A little B and a touch of K:

$B \to X_s \ell^+ \ell^-$ and $K^+ \to \pi^+ \nu \bar{\nu}$

Enrico Lunghi
Indiana University & CERN

Zürich Phenomenology Workshop
January 7, 2024

Huber, EL, Misiak, Wyler	hep-ph/0512066 (NPB)
Huber, Hurth, EL	1503.04849 (JHEP)
Huber, Hurth, Jenkins, EL, Qin, Vos	1908.07507 (JHEP)
Huber, Hurth, Jenkins, EL, Qin, Vos	2007.04191 (JHEP)
Huber, Hurth, Jenkins, EL, Qin, Vos	2306.03134 (JHEP)
Huber, Hurth, Jenkins, EL, Qin, Vos	2404.03517 (JHEP)
Gambino, EL, Schacht	2408.11190 (JHEP)

Outline

- Status of exclusive $b \to s\ell\ell$ anomalies

- Theory of inclusive $B \to X_s\ell\ell$ decays
  OPE and its breakdown at large m_X
  Krüger-Sehgal description of $c\bar{c}$ resonances
  Non-local power corrections at low $m_{\ell\ell}$
  QED radiation
  Weak annihilation

- Phenomenology of inclusive $b \to s\ell\ell$ decays
  SM predictions
  Experimental results (BaBar, Belle, **Belle-II**, **LHCb**),
  New Physics reach, comparison with exclusive

- Taming uncertainties in $K^\pm \to \pi^\pm\nu\bar{\nu}$

Introduction: operators

SM operator basis ($q = d, s$):

$$\mathcal{L}_{eff} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{tq}^* \left[\sum_{i=1}^{10} C_i Q_i + \frac{V_{ub} V_{uq}^*}{V_{tb} V_{tq}^*} \sum_{i=1}^{2} C_i (Q_i - Q_i^u) + \underbrace{\sum_{i=3}^{6} C_{iQ} Q_{iQ} + C_b Q_b}_{\text{for QED corrections}} \right]$$

- Semileptonic

$$Q_9 = (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\ell} \gamma_\mu \ell_L)$$

$$Q_{10} = (\bar{q}_L \gamma_\mu b_L) \sum (\bar{\ell} \gamma_\mu \gamma_5 \ell_L)$$

- Magnetic & chromo-magnetic

$$Q_7 = \frac{e}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} b_R) F_{\mu\nu}$$

$$Q_8 = \frac{g_s}{16\pi^2} m_b (\bar{q}_L \sigma^{\mu\nu} T^a b_R) G^a_{\mu\nu}$$

- Current-current

$$Q_1 = (\bar{q}_L \gamma_\mu T^a c_L)(\bar{c}_L \gamma_\mu T^a b_L)$$

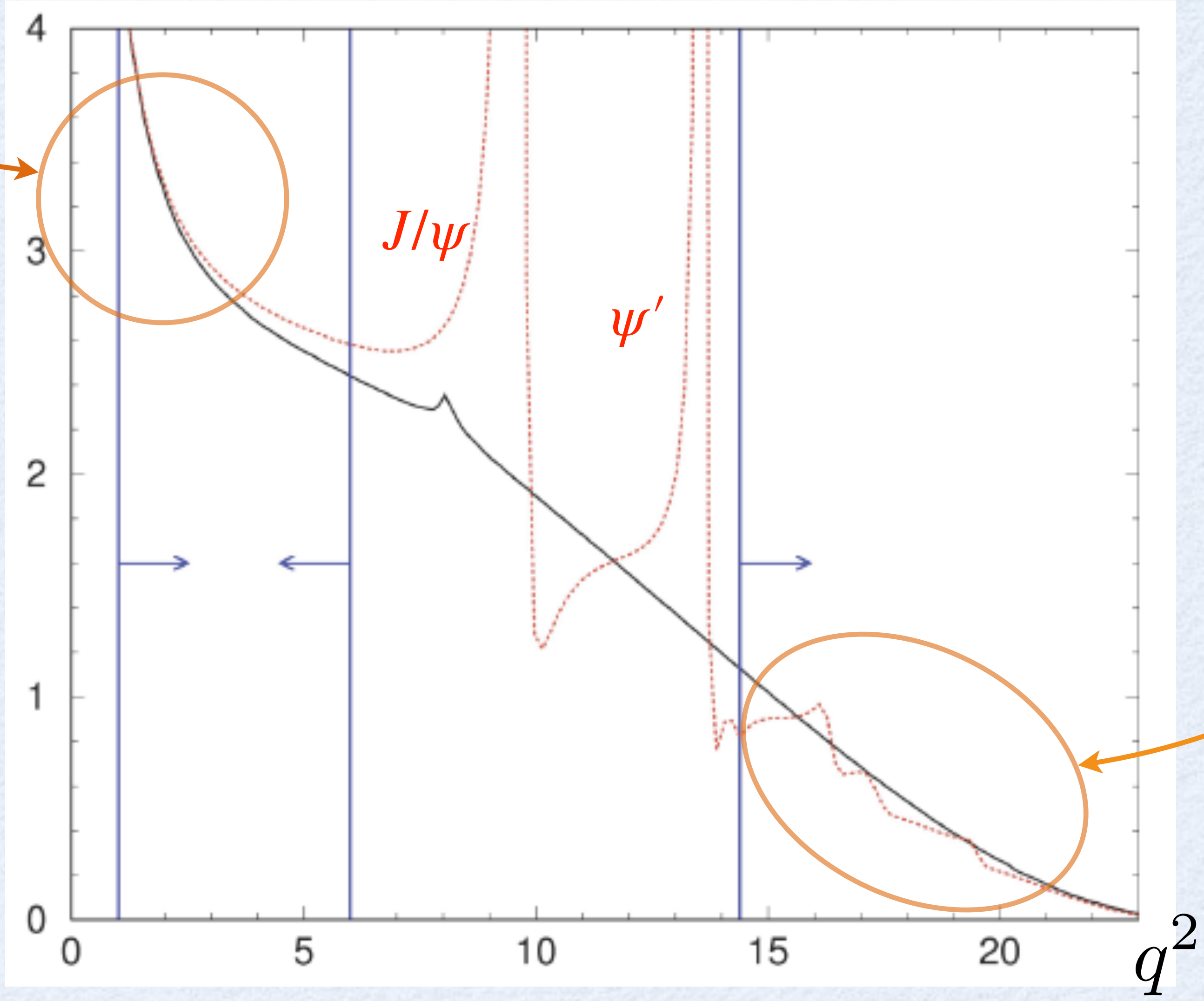
$$Q_2 = (\bar{q}_L \gamma_\mu c_L)(\bar{c}_L \gamma_\mu b_L)$$

$$Q_1^u = (\bar{q}_L \gamma_\mu T^a u_L)(\bar{u}_L \gamma_\mu T^a b_L)$$

$$Q_2^u = (\bar{q}_L \gamma_\mu u_L)(\bar{u}_L \gamma_\mu b_L)$$

The $V_{ub} V_{uq}^*$ contribution is small for $b \to s\ell\ell$ but important for $b \to d\ell\ell$

Introduction: typical spectrum



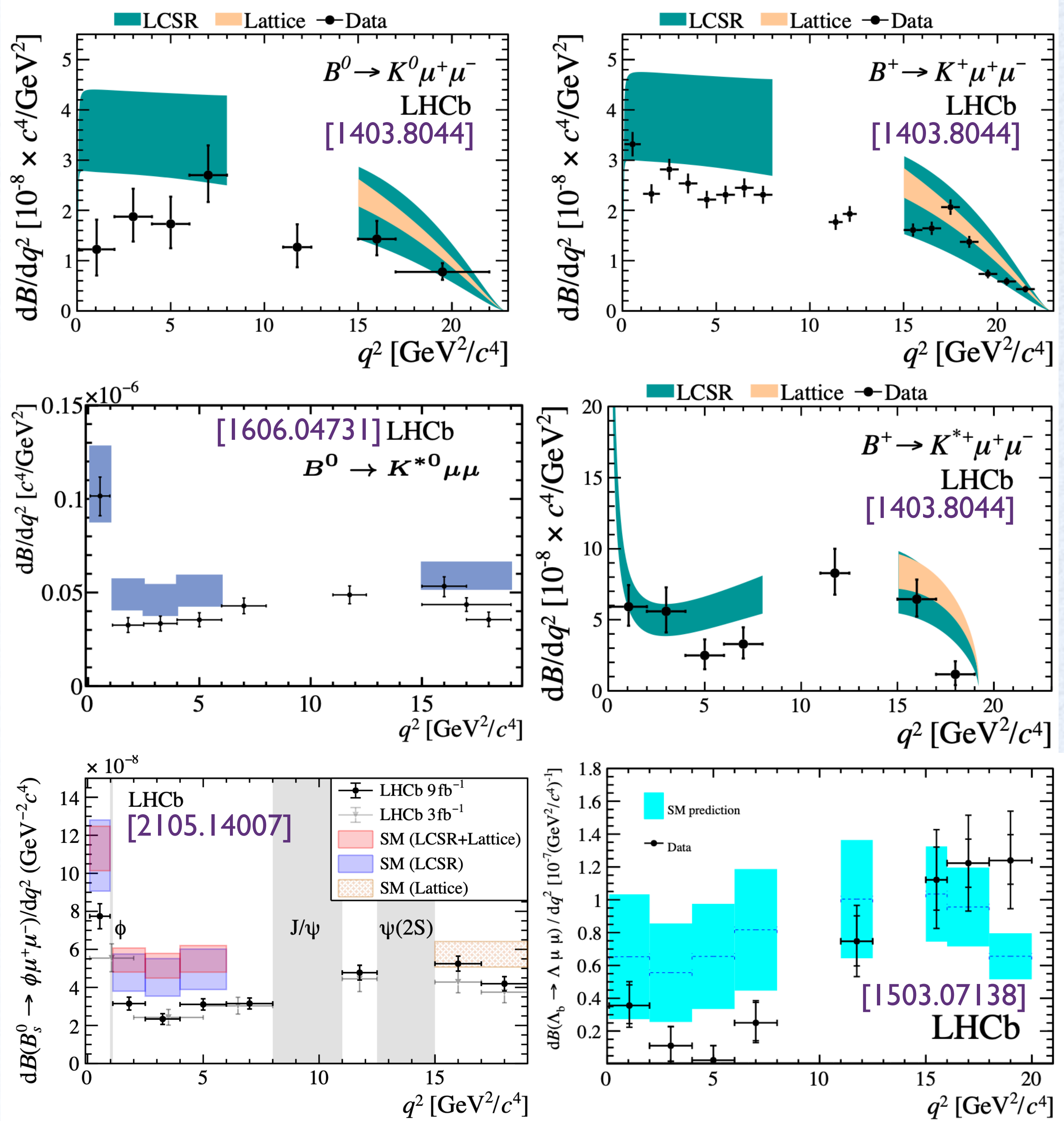
photon pole
(only K^* and $X_{s,d}$)

broad resonances
at high-q^2

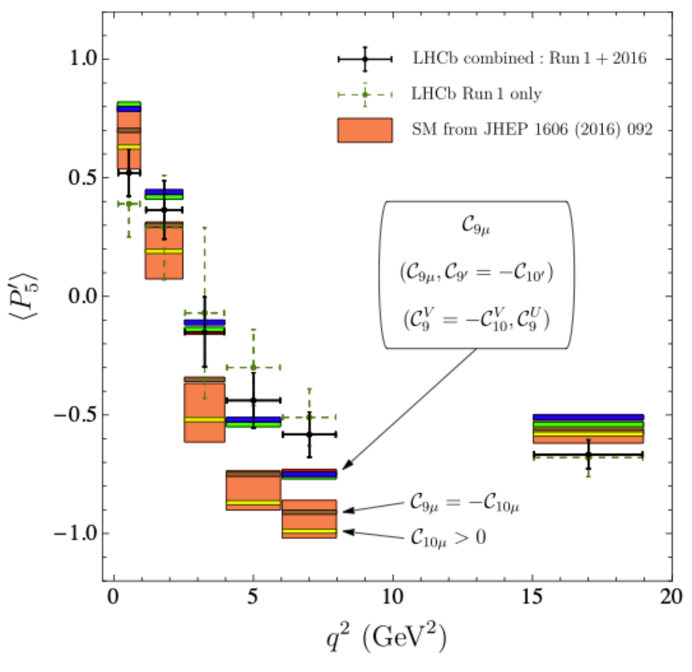
- Intermediate charmonium resonances contribute via:
  $B \to (K, K^*, X_s)\, \psi_{c\bar{c}} \to (K, K^*, X_s)\, \ell^+\ell^-$
- Contributions of J/ψ and ψ' have to be dropped
- Theory at low-q^2 and high-q^2 presents different challenges

Exclusive modes: anomalies

Branching Ratios:

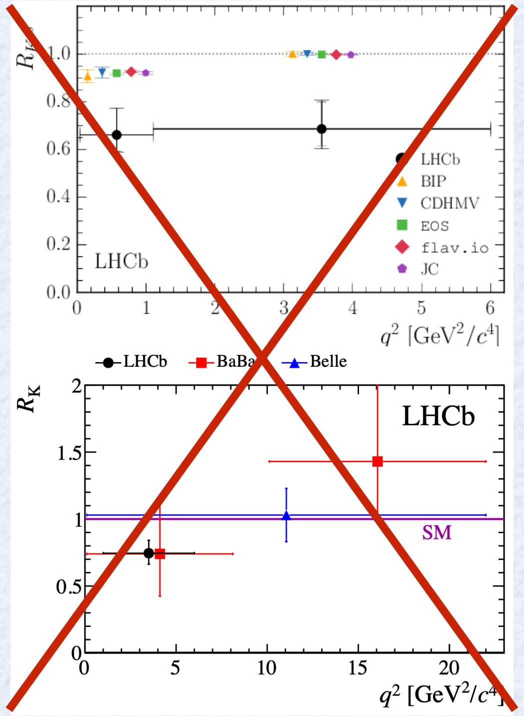


Angular observables:



[Alguero et al, 1903.09578]

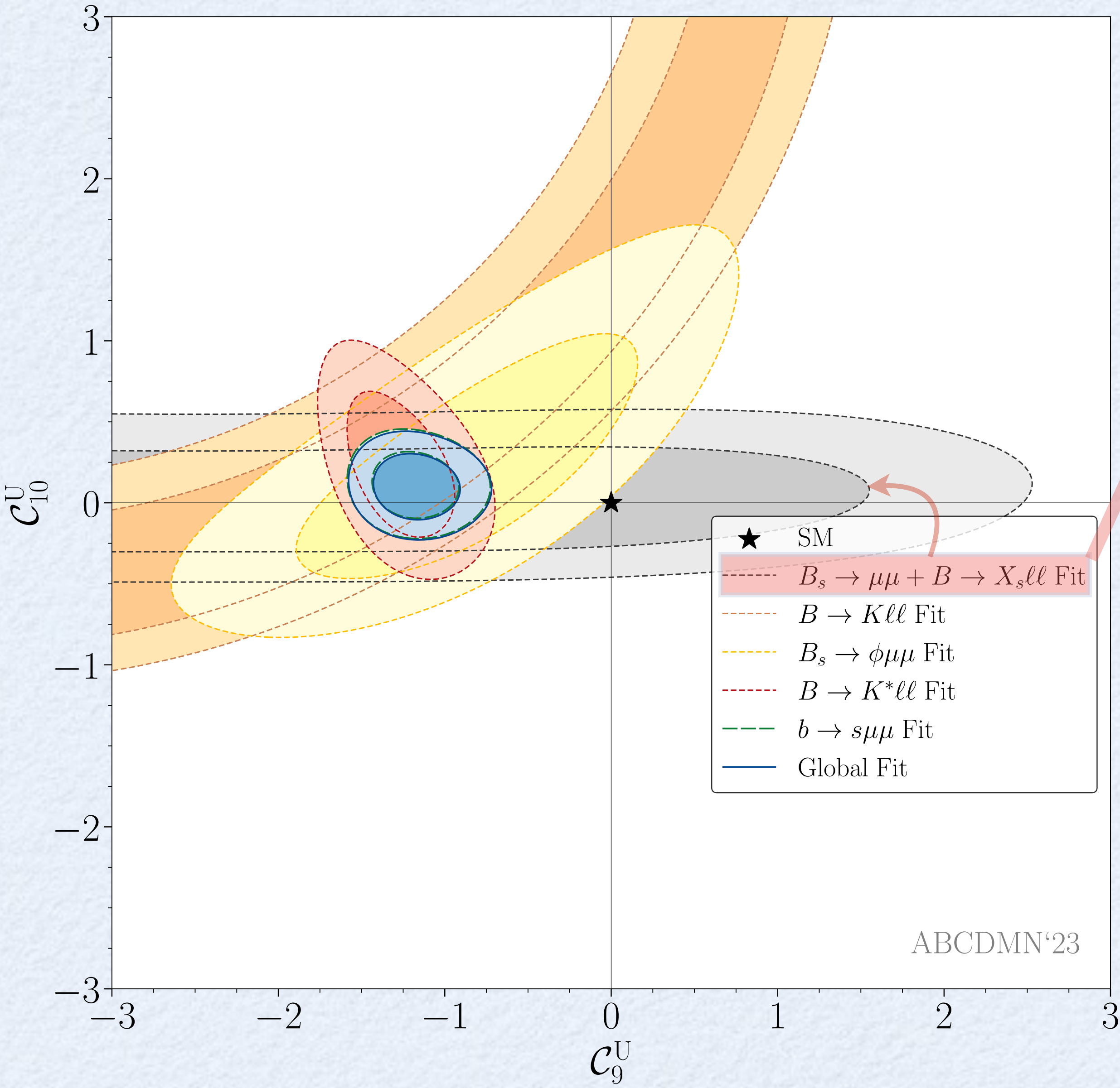
LFUV ratios:



Exclusive modes: global fits

Scenario		Best-fit point	1σ	$\mathrm{Pull_{SM}}$	p-value
$b \to s\ell^+\ell^-$	$\mathcal{C}_9^{\mathrm{U}}$	-1.17	$[-1.33, -1.00]$	5.8	39.9 %
$b \to s\ell^+\ell^-$	$\mathcal{C}_9^{\mathrm{U}}$ $\mathcal{C}_{10}^{\mathrm{U}}$	-1.18 $+0.10$	$[-1.35, -1.00]$ $[-0.04, +0.23]$	5.5	39.1 %

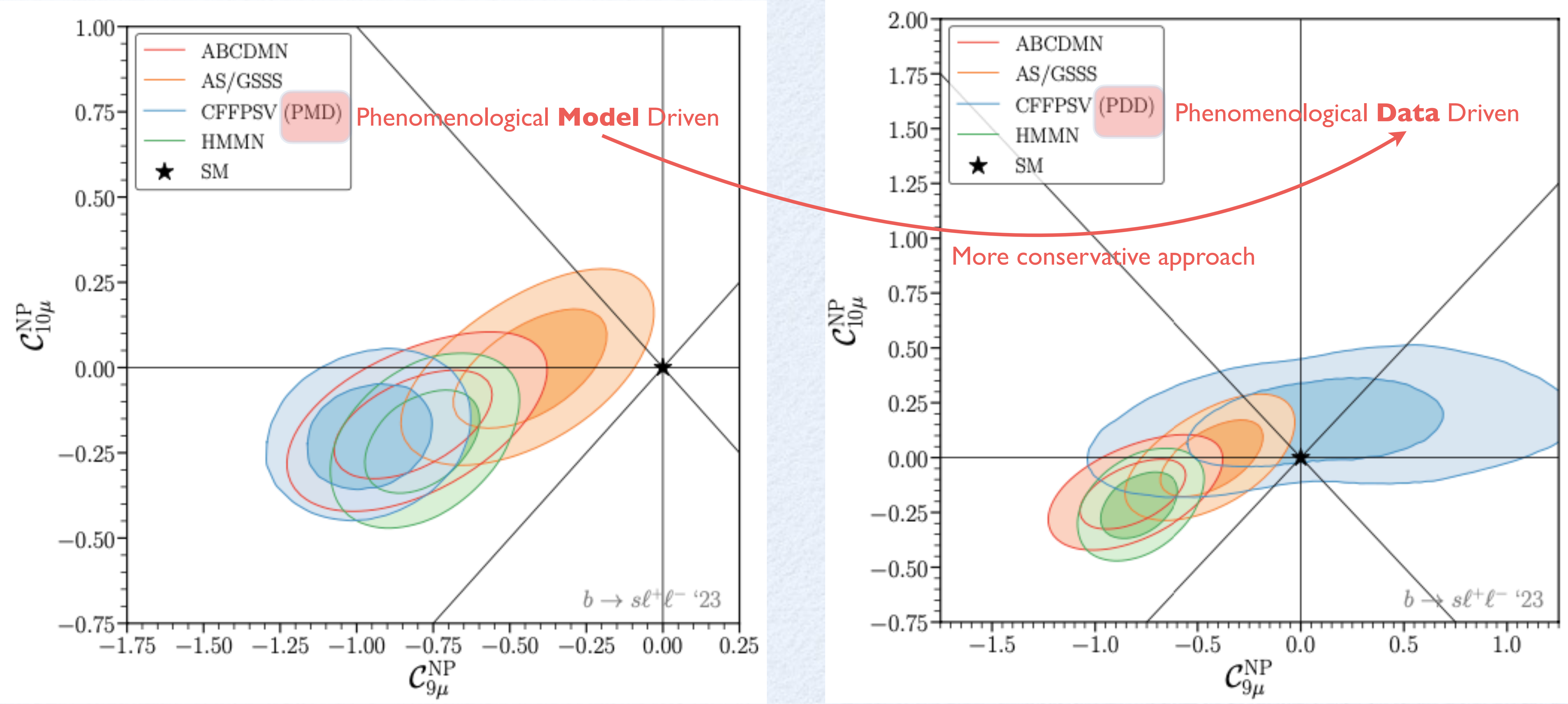
[Capdevila et al, 2309.01311]



- Low-q^2 $B \to X_s\ell\ell$ inclusive constraints are consistent with the SM (more on this later)
- Many global fitters:
  - ✦ ABCDMN [Algueró et al, 2304.07330]
  - ✦ AS/GSSS [Altmannshofer et al, 2212.10497]
  - ✦ CFFPSV [Ciuchini et al, 2212.10516]
  - ✦ HMMN [Hurth et al, 2104.10058]
  - ✦ GRvDV [Gubernari et al, 2206.03797]

Exclusive modes: global fits

- Good agreement between global fitters (if same sets of inputs are used *and* if treatment of unknown power corrections are similar):



Phenomenological **Model** Driven

Phenomenological **Data** Driven

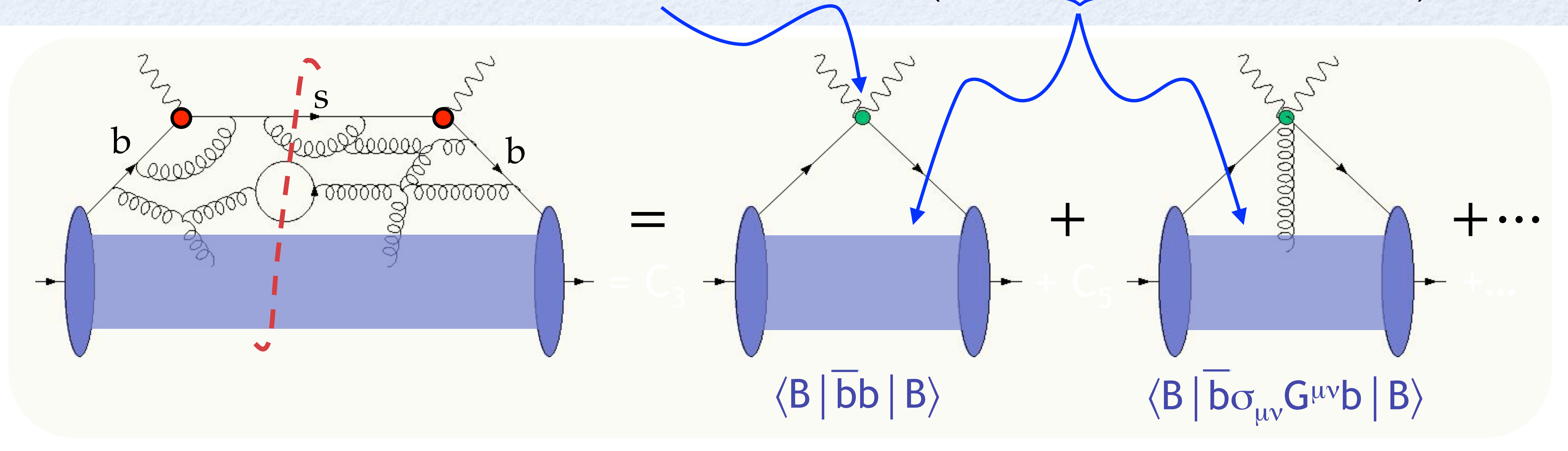
More conservative approach

[Capdevila et al, 2309.01311]

- After the loss LFUV ratios we need processes in which the same Wilson coefficients are tested and in which non-perturbative effects have a different nature: enters $B \to X_s \ell \ell$

Inclusive theory: OPE

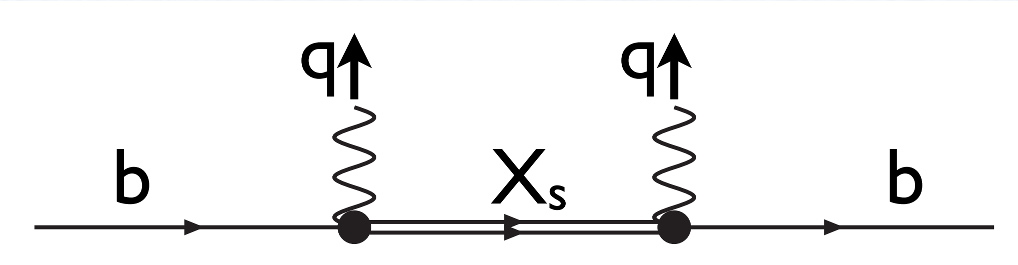
$$\Gamma\left[\bar{B} \to X_s \ell^+ \ell^-\right] = \underbrace{\Gamma\left[\bar{b} \to X_s \ell^+ \ell^-\right]} + O\left(\underbrace{\frac{\Lambda^2_{QCD}}{m_b^2}, \frac{\Lambda^3_{QCD}}{m_b^3}}, \frac{\Lambda^2_{QCD}}{m_c^2}, \dots\right)$$



- The leading power contribution to the width
  - corresponds to $b \to X_s \ell^+ \ell^-$
  - is most conveniently expressed as a series in α_s and $\kappa = \alpha_{\rm em}/\alpha_s$
  - is known *almost* up to and including $\alpha_s^3 \kappa^3$
    [the only missing contributions are proportional to the small κ and κ^2 terms in the $b \to X_s \ell\ell$ amplitude]

Inclusive theory: OPE failure at high-q^2

- The OPE breaks down at high-q^2:



$$\begin{aligned} p^2_{X_s} &= (p_b - q)^2 = m_b^2 + q^2 - 2 m_b q_0 \\ &< m_b^2 + q^2 - 2 m_b \sqrt{q^2} = \left(m_b - \sqrt{q^2}\right)^2 \end{aligned}$$

⇒ the OPE is an expansion in $\dfrac{\Lambda_{\rm QCD}}{m_b - \sqrt{q^2}}$ and breaks down at $q^2 \sim m_b^2$

- The breakdown of the OPE manifests as very large power corrections:
  - the poorly known matrix elements required to evaluate $1/m_b^3$ power corrections are responsible for the large uncertainty
  - possible progress from lattice-QCD [more on this later]

Inclusive theory: OPE failure at high-q^2

- Power corrections proportional to $|C_{9,10}|^2$ are identical to the power corrections which appear in $\bar{B}^0 \to X_u \ell \nu$

- Introduce a new observable obtained by normalizing the rate to the semileptonic rate with the same q^2 cut [Lee, Ligeti, Stewart, Tackmann]:

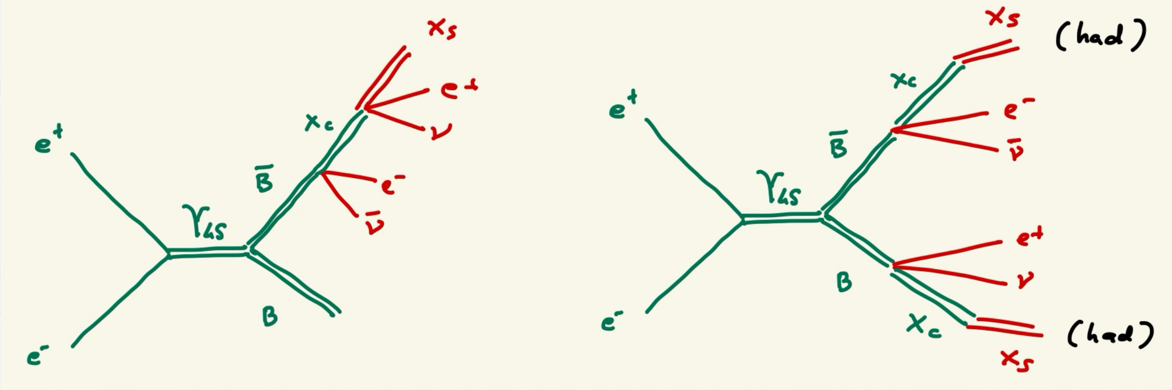
$$\mathcal{R}(q_0^2) = \frac{\int_{q_0^2}^{m_b^2} dq^2 \frac{\mathrm{d}\Gamma(\bar{B} \to X_s \ell^+ \ell^-)}{dq^2}}{\int_{q_0^2}^{m_b^2} dq^2 \frac{\mathrm{d}\Gamma(\bar{B}^0 \to X_u \ell \nu)}{dq^2}}$$

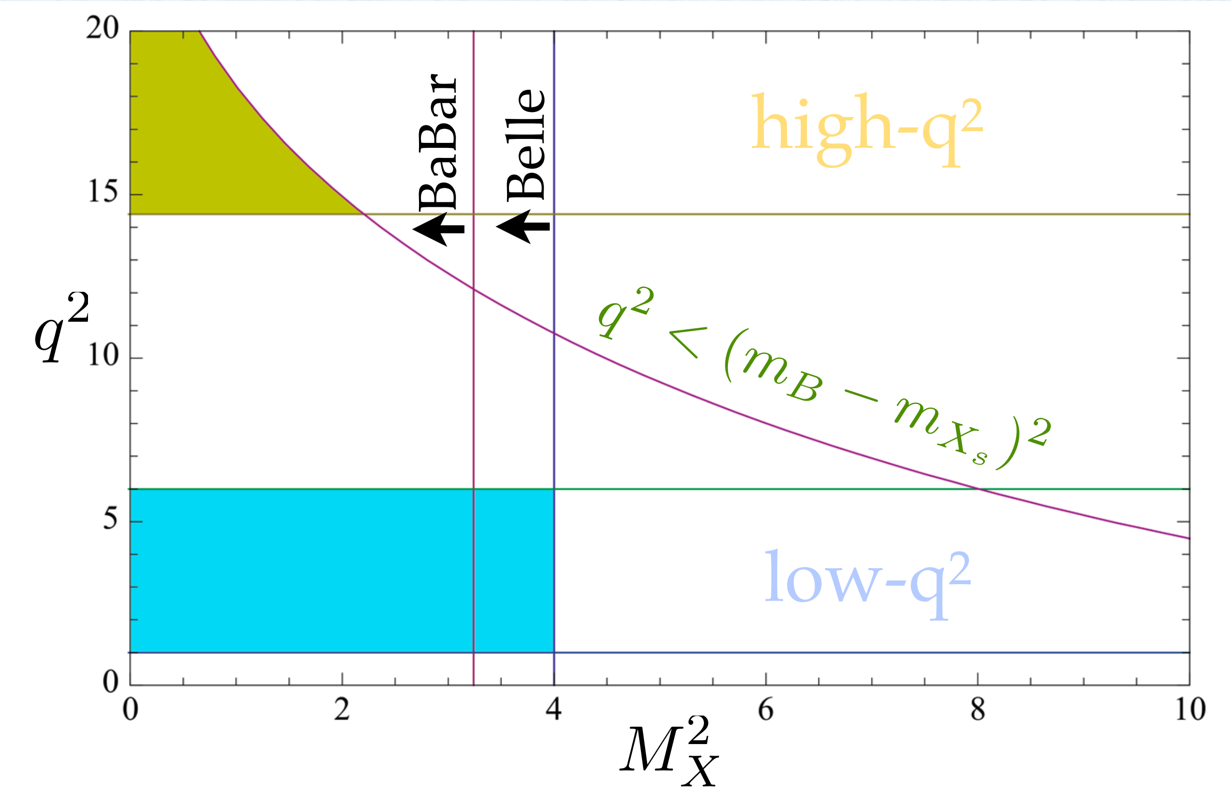
  - Non-perturbative effects associated to the breaking of the OPE in the $|C_{9,10}|^2$ terms cancels exactly against those in the denominator
  - Non-perturbative effects associated to other operators do not necessarily cancel

Inclusive theory: M_X cuts

- m_X cuts are required to suppress background from double semileptonic decays (both same side and opposite side):
  - $B \to (X_c \to X_s \ell^+ \nu)\ell^- \bar{\nu} = X_s \ell \ell$ + missing energy
  - $ee \to (B \to (X_c \to X_s)\ell^- \bar{\nu})(\bar{B} \to (X_c \to X_s)\ell^+ \nu) = X_s \ell \ell$ + missing energy

- These cuts introduce sensitivity to a hard collinear scale (of order 2 GeV) and the rate becomes dependent on the B meson shape function





- The high-q² region is unaffected
- Current BaBar and Belle analyses correct using a Fermi motion model
- Better modeling can be achieved within SCET and by using $B \to X_s \gamma$ and $B \to X_u \ell \nu$ data to extract the shape function

Inclusive theory: resonances

- Optical theorem:

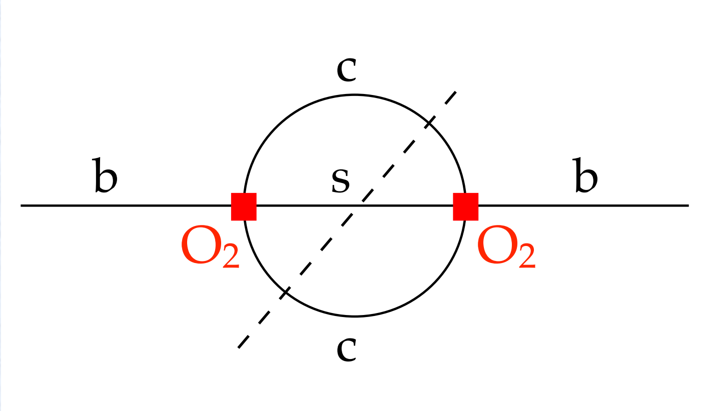
[Beneke, Buchalla, Neubert, Sachrajda]

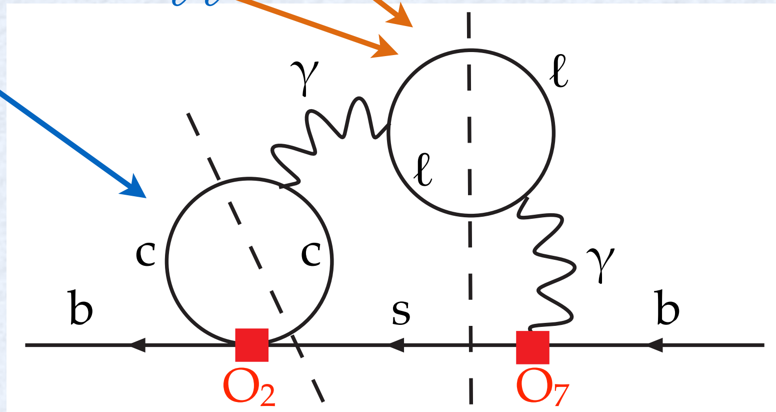
$$\mathrm{Im}\left[\sum_{ij}\langle\bar{B}|T\,Q_i(0)\,Q_j(x)|\bar{B}\rangle\right]\sim\Gamma(\bar{B}\to X_s)\neq\Gamma(\bar{B}\to X_s\ell^+\ell^-)$$

$b\to s(c\bar{c})_{\mathrm{had}}$

$b\to s(c\bar{c})_{\ell\ell}$

$b\to s\ell^+\ell^-$





$\mathrm{BR}(B\to X_s)\sim 10^{-2}$

$\mathrm{BR}(B\to X_s(J/\psi,\psi')\to X_s\ell\ell)\sim 10^{-4}$ ⟶ Experimental cuts

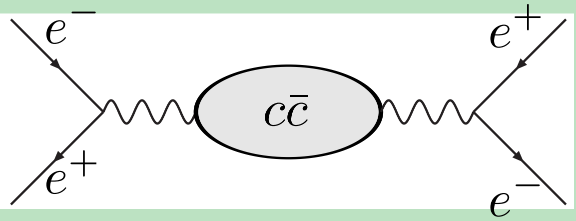
$\mathrm{BR}(B\to X_s\ell\ell)\sim 10^{-6}$ ⟶ Need to control charmonium contamination away from $\psi(1s,2s)$

