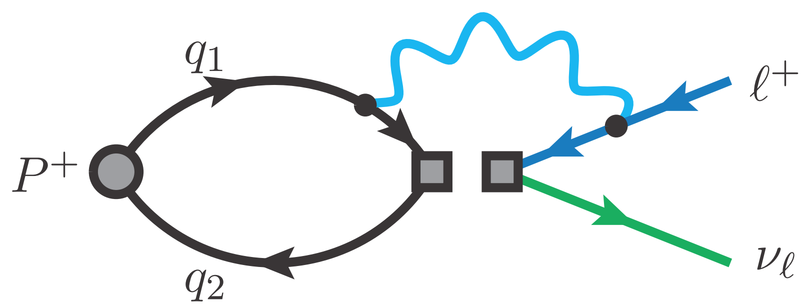
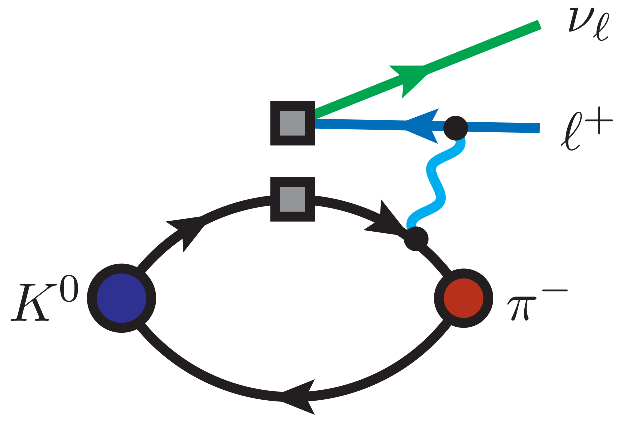
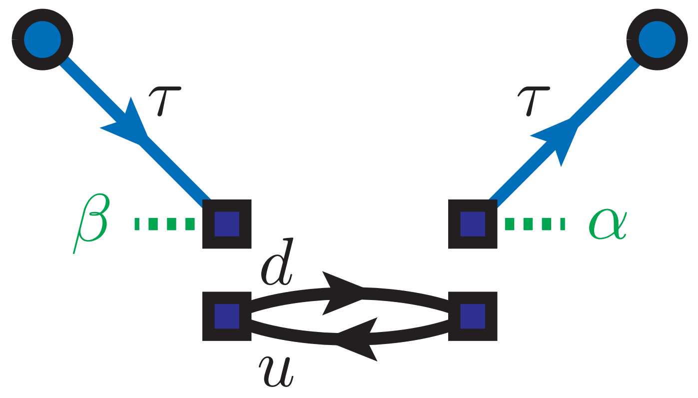
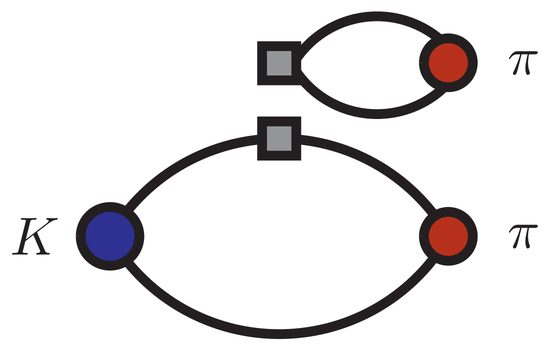
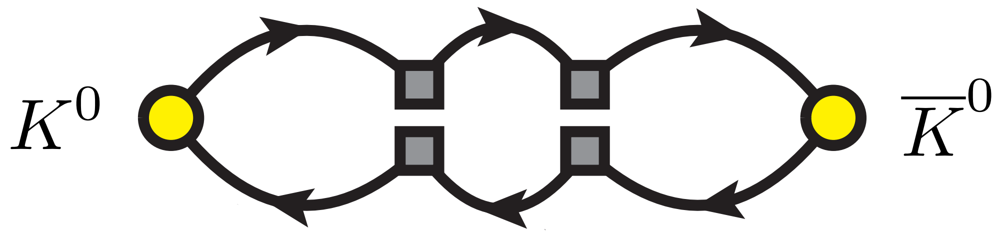










Precision flavour physics from the lattice

Matteo Di Carlo

8th January 2025

Funded by the European Union

Zürich Phenomenology Workshop 2025
Particle physics from low to high energies

CERN
Universität Zürich UZH
ETH zürich

Flavour physics

Flavour physics offers opportunities to test the Standard Model and probe new physics effects

