Grand Unification: early ideas and recent developments

Wilfried Buchmüller
DESY

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Embedding the SM into GUTs

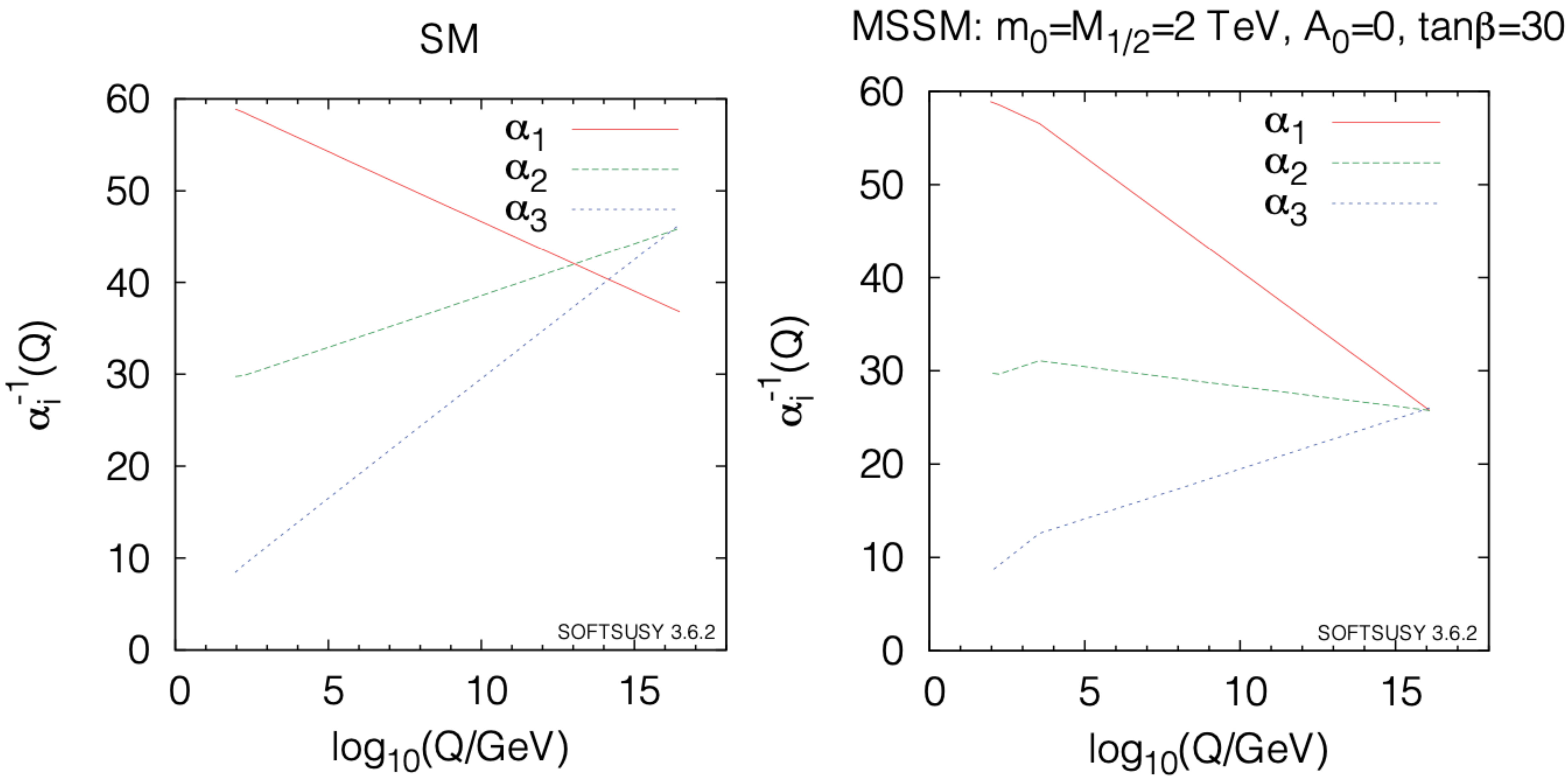
Forces and matter of the Standard Model naturally fit into the 'unification' (GUT) group $SU(5)$ [Georgi, Glashow '74], or larger extensions,

$$SU(5) = \begin{pmatrix} SU(3) & * \\ * & SU(2) \end{pmatrix} ,$$

with quarks and leptons represented by **10**, $\mathbf{5}^*$ and $(\mathbf{1} = \nu^{\mathbf{c}})$,

$$\begin{pmatrix} 0 & u^c & u^c & u & d \\ & 0 & u^c & u & d \\ & & 0 & u & d \\ & & & 0 & e^c \\ & & & & 0 \end{pmatrix} \oplus \begin{pmatrix} d^c \\ d^c \\ d^c \\ \nu \\ e \end{pmatrix} \oplus (\nu^c) \; .$$

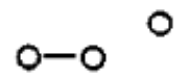
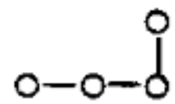
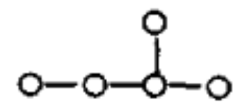
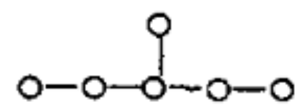
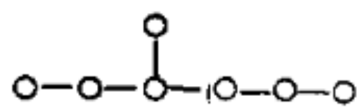
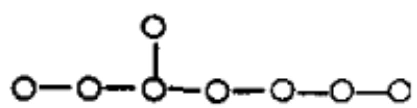
Hope: extrapolation of SM possible up to the GUT energy scale $\Lambda_{\mathrm{GUT}} \sim 10^{15}$ GeV where weak, electromagnetic and strong forces have equal strength.



[PDG Hebecker, Hisano, Nagata ’23, SOFTSUSY 3.6.2 Allanach et al.]

effective interaction strengths (couplings α_i) depend on energy and momentum transfer; “unification” at GUT scale in SM and supersymmetric extension; hint for supersymmetry? Search at LHC, ... !!

The "exceptional sequence"

Exceptional groups	Dynkin diagrams	Coset spaces $E_{n+1}/E_n \times U(1)$
$E_3 = SU(3) \times SU(2)$		
$E_4 = SU(5)$		$(3, 2)$ + c.c.
$E_5 = SO(10)$		10 + c.c.
E_6		$16 (= \bar{5} + 10 + 1)$ + c.c.
E_7		$27 (= 16 + 10 + 1)$ + c.c.
E_8		$56 (= 27 + \overline{27} + 1 + 1) + 1$ + c.c.

