



THEORY UNCERTAINTIES

Alexander Huss





THEORY UNCERTAINTIES IN HIGH-ENERGY **PRECISION MEASUREMENTS** AT THE **LHC**

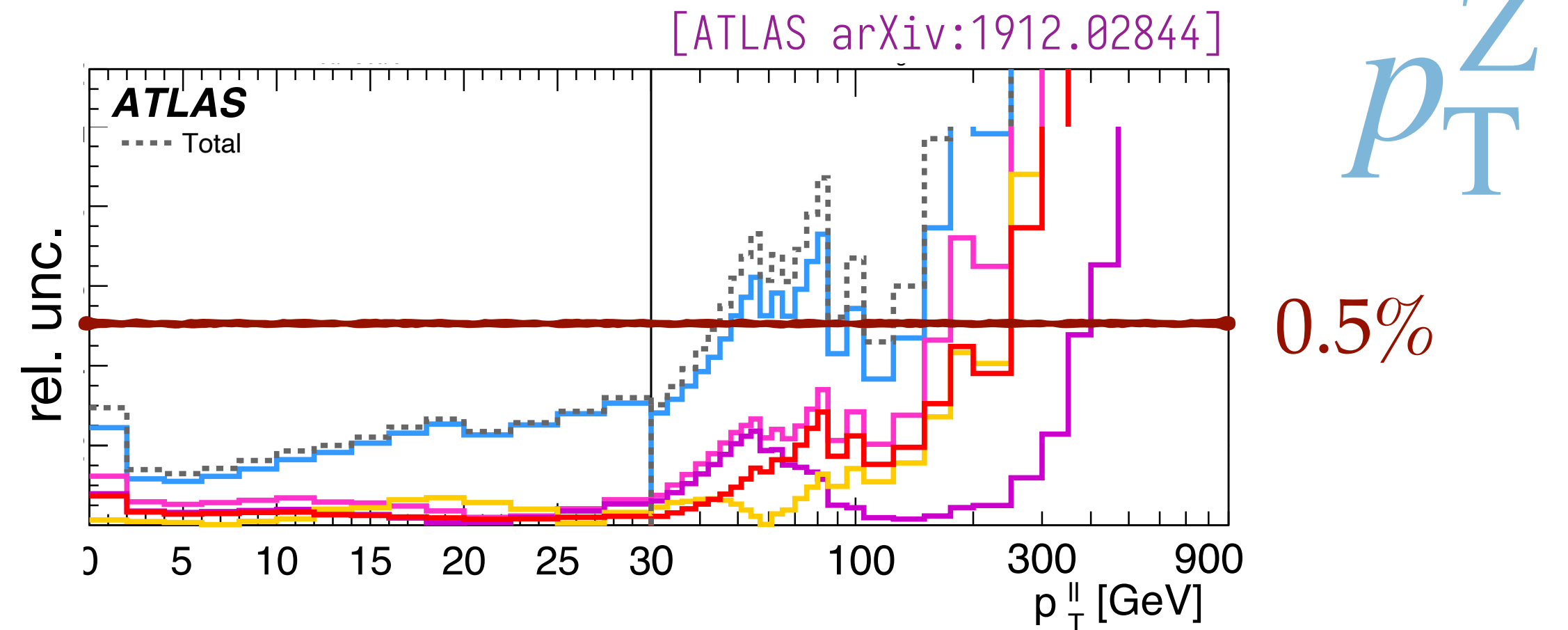
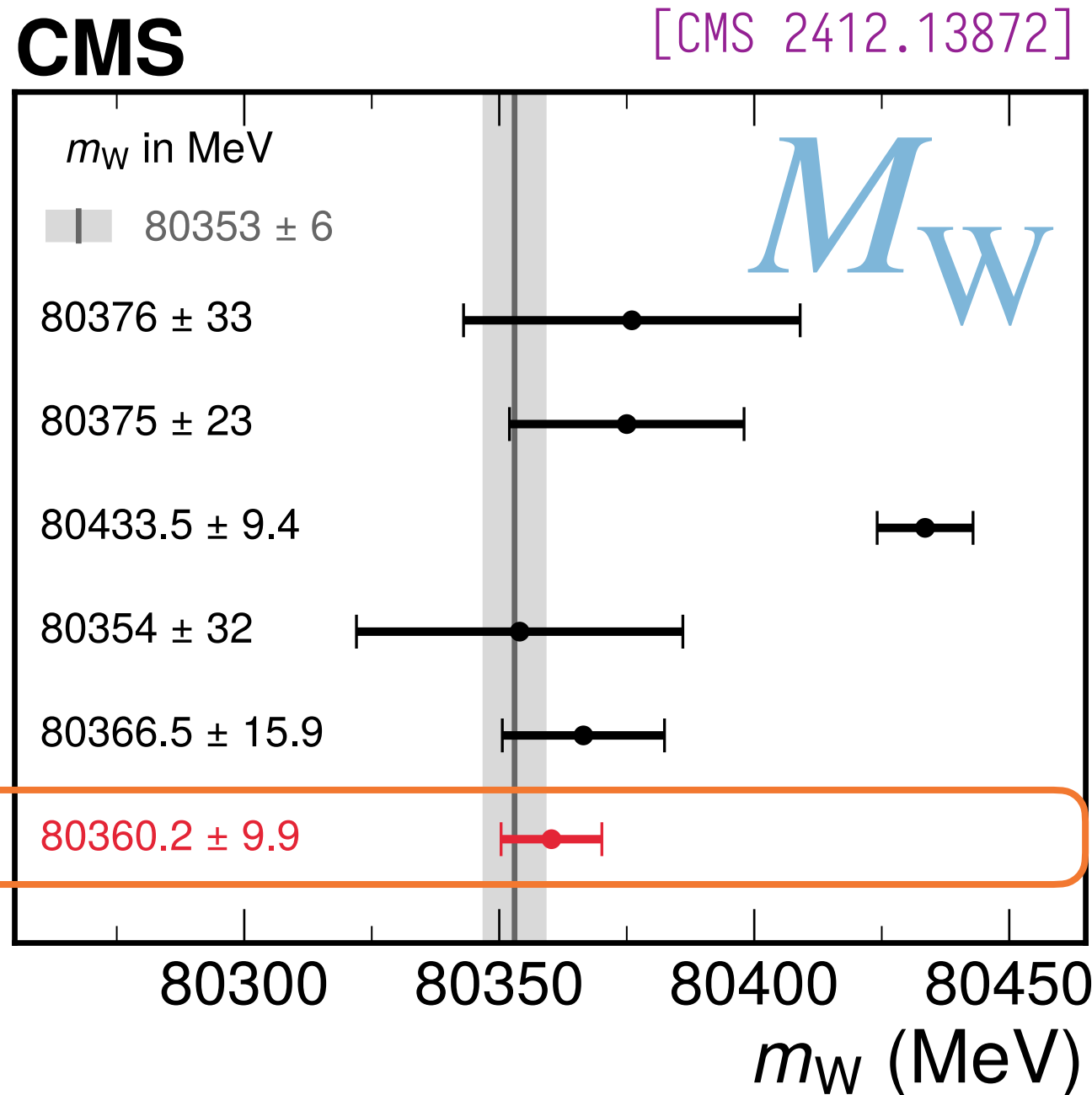
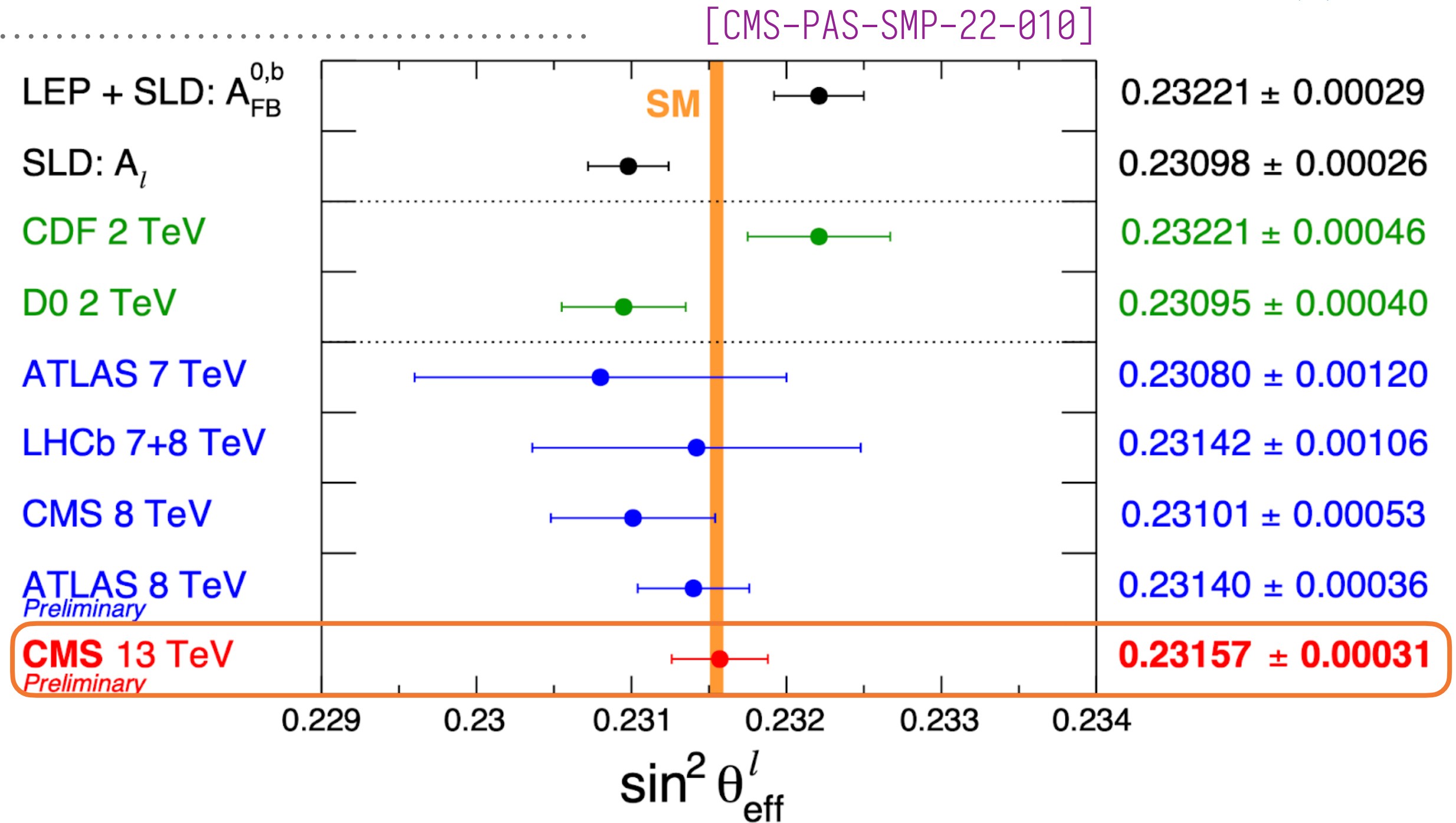
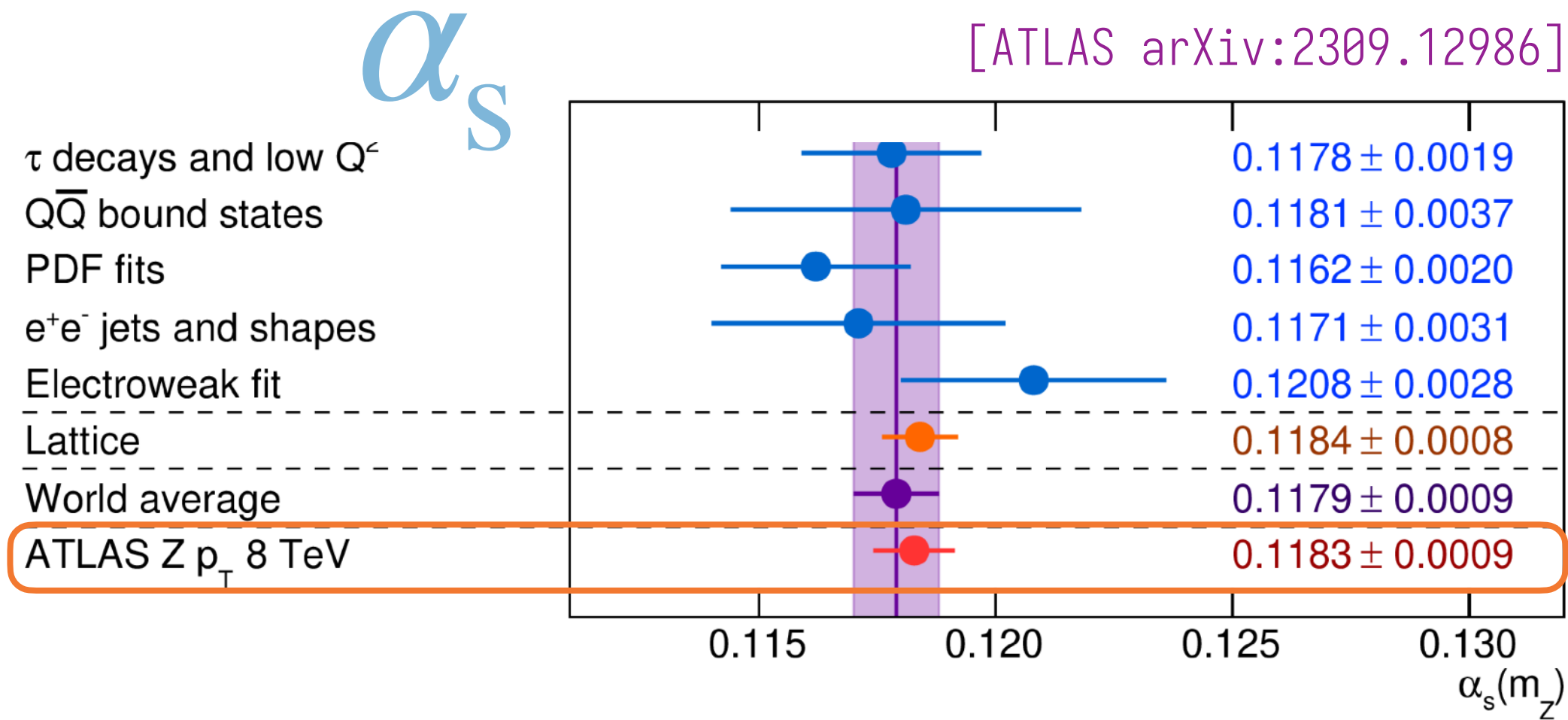
* I will focus on select recent developments & trends
which I find interesting / important / concerning / ...

Alexander Huss



LHC – FROM A DISCOVERY TO A PRECISION MACHINE

$$\sin^2 \theta_W$$



Electroweak fit
PRD 110 (2024) 030001
LEP combination
Phys. Rep. 532 (2013) 119
D0
PRL 108 (2012) 151804
CDF
Science 376 (2022) 6589
LHCb
JHEP 01 (2022) 036
ATLAS
arXiv:2403.15085
CMS
This work

WHAT IS THE **UNCERTAINTY** Δ_{TH} OF MY RESULT?

- increasingly urgent to address with $\Delta_{\text{EXP}} \searrow$ (\leftrightarrow HL-LHC)
 - ▶ what does 5σ mean if Δ_{TH} non-negligible?
 - ▶ interpretation of data in need for robust Δ_{TH} : precision measurements, PDF fits, ...
- various sources that contribute to Δ_{TH} :
 - ▶ Δ_{α_s} , Δ_{param} : parametric uncertainties \leftrightarrow exp. extraction
 - ▶ Δ_{PDF} : parton distribution functions (PDFs) \leftrightarrow fits to data, lattice, ...
 - ▶ $\Delta_{\text{non pert.}}$: intrinsic k_T , hadronisation, UE, ... \leftrightarrow TMD, parton showers, ...
 - ▶ $\Delta_{\text{pert.}}$: *missing higher-order* corrections \leftrightarrow conceptually tricky

OUTLINE

focus on dominant sources of Δ_{TH} in precision measurements

“QCD modelling”

- MOTIVATION
 1. Scale Variations
 2. Bayesian Estimates
 3. Theory Nuisance Parameters
 4. Parton Distribution Functions
- CONCLUSIONS

1 SCALE VARIATION

GENERAL IDEA & CONVENTIONS

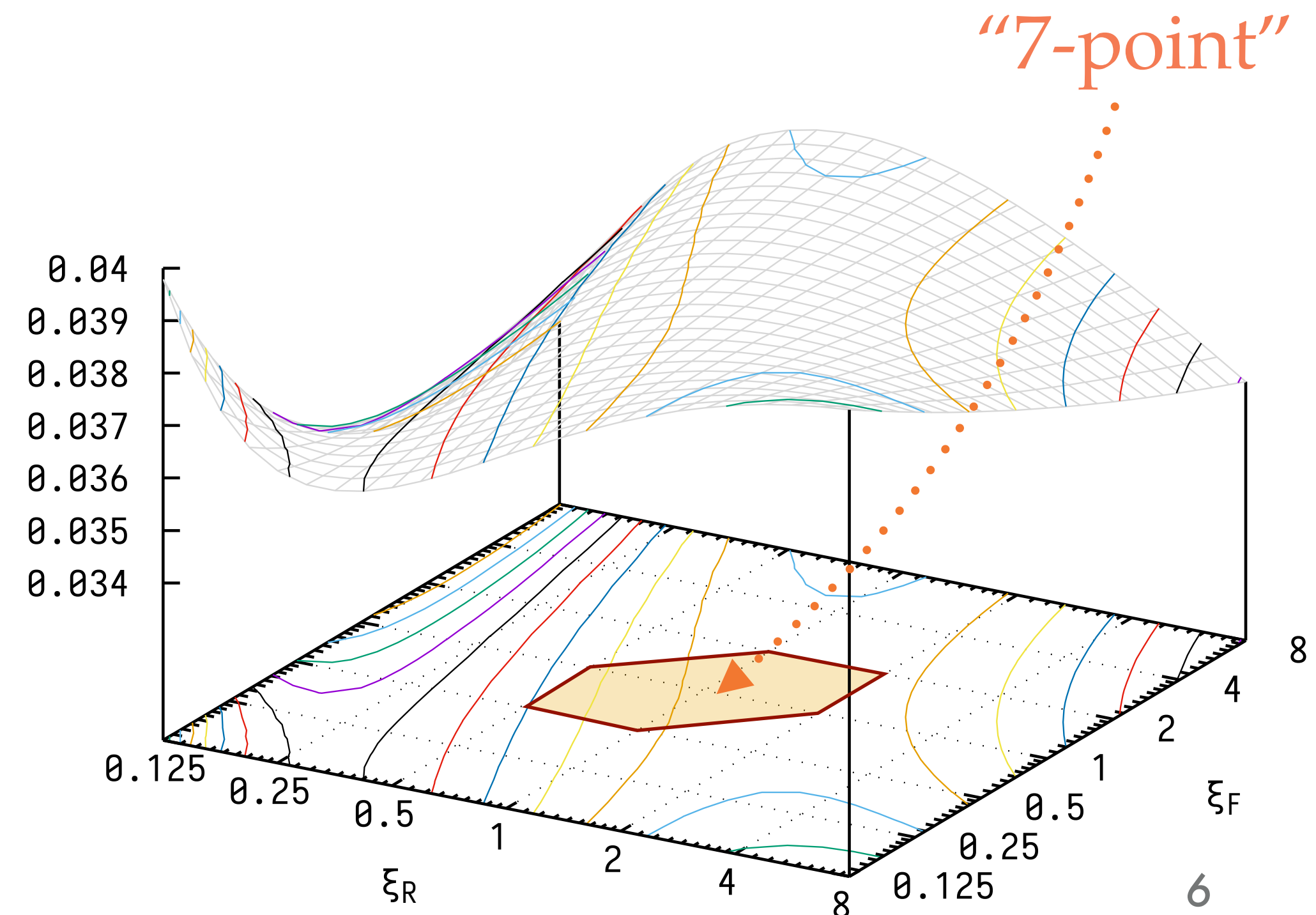
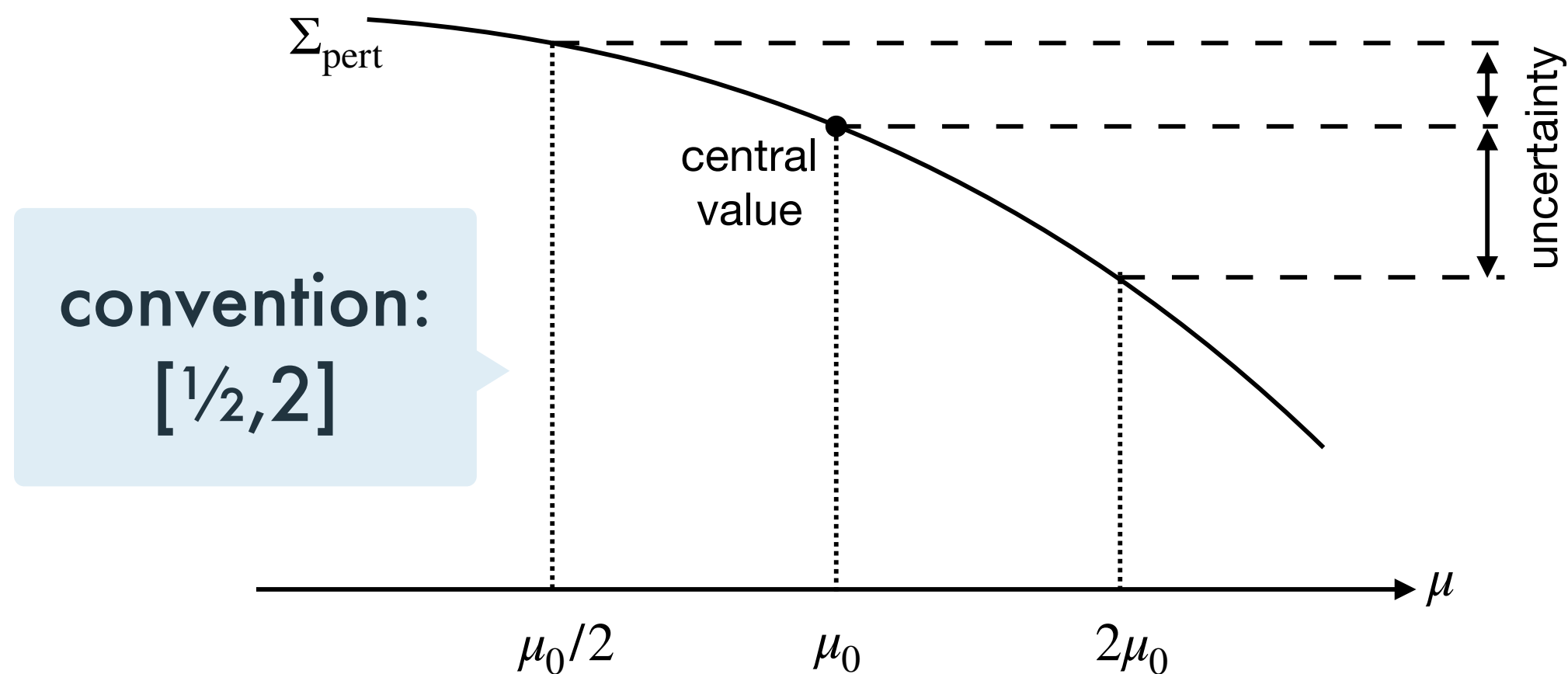
- approximation for an observable @ (next-to-)ⁿ leading order:

$$d\sigma^{\text{N}^n\text{LO}} = d\sigma^{(0)} + \alpha_s d\sigma^{(1)} + \dots + \alpha_s^n d\sigma^{(n)}$$