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

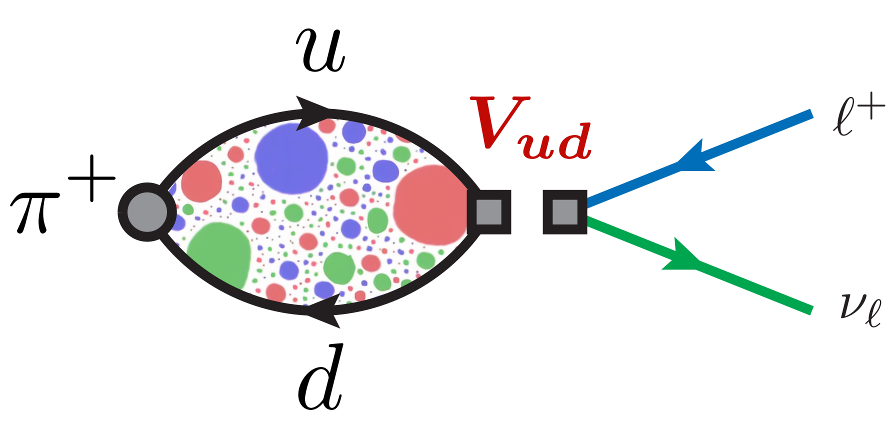
in the Standard Model:

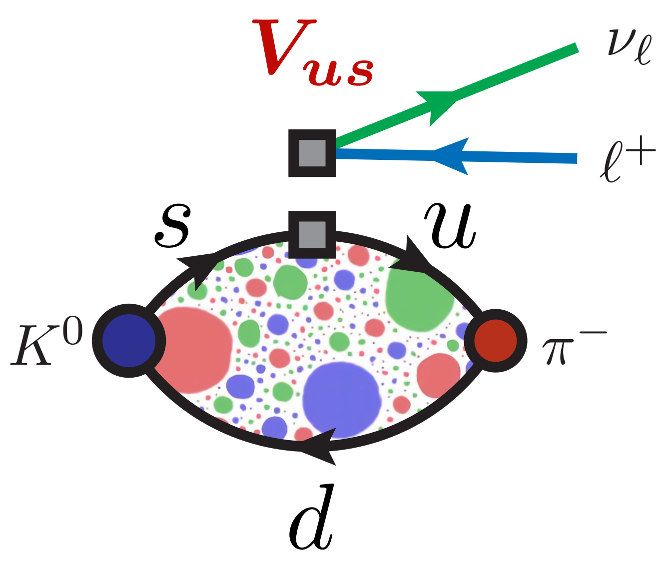
3 mixing angles + 1 CPV phase

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

Matrix elements can be extracted e.g. from leptonic and semileptonic decays of hadrons