Series of E_n groups emerges for **GUTs in higher dimensions**, …, coset spaces contain matter representations of GUT models; first step: 5d orbifold GUTs, $S^1/\mathbb{Z}_2$, SU(5) in bulk, GUT breaking at fixed point, doublet-triplet splitting [Kawamura; Hall, Nomura; Hebecker, March-Russell '01]; SO(10) in 6d; beyond: E8xE8 heterotic string or F-theory [review: Raby '17]

“True unification” in SO(10)

[Georgi ’75; Fritzsch, Minkowski `75]

matter fields: $\mathbf{16} = (\mathbf{q}, \mathbf{l}, \mathbf{u^c}, \mathbf{e^c}, \mathbf{d^c}, \nu^{\mathbf{c}})$

Higgs fields: $\mathbf{45}, \mathbf{10}, \mathbf{126}$ or $\mathbf{16}$

embedding of SM gauge group:

$$G_{SM} \subset SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \subset \ldots \subset SO(10)$$

Yukawa couplings: $\mathcal{L}_Y = h\, \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{10} + h'\, \mathbf{16} \cdot \mathbf{16} \cdot \mathbf{126}$

breaking of B-L/neutrino masses:

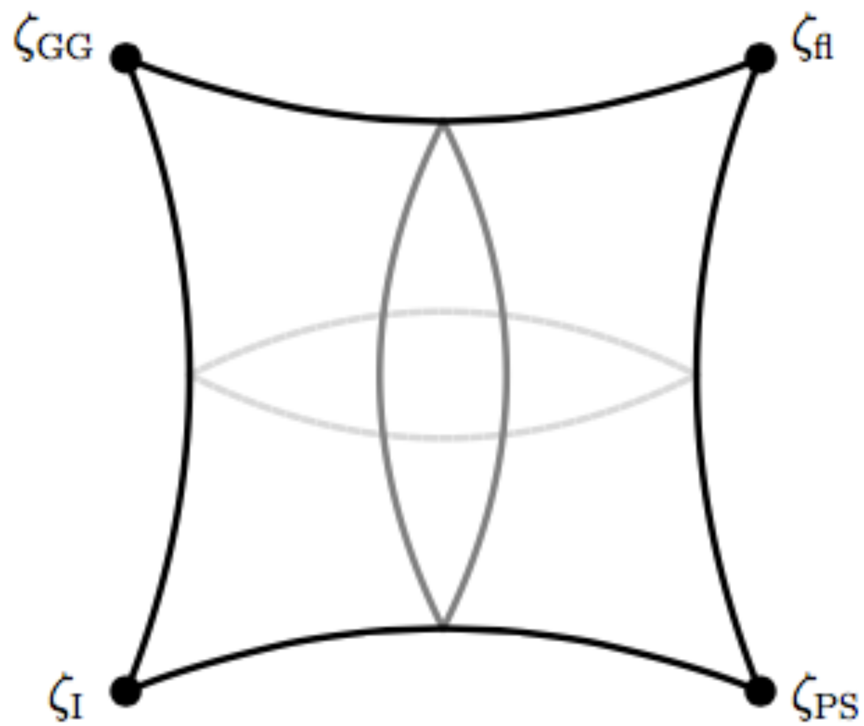
$$\mathbf{16} \cdot \mathbf{16} \cdot \langle \mathbf{126} \rangle \text{ or } \mathbf{16} \cdot \mathbf{16}\, \langle \mathbf{16} \rangle \langle \mathbf{16} \rangle$$

seesaw mechanism, leptogenesis …

SO(10) in six dimensions

[Asaka, WB, Covi '01; Hall, Nomura, Okui, Tucker-Smith, '01]

Consider SO(10) GUT group in 6d, broken at fixed points of orbifold $T^2/\mathbb{Z}_2$ to GG SU(5)xU(1), Pati-Salam SU(4)xSU(2)xSU(2) and flipped SU(5)xU(1), with SM group as intersection; gauge symmetries:



$$
\begin{aligned}
G_{\rm PS} &= SU(4) \times SU(2)_{\rm L} \times SU(2)_{\rm R} \\
&\supset SU(3) \times U(1)_{B-L} \times SU(2)_{\rm L} \times U(1)_{\rm R} \ , \\
G_{\rm GG} &= SU(5) \times U(1)_X \\
&\supset SU(3) \times SU(2)_{\rm L} \times U(1)_Y \times U(1)_X \ , \\
G_{\rm fl} &= SU(5)' \times U(1)_{X'} \\
&\supset SU(3) \times SU(2)_{\rm L} \times U(1)_Z \times U(1)_{X'} \ .
\end{aligned}
$$

unification of gauge **and** Higgs fields in bulk (N=2 SUSY), matter fields in bulk and/or at fixed points; different options for SUSY breaking (like split SUSY)

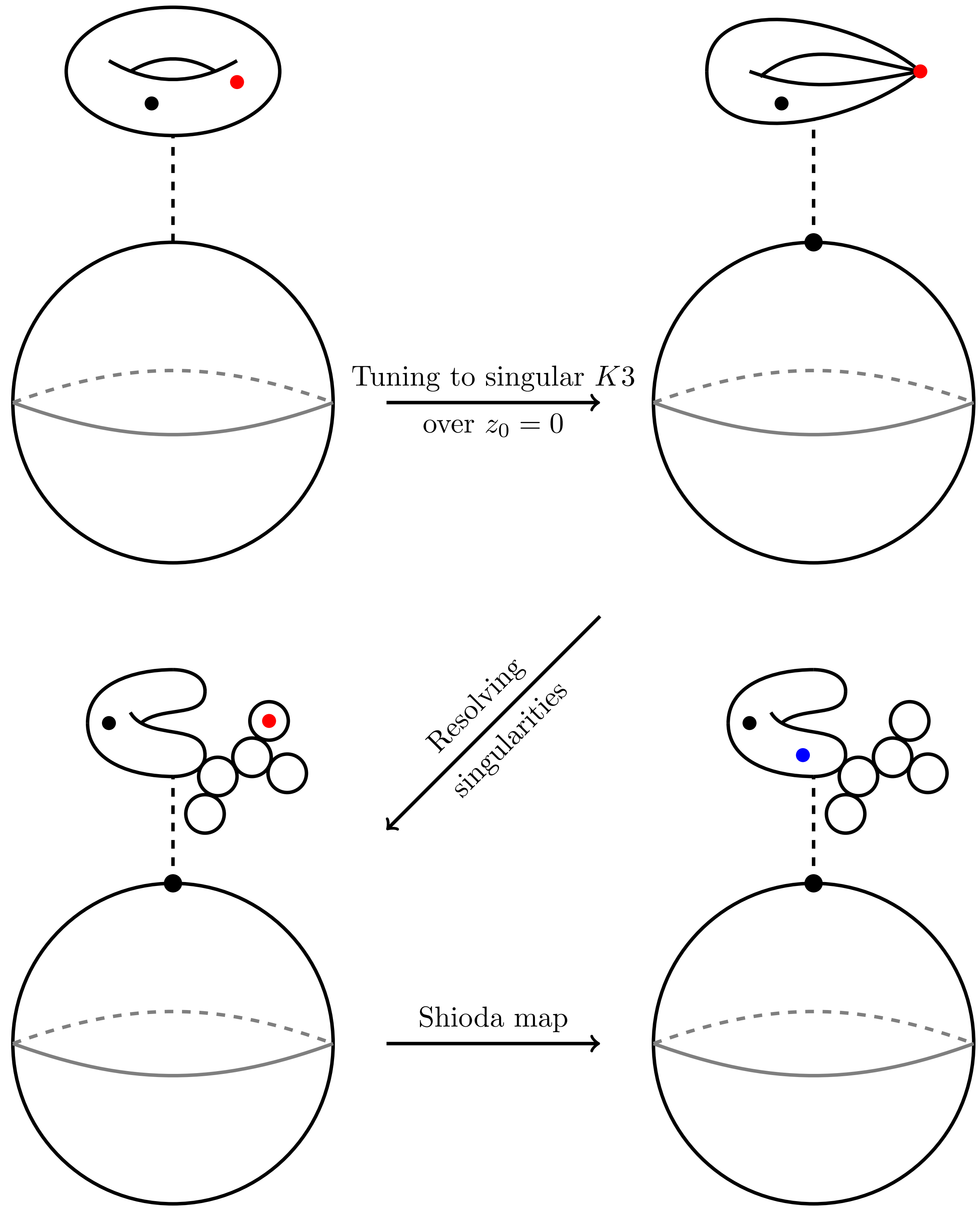
6d SO(10) F-theory vacua

[Vafa, Morrison ’96; Beasley, Heckman, Vafa ’08; review: Lin, Weigand ’14]