$\propto \alpha_s^{n_0}$

- truncation of series induces a sensitivity to terms of the next order

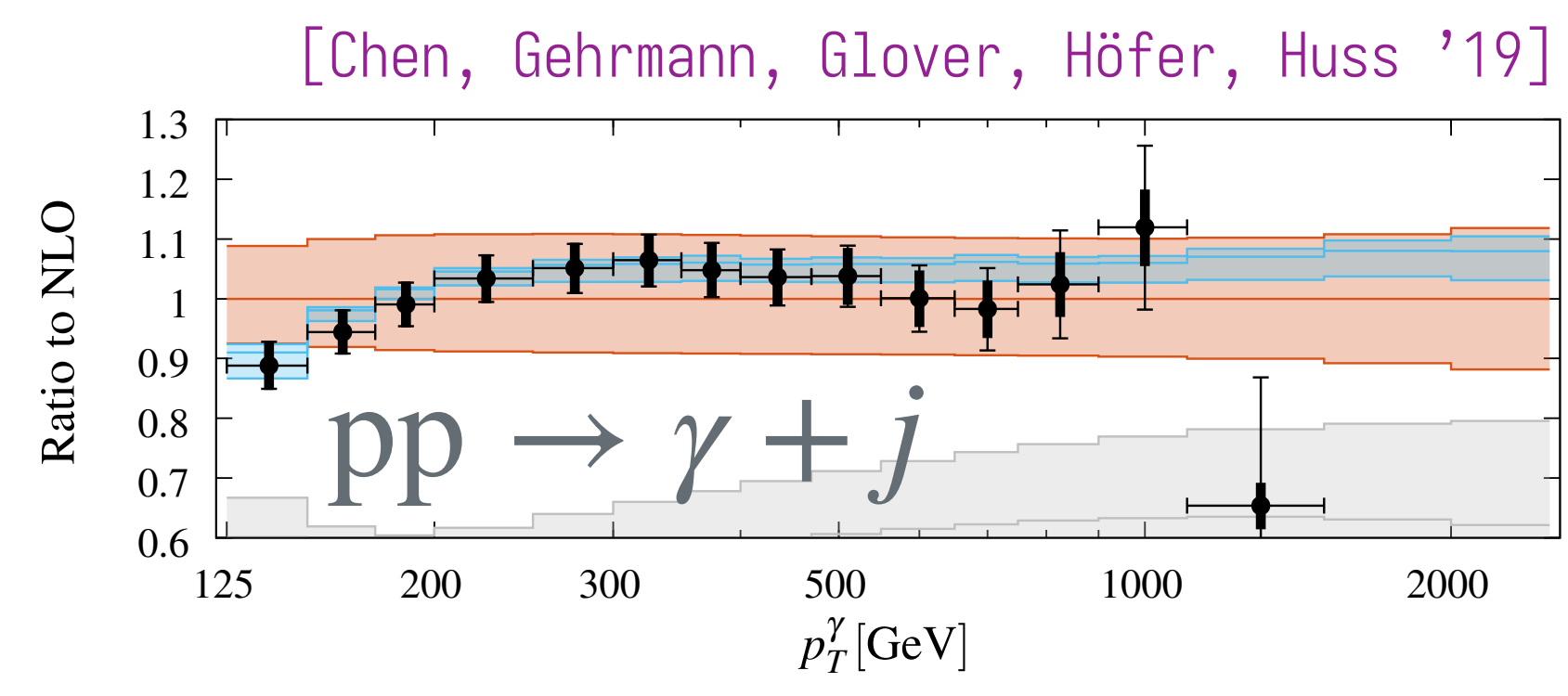
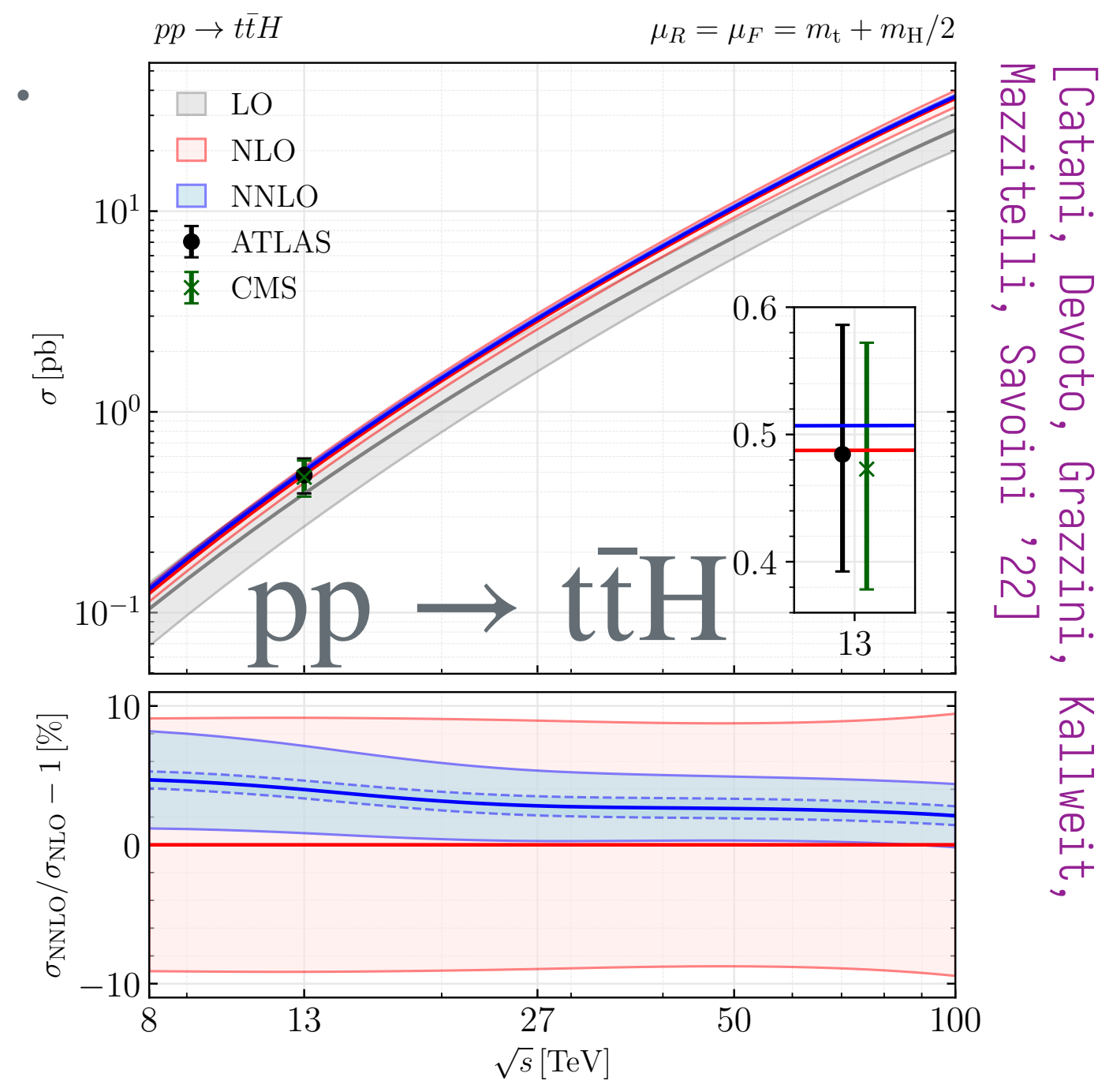
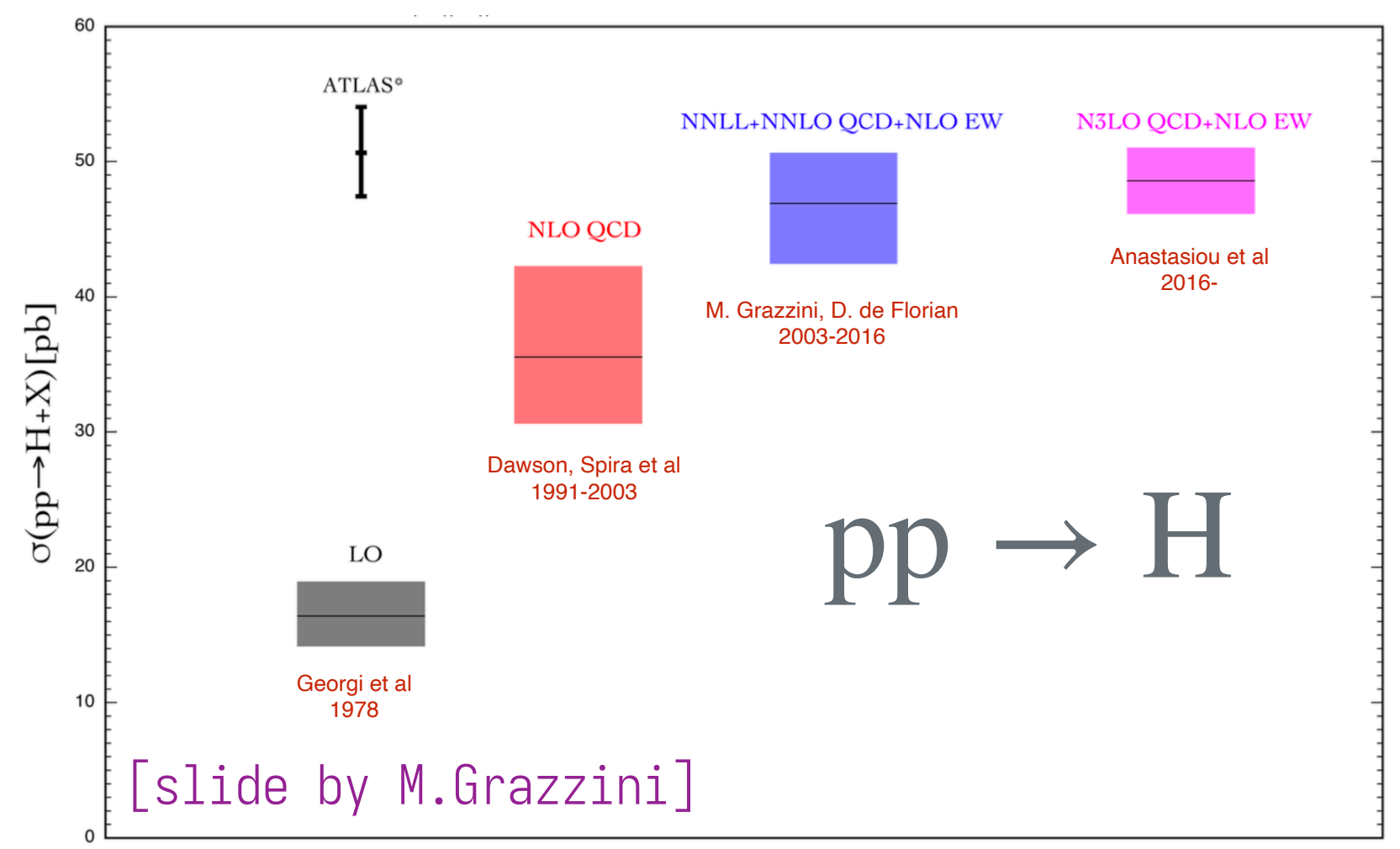
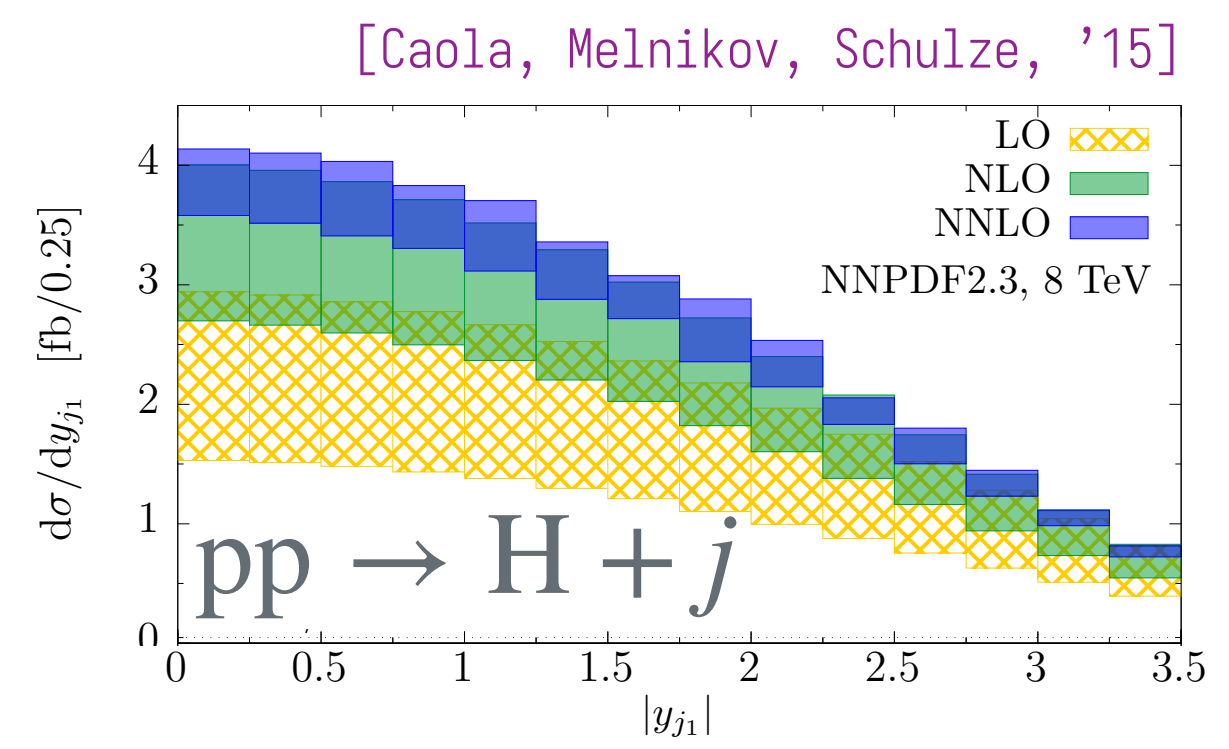
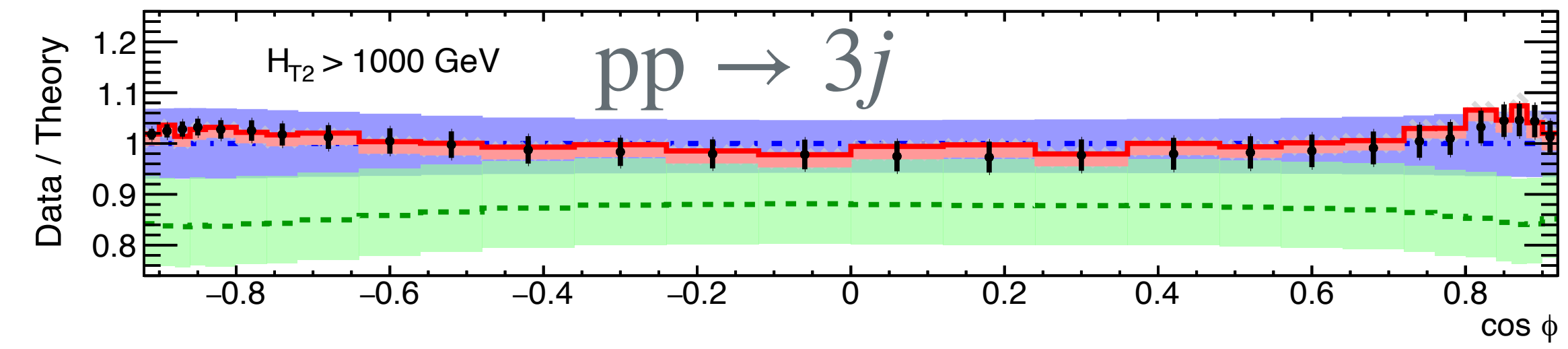
$$\mu \frac{d}{d\mu} \sigma^{\text{N}^n\text{LO}}(\mu) = \mathcal{O}(\alpha_s^{n_0+n+1}) = \mathcal{O}(\Delta_{\text{pert.}})$$



1 SCALE VARIATION

CREDIT WHERE CREDIT IS DUE

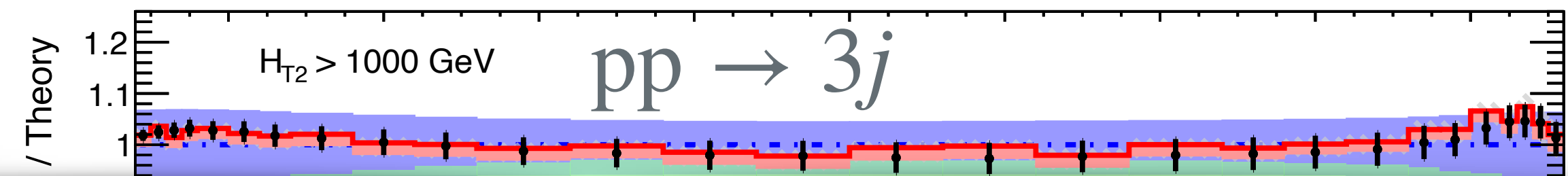
- quite neat & simple
- can work very well...
 - ... lot of experience if it doesn't
 - ↳ new channels, jet bins, "giant K factors", ratios, ...
 - ↳ strategies to mitigate



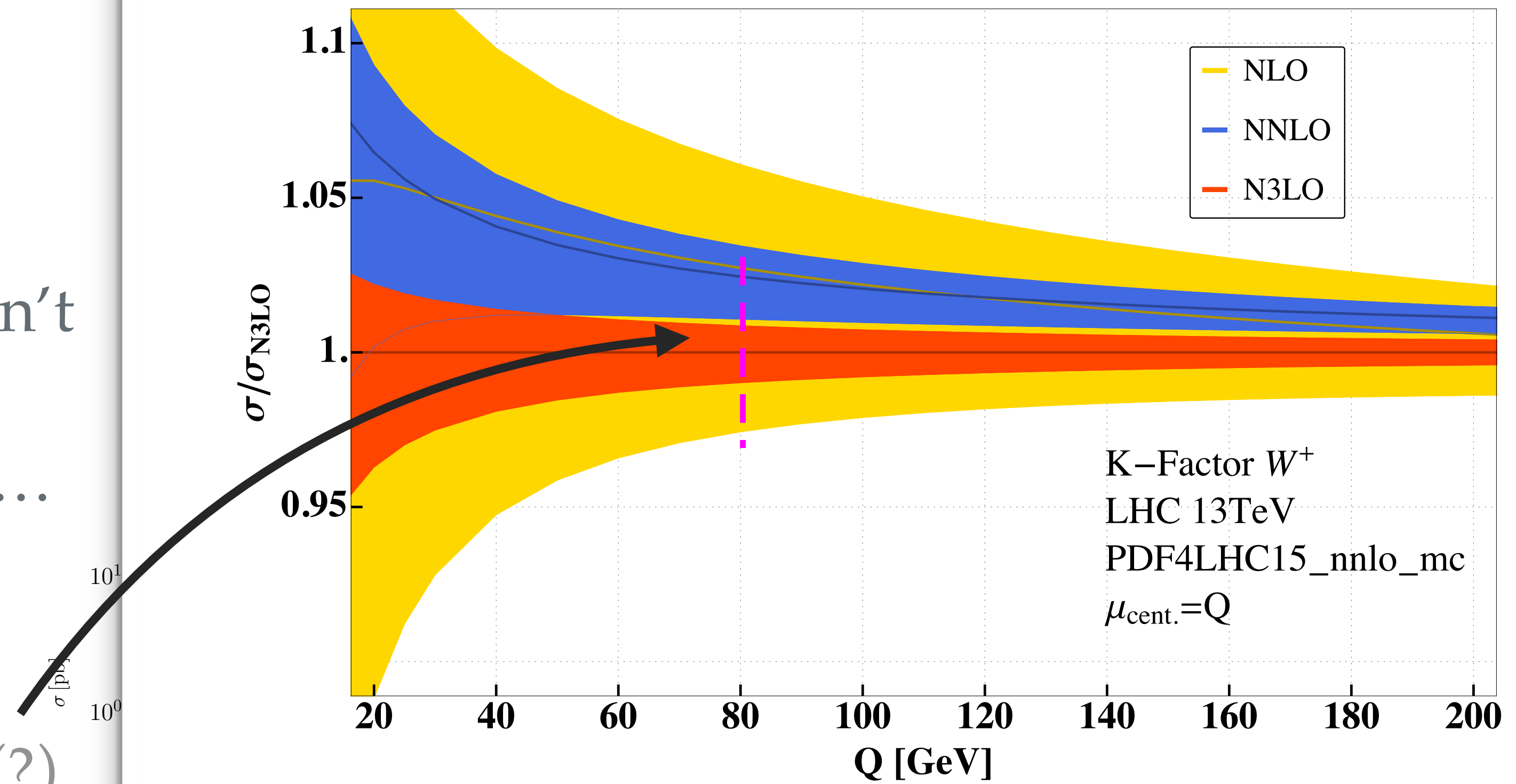
1 SCALE VARIATION

CREDIT WHERE CREDIT IS DUE

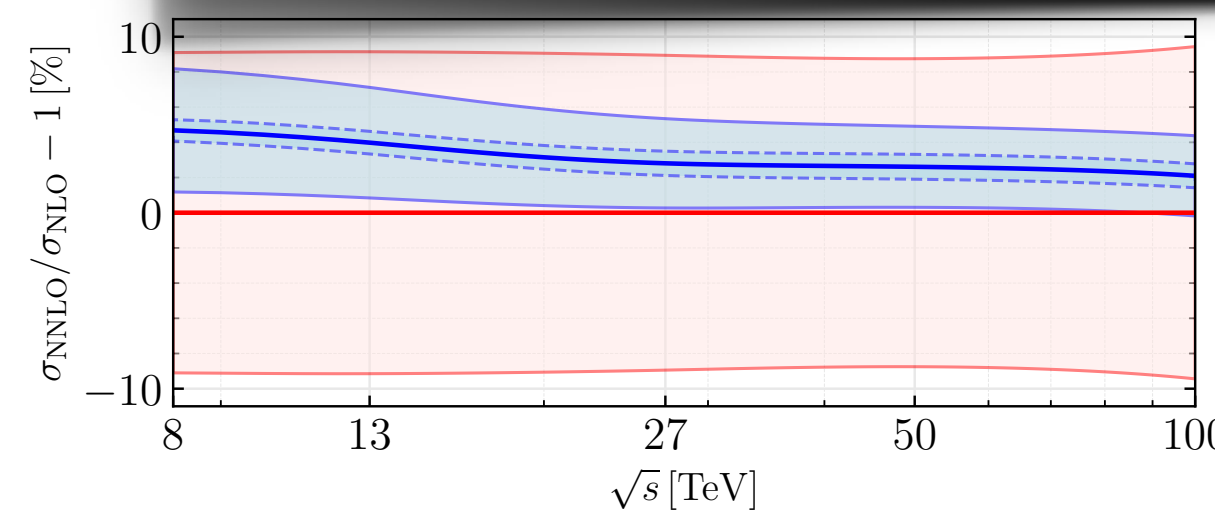
- quite neat & simple
- can work very well...
 - ... lot of experience if it doesn't
 - ↳ new channels, jet bins, "giant K factors", ratios, ...
 - ↳ strategies to mitigate
- failure of Δ_{scl} beyond repair(?)
 - ↔ difficult for any method



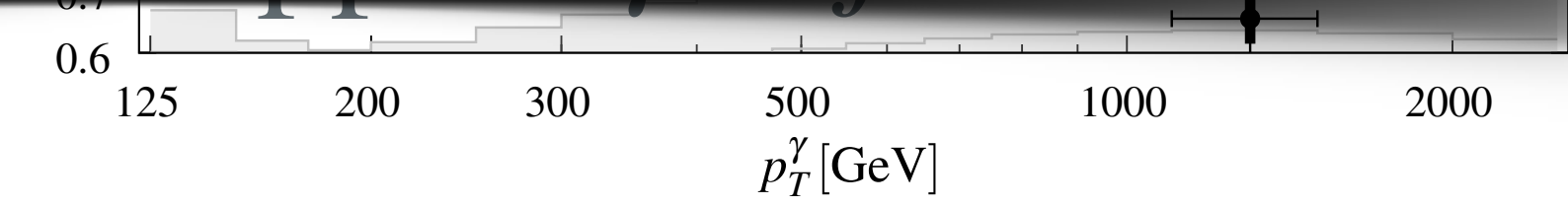
pp → W⁺ @ N3LO



[Duhr, Dulat, Mistlberger '20]



Kallweit,



1 SCALE VARIATION

MAIN ISSUES WITH SCALES

- 1 choice of the central scale μ_0
- 2 no probabilistic interpretation
- 3 no correlation model

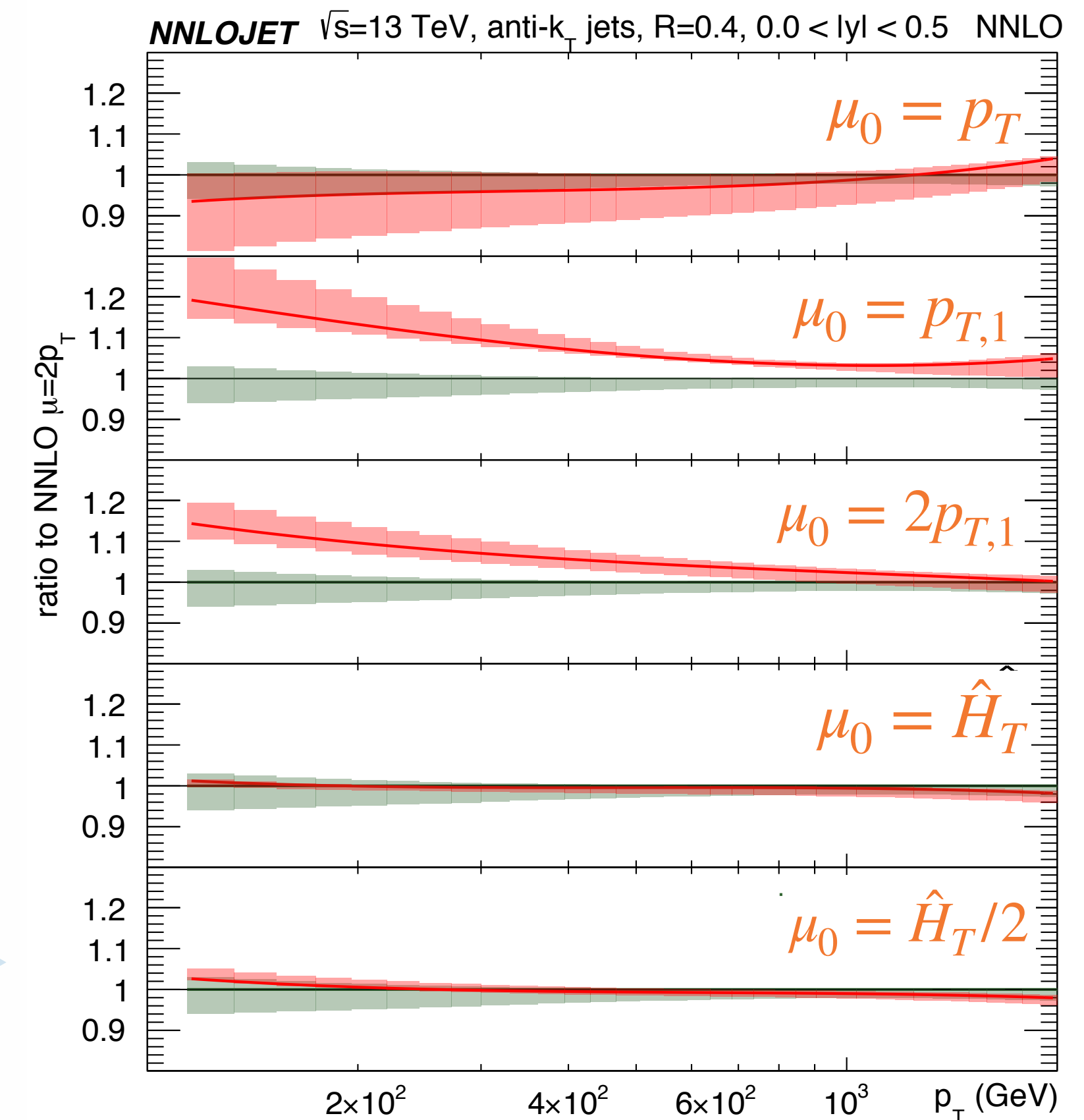
- fastest apparent convergence (FAC)
 - $\hookrightarrow \sigma^{(n)}(\mu_{\text{FAC}}) = 0$
- principle of minimal sensitivity (PMS)
 - $\hookrightarrow \left. \frac{\partial}{\partial \mu} \sigma^{(n)}(\mu) \right|_{\mu_{\text{PMS}}} = 0$
- BLM/PMC, ...

invoke some principle

incl. jets @ NNLO
w.r.t. $\mu_0 = 2p_T$

or combination:

- convergence
- band overlap
- stability
- ...