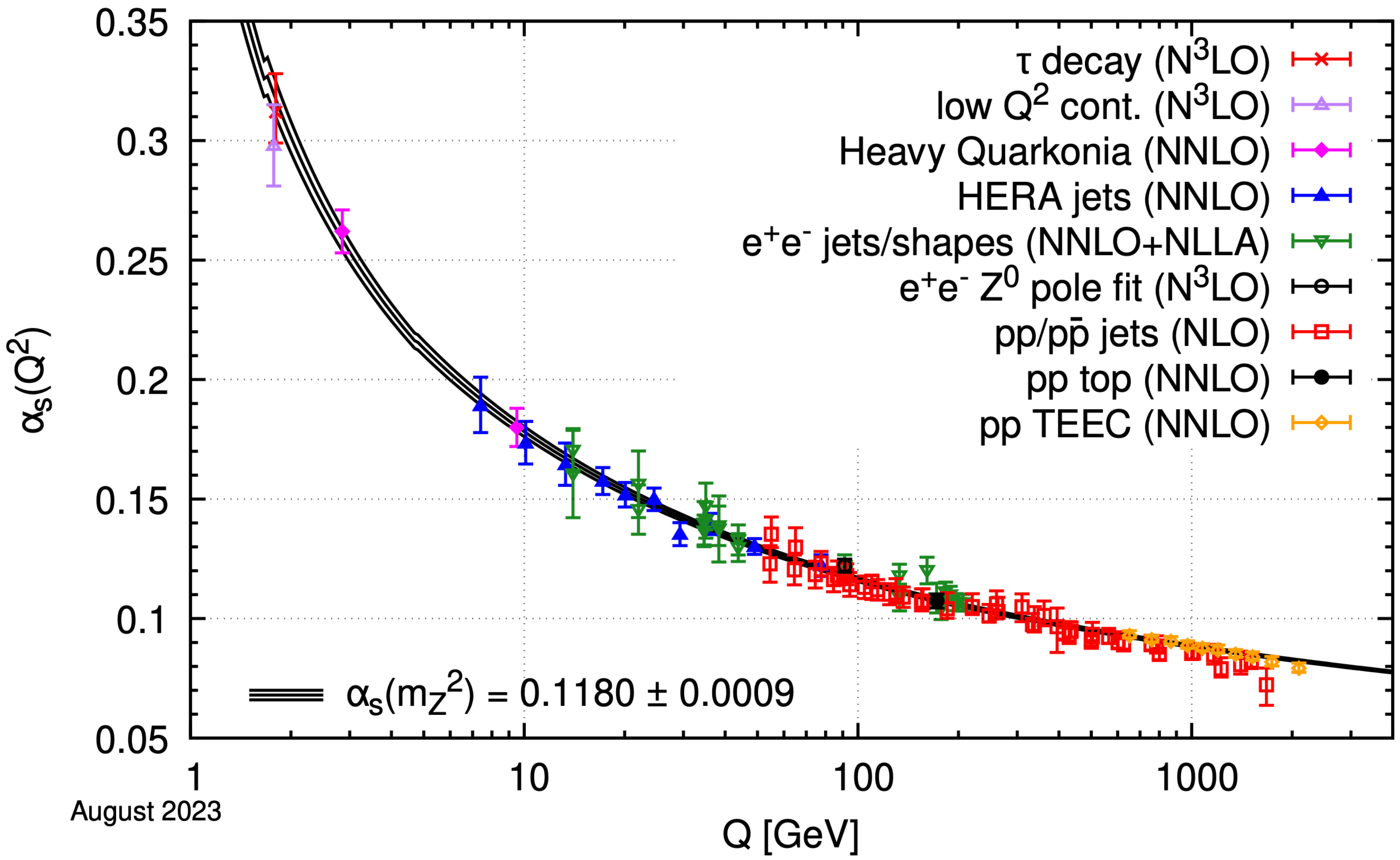
$$\underbrace{\frac{\Gamma\left[K \to \ell\nu_\ell(\gamma)\right]}{\Gamma\left[\pi \to \ell\nu_\ell(\gamma)\right]}}_{\text{experiments}} \propto \boxed{\left|\frac{V_{us}}{V_{ud}}\right|^2} \underbrace{\left(\frac{f_K}{f_\pi}\right)^2}_{\text{QCD}} \qquad \underbrace{\Gamma\left[K \to \pi\ell\nu_\ell(\gamma)\right]}_{\text{experiments}} \propto \boxed{|V_{us}|^2} \underbrace{|f_+^{K\pi}(0)|^2}_{\text{QCD}}$$