F-theory corresponds to version of 10d type-IIB string theory with D7-branes; complex axio-dilaton is treated geometrically as modular parameter of 2-torus, leads to 12d theory as starting point; supersymmetric compactifications require Calabi-Yau (CY) manifolds; first step: compactification on CY 2-fold K3, treated as fibration of torus over sphere; Kodaira classification:

ord (f)	ord (g)	ord (Δ)	singularity	nonabelian symmetry
≥ 0	≥ 0	0	none	none
0	0	n	A_{n-1}	$SU(n)$
≥ 1	1	2	none	none
1	≥ 2	3	A_1	$SU(2)$
≥ 2	2	4	A_2	$SU(3)$
2	3	$n+6$	D_{n+4}	$SO(8+2n)$
≥ 2	≥ 3	6	D_4	$SO(8)$
≥ 3	4	8	E_6	E_6
3	≥ 5	9	E_7	E_7
≥ 4	5	10	E_8	E_8

Table 1: Table of singularity types for elliptic surfaces and associated nonabelian symmetry groups.



from fibration of torus over sphere to K3 with resolved SO(10) singularity; classification for toric hypersurface fibrations [WB, Dierigl, Oehlman, Ruehle '17]

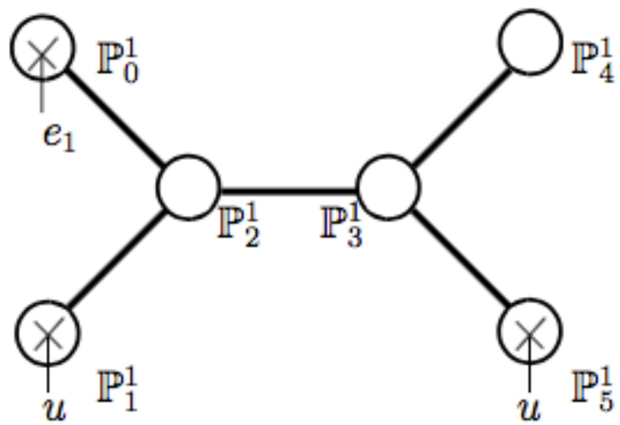
Resolution of singularities in terms of `spheres' whose intersection numbers can be computed [Bouchard, Skarke '03]; for SO(10) singularity one gets 5 $\mathbb{P}^1$s, calculation of intersection numbers yields SO(10) Cartan matrix:

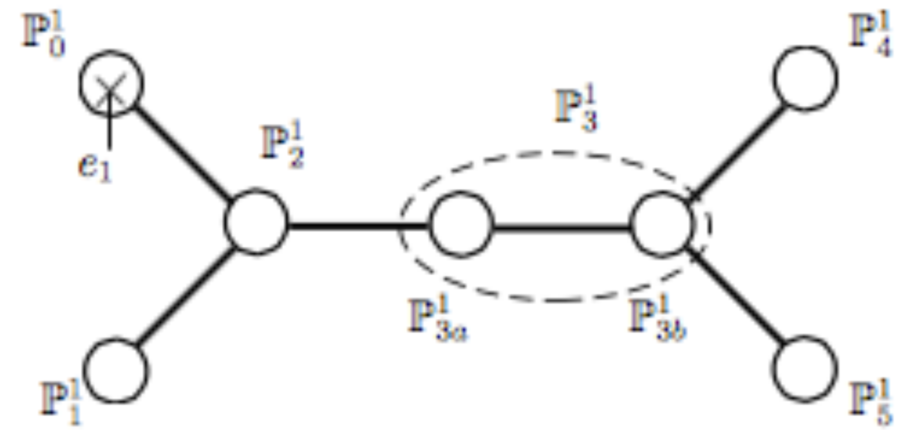
$$D_i \cdot \mathbb{P}^1_j = \mathbb{P}^1_i \cdot D_j = -(C_{SO(10)})_{ij}\,, \quad i,j = 1,\ldots 5\,,$$

$$C_{SO(10)} = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & -1 \\ 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 & 2 \end{pmatrix}$$

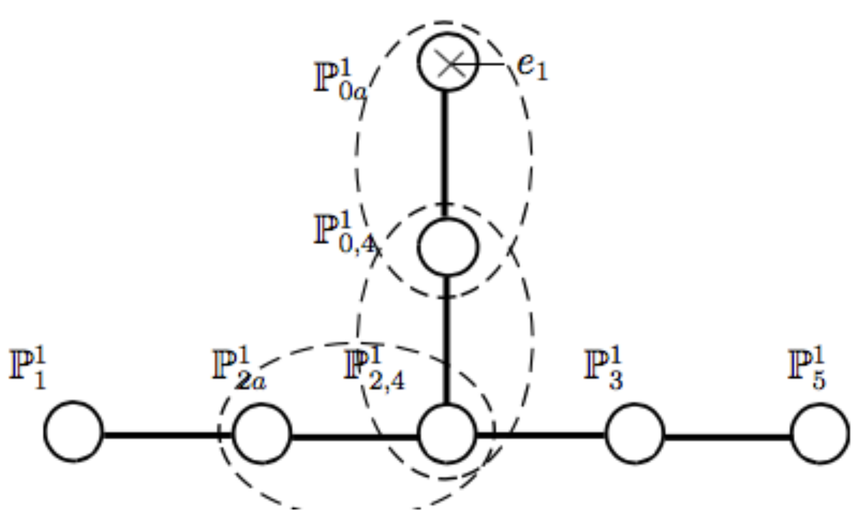
Hence, **SO(10) algebra from geometry** of elliptic fibration! Further tuning can yield stronger singularities and enhancement to SO(12) and E6 with 10 and 16's as `coset matter'

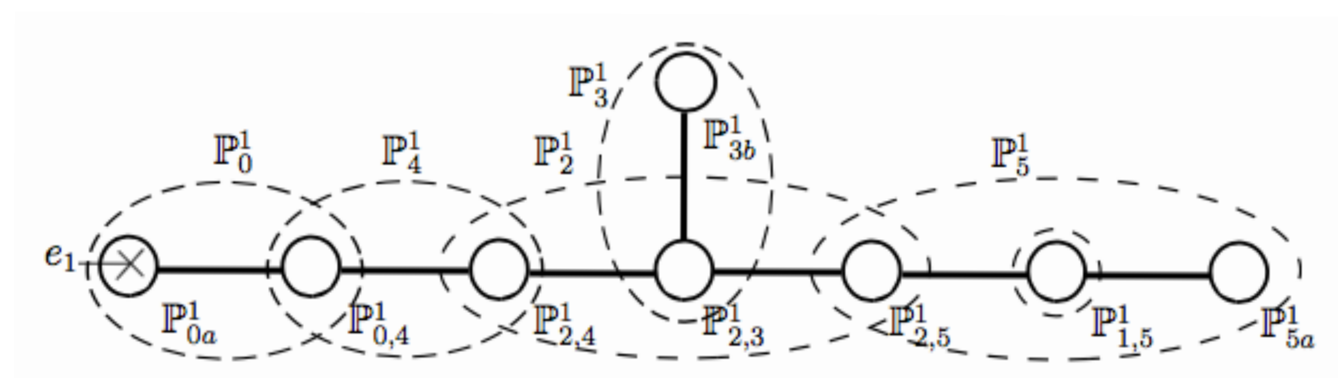
Intersection pattern at resolved SO(10) singularity and SO(12) enhancement:

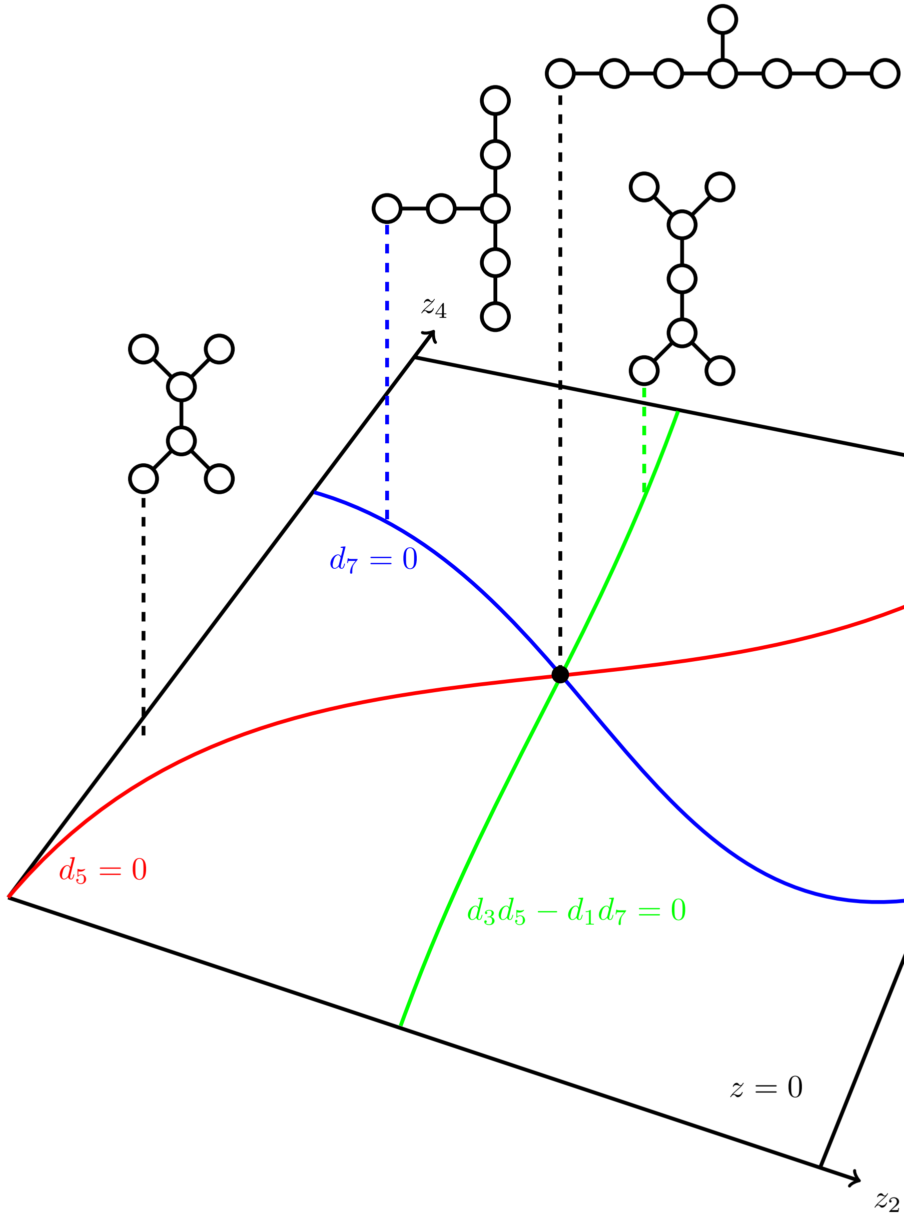




E6 enhancement with 16 matter, and E7 enhancement for **Yukawa coupling** :
$\mathbf{16_{3/4}16_{-1/4}10_{-1/2}}$







F-GUT: 12d

T^2 fibration over complex 3D base; singularities, effective field theory:

codim 1: GUT gauge group, 8d

codim 2: matter curves, 6d, including multiplicities

codim 3: matter curves intersect, Yukawa couplings, 4d

F-theory picture: 3 different matter curves intersecting at locus of Yukawa coupling in CY 4-fold (torus + base); unification of gauge, Higgs and matter fields (everything from geometry); **fundamental problem: SUSY breaking ...**

Nucleon decay

Generic prediction of GUTs: nucleon decay (dim-6 operators in non-SUSY models, in addition dim-5 operators in SUSY models); typical decay modes:

$$\text{non-SUSY SU(5)}: \quad p \to e^+\pi^0$$
$$\text{SUSY SU(5)}: \quad p \to \bar{\nu} K^+$$

$$\text{dim-6}: \quad \tau_p^{-1} \sim \frac{m_p^5}{M_{\rm GUT}^4}$$
$$\text{dim-5}: \quad \tau_p^{-1} \sim \frac{m_p^5}{M_{H_c}^2 m_{\rm SUSY}^2} \sim \frac{m_p^5}{M_{\rm GUT}^2 m_{\rm SUSY}^2}$$

dim-5 contribution dangerous, but strongly model dependent; in principle rich flavour structure in decays,

