Geometric approaches to SMEFT and HEFT

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COMETA

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The scalar sector of the SM

4 scalars
- 3 Goldstones π_i. $i = 1, 2, 3,$ needed for W, Z masses
- 1 scalar ϕ controlling EWSB

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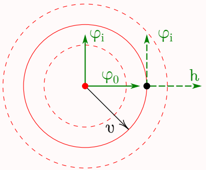
SM(EFT) packages them in a $SU(2)$ doublet $H = \frac{1}{\sqrt{2}}\begin{pmatrix}\pi_2 + i\pi_1 \\ \phi - i\pi_3\end{pmatrix}$ $\Rightarrow$ $H^\dagger H = \frac{1}{2}\left(\phi^2 + |\vec{\pi}|^2\right)$

under $SU(2)_L \times SU(2)_R$

$$\pi_i \mapsto \pi_i + \frac{\alpha_L^i - \alpha_R^i}{2}\,\phi + \varepsilon^{ijk}\,\pi_j\,\frac{\alpha_L^k + \alpha_R^k}{2}$$

$$\phi \mapsto \phi - \frac{\alpha_L^i - \alpha_R^i}{2}\,\pi_i$$

linear, but mixing fields



EWSB requires $\langle H^\dagger H\rangle = \frac{v^2}{2}$ and one can choose $\langle\phi\rangle = v$
$\langle\pi_i\rangle = 0$

The scalar sector of the SM

4 scalars
- 3 Goldstones π_i. $i = 1,2,3$, needed for W, Z masses
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to describe EWSB: polar coordinates more convenient $H = \frac{\phi}{\sqrt{2}} \exp\left[\frac{i\sigma_i\pi_i}{v}\right]$ $\Rightarrow$ $H^\dagger H = \frac{1}{2}\phi^2$

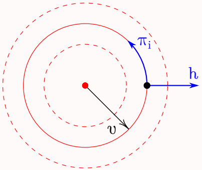
under $SU(2)_L \times SU(2)_R$

$$\pi_i \mapsto \pi_i \left[1 + \frac{\alpha_L^i - \alpha_R^i}{2}\frac{\pi_i}{|\vec{\pi}|}\left(\frac{v}{|\vec{\pi}|} - \cot\frac{|\vec{\pi}|}{v} + \dots\right)\right] + \varepsilon^{ijk}\,\pi_j\,\frac{\alpha_L^k + \alpha_R^k}{2}$$

$$\phi \mapsto \phi$$

nonlinear, but ϕ pure singlet

EWSB unambiguously requires $\langle\phi\rangle = v$



EFT extensions of the SM

SMEFT ▸ uses H doublet as building block

▸ expands in canonical dimensions (H/Λ), around $H = 0$

Buchmüller,Wyler 1986
Grzadkowski+ 2010

HEFT ▸ uses h and $\mathbf{U} = \exp(i\sigma_i \pi_i / v)$ as two independent building blocks

▸ expands around EW vacuum in (D_μ/Λ), not in h/v nor $\mathbf{U}$

Feruglio 1993, Grinstein,Trott 2007
Buchalla+, Gavela+ 2012, 2013
IB+ 2016, Sun+ 2022 ...

☞ expansions are different, and scalar representations are related by a field redefinition

$$H = \frac{v+h}{\sqrt{2}} \mathbf{U} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \mathbf{U} = \frac{1}{\sqrt{H^\dagger H}} \left(\tilde{H}\ H\right), \quad h = \sqrt{2H^\dagger H} - v$$

- it is always possible to rewrite SMEFT as HEFT
- rewriting HEFT as SMEFT requires that the theory contains $H^\dagger H = 0$ as a reachable point and that it is analytical there

Falkowski,Rattazzi 1902.05936
Alonso,Jenkins,Manohar 1605.03602
Cohen,Craig,Lu,Sutherland 2008.08597

SM $\subset$ SMEFT $\subset$ HEFT

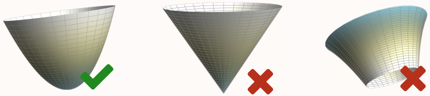
Geometrical descriptions of SMEFT/HEFT

Geometrical methods are introduced in this context to obviate field redefinition ambiguities.

