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Zurich Phenomenology Workshop

Istituto Nazionale di Fisica Nucleare

Kaon decays

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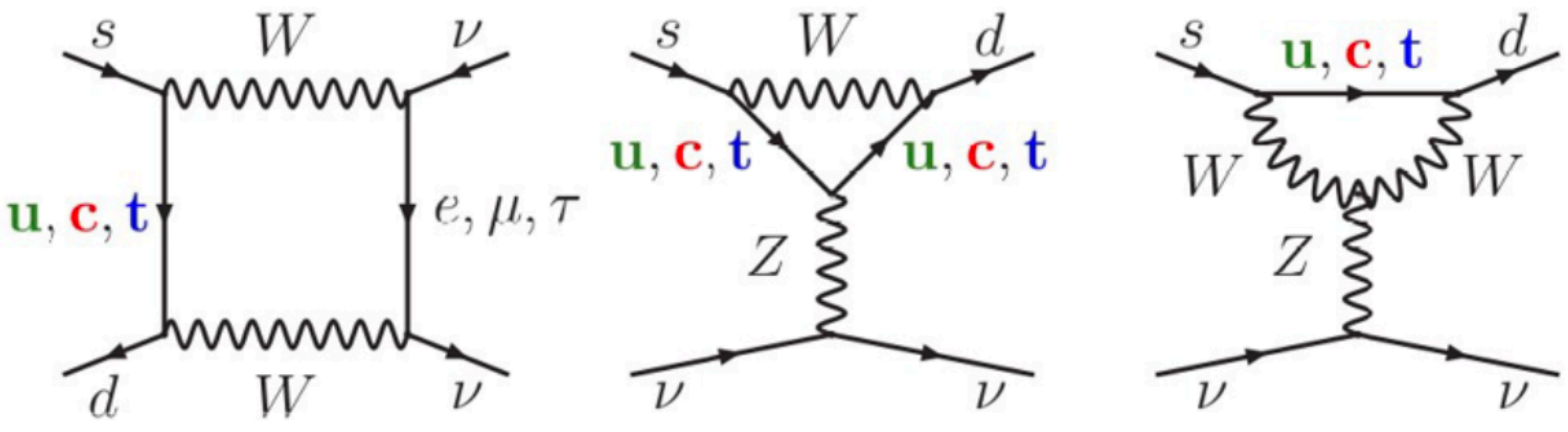
Predictions fofor the rare kaon decays
from QCD in the limit of a large number of colour
With M. Knecht and S.Neshatpour
e-Print: 2409.08568 [hep-ph] MDPI

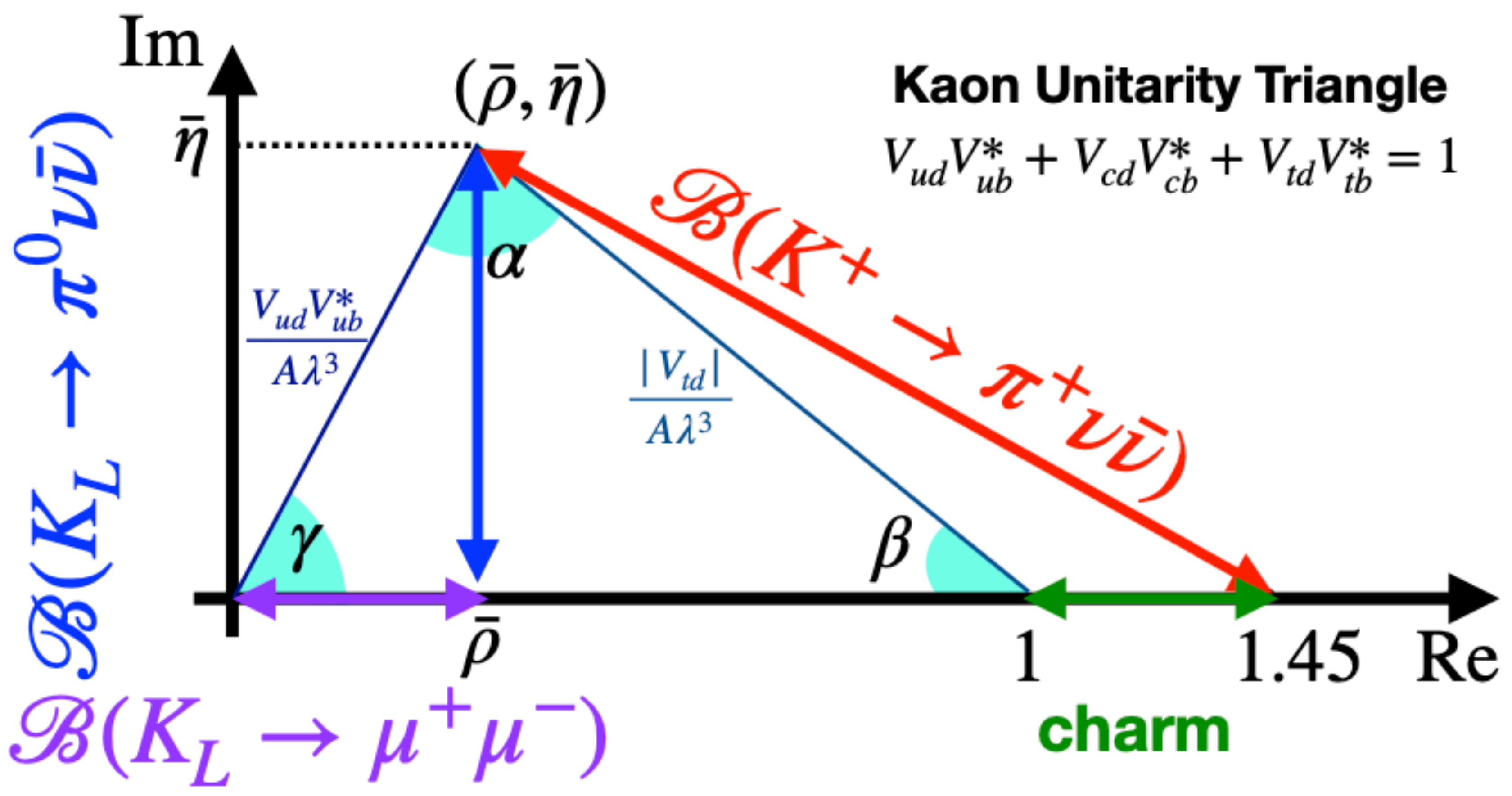
Outline

- K->π $\nu\overline{\nu}$
- Non-leptonic kaon decays
- K->πll
- conclusions

$K \to \pi\nu\bar{\nu}$: Precision test of the Standard Model

SM: Z-penguin & box diagrams





- $\mathscr{B}(K \to \pi\nu\bar{\nu})$ **highly suppressed** in SM
  - GIM mechanism & maximum CKM suppression $s \to d$ transition: $\sim \frac{m_t}{m_W} \left| V^*_{ts} V_{td} \right|$
- Theoretically clean ⇒ high precision SM predictions
  - Dominated by short distance contributions.
  - Hadronic matrix element extracted from $\mathscr{B}(K \to \pi^0 \ell^+ \nu_\ell)$ decays via isospin rotation.

Mode	SM Branching Ratio [1]	SM Branching Ratio [2]	Experimental Status
$K^+ \to \pi^+\nu\bar{\nu}$	$(8.60 \pm 0.42) \times 10^{-11}$	$(7.86 \pm 0.61) \times 10^{-11}$	$(10.6 \pm 4.0) \times 10^{-11}$ NA62 16−18
$K_L \to \pi^0\nu\bar{\nu}$	$(2.94 \pm 0.15) \times 10^{-11}$	$(2.68 \pm 0.30) \times 10^{-11}$	$< 2 \times 10^{-9}$ KOTO (2021 data)

^Recent SM calculations [1:Buras et al. EPJC 82 (2022) 7, 615][2:D'Ambrosio et al. JHEP 09 (2022) 148]
(Differences in SM calculations from choice of CKM parameters: see [Eur.Phys.J.C 84 (2024) 4, 377])

Conclusions

- New study of $K^+ \to \pi^+ \nu\bar{\nu}$ decay using NA62 2021—22 dataset:
  - Improved signal yield per SPS spill by 50%.
  - $N_{bg} = 11.0^{+2.1}_{-1.9}$, $N_{obs} = 31$
  - $\mathscr{B}_{21-22}(K^+ \to \pi^+ \nu\bar{\nu}) = \left(16.0^{+5.0}_{-4.5}\right) \times 10^{-11} = \left(16.0 \left({}^{+4.8}_{-4.2}\right)_{\text{stat}} \left[{}^{+1.4}_{-1.3}\right]_{\text{syst}}\right) \times 10^{-11}$
- Combining with 2016—18 data for full 2016—22 results:
  - $N_{bg} = 18^{+3}_{-2}$, $N_{obs} = 51$ (using 9+6 categories for BR extraction)
  - $\mathscr{B}_{16-22}(K^+ \to \pi^+ \nu\bar{\nu}) = \left(13.0^{+3.3}_{-2.9}\right) \times 10^{-11} = \left(13.0 \left({}^{+3.0}_{-2.7}\right)_{\text{stat}} \left[{}^{+1.3}_{-1.2}\right]_{\text{syst}}\right) \times 10^{-11}$
  - Background-only hypothesis rejected with significance Z>5.
- **First observation of $K^+ \to \pi^+ \nu\bar{\nu}$ decay: BR consistent with SM prediction within 1.7σ**
  - **Need full NA62 data-set to clarify SM agreement or tension.**

2023—LS3 data-set collection & analysis in progress...