The strong coupling constant

see A.Ramos talk



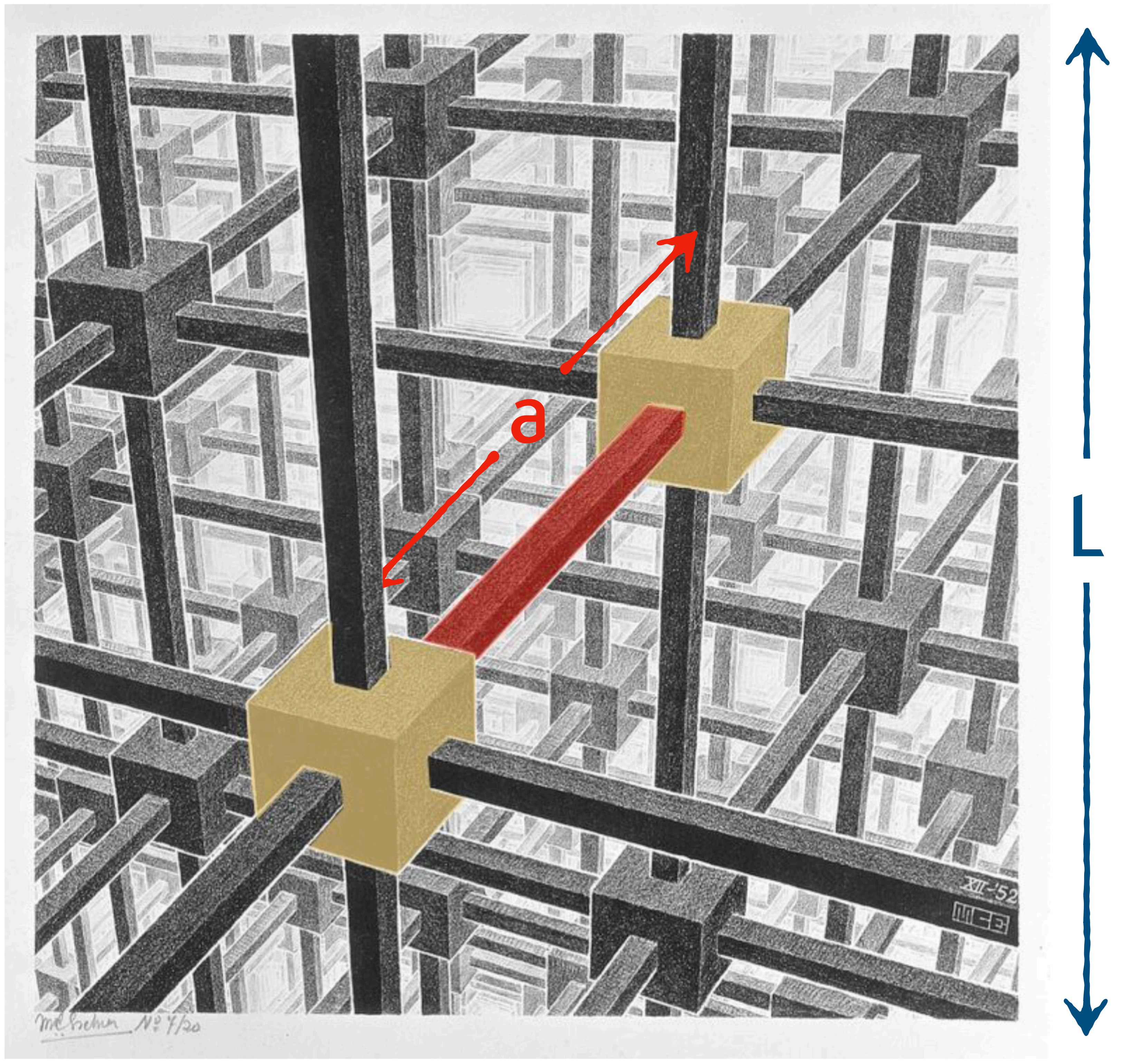
PDG, PRD 110 (2024) 3

- The strong coupling constant $\alpha_s(Q^2)$ runs with the energy Q
- At high energies $Q \sim m_Z$ the coupling is small:
  - perturbative expansion
  - quarks are **asymptotically free**
- At small energies $Q \sim \Lambda_{\text{QCD}}$ the coupling is strong:
  - non-perturbative
  - quarks are **confined**

Lattice QCD

Key points

- A **regularisation of QCD:**
  - the lattice spacing a is the UV cutoff
- **Finite-sized Euclidean spacetime**
  - rigorous and computable definition of path integrals using Monte Carlo methods
- Allows for **first-principle numerical calculations**
  - physical results obtained by taking continuum and infinite-volume limits



M.C. Escher, "Cubic space division" (1953)