Inclusive theory: resonant color singlet production

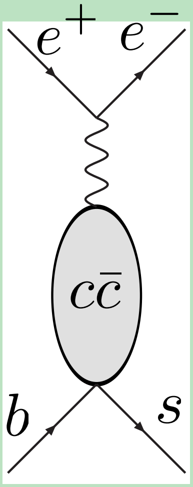
- Kruger-Sehgal mechanism:

$$R_{\rm had}^{c\bar{c}} = \frac{\sigma(e^+e^- \to c\bar{c}\text{ hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$=$$



$$\langle O_2 \rangle =$$

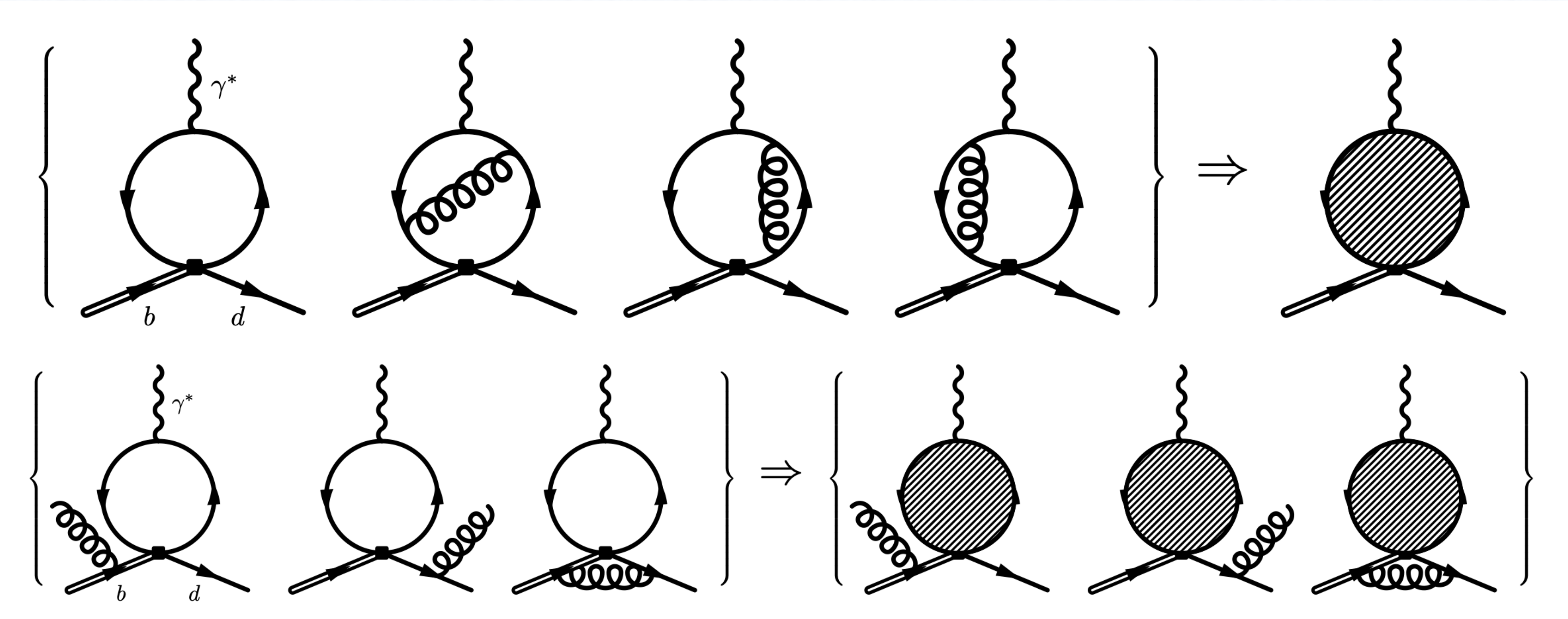


$$\mathrm{Im}[h_c] = \frac{\pi}{3} R_{\rm had}$$

$$\mathrm{Re}[h_c] = \mathrm{Re}[h_c(s_0)] + \frac{s - s_0}{\pi} \int_0^\infty \frac{\mathrm{Im}[h_q(t)]}{(t-s)(t-s_0)} dt$$

$\downarrow$ perturbative for $s_0 \sim -\mu_b^2$

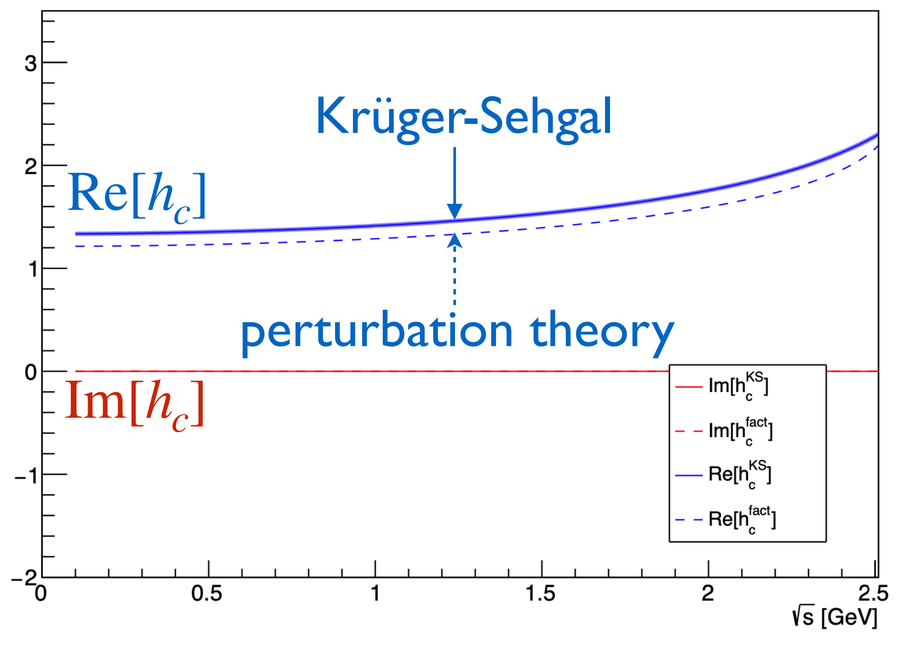
- We can include NLO effects [separation of two-loop perturbative functions provided by de Boer]



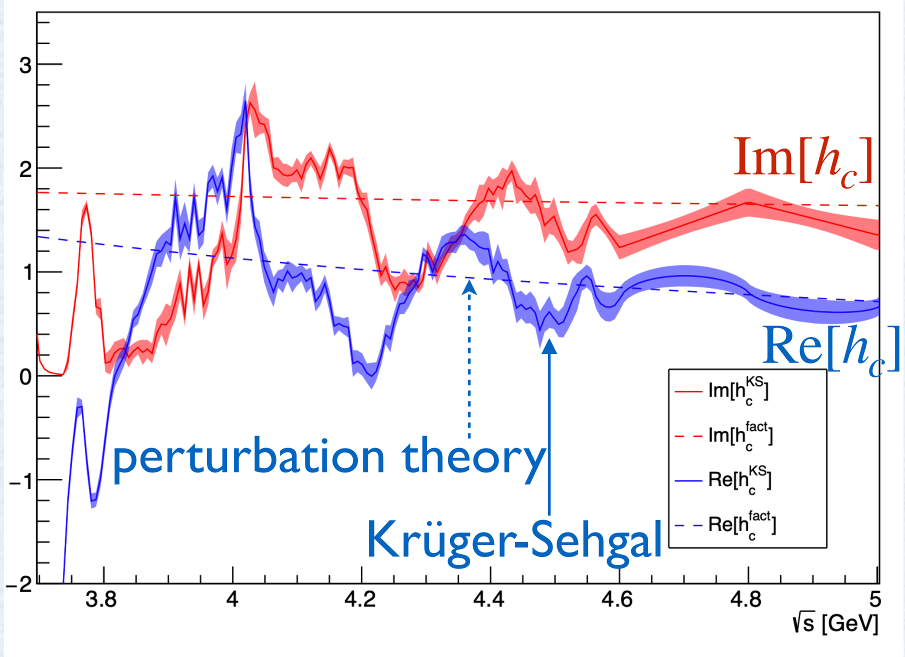
Inclusive theory: resonant color singlet production

- We use R_{had} data [BESII, BaBar, ALEPH; Keshavarzi, Nomura, Teubner] and perturbation theory (program `rhad`) for asymptotically large s [Harlander, Steinhauser]

Low-q^2



High-q^2



$[s = q^2]$

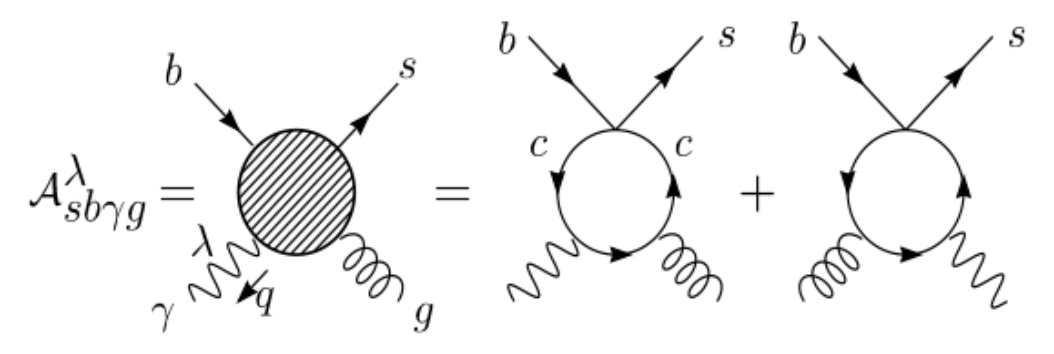
- Impact at low-q^2 is small ($\simeq 2\,\%$)

  Perturbation theory and dispersive approaches agree because below threshold we are mostly sensitive to the total integral over R_{had} which is well described in perturbation theory

- Impact at high-q^2 region is large ($\simeq -10\,\%$)

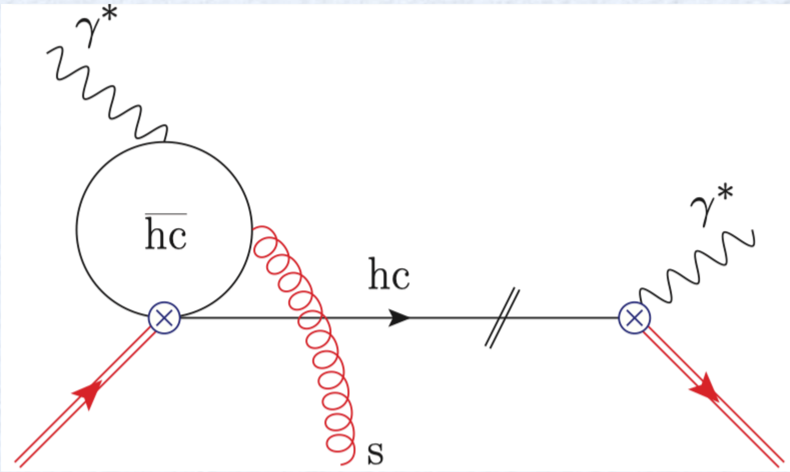
Inclusive theory: non-resonant color octet

- Non-resonant color octet effects at high-q² can be calculated in perturbation theory and it scales as $\Lambda^2_{\mathrm{QCD}}/q^2$ [Buchalla, Isidori, Rey]:



$$\Longrightarrow \frac{\langle B|\bar{b}\sigma_{\mu\nu}G^{\mu\nu}b|B\rangle}{m_c^2}\left(-6\frac{m_c^2}{q^2}\right) \sim \frac{\lambda_2}{q^2}$$

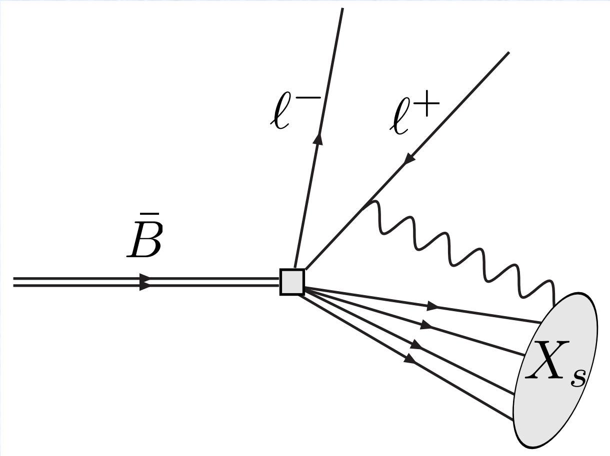
- At low-q² and with a cut on m_X the charm loop is hard-collinear and needs to be treated using SCET [Hurth, Benzke, Fickinger, Turczyk]:

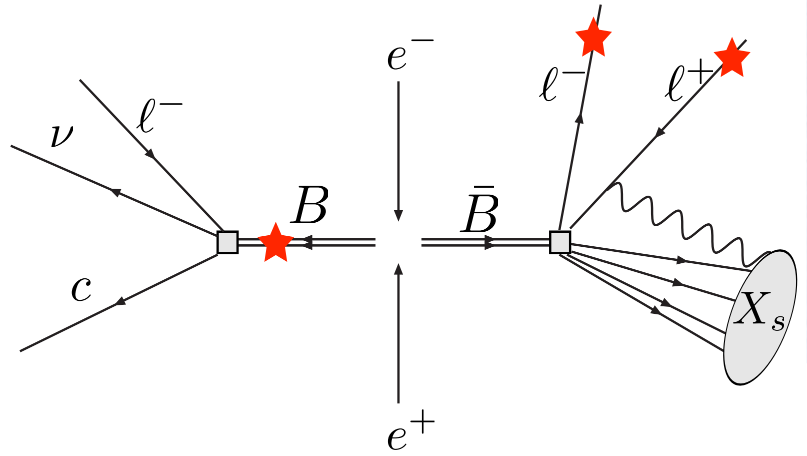


  - Power corrections remain non-local after m_X cut is released ⇒ so-called resolved contributions
  - Depend on mostly unknown subleading B shape functions
  - Work in progress on explicit estimate [Benzke, Hurth, Turczyk]
  - For the time being, we use rough estimates to asses an irreducible uncertainty of about 5%

QED radiation

- The rate is proportional to $\alpha_{\rm em}^2(\mu)$. Without QED corrections the scale μ is undetermined $\to \pm 4\,\%$ uncertainty
- Focus on corrections enhanced by large logarithms:
  - $\alpha_{\rm em} \log(m_W/m_b) \sim \alpha_{\rm em}/\alpha_s$ [WC, RG running] [Bobeth,Gambino,Gorbahn,Haisch]
  - $\alpha_{\rm em} \log(m_\ell/m_b)$ [Matrix Elements]
- Fate of photons emitted by the final state leptons (especially electrons):





- At B-factories most but not all of these photons are included in the X_s system: $\Rightarrow$ some collinear QED logs survive
- At LHCb all photons emitted by the charged leptons are recovered (physically and using PHOTOS) and included in the lepton 4-momentum: $\Rightarrow$ all collinear QED logs must not be included

Inputs

$\alpha_s(M_z) = 0.1181(11)$	$m_e = 0.51099895 \text{ MeV}$
$\alpha_e(M_z) = 1/127.955$	$m_\mu = 105.65837 \text{ MeV}$
$s_W^2 \equiv \sin^2 \theta_W^{\overline{\text{MS}}} = 0.2312$	$m_\tau = 1.77686 \text{ GeV}$
$\lvert V_{ts}^* V_{tb}/V_{cb}\rvert^2 = 0.96403(87)$	$\overline{m}_c(\overline{m}_c) = 1.275(25) \text{ GeV}$
$\lvert V_{ts}^* V_{tb}/V_{ub}\rvert^2 = 123.5(5.3)$	$m_b^{1S} = 4.691(37) \text{ GeV}$
$\lvert V_{td}^* V_{tb}/V_{cb}\rvert^2 = 0.04195(78)$	$\lvert V_{us}^* V_{ub}/(V_{ts}^* V_{tb})\rvert = 0.02022(44)$
$\lvert V_{td}^* V_{tb}/V_{ub}\rvert^2 = 5.38(26)$	$\arg\left[V_{us}^* V_{ub}/(V_{ts}^* V_{tb})\right] = 115.3(1.3)^\circ$
$\mathcal{B}(\bar{B} \to X_c e \bar{\nu})_{\text{exp}} = 0.1065(16)$	$\lvert V_{ud}^* V_{ub}/(V_{td}^* V_{tb})\rvert = 0.420(10)$
$m_B = 5.2794 \text{ GeV}$	$\arg\left[V_{ud}^* V_{ub}/(V_{td}^* V_{tb})\right] = -88.3(1.4)^\circ$
$M_Z = 91.1876 \text{ GeV}$	$m_{t,\text{pole}} = 173.1(0.9) \text{ GeV}$
$M_W = 80.379 \text{ GeV}$	$C = 0.568(7)(10)$
$\mu_b = 5^{+5}_{-2.5} \text{ GeV}$	$\mu_0 = 120^{+120}_{-60} \text{ GeV}$
$f_{\text{NV}} = (0.02 \pm 0.16) \text{ GeV}^3$	$\lambda_2^{\text{eff}} = 0.111(18) \text{ GeV}^2$
$f_{\text{V}} - f_{\text{NV}} = (0.041 \pm 0.052) \text{ GeV}^3$	$\lambda_1 = -0.314(56) \text{ GeV}^2$
$[\delta f]_{\text{SU(3)}} = (0 \pm 0.04) \text{ GeV}^3$	$\rho_1 = 0.080(31) \text{ GeV}^3$
$[\delta f]_{\text{SU(2)}} = (0 \pm 0.004) \text{ GeV}^3$	

References for all inputs can be found in:
2007.04191
2404.03517 ($\lambda_{1,2}$ and ρ_1)

Dominant uncertainties at high-q^2

Inputs: HQET matrix elements

- Power corrections affects mainly high-q² where the OPE breaks down:

$$\lambda_1 \equiv \frac{1}{2m_B}\langle B|\bar{b}_v(iD)^2 b_v|B\rangle$$

$$\lambda_2 \equiv \frac{1}{12m_B}\langle B|\bar{b}_v(-i\sigma_{\mu\nu})G^{\mu\nu}b_v|B\rangle$$

$$\rho_1 \equiv \frac{1}{2m_B}\langle B|\bar{b}_v iD_\mu(iv\cdot D)iD^\mu b_v|B\rangle$$

$$\rho_2 \equiv \frac{1}{6m_B}\langle B|\bar{b}_v iD^\mu(iv\cdot D)iD^\nu(-i\sigma_{\mu\nu})b_v|B\rangle$$

- Extracted in the kinetic scheme from moments of the $B \to X_c \ell \nu$ spectrum [Gambino, Healey, Turczyk]
- Converted to the pole scheme
- In $b \to s\ell\ell$ λ_2 and ρ_2 appear in the combination $\lambda_2^{\rm eff} \equiv \lambda_2 - \frac{\rho_2}{m_b}$

$$f_q^{0,\pm} \equiv \frac{1}{2m_B}\langle B^{0,\pm}|Q_1^q - Q_2^q|B^{0,\pm}\rangle$$

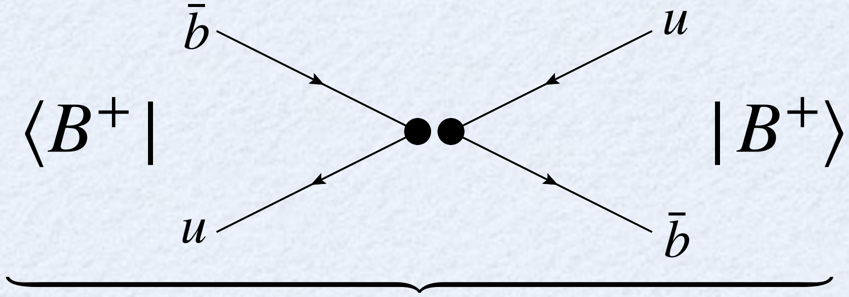
$$Q_1^q = \bar{b}_v\gamma_\mu(1-\gamma_5)q\ \bar{q}\gamma^\mu(1-\gamma_5)b_v\ ,$$

$$Q_2^q = \bar{b}_v(1-\gamma_5)q\ \bar{q}(1+\gamma_5)b_v\ .$$

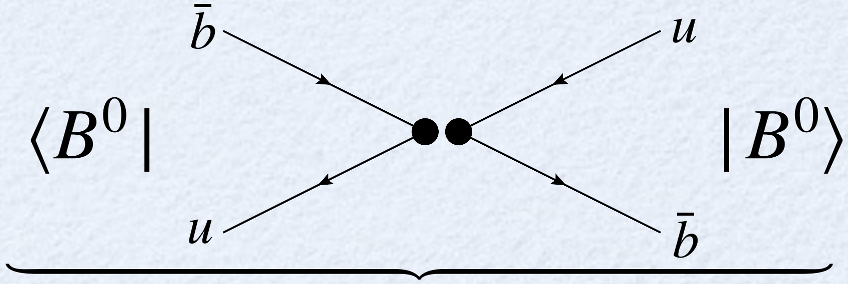
- Weak annihilation contributions ($q = u, d, s$ is the flavor of the spectator quark)

Inputs: Weak Annihilation

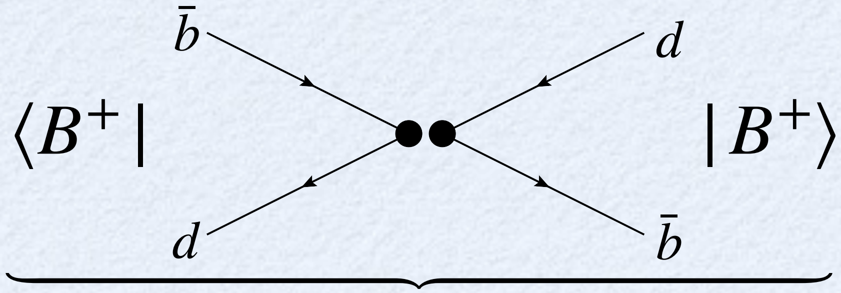
- In the isospin SU(3) limit there are only two WA matrix elements:



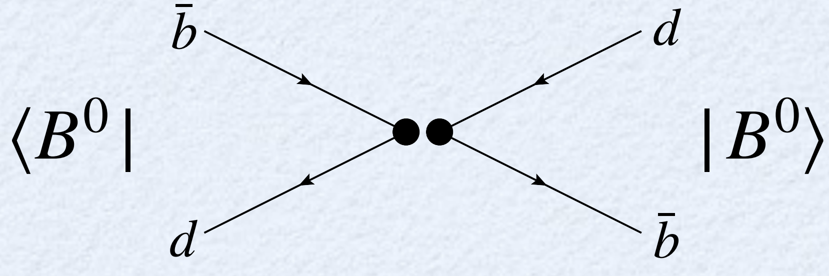
$f_u^{\pm} \equiv f_V$



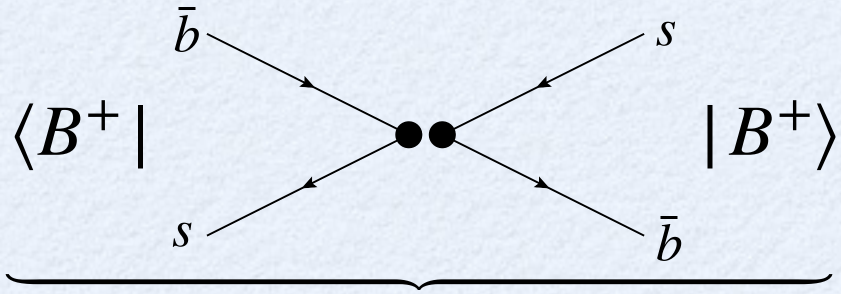
$f_u^{0} \equiv f_{NV}$



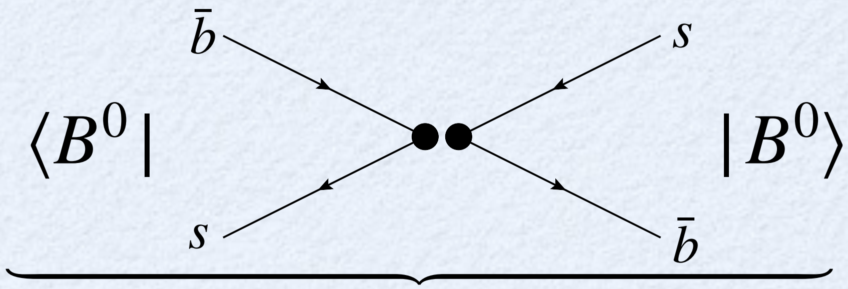
$f_d^{\pm} = f_{NV}$ [$SU(2)$]



$f_d^{0} = f_V$ [$SU(2)$]



$f_s^{\pm} = f_{NV}$ [$SU(3)$]



$f_s^{0} = f_{NV}$ [$SU(3)$]

Inputs: Weak Annihilation

- In the isospin SU(3) limit there are only two WA matrix elements:

$$f_{\rm V} \equiv f_u^{\pm} \overset{SU(2)}{=} f_d^0$$

$$f_{\rm NV} \equiv f_u^0 \overset{SU(2)}{=} f_d^{\pm} \overset{SU(3)}{=} f_s^0 \overset{SU(2)}{=} f_s^{\pm}$$

- Numerically we adopt upper limits extracted from $D^{0,\pm}$ and D_s decays rescaled by a factor $m_B f_B^2/(m_D f_D^2)$ [following the analysis of Gambino, Kamenik]
- We found that $f_{\rm NV}$ and $f_{\rm NV} - f_{\rm V}$ are mostly uncorrelated
- We estimate SU(2) and SU(3) breaking effects following [Ligeti, Tackmann]
- Taking into account the adopted normalizations, we need:

$$\mathscr{B}(B \to X_s \ell^+ \ell^-) \sim \frac{\Gamma(B \to X_s \ell^+ \ell^-)}{\Gamma(B \to X_u \ell \nu)} \Longrightarrow \begin{cases} f_s = (f_s^{\pm} + f_s^0)/2 = f_{\rm NV} \\ f_u = (f_u^{\pm} + f_u^0)/2 = (f_{\rm V} + f_{\rm NV})/2 \end{cases}$$

$$\mathscr{R}(s_0, B \to X_s \ell^+ \ell^-) \sim \frac{\Gamma(B \to X_s \ell^+ \ell^-)}{\Gamma(B^0 \to X_u \ell \nu)} \Longrightarrow \begin{cases} (f_s + f_u^0)/2 = f_{\rm NV} \\ f_s - f_u^0 = [\delta f]_{SU(3)} \end{cases}$$

$B \to X_s \ell\ell$: Error breakdown

- Low-q^2 branching ratio

$$\begin{aligned}\mathcal{B}[1,6]_{\mu\mu} &= (17.29 \pm 0.76_{\text{scale}} \pm 0.19_{m_t} \pm 0.39_{C,m_c} \pm 0.20_{m_b} \pm 0.09_{\alpha_s} \pm 0.02_{\text{CKM}} \pm 0.26_{\text{BR}_{\text{sl}}} \\ &\quad \pm 0.12_{\lambda_2} \pm 0.86_{\text{resolved}}) \times 10^{-7} \\ &= (17.29 \pm 1.28) \times 10^{-7} \quad [7.4\%]\end{aligned}$$

- High-q^2 branching ratio

$$\begin{aligned}\mathcal{B}[>14.4]_{\text{no QED}} &= (2.59 \pm 0.21_{\text{scale}} \pm 0.03_{m_t} \pm 0.05_{C,m_c} \pm 0.19_{m_b} \pm 0.004_{\alpha_s} \pm 0.002_{\text{CKM}} \\ &\quad \pm 0.04_{\text{BR}_{\text{sl}}} \pm 0.10_{\lambda_2} \pm 0.26_{\rho_1} \pm 0.54_{f_{u,s}}) \cdot 10^{-7} \\ &= (2.59 \pm 0.68) \times 10^{-7} \quad [26\%]\end{aligned}$$

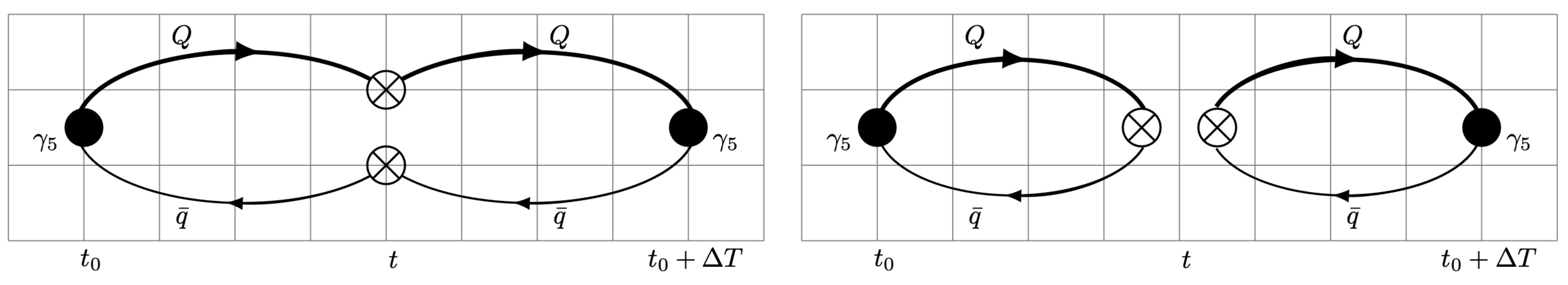
- $\mathcal{R}(s_0) = \Gamma_{s>s_0}(\bar{B} \to X_s \ell\ell)/\Gamma_{s>s_0}(\bar{B}^0 \to X_u \ell\nu)$

$$\begin{aligned}\mathcal{R}(14.4)_{\text{no QED}} &= (26.02 \pm 0.42_{\text{scale}} \pm 0.30_{m_t} \pm 0.11_{C,m_c} \pm 0.10_{m_b} \pm 0.12_{\alpha_s} \pm 1.12_{\text{CKM}} \\ &\quad \pm 0.05_{\lambda_2} \pm 0.33_{\rho_1} \pm 1.20_{f_{u,s}}) \times 10^{-4} \\ &= (26.02 \pm 1.76) \times 10^{-4} \quad [6.8\%]\end{aligned}$$

power corrections errors drops from 23.5% to 4.8%!

Inputs: Weak Annihilation

- Promising progress on a direct determination of some weak annihilation operators from lattice-QCD:



[Black, Harlander, Lange, Rago, Shindler, Witzel]

$$O_1^{cs} = (\bar{c}\gamma_\mu(1-\gamma_5)s)\,(\bar{s}\gamma_\mu(1-\gamma_5)c)$$

$$B_1^{\Delta c=0} = \frac{\langle D_s | O_1^{cs} | D_s \rangle}{m_{D_s}^2 f_{D_s}} \implies f_V$$

$$B_1^{\Delta c=0,\overline{\mathrm{MS}}}(3\ \mathrm{GeV}) = 1.105(13)$$ [preliminary]

$B \to X_s \ell\ell$: Experimental Status

- B-factories
  - Branching ratios measured as sum of exclusive modes from BaBar (424 fb^{-1}) and Belle (140 fb^{-1}):

$$\mathrm{BR}(\bar{B} \to X_s \ell\ell)^{exp}_{low} = \left(15.8 \pm 3.7\right) \times 10^{-7} \qquad \delta_{\mathrm{exp}} = 23\,\% \qquad q^2 \in [1,6]\ \mathrm{GeV}^2$$

$$\mathrm{BR}(\bar{B} \to X_s \ell\ell)^{exp}_{high} = (4.8 \pm 1.0) \times 10^{-7} \qquad \delta_{\mathrm{exp}} = 21\,\% \qquad q^2 > 14.4\ \mathrm{GeV}^2$$

  - Forward backward asymmetry from Belle ($772 \times 10^6\ B\overline{B}$ pairs):

$$\bar{A}^{\mathrm{exp}}_{\mathrm{FB}} = \begin{cases} 0.34 \pm 0.24 \pm 0.02 & q^2 \in [0.2,4.3]\mathrm{GeV}^2 \\ 0.04 \pm 0.31 \pm 0.05 & q^2 \in [4.3,7.3(8.1)]\mathrm{GeV}^2 \end{cases}$$

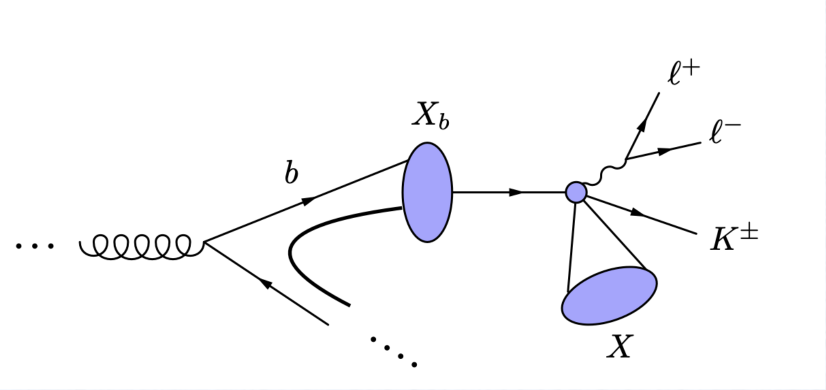
  - Still waiting for Belle analysis using full dataset!
  - BaBar and Belle **include some** collinear photons in the definition of q^2: need to compare with calculation which includes QED radiation (there are small corrections which can be only estimated using the PHOTOS Monte Carlo)
  - Belle-II (as far as we know) is following the same strategy

$B \to X_s \ell \ell$: Experimental Status

- LHCb
  - Until recently the “lore” was that LHCb cannot perform inclusive measurements
  - At **low-q^2** there have been proposals to measure the inclusive rate from:
    - $B^{0,+} \to \mu^+\mu^- K^+ + n\pi^\pm$ (only charged particles) and use isospin to reconstruct the full inclusive rate (this is very similar to current B-factories measurements)

      [Koppenburg, CERN-THESIS-2002-010]
    - $X_b \to K^+\mu^+\mu^- X$, use isospin to reconstruct the X_s system and subtract B_s and Λ_b modes

      [Amhis, Owen, 2106.15943]



[diagram courtesy of J. Jenkins]

  - At **high-q^2** the inclusive rate is dominated by the K, K^* and $K\pi$ modes

    [Buchalla, Isidori, 9801456]
    - Use exclusive LHCb measurements of these three modes (supplemented by Isospin rescaling) to produce an “effective” inclusive measurement at high-q^2.