QCD at work: Short Distance expansion for weak interaction

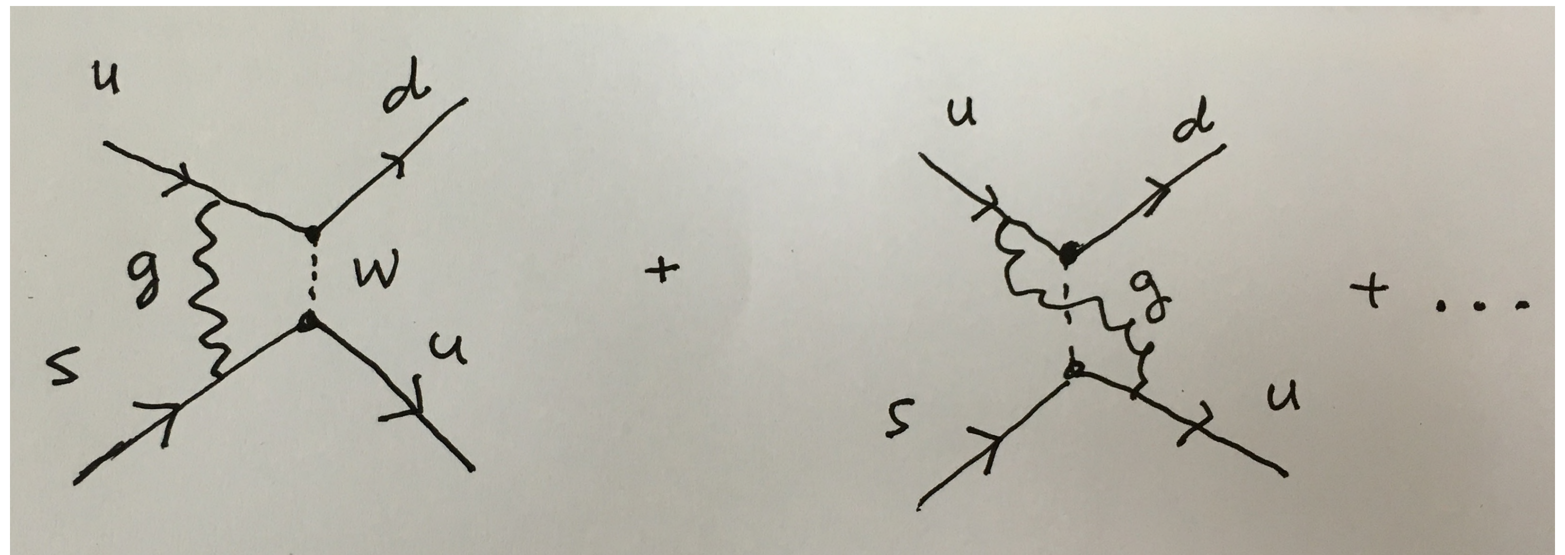
- Fermi lagrangian: description of the ΔS=1 weak lagrangian, in particular the explanation of ΔI =1/2 rule

$$\frac{A(K^+ \to \pi^+\pi^0)}{A(K_S \to \pi^+\pi^-)} \sim \frac{1}{22}$$

- Wilson suggestion (Feynman) , short distance expansion

$$-\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* C_- (\overline{s}_L \gamma^\mu u_L)(\overline{u}_L \gamma_\mu d_L)$$

- Gaillard Lee, Altarelli Maiani: right direction but not fully understood (Long distance?)



$K \to 2\pi$ Isospin Amplitudes

$$A[K^0 \to \pi^+\pi^-] \equiv A_0\, e^{i\chi_0} + \frac{1}{\sqrt{2}} A_2\, e^{i\chi_2}$$

$$A[K^0 \to \pi^0\pi^0] \equiv A_0\, e^{i\chi_0} - \sqrt{2}\, A_2\, e^{i\chi_2}$$

$$A[K^+ \to \pi^+\pi^0] \equiv \frac{3}{2} A_2^+\, e^{i\chi_2^+}$$

$$A_0\, e^{i\chi_0} = \mathcal{A}_{1/2}$$

$$A_2\, e^{i\chi_2} = \mathcal{A}_{3/2} + \mathcal{A}_{5/2}$$

$$A_2^+\, e^{i\chi_2^+} = \mathcal{A}_{3/2} - \frac{2}{3} \mathcal{A}_{5/2}$$

1) $\Delta I = 1/2$ Rule: $\omega \equiv \frac{\mathrm{Re}\,(A_2)}{\mathrm{Re}\,(A_0)} \approx \frac{1}{22}$

2) Strong Final State Interactions: $\chi_0 - \chi_2 \approx \delta_0 - \delta_2 \approx 45^\circ$

$\mathbf{O(p^2)} \quad \chi\mathbf{PT}$

$$\mathcal{L}_2^{\Delta S=1} = G_8\, F^4\, \langle \lambda L_\mu L^\mu \rangle \; + \; G_{27}\, F^4 \left(L_{\mu 23}\, L^\mu_{11} + \frac{2}{3}\, L_{\mu 21}\, L^\mu_{13} \right)$$

$$G_R \equiv -\frac{G_F}{\sqrt{2}}\, V_{ud} V^*_{us}\; g_R \quad ; \quad L_\mu = -iU^\dagger D_\mu U \quad ; \quad \lambda \equiv \tfrac{1}{2}\,\lambda_{6-i7} \quad ; \quad U \equiv \exp\left\{ i\sqrt{2}\,\Phi/F \right\}$$

$$\mathcal{A}_{1/2} = \sqrt{2}\, F_\pi \left(G_8 + \frac{1}{9}\, G_{27} \right) (M_K^2 - M_\pi^2)$$

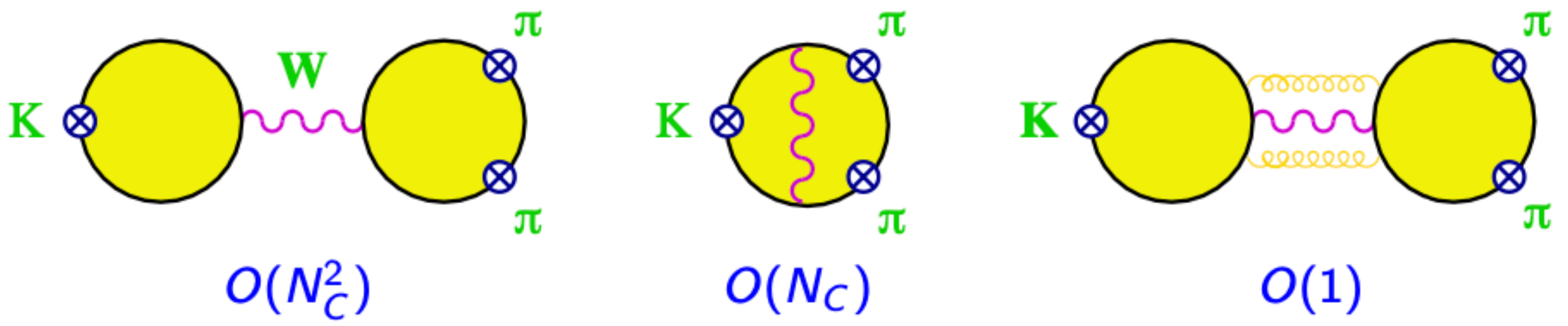
$$\mathcal{A}_{3/2} = \frac{10}{9}\, F_\pi\; G_{27}\, (M_K^2 - M_\pi^2)$$

$$\mathcal{A}_{5/2} = 0 \qquad ; \qquad \delta_0 = \delta_2 = 0$$

$$[\Gamma(K \to 2\pi) + \delta_I]_{\rm Exp} \quad \Longrightarrow \quad |g_8| \approx 5.1 \quad ; \quad |g_{27}| \approx 0.29$$

Weak Currents Factorize at Large N_C

Bardeen Buras Gerard '90



$$A[K^0 \to \pi^0\pi^0] = 0 \quad \Rightarrow \quad A_0 = \sqrt{2}\, A_2$$

No $\Delta I = \frac{1}{2}$ enhancement at leading order in $1/N_C$

- **Multiscale problem:** **OPE** $\frac{1}{N_C} \log\left(\frac{M_W}{\mu}\right) \sim \frac{1}{3} \times 4$

Short-distance logarithms must be summed

- **Large χPT logarithms:** **FSI** $\frac{1}{N_C} \log\left(\frac{\mu}{M_\pi}\right) \sim \frac{1}{3} \times 2$