$\rightarrow$ *what are the fundamental differences between the two EFTs?*
$\rightarrow$ *can phenomenological signatures of HEFT be identified?*

☞ a formalism to write scattering amplitudes independently of the field choice

- **model-independent** results, can be evaluated for different theories without recomputing diagrams
  true for amplitudes. applications also to RGEs and matching Jenkins,Manohar,Naterop,Pagés 2308.06315, 2310.19883 Li,Lu,Zhang 2411.04173
- allow a characterization of theories that *cannot* be matched onto SMEFT Cohen,Craig,Lu,Sutherland 2008.08597



this talk: **scalar** sector only applications with gauge fields and fermions: Helset+ 2212.03253, 2210.08000, 2307.03187 Pilaftsis+ 2006.05831, 2307.01126

"Standard" geometric description of SMEFT/HEFT

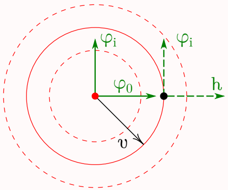
the 4 scalar fields can be seen as coordinates on 4D manifold

Alonso,Jenkins,Manohar 1511.00724,1605.03602

SMEFT $\sim$ **cartesian** coord.

$$(\mathbb{R}^4) \qquad \vec{\phi} = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{pmatrix}$$

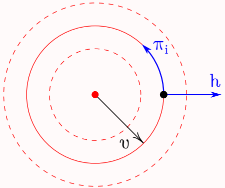
$$(SU(2)) \qquad H = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_2 + i\phi_1 \\ \phi_4 - i\phi_3 \end{pmatrix}$$



HEFT $\sim$ **polar** coord.

$$\vec{\phi} = (v + h) \exp\left[\frac{2\pi^i t_i}{v}\right] \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\mathbf{U} = \exp\left[\frac{\pi^i \sigma_i}{v}\right]$$



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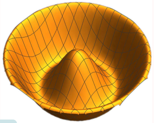

- field redefinition $\leftrightarrow$ change of coordinates
- physics can be associated to geometry of the field space, independent of coordinates

Physics – Geometry connection

The kinetic term corresponds to a metric in field space

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi^i\partial^\mu\phi^j g_{ij}(\phi) + \ldots$$

it captures **all operators with 2 derivatives**, up to arbitrary dimensions. e.g.

$$\begin{aligned}
\partial_\mu H^\dagger\partial^\mu H\,(H^\dagger H)^n &= \frac{1}{2}\partial_\mu\vec{\phi}\cdot\partial^\mu\vec{\phi}\ \frac{|\vec{\phi}|^{2n}}{2^n} &\to\quad g_{ij} &= \delta_{ij}\,\frac{|\vec{\phi}|^{2n}}{2^n}\\
H^\dagger H\,\Box\,(H^\dagger H) &= -(\vec{\phi}\cdot\partial_\mu\vec{\phi})^2 &\to\quad g_{ij} &= -2\phi_i\phi_j\\
(iH^\dagger\partial_\mu H - i\partial_\mu H^\dagger H)^2 &= 4(\partial_\mu\vec{\phi}\ t_{3R}\ \vec{\phi})^2 &\to\quad g_{ij} &= 8(t_{3R}\phi)_i(t_{3R}\phi)_j
\end{aligned}$$

amputated, on-shell amplitudes are covariant → func. of covariant geometric properties at vacuum

$$\mathcal{A}(\phi_i\phi_j \to \phi_k\phi_l) = R_{ijkl}s_{ik} + R_{ikjl}s_{ij}$$

S-matrices are obtained contracting them with vielbeins (external wavefunctions) ⇒ invariant

Cohen+ 2202.06965, 2312.06748

Two main issues

A. terms with **more** (or less!) than 2 derivatives are not described by geometry

B. the formalism is not invariant under field redefinitions involving **derivatives**, eg.

$$\phi_i \to \phi_i + c\frac{\Box\phi_i}{\Lambda^2}$$

which are used for operator basis reduction (EOMs)

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☞ attempts at solving with Lagrange geometry, functional geometry, geometry-kinematics duality

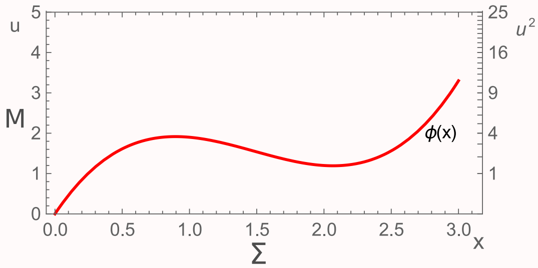
Craig+ 2305.09722, 2307.15742, 2202.06965, Cohen+ 2312.06748, 2410.21378, Cheung+ 2202.06972