[Isidori, Polonsky, Tinari, 2305.03076]

[Huber, Hurth, Jenkins, EL, Qin, Vos, 2404.03517]

$B \to X_s \ell\ell$: BR at $q^2 > 15\ \mathrm{GeV}^2$ from LHCb

- For $q^2 > 15\ \mathrm{GeV}^2$ we have $M_{X_s} < 1.41\ \mathrm{GeV}$ the $\mathcal{B}(B \to X_s \mu^+\mu^-)$ rate is saturated by the $X_s = K^{(*)}$, $(K\pi)_{\text{S-wave}}$, $K\pi\pi$ and $K(n\pi)_{n>2}$ modes with progressively smaller contributions
- Dominant $K^{(*)}$ modes [LHCb, 1403.8044]:

$$\begin{aligned} \mathcal{B}(B^+ \to K^+\mu\mu)[>15] &= (0.85 \pm 0.05) \times 10^{-7} \\ \mathcal{B}(B^0 \to K^0\mu\mu)[>15] &= (0.67 \pm 0.12) \times 10^{-7} \\ \mathcal{B}(B^+ \to K^{*+}\mu\mu)[>15] &= (1.58 \pm 0.32) \times 10^{-7} \\ \mathcal{B}(B^0 \to K^{*0}\mu\mu)[>15] &= (1.74 \pm 0.14) \times 10^{-7} \end{aligned}$$

$$\underbrace{\begin{aligned} &\Longrightarrow \mathcal{B}(\bar{B} \to K\mu\mu)[>15] = (0.82 \pm 0.05) \times 10^{-7} \\ &\Longrightarrow \mathcal{B}(\bar{B} \to K^*\mu\mu)[>15] = (1.72 \pm 0.13) \times 10^{-7} \end{aligned}}_{\mathcal{B}(\bar{B} \to K^{(*)}\mu\mu)[>15] = (2.54 \pm 0.14) \times 10^{-7}}$$

$\mathcal{B}(\bar{B} \to K^{(*)}\mu\mu)[>14.18] = (4.23 \pm 0.39) \times 10^{-7}$ [BaBar+Belle]

- $(K\pi)_{\text{S-wave}}$ contribution [LHCb, 1606.04731]:

$$F_S = \frac{\mathcal{B}(B \to (K^+\pi^-)_{J=0}\mu\mu)}{\mathcal{B}(B \to (K^+\pi^-)_{J=0}\mu\mu) + \mathcal{B}(B \to (K^+\pi^-)_{J=1}\mu\mu)} \Longrightarrow F_S\begin{pmatrix} 15 < q^2 < 19 \\ 0.64 < M_X < 1.20 \end{pmatrix} = 0.019^{+0.030}_{-0.025} \pm 0.015$$

Using isospin: $\mathcal{B}(\bar{B} \to (K\pi)_J \ell^+\ell^-) = \mathcal{B}(\bar{B} \to (K^+\pi^-)_J \ell^+\ell^-) \times \begin{cases} \frac{3}{2} & J = 0 \\ 1 & J = 1 \end{cases}$

$$\Longrightarrow \mathcal{B}(\bar{B} \to (K\pi)_S\mu\mu)[>15] = \frac{3}{2}\frac{F_S}{1 - F_S}\mathcal{B}(\bar{B} \to K^{*0}\mu\mu)[>15] = (0.05 \pm 0.09) \times 10^{-7}$$

[to be compared with the χ_{PT} estimate of the same quantity: $(0.58 \pm 0.25) \times 10^{-7}$]

$B \to X_s \ell\ell$: BR at $q^2 > 15\ \mathrm{GeV}^2$ from LHCb

- $K\pi\pi$ contribution [LHCb, 1408.1137]:

$$\frac{\mathscr{B}(B^+ \to K^+\pi^+\pi^- \mu\mu)[14.18 < q^2 < 19]}{\Delta q^2} = \left(0.10^{+0.08}_{-0.06} \pm 0.01\right) \times 10^{-8}\ \mathrm{GeV}^{-2}$$

Assuming that the $K\pi\pi$ mode is dominated by $\pi\pi$ in S wave and using isospin we obtain:

$$\mathscr{B}(\bar{B} \to K\pi\pi\,\mu\mu)[>15] \simeq \mathscr{B}(\bar{B} \to K(\pi\pi)_S \ell^+\ell^-) = \mathscr{B}(B^+ \to K^+(\pi^+\pi^-)_S \ell^+\ell^-) \times \frac{3}{2} = (0.06 \pm 0.04) \times 10^{-7}$$

[where we simply multiply the differential measurement in [14.18,19] to the [15,19] bin we need]

- $K(n\pi)_{n>2}$ contributions are further suppressed and we estimate them as
  $\mathscr{B}(\bar{B} \to K(n\pi)_{n>2}\mu\mu)[>15] \simeq (0.00 \pm 0.04) \times 10^{-7}$
  where the uncertainty is simply lifted from the $K\pi\pi$ mode.

- The complete $K(n\pi)$ contribution is $\mathscr{B}(\bar{B} \to K(n\pi)\mu\mu)[>15] = (0.10 \pm 0.10) \times 10^{-7}$ and accounts for only about $5\,\%$ of the inclusive rate at high-q^2

- Combining $K^{(*)}$ and $K(n\pi)$ modes we finally obtain:

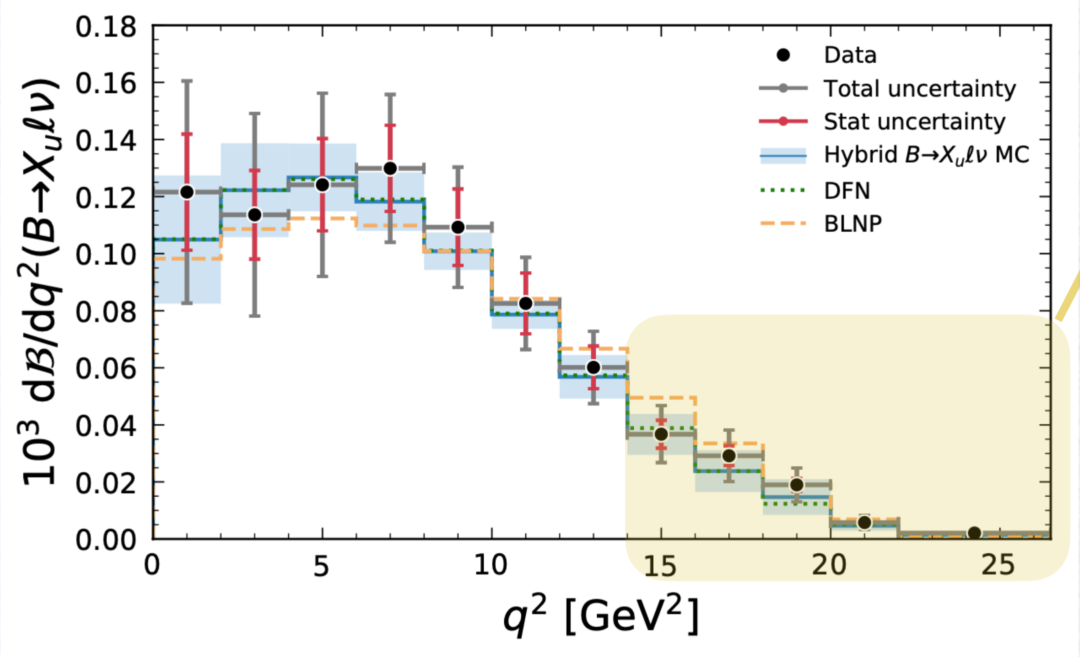
$$\mathscr{B}(\bar{B} \to X_s\mu\mu)[>15] = (2.65 \pm 0.17) \times 10^{-7}$$

Result obtained in collaboration
with G. Isidori, Z, Polonsky and A. Tinari

$B \to X_s \ell\ell$: SM predictions

- Using the Belle [2107.13855] measurement of the $B \to X_u \ell\nu$ q^2 spectrum we can convert our SM prediction for the ratio $\mathscr{R}(q_0^2)$ into a "experiment assisted prediction" for the high-q^2 branching ratio.
- This SM prediction can be used to discuss compatibility with the exclusive anomalies under the assumption of no New Physics in $B \to X_u \ell\nu$

Belle, 711 fb^{-1}



$$\mathscr{B}(\bar{B} \to X_u \ell \bar{\nu})[>14.4]_{\text{exp}} = (1.76 \pm 0.32) \times 10^{-4} \quad [18.2\%]$$

$$\mathscr{B}(\bar{B} \to X_u \ell \bar{\nu})[>15]_{\text{exp}} = (1.52 \pm 0.28) \times 10^{-4} \quad [18.4\%]$$

$$\Downarrow$$

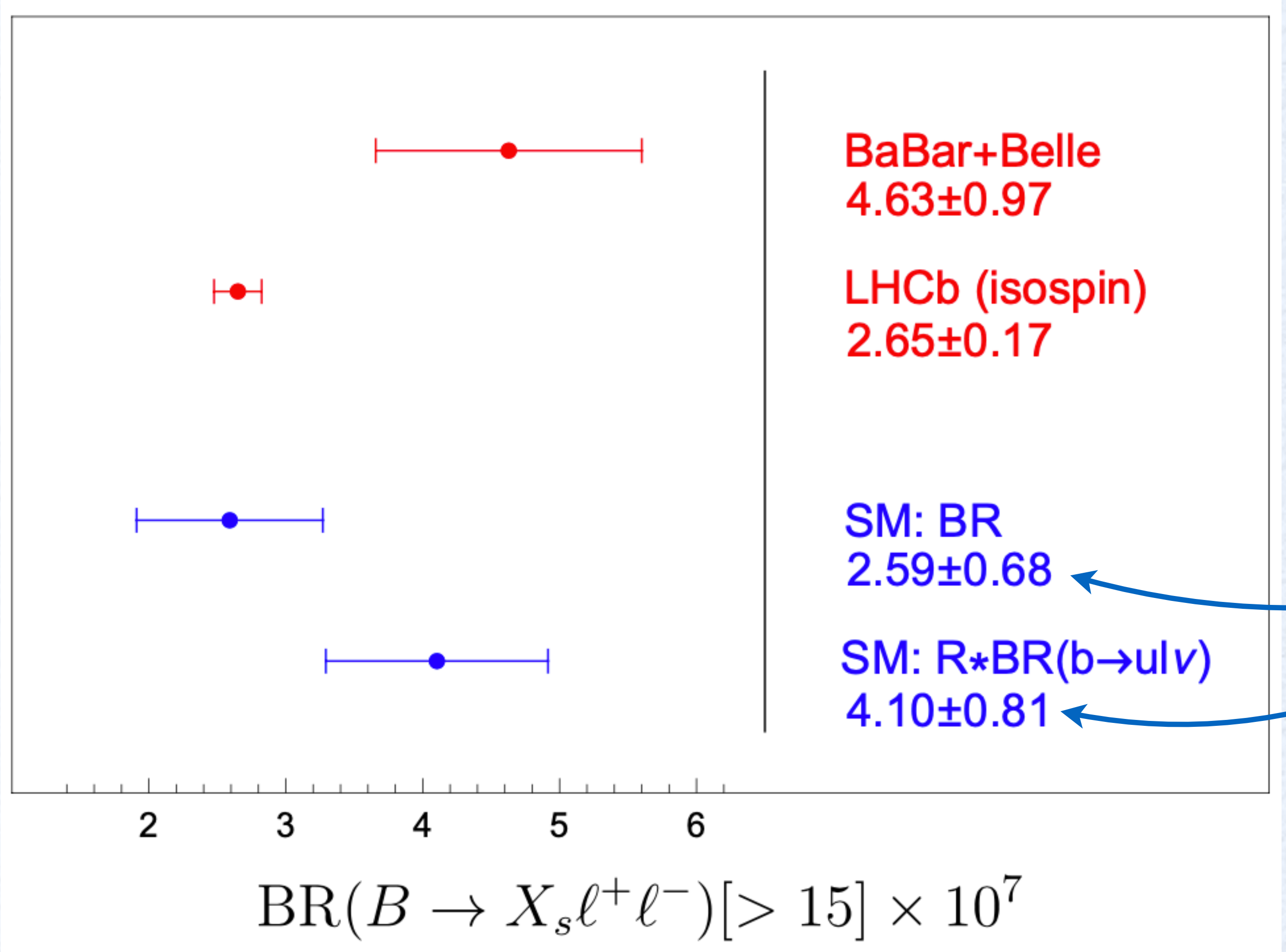
$$\mathscr{B}[>14.4]_{\text{SM},\mathscr{R}} = \mathscr{R}(14.4) \times \mathscr{B}(B \to X_u \ell \bar{\nu})[>14.4]_{\text{exp}}$$
$$= (4.58 \pm 0.89) \times 10^{-7} \quad [19.4\%]$$

$$\mathscr{B}[>15]_{\text{SM},\mathscr{R}} = \mathscr{R}(15) \times \mathscr{B}(B \to X_u \ell \bar{\nu})[>15]_{\text{exp}}$$
$$= (4.10 \pm 0.81) \times 10^{-7} \quad [19.8\%]$$

- The total uncertainty is dominated by the $B \to X_u \ell\nu$ partial rate

$B \to X_s \ell\ell$: theory vs experiment

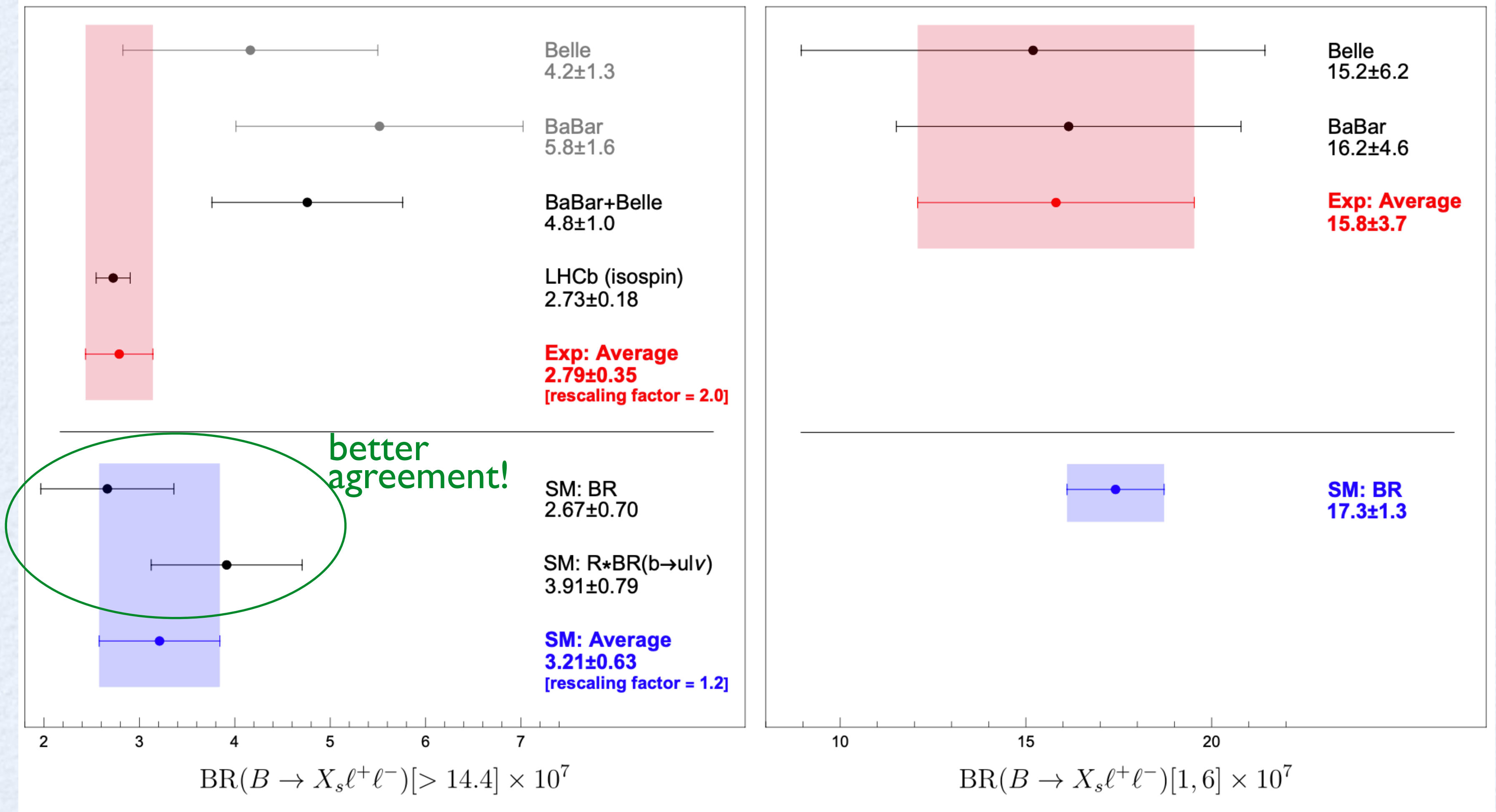
- Let's begin putting all this information together by "rescaling" the BaBar and Belle inclusive measurements to something that can be directly compared to the LHCb one:
  BaBar (with QED, $q_0^2 = 14.2$), Belle (with QED, $q_0^2 = 14.4$), LHCb (no QED, $q_0^2 = 15$).
- The required rescaling factors are: $\left(\frac{\mathcal{B}[>14.4]_{\text{with QED}}}{\mathcal{B}[>14.2]_{\text{with QED}}}\right)_{\text{SM}} = 0.96$ and $\left(\frac{\mathcal{B}[>15]_{\text{no QED}}}{\mathcal{B}[>14.4]_{\text{with QED}}}\right)_{\text{SM}} = 0.97$



- The picture that emerges is not clear: there are tensions between the two experimental and the two theoretical determinations!
- The two SM predictions are dominated by power corrections and by the experimental $b \to u \ell \nu$ rate, respectively.

$B \to X_s \ell \ell$: theory vs experiment

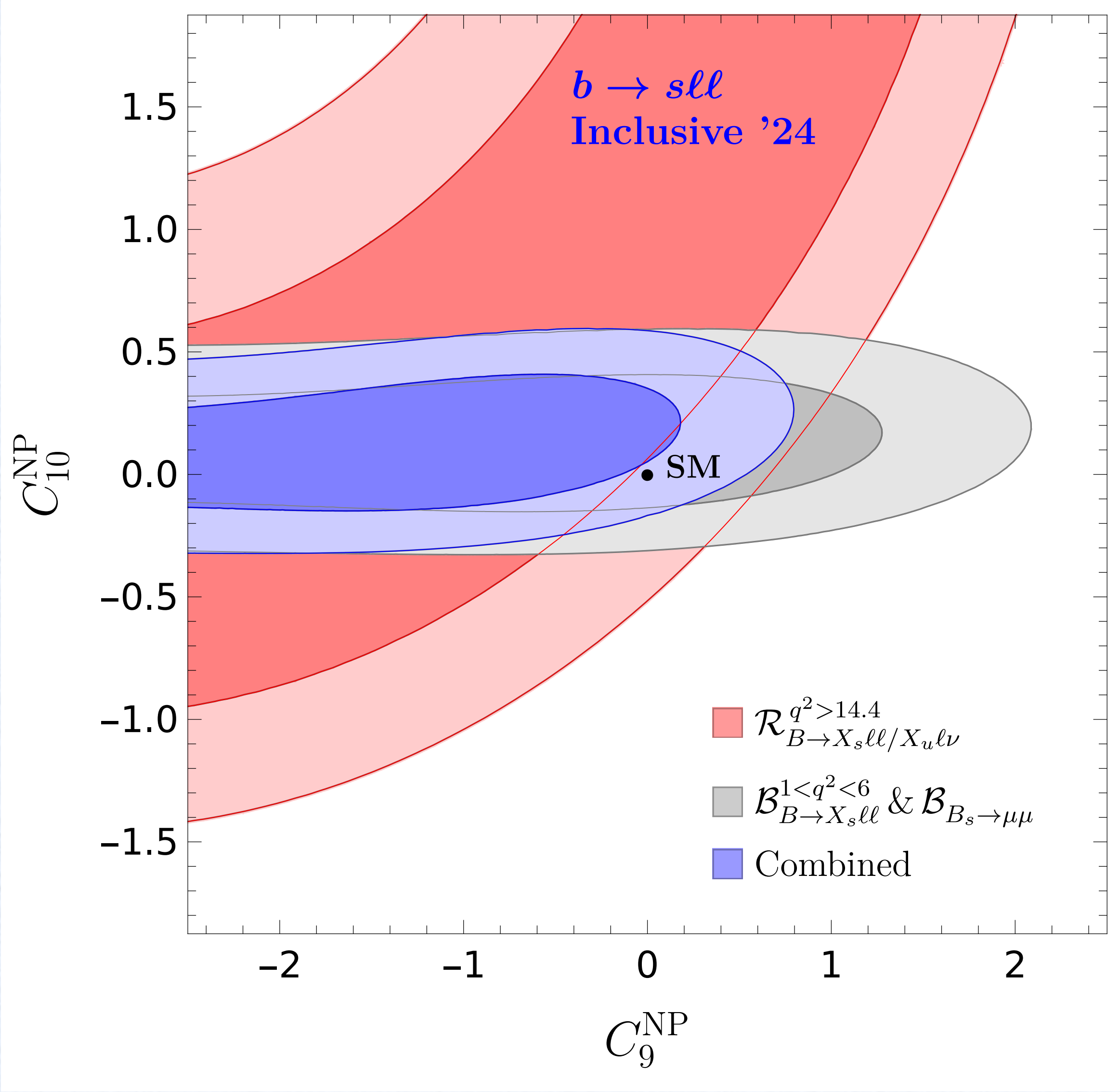
- All results corresponding to a cut at 14.4 GeV^2 (the LHCb "measurement" has been rescaled)
- A lower q^2 cut corresponds to a larger hadronic phase space for which the heavy quark expansion is expected to be under better control



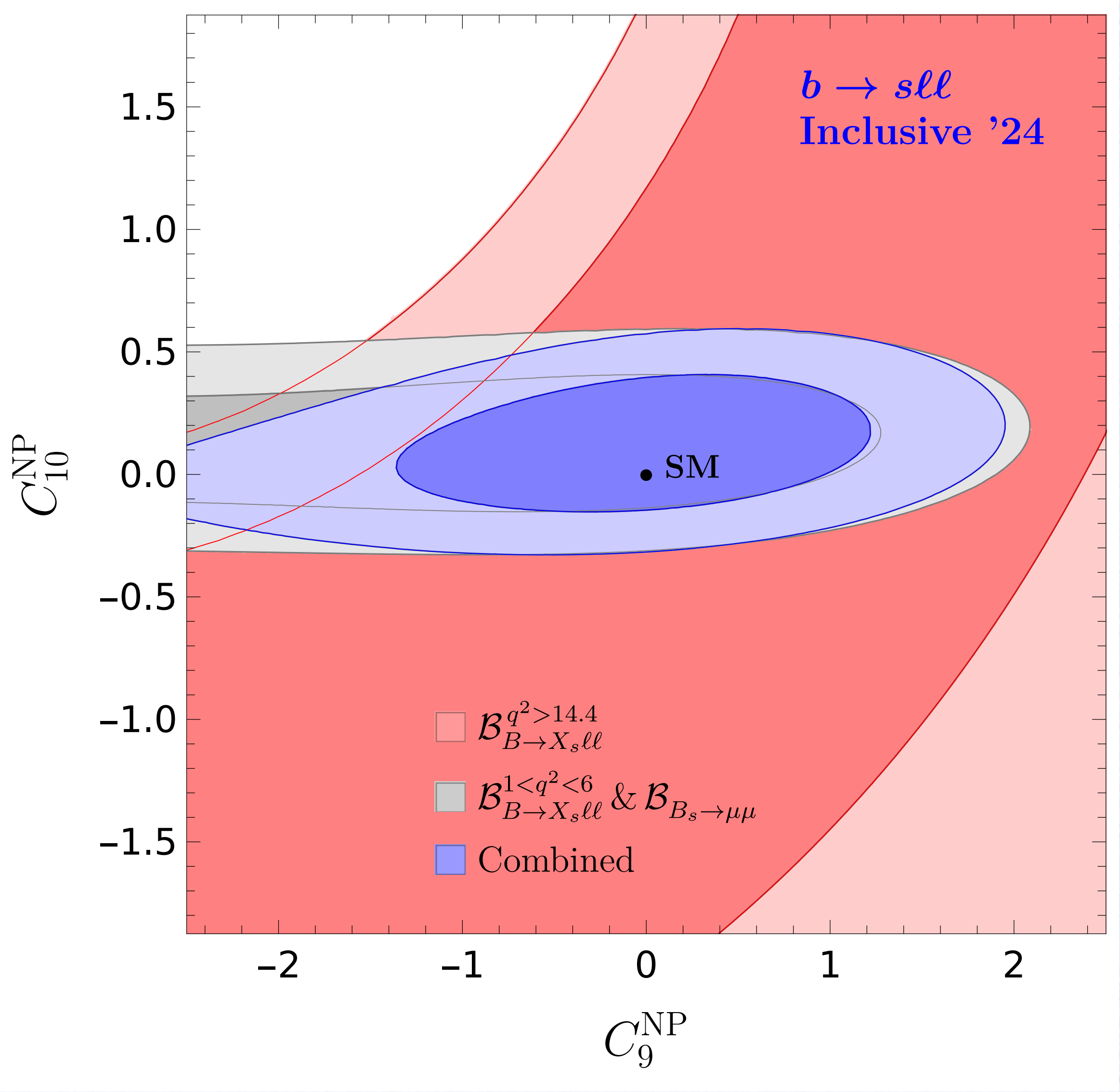
- Average Exp and SM determinations (in the latter case we include a $20\,\%$ correlation)
- The experimental average requires a PDG rescaling factor of 2!
- Keeping in mind several provisos we see that currently there seems to be good agreement

Current constraints

- Exp inputs: $\mathrm{BR}(B_s \to \mu\mu)$, $\mathrm{BR}(B \to X_s\ell\ell)^{\mathrm{B-factories}}_{\mathrm{low}}$, $\mathrm{BR}(B \to X_s\ell\ell)^{\mathrm{LHCb}}_{\mathrm{high}}$, $\mathrm{BR}(B \to X_u\ell\nu)^{\mathrm{Belle}}_{\mathrm{high}}$



using $\mathscr{R}[14.4]_{\mathrm{SM}}$

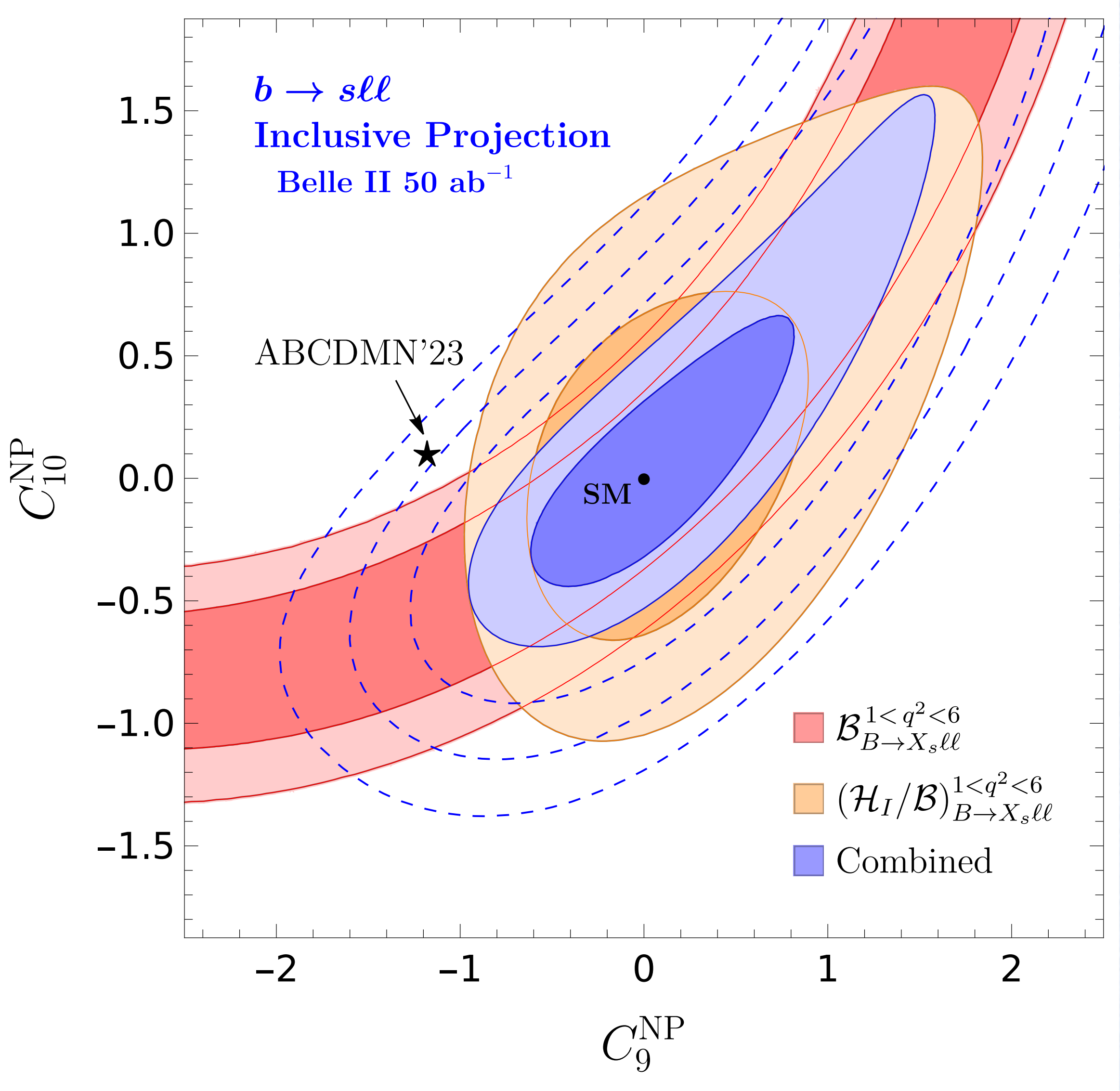


using $\mathrm{BR}(B \to X_s\ell\ell)_{\mathrm{high}}$

- High-q^2 constraints are of similar strength because of large error on $\mathrm{BR}(B \to X_u\ell\nu)^{\mathrm{Belle}}_{\mathrm{high}}$
- Overall picture is of agreement with the SM

Future constraints: low-q^2 (Belle II)

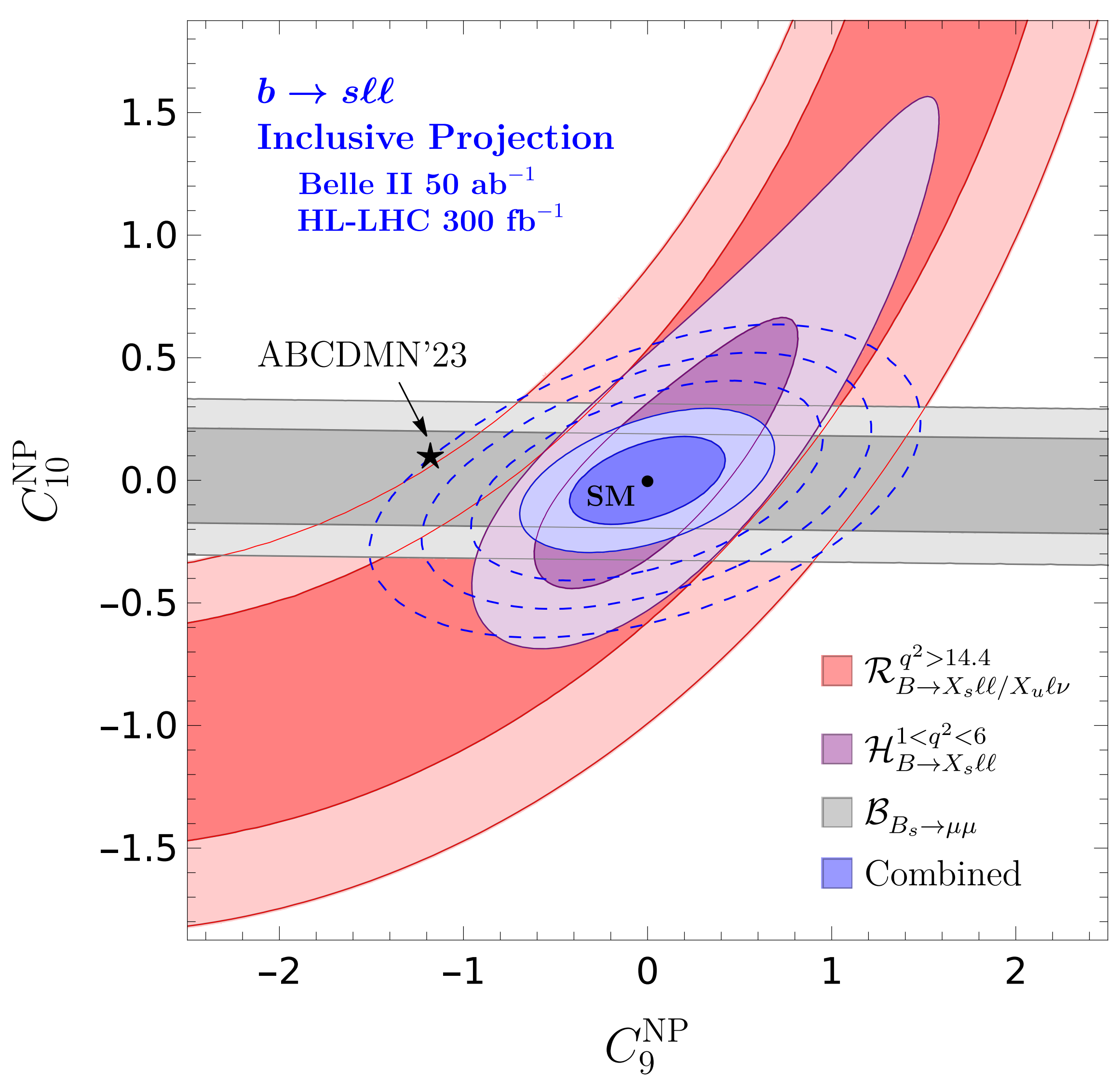
- Projected reach of Belle II with $50\ \mathrm{ab}^{-1}$ of integrated luminosity



- Focus on low-q^2 where the inclusive OPE is better behaved
- Use of normalized angular observables ($H_I/\mathscr{B}$), lowers impact of the M_X cut in the low-q^2 region
- Dashed contours correspond to 3σ, 4σ and 5σ
- ★ is the exclusive best fit from ABCDMN'23
- Low-q^2 observables at Belle-II should be able to confirm current anomalies at 4σ

Future constraints: including high-q^2

- Projected reach of Belle II with $50\ \mathrm{ab}^{-1}$ and of LHCb with $300\ \mathrm{fb}^{-1}$ of integrated luminosity



- We assume $\delta(B_s \to \mu\mu) = 4.8\,\%$ corresponding to $300\ \mathrm{fb}^{-1}$ at the HL-LHC

- Projected uncertainty on $\mathscr{R}(14.4)$ is obtained combining:

$$\delta\mathscr{B}_{bs\ell\ell}[>14.4] = \sqrt{(2.6\%_{\mathrm{stat}})^2 + (3.9\%_{\mathrm{syst}})^2} = 4.7\,\%$$

$$\delta\mathscr{B}_{bu\ell\nu}[>14.4] = 5.2\,\%$$

$$\Longrightarrow \delta\mathscr{R}(14.4) = 7.0\,\%$$

- Inclusion of high-q^2 observables allows to confirm the exclusive anomalies at the 5σ level

$K^+ \to \pi^+ \nu\bar{\nu}$

- The very rare Kaon decays $K^\pm \to \pi^\pm \nu\bar{\nu}$ and $K_L \to \pi^0 \nu\bar{\nu}$ are extremely clean and can be sensitive to new physics whose contributions to B physics observables are overwhelmed by QCD uncertainties.