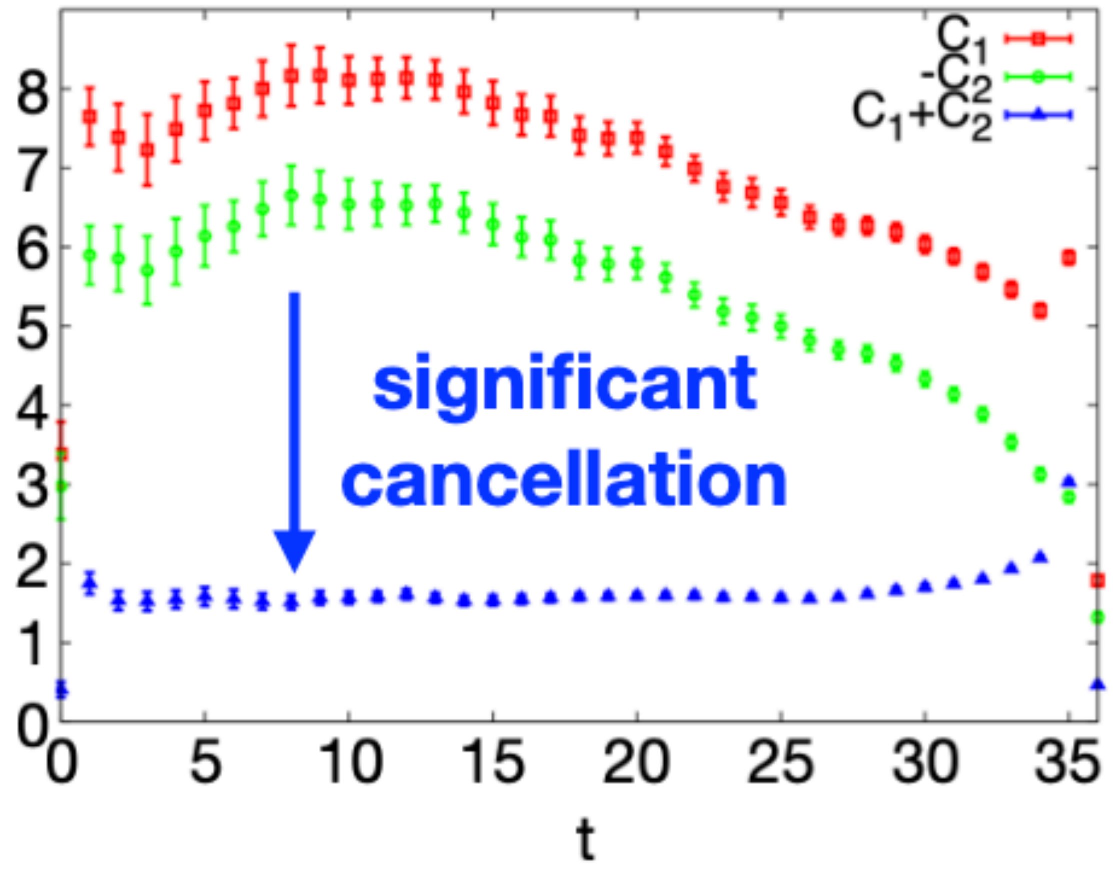
Infrared logarithms must also be included $[\delta_I \sim O(1/N_C)\, ,\ \delta_0 - \delta_2 \approx 45^\circ]$

The ΔI = 1/2 rule

- Experimental fact

$$\frac{\text{Re}\,A_0}{\text{Re}\,A_2} = 22.45(6)$$: large suppression of $\Delta I = 3/2$ (A_2) mode

- Significant suppression of $\text{Re}A_2$ (2012/2015)
  - C_1, C_2 contributions of different color structure to $K \to \pi\pi$ correlation function most significant to $\text{Re}A_2$
  - Naïvely $C_1 = -3C_2$ based on color counting
  - Significant cancellation at physical m_π observed



RBC/UKQCD, PRD91,074502 (2015)

- Lattice confirmation of the ΔI = 1/2 rule with the result for A_0 (2020)

$$\frac{\text{Re}A_0}{\text{Re}A_2} = 19.9(2.3)_{\text{stat}}(4.4)_{\text{sys}}$$

Some Daniel's papers

Kambor Missimer Wyler

- Quantify Long Distance Contributions and weak chiral lagrangian : classify the the O(p4) conterterm structures and K->2π/3π fit

Weak interaction

The symmetry of the short distance hamiltonian $-\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* C_- (\overline{s}_L \gamma^\mu u_L)(\overline{u}_L \gamma_\mu d_L)$

described in CHPT

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}^2_{\Delta S=1} + \mathcal{L}^4_{\Delta S=1} + \cdots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K\to 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+\to\pi^+\gamma\gamma, K\to\pi l^+ l^-} + \cdots$$

VMD not as successful, in particular for K-3pi, where in principle large VMD important

K->2 pi/3pi fit

Kambor Missimer Wyler, '90s

$$\mathcal{M}(K_L \to \pi^+\pi^-\pi^0) = \alpha_1 - \beta_1 u + (\zeta_1 + \xi_1)u^2 + \frac{1}{3}(\zeta_1 - \xi_1)v^2$$

$$\mathcal{M}(K_L \to \pi^0\pi^0\pi^0) = -3\alpha_1 - \zeta_1(3u^2 + v^2) \,,$$

$$\mathcal{M}(K^+ \to \pi^+\pi^+\pi^-) = 2\alpha_1 + \beta_1 u + (2\zeta_1 - \xi_1)u^2 + \frac{1}{3}(2\zeta_1 + \xi_1)v^2 \,,$$

$$\mathcal{M}(K^+ \to \pi^+\pi^0\pi^0) = -\alpha_1 + \beta_1 u - (\zeta_1 + \xi_1)u^2 - \frac{1}{3}(\zeta_1 - \xi_1)v^2 \,,$$

$$\alpha_1 = \alpha_1^{(0)} - \frac{2g_8}{27 f_K f_\pi} m_K^4 \{(k_1 - k_2) + 24\mathcal{L}_1\} \,,$$

$$\beta_1 = \beta_1^{(0)} - \frac{g_8}{9 f_K f_\pi} m_\pi^2 m_K^2 \{(k_3 - 2k_1) - 24\mathcal{L}_2\} \,,$$

$$\zeta_1 = -\frac{g_8}{6 f_K f_\pi} m_\pi^4 \{k_2 - 24\mathcal{L}_1\} \,,$$

$$\xi_1 = -\frac{g_8}{6 f_K f_\pi} m_\pi^4 \{k_3 - 24\mathcal{L}_2\} \,,$$

Table 1

The values of the amplitudes in eqs. (4) and (5) obtained from fits to experiment are shown in the first two columns. Our value of $\delta_2-\delta_0$ is obtained from $K\to2\pi$ decays alone, while some additional constraints were used in ref. [8]. The $K\to3\pi$ amplitudes $\alpha_1, ..., \xi_3'$ are in units of 10^{-8}. The results of lowest and next-to-lowest order chiral perturbation theory are displayed in the two columns to the right.

	Devlin and Dickey	Our fit	Lowest order	Order p^4
$a_{1/2}$ [keV]	0.4687 ± 0.0006	0.4699 ± 0.0012	0.4698	0.4698
$a_{3/2}$ [keV]	0.0210 ± 0.0001	0.0211 ± 0.0001	0.0211	0.0211
$\delta_2-\delta_0$ (deg)	-45.6 ± 5	-61.5 ± 4	0	-29
α_1	91.46 ± 0.24	91.71 ± 0.32	74.0	91.8
α_3	-7.14 ± 0.36	-7.36 ± 0.47	-4.1	-7.6
β_1	-25.83 ± 0.41	-25.68 ± 0.27	-16.5	-25.6
β_3	-2.48 ± 0.48	-2.43 ± 0.41	-1.0	-2.5
γ_3	2.51 ± 0.36	2.26 ± 0.23	1.8	2.5
ζ_1	-0.37 ± 0.11	-0.47 ± 0.15	–	-0.6
ζ_3	–	-0.21 ± 0.08	–	-0.02
ξ_1	-1.25 ± 0.12	-1.51 ± 0.30	–	-1.5
ξ_3	–	-0.12 ± 0.17	–	-0.05
ξ_3'	–	-0.21 ± 0.51	–	-0.08
χ^2/DOF	12.8/3	10.3/2	4121/5	37/13

K->3pi fit/K-lifetimes to be remeasured

arxiv 2209.02143

G.D. Knecht Neshtapour

amplitude coefficient	Devlin et *al.* (Ref. [3])	Kambor et *al.* (Ref. [4])	Bijnens et *al.* (Ref. [5])	Our scaled fit	SF
α_1	91.4 ± 0.24	91.71 ± 0.32	93.16 ± 0.36	92.80 ± 0.64	2.9
α_3	-7.14 ± 0.36	-7.36 ± 0.47	-6.72 ± 0.46	-7.45 ± 0.79	3.2
β_1	-25.83 ± 0.41	-25.68 ± 0.27	-27.06 ± 0.43	-26.46 ± 0.22	1.6
β_3	-2.48 ± 0.48	-2.43 ± 0.41	-2.22 ± 0.47	-2.50 ± 0.29	1.6
γ_3	2.51 ± 0.36	2.26 ± 0.23	2.95 ± 0.32	2.78 ± 0.10	1.0
ζ_1	-0.37 ± 0.11	-0.47 ± 0.15	-0.40 ± 0.19	-0.11 ± 0.03	1.7
ζ_3	—	-0.21 ± 0.08	-0.09 ± 0.10	-0.05 ± 0.03	1.8
ξ_1	-1.25 ± 0.12	-1.51 ± 0.30	-1.83 ± 0.30	-1.20 ± 0.13	1.7
ξ_3	—	-0.12 ± 0.17	-0.17 ± 0.16	0.10 ± 0.10	1.6
ξ'_3	—	-0.21 ± 0.51	-0.56 ± 0.42	-0.07 ± 0.16	1.8
χ^2/dof	12.8/3	10.3/2	5.4/5	5.18/5 (30.66/5)	