Jet bundle geometry

Fibre bundle picture

we want our language to include derivatives $\partial_\mu \phi(x) \rightarrow$ keep ϕ dependence on x manifest!

natural structure: **fibre bundle** Alminawi,IB,Davighi 2308.00017



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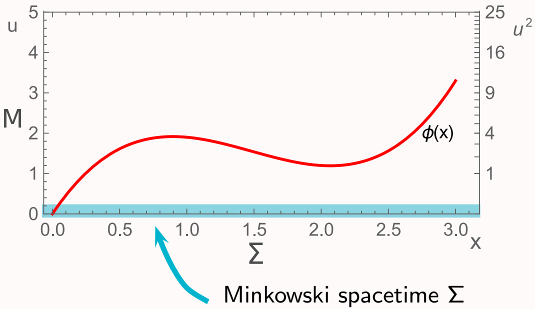

Minkowski spacetime Σ

w/ coord x^μ, metric η

$\equiv$ **base space**

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field config. space M

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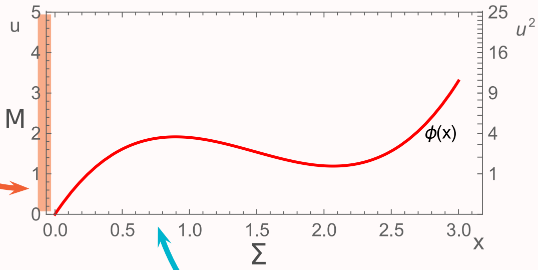

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locally: $E_x = \Sigma_x \times M$

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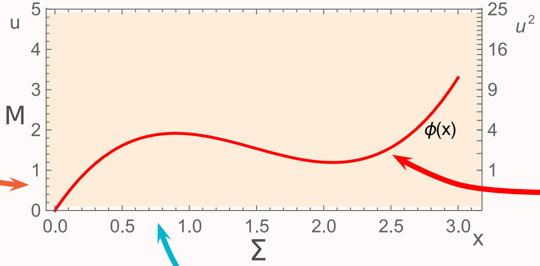

field $\phi^i(x) : \Sigma \to M$

$\equiv$ **section** of E over Σ

$\neq u^i$

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natural structure: **fibre bundle** (E, Σ, π) Alminawi,IB,Davighi 2308.00017

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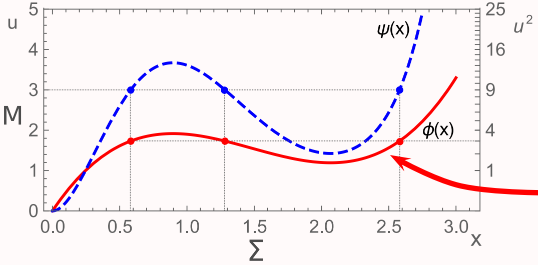
field redefinition

=

change of section

if non-derivative:
equivalent to **diffeo**
$f : E \to E$



field $\phi^i(x) : \Sigma \to M$
$\equiv$ **section** of E over Σ
$\neq u^i$

example: $(\phi \to \psi = \phi^2$ with fixed $u) \equiv (u \to u^2$ with fixed $\phi)$

Fibre bundle metric → ∂^2 scalar Lagrangian

the fibre bundle is a Riemannian manifold, on which we can build a **metric**

$$g = \begin{pmatrix} dx^\mu & du^i \end{pmatrix} \begin{pmatrix} g_{\mu\nu} & g_{\mu j} \\ g_{\nu i} & g_{ij} \end{pmatrix} \begin{pmatrix} dx^\nu \\ du^j \end{pmatrix} = g_{\mu\nu} dx^\mu dx^\nu + 2 g_{\mu i} dx^\mu du^i + g_{ij} du^i du^j$$

Poincaré invariance ⇒ g_{IJ} depend on u^i but *not* on x^μ, $g_{\mu i} \equiv 0$

⚠ $g_{\mu\nu} \neq \eta_{\mu\nu}$

Metric to Lagrangian.

g is a (0,2) tensor on E

↓ pull back from E to Σ along $\phi(x)$

(0,2) tensor on Σ

↓ contract with $\eta^{-1}/2$

scalar object on Σ = $\mathscr{L}$

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$u^i \to \phi^i(x)$
$du^i \to \partial_\rho \phi^i(x)$
$dx^\mu \to \delta^\mu_\rho$

$$\mathscr{L} = \eta^{\rho\sigma} \left[\frac{1}{2} g_{\rho\sigma}(\phi) + \frac{1}{2} g_{ij}(\phi) \partial_\rho \phi^i \partial_\sigma \phi^j \right]$$