- Experiment [NA62, 2412.12015]

$$\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{exp}} = \left(13.0^{+3.0}_{-2.7}|_{\mathrm{stat}} \pm 1.3_{\mathrm{syst}}\right) \times 10^{-11}$$

- SM prediction [Kaons@CERN 2023, 2311.02923 (UTfit inputs)]

$$\begin{aligned}\mathrm{BR}(K^+ \to \pi^+\nu\bar{\nu})_{\mathrm{SM}} &= \kappa_+(1+\Delta_{\mathrm{EM}})\left[\left(\frac{\mathrm{Im}\lambda_t}{\lambda^5}X(x_t)\right)^2 + \left(\frac{\mathrm{Re}\lambda_t}{\lambda^5}X(x_t) + \frac{\mathrm{Re}\lambda_c}{\lambda}(P_c + \delta P_{c,u})\right)^2\right] \\ &= \left(8.38 \pm \underset{\downarrow}{0.17_{\mathrm{pert}}} \pm 0.36_{\mathrm{CKM}} \pm 0.17_{\mathrm{param}} \pm \underset{\downarrow}{0.25_{\delta P_{c,u}}}\right) \times 10^{-11}\end{aligned}$$

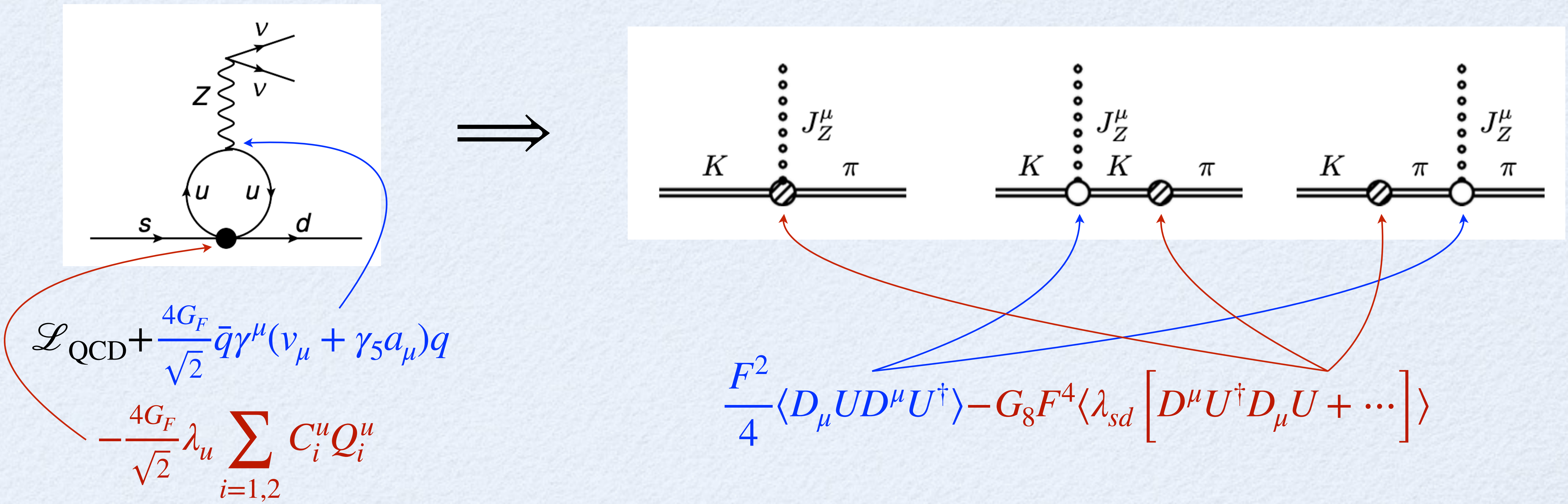
requires NNLO$_{\mathrm{QCD}}$

dominant non-parametric uncertainty

- Small 1.5σ tension

$K^+ \to \pi^+ \nu\bar{\nu}$

- The $\delta P_{u,c}$ contribution stems from non-perturbative up-quark loop contributions
- While we wait for first principle lattice-QCD calculations we need to rely on ChiPT:
  [Isidori, Mescia, Smith, 0503107]



$$\mathscr{L}_{\rm QCD} + \frac{4G_F}{\sqrt{2}}\bar{q}\gamma^\mu(v_\mu + \gamma_5 a_\mu)q$$

$$-\frac{4G_F}{\sqrt{2}}\lambda_u \sum_{i=1,2} C_i^u Q_i^u$$

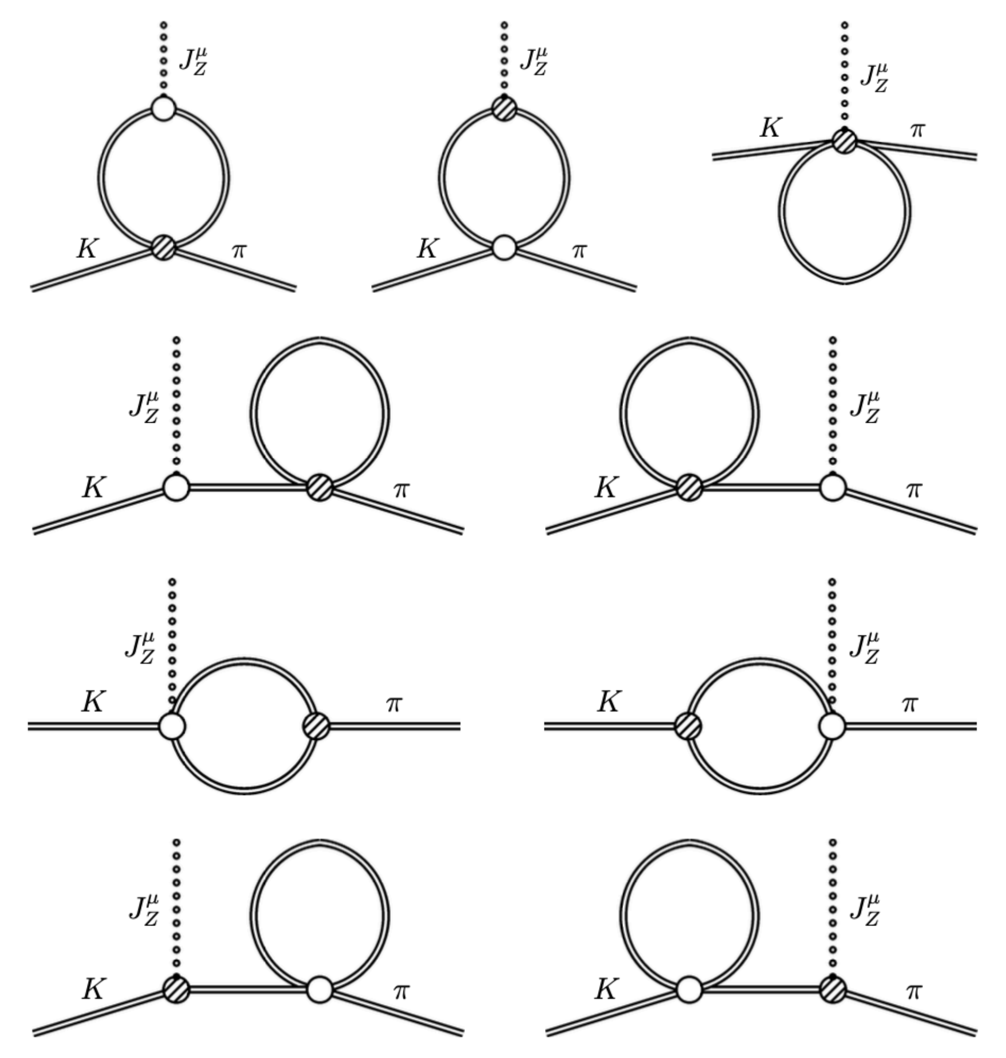
$$\frac{F^2}{4}\langle D_\mu U D^\mu U^\dagger\rangle - G_8 F^4 \langle \lambda_{sd}\left[D^\mu U^\dagger D_\mu U + \cdots\right]\rangle$$

- The tree-level diagrams above (including also $W^\pm$ exchange boxes) yield:

$$\delta P_{c,u} = \frac{1}{3}\sum_{\ell=e,\mu}\langle P_Z(q^2) + P_{WW}^{\ell}(q^2)\rangle = \frac{\pi^2 F^2}{\lambda^4 M_W^2}\left[\frac{4|G_8|}{\sqrt{2}\lambda G_F} - \frac{4}{3}\right]$$

$K^+ \to \pi^+ \nu \bar{\nu}$

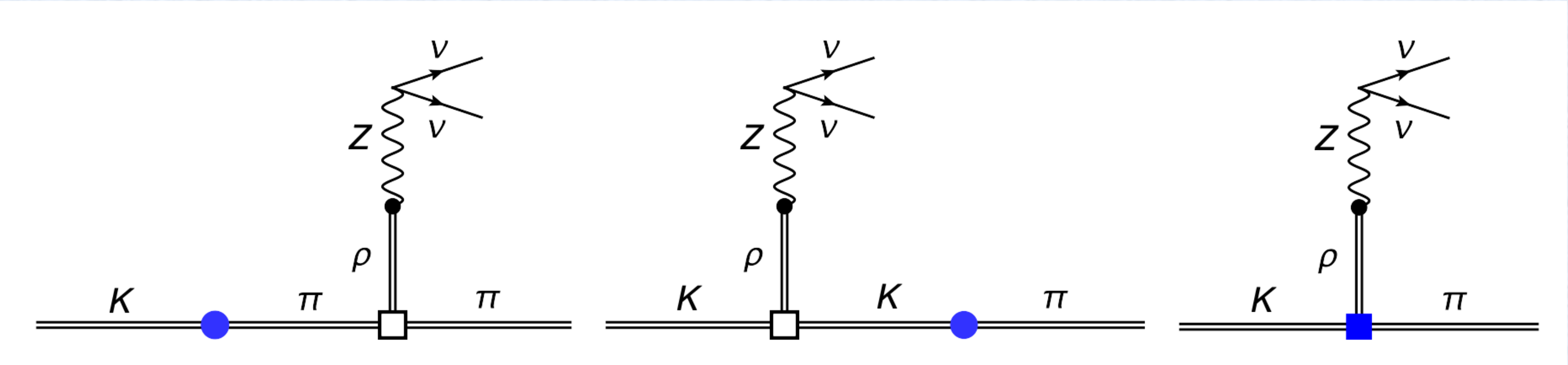
- At the one-loop level there are many diagrams for which not all counter-terms are known

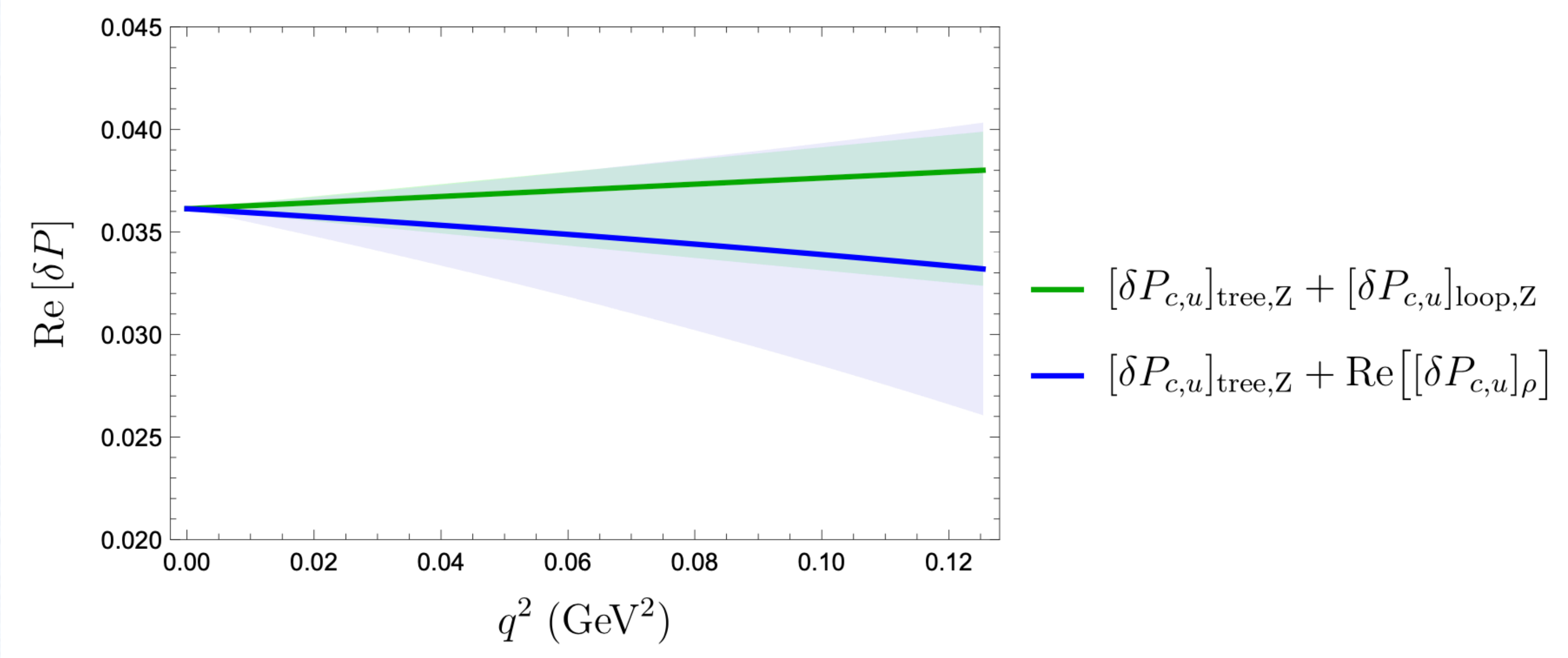


- To take into account these missing corrections Isidori et al. assigned a 50% uncertainty to the tree-level central value
- The effective coupling $G_8 \equiv -V_{ud}V_{us}^* G_F/\sqrt{2}g_8$ can be determined by NLO fits to $K \to \pi\pi$ amplitudes [Cirigliano, Gisbert, Pich, Rodríguez-Sànchez, 1911.01359]:
  $g_8 = 3.58 \pm 0.14$
- Using the updated g_8 one gets:
  $\delta P_{c,u} = 0.03(1 \pm 0.5) = 0.030 \pm 0.015$
  [using inputs from [0503107] one gets 0.04 ± 0.02 which is the value commonly adopted]

$K^+ \to \pi^+ \nu\bar{\nu}$

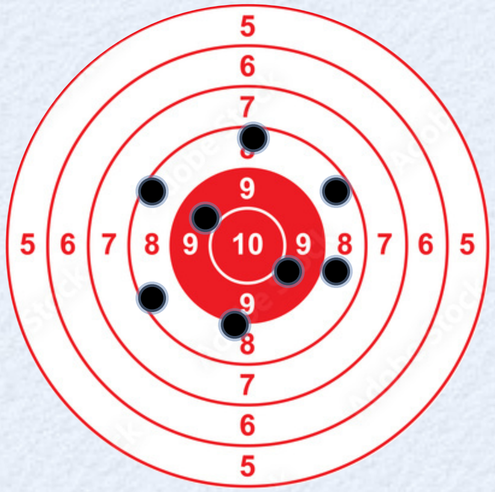
- In [Ecker, Kambor, Wyler NPB 394 (1993) 101] it was shown that vector meson contributions are sufficient to reproduce the observed values of all low-energy constants
- Inspired by this result we considered **resonant ChiPT** (in terms of antisymmetric spin $= 1$ fields) and used the so-called weak deformation model to estimate the $\Delta S = 1$ matching: [Lunghi, Soni, 2408.11190]





- We find that ρ meson effects tend to be much smaller than the conservative $\pm 50\,\%$ estimate adopted in the literature
- Even stronger motivation to get a first principle lattice-QCD estimate!
- Stay tuned for a dispersive approach to this calculation [Bansal, Jenkins, Winney]

Conclusions



$b \to s\ell\ell$
inclusive

$b \to s\ell\ell$
exclusive

$K \to \pi\nu\bar{\nu}$

R_K, R_{K^*} 🙁

[Idea curtesy of Maria Ubiali]

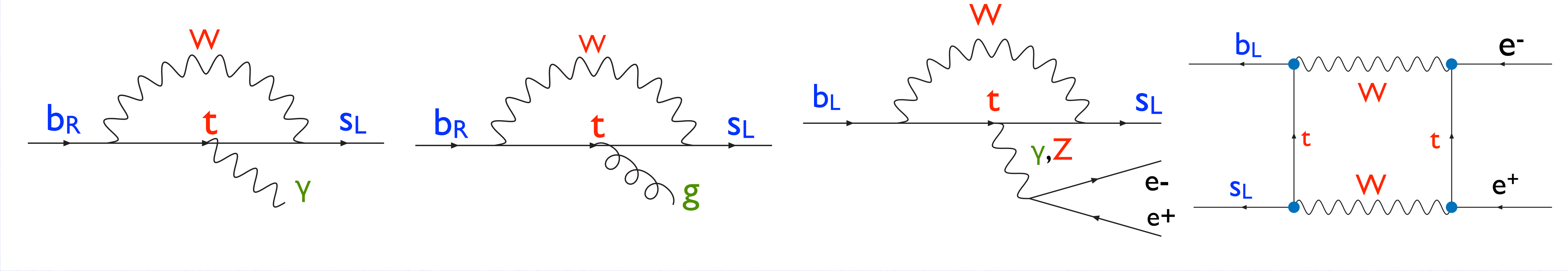
CREDITS


Flumserberg
Monselice
Rivello
©2009 Google - Map data ©2009 Tele A

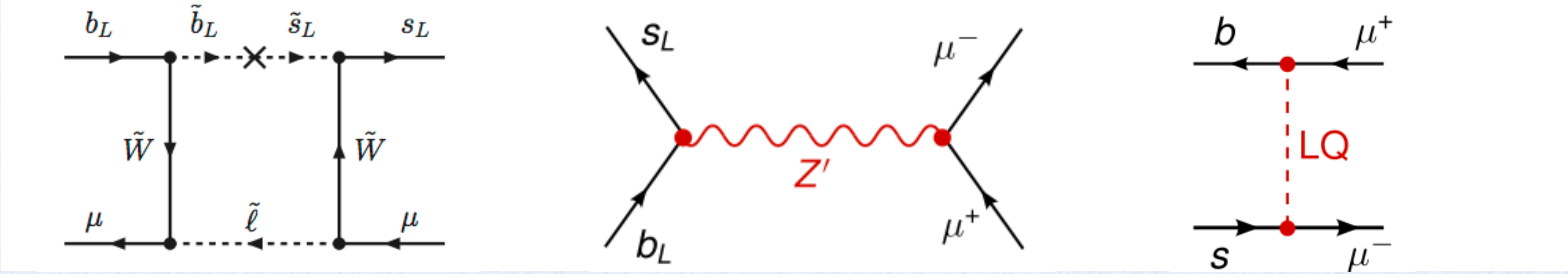
BACKUP

SM vs New Physics

SM contributions:



NP contributions:

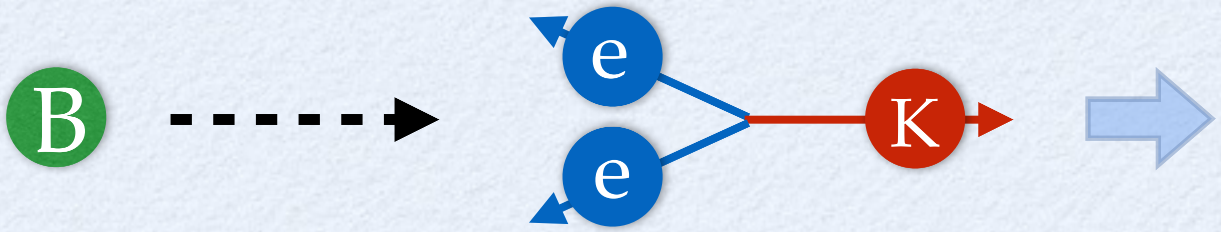


Exclusive modes: theoretical frameworks

- The central problem is the calculation of matrix elements:

$$\langle K^{(*)}\ell\ell|O(y)|B\rangle \;\approx\; \langle K^{(*)}|T\,J^{\rm em}_\mu(x)\,O(y)|B\rangle$$

- At low-q^2 the $K^{(*)}$ has large energy (large recoil):

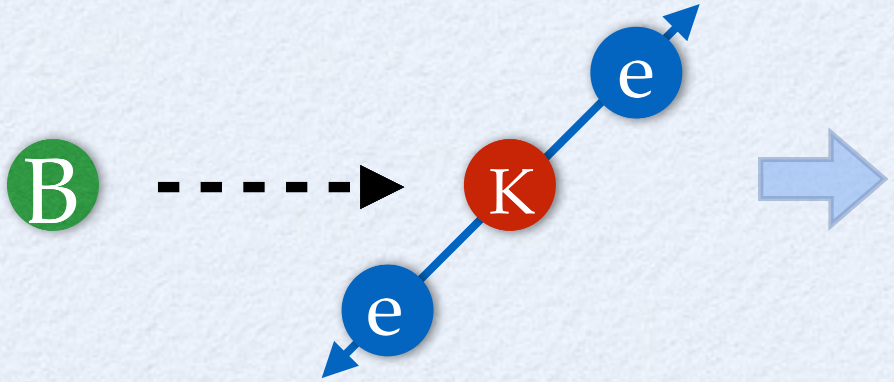


Soft-Collinear Effective Theory

The large energy of the $K^{(*)}$ introduces three scales: m_b^2, Λm_b and Λ^2:

$$\langle K^{(*)}|TJ^{\rm em}_\mu(x)O(y)|B\rangle \sim C \times [\text{ Form Factor} + \phi_B \star J \star \phi_{K^{(*)}}] + O\left(\Lambda/m_b\right)$$

- At high-q^2 the $K^{(*)}$ does not recoil:



$$(x-y)^2 \sim \frac{1}{q^2} \sim \frac{1}{m_b^2}$$

local OPE

$$\langle K^{(*)}|TJ^{\rm em}_\mu(x)O(y)|B\rangle \sim C \times [\text{ Form Factor}] + O\left(\Lambda/m_b\right)$$

Exclusive modes: issues

- Form factors
  - lattice QCD (high-q^2): $B \to K$ complete, $B \to K^*$ and $B_s \to \phi$ ongoing

    [see the recently completed FLAG 2024 review, 2411.04268]
  - LCSR (low-q^2): some uncertainties have to be ball-parked (power corrections, …) but get access to all form factors (including baryons)

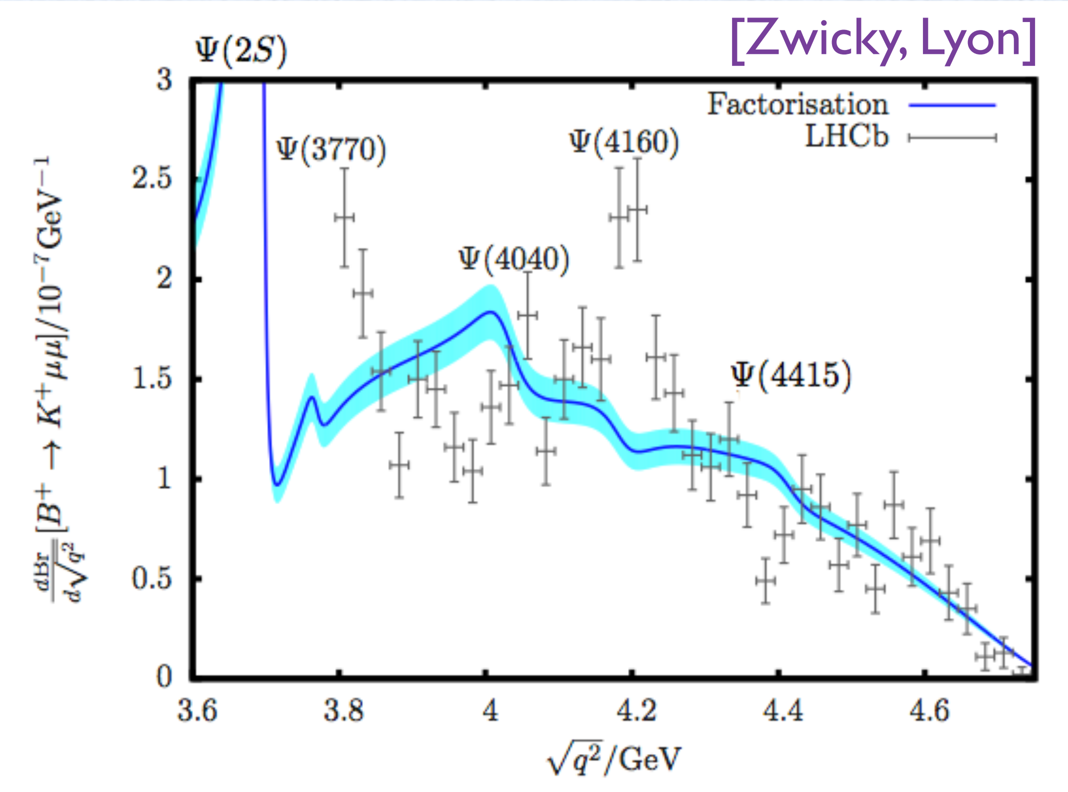
- Power corrections
  - Presently incalculable
  - In global fits they are taken into account via nuisance parameters
  - If no form factors relations are used, their impact is not expected to be too large because they are essentially confined to the the matrix element $\langle K^{(*)} | T J_\mu^{\rm em} O_2 | B \rangle$

    [see, for instance, the $B \to K$ analysis presented in Fermilab/MILC and EL, 1903.10434]
  - If form factors relations are used ⇒ construct "clean observables" (e.g. P_5')
  - A lot of recent work on extracting information on power corrections using dispersion techniques

    [see, for instance, Gubernari, van Dyk, Virto, 2011.09813 Mutke, Hoferichter, Kubis, 2406.14608]

Exclusive modes: issues

- Resonances at high-q^2
  - Unsurprisingly naive factorization fails to reproduce the resonant pattern observed in $B \to K\mu\mu$ at high-q^2
  - The OPE and quark-hadron duality lead to a reliable prediction for the integrated high-q^2 branching ratio [Beylich, Buchalla]
  - Within naive factorization the contribution of the “wiggles” is non-negligible
  - This has led to some uneasiness about our ability to use the high-q^2 region effectively



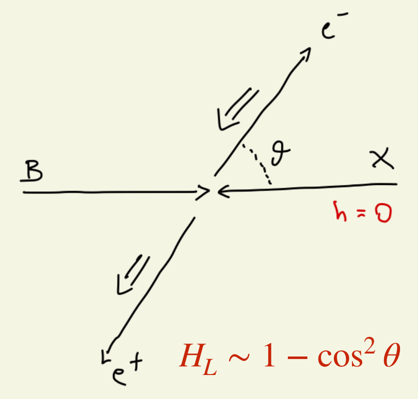
[Zwicky, Lyon]

- All these anomalies need confirmation at Belle II
  - different systematics
  - access to more observables (inclusive modes)

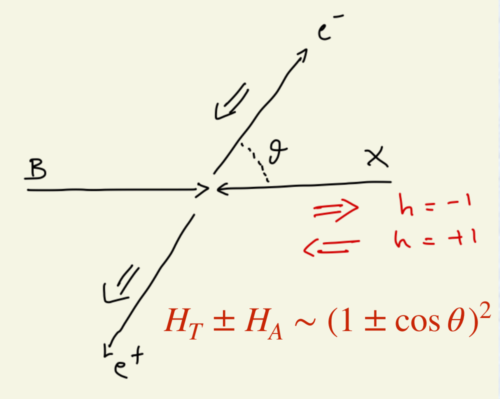
Inclusive theory: observables

- Structure of the differential decay width:

$$\frac{d^2\Gamma^{X_s}}{dq^2\, d\cos\theta_\ell} = \frac{3}{8}\left[(1+\cos^2\theta_\ell)\, H_T + 2(1-\cos^2\theta_\ell)\, H_L + 2\cos\theta_\ell\, H_A\right]$$



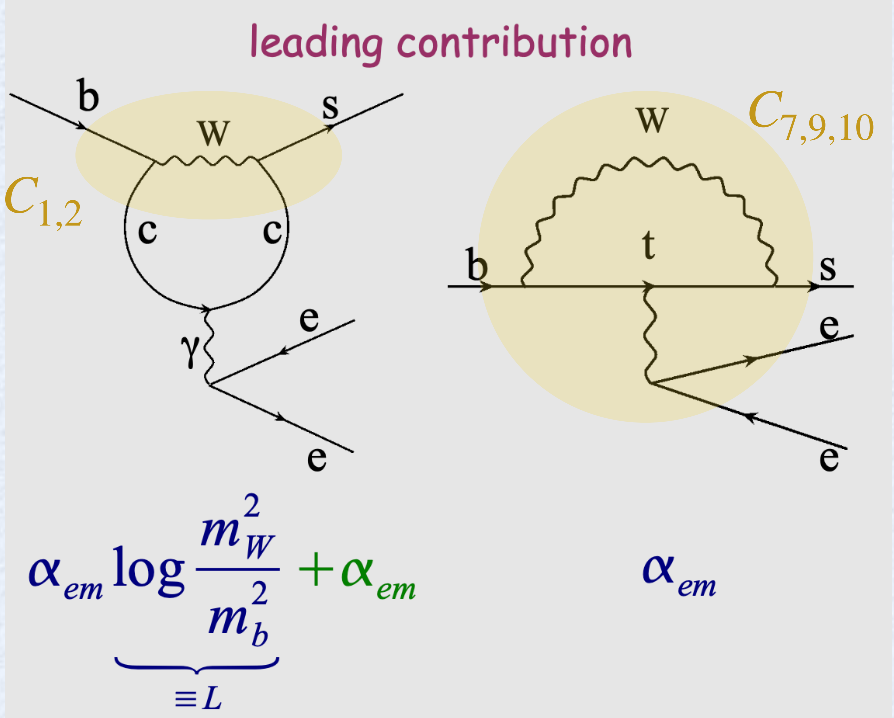
$$\begin{aligned} H_T &\sim 2\hat{s}(1-\hat{s})^2\left[\left|C_9 + \frac{2}{\hat{s}}C_7\right|^2 + |C_{10}|^2\right] \\ H_L &\sim (1-\hat{s})^2\left[|C_9 + 2C_7|^2 + |C_{10}|^2\right] \\ H_A &\sim -4\hat{s}(1-\hat{s})^2 \mathrm{Re}\left[C_{10}\left(C_9 + 2\frac{m_b^2}{q^2}C_7\right)\right] \end{aligned}$$

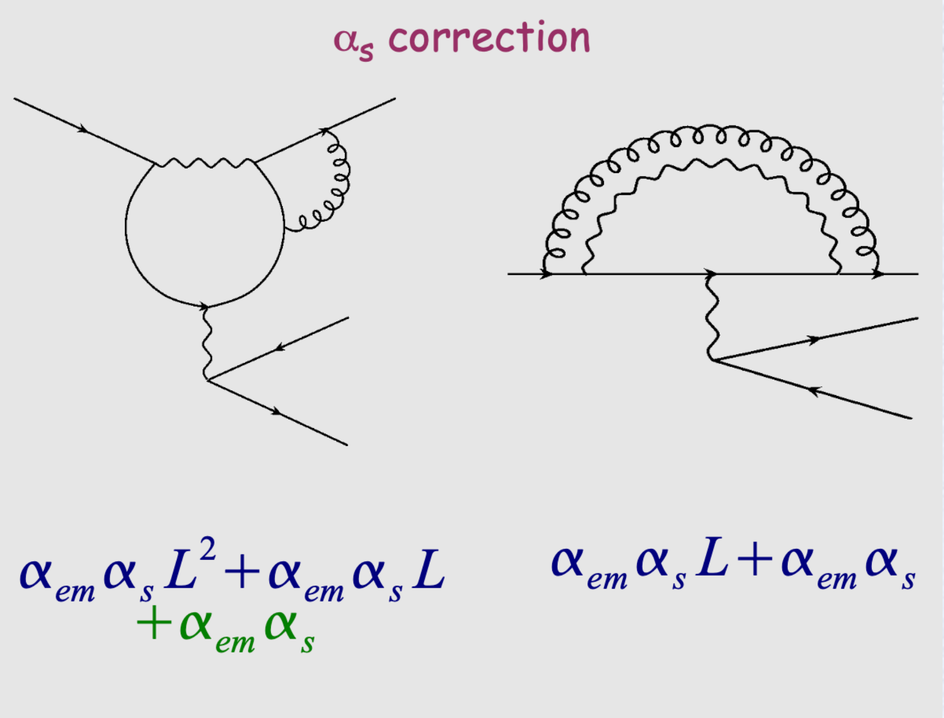


- In the SM H_A is not suppressed by the lepton mass
- There are similar contributions from non-SM operators but there is no interference between $V + A$ and $V - A$ structures
- At low-q^2 ($\hat{s} < 0.3$) H_T is suppressed ($C_7 < 0$)

Inclusive theory: leading power ($b \to X_s \ell \ell$)

- The perturbative expansion has two peculiar features:
  - the amplitude is proportional to $\alpha_{\rm e}(\mu)$
  - the one-loop matrix element of $O_{1,2}$ is “super-leading”





$C_1\langle O_1\rangle + C_2\langle O_2\rangle$ $\qquad$ $C_7\langle O_7\rangle + C_9\langle O_9\rangle + C_{10}\langle O_{10}\rangle$

$$\eta \equiv \frac{\alpha_s(\mu_0)}{\alpha_s(\mu_b)} = 1 + \beta_s^{(00)}\frac{\alpha_s(\mu_0)}{4\pi}\log\frac{\mu_b^2}{\mu_0^2} \sim O(1) \Longrightarrow \log\frac{\mu_b^2}{\mu_0^2} \sim \frac{1}{\alpha_s(\mu_0)}$$

$$\frac{\alpha_e(\mu_0)}{\alpha_e(\mu_b)} = 1 + \beta_e^{(00)}\frac{\alpha_e(\mu_0)}{4\pi}\log\frac{\mu_b^2}{\mu_0^2} \sim 1 + \frac{\alpha_e(\mu_0)}{\alpha_s(\mu_0)}$$

Expansion in α_s and $\kappa = \alpha_e/\alpha_s$

Inclusive theory: leading power ($b \to X_s \ell \ell$)

- Structure of the amplitude ($\kappa = \alpha_{\rm em}/\alpha_s$ and $\tilde{\alpha}_s = \alpha_s/4\pi$):

$$A = \kappa \left[A_{\rm LO} + \tilde{\alpha}_s A_{\rm NLO} + \tilde{\alpha}_s^2 A_{\rm NNLO} + \tilde{\alpha}_s^3 A_{\rm N^3LO} + O(\tilde{\alpha}_s^4)\right]$$

$$+\kappa^2 \left[A^{\rm em}_{\rm LO} + \tilde{\alpha}_s A^{\rm em}_{\rm NLO} + \tilde{\alpha}_s^2 A^{\rm em}_{\rm NNLO} + \tilde{\alpha}_s^3 A^{\rm em}_{\rm N^3LO} + O(\tilde{\alpha}_s^4)\right] + O(\kappa^3)$$

with $A^{\rm em}_{\rm LO} \lesssim A_{\rm LO} \sim 0.03$ and $A_{\rm NLO} \sim 4$

include only terms enhanced by $m_t^2/(M_W^2 \sin^2\theta_W)$ and $\log(m_b^2/m_\ell^2)$

- Decay width:

$$|A|^2 = \kappa^2 \left[A^2_{\rm LO} + \tilde{\alpha}_s 2 A_{\rm LO} A_{\rm NLO} + \tilde{\alpha}_s^2 A^2_{\rm NLO}\right]$$

$$+\kappa^2 \left[\tilde{\alpha}_s^2 A_{\rm LO} A_{\rm NNLO} + \alpha_s^3 \left(2 A_{\rm NLO} A_{\rm NNLO} + 2 A_{\rm LO} A_{\rm N^3LO}\right)\right]$$

$$+\kappa^3 \Big[2 A_{\rm LO} A^{\rm em}_{\rm LO} + \tilde{\alpha}_s \left(2 A_{\rm NLO} A^{\rm em}_{\rm LO} + 2 A_{\rm LO} A^{\rm em}_{\rm NLO}\right)$$

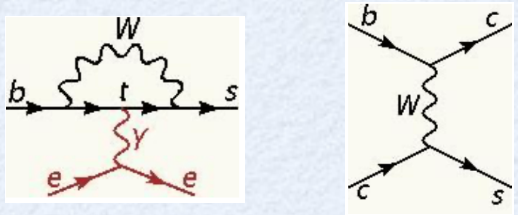
$$+\tilde{\alpha}_s^2 \left(2 A_{\rm NLO} A^{\rm em}_{\rm NLO} + 2 A_{\rm NNLO} A^{\rm em}_{\rm LO} + A_{\rm LO} A^{\rm em}_{\rm NNLO}\right)$$

$$+\tilde{\alpha}_s^3 \left(2 A_{\rm NLO} A^{\rm em}_{\rm NNLO} + 2 A_{\rm NNLO} A^{\rm em}_{\rm NLO} + 2 A_{\rm N^3LO} A^{\rm em}_{\rm LO} + 2 A_{\rm LO} A^{\rm em}_{\rm N^3LO}\right)\Big] + O(\kappa^4)$$

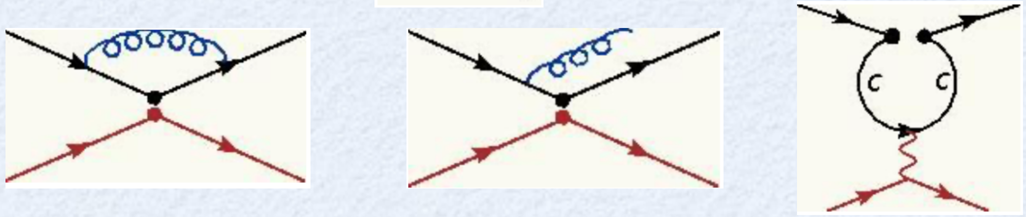
Inclusive theory: leading power ($b \to X_s \ell \ell$)

- QCD at NLO ($A_{\rm LO}, A_{\rm NLO}$)

WCs:



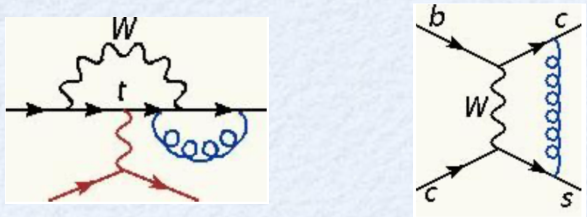
MEs:



Misiak
Buras, Münz

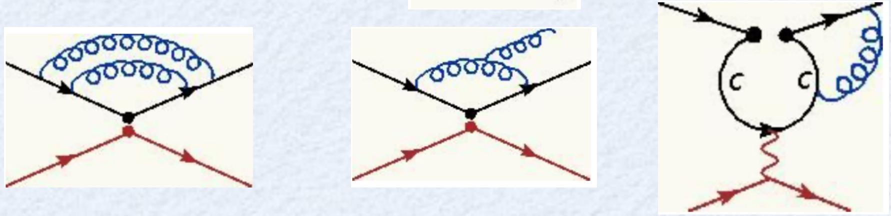
- QCD at NNLO ($A_{\rm NNLO}$)

WCs:



Bobeth, Misiak, Urban

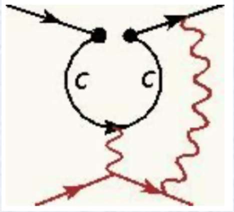
MEs:



Asatrian, Asatryan, Greub Walker
Ghinculov, Hurth, Isidori, Yao
Bobeth, Gambino, Gorbahn, Haisch
de Boer

- QED at NLO ($A^{\rm em}_{\rm LO}, A^{\rm em}_{\rm NLO}$)

WCs:



Bobeth, Gambino, Gorbahn, Haisch

MEs:

Huber, Lunghi, Misiak, Wyler

Inclusive theory: m_b scheme and normalization

- b quark mass scheme
  - $\Gamma(b \to X_s \ell\ell)$ is a renormalon free observable but m_b^{pole} is not

    [see e.g.: Beneke - Renormalons]
  - These spurious renormalon ambiguities can be removed by analytically converting m_b^{pole} to a short distance scheme (e.g. m_b^{1S} or m_b^{kin})
  - We adopt the $1S$ scheme using the Upsilon expansion [Hoan, Ligeti, Manohar]
- Choice of normalization
  - In order to remove an overall m_b^5 prefactor the rate is usually normalized to either the total $B \to X_u \ell \nu$ or $B \to X_c \ell \nu$ rate.
  - We adopt the former:

$$\Gamma(B \to X_s \ell\ell) = \text{BR}(B \to X_c \ell \nu) \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{1}{C} \frac{\Phi_{\ell\ell}}{\Phi_u}$$

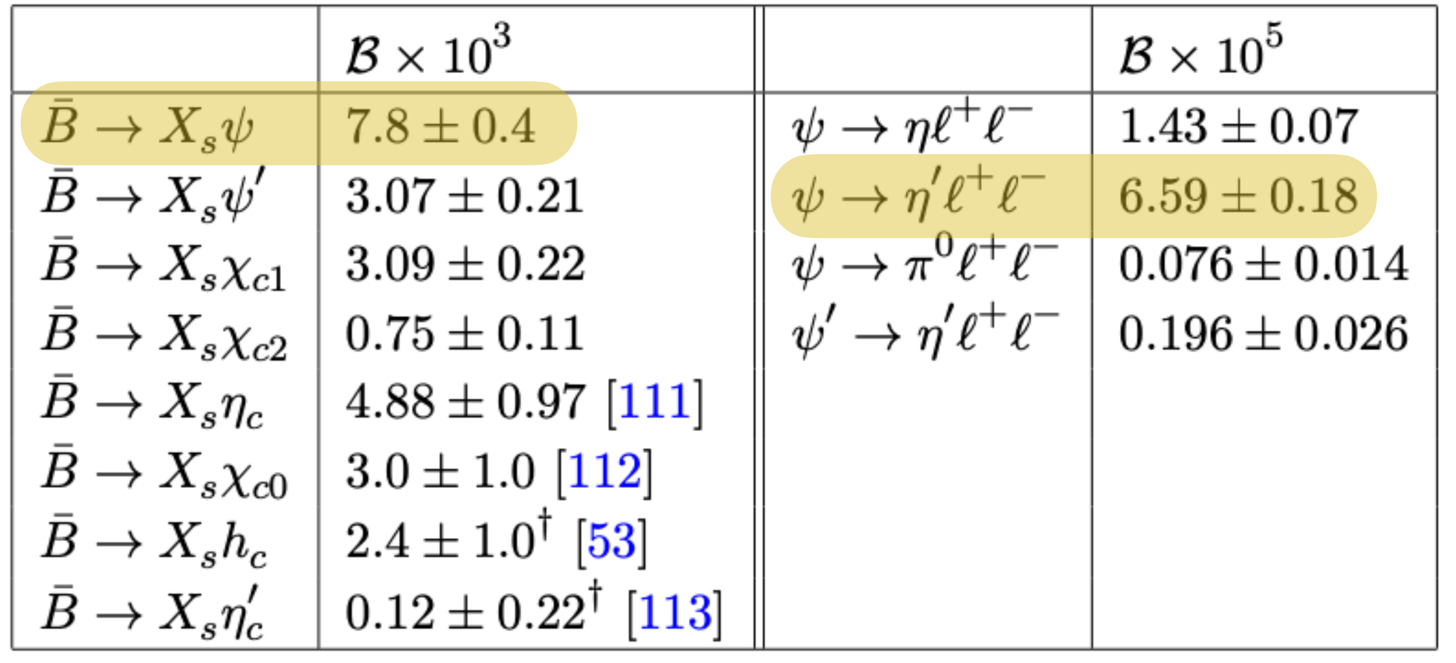
where $C = \left| \frac{V_{ub}}{V_{cb}} \right|^2 \frac{\Gamma(B \to X_c \ell \nu)}{\Gamma(B \to X_u \ell \nu)}$ and $\Phi_{\ell\ell,u}$ are free of CKM angles.