Needed also for various SM Kaon assessments

Vector Meson Dominance in the strong sector

Ecker, Gasser, de Rafael, Pich

L_i	L_i expts	V	A	Total (Scalar incl.)	Total QCD rel. incl.
L_1	0.4 ± 0.3	0,6	0	0,6	0,9
L_2	1.4 ± 0.3	1,2	0	1,2	1,8
L_3	-3.5 ± 1.1	-3,6	0	-3,0	-4,9
L_4	-0.3 ± 0.5	0	0	0	0
L_5	1.4 ± 0.5	0	0	1,4	1,4
L_6	-0.2 ± 0.3	0	0	0	0
L_7	-0.4 ± 0.2	0	0	-0,3	-0,3
L_8	0.9 ± 0.3	0	0	0,9	0,9
L_9	6.9 ± 0.7	6,9	0	6,9	7,3
L_{10}	-5.5 ± 0.7	-10	4	-6,0	-5,5

QCD inspired relations relations

$$F_V = 2G_V = \sqrt{2} f_\pi$$

$$F_A = f_\pi$$

$$M_A = \sqrt{2} M_V$$

KSFR: $G_V = \sqrt{2}\ F_\pi$ determined by dominance of pion, V,A to recover QCD short distance constraints

$$L_1^V = \frac{L_2^V}{2} = -\frac{L_3^V}{6} = \frac{G_V^2}{8M_V^2}, \qquad L_9^V = \frac{F_V G_V}{2M_V^2}, \qquad L_{10}^{V+A} = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}$$

QCD inspired relations relations

$$L_1^V = L_2^V/2 = -L_3^V/6 = L_9^V/8 = -L_{10}^{V+A}/6 = f_\pi^2/(16M_V^2)$$

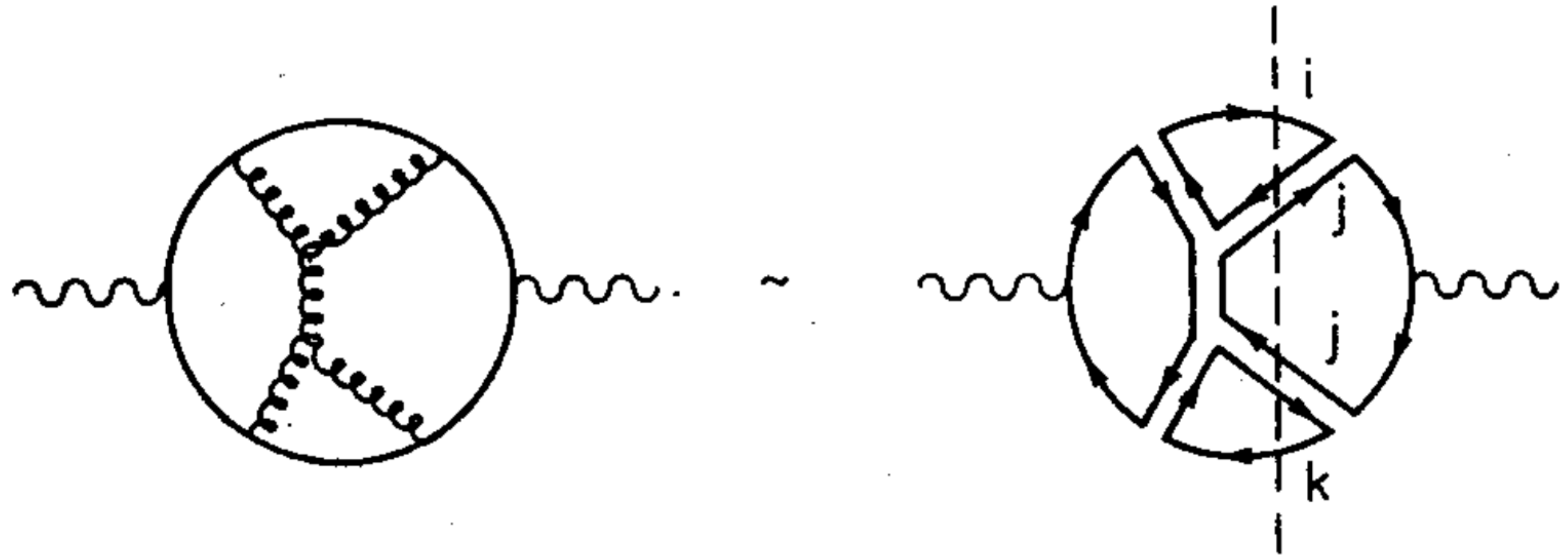


Fig. 14. A typic al contribution to the two-point function in the large-N limit. A cut reveals the intermediate states

$$\langle J(q)J(-q)\rangle = \sum_n \frac{a_n^2}{q^2 - m_n^2} .$$

$$\langle J(q)J(-q)\rangle \underset{q_2\to\infty}{\simeq} \log q^2$$

Not only a bookkeeping but predictive already

π	2π	3π	N_i
$\pi^+\gamma^*$	$\pi^+\pi^0\gamma^*$		$N_{14}^r - N_{15}^r$
$\pi^0\gamma^*\ (S)$	$\pi^0\pi^0\gamma^*\ (L)$		$2N_{14}^r + N_{15}^r$
$\pi^+\gamma\gamma$	$\pi^+\pi^0\gamma\gamma$		$N_{14} - N_{15} - 2N_{18}$
	$\pi^+\pi^-\gamma\gamma\ (S)$		”
	$\pi^+\pi^0\gamma$	$\pi^+\pi^+\pi^-\gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$
	$\pi^+\pi^-\gamma\ (S)$	$\pi^+\pi^0\pi^0\gamma$	”
		$\pi^+\pi^-\pi^0\gamma\ (L)$	”
		$\pi^+\pi^-\pi^0\gamma\ (S)$	$7(N_{14}^r - N_{16}^r) + 5(N_{15}^r + N_{17})$
	$\pi^+\pi^-\gamma^*\ (L)$		$N_{14}^r - N_{15}^r - 3(N_{16}^r - N_{17})$
	$\pi^+\pi^-\gamma^*\ (S)$		$N_{14}^r - N_{15}^r - 3(N_{16}^r + N_{17})$
	$\pi^+\pi^0\gamma^*$		$N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17})$
	$\pi^+\pi^-\gamma\ (L)$	$\pi^+\pi^-\pi^0\gamma\ (S)$	$N_{29} + N_{31}$
		$\pi^+\pi^+\pi^-\gamma$	”
	$\pi^+\pi^0\gamma$	$\pi^+\pi^0\pi^0\gamma$	$3N_{29} - N_{30}$
		$\pi^+\pi^-\pi^0\gamma\ (S)$	$5N_{29} - N_{30} + 2N_{31}$
		$\pi^+\pi^-\pi^0\gamma\ (L)$	$6N_{28} + 3N_{29} - 5N_{30}$