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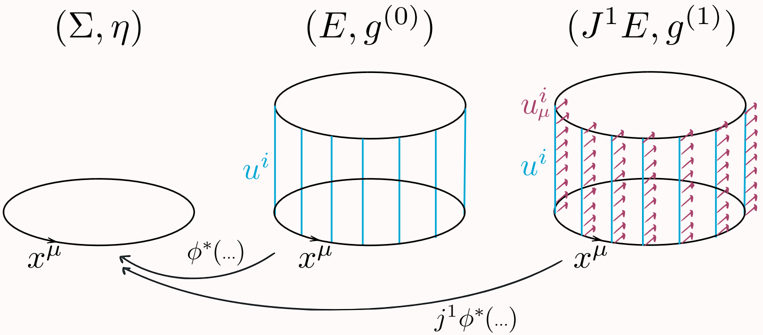
Metric to Lagrangian.

$$\mathscr{L} = \underbrace{\frac{1}{2} \eta^{\rho\sigma} g_{\rho\sigma}(\phi)}_{\equiv -V(\phi)} + \underbrace{\frac{1}{2} g_{ij}(\phi) \partial_\rho \phi^i \partial^\rho \phi^j}_{\text{same as usual geo}}$$

geometric interpretation of the scalar potential!

Jet bundles

Saunders 1989. see also Craig,Lee 2307.15742



$j^i_x\phi$ $=$ r-jet of ϕ at x $=$ equivalence class containing sections identical up to r-th derivative

J^rE $=$ r-jet bundle $= \{j^r_x\phi | x \in \Sigma, \phi \in \Gamma_x(\pi)\}$ is a differentiable manifold. we use only J^1E

1-jet bundle metric $\to$ ∂^4 scalar Lagrangian

the 1-jet bundle is a Riemannian manifold, on which we can build a **metric**

$$g^{(1)} = \begin{pmatrix} dx^\mu & du^i & du^i_\mu \end{pmatrix} \begin{pmatrix} g_{\mu\nu} & g_{\mu j} & g^\nu_{\mu j} \\ g_{\nu i} & g_{ij} & g^\nu_{ij} \\ g^\mu_{\nu i} & g^\mu_{ij} & g^{\mu\nu}_{ij} \end{pmatrix} \begin{pmatrix} dx^\nu \\ du^j \\ du^j_\nu \end{pmatrix}$$

$$= g_{\mu\nu} dx^\mu dx^\nu + 2 g_{\mu i} dx^\mu du^i + 2 g^\nu_{\mu j}\, dx^\mu du^j_\nu + g_{ij}\, du^i du^j + 2 g^\nu_{ij}\, du^i du^j_\nu + g^{\mu\nu}_{ij}\, du^i_\mu du^j_\nu$$

Poincaré invariance $\Rightarrow$ g_{IJ} depend on u^i, u^i_μ but *not* on x^μ

Metric to Lagrangian.

$g^{(1)}$ is a (0,2) tensor on $J^1 E$

$\downarrow$ pull back from $J^1 E$ to Σ along $j^1\phi(x)$

(0,2) tensor on Σ

$\downarrow$ contract with $\eta^{-1}/2$

scalar object on Σ = $\mathscr{L}$

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$$= g_{\mu\nu} dx^\mu dx^\nu + 2 g_{\mu i} dx^\mu du^i + 2 g^\nu_{\mu j} dx^\mu du^j_\nu + g_{ij} du^i du^j + 2 g^\nu_{ij} du^i du^j_\nu + g^{\mu\nu}_{ij} du^i_\mu du^j_\nu$$

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scalar object on $\Sigma = \mathscr{L}$

$u^i \to \phi^i(x)$ $\quad u^i_\mu \to \partial_\mu \phi^i(x)$

$du^i \to \partial_\rho \phi^i(x)$ $\quad du^i_\mu \to \partial_\rho \partial_\mu \phi^i(x)$

$dx^\mu \to \delta^\mu_\rho$

↓

$\mathscr{L}$

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Metric to Lagrangian.