1 SCALE VARIATION

MAIN ISSUES WITH SCALES

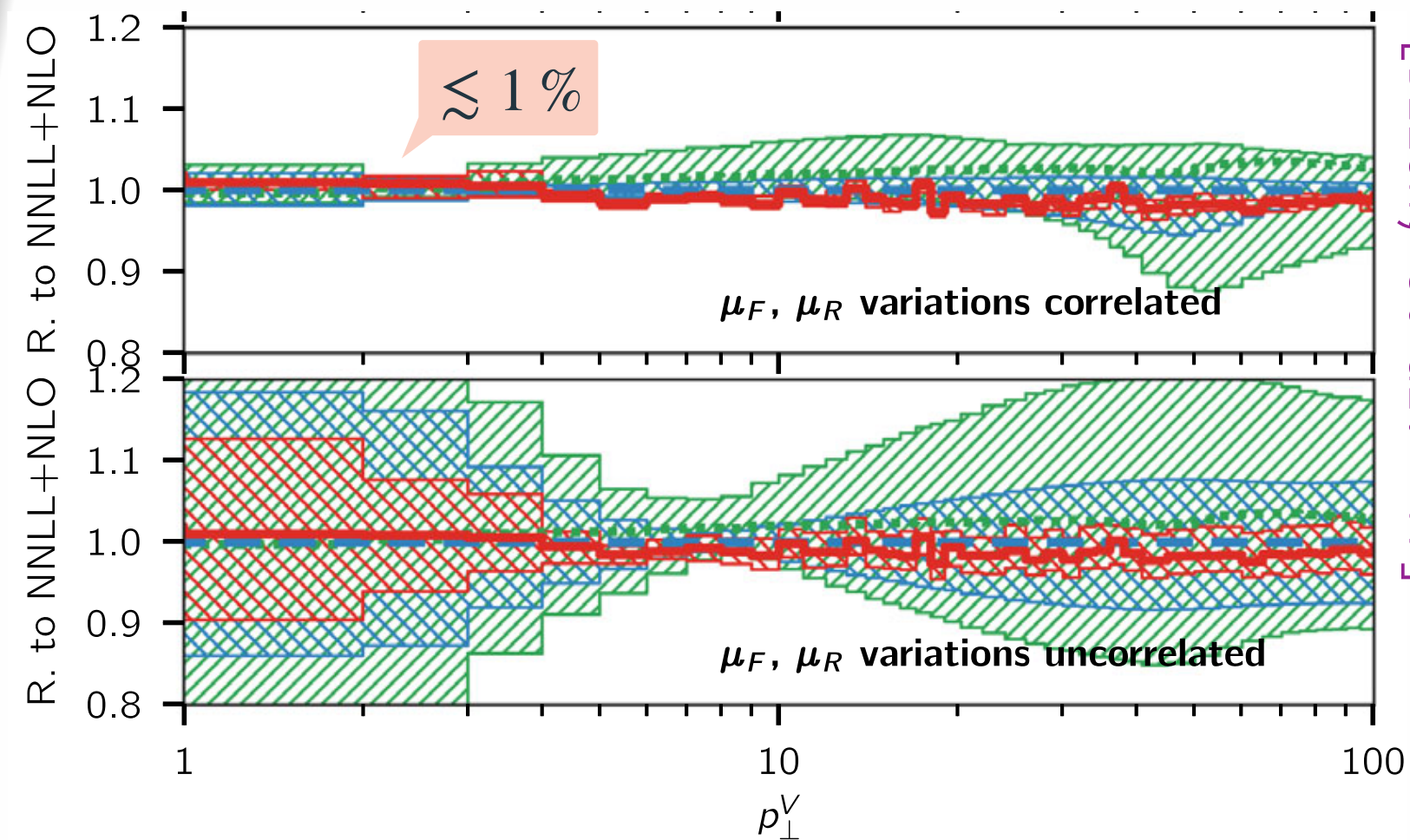
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meaning of the envelope?

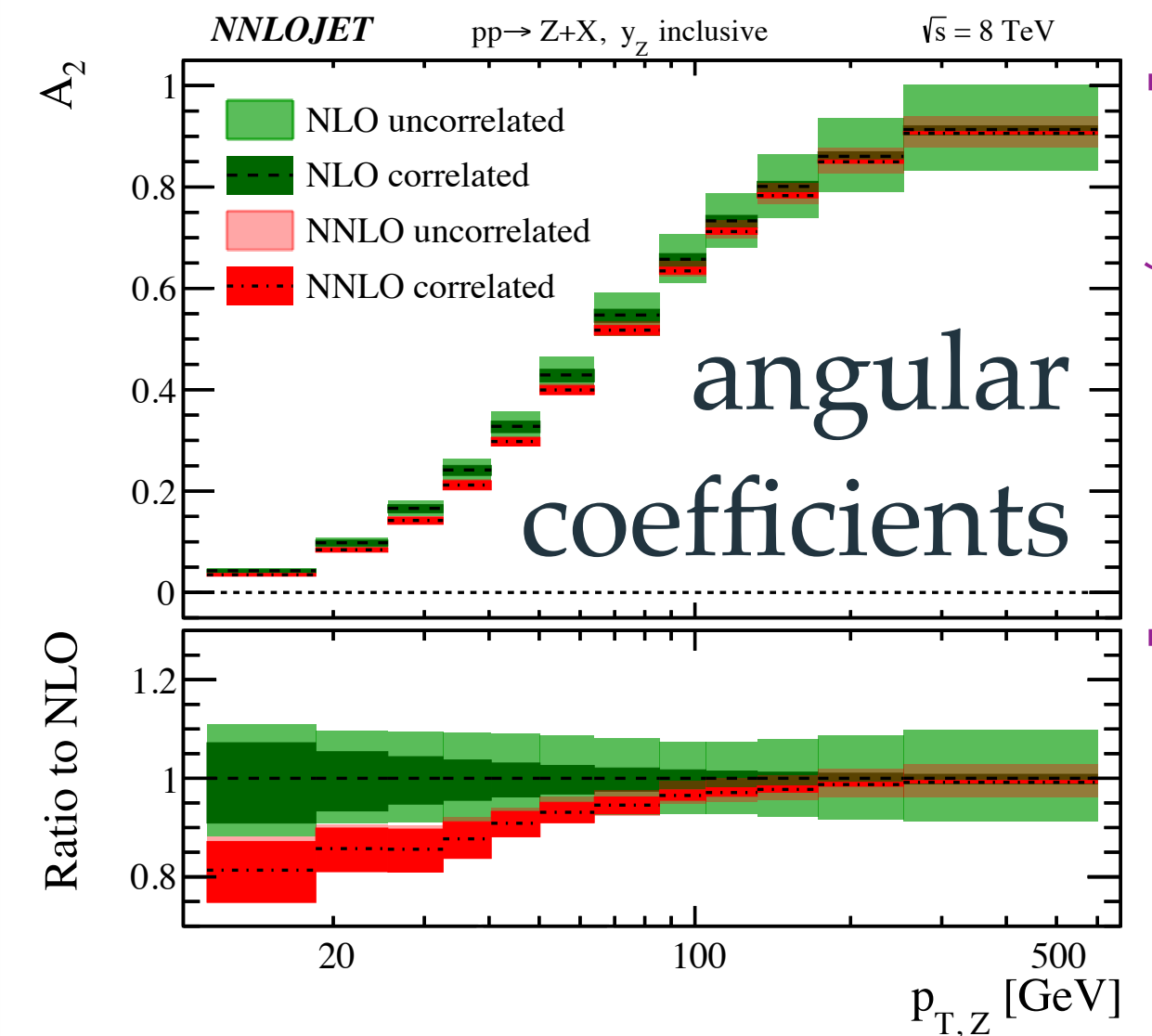
box , Gaussian , ...

to correlate, or not to correlate

$$\frac{\frac{1}{\sigma^Z} \left(\frac{d\sigma^Z}{dp_T^Z} \right)}{\frac{1}{\sigma^{W^+}} \left(\frac{d\sigma^{W^+}}{dp_T^{W^+}} \right)}$$



[Bizon, et al. '19]



[Gauld, et al. '17]

1 SCALE VARIATION

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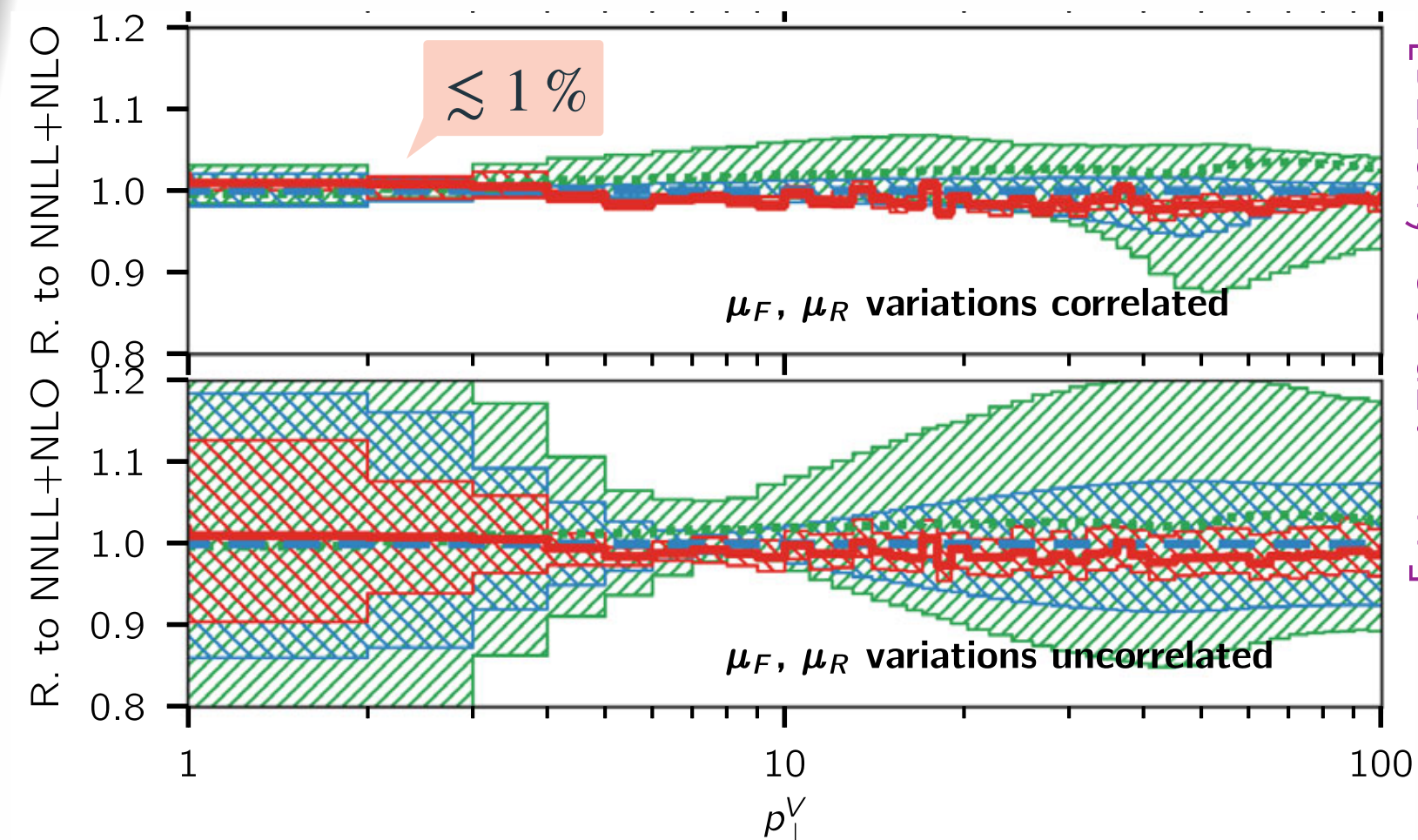
new approaches try to mainly address 2 & 3
1 remains a challenge ∇

meaning of the envelope?

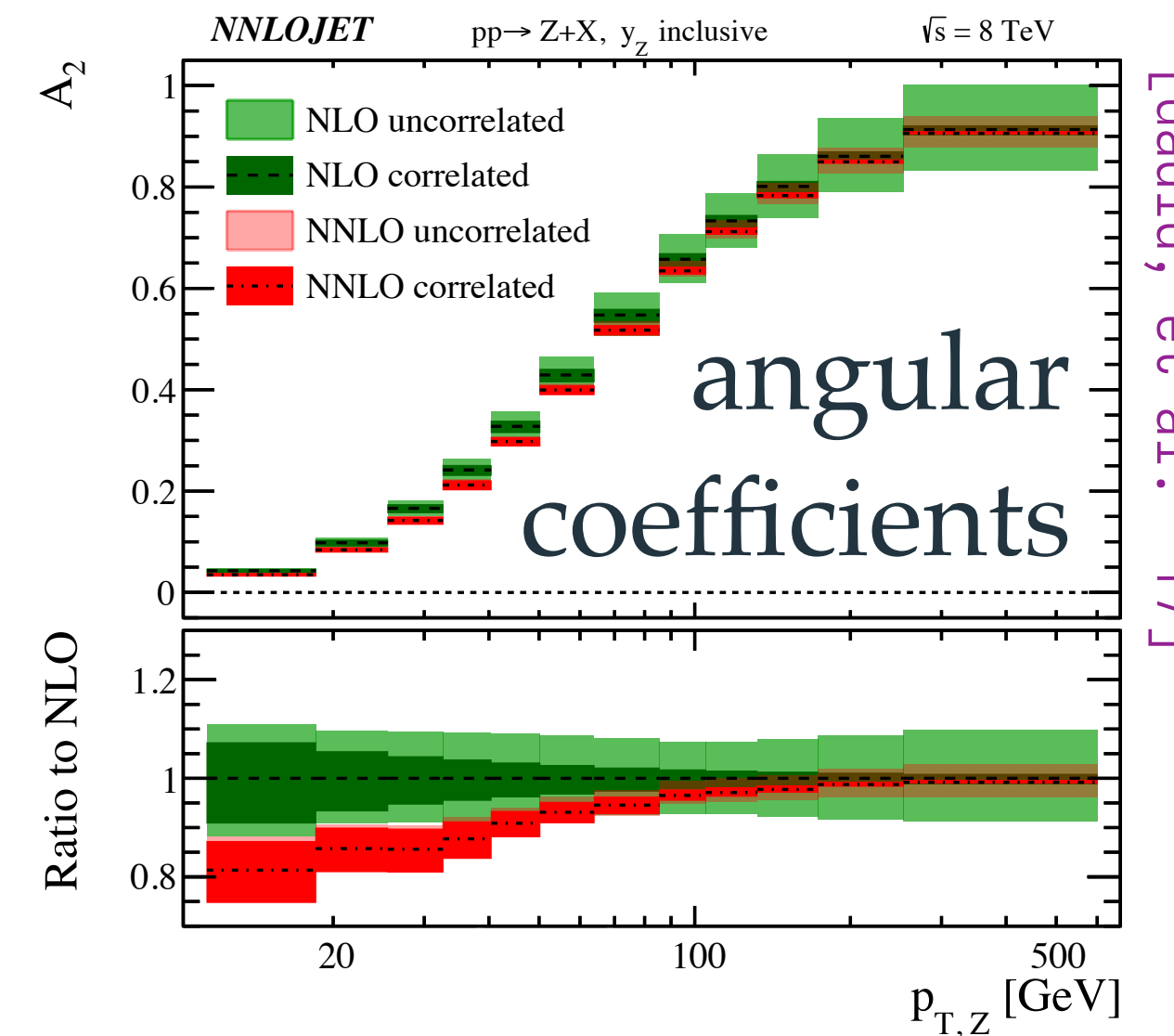
box , Gaussian , ...

to correlate, or not to correlate

$$\frac{\frac{1}{\sigma^Z} \left(\frac{d\sigma^Z}{dp_T^Z} \right)}{\frac{1}{\sigma^{W^+}} \left(\frac{d\sigma^{W^+}}{dp_T^{W^+}} \right)}$$



[Bizon, et al. '19]



[Gauld, et al. '17]

2 BAYESIAN ESTIMATES

[Cacciari, Houdeau '11], [Bonvini '20]
[Duhr, Huss, Mazeliauskas, Szafron '21]

GENERAL IDEA

- consider the perturbative series as a *sequence of dimensionless numbers*

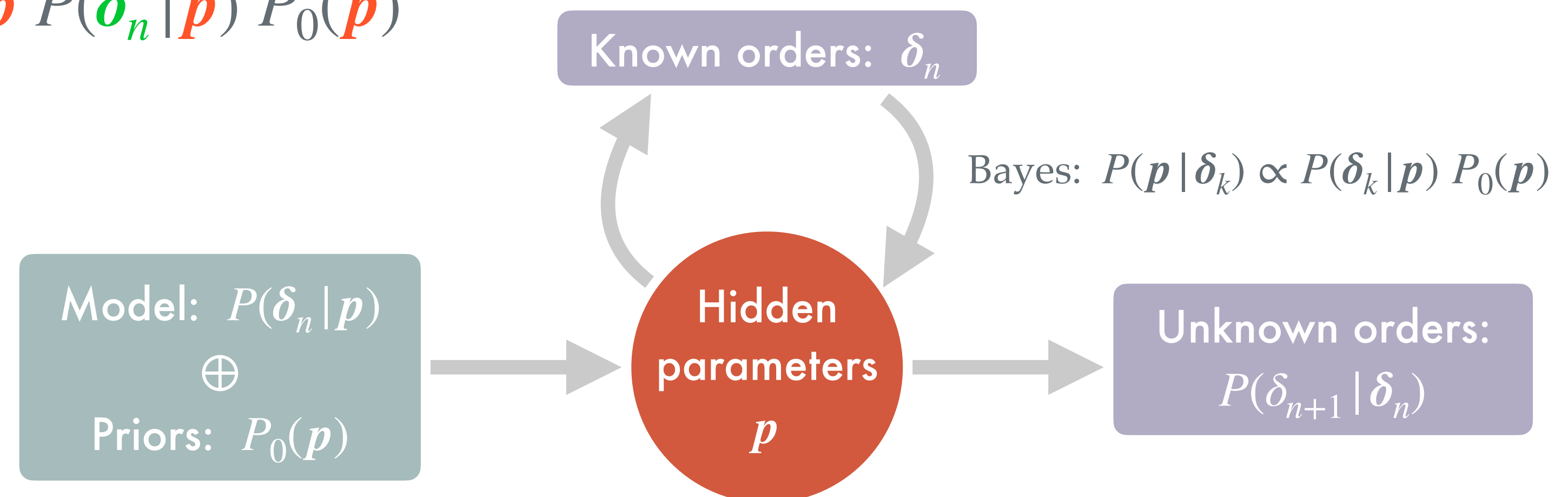
$$d\sigma = d\sigma^{(0)} [1 + \delta_1 + \delta_2 + \dots] \rightsquigarrow \delta_k = \mathcal{O}(\alpha_s^k)$$

- Q:** after observing $\delta_n \equiv (\delta_0, \delta_1, \dots, \delta_n)$, prob. to observe δ_{n+1} ?

$$P(\delta_{n+1} | \delta_n) = \frac{P(\delta_{n+1}, \delta_n)}{P(\delta_n)} = \frac{\int d^m \mathbf{p} P(\delta_{n+1}, \delta_n | \mathbf{p}) P_0(\mathbf{p})}{\int d^m \mathbf{p} P(\delta_n | \mathbf{p}) P_0(\mathbf{p})}$$

$$P(A, B) = P(A | B) P(B)$$

$$P(A) = \int dB P(A, B)$$



2 BAYESIAN ESTIMATES

THE CH MODEL [Cacciari, Houdeau '11]

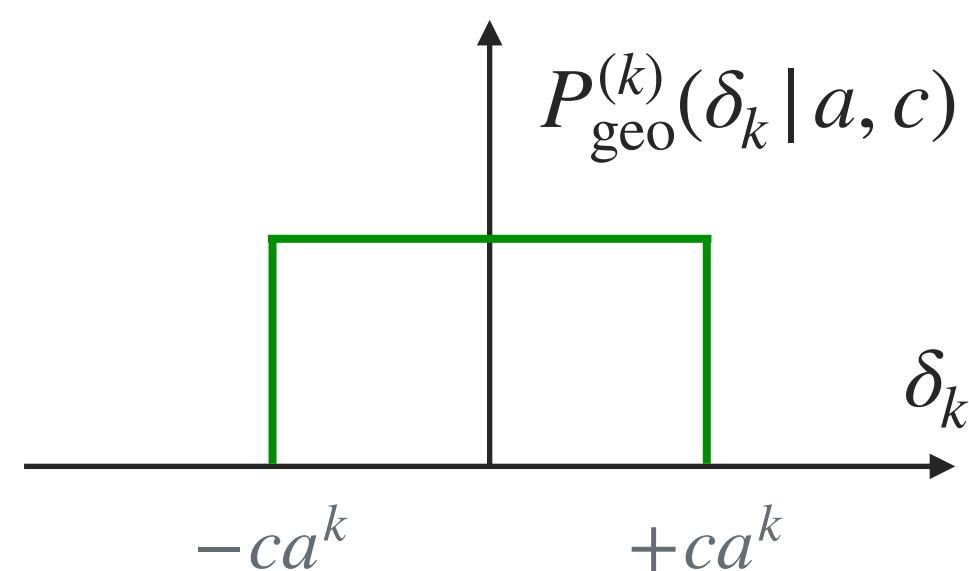
- pert. exp. $\delta_k = c_k \alpha_s^k$ bounded by geometric series: $|c_k| \leq \bar{c} \quad \forall k \iff 1 \text{ param: } \bar{c}$
 α_s at what scale? why not: $\alpha_s/\pi, \alpha_s/(4\pi), \alpha_s \ln^2(v), \alpha_s \ln(v), \dots$?

THE GEOMETRIC MODEL [Bonvini '20]

- let the model learn the expansion parameter: $|\delta_k| \leq c a^k \quad \forall k \iff 2 \text{ params: } a, c$

model:

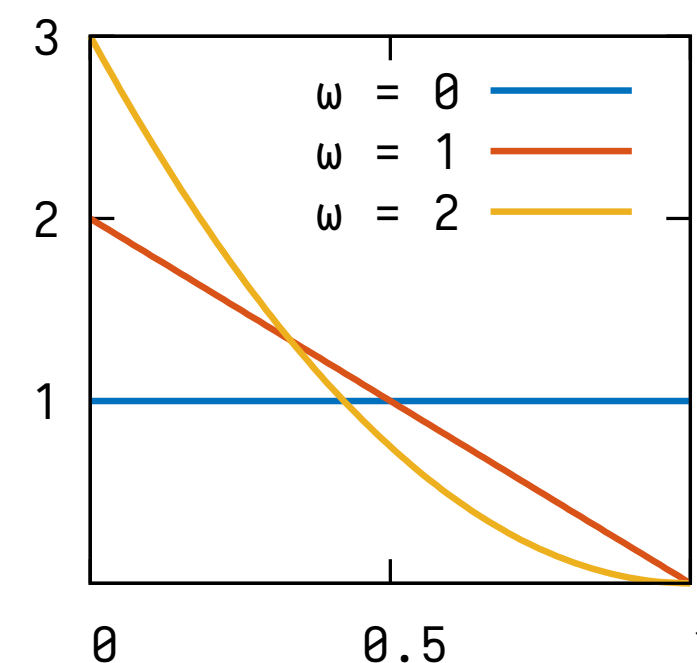
$$P_{\text{geo}}^{(k)}(\delta_k | a, c) = \frac{1}{2ca^k} \Theta\left(c - \frac{|\delta_k|}{a^k}\right)$$



priors:

$$P_0(a) = (1 + \omega) (1 - a)^\omega \Theta(|a| < 1)$$

$$P_0(c) = \frac{\varepsilon}{c^{1+\varepsilon}} \Theta(c - 1)$$

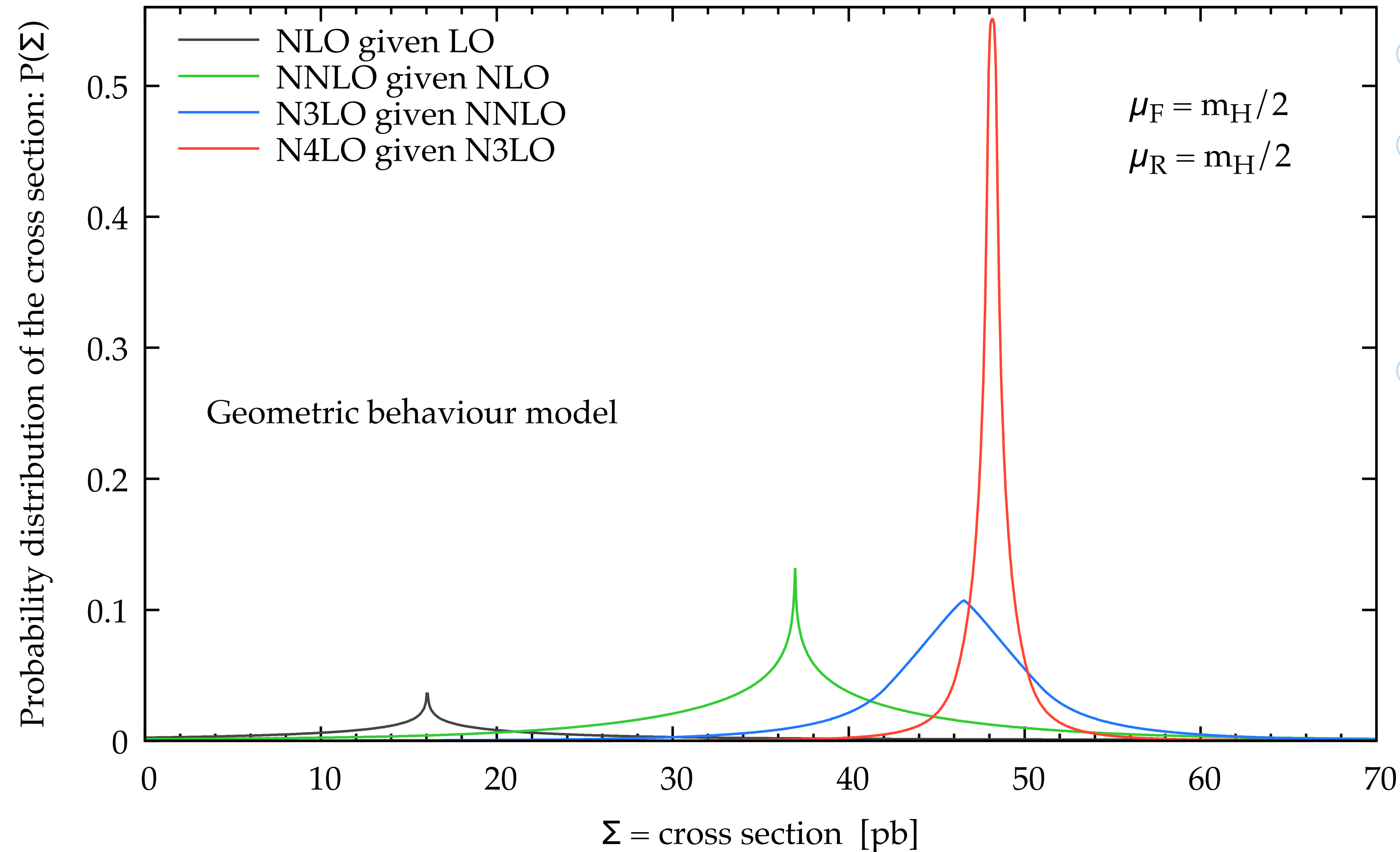


$dc/c \sim d \ln(c)$
(ε : regulator)

2 BAYESIAN ESTIMATES

[Bonvini '20]