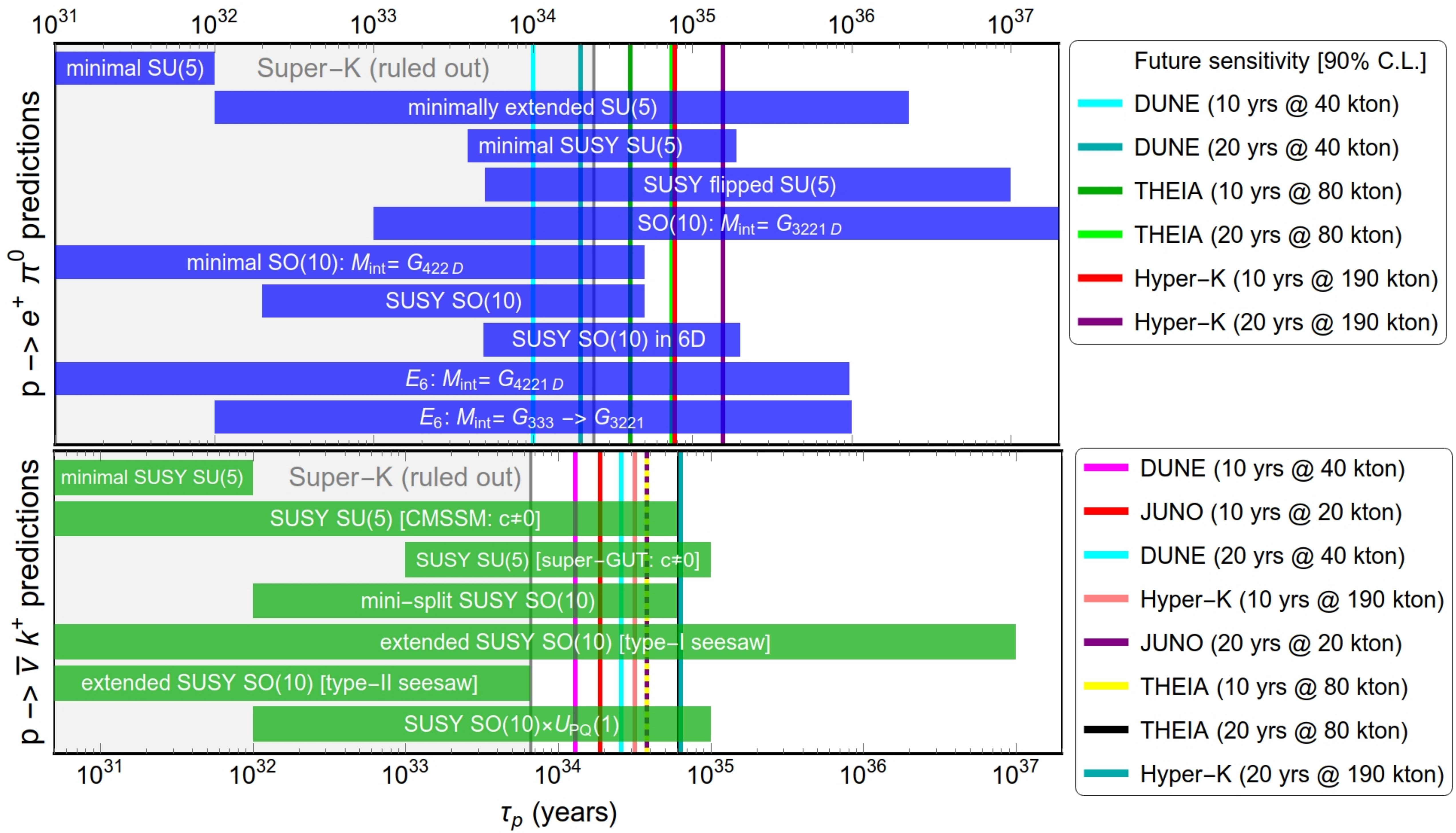
$$p \to e^+\pi^0,\ \mu^+\pi^0,\ e^+K^0,\ \mu^+K^0, \bar{\nu}\pi^+,\ \bar{\nu}K^+, \ldots$$

branching ratios strongly depend on GUT model!

SEARCHES in 1980's: Kamiokande, IMB (water Cherenkov, no proton decay but supernovae neutrinos!); present upper bounds from Super-K (20 kton, started 1996, discovery of atmospheric neutrino oscillations!); under construction/proposed: DUNE (40 kton LAr), Hyper-K (190 kton, water Cherenkov), JUNO (20 kton liquid scintillator); THEIA (80 kton liquid scintillator); current bounds $\sim 10^{34}\mathrm{yrs}$

Modes (partial lifetime)	Current limit [90% CL] (10^{34} years)	Future Sensitivity [90% CL] (10^{34} years)
$\tau_p\ (p \to e^+\pi^0)$	Super-K: 2.4 [55]	Hyper-K (1900 kton-yrs): 7.8 [56] DUNE (400 kton-yrs): ~1.0 [57] THEIA (800 kton-yrs): 4.1
$\tau_p\ (p \to \mu^+\pi^0)$	Super-K: 1.6 [55]	Hyper-K (1900 kton-yrs): 7.7 [56]
$\tau_p\ (p \to \overline{\nu}K^+)$	Super-K: 0.66 [58]	Hyper-K (1900 kton-yrs): 3.2 [56] DUNE (400 kton-yrs): 1.3 [59] JUNO (200 kton-yrs): 1.9 [60] THEIA (800 kton-yrs) 3.8
$\tau_p\ (p \to \overline{\nu}\pi^+)$	Super-K: 0.039 [61]	–

[Dev et al. `22]



[Dev et al. `22]

next 10-20 yrs: bounds will improve by factor 5 to 10, decisive tests for `best models'!

Neutrino masses

In GUTs small neutrino masses due to seesaw mechanism (Dirac neutrino masses related to quark/lepton masses, RH neutrino masses from B-L breaking at GUT scale:

$$m_\nu = -m_D \frac{1}{M} m_D^T$$

With 3rd generation couplings ~ 1, and $M \propto v_{B-L} \sim \Lambda_{\rm GUT} \sim 10^{15}\ {\rm GeV}$ observed neutrino mass scale:

$$m_3 \sim \frac{v_{\rm EW}^2}{v_{B-L}} \sim 0.01\ {\rm eV}$$

Evidence for connection between GUTs and neutrino masses?
Low-scale seesaw? MeV/GeV RH neutrino masses? Collider searches?
→ GUT connection more compelling for neutrino mass matrices

Example: minimal SO(10) [WB, Wyler '01]; needed for neutrino mixing: connection between generations, e.g. (discrete) family symmetry, texture …; fermion masses from coupling of 16-plets to Higgs multiplets:

$$\mathcal{L} = h_u \mathbf{16}\ \mathbf{16}\ H_1(\mathbf{10}) + h_d \mathbf{16}\ \mathbf{16}\ H_2(\mathbf{10}) + h_N \mathbf{16}\ \mathbf{16}\ \Phi(\mathbf{126})$$

$$m_u = m_D\ , \quad m_d = m_e$$

use texture from quark sector [Fritzsch, Xing '00; Rosenfeld, Rosner '01], assume independence of Higgs representation,

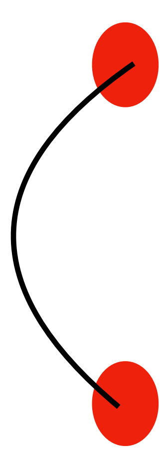
$$m_{u,d} \propto \begin{pmatrix} 0 & \epsilon^3 e^{i\phi} & 0 \\ \epsilon^3 e^{i\phi} & \rho\epsilon^2 & \eta\epsilon^2 \\ 0 & \eta\epsilon^2 & e^{i\psi} \end{pmatrix},\ M = \begin{pmatrix} 0 & M_{12} & 0 \\ M_{12} & M_{22} & M_{23} \\ 0 & M_{23} & M_{33} \end{pmatrix}$$

with $\epsilon \ll 1; M_{12} \ll M_{22} \sim M_{23} \ll M_{33}$; LMA for hierarchical RHNs, leptogenesis OK; recent detailed study with many refs. [Babu, di Bari, Fong, Saad '24]; **consistency between hierarchical Dirac and RHN masses, GUT scale seesaw and large ν-mixings remarkable, hint for GUTs!**

SO(10) & gravitational waves

Formation of topological defects generic feature of **cosmological GUT phase transitions** [Kibble '76]: strings, monopoles etc. Hot topic after new PTA data in June '23, evidence for stochastic gravitational background in nHz band - SMBHB or cosmological origin, in particular **cosmic strings**? [Shafi et al, King et al, Antusch et al, Ellis et al, …, WB, Domcke, Schmitz (following slides)]. PTA data disfavor stable strings but favor metastable strings (or superstrings). Cosmic string networks can form by Kibble mechanism and decay by nucleation of monopoles.