Inclusive theory: resonances

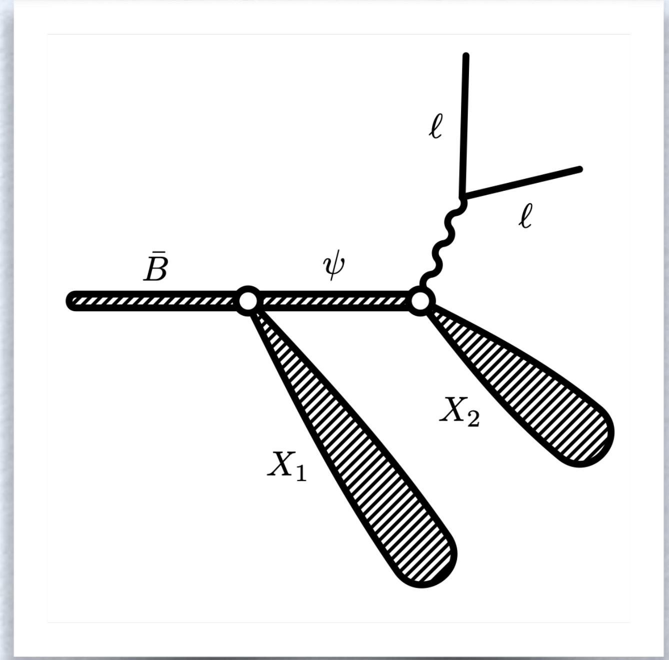
- The charmonium in $B \to X_s(\psi_{cc} \to \ell\ell)$ can be produced by an underlying color singlet and color octet quark transition:
  - the color singlet contribution is modeled exactly over the whole q^2 spectrum using R_{had} data for both on- and off-shell charmonium (Krüger-Sehgal mechanism)
  - off-shell color octet effects at high-q^2 are taken into account by $1/m_c^2$ corrections [Voloshin; Buchalla, Isidori, Rey]
  - off-shell color octet effects at low-q^2 can be described within SCET and yield so-called resolved contributions which at present can only be estimated

- Cascade decays $B \to X_s(\psi_{cc} \to X_s'\ell\ell)$:
  - on-shell effects do not interfere and can be measured and subtracted from the experimental measurement or added to the theory prediction (luckily they turn out to have negligible impact)

Inclusive theory: cascades

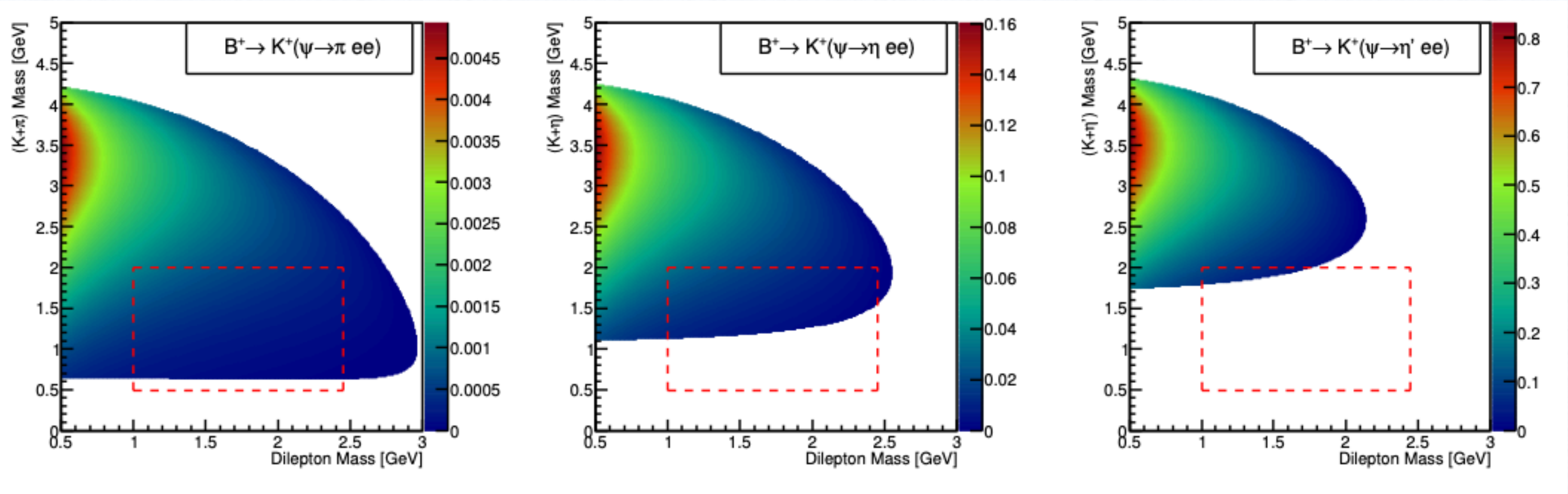


	$\mathcal{B} \times 10^3$		$\mathcal{B} \times 10^5$
$\bar{B} \to X_s \psi$	7.8 ± 0.4	$\psi \to \eta \ell^+ \ell^-$	1.43 ± 0.07
$\bar{B} \to X_s \psi'$	3.07 ± 0.21	$\psi \to \eta' \ell^+ \ell^-$	6.59 ± 0.18
$\bar{B} \to X_s \chi_{c1}$	3.09 ± 0.22	$\psi \to \pi^0 \ell^+ \ell^-$	0.076 ± 0.014
$\bar{B} \to X_s \chi_{c2}$	0.75 ± 0.11	$\psi' \to \eta' \ell^+ \ell^-$	0.196 ± 0.026
$\bar{B} \to X_s \eta_c$	4.88 ± 0.97 [111]		
$\bar{B} \to X_s \chi_{c0}$	3.0 ± 1.0 [112]		
$\bar{B} \to X_s h_c$	$2.4 \pm 1.0^{\dagger}$ [53]		
$\bar{B} \to X_s \eta_c'$	$0.12 \pm 0.22^{\dagger}$ [113]		



- For instance, the η' contribution alone yields a contribution which is of the same order as the short distance $b \to s\ell\ell$:
  $\mathrm{BR}(B \to X_s J/\psi)\mathrm{BR}(J/\psi \to \eta'\ell\ell) = 5.1 \times 10^{-7}$

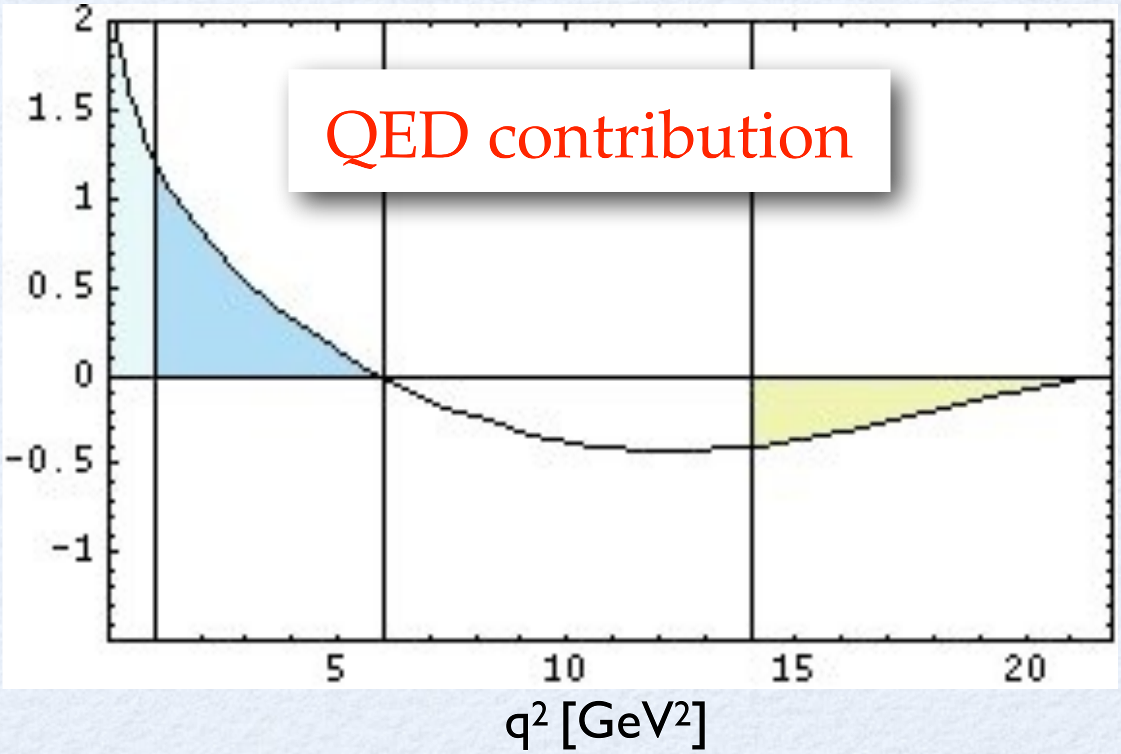
Inclusive theory: cascades



- After imposing $m_X < 2\ \mathrm{GeV}$ this background becomes $\ll 1\,\%$!

QED radiation

- The rate is proportional to $\alpha_{\rm em}^2(\mu^2)$. Without QED corrections the scale μ is undetermined → ± 4% uncertainty
- Focus on corrections enhanced by large logarithms:
  - $\alpha_{\rm em} \log(m_W/m_b) \sim \alpha_{\rm em}/\alpha_s$ [WC, RG running] [Bobeth,Gambino,Gorbahn,Haisch]
  - $\alpha_{\rm em} \log(m_\ell/m_b)$ [Matrix Elements]
- The differential rate is not IR safe with respect to photon emission the results in the presence of a physical collinear logarithm, $\log(m_\ell/m_b)$



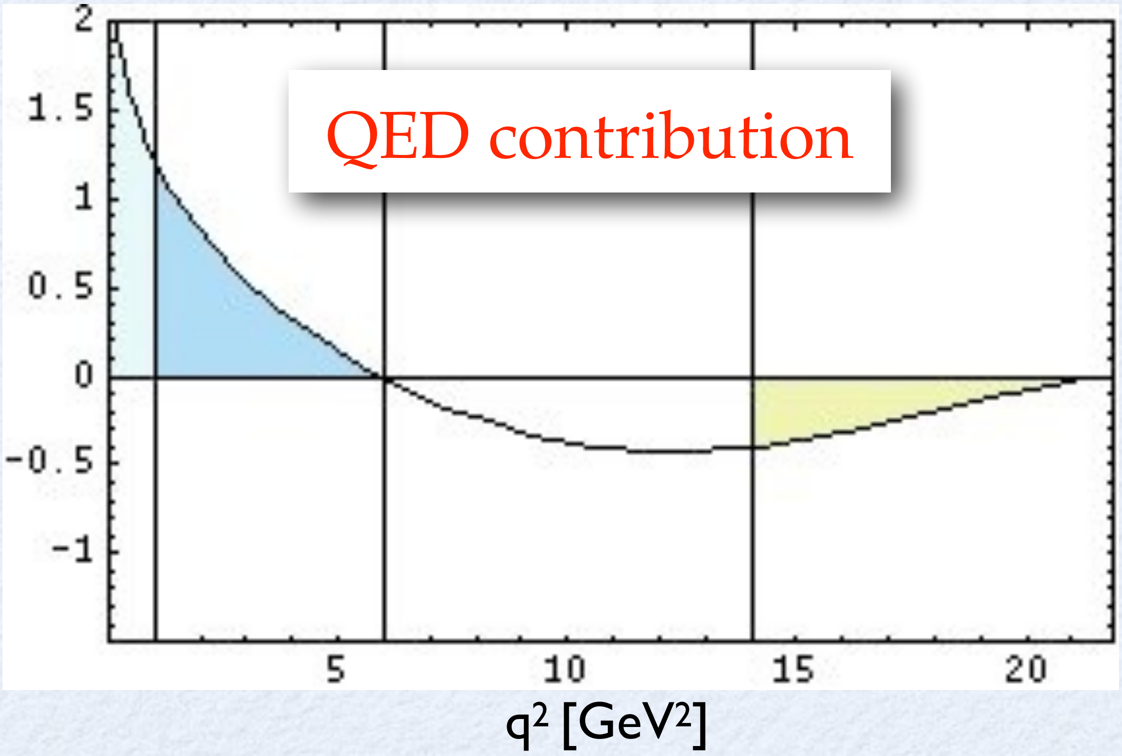
$$\text{virtual} = \frac{A_{\rm soft+collinear}}{\epsilon^2} + \frac{B_{\rm collinear} + B_{\rm soft}}{\epsilon} + C$$

$$\text{real} = -\frac{A_{\rm soft+collinear}}{\epsilon^2} - \frac{B'_{\rm collinear} + B_{\rm soft}}{\epsilon} + C'$$

$$\int dq^2 \left(B_{\rm collinear} - B'_{\rm collinear}\right) = 0$$

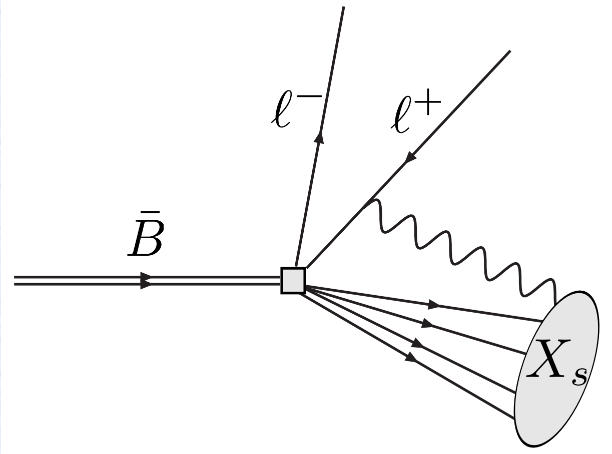
QED radiation

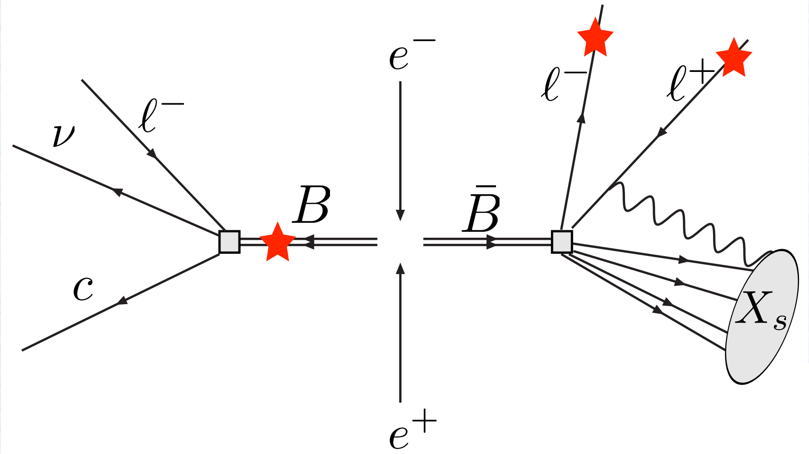
- The rate is proportional to $\alpha_{\rm em}^2(\mu)$. Without QED corrections the scale μ is undetermined $\rightarrow \pm 4\,\%$ uncertainty
- Focus on corrections enhanced by large logarithms:
  - $\alpha_{\rm em} \log(m_W/m_b) \sim \alpha_{\rm em}/\alpha_s$ [WC, RG running] [Bobeth,Gambino,Gorbahn,Haisch]
  - $\alpha_{\rm em} \log(m_\ell/m_b)$ [Matrix Elements]
- The differential rate is not IR safe with respect to photon emission the results in the presence of a physical collinear logarithm, $\log(m_\ell/m_b)$



QED radiation: theory vs experiment

- Photons emitted by the final state leptons (especially electrons) should be technically included in the X_s system:

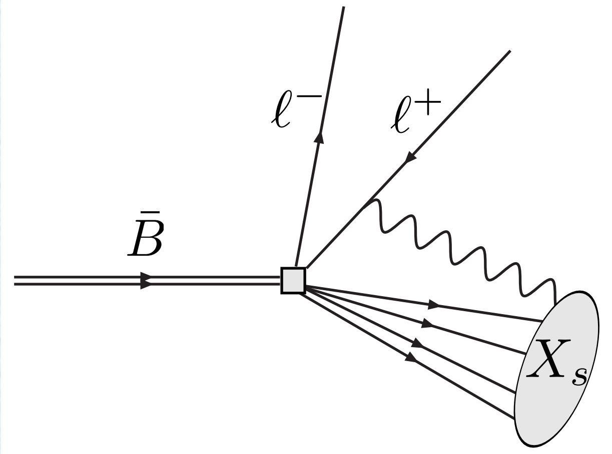


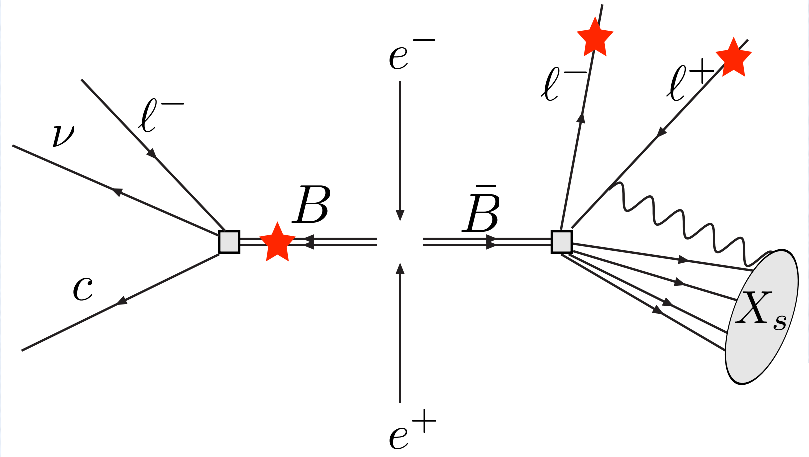


- This implies large α_{em} log(m_e/m_b) at low and high-q^2:
  the logs cancel in the total rate that is however inaccessible (resonances)
- At BaBar and Belle most but not all of these photons are included in the X_s system:
  Need Monte Carlo studies (EVTGEN+PHOTOS) to find the correction factor
- At LHCb all photons emitted by the charged leptons are recovered (physically and using PHOTOS) and included in the lepton 4-momentum ⇒ collinear QED logs must not be included

QED radiation: theory vs experiment

- Photons emitted by the final state leptons (especially electrons) should be technically included in the X_s system:





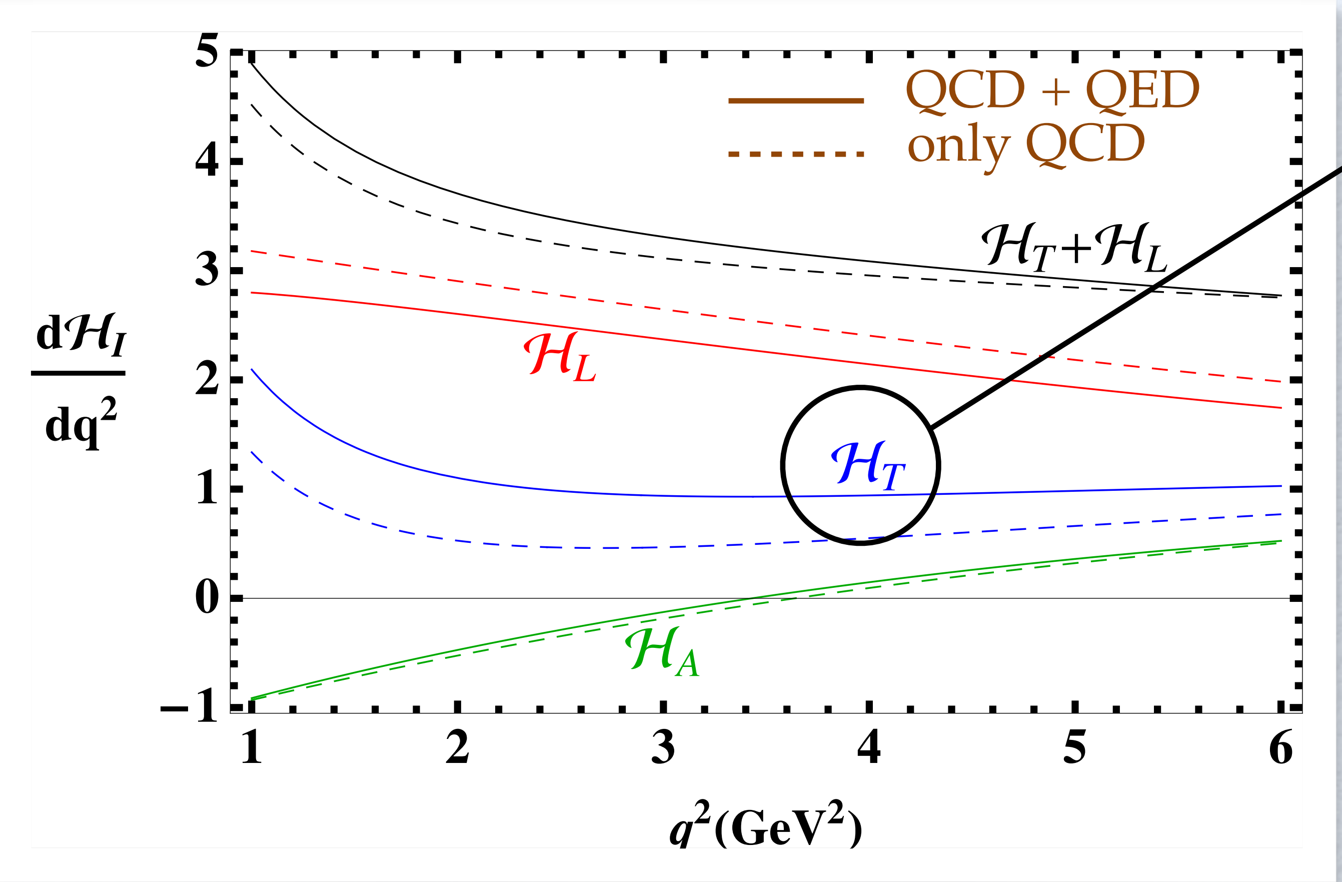
- This implies large α_{em} $\log(m_e/m_b)$ at low and high-q^2
- The logs cancel in the total rate that is however inaccessible (resonances)
- At BaBar and Belle most but not all of these photons are included in the X_s system
- Need Monte Carlo studies (EVTGEN+PHOTOS) to find the correction factor:

$$\frac{\left[\mathcal{B}_{ee}^{\rm low}\right]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\rm coll}}}}{\left[\mathcal{B}_{ee}^{\rm low}\right]_{q=p_{e^+}+p_{e^-}}}-1 = 1.65\%$$

$$\frac{\left[\mathcal{B}_{ee}^{\rm high}\right]_{q=p_{e^+}+p_{e^-}+p_{\gamma_{\rm coll}}}}{\left[\mathcal{B}_{ee}^{\rm high}\right]_{q=p_{e^+}+p_{e^-}}}-1 = 6.8\%$$

QED radiation: size of the effect

- Impact of collinear photon radiation is huge on some observables
- Cross check with Monte Carlo study (EVTGEN + PHOTOS)

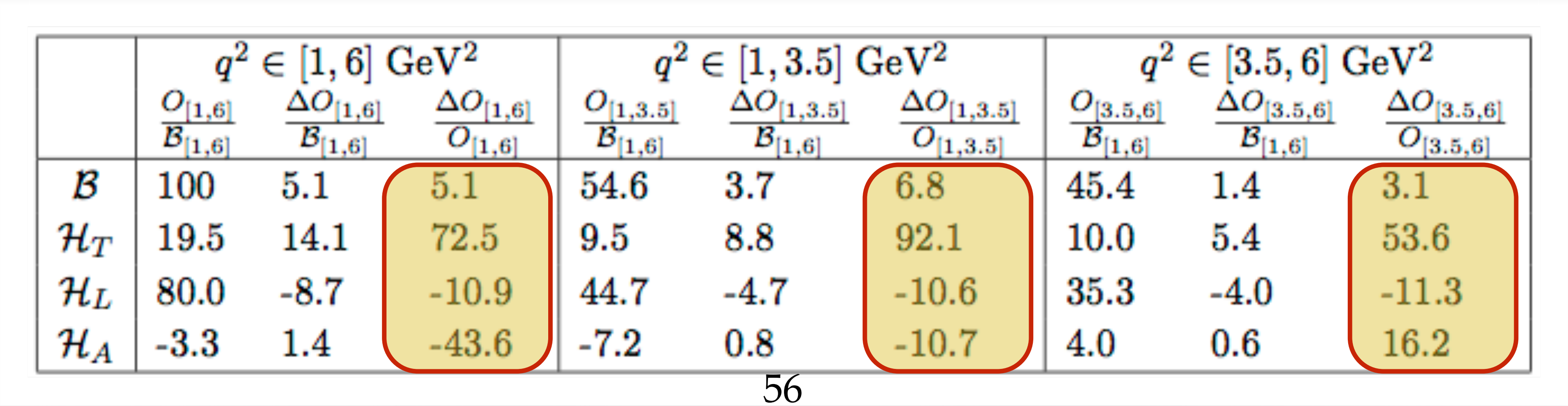


Shift on H_T is $\sim 70\,\%$!

$\mathrm{H_T}$ is smaller than $\mathrm{H_L}$ ($\hat{s} < 0.3$ and $C_7 < 0$):

$$H_T \sim 2\hat{s}(1-\hat{s})^2\left[|C_9 + \frac{2}{\hat{s}}C_7|^2 + |C_{10}|^2\right]$$

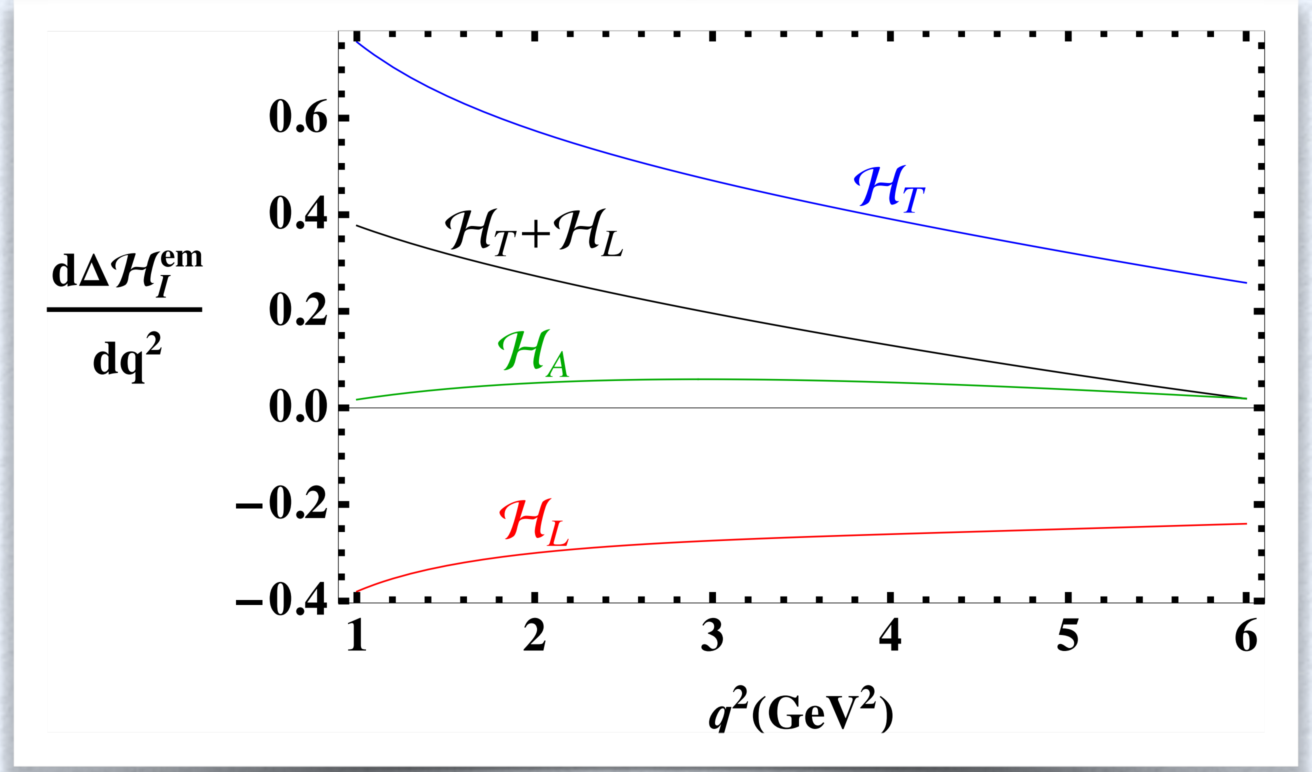
$$H_L \sim (1-\hat{s})^2\left[|C_9 + 2C_7|^2 + |C_{10}|^2\right]$$



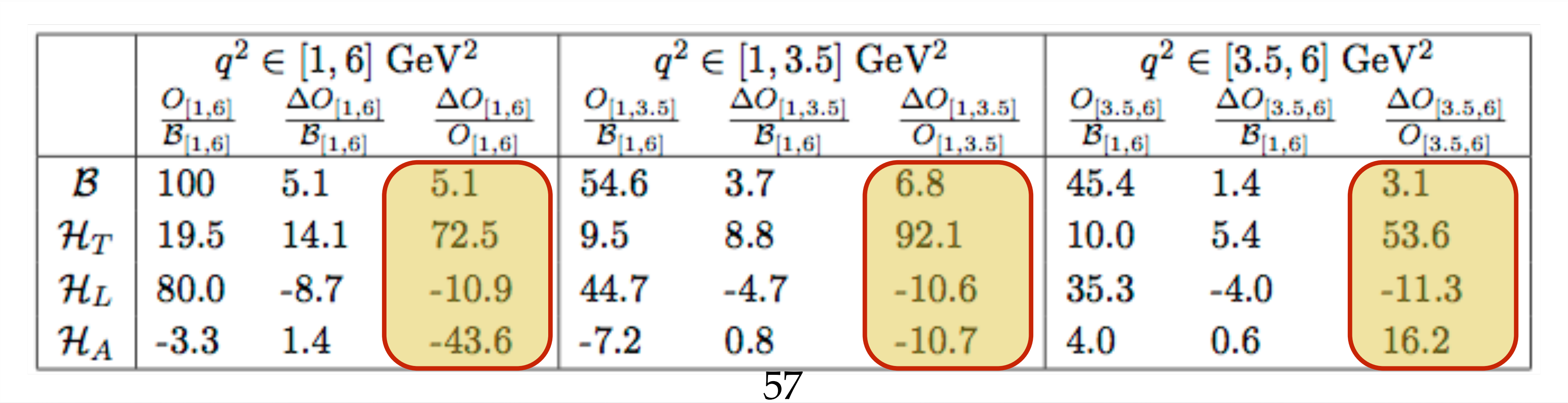
	$q^2 \in [1,6]$ GeV^2			$q^2 \in [1,3.5]$ GeV^2			$q^2 \in [3.5,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$	$\frac{O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,3.5]}}{O_{[1,3.5]}}$	$\frac{O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[3.5,6]}}{O_{[3.5,6]}}$
$\mathcal{B}$	100	5.1	5.1	54.6	3.7	6.8	45.4	1.4	3.1
$\mathcal{H}_T$	19.5	14.1	72.5	9.5	8.8	92.1	10.0	5.4	53.6
$\mathcal{H}_L$	80.0	-8.7	-10.9	44.7	-4.7	-10.6	35.3	-4.0	-11.3
$\mathcal{H}_A$	-3.3	1.4	-43.6	-7.2	0.8	-10.7	4.0	0.6	16.2

QED radiation: size of the effect

- We calculated the effect of collinear photon radiation and found large effects on some observables



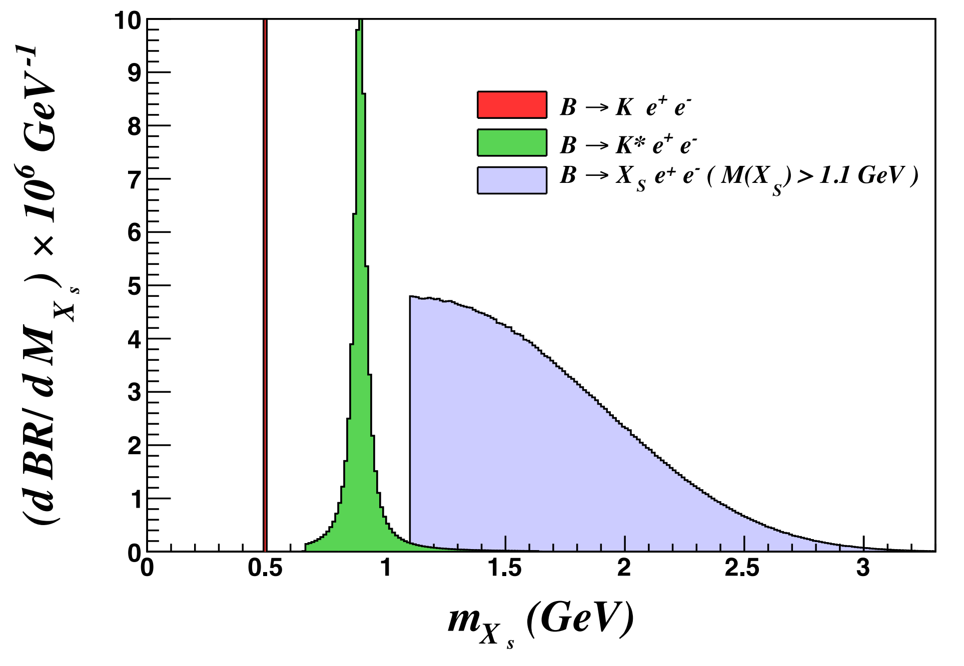
Size of QED contributions to the H_T and H_L is similar