$K^+ \to \pi^+ l^+ l^-$

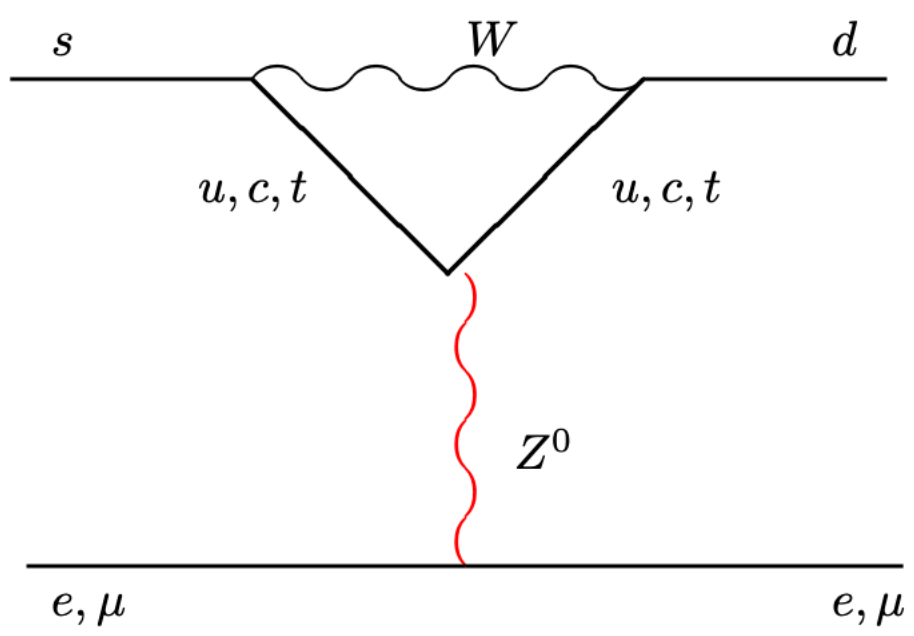
$K_S \to \pi^0 l^+ l^-$

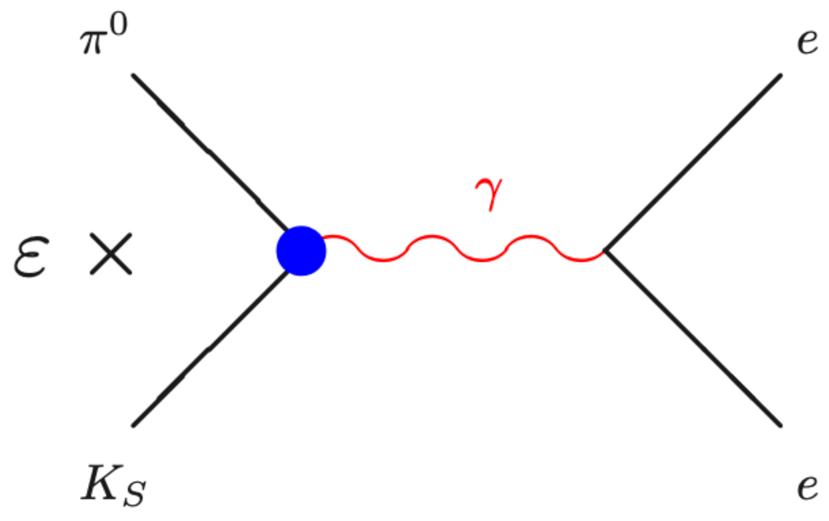
Vectors and axials

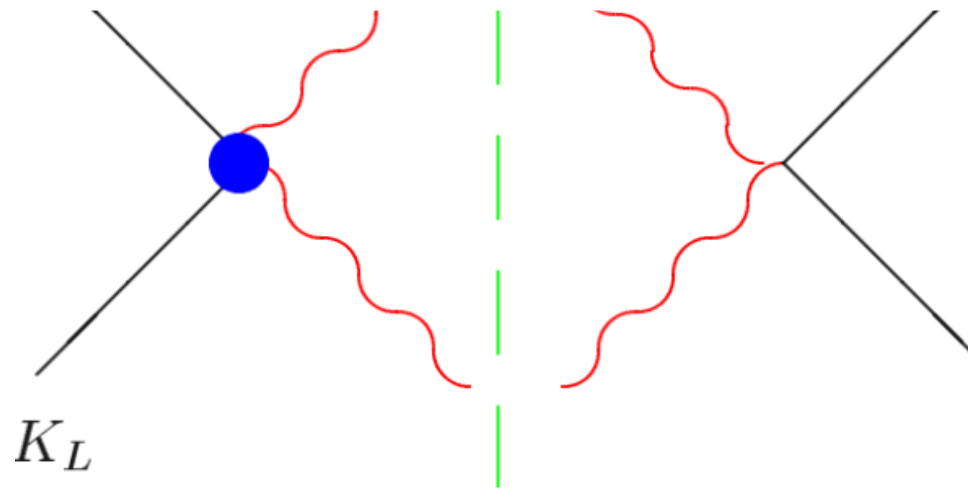
Counterterm combination	Processes	VMD weak coupling
$N_{14}^r - N_{15}^r$	$K^+ \to \pi^+ \gamma^*$ $K^+ \to \pi^+ \pi^0 \gamma^*$	$-0.020\,\eta_V\ + 0.004\,\eta_A$
$2N_{14}^r + N_{15}^r$	$K_S \to \pi^0 \gamma^*$	$0.08\,\eta_V$
$N_{14} - N_{15} - 2N_{18}$	$K^+ \to \pi^+ \gamma\gamma$ $K^+ \to \pi^+ \pi^0 \gamma\gamma$	$-0.01\,\eta_A$
$N_{14} - N_{15} - N_{16} - N_{17}$	$K^+ \to \pi^+ \pi^0 \gamma$ $K_S \to \pi^+ \pi^- \gamma$	$-\,0.010\,\eta_A$
$N_{14}^r - N_{15}^r - 3(N_{16}^r - N_{17})$	$K_L \to \pi^+ \pi^- \gamma^*$	$-0.004\,\eta_V\ + 0.018\,\eta_A$
$N_{14}^r - N_{15}^r - 3(N_{16}^r + N_{17})$	$K_S \to \pi^+ \pi^- \gamma^*$	$0.05\,\eta_V\ - 0.04\,\eta_A$
$N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17})$	$K^+ \to \pi^+ \pi^0 \gamma^*$	$0.12\,\eta_V\ + 0.01\,\eta_A$
$N_{29} + N_{31}$	$K_L \to \pi^+ \pi^- \gamma$	$0.005\,\eta_V\ + 0.003\,\eta_A$
$3N_{29} - N_{30}$	$K^+ \to \pi^+ \pi^0 \gamma$	$-0.005\,\eta_V\ - 0.003\,\eta_A$

Observation hidden by other effects: different analysis maybe useful (Kaon charge radius)

NA48 has a good chance







$$\mathrm{BR}(K_L \to \pi^0 \ell\bar{\ell}) = \left(C^\ell_{\mathrm{dir}} \pm C^\ell_{\mathrm{int}}|a_S| + C^\ell_{\mathrm{mix}}|a_S|^2 + C^\ell_{\gamma\gamma}\right) \cdot 10^{-12}$$

$$|a_S| = 1.20 \pm 0.20$$

	C^ℓ_{dir}	C^ℓ_{int}	C^ℓ_{mix}	$C^\ell_{\gamma\gamma}$
$\ell = e$	$(4.62 \pm 0.24)(w_{7V}^2 + w_{7A}^2)$	$(11.3 \pm 0.3)w_{7V}$	14.5 ± 0.5	≈ 0
$\ell = \mu$	$(1.09 \pm 0.05)(w_{7V}^2 + 2.32 w_{7A}^2)$	$(2.63 \pm 0.06)w_{7V}$	3.36 ± 0.20	5.2 ± 1.6

Isidori Mescia, Smith

Mescia, Smith, Trine

$$\mathrm{BR}^{\mathrm{SM}}(K_L \to \pi^0 e\bar{e}) = 3.46^{+0.92}_{-0.80}\,\left(1.55^{+0.60}_{-0.48}\right) \times 10^{-11}$$

$$\mathrm{BR}^{\mathrm{SM}}(K_L \to \pi^0 \mu\bar{\mu}) = 1.38^{+0.27}_{-0.25}\,\left(0.94^{+0.21}_{-0.20}\right) \times 10^{-11}$$

$$w_{7A,7V} = \mathrm{Im}\left(\lambda_t y_{7A,7V}\right) / \,\mathrm{Im}\,\lambda_t$$

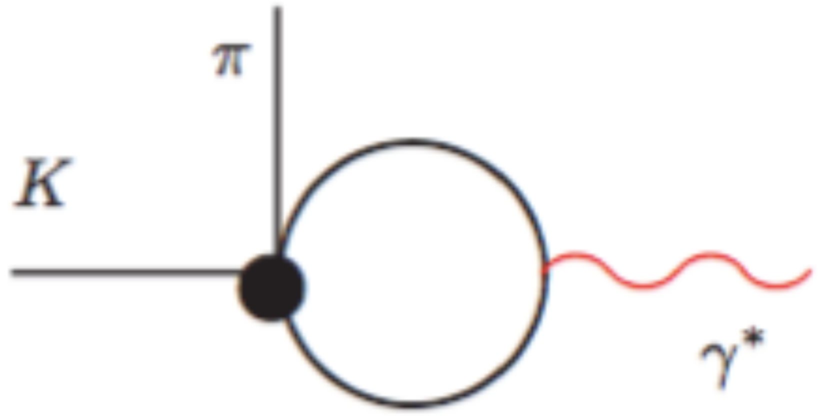
Ecker, Pich, de Rafael

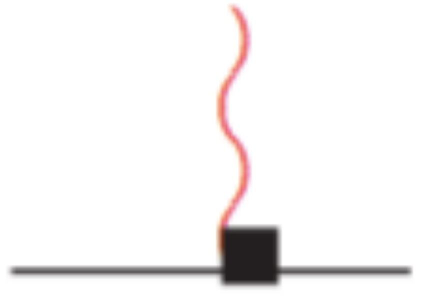
$\mathcal{O}(p^4)$ CHPT :

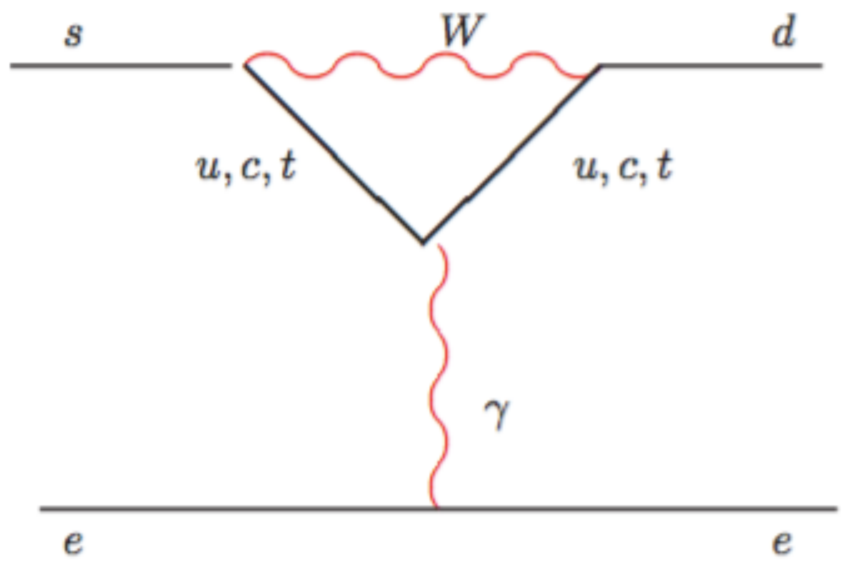
$$K^\pm \to \pi^\pm l\bar{l} \quad K_S \to \pi^0 l\bar{l}$$