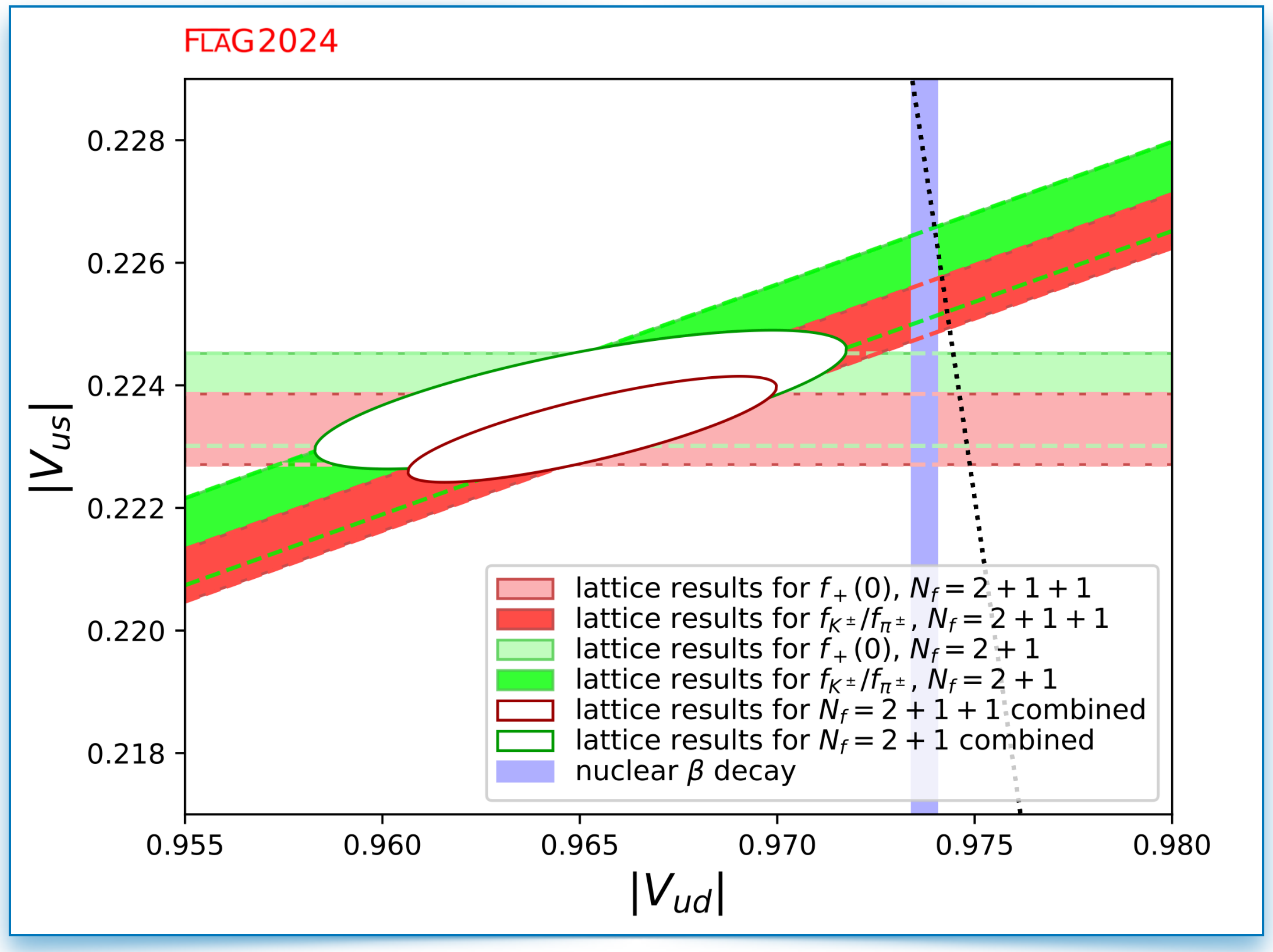
Precise predictions $\leftrightarrow$ good control of **statistical & systematic