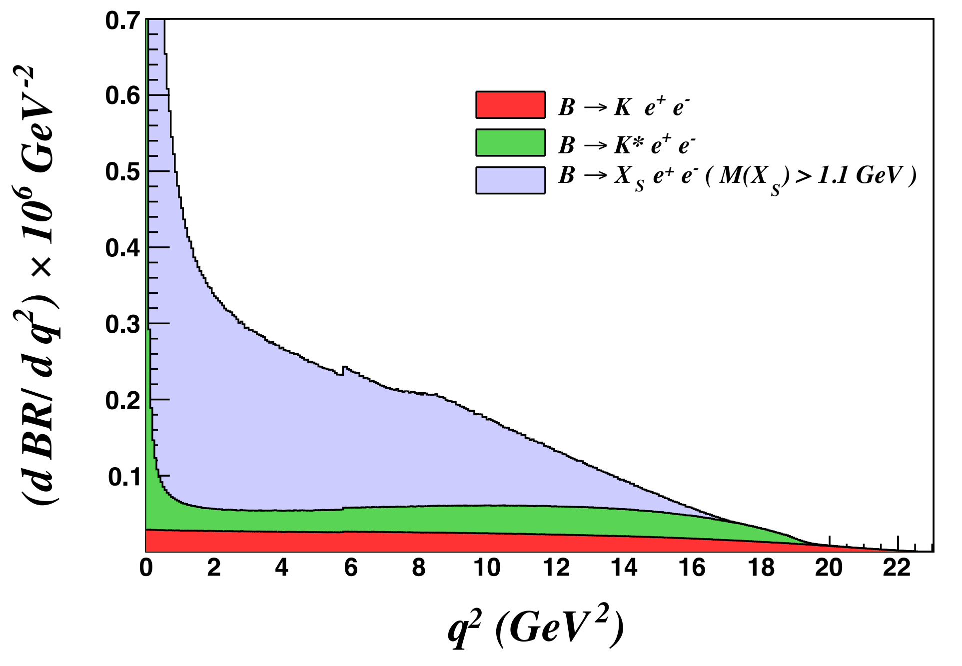


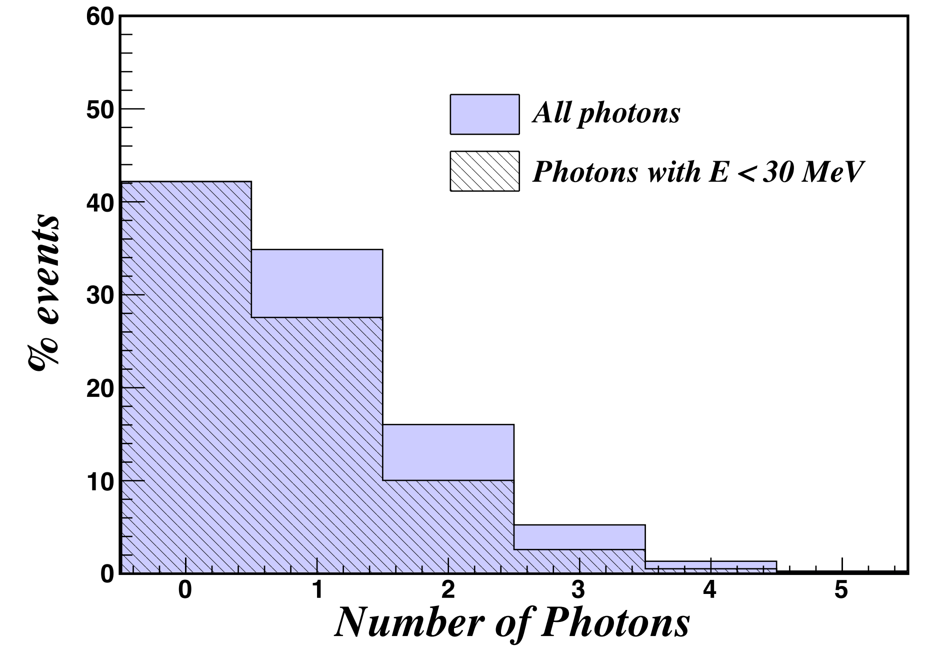
	$q^2 \in [1,6]$ GeV^2			$q^2 \in [1,3.5]$ GeV^2			$q^2 \in [3.5,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$	$\frac{O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,3.5]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,3.5]}}{O_{[1,3.5]}}$	$\frac{O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[3.5,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[3.5,6]}}{O_{[3.5,6]}}$
$\mathcal{B}$	100	5.1	5.1	54.6	3.7	6.8	45.4	1.4	3.1
$\mathcal{H}_T$	19.5	14.1	72.5	9.5	8.8	92.1	10.0	5.4	53.6
$\mathcal{H}_L$	80.0	-8.7	-10.9	44.7	-4.7	-10.6	35.3	-4.0	-11.3
$\mathcal{H}_A$	-3.3	1.4	-43.6	-7.2	0.8	-10.7	4.0	0.6	16.2

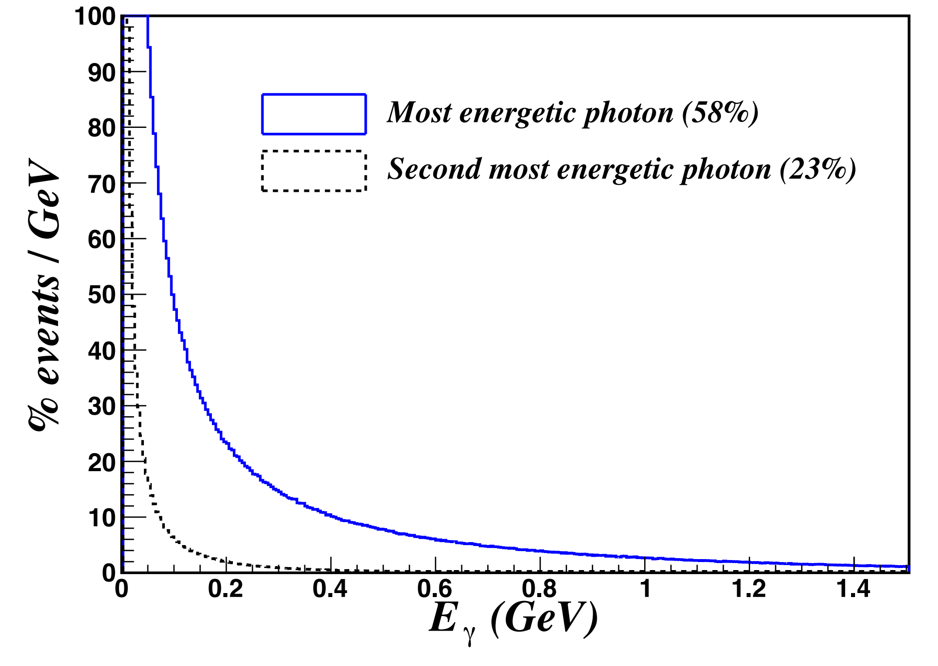
QED radiation: Monte Carlo check

- EM effects have been calculated analytically and cross checked against Monte Carlo generated events (EVTGEN + PHOTOS)
[Many thanks to K. Flood, O. Long and C. Schilling]



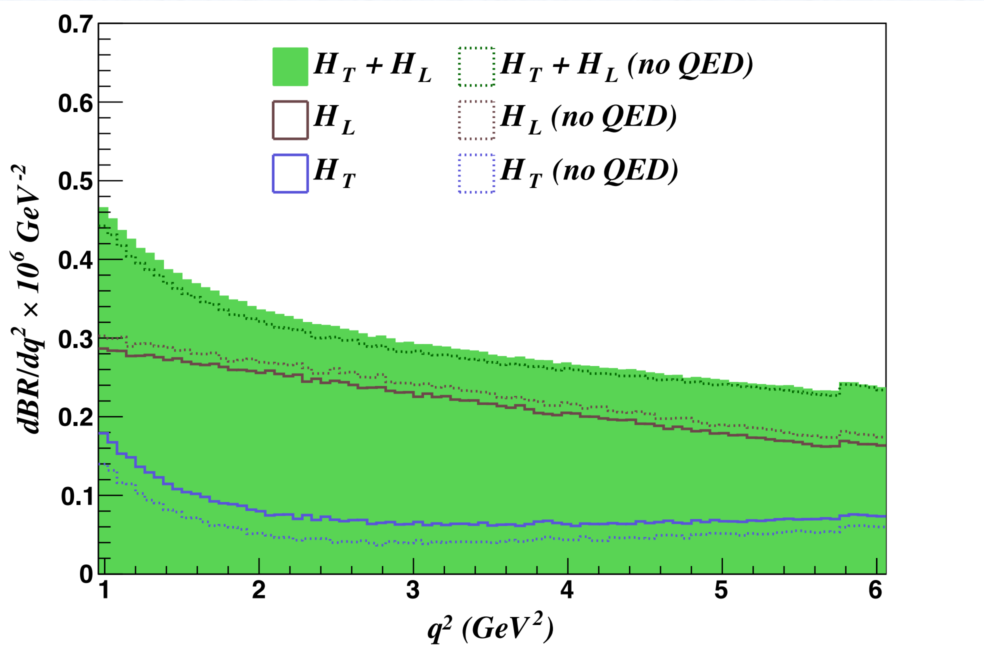


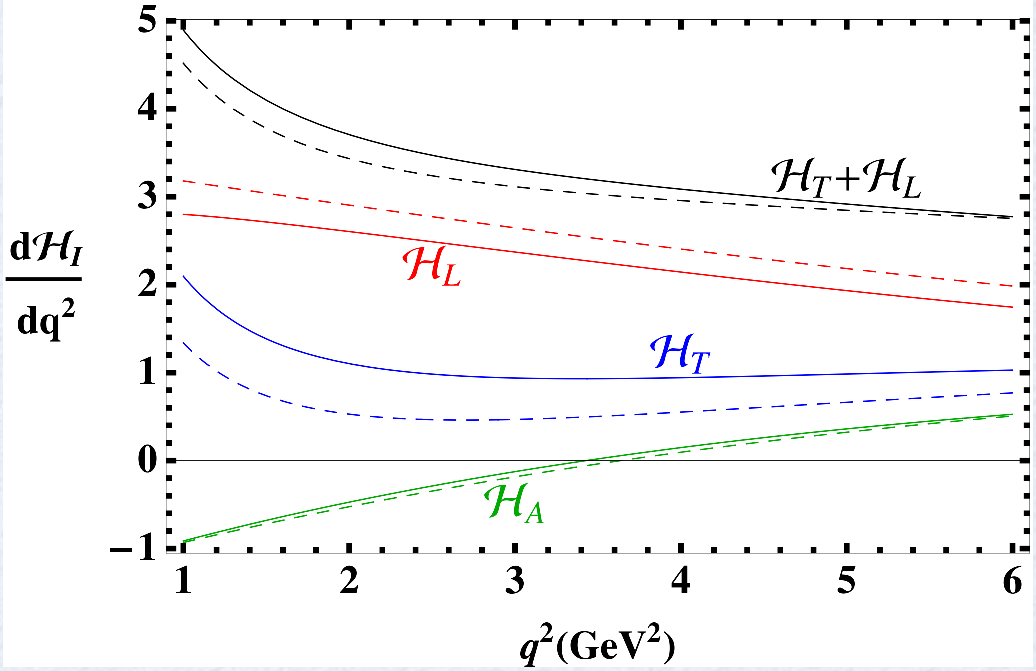


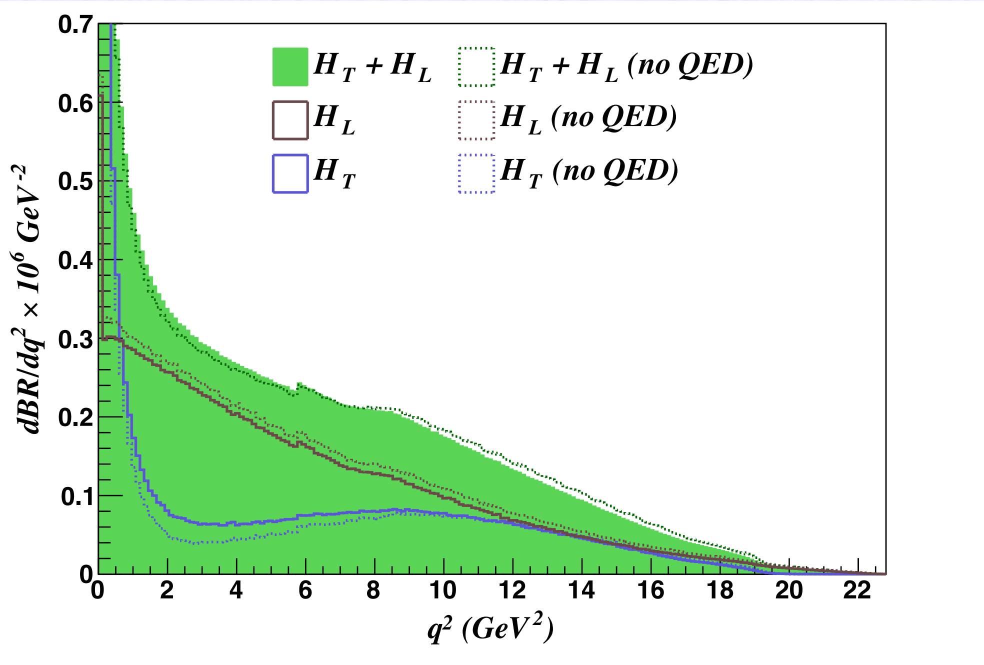


QED radiation: Monte Carlo check

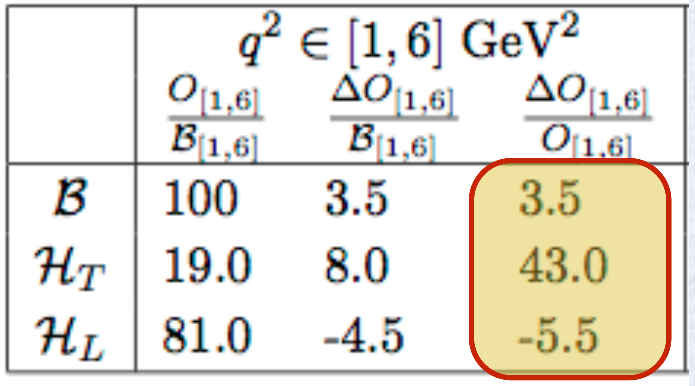
- The Monte Carlo study reproduces the main features of the analytical results





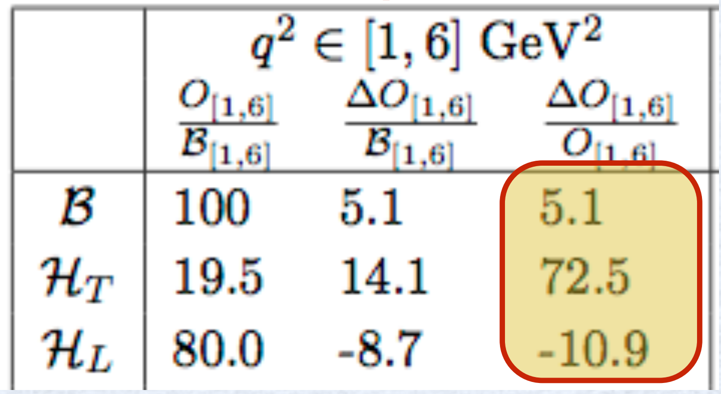


Monte Carlo:



	$q^2 \in [1,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$
$\mathcal{B}$	100	3.5	3.5
$\mathcal{H}_T$	19.0	8.0	43.0
$\mathcal{H}_L$	81.0	-4.5	-5.5

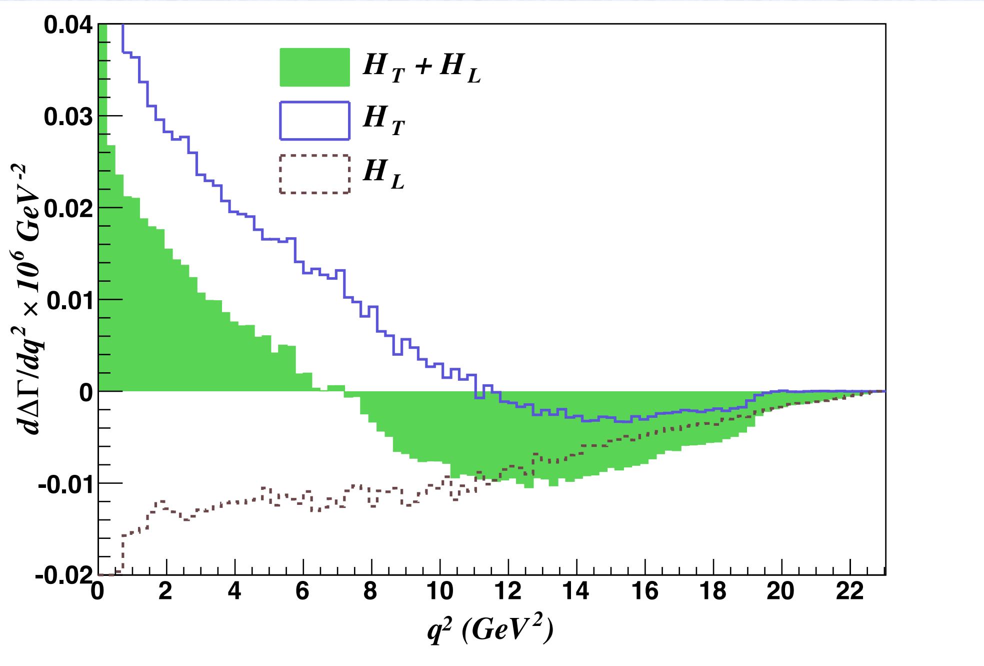
Analytical:

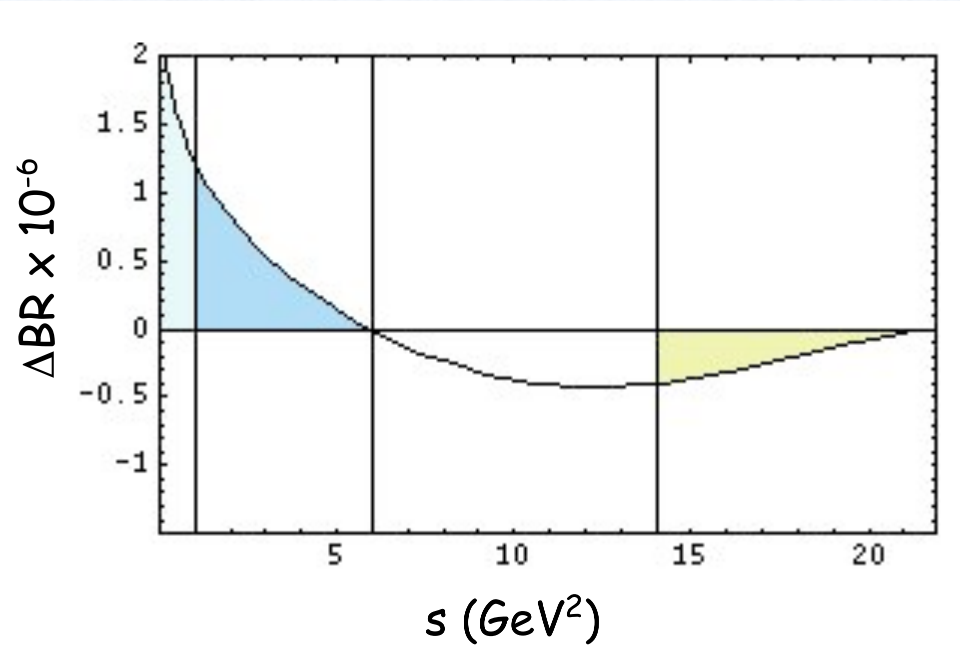


	$q^2 \in [1,6]$ GeV^2		
	$\frac{O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{\mathcal{B}_{[1,6]}}$	$\frac{\Delta O_{[1,6]}}{O_{[1,6]}}$
$\mathcal{B}$	100	5.1	5.1
$\mathcal{H}_T$	19.5	14.1	72.5
$\mathcal{H}_L$	80.0	-8.7	-10.9

QED radiation: Monte Carlo check

- The Monte Carlo study reproduces the main features of the analytical results:





- Take home points on QED radiation and treatment of photons:
  - Large impact (up to $70\,\%$ for H_T)
  - Strong dependence on the observable (e.g. H_T) and on the shape of the spectrum (as shown by the comparison between theory and EVTGEN+PHOTOS)
- Experimental strategies:
  - be as inclusive as possible (i.e. include photons in X_s system)
  - "remove" collinear photons effects with PHOTOS (be wary of dependence on the shape of the EVTGEN generated spectrum)

$B \to X_s \ell\ell$: SM predictions

- Current LHCb measurements of all modes that enter the sum-over-exclusive determination of the inclusive branching ratio at high-q^2 use PHOTOS to "eliminate" QED effects.
- BaBar and Belle measurements, as well as and current Belle-II analysis strategies include certain collinear photons in the definition of the q^2.
- Breakdown of uncertainties (for the no-QED LHCb situation) for branching ratios and ratio $\mathcal{R}$ for two values of the q^2 cut:

$$\begin{aligned}\mathcal{B}[>14.4] &= (3.04 \pm 0.25_{\rm scale} \pm 0.03_{m_t} \pm 0.04_{C,m_c} \pm 0.22_{m_b} \pm 0.005_{\alpha_s} \pm 0.003_{\rm CKM} \pm 0.05_{\rm BR_{sl}} \pm 0.25_{\rho_1} \pm 0.11_{\lambda_2} \pm 0.54_{f_{u,s}}) \times 10^{-7}\\ &= (3.04 \pm 0.69) \times 10^{-7} \quad [22.7\%]\\ \mathcal{B}[>15] &= (2.59 \pm 0.21_{\rm scale} \pm 0.03_{m_t} \pm 0.05_{C,m_c} \pm 0.19_{m_b} \pm 0.004_{\alpha_s} \pm 0.002_{\rm CKM} \pm 0.04_{\rm BR_{sl}} \pm 0.26_{\rho_1} \pm 0.10_{\lambda_2} \pm 0.54_{f_{u,s}}) \times 10^{-7}\\ &= (2.59 \pm 0.68) \times 10^{-7} \quad [26.3\%]\end{aligned}$$

$$\begin{aligned}\mathcal{R}(14.4) &= (26.02 \pm 0.42_{\rm scale} \pm 0.30_{m_t} \pm 0.11_{C,m_c} \pm 0.10_{m_b} \pm 0.12_{\alpha_s} \pm 1.12_{\rm CKM} \pm 0.33_{\rho_1} \pm 0.05_{\lambda_2} \pm 1.20_{f_{u,s}}) \times 10^{-4}\\ &= \ (26.02 \pm 1.76) \times 10^{-4} \quad [6.8\%]\\ \mathcal{R}(15) &= (27.00 \pm 0.25_{\rm scale} \pm 0.30_{m_t} \pm 0.11_{C,m_c} \pm 0.17_{m_b} \pm 0.15_{\alpha_s} \pm 1.16_{\rm CKM} \pm 0.37_{\rho_1} \pm 0.07_{\lambda_2} \pm 1.43_{f_{u,s}}) \times 10^{-4}\\ &= (27.00 \pm 1.94) \times 10^{-4} \quad [7.2\%]\end{aligned}$$

- Results at $q^2 > 14.4\ \mathrm{GeV}^2$ have smaller uncertainties and can be extracted with larger data sets

$B \to X_s \ell \ell$: SM predictions

- Current LHCb measurements of all modes that enter the sum-over-exclusive determination of the inclusive branching ratio at high-q^2 use PHOTOS to “eliminate” QED effects.
- BaBar and Belle measurements, as well as and current Belle-II analysis strategies include certain collinear photons in the definition of the q^2.
- Complete SM predictions

LHCb (no collinear photons)

q^2 range [GeV^2]	$[1, 6]$	$[1, 3.5]$	$[3.5, 6]$
$\mathcal{B}$ $[10^{-7}]$	16.87 ± 1.25	9.17 ± 0.61	7.70 ± 0.65
$\mathcal{H}_T$ $[10^{-7}]$	3.14 ± 0.25	1.49 ± 0.09	1.65 ± 0.17
$\mathcal{H}_L$ $[10^{-7}]$	13.65 ± 1.00	7.63 ± 0.54	6.02 ± 0.49
$\mathcal{H}_A$ $[10^{-7}]$	-0.27 ± 0.21	-1.08 ± 0.08	0.81 ± 0.16

q^2 range [GeV^2]	> 14.4	> 15
$\mathcal{B}$ $[10^{-7}]$	3.04 ± 0.69	2.59 ± 0.68
$\mathcal{R}(q_0^2)$ $[10^{-4}]$	26.02 ± 1.76	27.00 ± 1.94

B-factories (with QED, e/μ average)

q^2 range [GeV^2]	$[1, 6]$	$[1, 3.5]$	$[3.5, 6]$
$\mathcal{B}$ $[10^{-7}]$	17.41 ± 1.31	9.58 ± 0.65	7.83 ± 0.67
$\mathcal{H}_T$ $[10^{-7}]$	4.77 ± 0.40	2.50 ± 0.18	2.27 ± 0.22
$\mathcal{H}_L$ $[10^{-7}]$	12.65 ± 0.92	7.085 ± 0.48	5.56 ± 0.45
$\mathcal{H}_A$ $[10^{-7}]$	-0.10 ± 0.21	-0.989 ± 0.080	0.89 ± 0.16

q^2 range [GeV^2]	> 14.4
$\mathcal{B}$ $[10^{-7}]$	2.66 ± 0.70
$\mathcal{R}(q_0^2)$ $[10^{-4}]$	22.27 ± 1.83

Inputs: Weak Annihilation

- In the isospin SU(3) limit there are only two WA matrix elements:

$$f_{\rm V} \equiv f_u^{\pm} \overset{SU(2)}{=} f_d^0$$

$$f_{\rm NV} \equiv f_u^0 \overset{SU(2)}{=} f_d^{\pm} \overset{SU(3)}{=} f_s^0 \overset{SU(2)}{=} f_s^{\pm}$$

- Numerically we adopt upper limits extracted from $D^{0,\pm}$ and D_s decays rescaled by a factor $m_B f_B^2/(m_D f_D^2)$ [following the analysis of Gambino, Kamenik]

- We found that $f_{\rm NV}$ and $f_{\rm NV} - f_{\rm V}$ are mostly uncorrelated

- We estimate SU(2) and SU(3) breaking effects following [Ligeti, Tackmann]

- Taking into account the adopted normalizations, we need:

$$\mathscr{B}(B \to X_s \ell^+\ell^-) \sim \frac{\Gamma(B \to X_s \ell^+\ell^-)}{\Gamma(B \to X_u \ell \nu)} \Longrightarrow \begin{cases} f_s = (f_s^{\pm} + f_s^0)/2 = f_{\rm NV} \\ f_u = (f_u^{\pm} + f_u^0)/2 = (f_{\rm V} + f_{\rm NV})/2 \end{cases}$$

$$\mathscr{R}(s_0, B \to X_s \ell^+\ell^-) \sim \frac{\Gamma(B \to X_s \ell^+\ell^-)}{\Gamma(B^0 \to X_u \ell \nu)} \Longrightarrow \begin{cases} (f_s + f_u^0)/2 = f_{\rm NV} \\ f_s - f_u^0 = [\delta f]_{SU(3)} \end{cases}$$

$$\begin{matrix} \mathscr{B}(B \to X_d \ell^+\ell^-) \\ \mathscr{R}(s_0, B \to X_d \ell^+\ell^-) \end{matrix} \Longrightarrow \begin{cases} (f_d + f_u)/2 = (f_{\rm V} + f_{\rm NV})/2 \\ f_d - f_u = [\delta f]_{SU(2)} \end{cases}$$

$B \to X_s \ell \ell$: new observables

- At leading order in QED and at all orders in QCD, the double differential width is a quadratic polynomial: $\Gamma \sim a \ \cos^2\theta + b \ cos\theta + c$
- Γ receives non polynomial log-enhanced QED corrections
- We can build new observables by projecting out with Legendre polynomials:

$$H_I(q^2) = \int_{-1}^{1} \frac{d^2\Gamma}{dq^2dz} W_I(z) dz$$

$$W_T = \frac{2}{3}P_0(z) + \frac{10}{3}P_2(z)$$

$$W_L = \frac{1}{3}P_0(z) - \frac{10}{3}P_2(z)$$

$$W_A = \frac{4}{3}\mathrm{sign}(z)$$

$$W_3 = P_3(z)$$

$$W_4 = P_4(z)$$

new observables

Current constraints

- We begin with the current constraints in the $[C_9^{\rm NP}, C_{10}^{\rm NP}]$ plane
- Experimental measurements:

$$\mathscr{B}[1{,}6]_{\rm exp} = (15.8 \pm 3.7) \times 10^{-7}$$

$$\mathscr{B}[\,>14.4]_{\rm exp} = (2.79 \pm 0.35) \times 10^{-7}$$

$$\mathscr{B}(\bar{B} \to X_u \ell \bar{\nu})[\,>14.4]_{\rm exp} = (1.76 \pm 0.32) \times 10^{-4}$$

$$\mathscr{B}(B_s \to \mu\mu)_{\rm exp} = (3.45 \pm 0.29) \times 10^{-9}$$

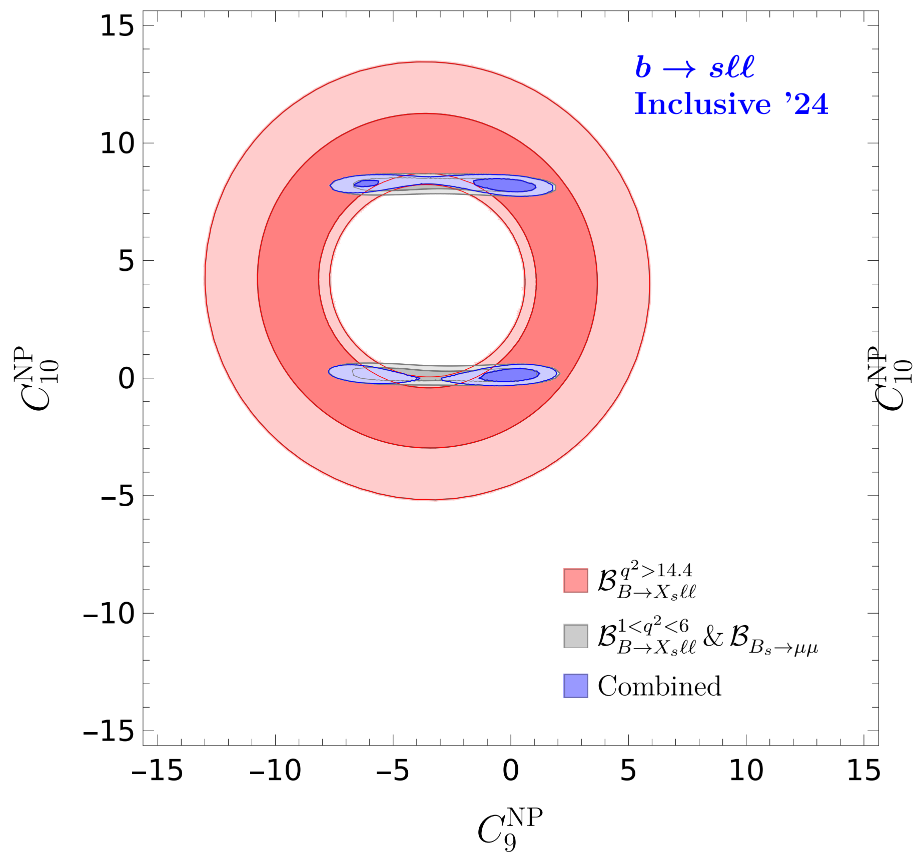
- SM predictions:

$$\mathscr{B}[1{,}6]_{\rm SM} = (17.3 \pm 1.3) \times 10^{-7}$$

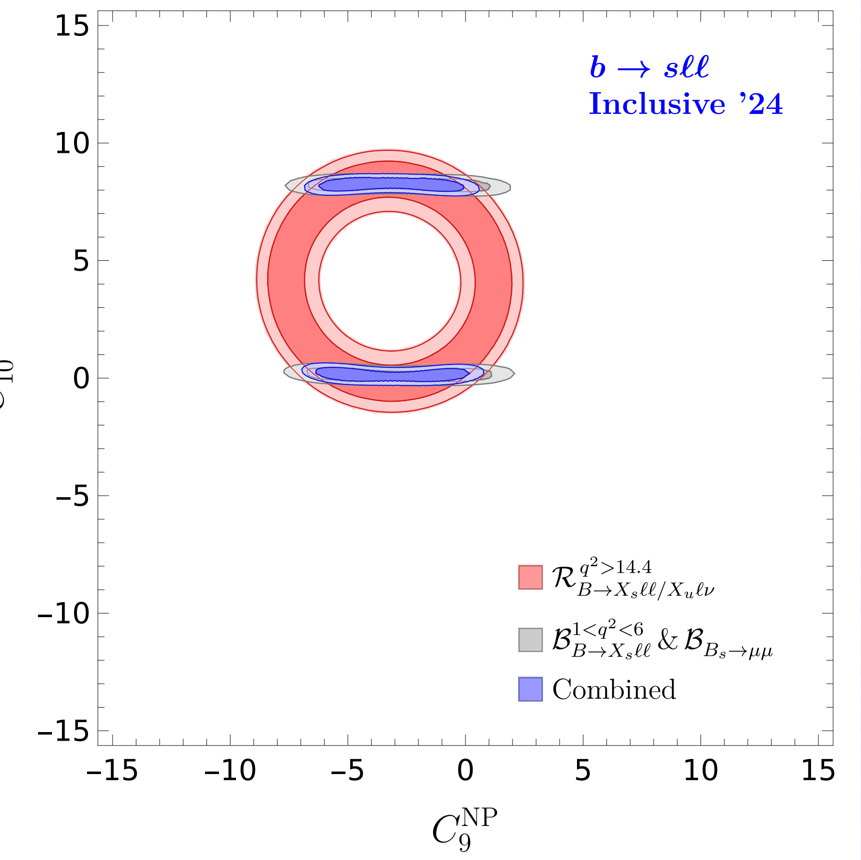
$$\mathscr{B}[\,>14.4]_{\rm SM} = (2.67 \pm 0.70) \times 10^{-7}$$

$$\mathscr{R}[14.4]_{\rm SM} = (26.02 \pm 1.76) \times 10^{-4}$$

$$\mathscr{B}(B_s \to \mu\mu)_{\rm SM} = (3.66 \pm 0.14) \times 10^{-9}$$



$\mathscr{B}[\,>14.4]_{\rm exp}$ & $\mathscr{B}[\,>14.4]_{\rm SM}$



$\mathscr{B}[\,>14.4]_{\rm exp}$, $\mathscr{R}[14.4]_{\rm SM}$
& $\mathscr{B}(\bar{B} \to X_u \ell \bar{\nu})[\,>14.4]_{\rm exp}$

- $B_s \to \mu\mu$ constrains $C_{10}^{\rm NP}$
- High-q^2 constraints from $\mathscr{B}$ and $\mathscr{R}$ are of similar strength because of the large uncertainty from $\bar{B} \to X_u \ell \bar{\nu}$
- Overall picture is of agreement with the SM

$B \to X_s \ell \ell$: BR at $q^2 > 15 \text{ GeV}^2$ from LHCb

- Inclusive measurements are the only direct test of the $b \to s\mu\mu$ anomalies which do not suffer from the same hadronic uncertainties that afflict the exclusive modes
- We strongly encourage the LHCb collaboration to present an inclusive high-q^2 measurement
- Aspects of the above derivation of $\mathscr{B}(\bar{B} \to X_s \mu\mu)[>15]$ that can be improved:
  - We do not have correlations between the various K and K^* modes
  - The $K^*(1410)$ and $K^*(1430)$ resonances lie above the kinematical threshold ($M_{X_s} < 1.41 \text{ GeV}$) but their tails can contribute to the $K\pi$ and $K\pi\pi$ modes
  - More precise integration of the $K\pi\pi$ modes
  - More serious estimate of $K(n\pi)_{n>2}$ modes
  - Present LHCb measurements are for $q^2 > q^2_{\text{cut}} = 15 \text{ GeV}^2$ but the heavy-mass expansion and the integrated spectrum has an effective expansion in inverse powers of $m_b(1 - \sqrt{q^2_{\text{cut}}/m_b^2})$; hence it would be preferable to consider $q^2 > q^2_{\text{cut}} = 14.4 \text{ GeV}^2$