$\mathcal{O}(p^4)$

loops +CT







Short distance

electrons and μ's in the final state

observables

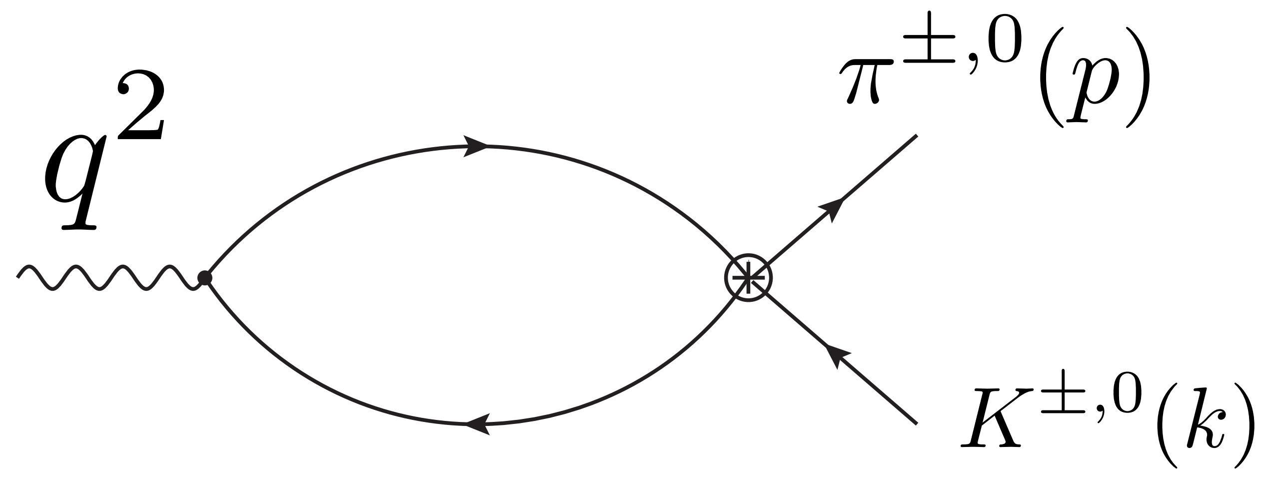
$$\left\{ \begin{array}{l} R = \dfrac{\Gamma(K^+ \to \pi^+\mu^+\mu^-)}{\Gamma(K^+ \to \pi^+e^+e^-)} \\ \text{lepton spectrum} \quad z = \dfrac{q^2}{M_K^2} \end{array} \right.$$

'97 Initial data inconsistency e and μ's: LFUV?

GD,Ecker,Isidori,Portoles

General consideration on the form factor

Lorentz and gauge invariance tell us on the structure of the amplitude and ff



$$z = \frac{q^2}{M_K^2}$$

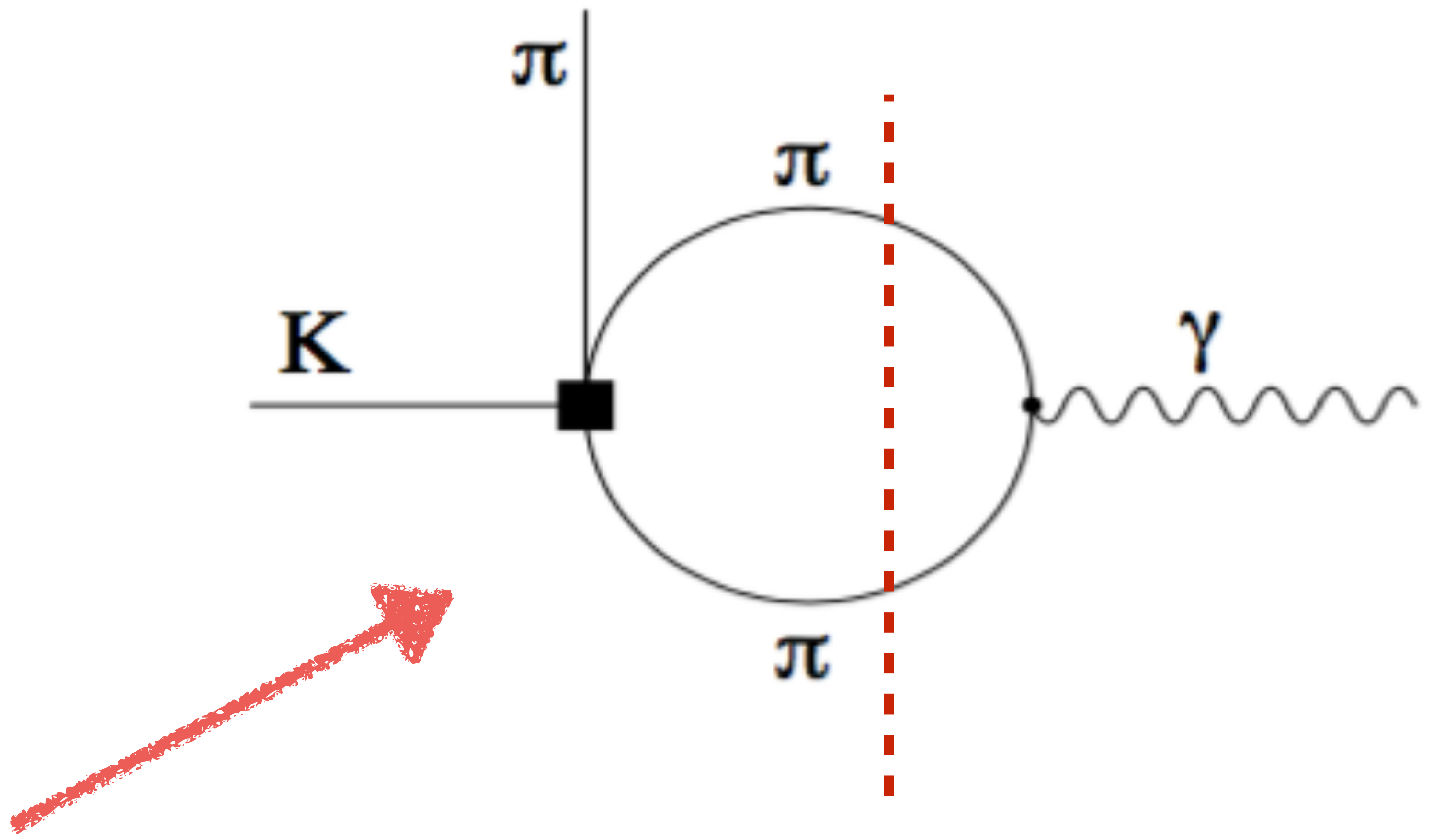
$$i \int d^4x e^{iqx} \langle \pi(p) | T \left\{ J^\mu_{\rm elm}(x) \mathcal{L}_{\Delta S=1}(0) \right\} | K(k) \rangle = \frac{W(z)}{(4\pi)^2} \left[z(k+p)^\mu - (1 - r_\pi^2) q^\mu \right]$$

$$W_i(z) = G_F M_K^2 W_i^{\rm pol}(z) + W_i^{\pi\pi}(z)$$

GD,Ecker,Isidori,Portoles

$$W_i^{\rm pol}(z) = a_i + b_i z \qquad (i = +, S)$$

$$W_i^{\pi\pi}(z)$$



$$A(K^+ \to \pi^+\pi^+\pi^-) = \alpha_0 + \alpha_+ Y + \gamma(Y^2 + X^2/3) + \beta_+(Y^2 - X^2/3) \,,$$
$$A(K_S \to \pi^+\pi^-\pi^0) = b_2 X - d_2 XY \,,$$

$$s_i = (k - p_i)^2 \,, \qquad s_0 = \frac{1}{3}(s_1 + s_2 + s_3) \,, \qquad X = \frac{s_1 - s_2}{M_\pi^2} \,, \qquad Y = \frac{s_3 - s_0}{M_\pi^2} \,,$$

$$W_i(z) = G_F M_K^2 W_i^{\rm pol}(z) + W_i^{\pi\pi}(z)$$

$$W_i^{\rm pol}(z) = a_i + b_i z \qquad (i = +, S)$$

$$W_i^{\pi\pi}(z) = \frac{1}{r_\pi^2}\left[\alpha_i + \beta_i \frac{z - z_0}{r_\pi^2}\right] F(z)\,\chi(z)$$

$$z_0 = 1/3 + r_\pi^2$$

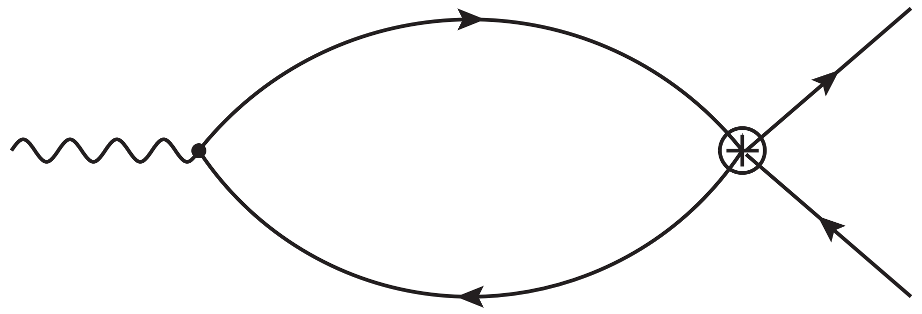
$$\chi(z) = \frac{4}{9} - \frac{4r_\pi^2}{3z} - \frac{1}{3}(1 - \frac{4r_\pi^2}{z})G(z/r_\pi^2)$$

$$F(z) = 1 + z/r_V^2$$

$K \to 3\pi$ slopes

$$A(K^+ \to \pi^+\pi^+\pi^-) = \alpha_0 + \alpha_+ Y + \gamma(Y^2 + X^2/3) + \beta_+(Y^2 - X^2/3)\,,$$

$$A(K_S \to \pi^+\pi^-\pi^0) = b_2 X - d_2 XY\,,$$



$K \to \pi\gamma^*$: Experimental situation

exp.	mode	number of events	a_+	b_+
BNL-E865	$K^+ \to \pi^+ e^+ e^-$	$10\,300$	$-0.587(10)$	$-0.655(44)$
NA48/2	$K^\pm \to \pi^\pm e^+ e^-$	$7\,253$	$-0.578(16)$	$-0.779(66)$
NA48/2	$K^\pm \to \pi^\pm \mu^+ \mu^-$	3120	$-0.575(39)$	$-0.813(145)$
NA62	$K^+ \to \pi^+ \mu^+ \mu^-$	27679	$-0.575(13)$	$-0.722(43)$