Decay rate per string unit length [Preskill, Vilenkin '92, …]:



$$\Gamma_d = \frac{\mu}{2\pi}\exp\left(-\pi\kappa\right) \ , \quad \kappa = \frac{m_M^2}{\mu}$$

determined by string tension and monopole mass

SO(10) breaking allows formation of monopoles and strings (one more U(1) beyond SM needed, i.e. B-L); early example [Kibble, Lazarides, Shafi '82]:

$$\mathrm{SO}(10) \overset{\mathbf{45}}{\to} \mathrm{SU}(5) \times \mathrm{U}(1) \overset{\mathbf{45 \oplus 126}}{\to} \mathrm{G}_{SM} \times \mathbb{Z}_2$$

first breaking yields monopoles, second breaking yields stable strings, **non-trivial homotopy group** $\pi_1(\mathcal{M}) = \mathbb{Z}_2$; similar example:

$$\mathrm{SO}(10) \overset{\mathbf{45}}{\to} \mathrm{SU}(5) \times \mathrm{U}(1) \overset{\mathbf{45 \oplus 16}}{\to} \mathrm{G}_{SM}$$

first breaking yields monopoles, second breaking yields metastable strings, **trivial homotopy group** $\pi_1(\mathcal{M}) = I$

Note: 16-plet appears in exceptional sequence, 126-plet does not appear in exceptional sequence! Homotopy group crucial in analyzing topological defects; see [Vilenkin, Shellard '2000; Lazarides, Shafi, …'18, …; Murayama, …'21, …]

Minimal model for MCS

Simplest example: supersymmetric breaking of $SU(2)_R \times U(1)_{B-L}$, embedded in SO(10):

$$\mathrm{G}_{SM} \subset \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{B-L} \subset \ldots \subset \mathrm{SO}(10)$$

fields $U \sim (3,0)\,,\; S \sim (2,q)\,,\; S_c \sim (\bar{2},-q)$; Kahler and superpotential:

$$K = U^\dagger e^{2gV} U + S^\dagger e^{2\left(g\tilde{V}+g'qV'\right)} S + S_c^\dagger e^{-2\left(g\tilde{V}+g'qV'\right)} S_c + \phi^\dagger \phi + \phi'^\dagger \phi'\,,$$

$$P = \frac{1}{8}\,\mathrm{tr}\,[WW] + \frac{1}{4} W'W' + 2h\, S_c^T\, \tilde{U}\, S$$
$$+ \frac{\lambda'}{2}\phi'\left(\frac{v_u^2}{2} - U^T U\right) + \lambda\phi\left(v_s^2 - S_c^T S\right) - h v_u\, S_c^T S$$

singlet fields ϕ and ϕ' play role of **inflatons** in two stages of SUSY **hybrid inflation** (dilution of monopoles, primordial density fluctuations)

VEVs breaking SU(2) to U(1) (monopoles) and U(1)xU(1) to U(1) (strings):

$$U^a = \frac{v_u}{\sqrt{2}}\,\delta_{a3}\,, \qquad S = S_c = v_s \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

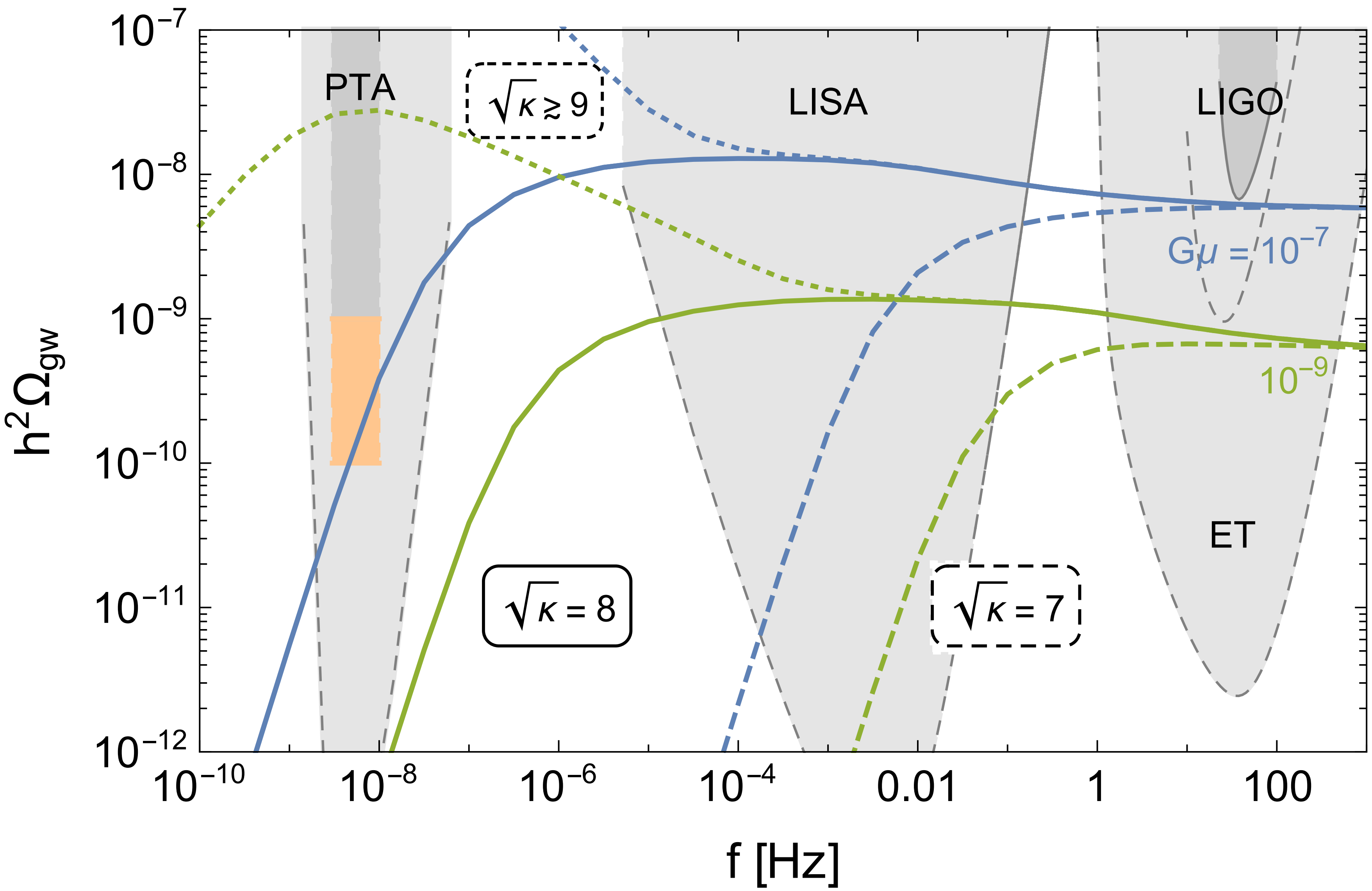
monopole mass and string tension:

$$m_M \simeq \frac{4\pi v_u}{g}\,, \quad \mu \simeq 2\pi v_s^2$$

decay parameter:

$$\kappa = \frac{m_M^2}{\mu} \sim \frac{8\pi}{g^2}\frac{v_u^2}{v_s^2}$$

analysis of early NANOGraph data: signal can be explained in terms of **metastable cosmic strings** for $\sqrt{\kappa} \simeq 8$ [WB, Domcke, Schmitz '20]; corresponds to similar symmetry breaking scales $v_u \sim v_s$ (fine tuning?); previously MCSs were always considered as effectively stable! Considerable theoretical uncertainty of string decay rate, detailed calculation by [Chitose, Ibe, Nakayama, Shirai, Watanabe '23]