Higgs production in gluon fusion at LHC 13 TeV, $m_H = 125$ GeV



THE GEOMETRIC MODEL

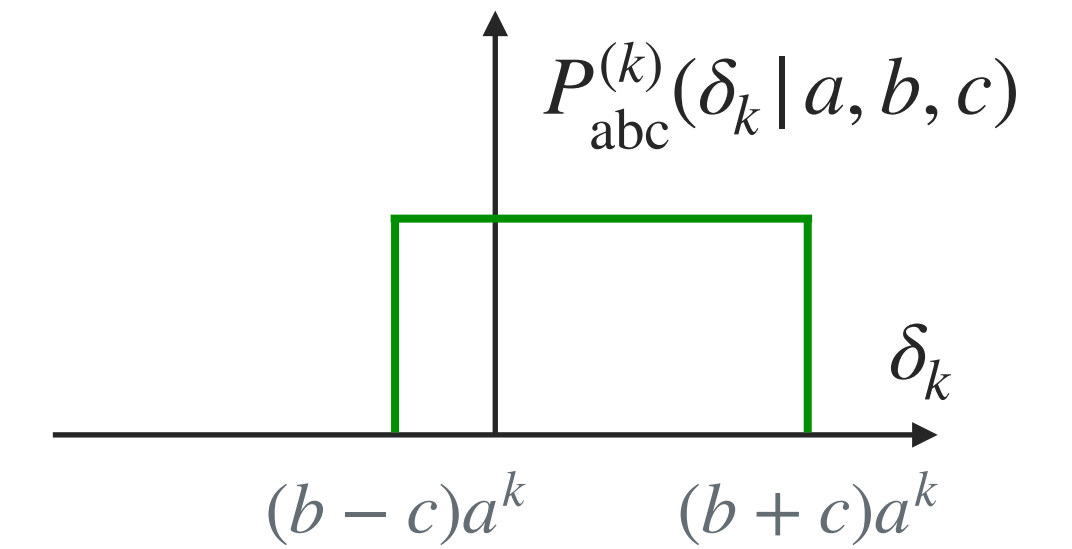
- full prob. dist. $P(\sigma)$
- broad* lower orders:
 - stronger dependence on priors P_0
- narrow* higher orders:
 - inference of hidden parameters

2 BAYESIAN ESTIMATES

[Duhr, Huss, Mazeliauskas, Szafron '21]

THE ABC MODEL

- allow for a bias & alt. series: $b - c \leq \frac{\delta_k}{a^k} \leq b + c \quad \forall k$



INCORPORATION OF SCALES

- scale marginalisation (sm)

$$\int d\mu P(\delta_{n+1} | \delta_n; \mu) \underbrace{P(\mu | \delta_n)}_{\propto P(\delta_n; \mu) P_0(\mu)}$$

- scale average (sa)

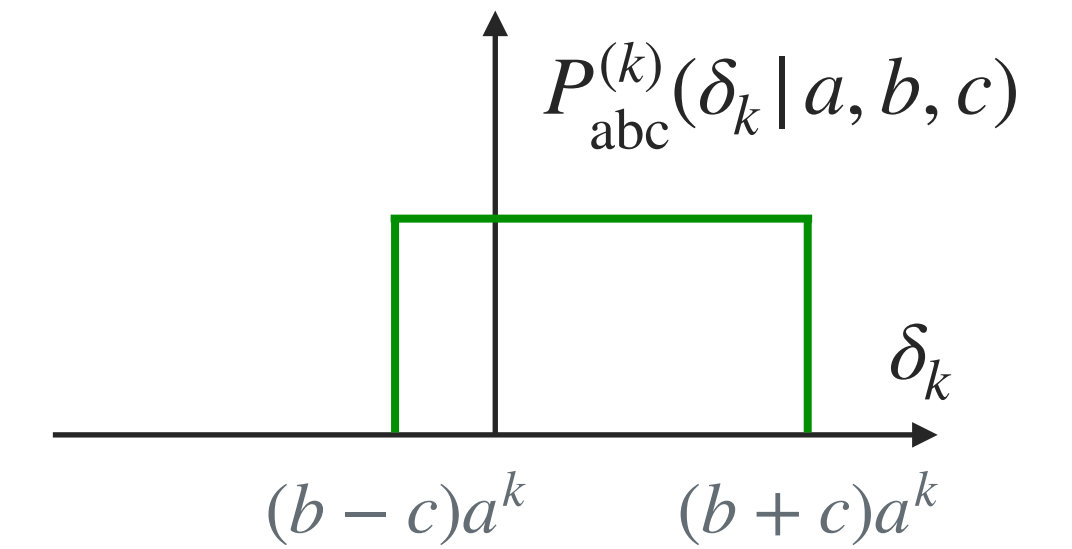
$$\int d\mu \underbrace{w(\mu)}_{\text{weight}} P(\delta_{n+1} | \delta_n; \mu) \quad \frac{1}{2\mu \ln F} \Theta\left(\ln F - \left|\ln\left(\frac{\mu}{\mu_0}\right)\right|\right)$$

2 BAYESIAN ESTIMATES

[Duhr, Huss, Mazeliauskas, Szafron '21]

THE ABC MODEL

- allow for a bias & alt. series: $b - c \leq \frac{\delta_k}{a^k} \leq b + c \quad \forall k$



INCORPORATION OF SCALES

- scale marginalisation (sm)

$$\Leftrightarrow P_{\text{sm}}(\delta_{n+1} | \delta_n)$$

peaks at μ_{FAC}

$$\int d\mu P(\delta_{n+1} | \delta_n; \mu) \underbrace{P(\mu | \delta_n)}_{\text{weight}} \propto P(\delta_n; \mu) \underbrace{P_0(\mu)}_{\text{prior}}$$

- scale average (sa)

$$\Leftrightarrow P_{\text{sa}}(\delta_{n+1} | \delta_n)$$

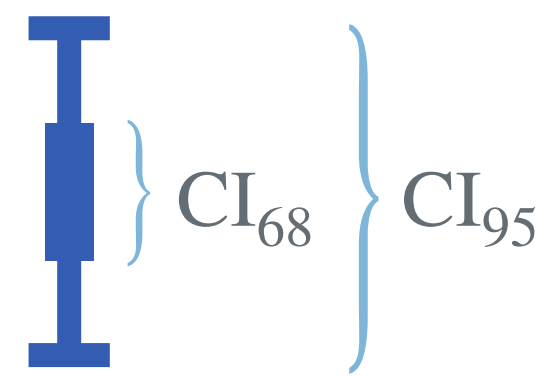
peaks at μ_{PMS}

$$\int d\mu \underbrace{w(\mu)}_{\text{weight}} P(\delta_{n+1} | \delta_n; \mu) \quad \frac{1}{2\mu \ln F} \Theta\left(\ln F - \left|\ln\left(\frac{\mu}{\mu_0}\right)\right|\right)$$

scale interpretation

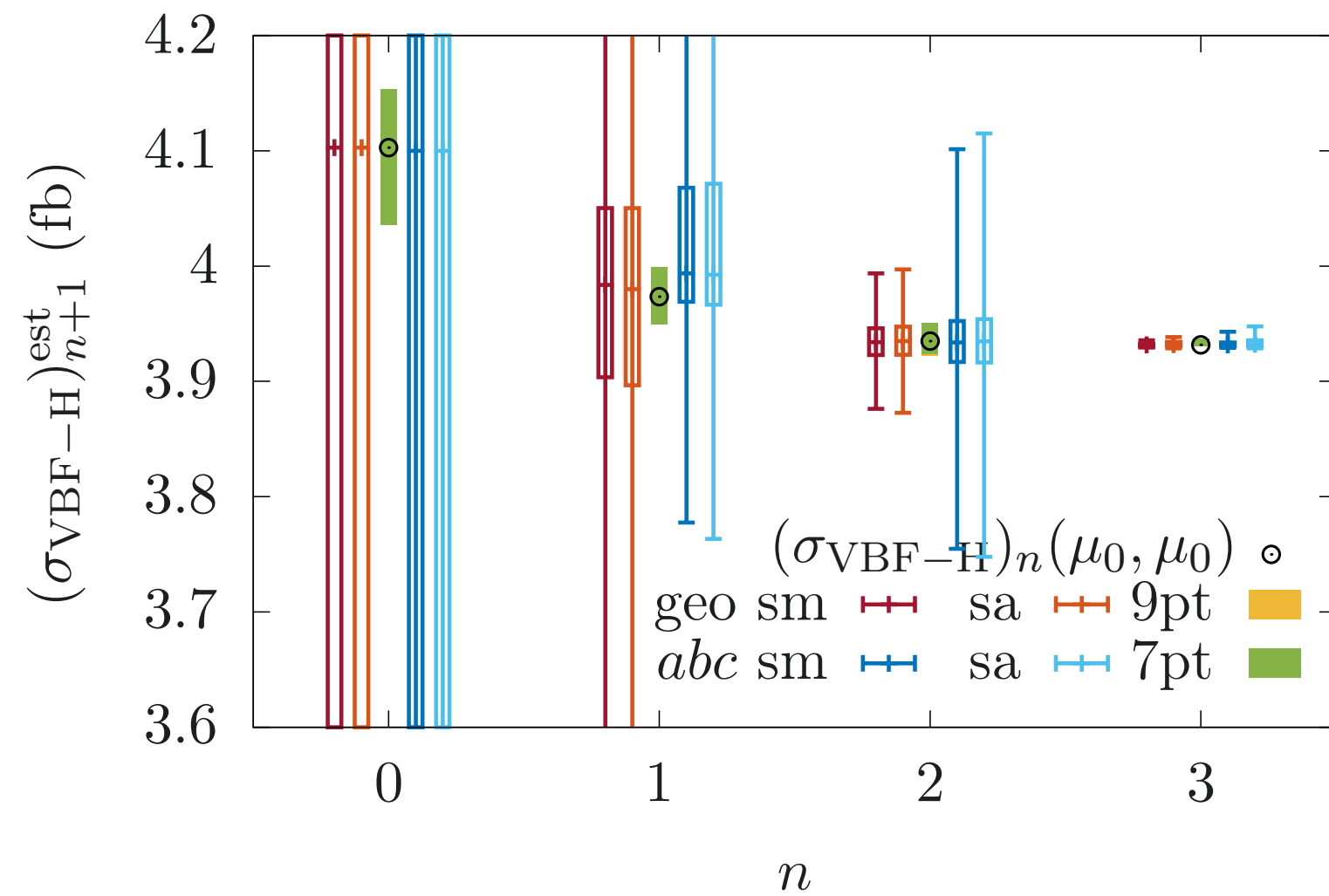
\Leftrightarrow prescription

2 BAYESIAN ESTIMATES



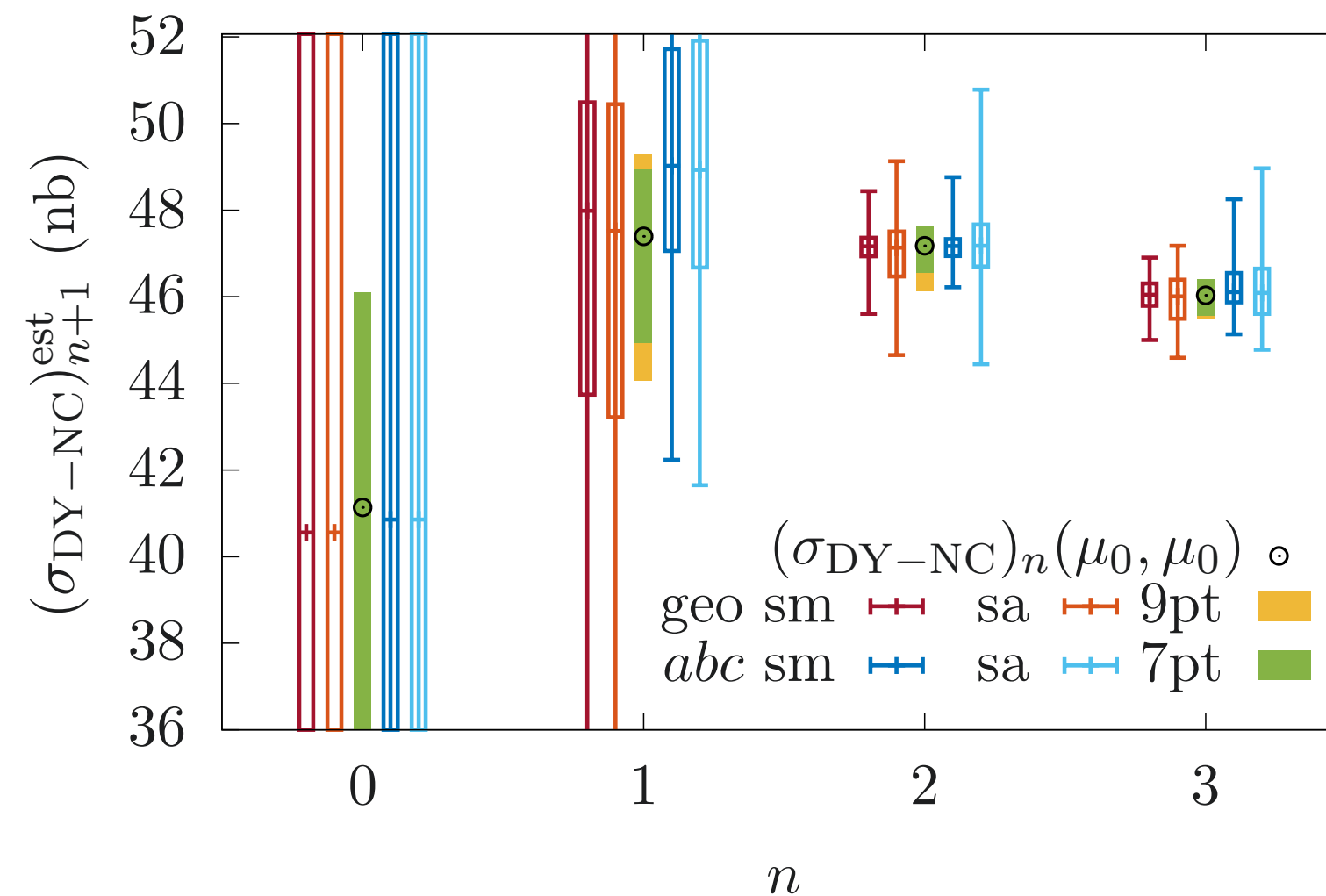
[Duhr, Huss, Mazeliauskas, Szafron '21]

VBF-H



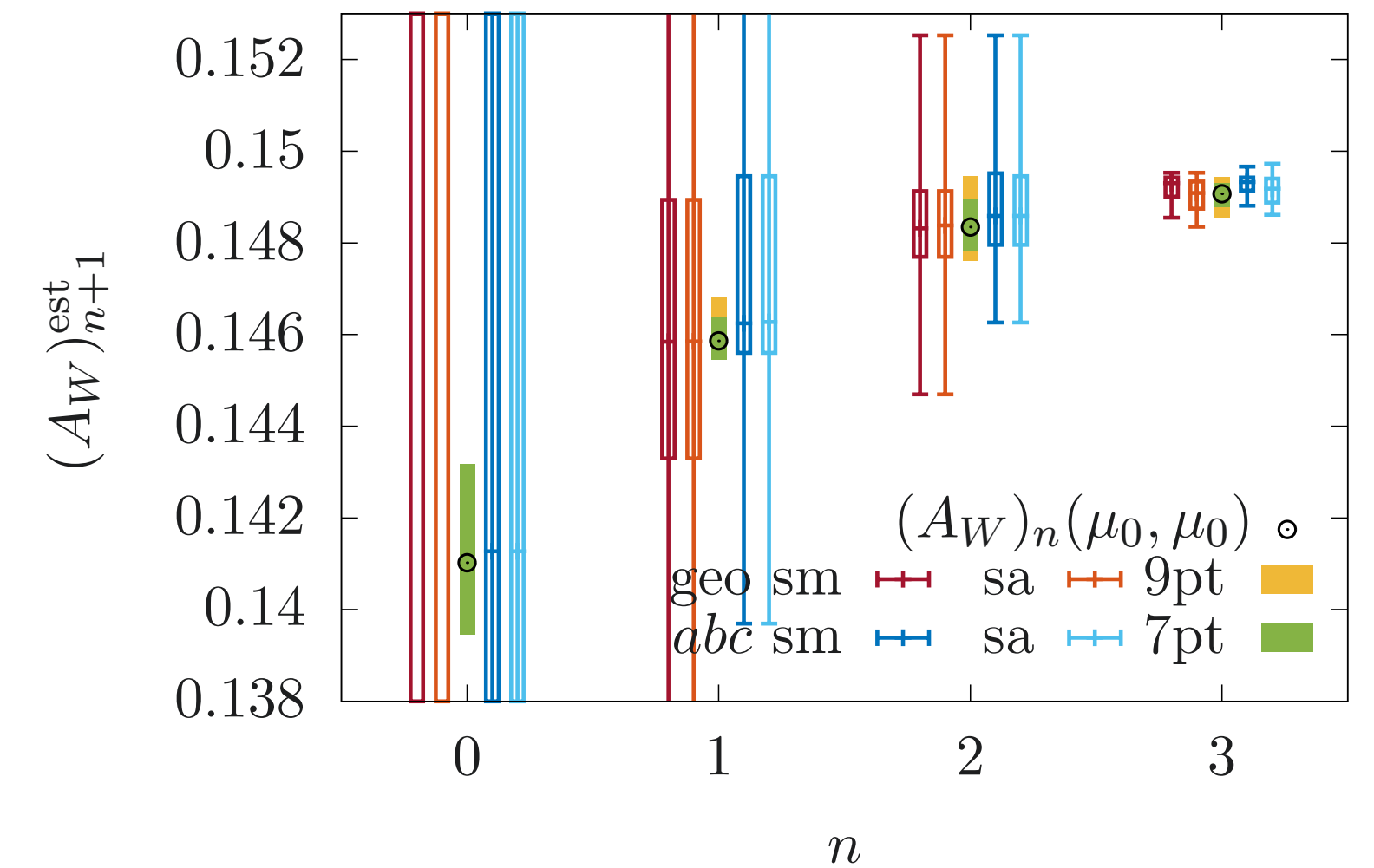
- $n < 2$: CI_{68} bigger than 9pt
- $\delta_1 < 0 \rightsquigarrow abc$ alternating
- $n > 2$: all prescriptions similar

DY-NC



- δ_3 is large and outside of 9pt!
- similar unc.: $sa \simeq 9pt$
- $n = 2$: $sm \ll$ others (μ_{FAC})
- $n = 3$: all prescriptions similar

$$A_W = \frac{W^+ - W^-}{W^+ + W^-}$$



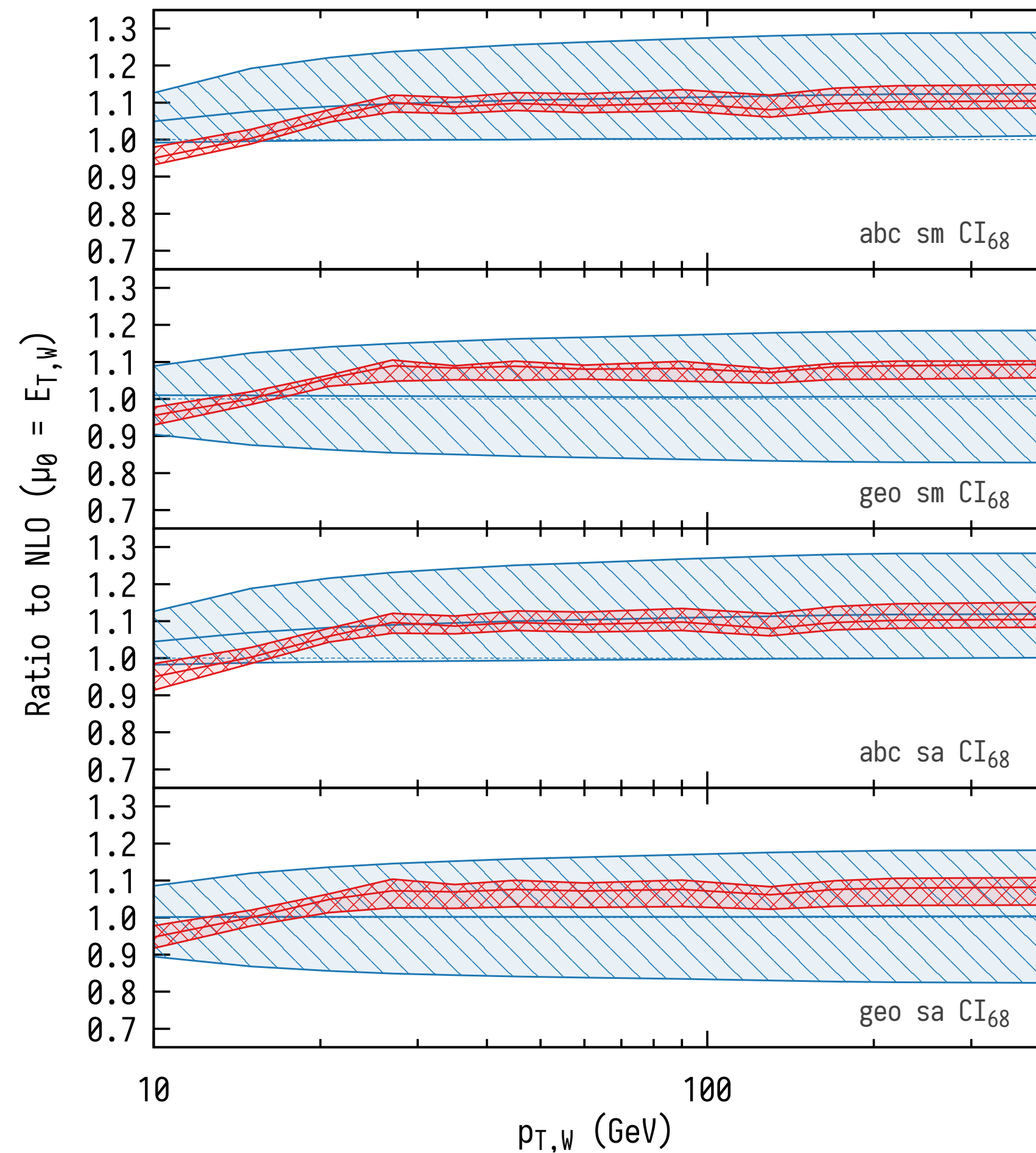
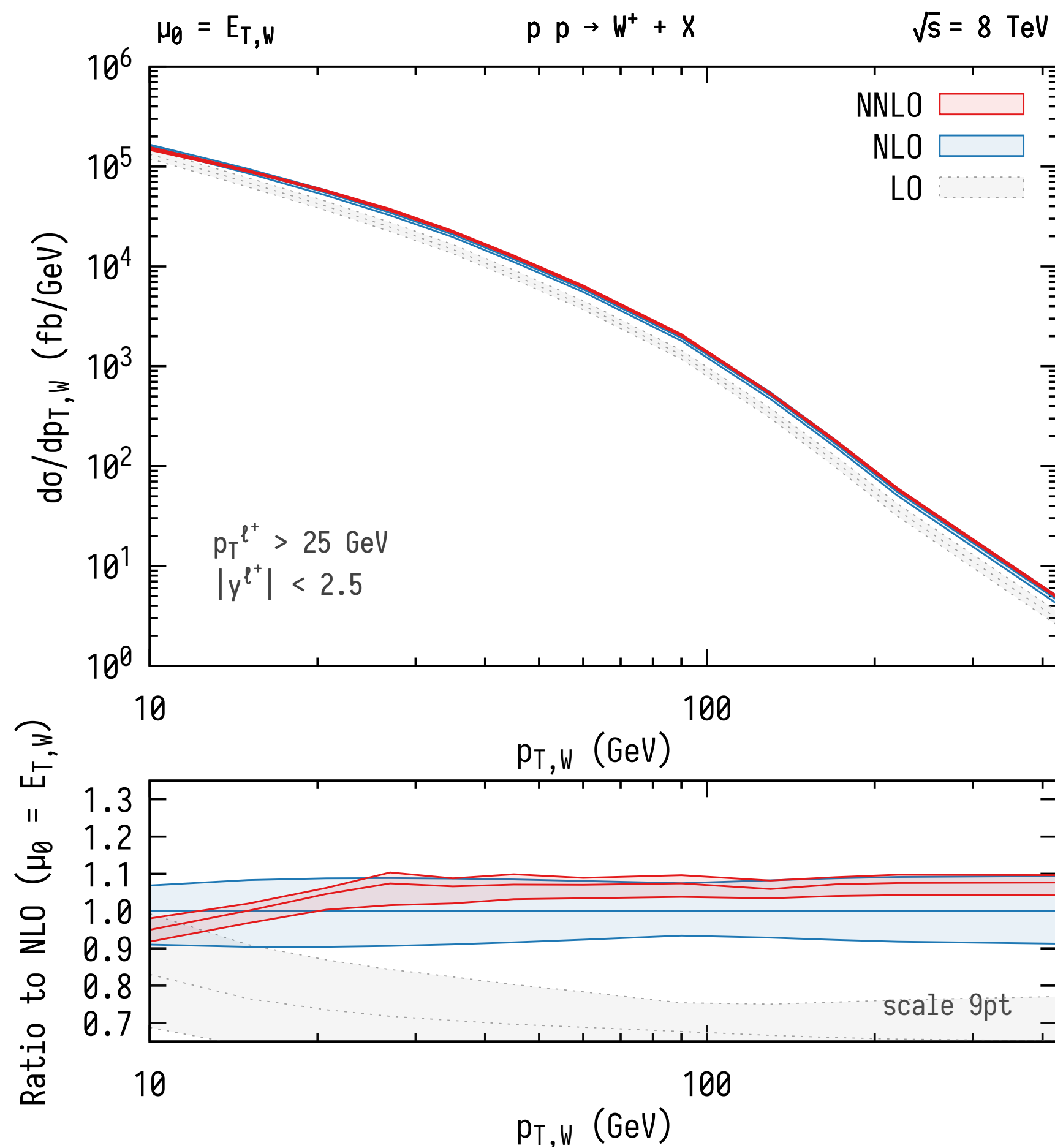
- large cancellations in the ratio
- $n < 2$: 9pt performs poorly
- $(A_W)_n \nearrow$ (anticipated by abc)
- size: $abc \lesssim$ others

overall: not radically different estimates for $\Delta_{\text{pert.}}$ ($n \geq 2$)

2 BAYESIAN ESTIMATES

[Duhr, Huss, Mazeliauskas, Szafron '21]

DIFFERENTIAL DISTRIBUTIONS



- $n < 2$:
 - CI_{68} bigger than 9pt
 - *abc* captures pos. shift
- $n = 2$:
 - almost identical bands
 - Δ_{MHO} very robust
- sm vs. sa
 - almost identical CI

3 THEORY NUISANCE PARAMETERS

[McGowan, Cridge, Harland-Lang, Thorne '23]
[Tackmann '24], [Lim, Poncelet '24]