uncertainties**

Flavour physics on the lattice

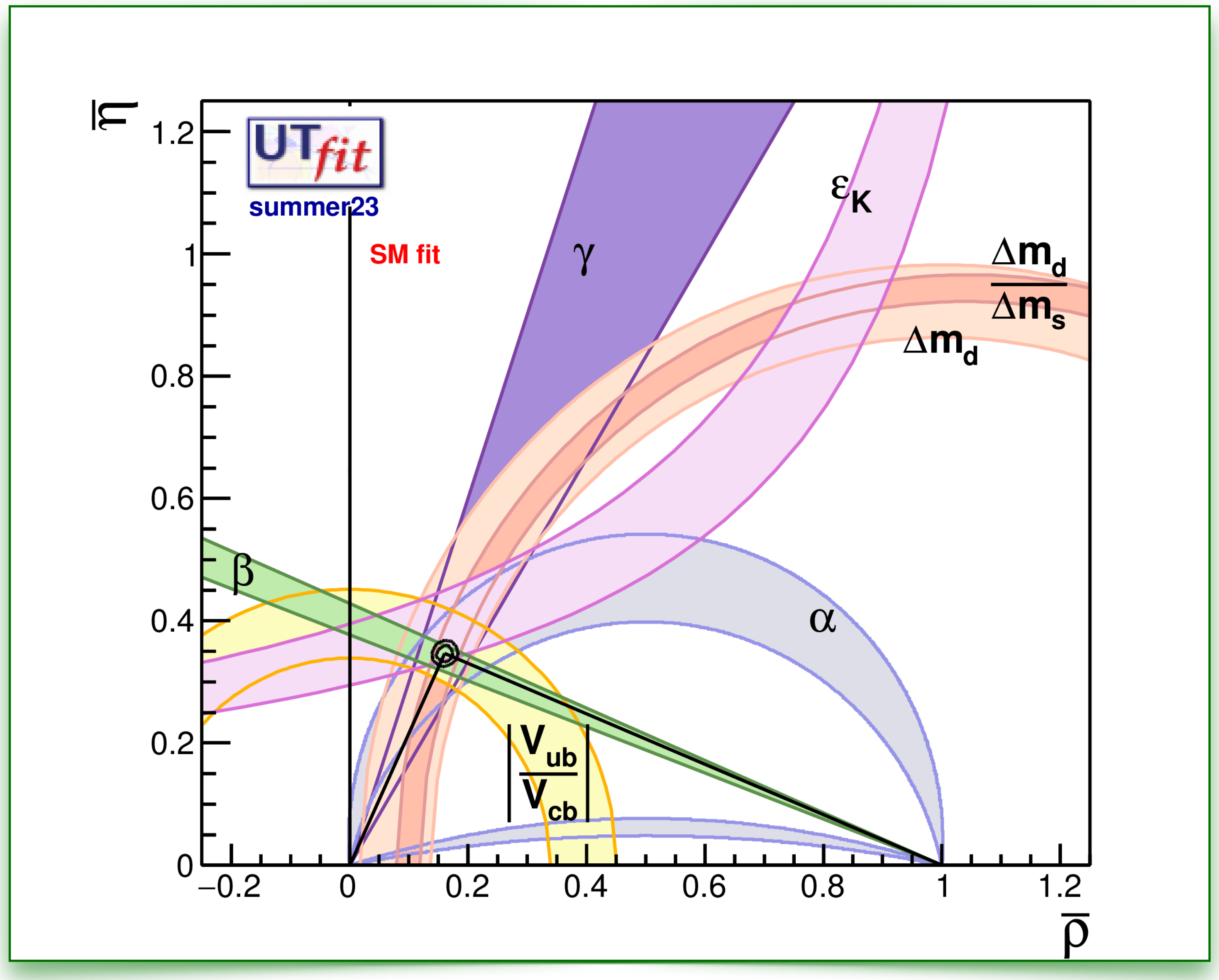
The plan of this talk:

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Cabibbo anomaly



CPV in neutral mesons



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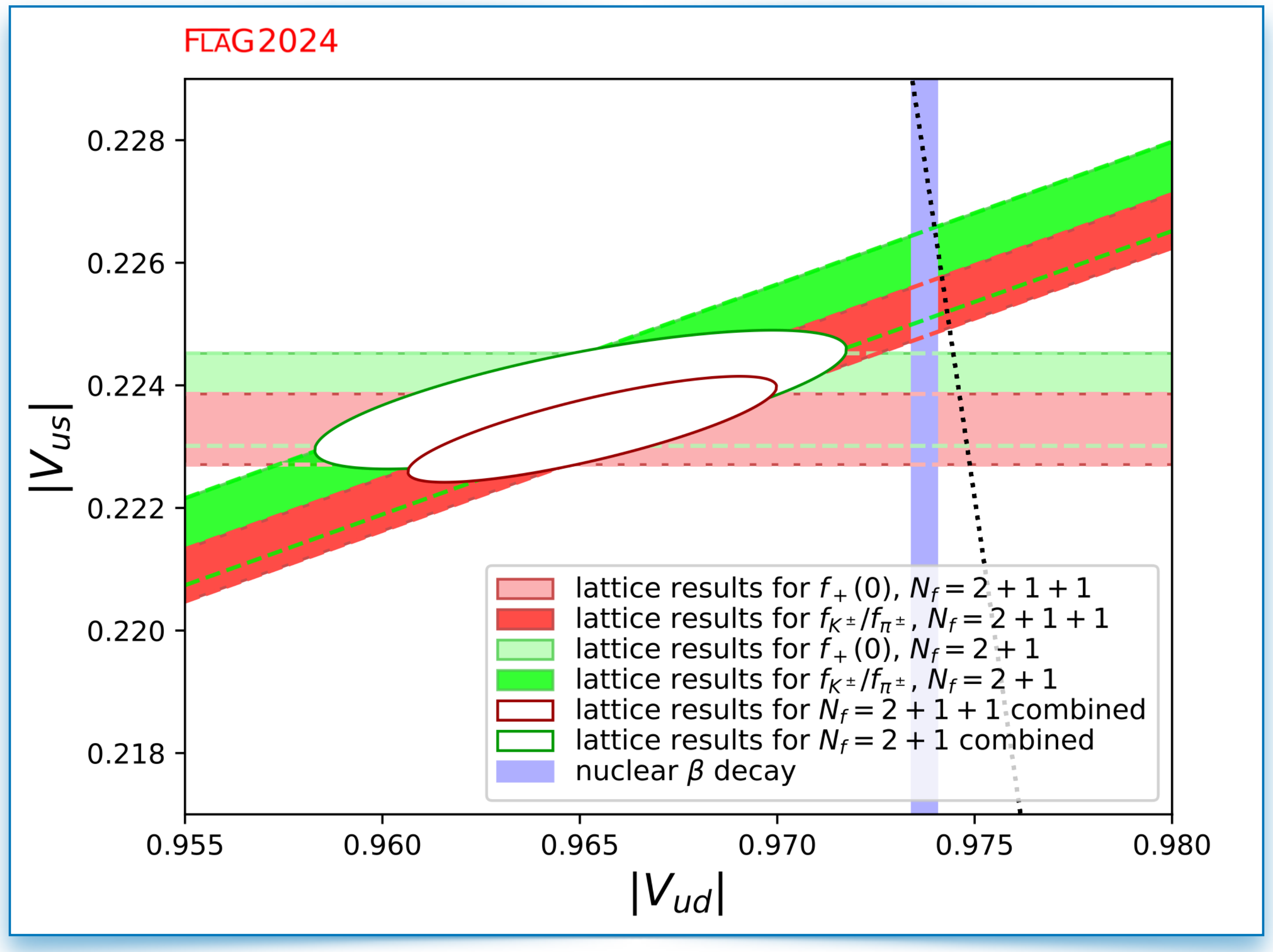
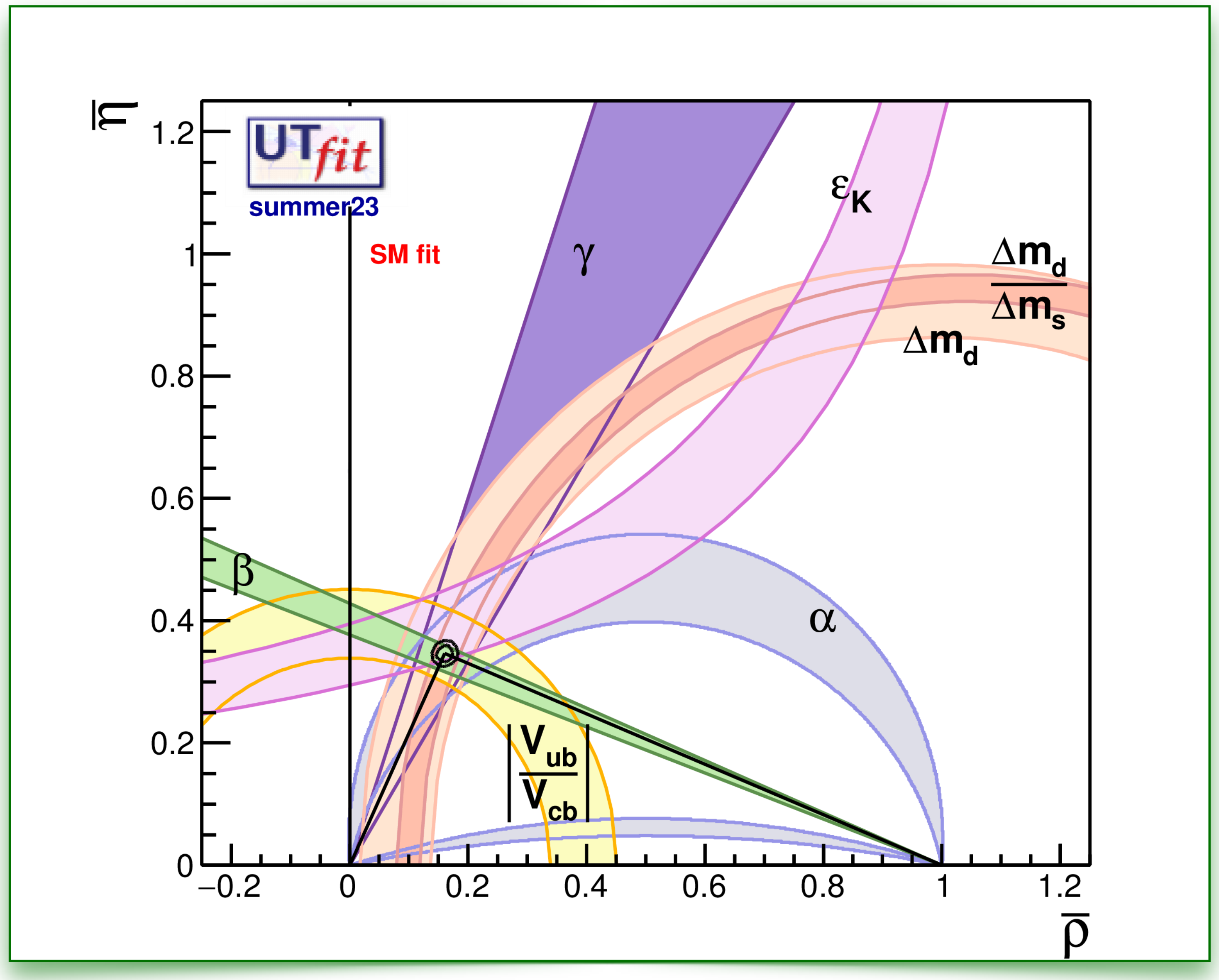
see M.Bordone talk