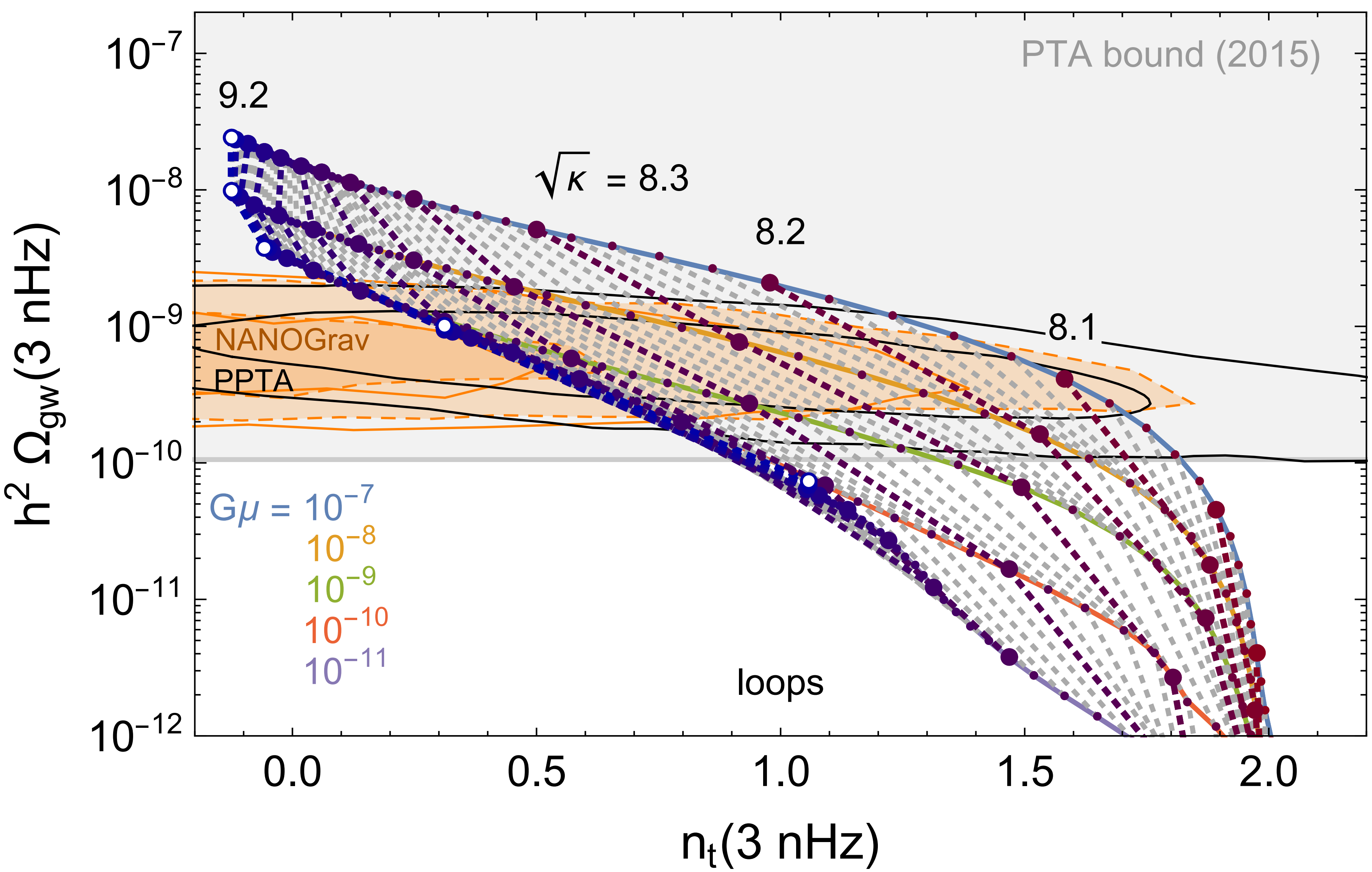
[WB, Domcke, Schmitz '21,'23]

calculation of GW spectrum complicated, several uncertainties; amplitude and spectral index determined by $v_u,\ v_s$:

$$\Omega_{\rm gw}^{\rm plateau} \simeq \frac{128\pi}{9} B\,\Omega_r \left(\frac{G\mu}{\Gamma}\right)^{1/2} \sim v_s\ , \quad n_t = d\ln\Omega_{\rm gw}/d\ln f$$

Detailed analysis of NANOgraph and PPTA data

data yield ellipse in amplitude-slope plane (NANOGraph & PPTA, 1σ & 2σ); observations larger than previous upper bound!