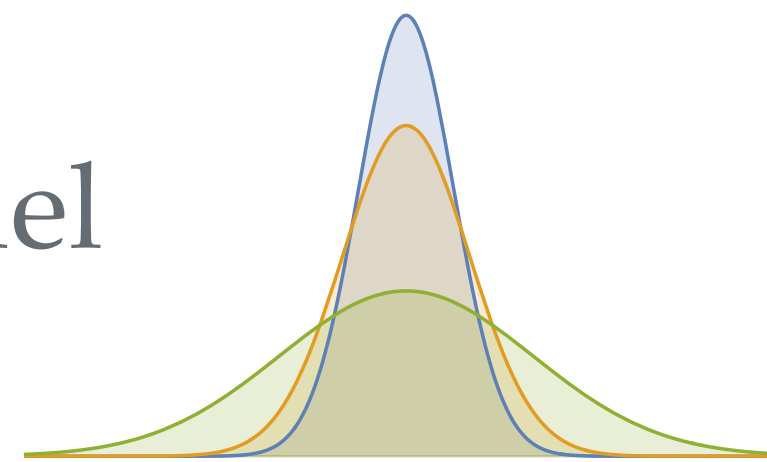
GENERAL IDEA & STEPS

- 1 parametrise the **unknown order** using nuisance parameters $\vec{\theta}$ (TNP)

$$d\sigma = d\sigma^{(0)} + \alpha_s d\sigma^{(1)} + \alpha_s^2 d\sigma_{\text{TNP}}^{(2)}(\vec{\theta})$$

$\mathcal{N} \theta$ (simplest case)

- 2 assign a probability distribution $P(\vec{\theta})$
 \hookrightarrow stat. interpretation & correlation model



- 3 possibility to constrain $\vec{\theta}$ using data

most interesting when we have information on the functional dependence of an observable
 \leftrightarrow correlations

3 THEORY NUISANCE PARAMETERS

[Tackmann '24]

RESUMMED PREDICTION

- 1 factorization in limit $p_T \rightarrow 0 \rightsquigarrow$ functional dependence known

$$\frac{d\sigma}{dp_T} = [H \otimes B_a \otimes B_b \otimes S](\alpha_s; L) + \mathcal{O}(p_T/Q) \quad L \equiv \ln(p_T/Q)$$

RGE

$$\mathcal{X}(\alpha_s; L) = \mathcal{X}(\alpha_s) \exp \int_0^L dL' \left\{ \Gamma(\alpha_s(L')) L' + \gamma_{\mathcal{X}}(\alpha_s(L')) \right\}$$

- boundary conditions:

$$\mathcal{X}(\alpha_s) = \mathcal{X}_0 + \alpha_s \mathcal{X}_1 + \alpha_s^2 \mathcal{X}_2 + \dots$$

- anomalous dimensions:

$$\Gamma(\alpha_s) = \alpha_s [\Gamma_0 + \alpha_s \Gamma_1 + \alpha_s^2 \Gamma_2 + \dots] \quad \gamma_{\mathcal{X}}(\alpha_s) = \alpha_s [\gamma_0 + \alpha_s \gamma_1 + \alpha_s^2 \gamma_2 + \dots]$$

3 THEORY NUISANCE PARAMETERS

[Tackmann '24]

RESUMMED PREDICTION

1' parametrise unknown resummation ingredients using nuisance parameters $\vec{\theta}$

▸ boundary conditions: $\mathcal{X} \in \{H, B_a, B_b, S\}$

$$\mathcal{X}_n = \mathcal{N}_{\mathcal{X}}^{(n)} \theta_{\mathcal{X}} \quad \underbrace{\hspace{10em}}_{\text{actually functions: } B_i(x_i)}$$

▸ anomalous dimensions:

(+ beta function β & splitting functions $P_{a \rightarrow b}$)

$$\Gamma_n = \mathcal{N}_{\Gamma}^{(n)} \theta_{\Gamma} \quad \gamma_{\mathcal{X},n} = \mathcal{N}_{\gamma_{\mathcal{X}}}^{(n)} \theta_{\gamma_{\mathcal{X}}}$$

○ implements a correlation model for the (low-ish) p_T spectrum

3 THEORY NUISANCE PARAMETERS

[Tackmann '24]

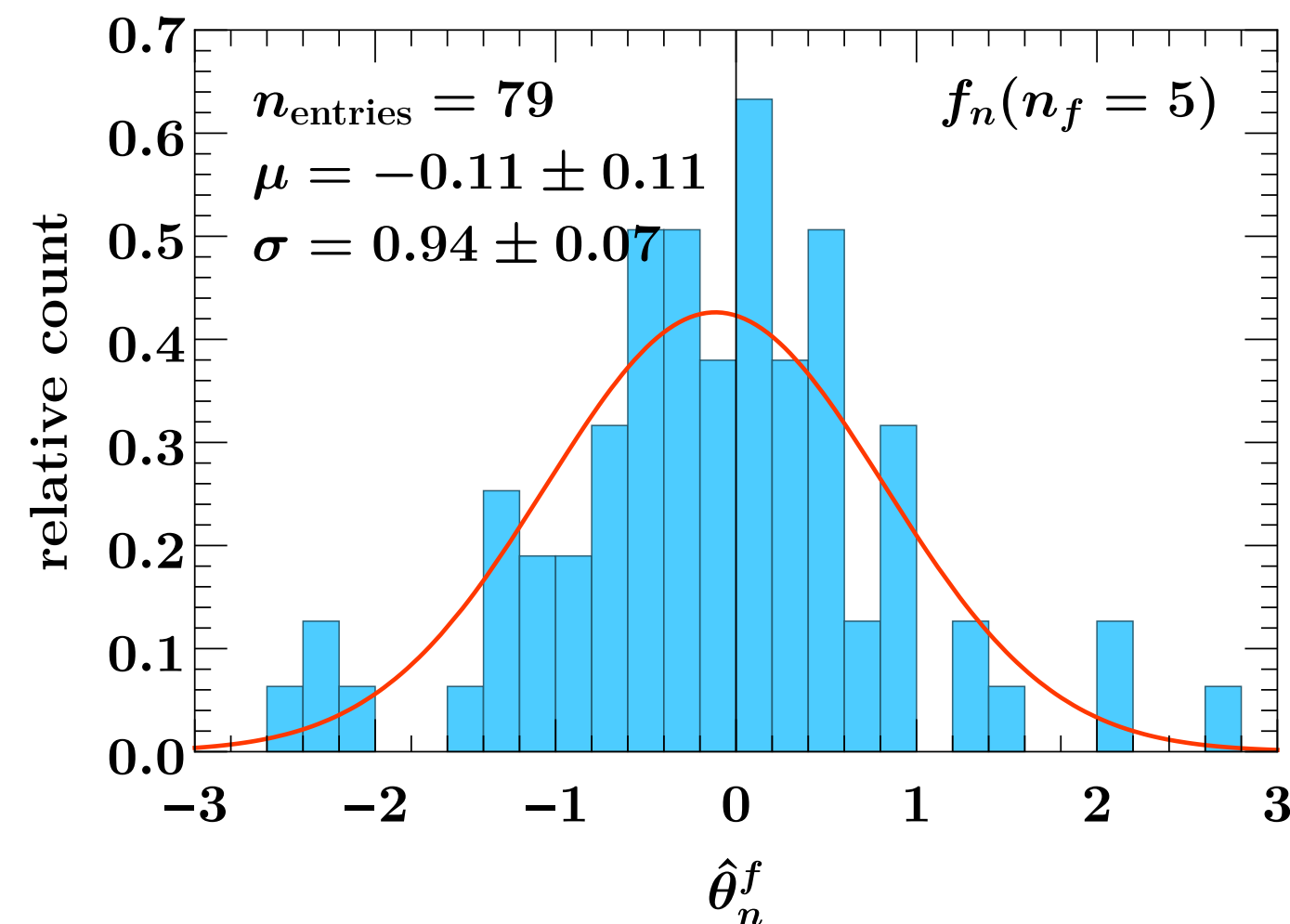
RESUMMED PREDICTION

selection bias?

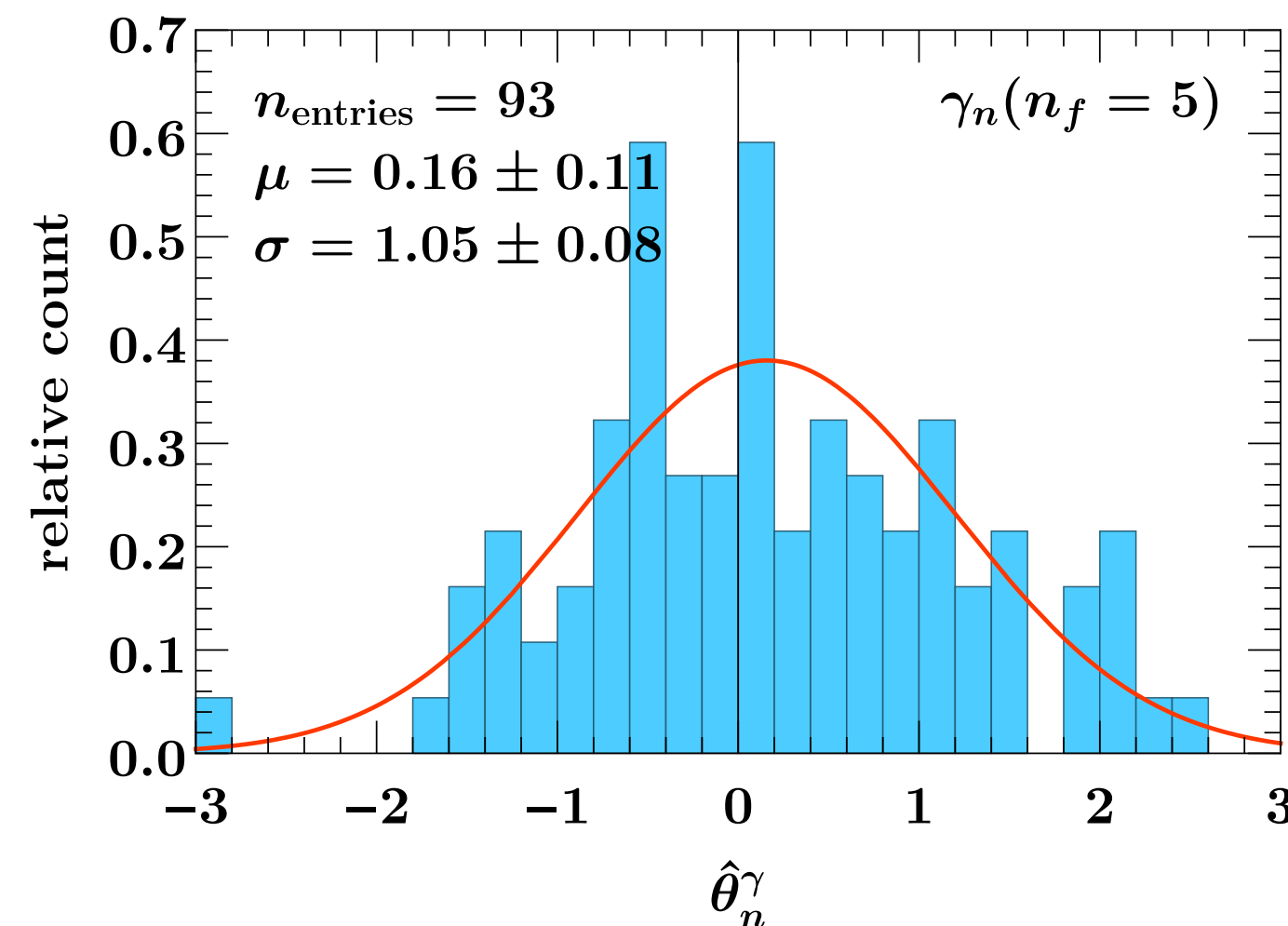
2 assign a probability distribution (statistics over \exists calculations)

\leftrightarrow assume universality in order n as well as processes / ingredients valid?

Matrix Elements



anomalous dimensions



CHOICES (\leftrightarrow ambiguities)

- scheme dependence (scale, ren. scheme, IR subtr., ...)
- parametrisation freedom ($\vec{\theta} \rightarrow \vec{\theta}'$: changes what is uncorrelated / independent)

$$\mathcal{N}_{\mathcal{X}}^{(n)} = 4^n C_r C_A^{n-1} (n-1)!$$

(& $\mathcal{X} \rightarrow \mathcal{X}^\delta$ to scale like $\mathcal{M}_{1 \rightarrow 1}$)

$$\mathcal{N}_{\Gamma}^{(n)} = 4^n C_r C_A^n$$

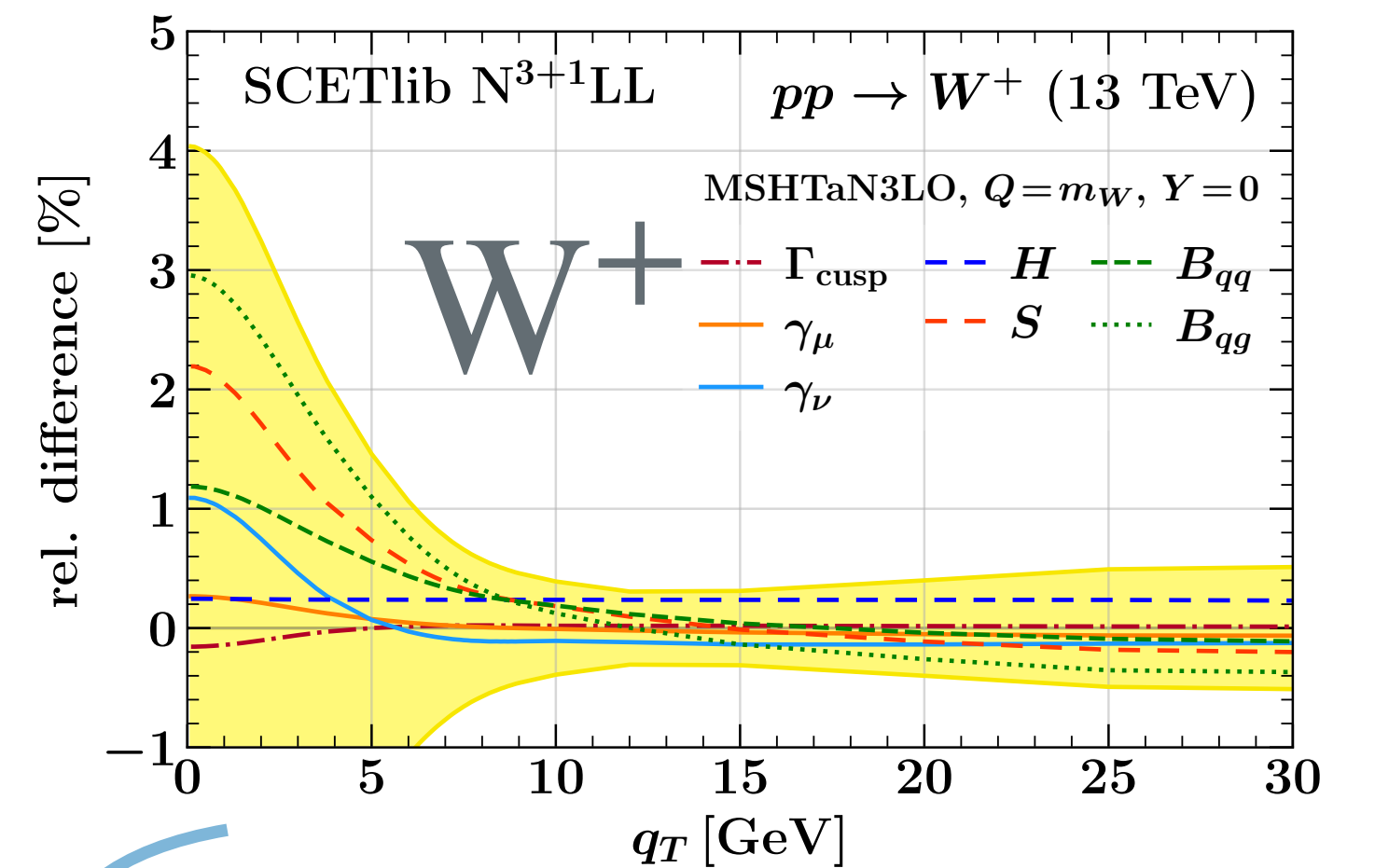
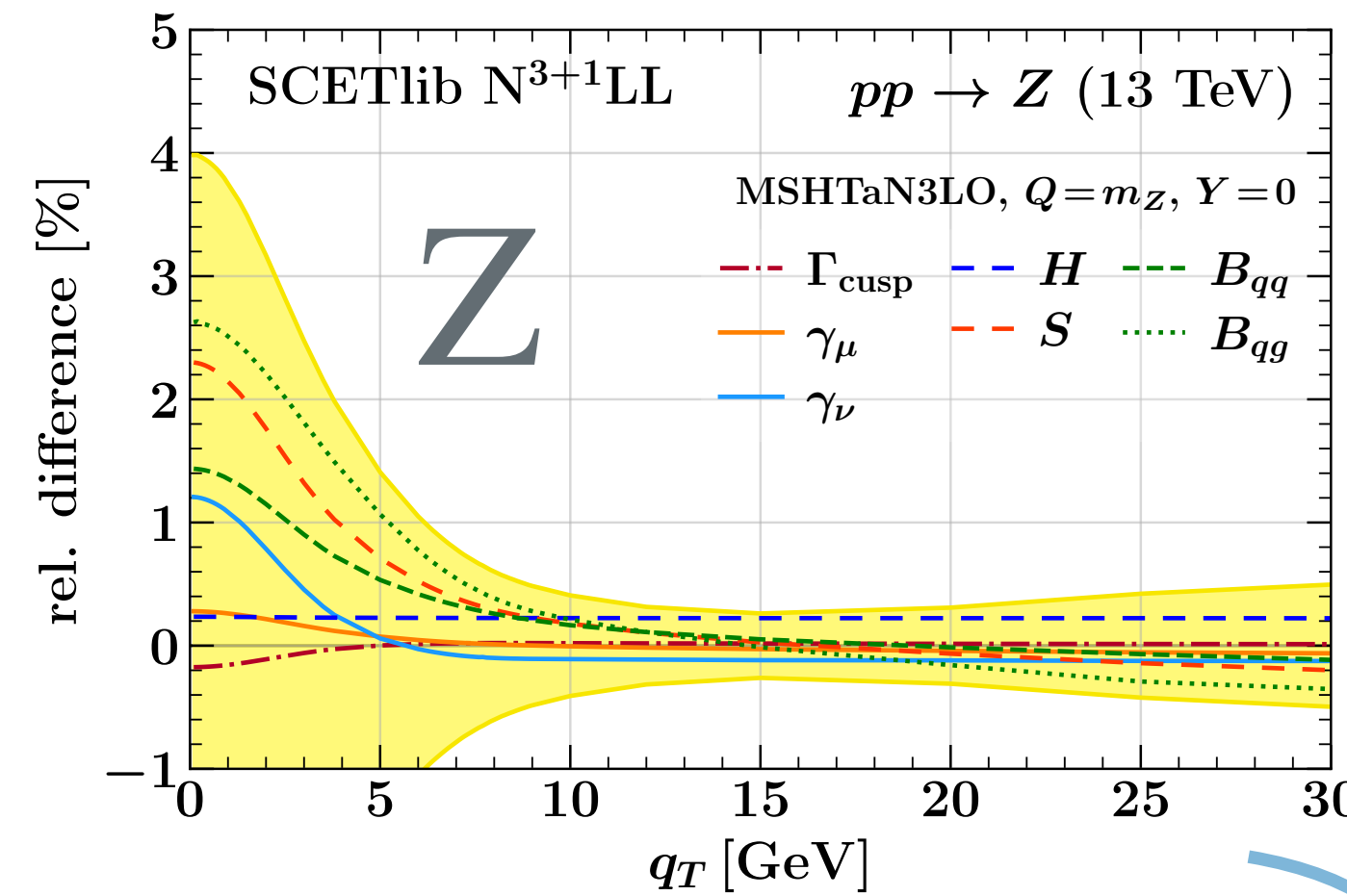
“massaging”

3 THEORY NUISANCE PARAMETERS

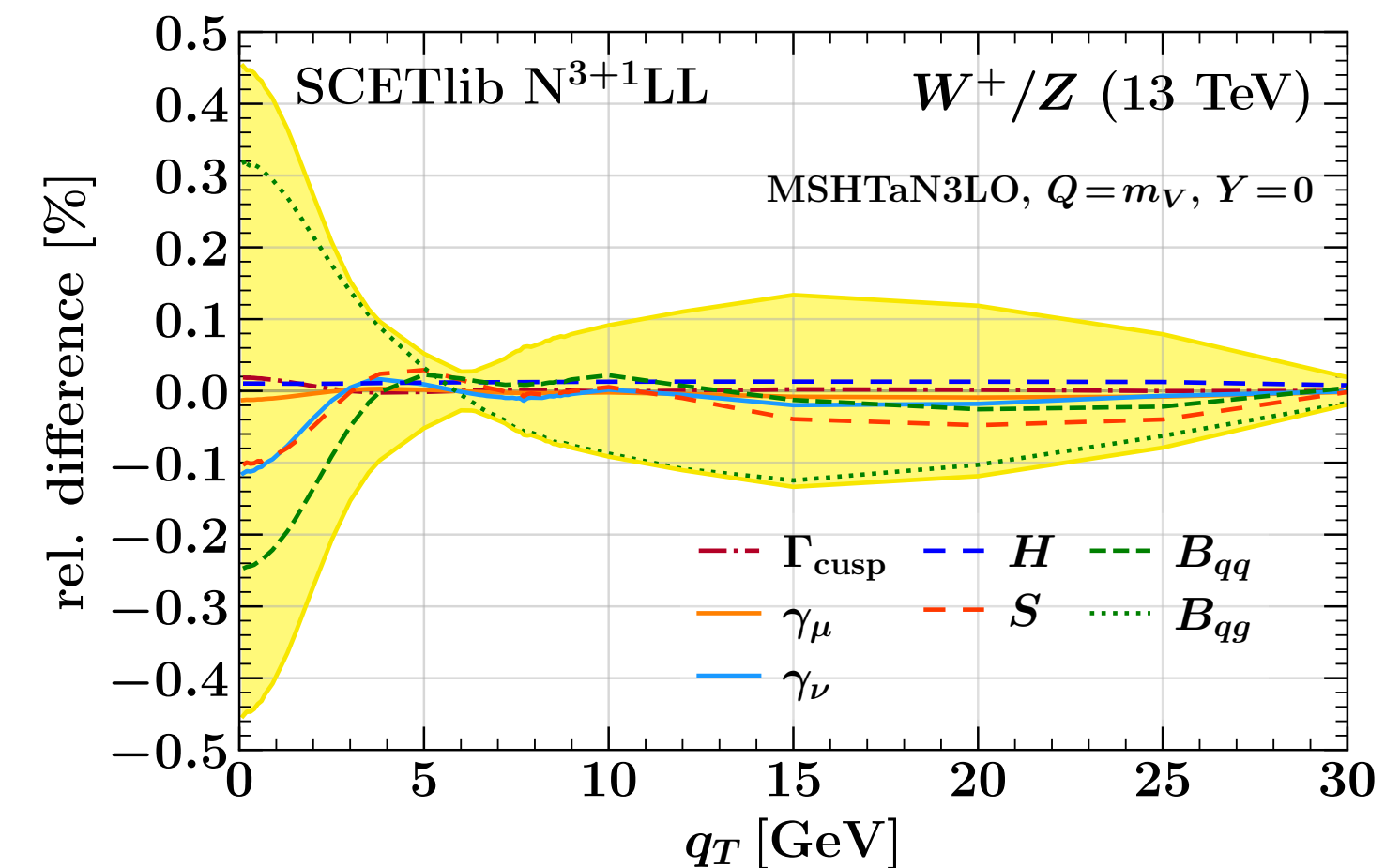
[Tackmann '24]

RESUMMED PREDICTION

- 2' 100% correlation of $\vec{\theta}$
 - ▶ large cancellation in W^+/Z
 - ▶ dominant residual uncertainties from B_{ab}
- similar absolute errors to Δ_{scl}
- valid at low p_T
 - ↪ requires matching @ high p_T (μ_R, μ_F)
- missing: non-pert. modelling



