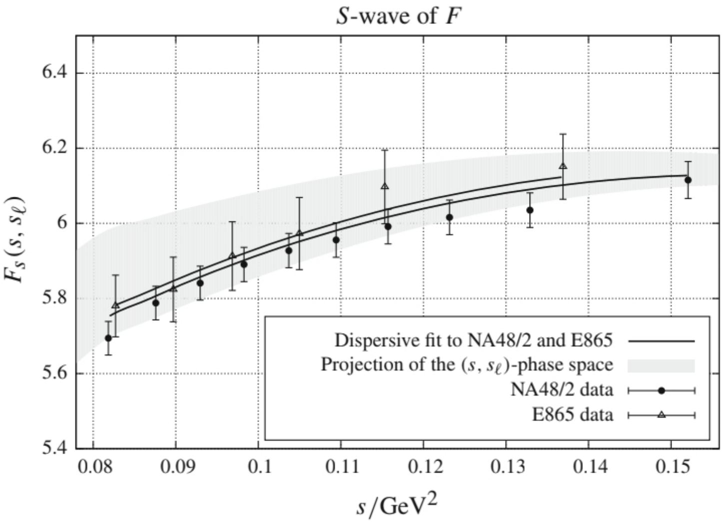
K_{e4} decays: fit results

GC, Passemar, Stoffer (15)



K_{e4} decays: fit results

GC, Passemar, Stoffer (15)



K_{e4} decays: fit results

GC, Passemar, Stoffer (15)

	NA48/2	NA48/2, ~~iso~~	NA48/2 and E865	NA48/2 and E865, ~~iso~~
$10^3 \cdot L_1^r$	0.69 (03)	0.71 (04)	0.62 (03)	0.64 (04)
$10^3 \cdot L_2^r$	1.88 (07)	1.80 (08)	1.79 (06)	1.70 (06)
$10^3 \cdot L_3^r$	−3.89 (13)	−3.93 (14)	−3.62 (11)	−3.60 (12)
$10^3 \cdot L_4^r$	≡0.04	≡0.04	≡0.04	≡0.04
$10^3 \cdot L_5^r$	≡0.84	≡0.84	≡0.84	≡0.84
$10^3 \cdot L_9^r$	≡5.93	≡5.93	≡5.93	≡5.93
χ^2	159.4	67.5	199.9	117.1
dof	27	27	39	39
χ^2/dof	5.9	2.5	5.1	3.0

K_{e4} decays: fit results

GC, Passemar, Stoffer (15)

	Ref. [45]	Ref. [45]	NA48/2	NA48/2 & E865	NA48/2	NA48/2 & E865
C_i^r	$\equiv 0$	BE14	$\equiv 0$	$\equiv 0$	BE14	BE14
$10^3 \cdot L_1^r$	0.67 (06)	0.53 (06)	0.34 (03)	0.28 (02)	0.33 (03)	0.27 (02)
$10^3 \cdot L_2^r$	0.17 (04)	0.81 (04)	0.42 (06)	0.35 (05)	0.95 (06)	0.89 (05)
$10^3 \cdot L_3^r$	−1.76 (21)	−3.07 (20)	−1.54 (14)	−1.25 (11)	−3.06 (14)	−2.80 (11)
$10^3 \cdot L_4^r$	0.73 (10)	$\equiv 0.3$	$\equiv 0.04$	$\equiv 0.04$	$\equiv 0.04$	$\equiv 0.04$
$10^3 \cdot L_5^r$	0.65 (05)	1.01 (06)	$\equiv 0.84$	$\equiv 0.84$	$\equiv 0.84$	$\equiv 0.84$
$10^3 \cdot L_6^r$	0.25 (09)	0.14 (05)	$\equiv 0.07$	$\equiv 0.07$	$\equiv 0.07$	$\equiv 0.07$
$10^3 \cdot L_7^r$	−0.17 (06)	−0.34 (09)	$\equiv -0.34$	$\equiv -0.34$	$\equiv -0.34$	$\equiv -0.34$
$10^3 \cdot L_8^r$	0.22 (08)	0.47 (10)	$\equiv 0.36$	$\equiv 0.36$	$\equiv 0.36$	$\equiv 0.36$
$10^3 \cdot L_9^r$			$\equiv 5.93$	$\equiv 5.93$	$\equiv 5.93$	$\equiv 5.93$
χ^2	26	1.0	81.3	128.7	52.5	91.2
dof	9		27	39	27	39
χ^2/dof	2.9		3.0	3.3	1.9	2.3

K_{e4} decays: fit results

GC, Passemar, Stoffer (15)

Table 7 Matching results for the LECs at NLO and NNLO. The scale is $\mu = 770$ MeV

	NLO	NNLO
$10^3 \cdot L_1^r(\mu)$	0.51 (02) (06)	0.69 (16) (08)
$10^3 \cdot L_2^r(\mu)$	0.89 (05) (07)	0.63 (09) (10)
$10^3 \cdot L_3^r(\mu)$	−2.82 (10) (07)	−2.63 (39) (24)

Nonleptonic and radiative K decays

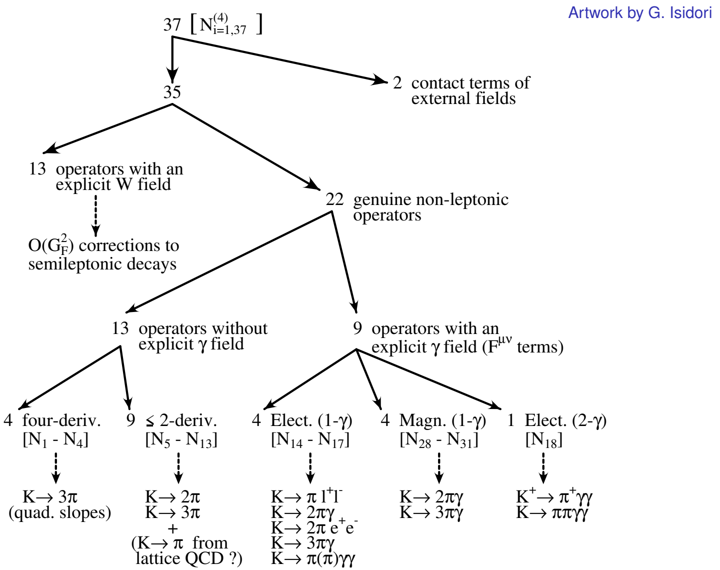
Rich phenomenology

Complex chiral Lagrangian

Kambor, Missimer, Wyler (90)

Ecker, Kambor, Wyler (93)

Nonleptonic and radiative K decays



Nonleptonic and radiative K decays

Rich phenomenology

Complex chiral Lagrangian

Kambor, Missimer, Wyler (90)

Ecker, Kambor, Wyler (93)

Convergence of the chiral series even more difficult to assess

Matching with a dispersive representation would be very interesting and useful

$\rightarrow$ talk by G. D'Ambrosio

Outline

Summary

- A detailed understanding of the SM at low energy is a fascinating challenge and we have the tools to tackle it
- Lattice, χPT and dispersion relations are complementary approaches, all necessary to reach that goal as in SU(2) χPT
- SU(3) χPT: despite huge progress, much remains to be done
- There is a clear physics case for new Kaon experiments
- Moreover, they would provide motivation to bring SU(3) χPT closer to the success level of SU(2) χPT