Future constraints

- Projected reach of Belle-II with $50\ \text{ab}^{-1}$ of integrated luminosity

$$\mathcal{O}_{\text{exp}} = \int \frac{d^2\mathcal{N}}{d\hat{s}dz}\, W[\hat{s},z]\, d\hat{s}\, dz$$

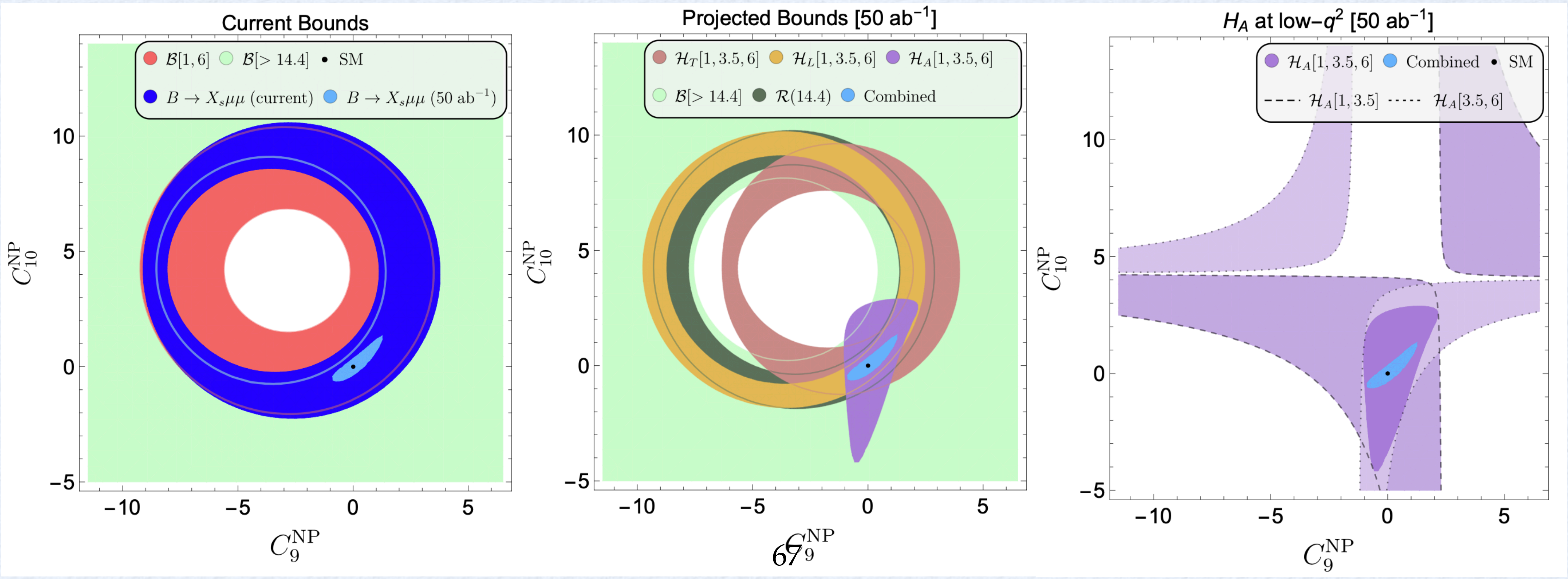
$$\delta\mathcal{O}_{\text{exp}} = \left[\int \frac{d^2\mathcal{N}}{d\hat{s}dz}\, W[\underset{\uparrow}{\hat{s}},z]^2\, d\hat{s}\, dz\right]^{\frac{1}{2}}$$

weight (Legendre polynomial)

projected statistical uncertainties

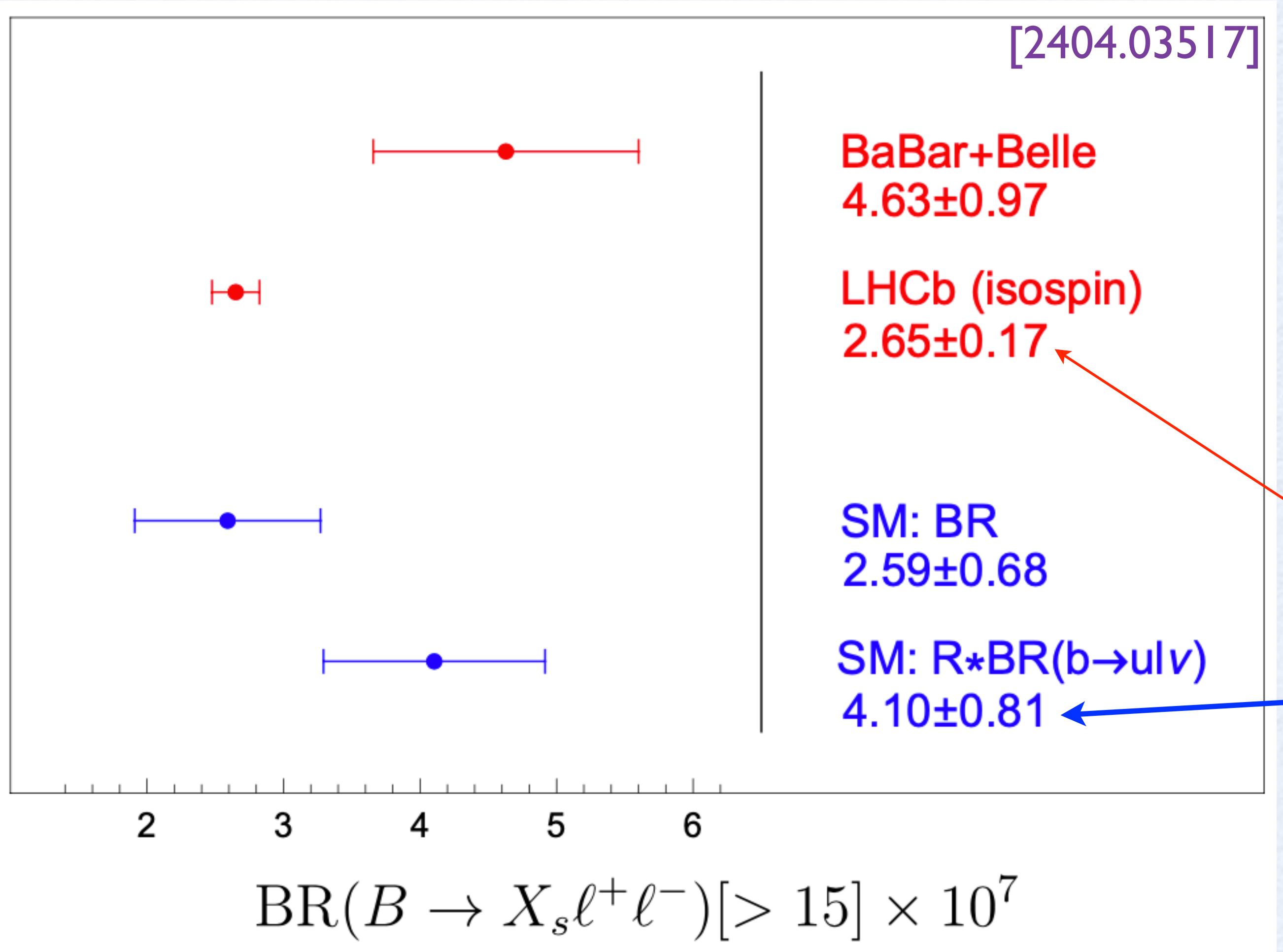
	$[1,3.5]$	$[3.5,6]$	$[1,6]$	>14.4
$\mathcal{B}$	3.1 %	2.6 %	2.0 %	2.6%
$\mathcal{H}_T$	24 %	15 %	13 %	-
$\mathcal{H}_L$	5.5 %	5.0 %	3.7 %	-
$\mathcal{H}_A$	40 %	33 %	- %	-
$\mathcal{H}_3$	240 %	140 %	120 %	-
$\mathcal{H}_4$	140 %	270 %	120 %	-
	+5.8 %			+3.9 %

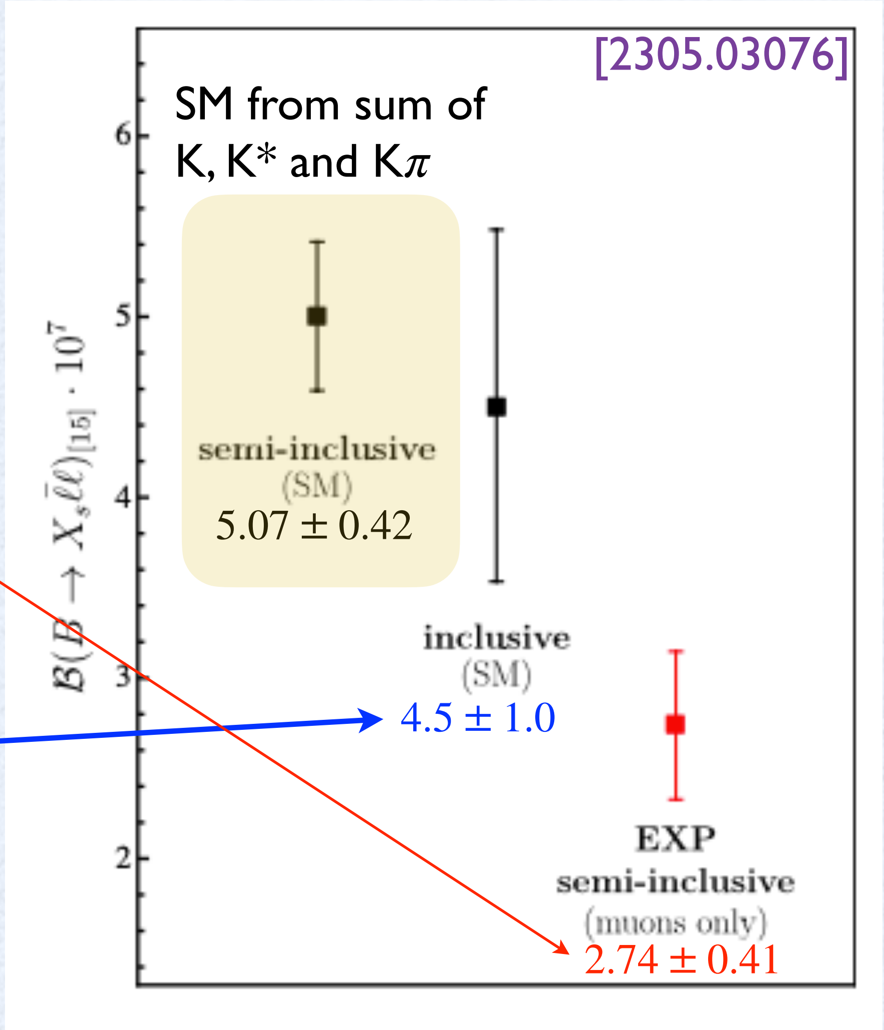
← estimated systematic uncertainties



$B \to X_s \ell \ell$: theory vs experiment

- Comparison with the Isidori, Polonsky, Tinari analysis [2305.03076]





After publication we collaborated and converged on a common determination

- Both analyses find a tension between the LHCb "measurement" and the SM from $\mathscr{R} \times \mathscr{B}_{b \to u \ell \nu}$
- The tension between the semi-inclusive (SM) and LHCb is a restatement of the anomalies
- The difference between the $\mathscr{R} \times \mathscr{B}_{b \to u \ell \nu}$ determinations originates from NLO $Q_{1,2} - Q_{7,9}$ interference $(-9\,\%)$ contributions and from long distance $c\bar{c}$ effects $(-4\,\%)$.

Current constraints

- We begin with the current constraints in the $[C_9^{\rm NP}, C_{10}^{\rm NP}]$ plane
- Experimental measurements:

$$\mathscr{B}[1{,}6]_{\rm exp} = (15.8 \pm 3.7) \times 10^{-7}$$

$$\mathscr{B}[>14.4]_{\rm exp} = (2.79 \pm 0.35) \times 10^{-7}$$

$$\mathscr{B}(\bar{B} \to X_u \ell \bar{\nu})[>14.4]_{\rm exp} = (1.76 \pm 0.32) \times 10^{-4}$$

$$\mathscr{B}(B_s \to \mu\mu)_{\rm exp} = (3.45 \pm 0.29) \times 10^{-9}$$

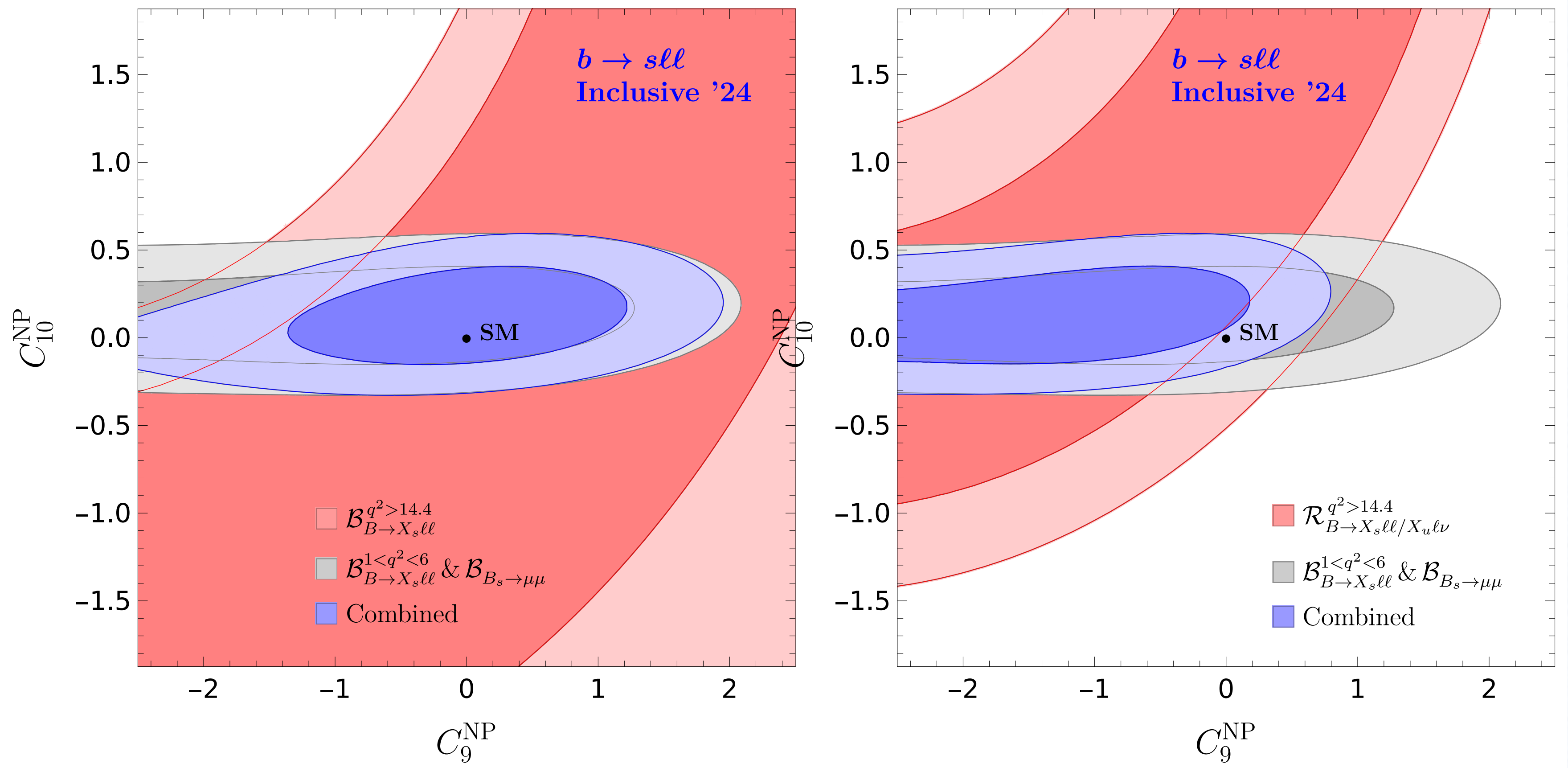
- SM predictions:

$$\mathscr{B}[1{,}6]_{\rm SM} = (17.3 \pm 1.3) \times 10^{-7}$$

$$\mathscr{B}[>14.4]_{\rm SM} = (2.67 \pm 0.70) \times 10^{-7}$$

$$\mathscr{R}[14.4]_{\rm SM} = (26.02 \pm 1.76) \times 10^{-4}$$

$$\mathscr{B}(B_s \to \mu\mu)_{\rm SM} = (3.66 \pm 0.14) \times 10^{-9}$$



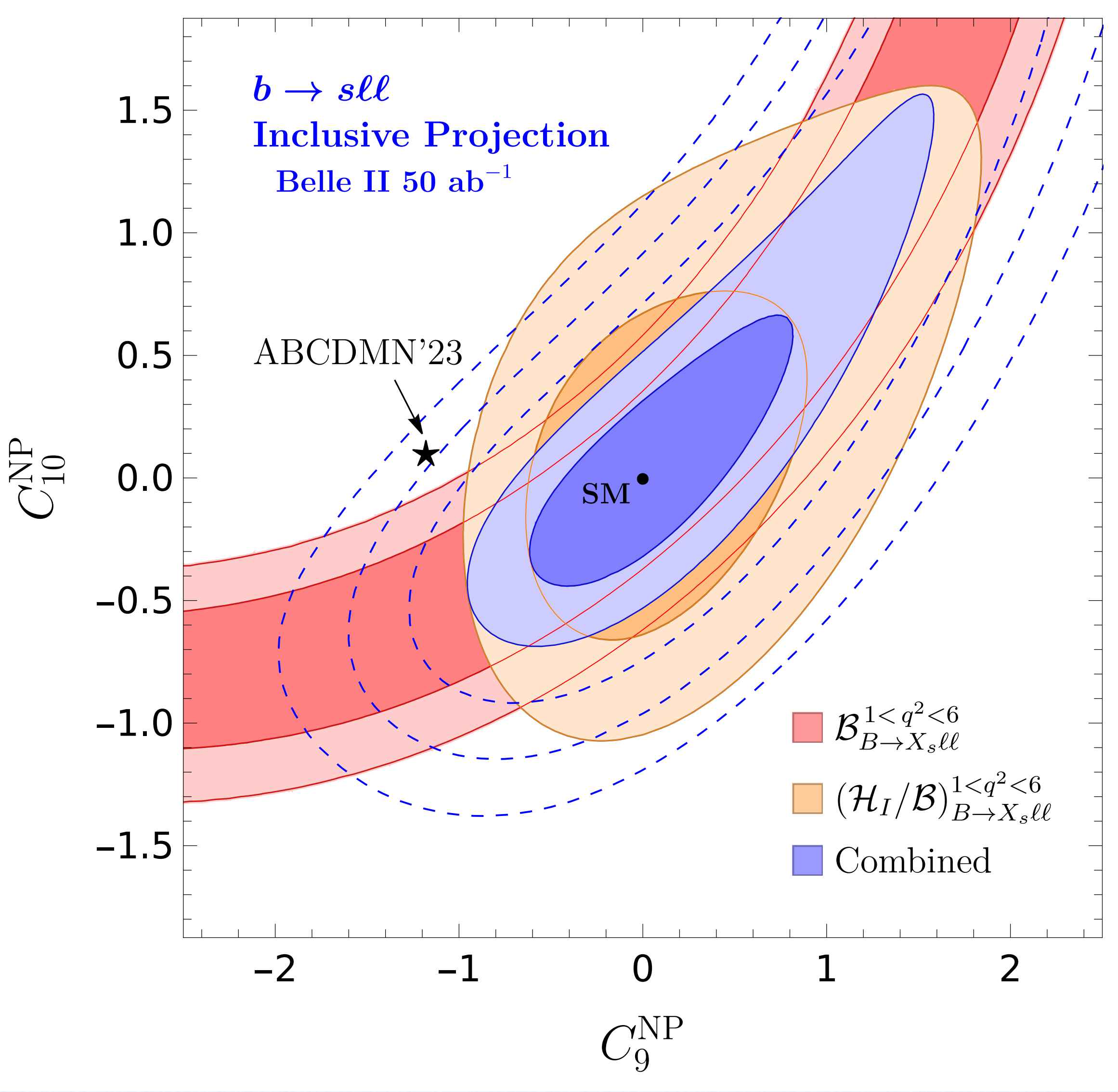
$\mathscr{B}[>14.4]_{\rm exp}$ & $\mathscr{B}[>14.4]_{\rm SM}$

$\mathscr{B}[>14.4]_{\rm exp}$, $\mathscr{R}[14.4]_{\rm SM}$ & $\mathscr{B}(\bar{B} \to X_u \ell \bar{\nu})[>14.4]_{\rm exp}$

- $B_s \to \mu\mu$ constrains $C_{10}^{\rm NP}$
- High-q^2 constraints from $\mathscr{B}$ and $\mathscr{R}$ are of similar strength because of the large uncertainty from $\bar{B} \to X_u \ell \bar{\nu}$
- Overall picture is of agreement with the SM

Future constraints: low-q^2 (Belle II)

- Projected reach of Belle II with $50\ \mathrm{ab}^{-1}$ of integrated luminosity



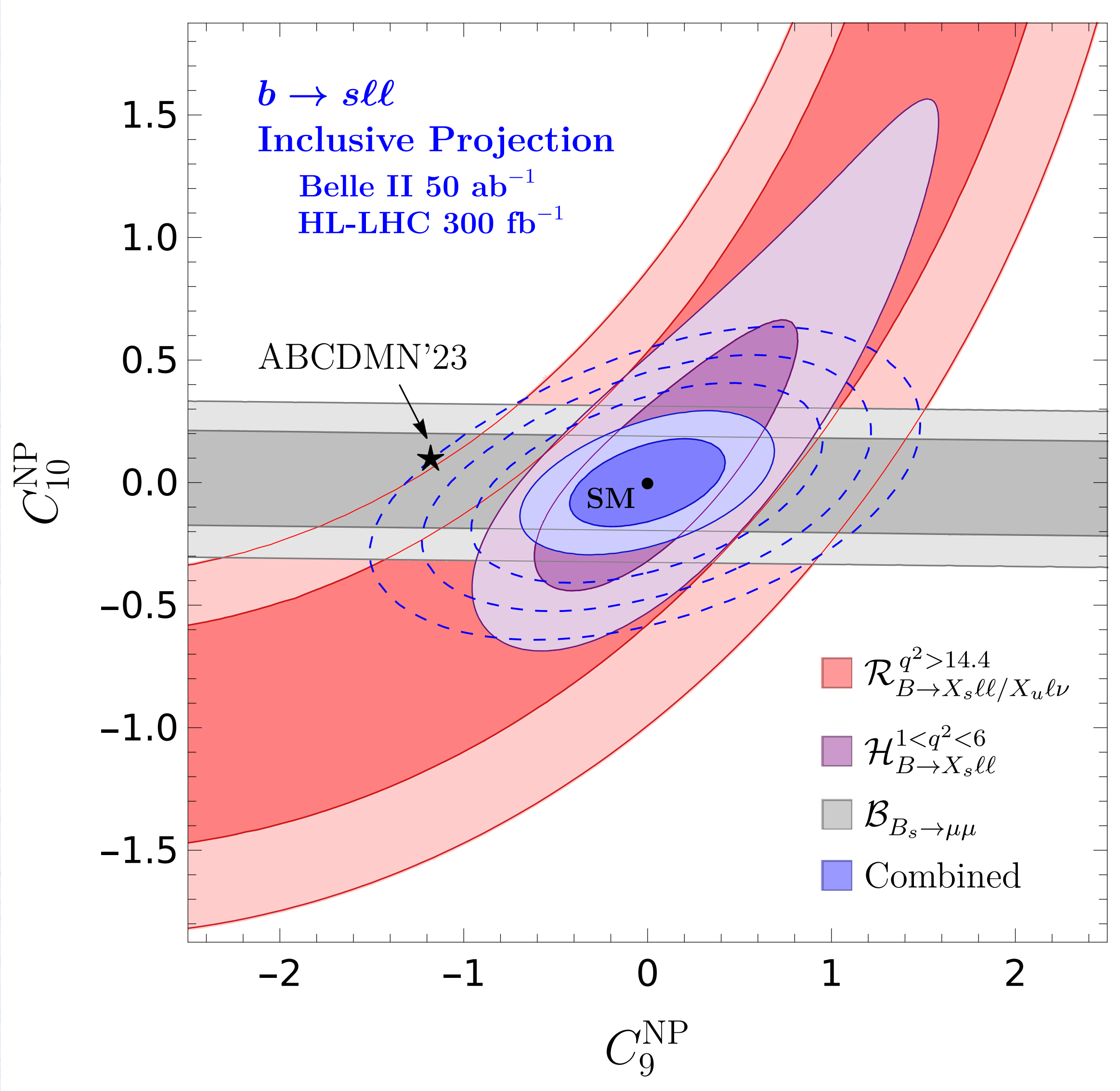
- Focus on low-q^2 where the inclusive OPE is better behaved
- Use of normalized angular observables $(H_I/\mathscr{B})$, lowers impact of the M_X cut in the low-q^2 region
- Constraints from low-q^2 rate and angular distributions are somewhat orthogonal
- We include, for reference, the exclusive best fit point from ABCDMN'23: $C_9^{\mathrm{NP}} = -1.18 \pm 0.18$ and $C_{10}^{\mathrm{NP}} = 0.10 \pm 0.13$

  [**A**lguero, **B**iswas, **C**apdevila, **D**escotes-Genon, **M**atias, **N**ovoa-Brunet, 2304.07330 and Capdevila FPCP2023]
- Dashed contours correspond to 3σ, 4σ and 5σ

- Low-q^2 observables at Belle-II should be able to confirm current anomalies at 4σ

Future constraints: including high-q^2 (Belle II, LHCb)

- Projected reach of Belle II with $50\ \mathrm{ab}^{-1}$ and of LHCb with $300\ \mathrm{fb}^{-1}$ of integrated luminosity



- We assume $\delta(B_s \to \mu\mu) = 4.8\,\%$ corresponding to $300\ \mathrm{fb}^{-1}$ at the HL-LHC

- The projected uncertainty on $\mathscr{R}(14.4)$ is obtained by combining:

$$\delta\mathscr{B}_{bs\ell\ell}[>14.4] = \sqrt{(3.9\%)^2 + (2.6\%)^2} = 4.7\,\%$$

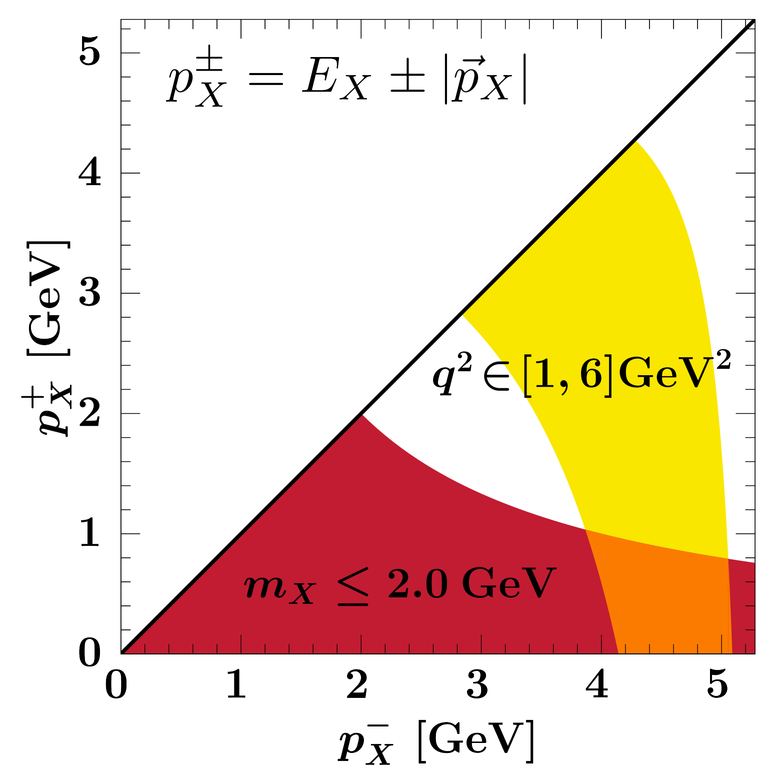
$$\delta\mathscr{B}_{bu\ell\nu}[>14.4] = 5.2\,\%$$

$$\Rightarrow\ \delta\mathscr{R}(14.4) = 7.0\,\%$$

- We see that the inclusion of high-q^2 observables allows to confirm the exclusive anomalies at the 5σ level

M_X cuts

- Kinematics:

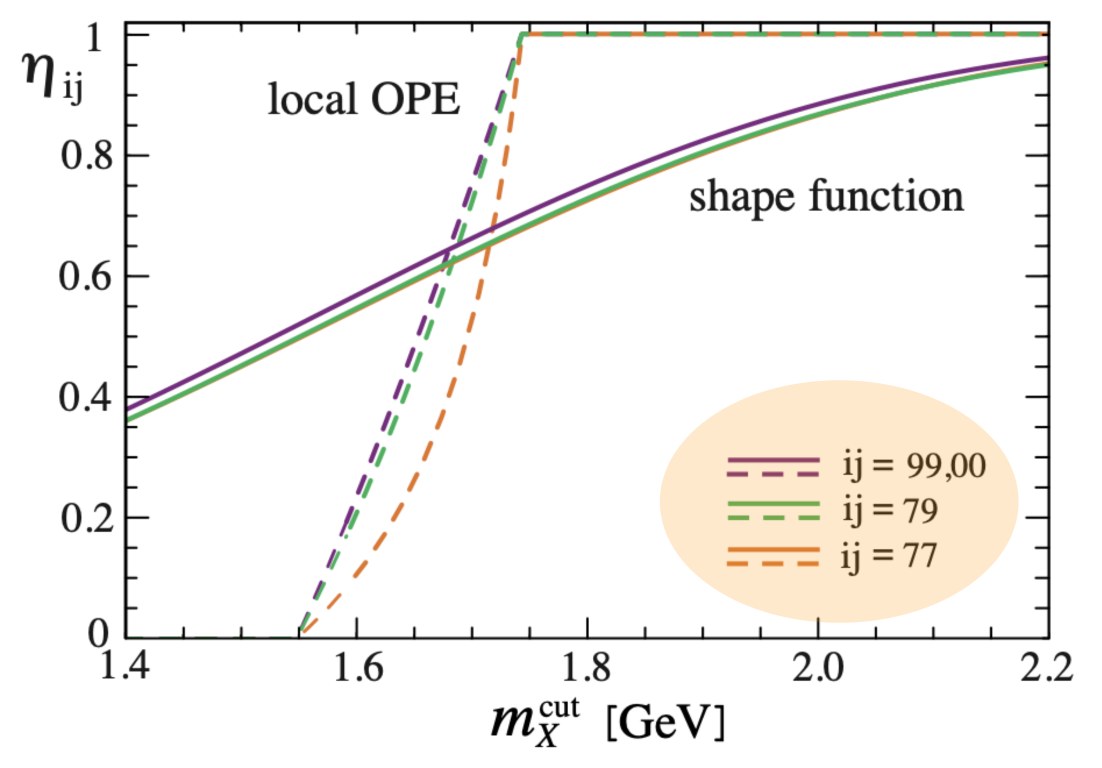


$$p_X^+ \ll p_X^- \Longrightarrow m_X^2 \ll E_X^2$$

X is hard-collinear:

$$\Lambda^2 \ll m_X^2 \sim \Lambda m_b \ll m_b^2$$

- The impact of the cuts is universal ($\eta = \Gamma_{\mathrm{cut}}/\Gamma$): [Lee, Ligeti, Stewart, Tackmann]



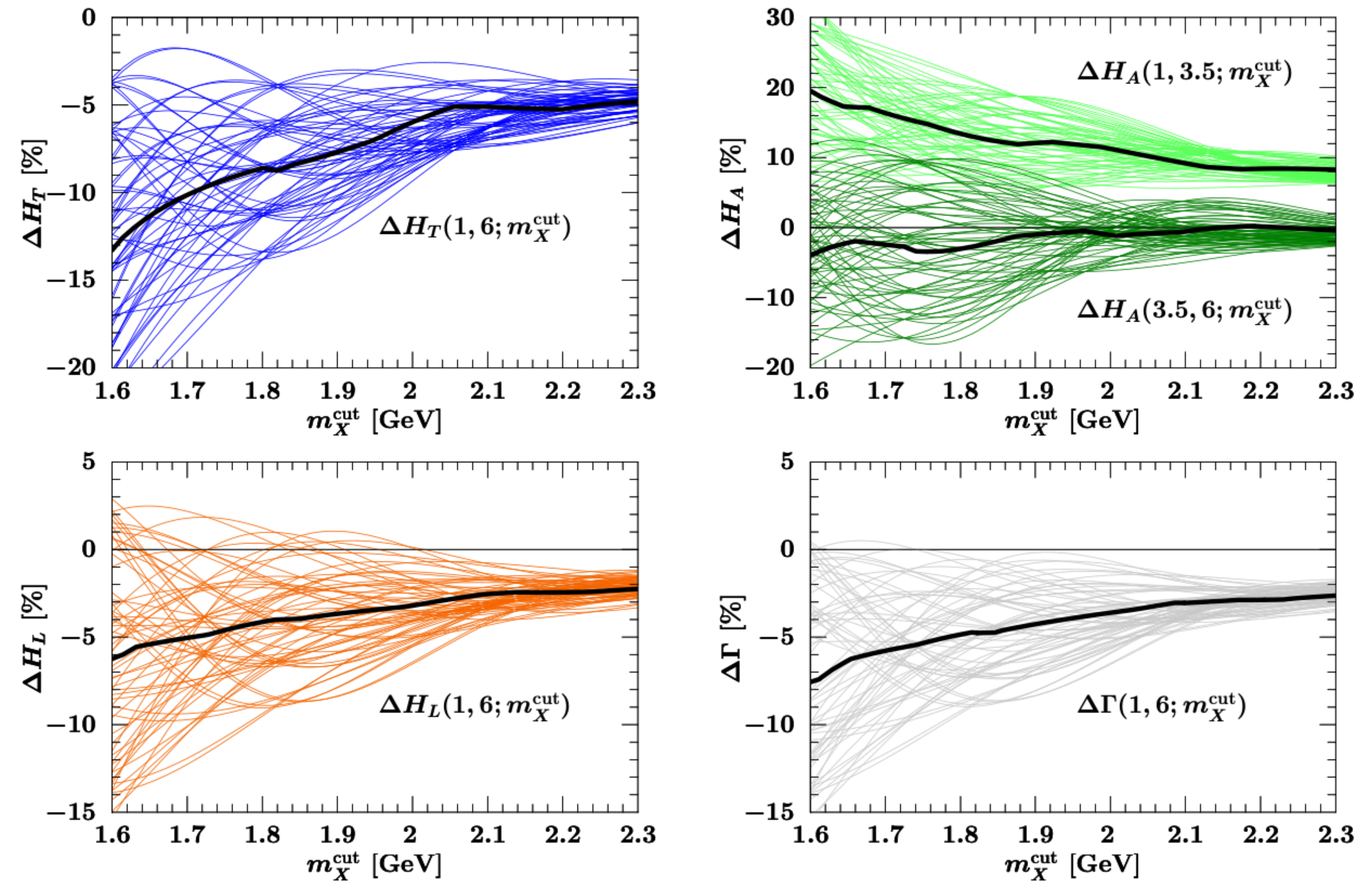
- Since the universality of the cuts extends to $B \to X_u \ell \nu$, the following ratio is minimally sensitive to the shape function modeling:

$$\frac{\Gamma(B \to X_s \ell \ell)_{\mathrm{cut}}}{\Gamma(B \to X_u \ell \nu)_{\mathrm{cut}}}$$

[same m_X cut]

M_X cuts: shape function modeling

- Current status of shape function modeling:
  [Lee, Ligeti, Stewart, Tackmann; Bell, Beneke, Huber, Li]



The same-color curves correspond to a sampling of potential shape functions

M_X cuts: shape function from $B \to X_s\gamma$

[Gambino, EL, Schacht - Work in progress]

- SCET at **leading power** shows that inclusive $b \to s\ell\ell$ and $b \to s\gamma$ depend on a **universal shape function**
- **Subleading** effects introduce dependence on subleading shape functions which destroy this universality (in particular the "effective" shape function that appears in $b \to s\ell\ell$ acquires a q^2 dependence
- As an alternative to SCET (and following the kinetic scheme analysis of $B \to X_c\ell\nu$) we write the $b \to s\gamma$ rate with a **Wilsonian cutoff** ($\mu \sim 1$ GeV):

$$\frac{d\Gamma}{dE_\gamma} = \int dk_+\, f(k_+,\mu)\frac{d\Gamma^{pert}}{dE_\gamma}\left(E_\gamma - \frac{k_+}{2}, \mu\right)$$

$$= \Gamma_0 \sum_{i\le j=1}^{8} C_i^{\text{eff}\,*}(\mu_b) C_j^{\text{eff}}(\mu_b) \int_{-\infty}^{\lambda} d\kappa\, F(\kappa,\mu) W_{ij}^{pert}(\xi-\kappa,\mu,\mu_b)$$