$$\mathscr{L} = \frac{1}{2}\eta^{\mu\nu} g_{\mu\nu} + g_{\mu i}\partial^\mu\phi^i + g^\nu_{\mu j}\partial^\mu\partial^\nu\phi^j + \frac{1}{2} g_{ij}\partial_\mu\phi^i\partial^\mu\phi^j + g^\nu_{ij}\partial_\rho\phi^i\partial^\rho\partial_\nu\phi^j + \frac{1}{2} g^{\mu\nu}_{ij}\partial_\rho\partial_\mu\phi^i\partial^\rho\partial_\nu\phi^j$$

a redundant basis of operators with up to 4 derivatives

Scalar Lagrangian from 1-jet bundle metric: 1 scalar case

retaining only terms leading to operators with **up to 4 derivatives**

$$\frac{g_{\mu\nu}}{\Lambda^4} = -\frac{\eta_{\mu\nu}}{2}V(u) + \left[\frac{u_\mu u_\nu}{\Lambda^4} + \frac{\eta_{\mu\nu}}{4}\frac{u_\rho u^\rho}{\Lambda^4}\right]\frac{J(u)}{2} + \left[\frac{u_\mu u_\nu}{\Lambda^4} + \frac{\eta_{\mu\nu}}{4}\frac{u_\rho u^\rho}{\Lambda^4}\right]\frac{u_\sigma u^\sigma}{\Lambda^4}\frac{K(u)}{2}$$

$$\frac{g_{\mu u}}{\Lambda^2} = \frac{u_\mu}{\Lambda^2}G(u) + \frac{u_\mu u_\rho u^\rho}{\Lambda^6}H(u)$$

$$g_{\mu u}^{\nu} = \delta_\mu^\nu E(u) + \frac{u^\nu u_\mu}{\Lambda^4}F_1(u) + \delta_\mu^\nu \frac{u_\rho u^\rho}{\Lambda^4}F_2(u)$$

$$g_{uu} = C(u) + \frac{u_\rho u^\rho}{\Lambda^4}D(u) \qquad \Lambda g_{uu}^{\mu} = \frac{u^\mu}{\Lambda}B(u) \qquad \Lambda^2 g_{uu}^{\mu\nu} = \eta^{\mu\nu}A(u)$$

pulls back to

$$\begin{aligned}\mathscr{L} = &\frac{1}{2}\partial_\mu\phi\partial^\mu\phi\,(C + 2G + J) - \Lambda(\Box\phi)E - \Lambda^4 V \\ &+ \frac{\partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi}{\Lambda^2}\frac{A}{2} + \frac{\partial_\mu\partial_\nu\phi\partial^\mu\phi\partial^\nu\phi}{\Lambda^3}(B + F_1) + \frac{(\Box\phi)(\partial_\mu\phi\partial^\mu\phi)}{\Lambda^3}F_2 + \frac{(\partial_\mu\phi\partial^\mu\phi)^2}{\Lambda^4}\frac{D + 2H + K}{2}\end{aligned}$$

Scalar Lagrangian from 1-jet bundle metric: 1 scalar case

retaining only terms leading to operators with **up to 4 derivatives**

$$\frac{g_{\mu\nu}}{\Lambda^4} = -\frac{\eta_{\mu\nu}}{2}V(u) + \left[\frac{u_\mu u_\nu}{\Lambda^4} + \frac{\eta_{\mu\nu}}{4}\frac{u_\rho u^\rho}{\Lambda^4}\right]\frac{J(u)}{2} + \left[\frac{u_\mu u_\nu}{\Lambda^4} + \frac{\eta_{\mu\nu}}{4}\frac{u_\rho u^\rho}{\Lambda^4}\right]\frac{u_\sigma u^\sigma}{\Lambda^4}\frac{K(u)}{2}$$

$$\frac{g_{\mu u}}{\Lambda^2} = \frac{u_\mu}{\Lambda^2}G(u) + \frac{u_\mu u_\rho u^\rho}{\Lambda^6}H(u)$$

$$g^\nu_{\mu u} = \delta^\nu_\mu E(u) + \frac{u^\nu u_\mu}{\Lambda^4}F_1(u) + \delta^\nu_\mu \frac{u_\rho u^\rho}{\Lambda^4}F_2(u)$$

$$g_{uu} = C(u) + \frac{u_\rho u^\rho}{\Lambda^4}D(u) \qquad \Lambda g^\mu_{uu} = \frac{u^\mu}{\Lambda}B(u) \qquad \Lambda^2 g^{\mu\nu}_{uu} = \eta^{\mu\nu}A(u)$$

pulls back to

$$\mathscr{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi\left(C + 2G + J - 2E'\right) - \Lambda^4 V$$

blue = can be removed via EOM

$$+\frac{\partial_\mu\partial_\nu\phi\partial^\mu\partial^\nu\phi}{\Lambda^2}\frac{A}{2} + \frac{\partial_\mu\partial_\nu\phi\partial^\mu\phi\partial^\nu\phi}{\Lambda^3}(B + F_1 - 2F_2) + \frac{(\partial_\mu\phi\partial^\mu\phi)^2}{\Lambda^4}\frac{D + 2H + K - 2F_2'}{2}$$