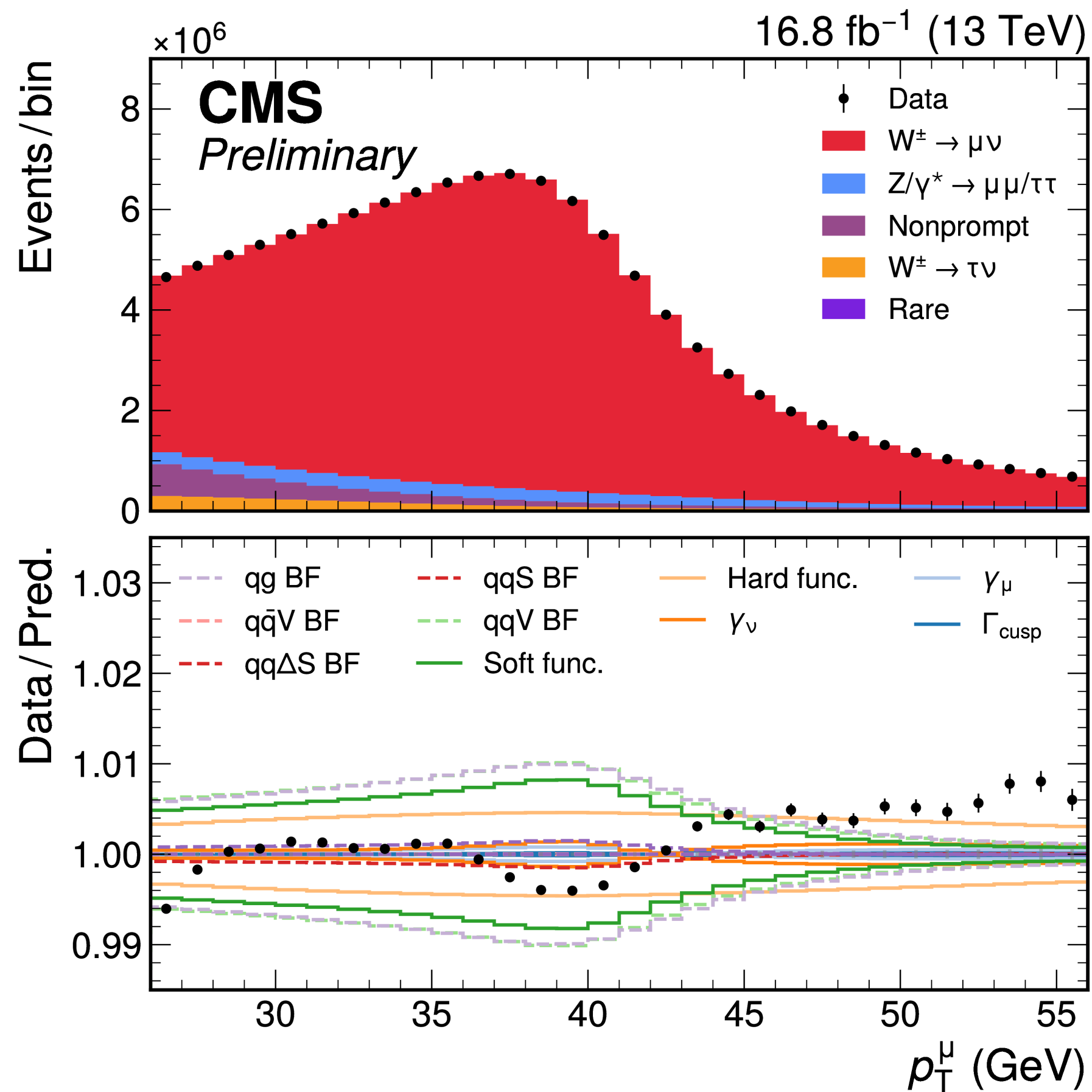
W^+/Z



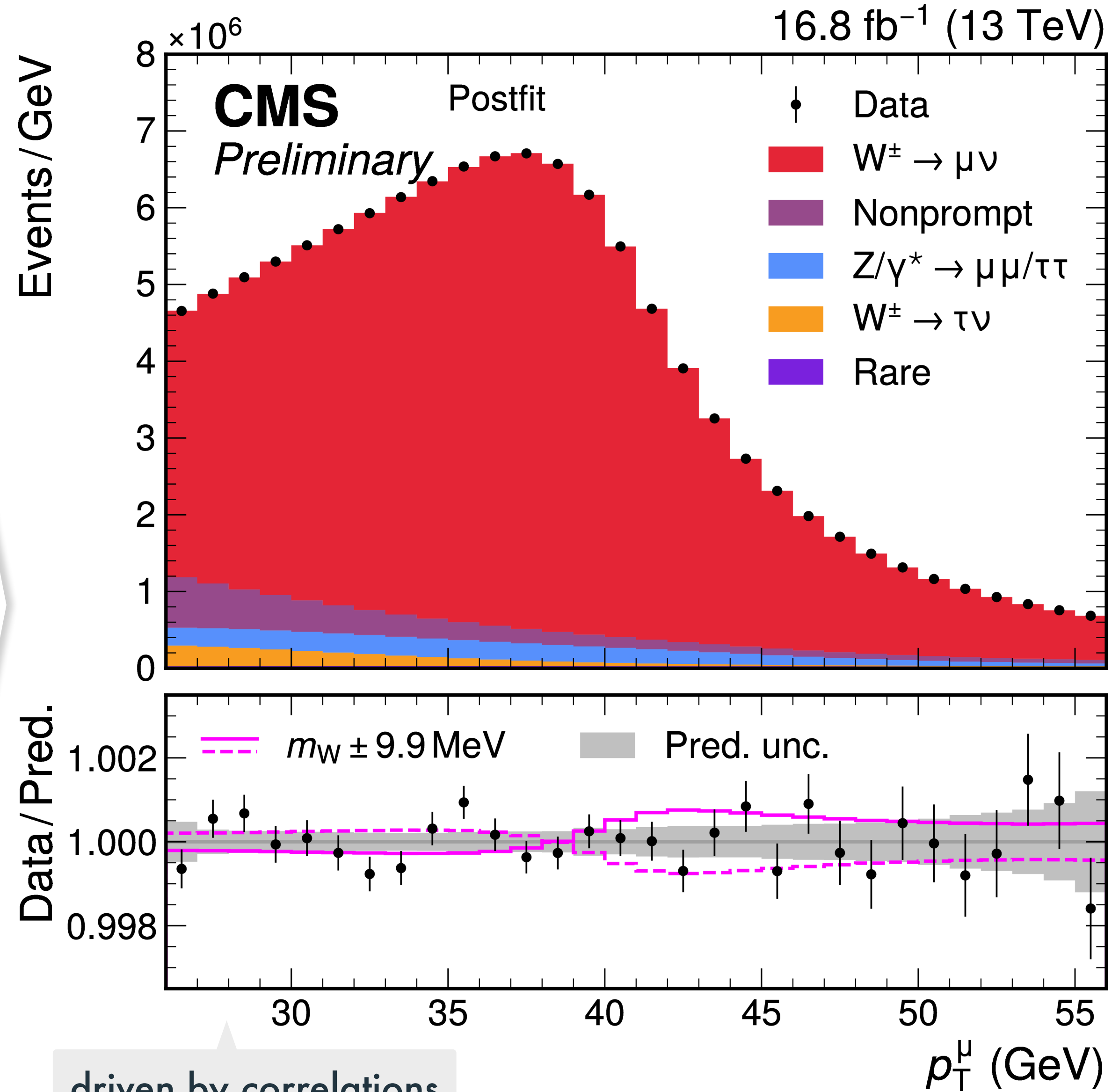
3 THEORY NUISANCE PARAMETERS

CMS M_W MEASUREMENT

3 constrain $\vec{\theta}$ using data



FIT



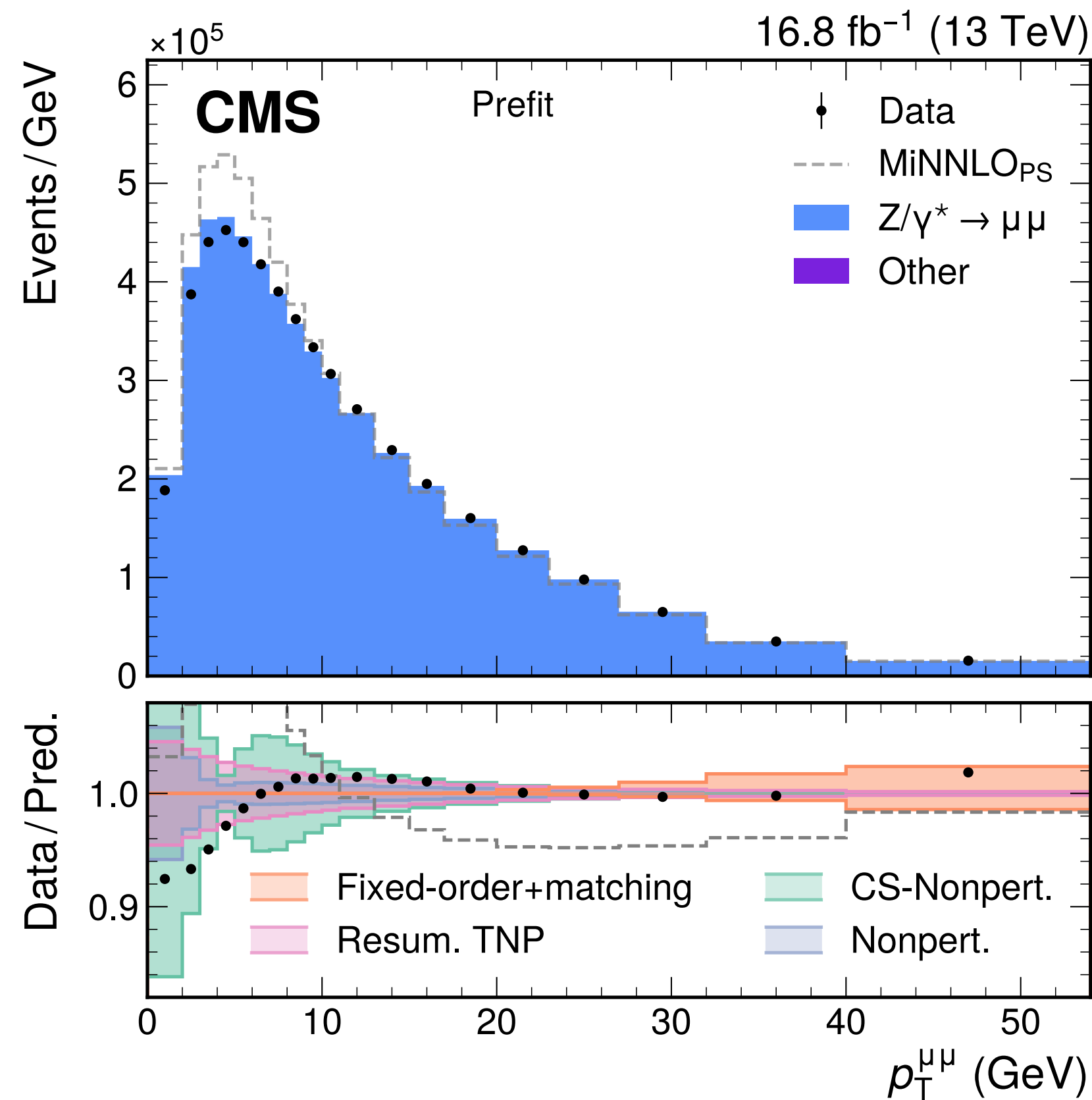
driven by correlations

[CMS arXiv:2412.13872 [hep-ex]]

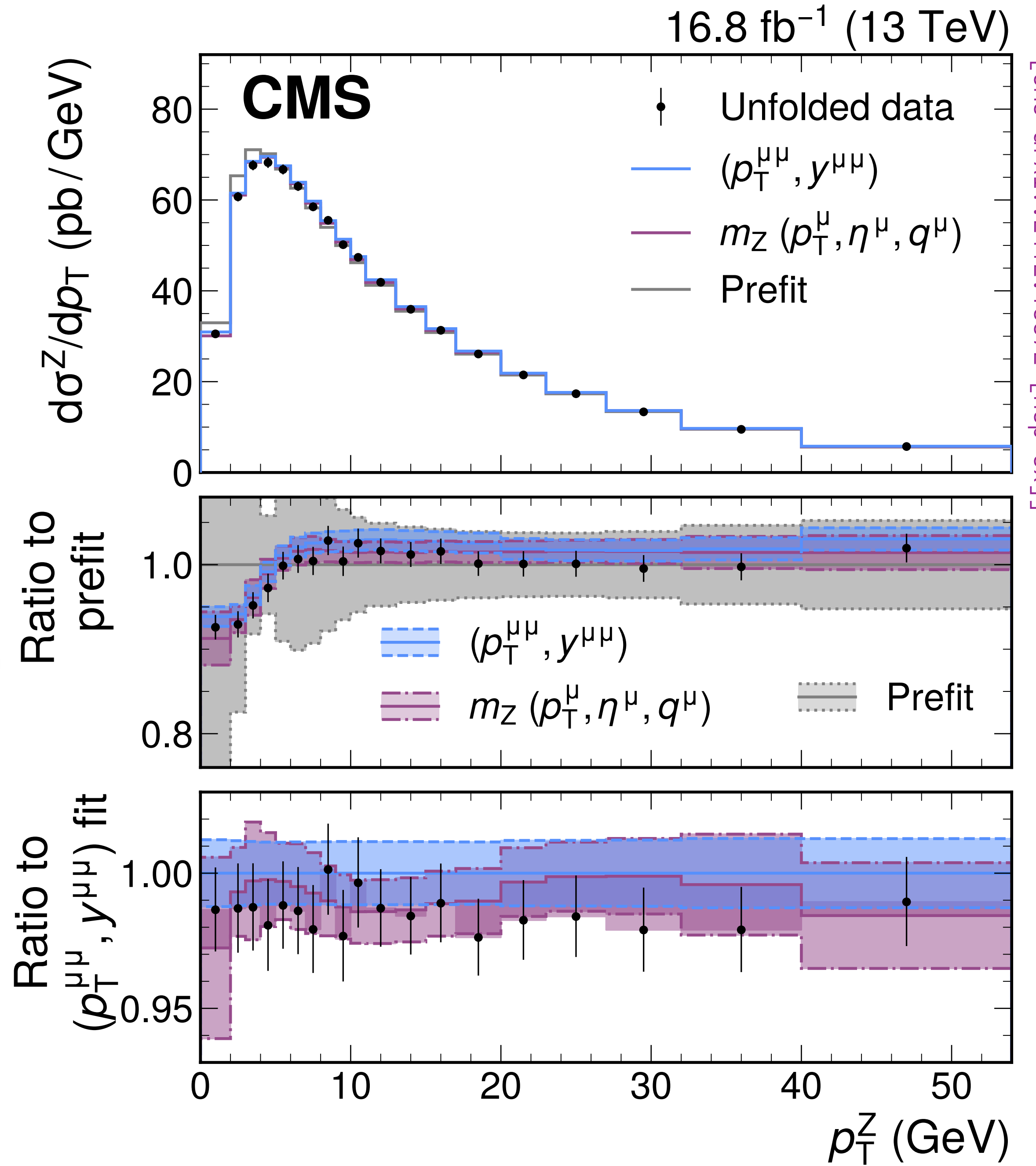
3 THEORY NUISANCE PARAMETERS

CMS M_W MEASUREMENT

3 constrain $\vec{\theta}$ using data



FIT



[CMS arXiv:2412.13872 [hep-ex]]

3 THEORY NUISANCE PARAMETERS

[McGowan, Cridge, Harland-Lang, Thorne '23]
[Lim, Poncelet '24]

BEYOND RESUMMED SPECTRA

- pull out the Born prediction and split off leading-colour factor at each order n

$$\frac{d\sigma}{dx} = \frac{d\sigma^{(0)}}{dx} \left[1 + \alpha_s N_c \bar{K}^{(1)}(x) + \alpha_s^2 N_c^2 \bar{K}^{(2)}(x) + \dots \right] \quad \bar{K}^{(n)}(x) \equiv \frac{1}{N_c^n} \frac{d\sigma^{(n)}/dx}{d\sigma^{(0)}/dx}$$

- parametrise first unknown order as superposition of lower-order $\bar{K}^{(n)}$

$$\bar{K}_{\text{TNP}}^{(n+1)}(\vec{\theta}; x) = \sum_{j=1}^n \underbrace{f_k^{(j)}(\vec{\theta}; x)}_{\text{polyn. modulation}} \bar{K}^{(j)}(x) \left\{ \begin{array}{l} f_k^{(j)}(\vec{\theta}; x) = \sum_{i=0}^k \theta_i^{(j)} \binom{k}{i} x^{k-i} (1-x)^i \quad (\text{Bernstein}) \\ f_k^{(j)}(\vec{\theta}; x) = \frac{1}{2} \sum_{i=0}^k \theta_i^{(j)} T_i(x) \quad (\text{Chebyshev}) \end{array} \right.$$

$P(\theta_i^{(j)})$: uniform in $[-1, +1]$, normal with $\sigma = 1$

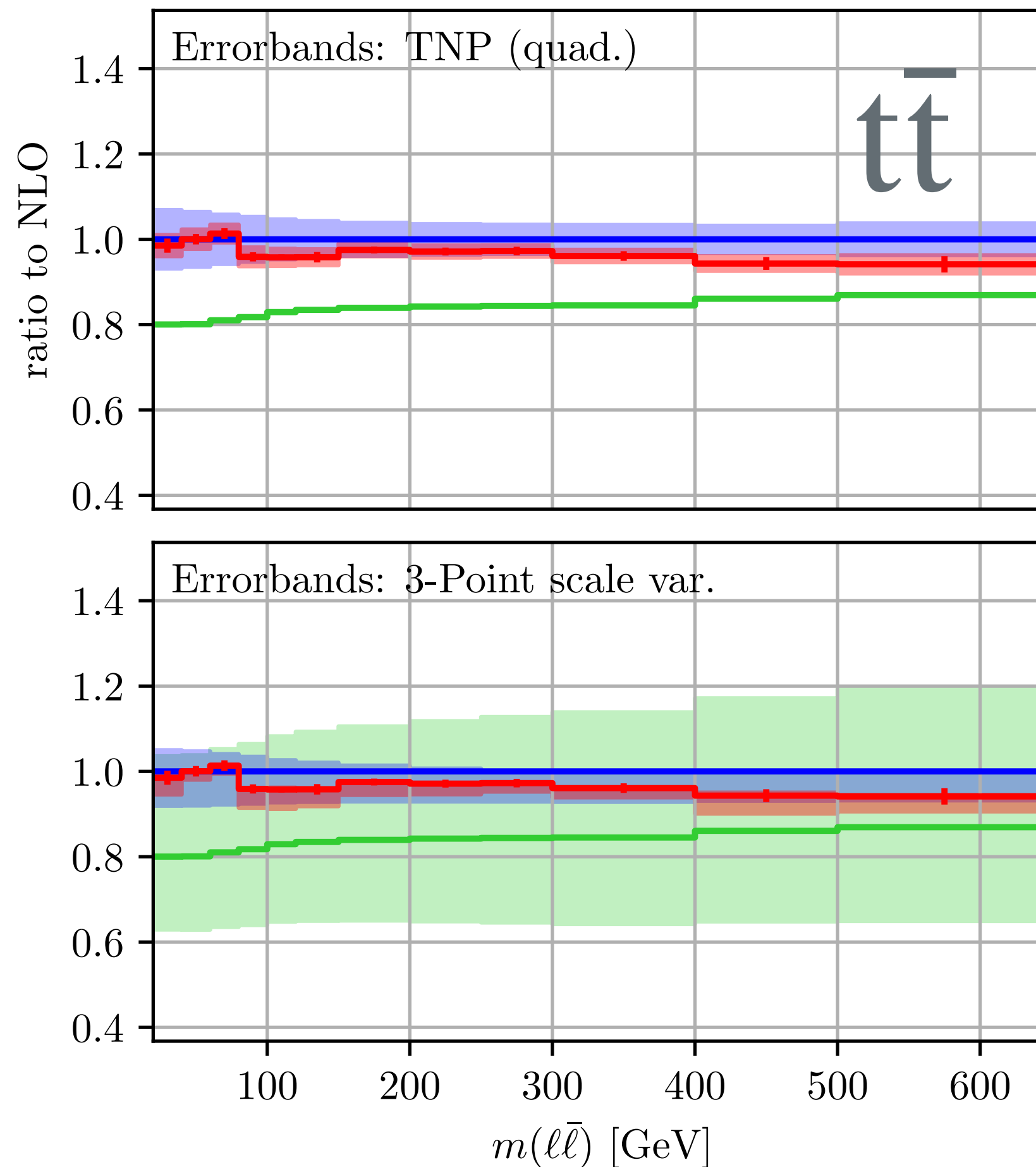
- $k = 0 \Rightarrow f_0^{(j)}(\vec{\theta}; x) \rightarrow \theta_0^{(j)} \Leftrightarrow$ approach to approx. N³LO in MSHT PDF fit

3 THEORY NUISANCE PARAMETERS

[Lim, Poncelet '24]

$pp \rightarrow t\bar{t} \rightarrow \ell\bar{\ell} 2b\text{-jets}$, LHC @ 13 TeV
 central scale: $\mu = H_T/4$
 Bernstein parameterisation (k=2)

TNP's:
 symmetric by
 construction



scale variation:
 3-point, *not* 7-point
 can be asymmetric

BEYOND RESUMMED SPECTRA

- LO: no uncertainty estimate
- NLO: effectively a $\sim \alpha_s N_c \mathcal{O}(1)$ modulation of NLO correction
- NNLO: first time an interplay between two independent $\bar{K}^{(1,2)}$

3 THEORY NUISANCE PARAMETERS

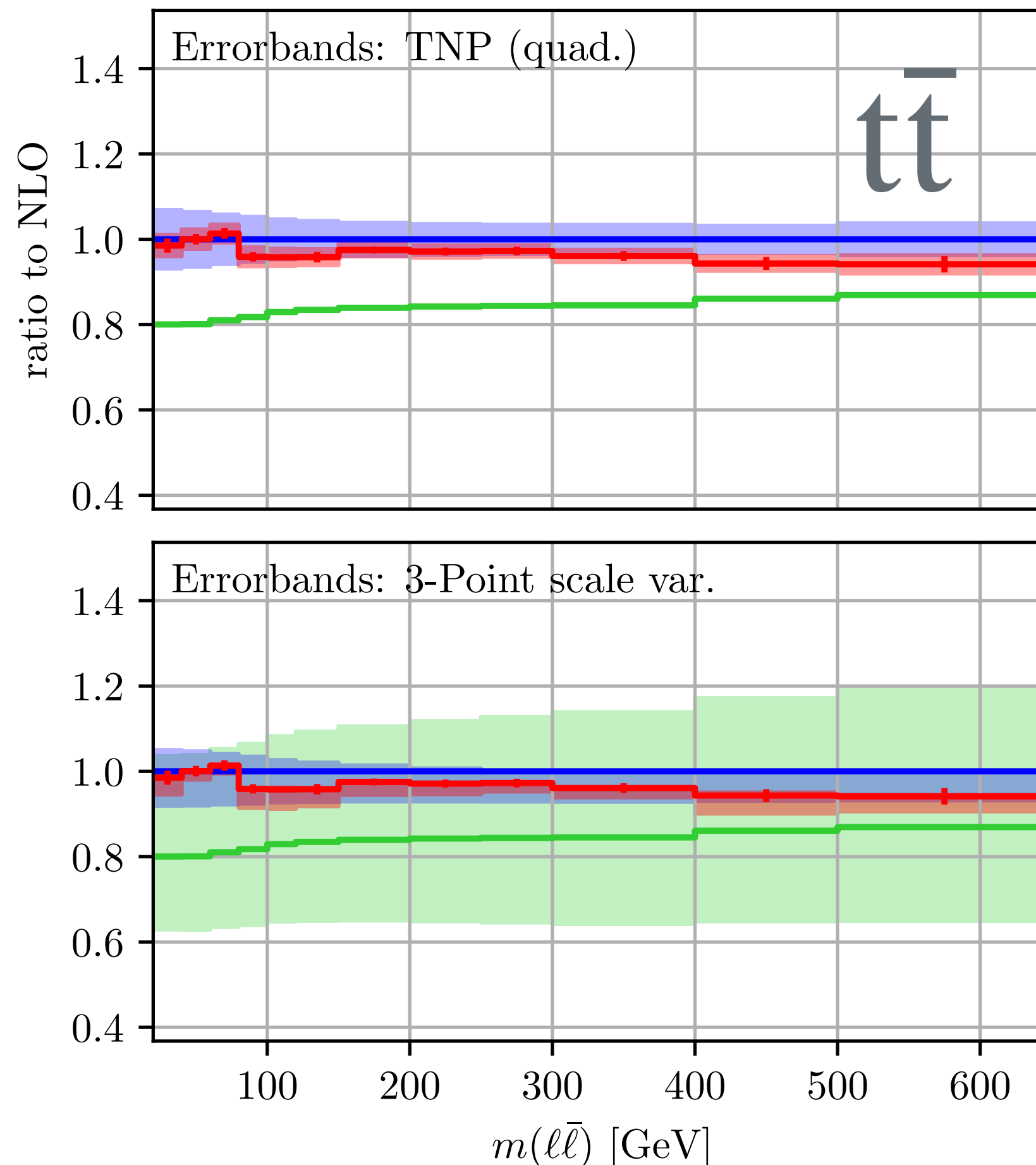
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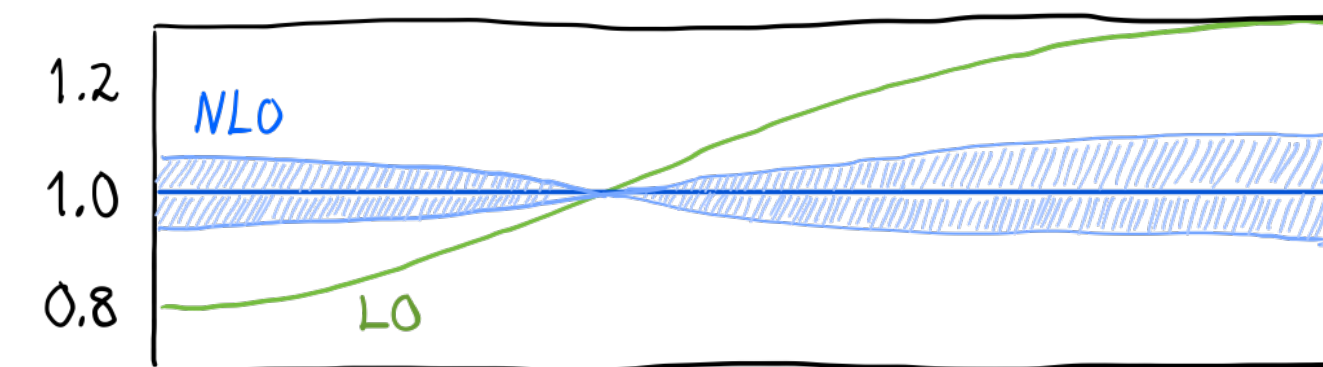
- ▶ how to map x ?
- ▶ multi-dim?
- ▶ ...

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- NLO can have artefacts?



LO–NLO cross
 $\Rightarrow \Delta_{\text{NLO}} = 0$?!

- similar errors @ NNLO

3 THEORY NUISANCE PARAMETERS

[Lim, Poncelet '24]

$pp \rightarrow e^+ \bar{\nu}_e \mu^- \bar{\nu}_\mu$ LHC @ 13 TeV eV

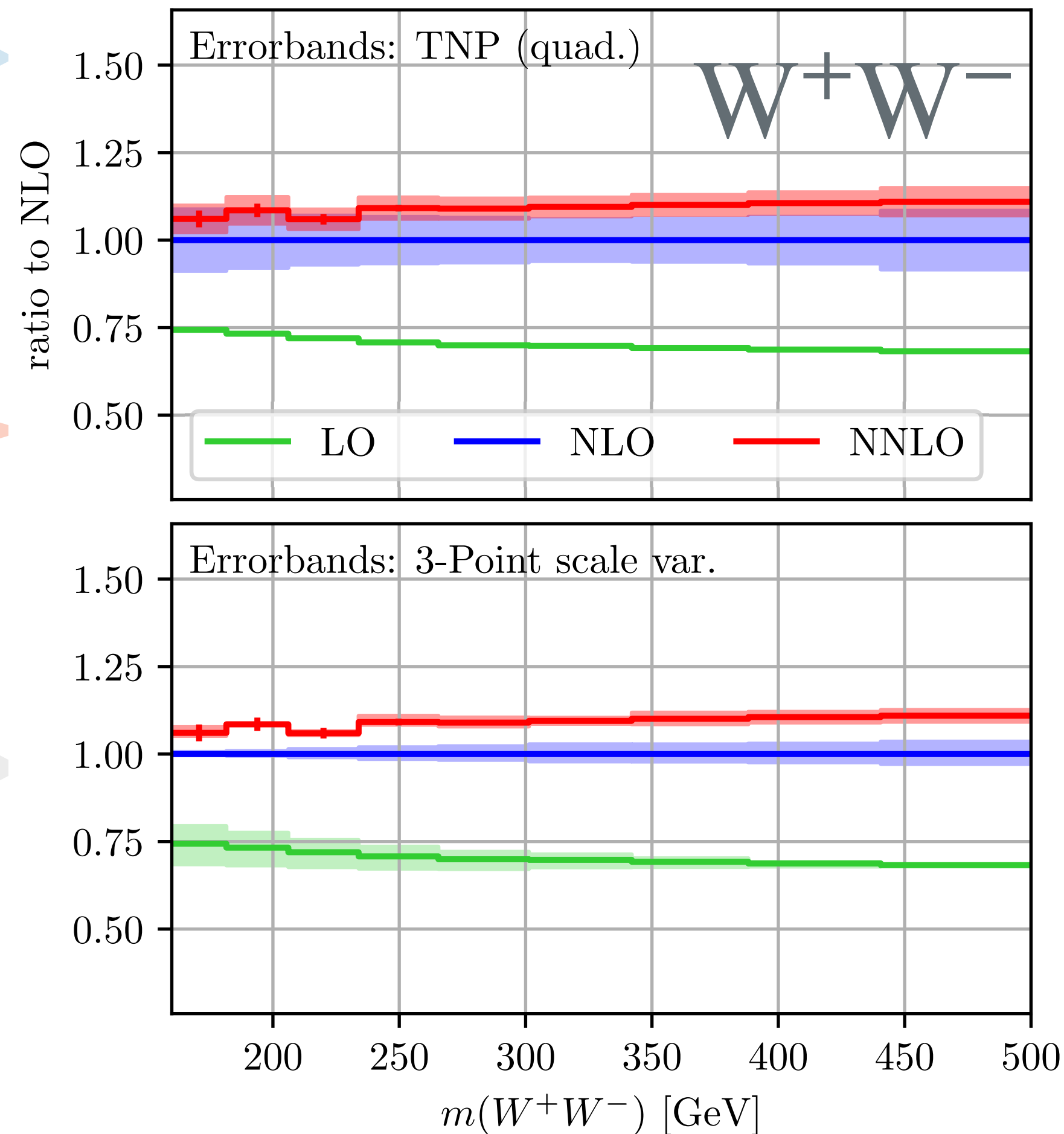
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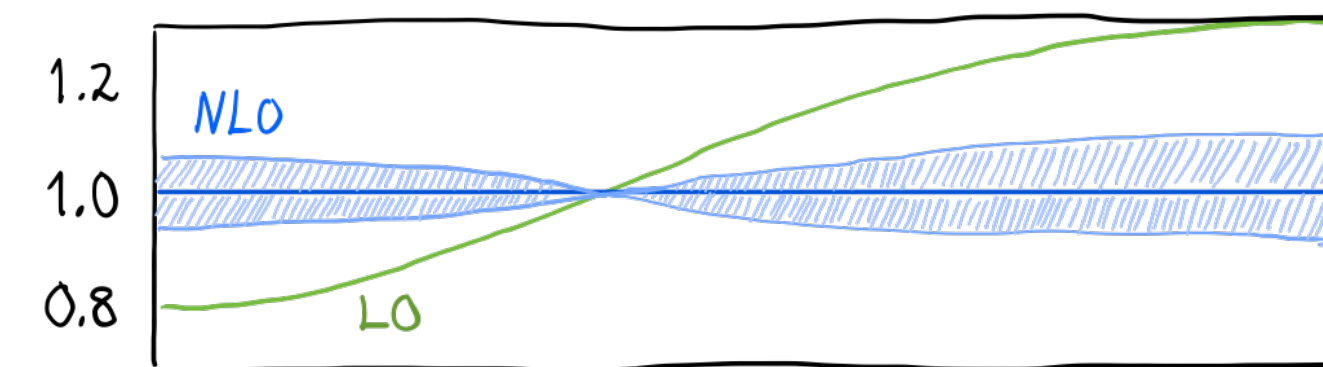
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- more robust? gg channel?