Shape Function in the kinetic scheme

where $F(\kappa,\mu) = m_b\, f(m_b\kappa,\mu)$ $\qquad \lambda = (m_B - m_b)/m_b$

$m_b = m_b^{\text{kin}}(\mu)$

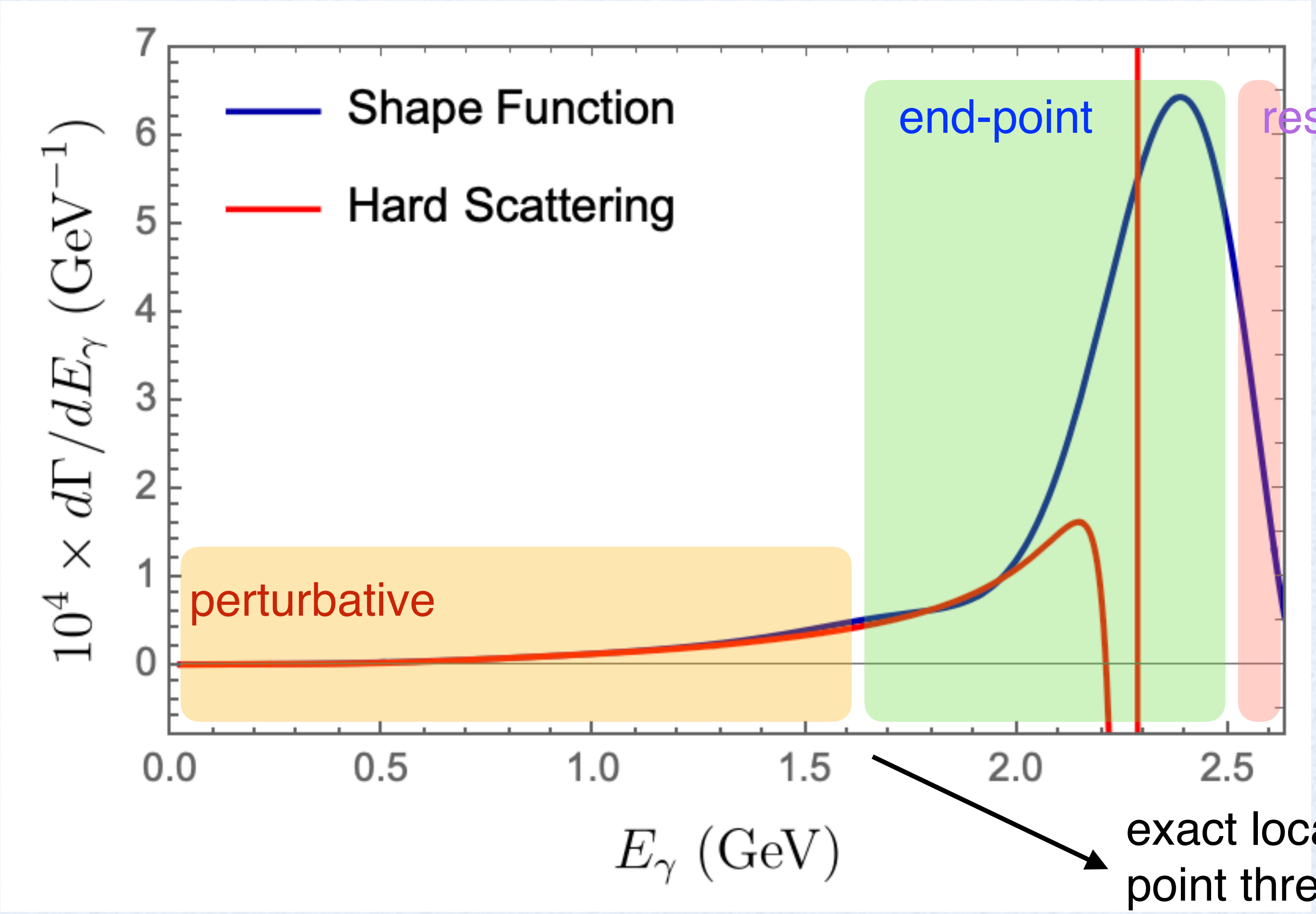
$\xi = 2E_\gamma/m_b$

$$\Gamma_0 = \frac{G_F^2 \alpha m_b^2 m_b^{\overline{\text{MS}}}(\mu_b)^2}{16\pi^4} |V_{tb}V_{ts}^*|^2$$

M_X cuts: shape function from $B \to X_s\gamma$

[Gambino, EL, Schacht - Work in progress]

- Shape function vs hard scattering spectra:



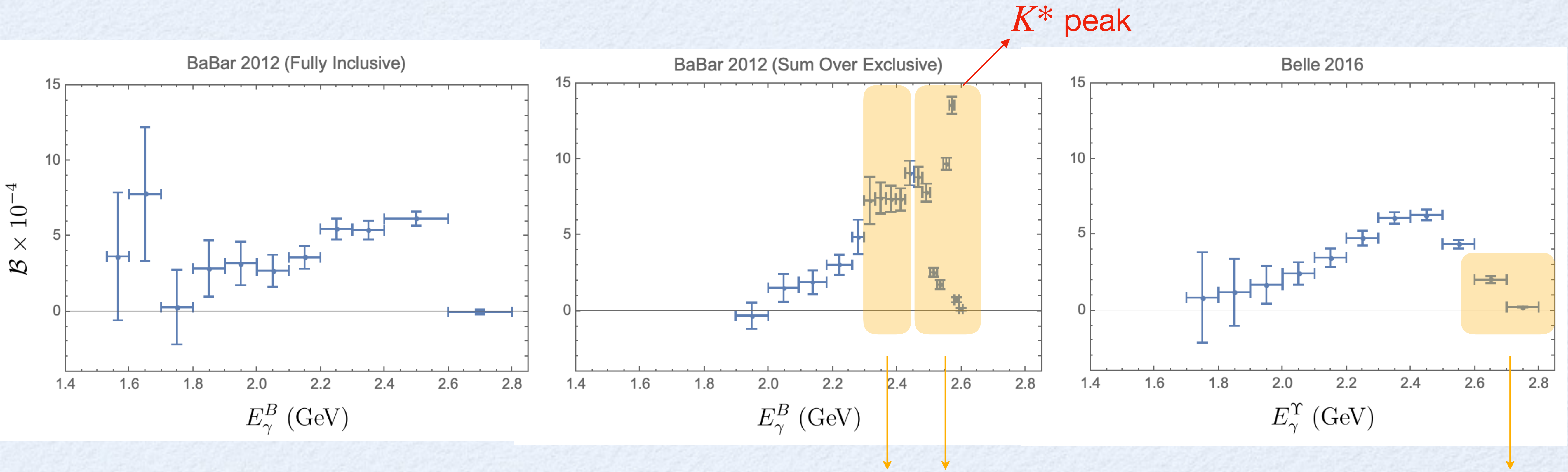
no Shape Function description: need to integrate over large enough E_γ range

exact location of the perturbative/end-point threshold depends on Shape Function

M_X cuts: shape function from $B \to X_s\gamma$

[Gambino, EL, Schacht - Work in progress]

- We considered data from 2012 BaBar fully inclusive and sum over exclusive analyses (in the B rest frame) and 2016 Belle results (in the $\Upsilon(4S)$ rest frame):



K^* peak

BaBar 2012 (Fully Inclusive)

BaBar 2012 (Sum Over Exclusive)

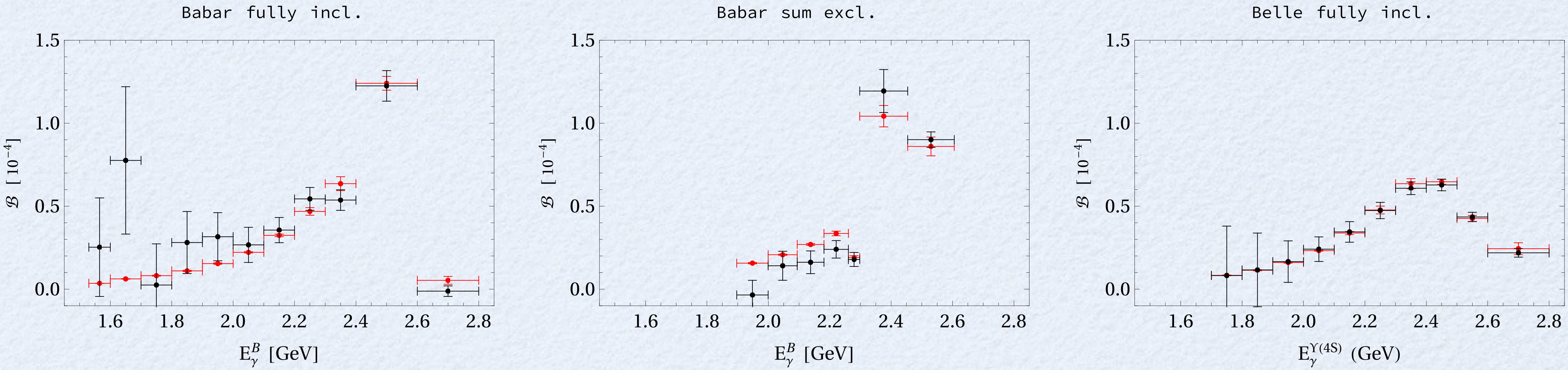
Belle 2016

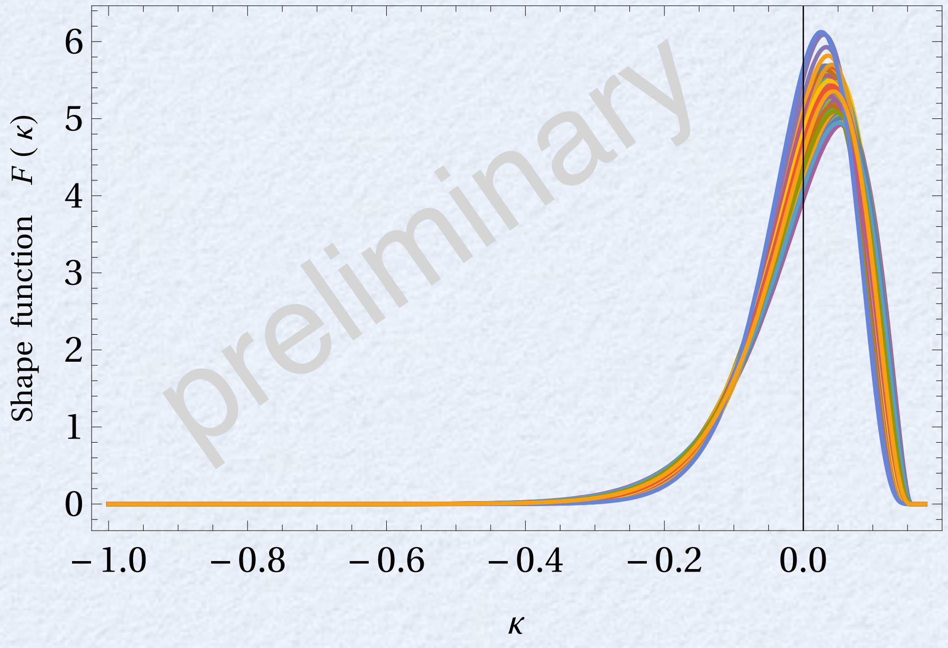
combined to remove sensitivity to the resonance region

M_X cuts: shape function from $B \to X_s\gamma$

[Gambino, EL, Schacht - Work in progress]

- Some very preliminary results:





- Some NNLO corrections missing
- Still working on training
- …

M_X cuts: shape function from $B \to X_s\gamma$

[Gambino, EL, Schacht - Work in progress]

- Implications for $B \to X_s \ell\ell$:
  - SF needed for extrapolation in and to improve the EvtGen Monte Carlo event generator which is the heart of Belle, BaBar and Belle II analyses.

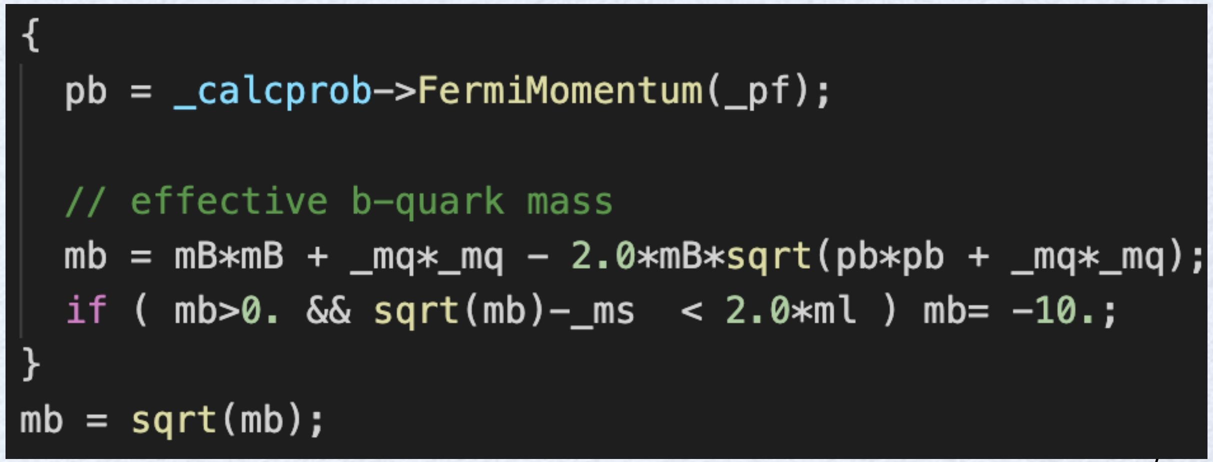
    [EvtGen: Ryd, Lange, Kuznetsova, Versille, Rotondo, Kirkby, Wuerthwein, Ishikawa; Maintained by J. Back, M. Kreps and T. Latham at University of Warwick]
  - Hadronic spectrum is based on the Fermi motion implementation presented in Ali, Hiller, Handoko, Morozumi hep-ph/9609449:

parton level with momentum dependent b mass

$$\frac{d\Gamma_B}{ds\,du\,dp} = \int du' \frac{m_b(p)^2}{m_B} p \left[\frac{4}{\sqrt{\pi p_F^3}} \exp(-p^2/p_F^2)\right] \left(u'^2 + 4m_b(p)^2 s\right)^{-1/2} \left[\frac{d\Gamma_b}{ds\,du}\right]_{m_b \to m_b(p)}$$

  - We need to urgently update the code!
  - Work in progress on the complete triple differential rate at $O(\alpha_s)$

    [T. Huber, T. Hurth, J. Jenkins, EL, in preparation]

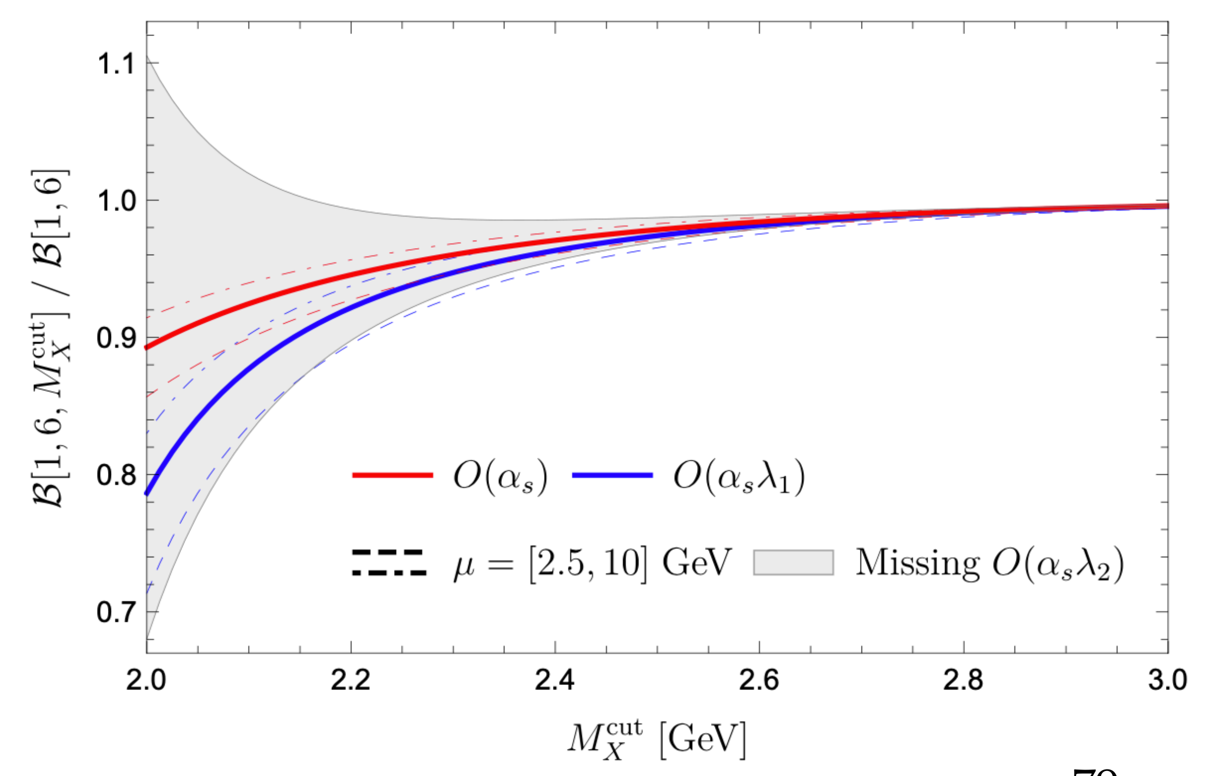


```
{
  pb = _calcprob->FermiMomentum(_pf);

  // effective b-quark mass
  mb = mB*mB + _mq*_mq - 2.0*mB*sqrt(pb*pb + _mq*_mq);
  if ( mb>0. && sqrt(mb)-_ms  < 2.0*ml ) mb= -10.;
}
mb = sqrt(mb);
```

M_X cuts: perturbative study

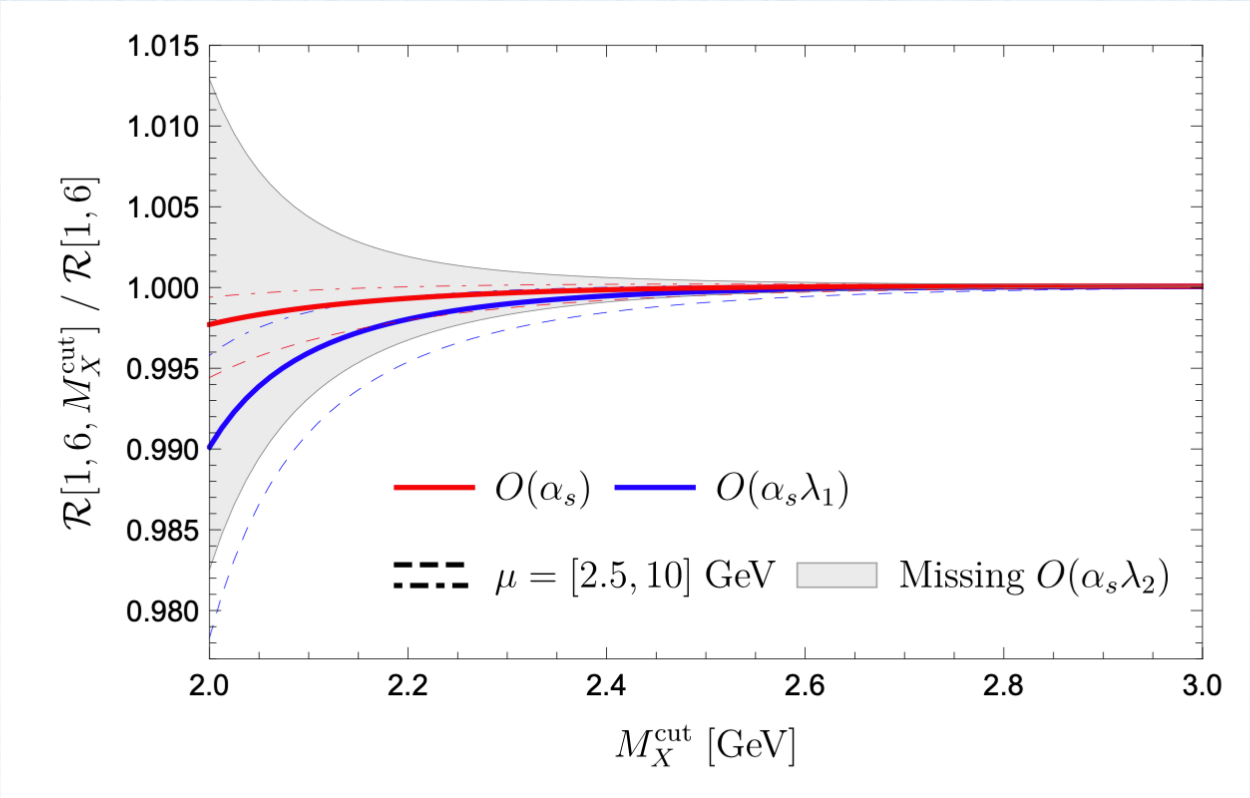
- We calculated the $B \to X_s \ell\ell$ M_X spectrum in perturbation theory at NLO including α_s and $\alpha_s\ \lambda_1/m_b^2$ corrections (using reparameterization invariance relations) [Huber, Hurth, Jenkins, EL, 2306.03134]
- The spectrum deviates develops a tail in M_X at $O(\alpha_s)$
- The $O(\alpha_s \mu_\pi^2)$ correction is necessary in order to asses the breakdown of the OPE
- The aim is to identify the minimum value of M_X^{cut} for which the perturbative calculation still holds (similar to a similar analysis for the photon energy spectrum in $B \to X_s \gamma$).



- A threshold can be tentatively set at $M_X^{\text{cut}} = 2.5$ GeV
- Experimental cuts are at $M_X^{\text{cut}} = 2$ GeV and they will require an extrapolation based on a Shape Function approach

M_X cuts: perturbative study

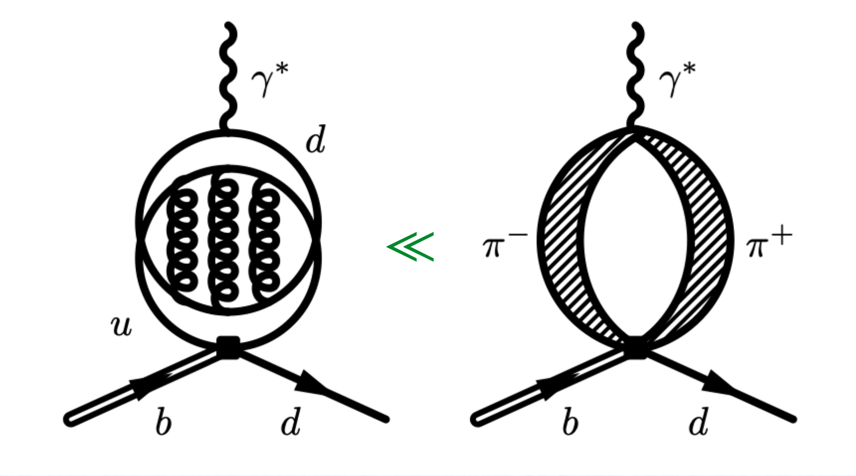
- The ratio of the low-q^2 branching ratio normalized to the $\bar{B} \to X_u \ell \bar{\nu}$ rate measured in the **same** q^2 range has much smaller power corrections: this suggests that the OPE for this quantity is much better behaved.



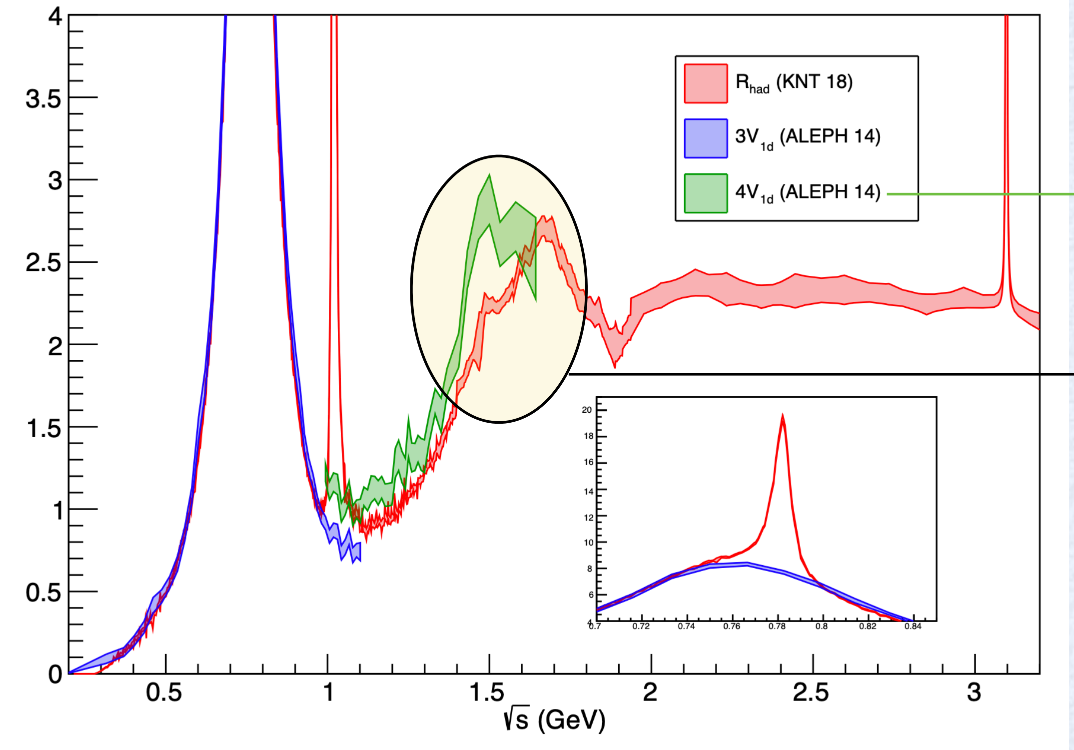
- The next step is to study the interpolation between the Shape Function region at small M_X and the perturbative region for $M_X > 2.5$ GeV

Inclusive theory: resonant color singlet production

- Considerable complications arise because we need to estimate $\langle J_q J_{q'} \rangle$ correlators with $q, q' = u, d, s$ whose relative size at low-q² is not described by perturbation theory at all



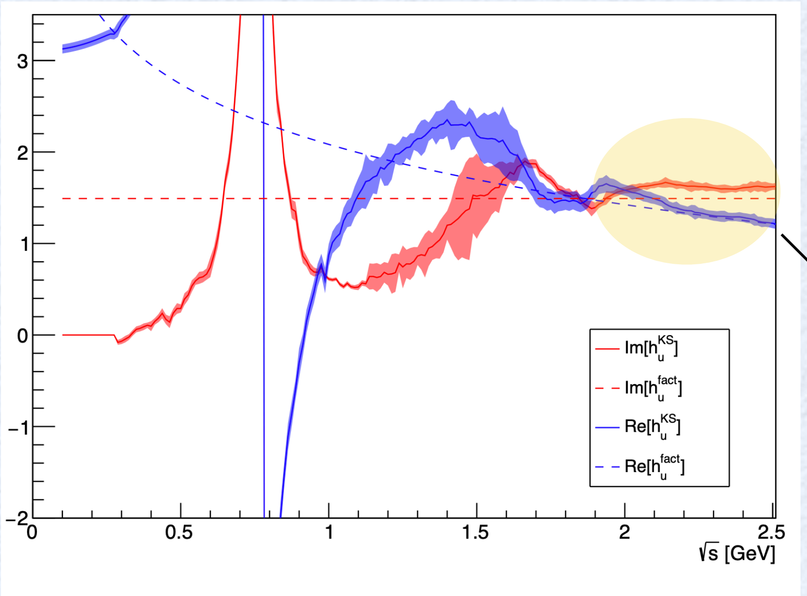
- Using both Isospin SU(2) and SU(3) we were able to express the $u\bar{u}, d\bar{d}$ and $s\bar{s}$ KS functions in terms of R_{had} and τ decay data only

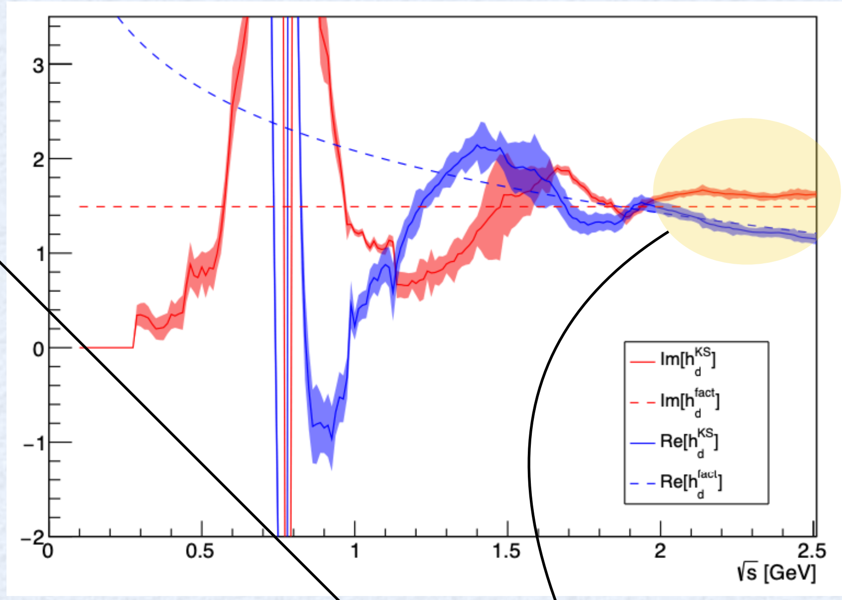


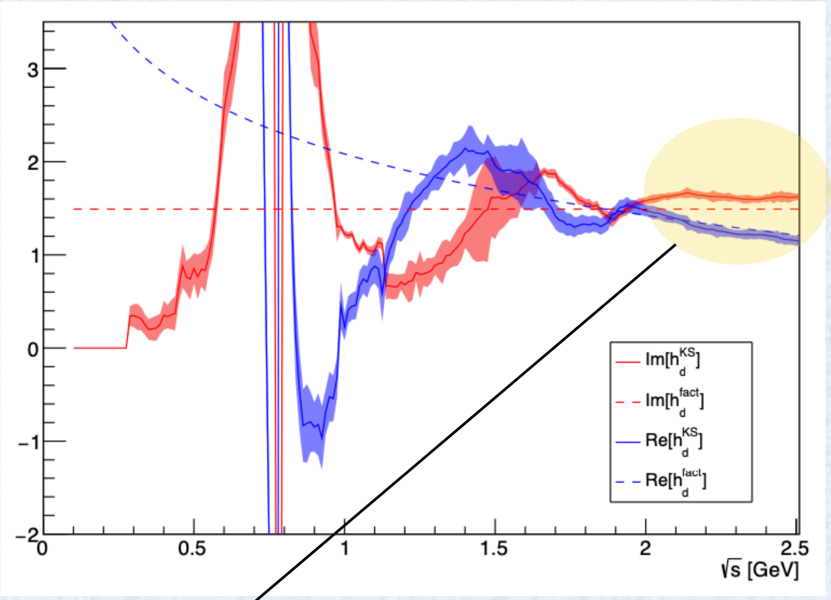
Rhad predicted from τ data using Isospin SU(3)

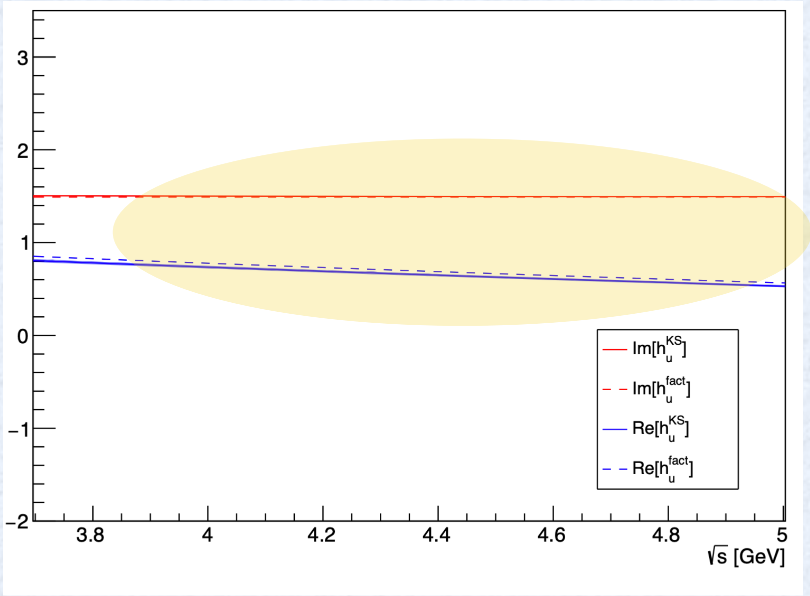
We use its deviation from the actual Rhad measurement (in red) as an estimate of SU(3) breaking effects

Inclusive theory: resonant color singlet production









Very good asymptotic agreement
with perturbation theory

$B \to X_d \ell \ell$: SM predictions

- Branching ratios

$$\begin{aligned}
\mathscr{B}[1,6]_{ee} &= (7.81 \pm 0.37_{\text{scale}} \pm 0.08_{m_t} \pm 0.17_{C,m_c} \pm 0.08_{m_b} \pm 0.04_{\alpha_s} \pm 0.15_{\text{CKM}} \\
&\quad \pm 0.12_{\text{BR}_{\text{sl}}} \pm 0.05_{\lambda_2} \pm 0.39_{\text{resolved}}) \cdot 10^{-8} \\
&= 7.81\ (1 \pm 7.8\%) \cdot 10^{-8} \\
\mathscr{B}[1,6]_{\mu\mu} &= 7.59\ (1 \pm 7.8\%) \cdot 10^{-8}
\end{aligned}$$

$$\begin{aligned}
\mathscr{B}[>14.4]_{ee} &= \ (0.86 \pm 0.12_{\text{scale}} \pm 0.01_{m_t} \pm 0.01_{C,m_c} \pm 0.08_{m_b} \pm 0.02_{\text{CKM}} \pm 0.02_{\text{BR}_{\text{sl}}} \\
&\quad \pm 0.06_{\lambda_2} \pm 0.25_{\rho_1} \pm 0.25_{f_{u,d}}) \cdot 10^{-8} \\
&= 0.86\ (1 \pm 45\%) \cdot 10^{-8} \\
\mathscr{B}[>14.4]_{\mu\mu} &= 1.00\ (1 \pm 39\%) \cdot 10^{-8}
\end{aligned}$$

- Scale and resolved uncertainties dominate at low-q^2 (hard to improve)
- Power corrections and scale uncertainties dominate at high-q^2

$B \to X_d \ell \ell$: SM predictions

- Ratio $\mathscr{R}(s_0)$

$$\begin{aligned}\mathscr{R}(14.4)_{ee} = \;& (0.93 \pm 0.02_{\text{scale}} \pm 0.01_{m_t} \pm 0.01_{C,m_c} \pm 0.002_{m_b} \pm 0.01_{\alpha_s} \pm 0.05_{\text{CKM}} \\ & \pm 0.004_{\lambda_2} \pm 0.06_{\rho_1} \pm 0.05_{f_{u,d}}) \times 10^{-4} \\ =\;& 0.93\;(1 \pm 9.7\%) \times 10^{-4}\end{aligned}$$

$$\mathscr{R}(14.4)_{\mu\mu} = 1.10\;(1 \pm 6.4\%) \times 10^{-4}$$

- Forward-backward asymmetry and zero-crossing

$$H_A[1,3.5]_{ee} = -0.41\;(1 \pm 9.8\%) \cdot 10^{-8}$$

$$H_A[3.5,6]_{ee} = 0.40\;(1 \pm 18\%) \cdot 10^{-8}$$

$$H_A[1,3.5]_{\mu\mu} = -0.44\;(1 \pm 9.1\%) \cdot 10^{-8}$$

$$H_A[3.5,6]_{\mu\mu} = 0.37\;(1 \pm 19\%) \cdot 10^{-8}$$

$$\begin{aligned}(q_0^2)_{ee} = \;& 3.28 \pm 0.11_{\text{scale}} \pm 0.001_{m_t} \pm 0.02_{C,m_c} \pm 0.05_{m_b} \\ & \pm 0.03_{\alpha_s} \pm 0.004_{\text{CKM}} \pm 0.001_{\lambda_2} \pm 0.06_{\text{resolved}} = 3.28 \pm 0.14\end{aligned}$$

$$(q_0^2)_{\mu\mu} = 3.39 \pm 0.14$$