Extension to higher derivatives

metric $g^{(r)}$ of a r-jet bundle $\longrightarrow$ **redundant** basis of operators with up to $2(r+1)$ deriv.

r-jet bundle has coordinates $y^I = (x^\mu, u^i, u^i_{\mu_1}, u^i_{\mu_1\mu_2}, \dots, u^i_{\mu_1\dots\mu_r})$

$$g^{(r)} = \begin{pmatrix} dx^\mu & du^i & du^i_{\mu_1} & \cdots & du^i_{\mu_1\dots\mu_r} \end{pmatrix} \begin{pmatrix} g_{\mu\nu} & g_{\mu j} & g^{\nu_1}_{\mu j} & \cdots & g^{\nu_1\dots\nu_r}_{\mu j} \\ g_{\nu i} & g_{ij} & g^{\nu_1}_{ij} & \cdots & g^{\nu_1\dots\nu_r}_{ij} \\ g^{\mu_1}_{\nu i} & g^{\mu_1}_{ij} & g^{\mu_1\nu_1}_{ij} & \cdots & g^{\mu_1\nu_1\dots\nu_r}_{ij} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ g^{\mu_1\dots\mu_r}_{\nu i} & g^{\mu_1\dots\mu_r}_{ij} & g^{\mu_1\dots\mu_r\nu_1}_{ij} & \cdots & g^{\mu_1\dots\mu_r\nu_1\dots\nu_r}_{ij} \end{pmatrix} \begin{pmatrix} dx^\nu \\ du^j \\ du^j_{\nu_1} \\ \cdots \\ du^j_{\nu_1\dots\nu_r} \end{pmatrix}$$

- arbitrary internal symmetries (or absence thereof) can always be implemented
- many redundancies! different metric entries mapping to same operators, IBP, EOM, diffeos. . .

Connection to scattering amplitudes

next step: express S-matrix elements as a function of geometric objects on the r-jet bundle

- on-shell amplitudes $\mathcal{A}_{i_1 \dots i_n}$ are expected to be **covariant in field indices**
- off-shell amplitudes and Feynman rules are **not** expected to transform covariantly in general
- covariant results are well established in usual geometric language for 2 derivative terms

Helset et al 2111.03045,2202.06972,2210.08000, Nagai et al 1904.07618, Cohen et al 2108.03240. . .

we start by working with the **0-jet bundle = fibre bundle** first

→ we should be able to reproduce the known results + a geo interpretation of the potential

Procedure:

1. derive Feynman rules from $\mathscr{L}(g)$. They will generally contain derivatives of $g_{IJ} \to \Gamma^I_{JK}$ etc
2. compute an on-shell amplitude
3. **evaluate at the vacuum** of the theory, defined by $g_{\mu\nu,i} = -\frac{\eta_{\mu\nu}}{2} \partial_i V(\phi) \equiv 0 \quad \forall i$

Fibre bundle geometry

Alminawi,IB,Davighi in progress

$\bar{X} \equiv X\big|_{\text{vacuum}} \qquad A_{,b} \equiv \partial_b A$

Christoffel symbols

$$\Gamma^{\mu}_{\nu\rho} = \Gamma^{\mu}_{ij} = \Gamma^{i}_{j\mu} = 0$$

$$\Gamma^{i}_{\mu\nu} = -\frac{g^{im}}{2} g_{\mu\nu,m}$$

$$\Gamma^{\mu}_{i\nu} = \frac{g^{\mu\rho}}{2} g_{\rho\nu,i}$$

$$\Gamma^{i}_{jk} = \frac{g^{im}}{2} \left[g_{jm,k} + g_{km,j} - g_{jk,m}\right]$$

evaluating at the **vacuum** of the theory: $\overline{g_{\mu\nu,i}} = -\eta_{\mu\nu} \overline{V_{,i}}/2 \equiv 0 \quad \forall i$

$$\bar{\Gamma}^{i}_{jk} = \frac{\bar{g}^{im}}{2} \left[\overline{g_{jm,k}} + \overline{g_{km,j}} - \overline{g_{jk,m}}\right]$$