4 PARTON DISTRIBUTION FUNCTIONS

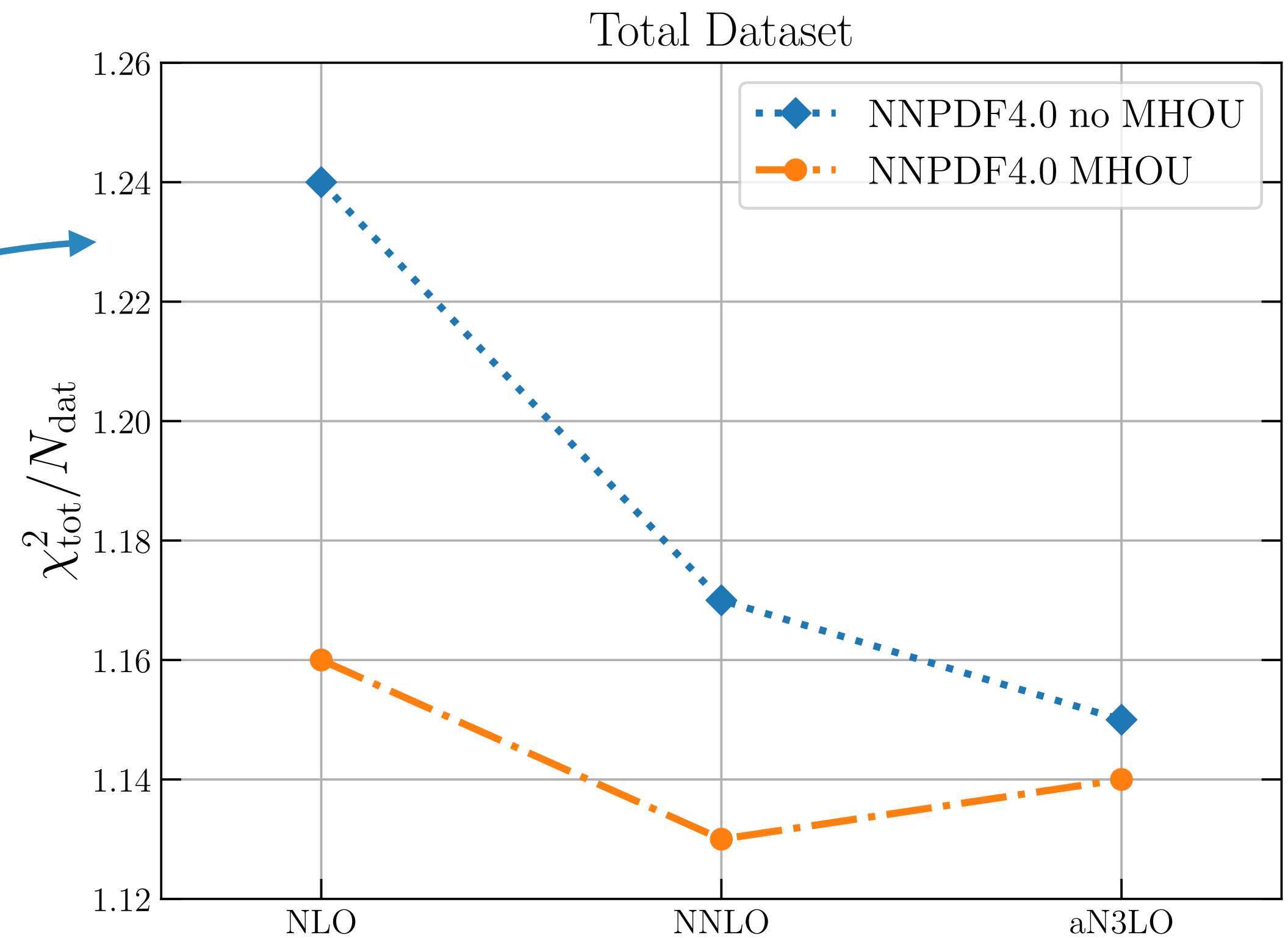
DATASET & TOLERANCES

- $\mathcal{O}(4000)$ datapoints in a global fits
 - ↳ inconsistencies between data, unknown/underestimated EXP/TH uncertainties, model, ...
- $\Delta\chi^2 = 1$ for 68% C.L. not suited
 - ↳ “tolerance” $T^2 = 10-30$

THEORY UNCERTAINTIES IN PDFs

- NNPDF4+ (scale variation)
- MSHT aN³LO (nuisance parameter)
- mandatory in aN³LO as (almost) no predictions available at this order

more stable χ^2 in the progression of the orders



[NNPDF 2402.18635]

4 PARTON DISTRIBUTION FUNCTIONS

ggH @ N³LO

- estimate ~ 1% error $\left(\Delta_{\text{NNLO}}^{\text{app}} \equiv \frac{1}{2} \left| \frac{\sigma_{\text{NNLO-PDF}}^{\text{NNLO}} - \sigma_{\text{NLO-PDF}}^{\text{NNLO}}}{\sigma_{\text{NNLO-PDF}}^{\text{NNLO}}} \right| \right)$
- ↪ actual shift much larger:

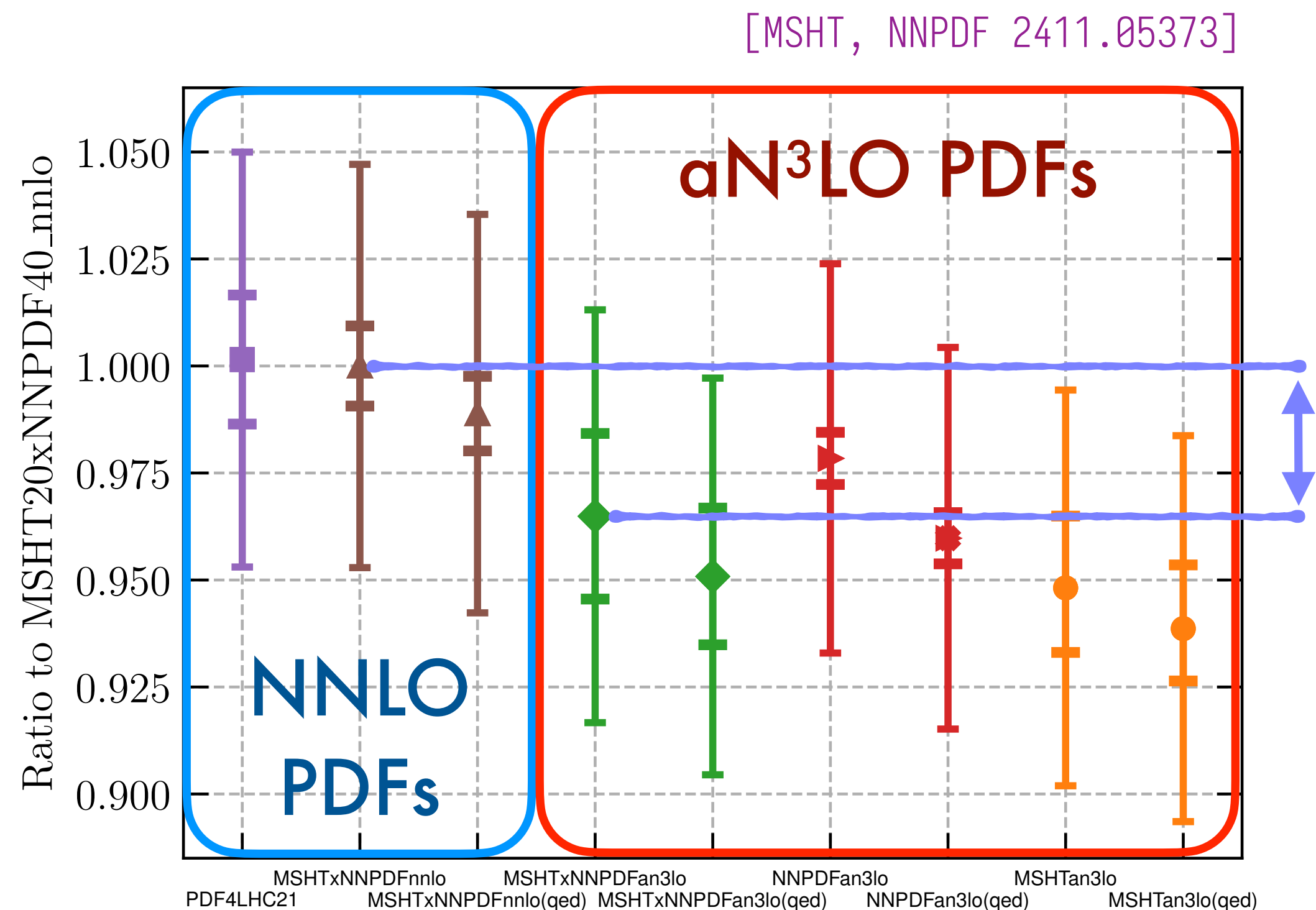
$$\sigma_{\text{ggH}}^{\text{N}^3\text{LO}}(\text{NNLO PDF})$$

$$= 48.3 \text{ pb}$$

3.8%

$$\sigma_{\text{ggH}}^{\text{N}^3\text{LO}}(\text{aN}^3\text{LO PDF})$$

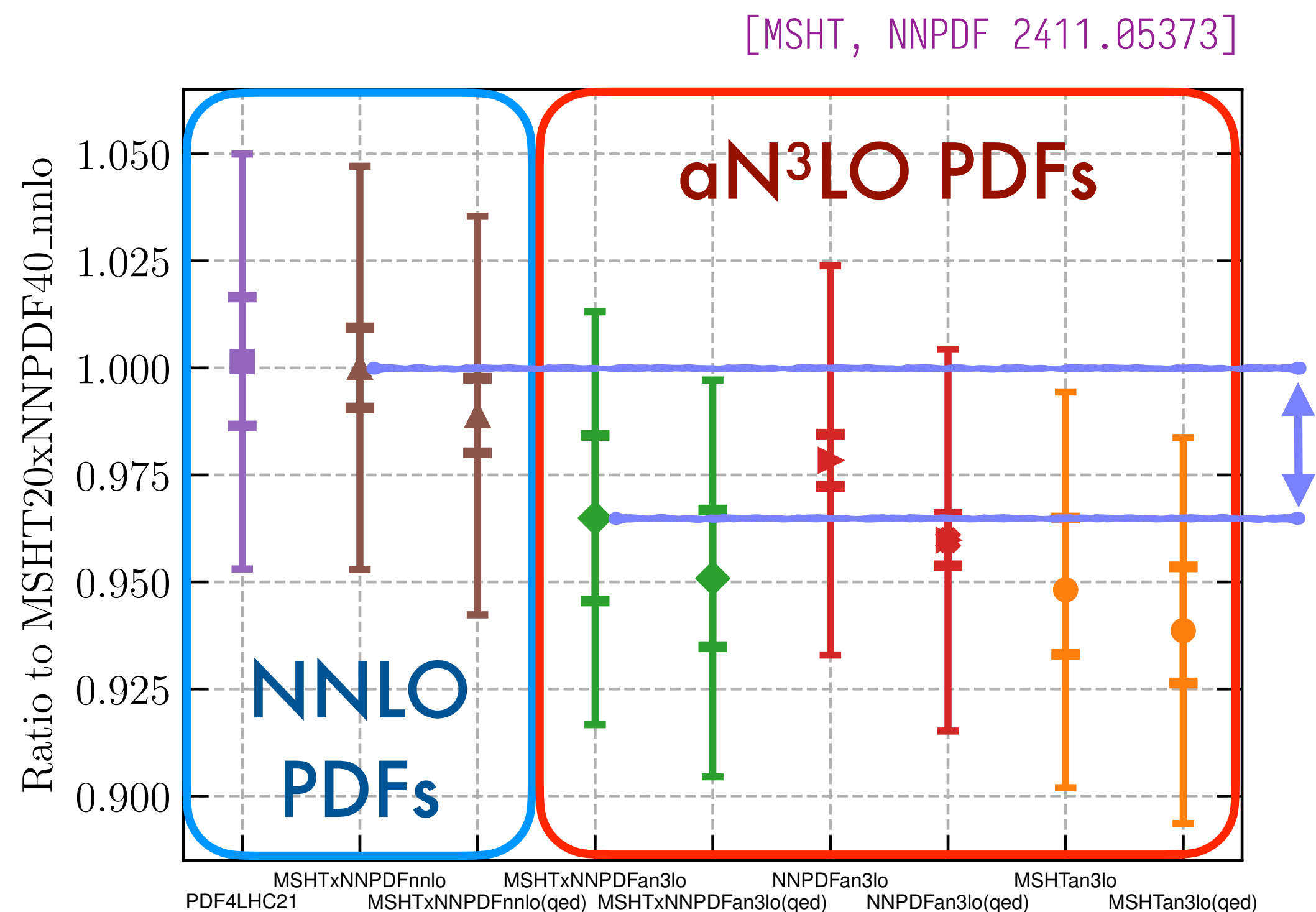
$$= 46.5 \text{ pb}$$



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3.8%

$$\sigma_{\text{ggH}}^{\text{NNLO}}(\text{NNLO PDF})$$

$$= 46.7 \text{ pb}$$

$$\sigma_{\text{ggH}}^{\text{N}^3\text{LO}}(\text{aN}^3\text{LO PDF})$$

$$= 46.5 \text{ pb}$$

0.4%

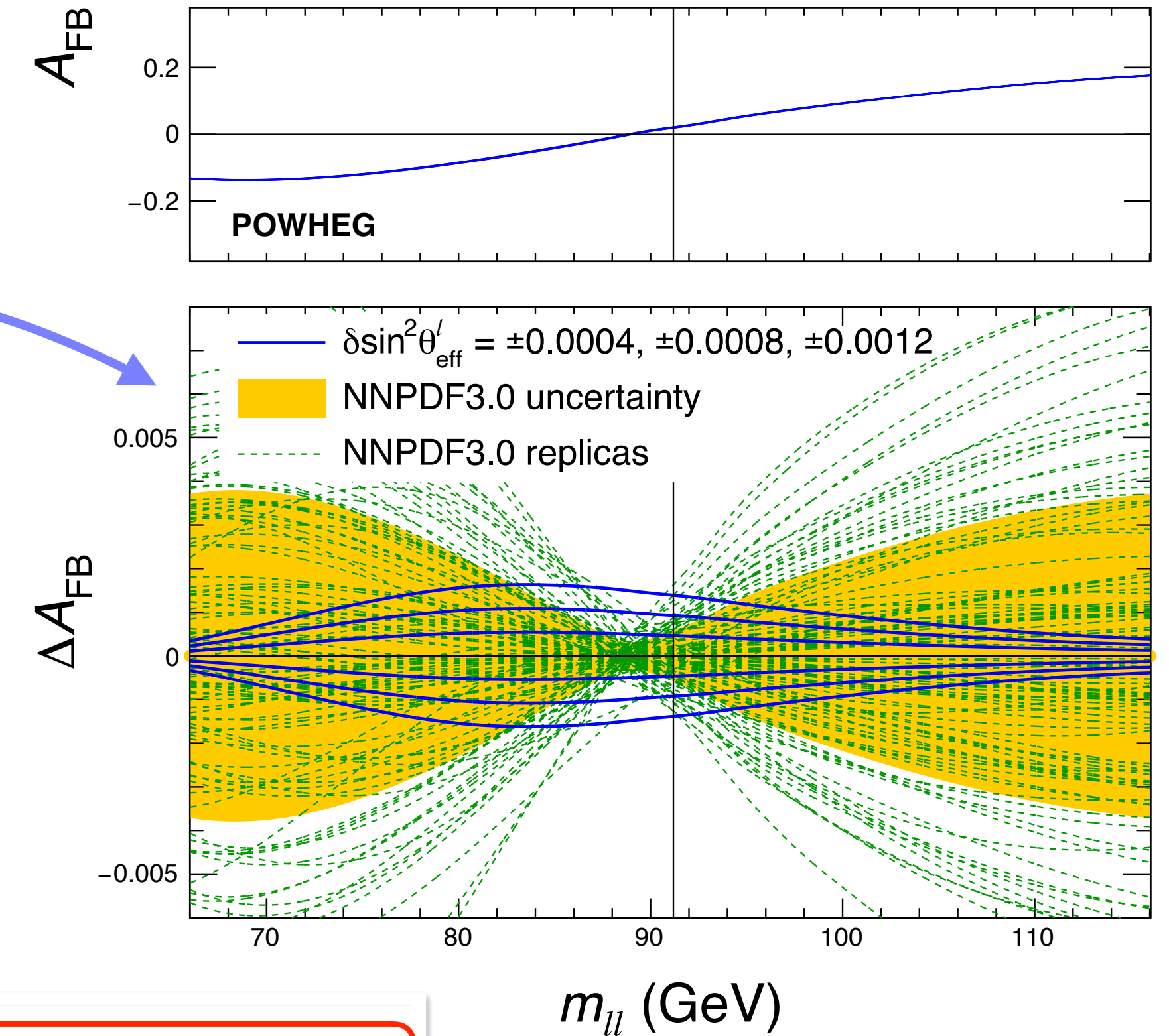
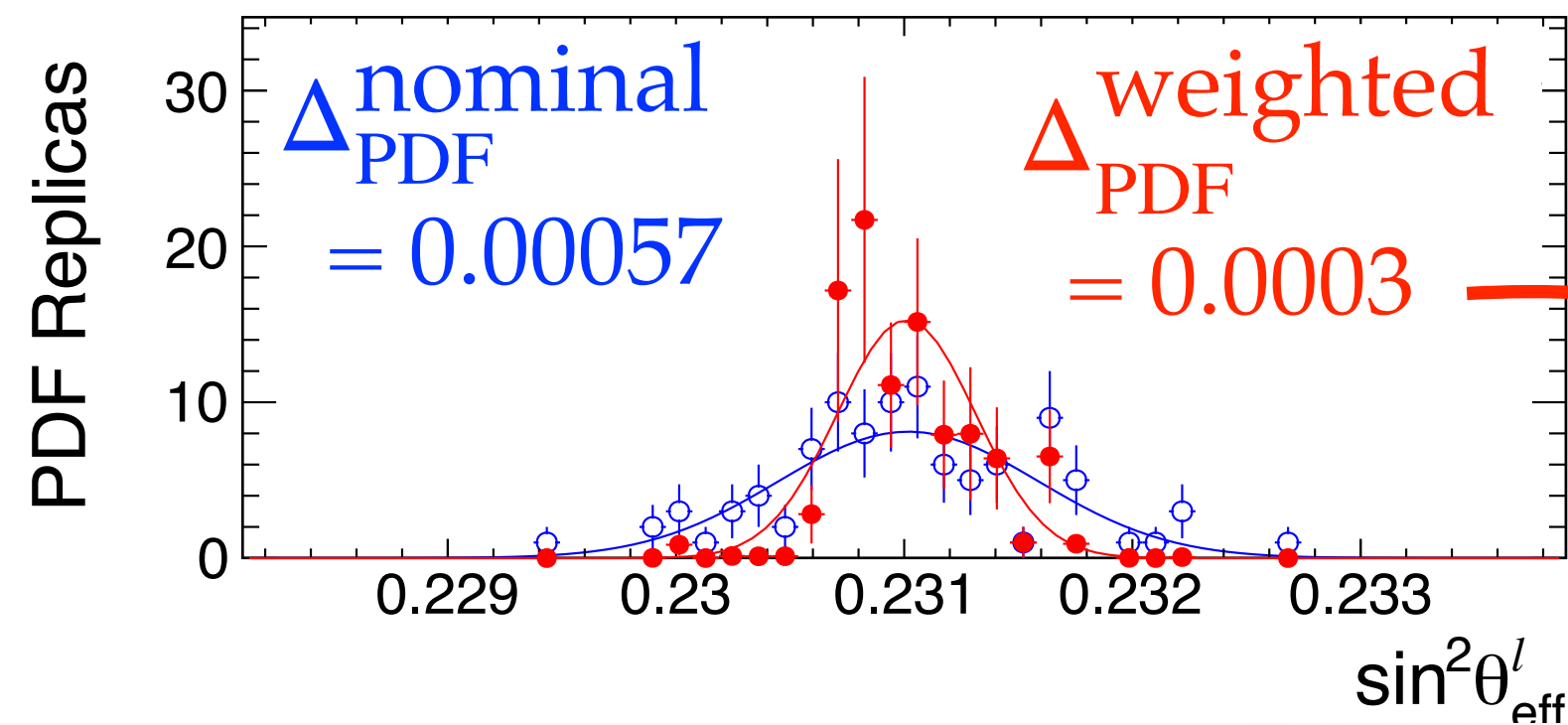
did we just see some artefacts from the mismatch between $\hat{\sigma}$ & PDFs? (uncompensated logs, ...)

4 PARTON DISTRIBUTION FUNCTIONS

[CMS 1806.00863]

PROFILING / REWEIGHTING

- PDF uncert. often among dominant sources
 - ⇒ exploit correlations to reduce impact
 - ⇒ often a huge reduction ($\sim 2\times$) in Δ_{PDF} !



$$\sin^2 \theta_{\text{eff}}^l = 0.23101 \pm 0.00036 \text{ (stat)} \pm 0.00018 \text{ (syst)} \pm 0.00016 \text{ (theo)} \pm 0.00031 \text{ (PDF)}$$

how sensible is that $\mathcal{O}(100)$ datapoints have such a big impact?
 ⇔ this data effectively carries a very high weight!

still almost 50% of Δ_{tot}

4 PARTON DISTRIBUTION FUNCTIONS

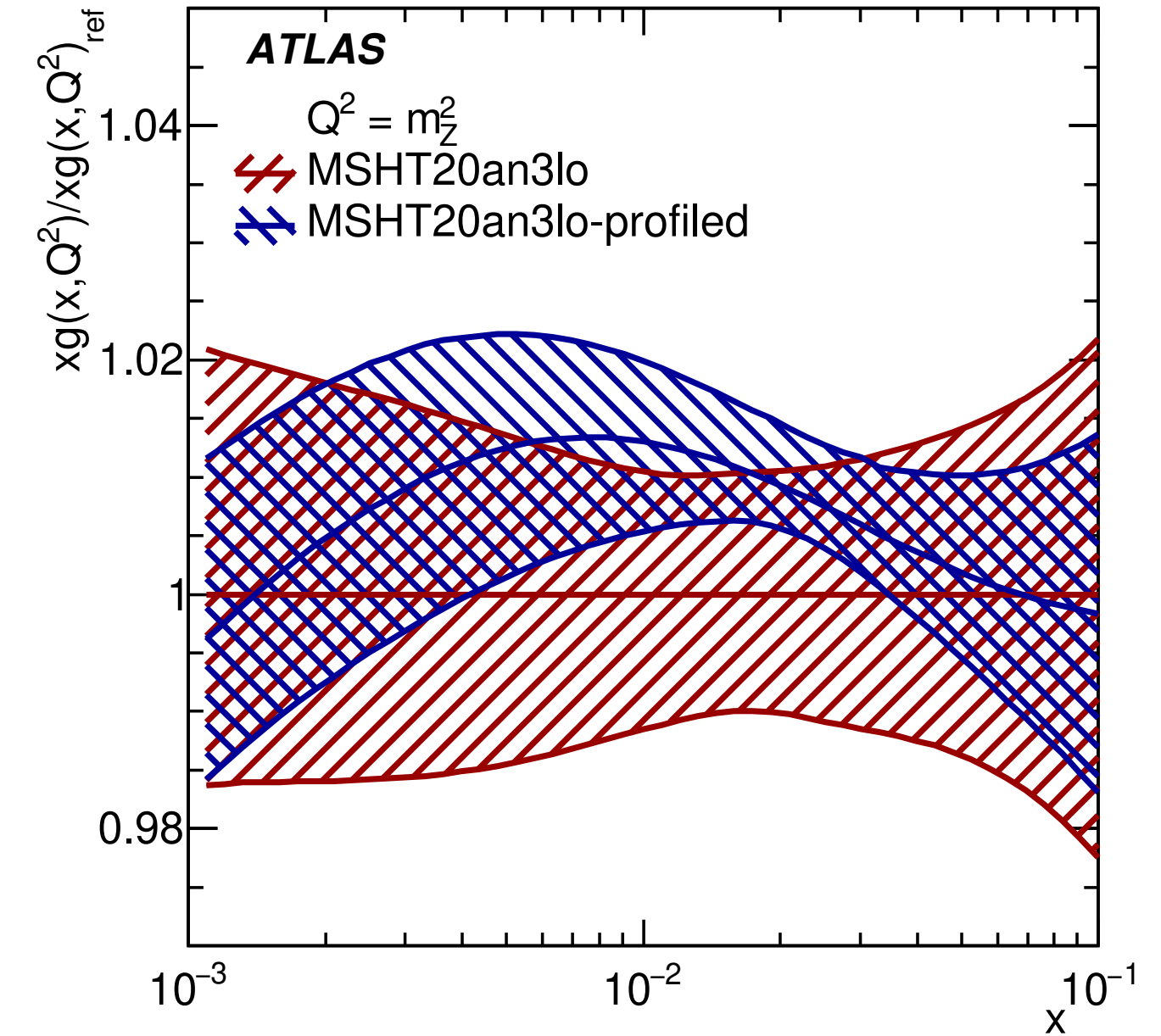
$$\alpha_s(m_Z) = 0.1183 \pm 0.0009$$

[ATLAS 2309.12986]

PROFILING / REWEIGHTING FOR α_s

- situation is even more delicate for α_s
 \leftrightarrow strong correlations between α_s & PDFs (g)
- quoted $\Delta_{\text{PDF}} = \pm 0.00051$

profiling may bias probing direction of factorised approx.