E. Cortina Gil et al. [NA62 Collaboration], JHEP 11, 011 (2022)

exp.	mode	number of events
NA48/1	$K_S \to \pi^0 e^+ e^-$	7
NA48/1	$K_S \to \pi^0 \mu^+ \mu^-$	6

$$a_S = -1.29(3.15) \quad b_S = +17.8(10.6)$$

or

$$a_S = +1.28(3.16) \quad b_S = -17.6(10.6)$$

G. D'Ambrosio, D. Greynat, MK, JHEP 02, 049 (2019)

- Neither the sign of a_S nor the sign of a_S/b_S are fixed by data

Calculation in large N QCD of the form factor for K->pi ll

PLAN A $K_S \to \pi^0 l^+ l^-$

$$i \int d^4x e^{iqx} \langle \pi(p) | T \left\{ J^{\mu}_{\rm elm}(x) \mathcal{L}_{\Delta S=1}(0) \right\} | K(k) \rangle = \frac{W(z)}{(4\pi)^2} \left[z(k+p)^{\mu} - (1-r_{\pi}^2) q^{\mu} \right]$$

$$k^2 = M_K^2 \,, \quad p^2 = M_\pi^2 \,, \quad q = k - p \,, \quad z = q^2/M_K^2 \,, \quad r_\pi = M_\pi/M_K \,,$$

$$i \int d^4x e^{iqx} \langle \pi(p) | T \left\{ J^{\mu}_{\text{elm}}(x) \mathcal{L}_{\Delta S=1}(0) \right\} | K(k) \rangle = \frac{W(z)}{(4\pi)^2} \left[z(k+p)^{\mu} - (1 - r_{\pi}^2) q^{\mu} \right]$$

$$k^2 = M_K^2 \ , \quad p^2 = M_\pi^2 \ , \quad q = k - p \ , \quad z = q^2/M_K^2 \ , \quad r_\pi = M_\pi / M_K \ ,$$

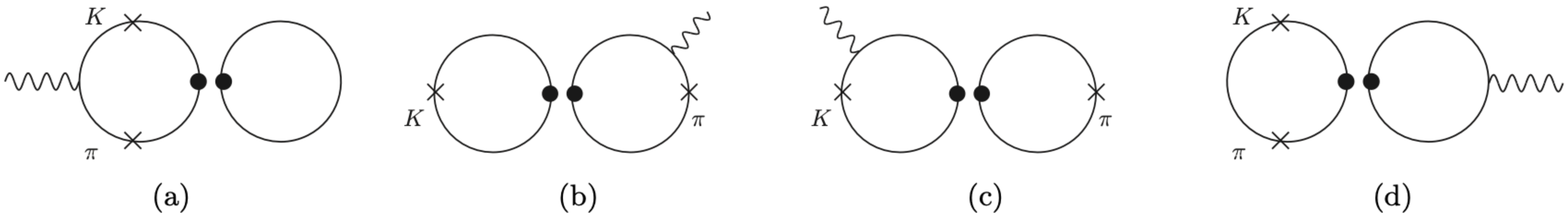
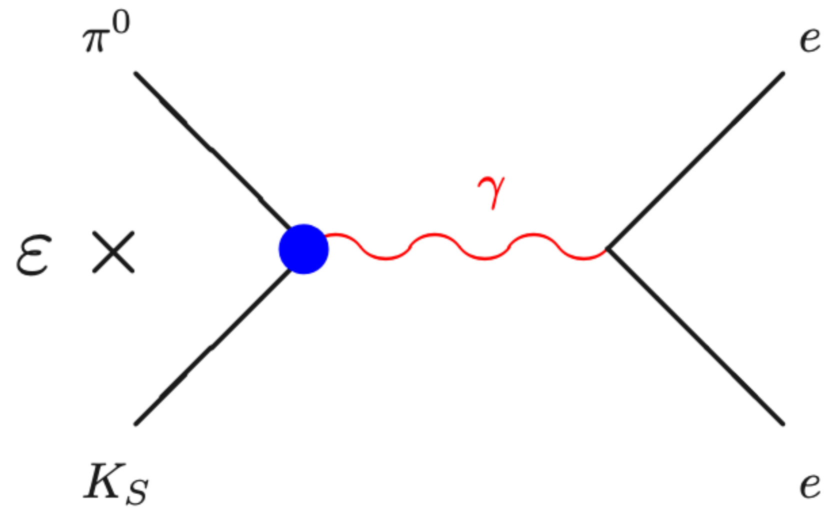
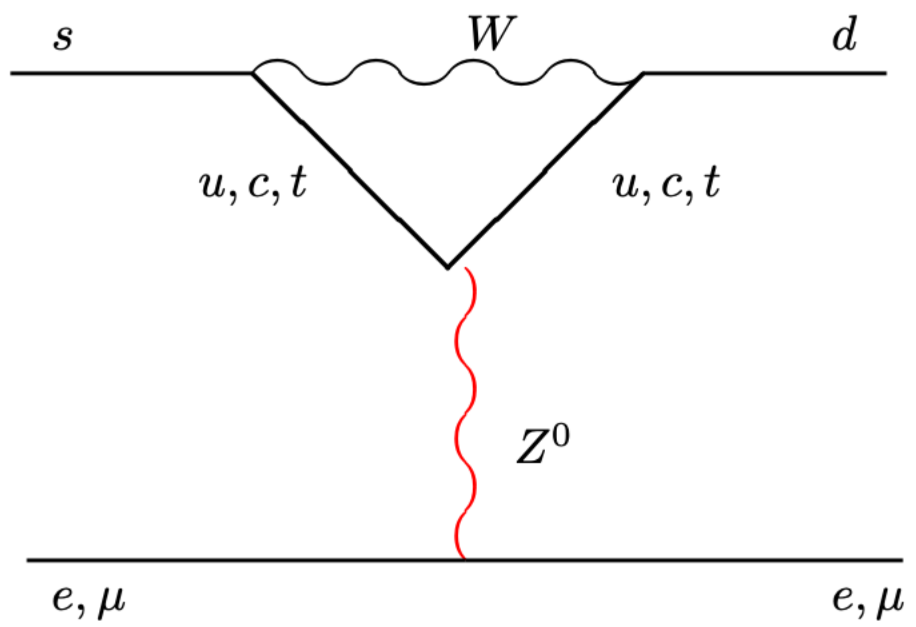
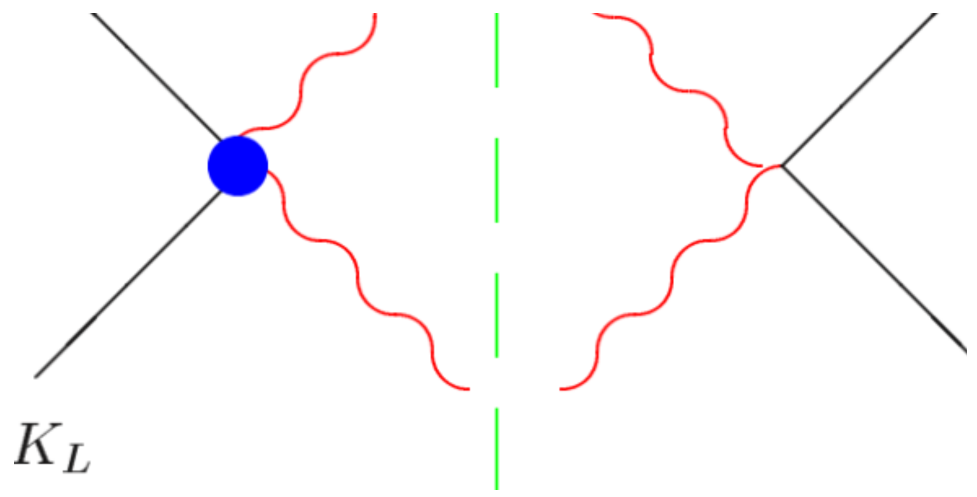


Figure 1: Potential diagrams contributing at leading order in large N_c $\left(\text{all at the order } (N_c)^2/(\sqrt{N_c})^2 = N_c\right)$. The dots represent the effective $\Delta S = 1$ operators.