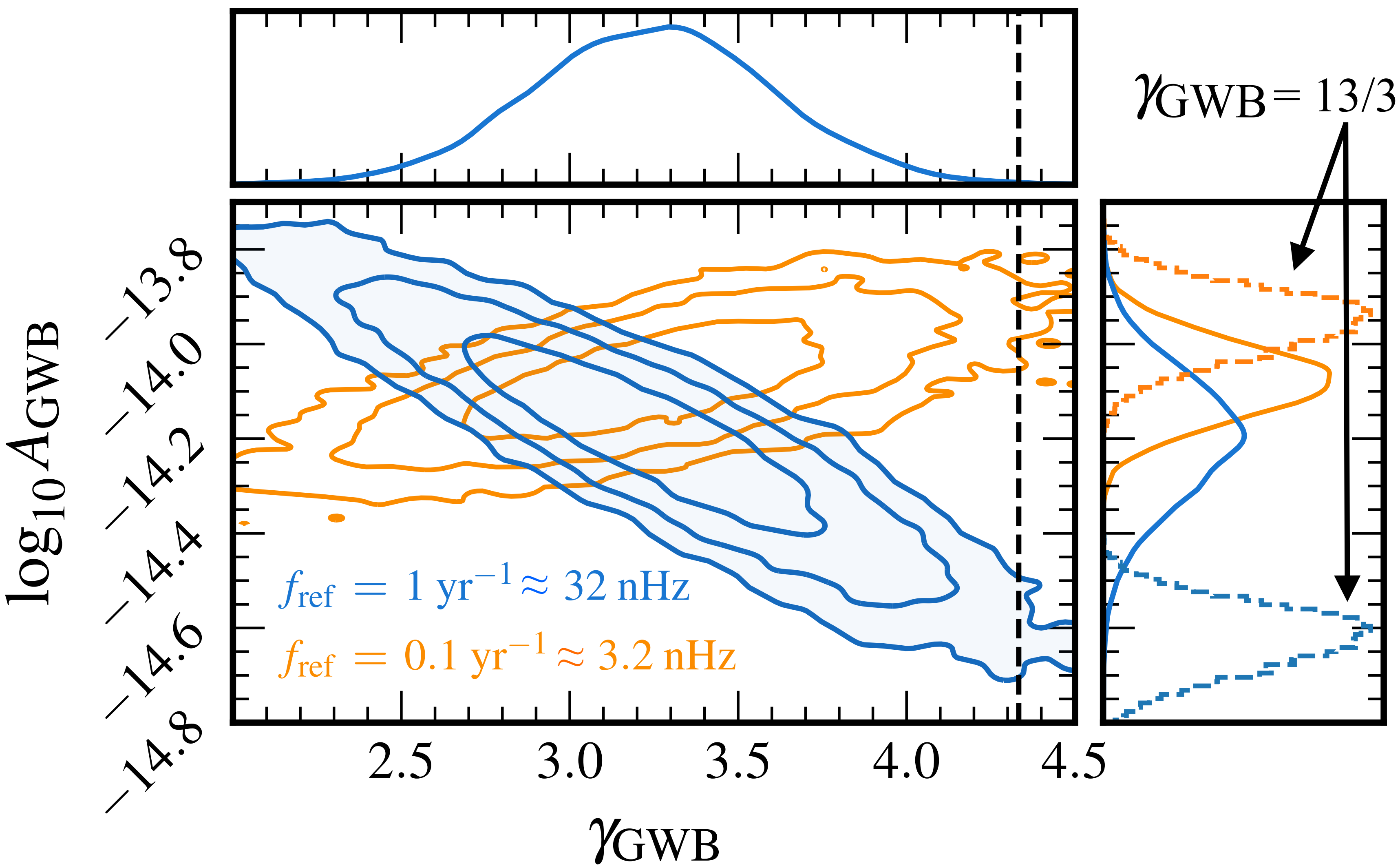


NANOGraph & PPTA ellipses, 1σ & 2σ; lines of constant Gμ and κ

[WB, Domcke, Schmitz '21]

successful MCS prediction for $2 \times 10^{-11} < G\mu < 2 \times 10^{-7},\ \sqrt{\kappa} \sim 8$

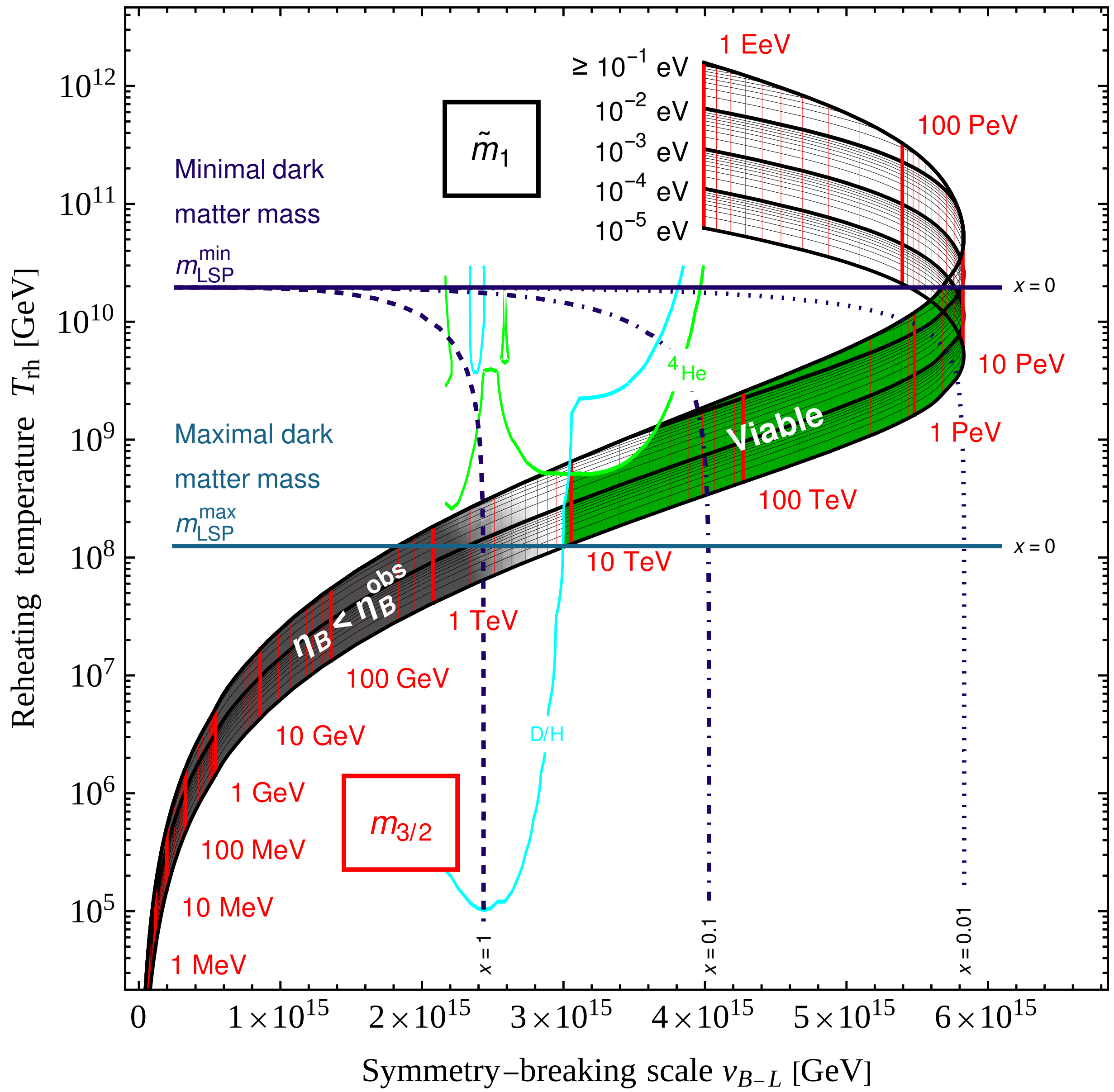
June 2023: evidence for **Hellings-Downs** spatial correlations (~4σ) and therefore gravitational wave signal, NANOGraph, PPTA, EPTA, CPTA, more precise determination of amplitude and slope, $\gamma = 5 - n_t$



NANOGraph
1σ, 2σ & 3σ

What is it? SMBHB (γ = 13/3, slightly disfavoured?), MCSs, stable strings (disfavoured), superstrings, global strings, inflation, domain walls, SMBHs … ? Time (more data) will tell

Consistent picture of cosmological $\mathrm{U}(1)_{\mathrm{B-L}}$ breaking, leptogenesis, hybrid inflation & DM possible/challenging:



GUT scale: $\sim 10^{15}$ GeV

SUSY scale: $10\ \mathrm{TeV} \ldots 10\ \mathrm{PeV}$

reheating temp: $\sim 10^{10}$ GeV

higgsino dark matter (gravitino decays)

fits PTA for $\sqrt{\kappa} \simeq 8$

[WB, Domcke, Murayama, Schmitz '19]

recent discussion of PTA data in context of SUSY SO(10) GUT, with many refs. in [Antusch, Hinze, Saad '24].

Summary

- Grand unification theoretically very attractive: unification of matter and gauge fields (and gravity) in higher dimensions

- Hints for grand unification: gauge coupling unification and neutrino masses

- (Expected) future evidence: proton decay and (hopefully) gravitational waves from cosmic strings


“Im übrigen ist das allgemeine Milieu in Zürich sehr schön”
Zürich
Pittsburg
New York
Bonn
Geneva
Zürich