all others $= 0$

Fibre bundle geometry

Alminawi,IB,Davighi in progress

$\bar{X} \equiv X\big|_{\text{vacuum}} \qquad A_{,b} \equiv \partial_b A$

Riemann tensors

$$R^i_{\mu\nu\rho} = R^\mu_{i\nu\rho} = R^\mu_{\nu i\rho} = R^\mu_{\nu\rho i} = R^i_{jk\mu} = R^i_{j\mu k} = R^i_{\mu jk} = R^\mu_{ijk} \equiv 0$$

$$\begin{aligned} R^\mu_{\nu\rho\sigma} &= \Gamma^\mu_{\rho m}\Gamma^m_{\nu\sigma} - \Gamma^\mu_{\sigma m}\Gamma^m_{\nu\rho} \\ R^\mu_{ij\nu} &= \Gamma^\mu_{i\nu,j} + \Gamma^\mu_{j\rho}\Gamma^\rho_{i\nu} - \Gamma^\mu_{m\nu}\Gamma^m_{ij} \\ R^i_{j\mu\nu} &= \Gamma^i_{\mu\rho}\Gamma^\rho_{\nu j} - \Gamma^i_{\nu\rho}\Gamma^\rho_{\mu j} \end{aligned}$$

$$\begin{aligned} R^i_{jkl} &= \Gamma^i_{jl,k} - \Gamma^i_{jk,l} + \Gamma^i_{km}\Gamma^m_{jl} - \Gamma^i_{lm}\Gamma^m_{jk} \\ R^\mu_{\nu ij} &= \Gamma^\mu_{\nu j,i} - \Gamma^\mu_{\nu i,j} + \Gamma^\mu_{i\rho}\Gamma^\rho_{\nu j} - \Gamma^\mu_{j\rho}\Gamma^\rho_{\nu i} \\ R^i_{\mu j\nu} &= \Gamma^i_{\mu\nu,j} - \Gamma^i_{\nu\rho}\Gamma^\rho_{\mu j} + \Gamma^i_{jm}\Gamma^m_{\mu\nu} \end{aligned}$$

evaluating at the **vacuum** of the theory: $\overline{g_{\mu\nu,i}} = -\eta_{\mu\nu}\overline{V_{,i}}/2 \equiv 0 \quad \forall i$

$$\begin{aligned} \bar{R}^\mu_{\nu\rho\sigma} &= \bar{R}^\mu_{\nu ij} = \bar{R}^i_{j\mu\nu} = 0 \\ \bar{R}^\mu_{ij\nu} &= \overline{\Gamma^\mu_{i\nu,j}} \end{aligned}$$

$$\begin{aligned} \bar{R}^i_{jkl} &= \overline{\Gamma^i_{jl,k}} - \overline{\Gamma^i_{jk,l}} + \bar{\Gamma}^i_{km}\bar{\Gamma}^m_{jl} - \bar{\Gamma}^i_{lm}\bar{\Gamma}^m_{jk} \\ \bar{R}^i_{\mu j\nu} &= \overline{\Gamma^i_{\mu\nu,j}} \end{aligned}$$

Fibre bundle geometry

Alminawi,IB,Davighi in progress

$\bar{X} \equiv X\big|_{\text{vacuum}} \qquad A_{,b} \equiv \partial_b A$

Covariant derivatives of the Riemann tensors.

The non-vanishing options are

$$\nabla_\alpha R^i_{\mu\nu\rho} \quad \nabla_\alpha R^\mu_{i\nu\rho} \quad \nabla_\alpha R^\mu_{\nu i\rho} \quad \nabla_a R^\mu_{\nu\rho\sigma} \quad \nabla_a R^\mu_{ij\nu} \quad \nabla_a R^\mu_{\nu ij}$$

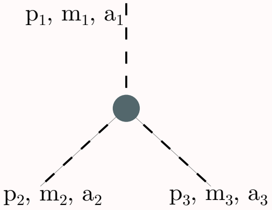
$$\nabla_\alpha R^i_{jk\mu} \quad \nabla_\alpha R^i_{\mu jk} \quad \nabla_\alpha R^\mu_{ijk} \quad \nabla_a R^i_{j\mu\nu} \quad \nabla_a R^i_{\mu j\nu} \quad \nabla_a R^i_{jkl}$$

evaluating at the **vacuum** of the theory: $\overline{g_{\mu\nu,i}} = -\eta_{\mu\nu}\overline{V_{,i}}/2 \equiv 0 \quad \forall i$

the only surviving objects are

$$\overline{\nabla_a R^i_{jkl}} \qquad \overline{\nabla_a R^i_{\mu j\nu}} \qquad \overline{\nabla_a R^\mu_{ij\nu}}$$