$$\mathrm{Br}(K_L \to \pi^0 \ell^+ \ell^-) = 10^{-12}\left[C_{\rm mix}^{(\ell)} + C_{\rm int}^{(\ell)} \frac{\mathrm{Im}\,\lambda_t}{10^{-4}} + C_{\rm dir}^{(\ell)} \left(\frac{\mathrm{Im}\,\lambda_t}{10^{-4}}\right)^2 + C_{\gamma^*\gamma^*}^{(\ell)}\right]$$

$$C_{\rm mix}^{(\ell)} = 10^{12} |\bar{\epsilon}|^2 \frac{\tau(K_S)}{\tau(K_L)} \mathrm{Br}(K_S \to \pi^0 \ell^+ \ell^-).$$

$$\mathcal{W}_{+,S}(s) = G_F(M_K^2 a_{+,S} + b_{+,S}\, s) + \mathcal{V}_{+,S}^{\pi\pi}(s)$$

$$a_{+,S}, b_{+,S} \sim \mathcal{O}(N_c), \quad \mathcal{V}_{+,S}^{\pi\pi}(s) \sim \mathcal{O}(N_c^0)$$

$$\mathcal{A}_S = -e^2 \bar{\mathrm{u}}(p_{\ell-})\gamma_\rho \mathrm{v}(p_{\ell+})(k+p)^\rho \times \frac{\mathcal{W}_S(s)}{16\pi^2 M_K^2}$$

$$\mathcal{W}_S(s) = \mathcal{W}_S^{\text{loc}}(s;\nu) + \mathcal{W}_S^{\text{non-loc}}(s;\nu)$$

local short-distance part

long- distance dominated, non-local component

$$\mathcal{L}_{\rm lept}^{|\Delta S|=1}(\nu) = -\frac{G_{\rm F}}{\sqrt{2}} V_{us} V_{ud} \left[C_{7V}(\nu) Q_{7V} + C_{7A} Q_{A} \right] + {\rm H.c.}$$

$$Q_{7V} = (\bar{\ell}\gamma_\mu \ell)[\bar{s}\gamma^\mu(1-\gamma_5)d]$$

$$Q_{7A} = (\bar{\ell}\gamma_\mu\gamma_5 \ell)[\bar{s}\gamma^\mu(1-\gamma_5)d]$$

$$j_\rho(x) = \frac{2}{3}(\bar{u}\gamma_\rho u)(x) - \frac{1}{3}[(\bar{d}\gamma_\rho d)(x) + (\bar{s}\gamma_\rho s)(x)]$$

$$\mathcal{L}^{|\Delta S|=1}_{\mathrm{non-lept}} = -\frac{G_\mathrm{F}}{\sqrt{2}} V_{us} V_{ud} \sum_{I=1}^{6} C_I(\nu) Q_I(\nu) + \mathrm{H.c.}$$

$$Q_1 = (\bar{s}^i u_j)_{V-A}(\bar{u}^j d_i)_{V-A},\ Q_2 = (\bar{s}^i u_i)_{V-A}(\bar{u}^j d_j)_{V-A}$$

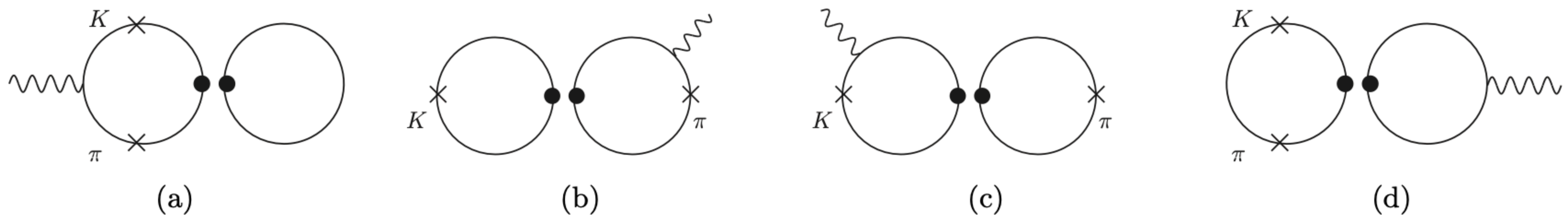


Figure 1: Potential diagrams contributing at leading order in large N_c $\big($all at the order $(N_c)^2/(\sqrt{N_c})^2 = N_c\big)$. The dots represent the effective $\Delta S = 1$ operators.

$$\langle \pi^0(p)|T\{j_\rho(0)Q_1(x)\}|K^0(k)\rangle =$$

$$i\int d^4x\, e^{iq\cdot x}\langle 0|T\{[\bar{u}\gamma_\mu u](x)[\bar{u}\gamma_\nu u](0)\}|0\rangle_{\overline{\rm MS}}$$

$$i\int d^4x\, e^{iq\cdot x}\langle 0|T\{[\bar{u}\gamma_\mu u](x)[\bar{u}\gamma_\nu u](0)\}|0\rangle_{\overline{\rm MS}} = (q_\mu q_\nu - q^2\eta_{\mu\nu})\Pi_{\overline{\rm MS}}(q^2;\nu)$$

$$\frac{\mathcal{W}_S(s)}{16\pi^2 M_K^2} = -\frac{G_F}{\sqrt{2}} V_{us} V_{ud} \Bigg\{ f_+(s) \Bigg[\frac{2}{3} C_1 \Pi_{\overline{\rm MS}}(s;\nu) + \frac{{\rm Re} C_{7V}(\nu)}{4\pi\alpha} \Bigg] + \frac{2}{3} C_1 \Bigg[\frac{F_\pi F_K M_K^2}{M_K^2 - M_\pi^2} \, \frac{F_d^{K^0}(s) + F_s^{K^0}(s)}{s} - \frac{F_\pi}{2} \mathcal{P}(s, M_\pi^2) - \frac{F_\pi}{2} \tilde{\mathcal{P}}(s, M_\pi^2) \Bigg] \Bigg\}$$

$$f_+(q^2) = \frac{M_{K^*}^2}{M_{K^*}^2 - q^2}, \quad F_s^{K^0}(q^2) = \frac{M_\phi^2}{q^2 - M_\phi^2}, \quad F_d^{K^0}(q^2) = \frac{M_\rho^2}{M_\rho^2 - q^2}$$

$$\Pi_{\overline{\mathrm{MS}}}(q^2;\nu) = \frac{f_\rho^2 M_\rho^2}{M_\rho^2 - q^2} + \frac{9 f_\omega^2 M_\omega^2}{M_\omega^2 - q^2} + \frac{N_c}{12\pi^2}\left\{ -\ln(M^2/\nu) + \frac{5}{3} - \psi\left(3 - \frac{q^2}{M^2}\right) \right\}$$

$$q^2 \to -\infty$$

$$\psi\left(3 - \frac{q^2}{M^2}\right) \sim \ln(-q^2/M^2) - \frac{5}{2}\frac{M^2}{q^2} + \mathcal{O}(M^4/q^4)$$

$$\psi\left(3 - \frac{q^2}{M^2}\right) = -\gamma_E + \frac{3}{2} + \sum_{n\geq 1} \frac{1}{n+2}\frac{q^2}{q^2 - M_n^2}$$

$$M^2 = \frac{16\pi^2}{5}\frac{3}{N_c}\, f_\rho^2 M_\rho^2$$

$$M^2 = \frac{16\pi^2}{5}\frac{3}{N_c}\, f_\rho^2 M_\rho^2 \sim (0.87\ \mathrm{GeV})^2$$

$$\mathrm{Br}(K_S \to \pi^0 e^+ e^-)|_{m_{ee}>165\ \mathrm{MeV}} = 2.9(1.0) \cdot 10^{-9}$$

$$\mathrm{Br}(K_S \to \pi^0 e^+ e^-) = 5.1(1.7) \cdot 10^{-9}$$

$$\mathrm{Br}(K_S \to \pi^0 \mu^+ \mu^-) = 1.3(0.4) \cdot 10^{-9}$$

$$\mathrm{Br}(K_S \to \pi^0 e^+ e^-)|_{m_{ee}>165\ \mathrm{MeV}} = (3.0^{+1.5}_{-1.2} \pm 0.2) \cdot 10^{-9}$$

$$\mathcal{W}_S(s) \sim G_{\mathrm{F}}[0.92 M_K^2 + 0.64 s + 0.39 s^2/M_K^2]$$

$$C_{\mathrm{int}}^{(e)} = +7.8(2.6)\,\frac{y_{7V}}{\alpha},\;\; C_{\mathrm{int}}^{(\mu)} = +1.9(0.6)\,\frac{y_{7V}}{\alpha}$$

$$y_{7V} > 0$$

Conclusion

Large N QCD still very inspirational and predictive

Kaon physics very important

K+->pi+ l l coming up

$$
\begin{aligned}
Q_3 &= (\bar{s}^i d_i)_{V-A} \sum_{q=u}^{s} (\bar{q}^j q_j)_{V-A} = \sum_{q=u}^{s} (\bar{s}^i q_j)_{V-A} (\bar{q}^j d_i)_{V-A}, \\
Q_4 &= (\bar{s}^i d_j)_{V-A} \sum_{q=u}^{s} (\bar{q}^j q_i)_{V-A} = \sum_{q=u}^{s} (\bar{s}^i q_i)_{V-A} (\bar{q}^j d_j)_{V-A}, \\
Q_5 &= (\bar{s}^i d_i)_{V-A} \sum_{q=u}^{s} (\bar{q}^j q_j)_{V+A} = -8 \sum_{q=u}^{s} (\bar{s}^i_L q_{jR}) (\bar{q}^j_R d_{iL}), \\
Q_6 &= (\bar{s}^i d_j)_{V-A} \sum_{q=u}^{s} (\bar{q}^j q_i)_{V+A} = -8 \sum_{q=u}^{s} (\bar{s}_L q_R) (\bar{q}_R d_L),
\end{aligned}
$$