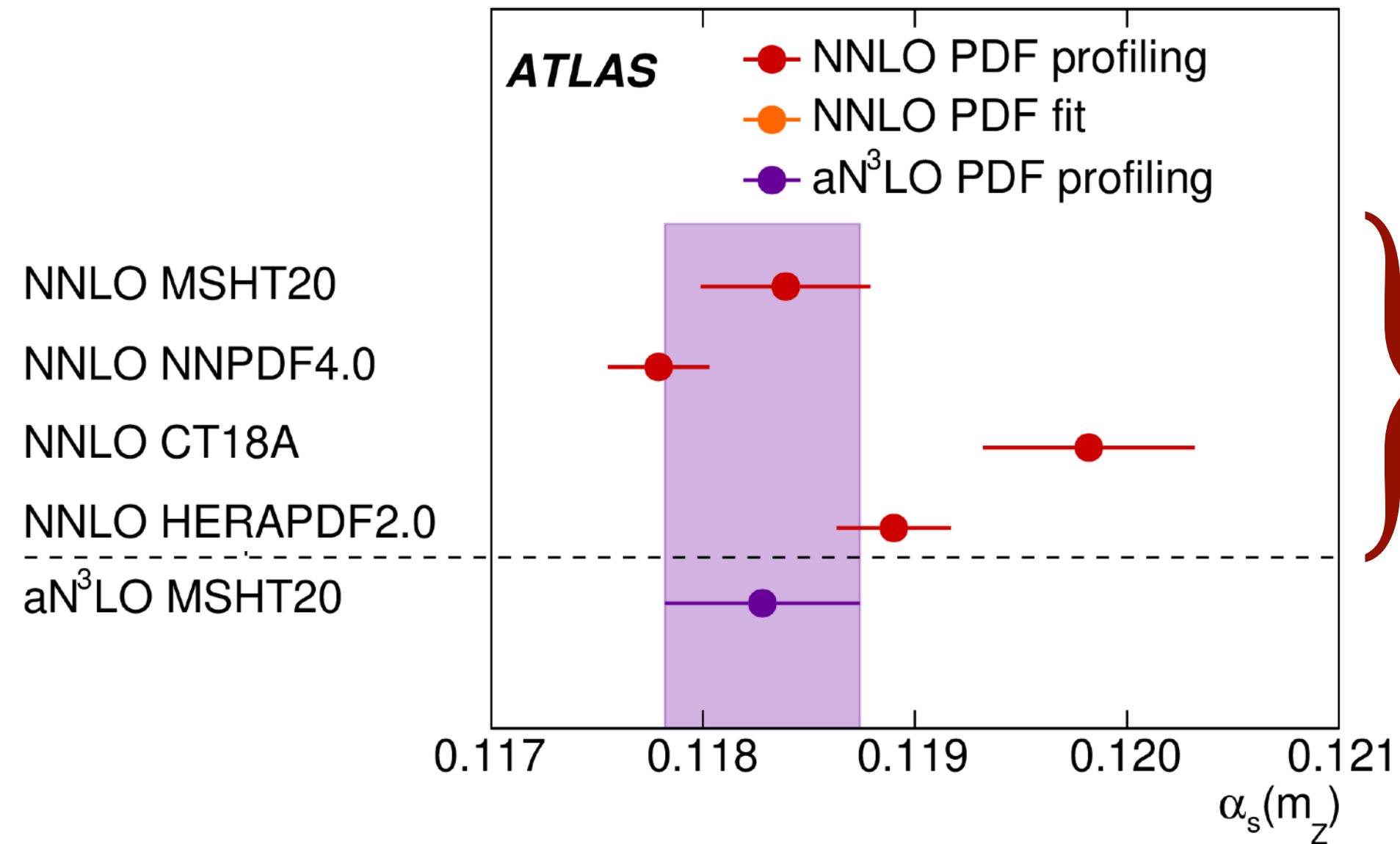


back-reaction to other measurements in the fit?

4x

spread in NNLO PDFs $\sim 0.00203 \simeq 2\Delta_{\text{tot}}$

underestimated unc. / unaccounted Δ_{model}



CONCLUSIONS & OUTLOOK

- Covered two dominant TH systematics in precision measurements
 - ↔ **QCD modelling** & **PDF uncertainties**.
 - no time to cover: EW corrections, PS, non-pert. uncertainties, ...
- Situation: $\Delta_{\text{EXP}} \ll \Delta_{\text{TH}}$ puts analyses in the position to constrain Δ_{TH} from data
 - ↪ prob. interpretation & correlation model crucial in this endeavour
 - **Bayesian estimates**: prob. interpretation (can also implement correlations)
 - **Theory nuisance parameters**: “physical” correlation model $\oplus P(\theta)$
- \nexists a “correct” $\Delta_{\text{pert.}}$ model (each approach relies on set of assumptions)
- **PDF reweighting / profiling** by now standard practice
 - can reduce nominal Δ_{PDF} by factors of 2–4
 - impact on global PDF fit picture needs to be better understood & potential missing model / methodology uncertainties?

relying on a single prescription is potentially dangerous!

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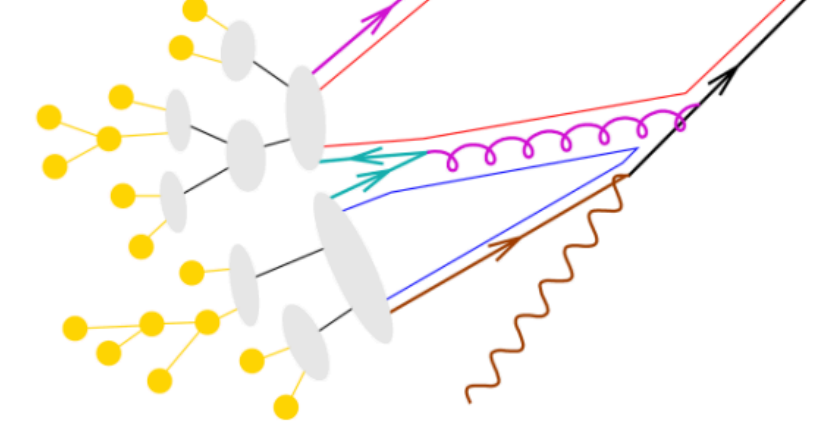
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Thank you!

BACKUP

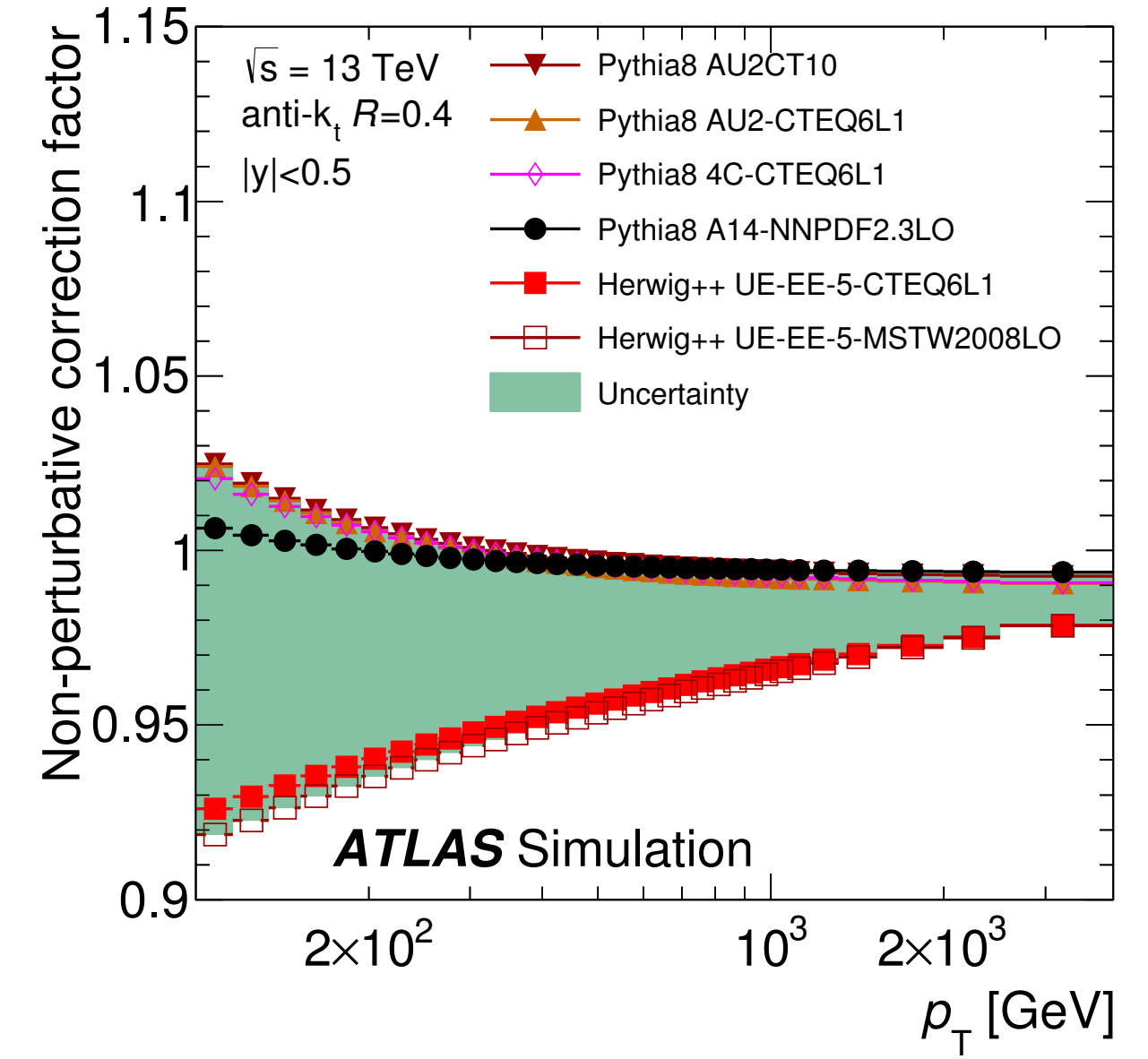
BACKUP

X PARTON SHOWERS

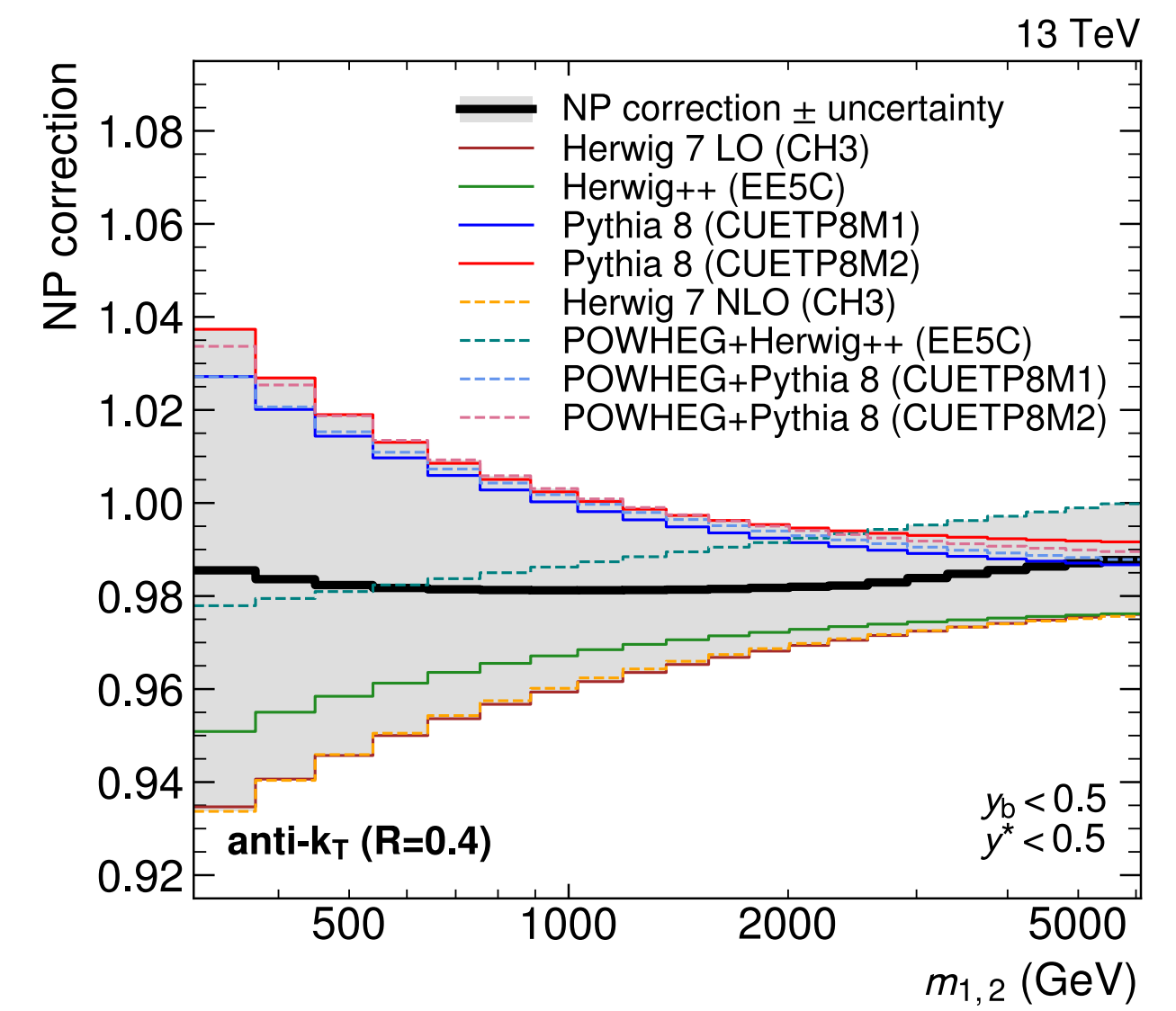


PERTURBATIVE ↔ NON-PERTURBATIVE

- Parton showers (PS) model all-order QCD emissions through the evolution from hard to soft scales
- no systematic approach (afaik) to assess *perturbative* uncertainties in PS
- tuning** of PS further entangles the pert. part with the non-pert. (NP) model
 - ↳ latter very sizeable
 - ↳ difficult to assess due to the notorious “Herwig vs. Pythia” effective two-point systematics



[ATLAS 1711.02692]

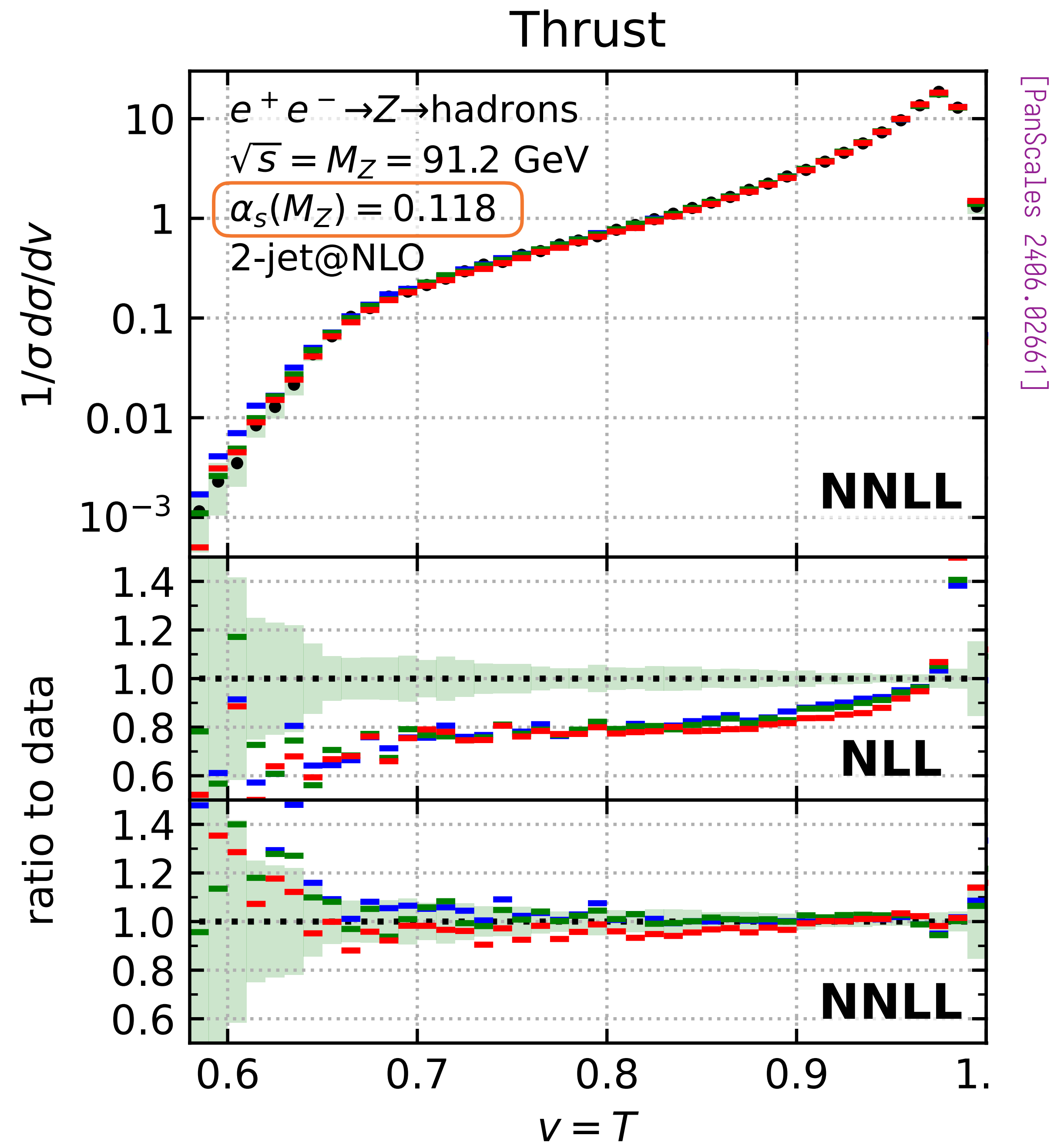


[CMS 2312.16669]

X PARTON SHOWERS

FORMAL ACCURACY

- tremendous progress in improving the formal accuracy of PS to
 - ↳ NLL [Alaric, Deductor, Herwig, Apollo, FHP, Panscales]
 - ↳ NNLL [PanScales 2406.02661]
- addresses long-standing issue with dipole showers that required an anomalously large $\alpha_s(M_Z) \gtrsim 0.130$
- NP effectively absorbing pert. effects?
 - ⇒ can formally accurate PS lead to better alignment in NP modelling?



WORK IN PROGRESS — CORRELATIONS

[AH, Mazeliauskas w.i.p]

- idea: if two bins show similar (opposite) perturbative behaviour
↪ two bins should be partially (anti-)correlated.

- we want: joint probability distribution $P(x, y)$ for two bins x & y
↪ preserve projections for compatibility:

$$P(x) = \int dy P(x, y) = \int dz P(x, z)$$

- ↪ hidden parameter $-1 < c < +1$ to smoothly implement the correlation
- possibilities: algorithmic “earth movers distance”; map $P(x)$ onto $P(y)$, ...
↪ can be done much simpler

WORK IN PROGRESS — CORRELATION MODEL

[AH, Mazeliauskas w.i.p]

- projections of multi-dim. Gaussians (+ correlation matrix) are again Gaussian
 \hookrightarrow map P_i onto Gaussians, implement correlations, map back

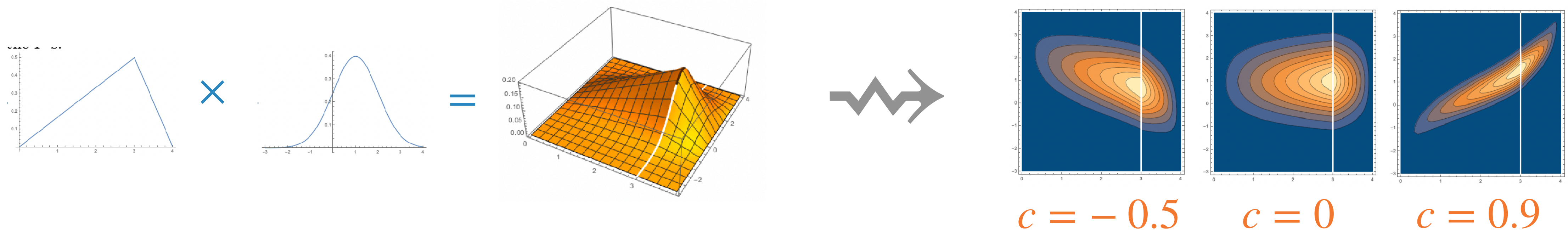
$$\begin{aligned}
 P(x, y) &= P_1(x)P_2(y) \\
 &\times \left. \frac{d\Phi^{-1}(\alpha)}{d\alpha} \right|_{\alpha=\Sigma_1(x)} \left. \frac{d\Phi^{-1}(\beta)}{d\beta} \right|_{\beta=\Sigma_2(y)} \\
 &\times \frac{1}{2\pi\sqrt{1-c^2}} \exp\left(-\frac{1}{2(1-c^2)} [\xi(x)^2 + \eta(y)^2 - c2\xi(x)\eta(y)]\right)
 \end{aligned}$$

$$\Sigma_i(x) = \int_{-\infty}^x dx' P_i(x')$$

$$\Phi^{-1}(p) = \sqrt{2}\text{Erf}^{-1}(-1 + 2p)$$

$$\xi(x) = \Phi^{-1}(\Sigma_1(x))$$

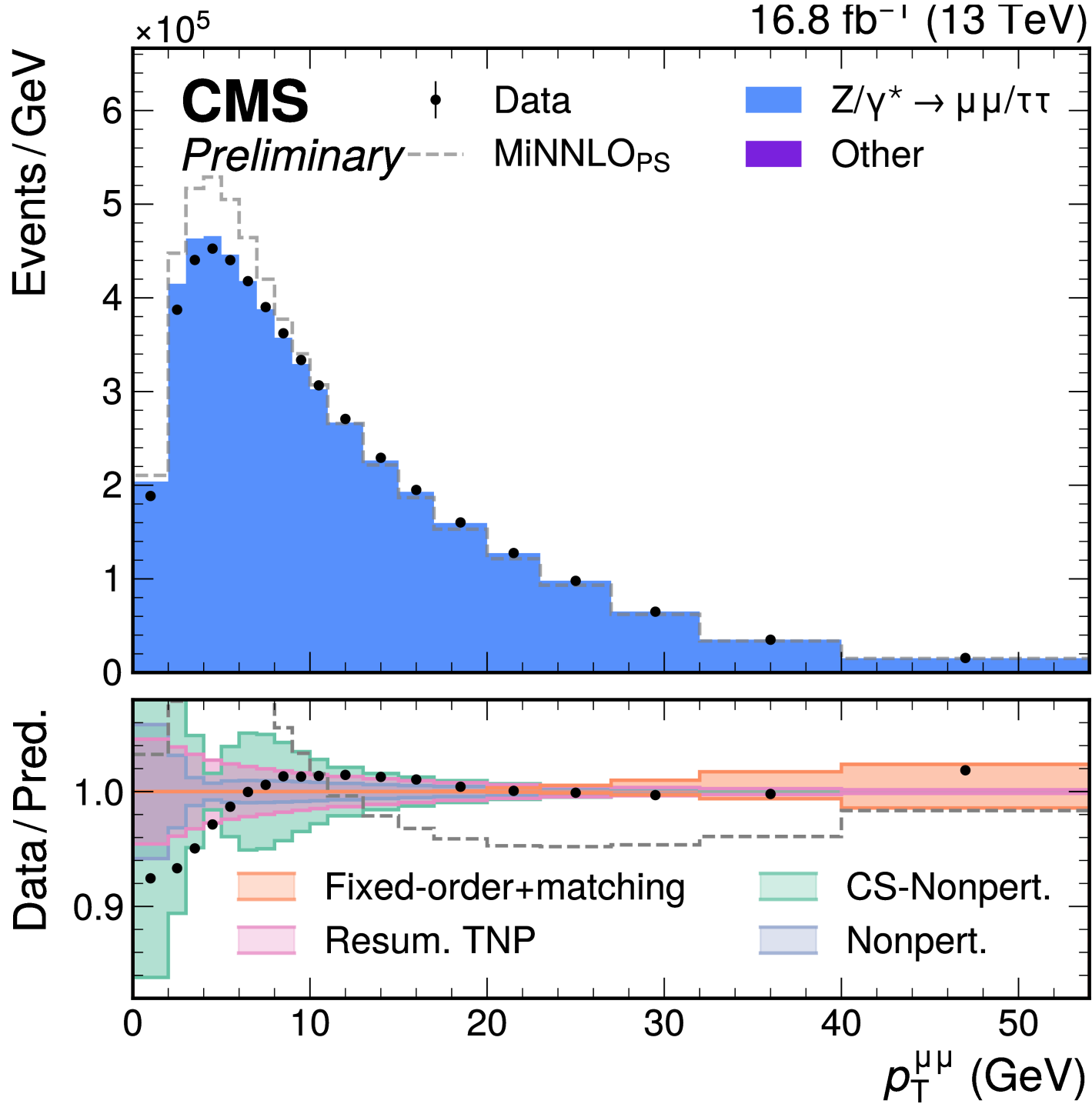
$$\eta(y) = \Phi^{-1}(\Sigma_2(y))$$



use inference to constrain c

PULL ± CONSTRAINTS ON TNPs

[slide by K. Long]



- Small pulls/constraints on TNPs
- Nonperturbative terms most important
 - Different behaviour of $\Lambda^{(2)}$ and CS terms due to degeneracy
- Consistent impact on p_T^Z