Three-point function from fibre bundle



$$= \frac{\overline{\nabla_{a_1} R^{\mu}_{a_3\mu a_2}} + \overline{\nabla_{a_2} R^{\mu}_{a_1\mu a_3}} + \overline{\nabla_{a_3} R^{\mu}_{a_1\mu a_2}}}{6}$$

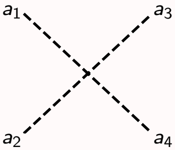
$$- \frac{1}{2}\left[(p_1^2 - m_1^2)\bar{\Gamma}_{a_1 a_2 a_3} + (p_2^2 - m_2^2)\bar{\Gamma}_{a_2 a_1 a_3} + (p_3^2 - m_3^2)\bar{\Gamma}_{a_3 a_2 a_1}\right]$$

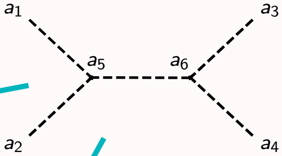
where, using $g_{\mu\nu} = -\frac{\eta_{\mu\nu}}{2} V(\phi)$ and $\overline{V_{,ij}} = m_i^2 \delta_{ij}$:

$$\frac{\overline{\nabla_{a_1} R^{\mu}_{a_3\mu a_2}} + \overline{\nabla_{a_2} R^{\mu}_{a_1\mu a_3}} + \overline{\nabla_{a_3} R^{\mu}_{a_1\mu a_2}}}{6} = \frac{1}{\bar{V}}\left[-\overline{V_{,a_1 a_2 a_3}} + m_1^2 \bar{\Gamma}^{a_1}_{a_2 a_3} + m_2^2 \bar{\Gamma}^{a_2}_{a_1 a_3} + m_3^2 \bar{\Gamma}^{a_3}_{a_1 a_2}\right]$$

agrees with e.g. Cohen et al 2108.03240

$2 \to 2$ scattering at tree-level in the fibre bundle





$$\frac{i}{48}\left(\overline{\nabla_{a_1}\nabla_{a_2}R^{\mu}_{a_3\mu a_4}} + \text{perm}_{1234}\right)$$

$$+\Bigg[-\frac{2i}{3}\overline{R^{\mu}_{a_1\nu a_2}R^{\nu}_{a_3\mu a_4}} + \frac{i}{3}s_{12}\left(\overline{R_{a_1a_4a_3a_2}}+\overline{R_{a_2a_4a_3a_1}}\right)$$

$$+\frac{i}{36}\frac{\overline{g^{a_5a_6}}}{s_{12}-m_5^2}\left(\overline{\nabla_{a_5}R^{\mu}_{a_1\mu a_2}}+\overline{\nabla_{a_1}R^{\mu}_{a_5\mu a_2}}+\overline{\nabla_{a_2}R^{\mu}_{a_1\mu a_5}}\right)\left(\overline{\nabla_{a_6}R^{\mu}_{a_3\mu a_4}}+\overline{\nabla_{a_3}R^{\mu}_{a_6\mu a_4}}+\overline{\nabla_{a_4}R^{\mu}_{a_3\mu a_6}}\right)$$

$$+(2\leftrightarrow 3)+(2\leftrightarrow 4)\Bigg]$$

☞ calculation of higher point amplitudes can be done via recursion relations + contact vertex

Summary and outlook

- different parameterizations are adopted for the scalar sector of the SM, all physically equivalent.

  when building EFT extensions of the SM, the H vs $h + \mathbf{U}$ field choice implemente in SMEFT and HEFT translate into different power countings and are such that

$$\text{HEFT} \supseteq \text{SMEFT} \supseteq \text{SM}$$

- geometrical methods were introduced to obviate field-redefinition ambiguities in SMEFT/HEFT comparisons, but are recently being developed independently, as **"theory-independent" parameterizations of scattering amplitudes**

- we proposed new geometrical description using field space bundles and their higher **jet bundles**
  - 👍 extends geometric interpretation to **scalar potential and higher-∂ terms**
  - 👍 the 0-jet description works! (covariant and not overly complex)

- plans down the line:
  - scattering amplitudes in 1-jet bundle formalism and interplay with derivative redefinitions
  - gauging and reconnection to EWSB and SM phenomenology