Cabibbo anomaly

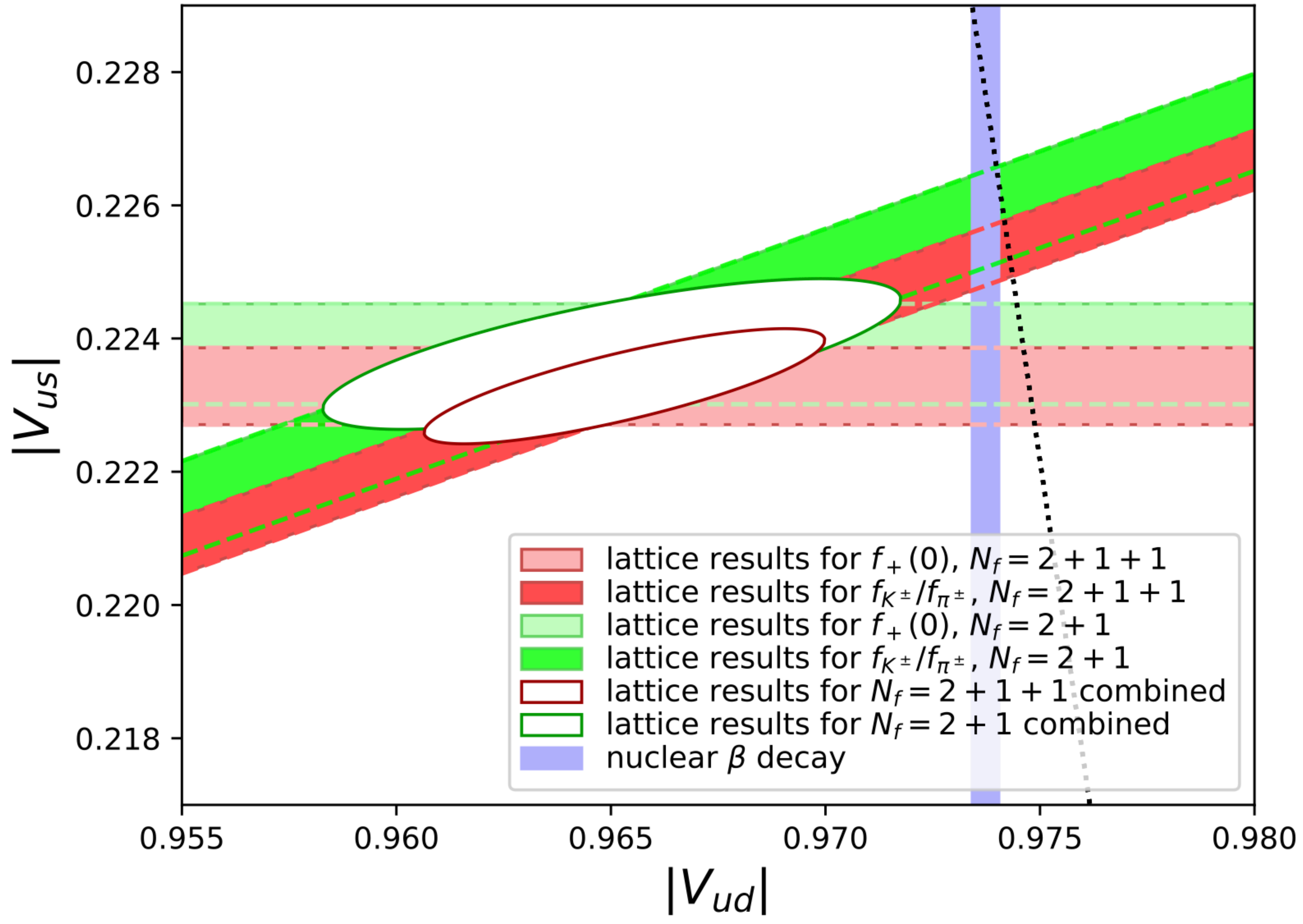


CPV in neutral mesons



The Cabibbo anomaly

FLAG2024 FLAG Review 2024, [arXiv:2411.04268]



$$\frac{|\boldsymbol{V_{us}}|}{|\boldsymbol{V_{ud}}|}\frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.27599(41)$$

$$|\boldsymbol{V_{us}}|\,|f_+^{K^0\pi^-}(0)| = 0.21654(41)$$

M.Moulson, PoS CKM2016 (2017)
PDG, PTET **2022** (2022)

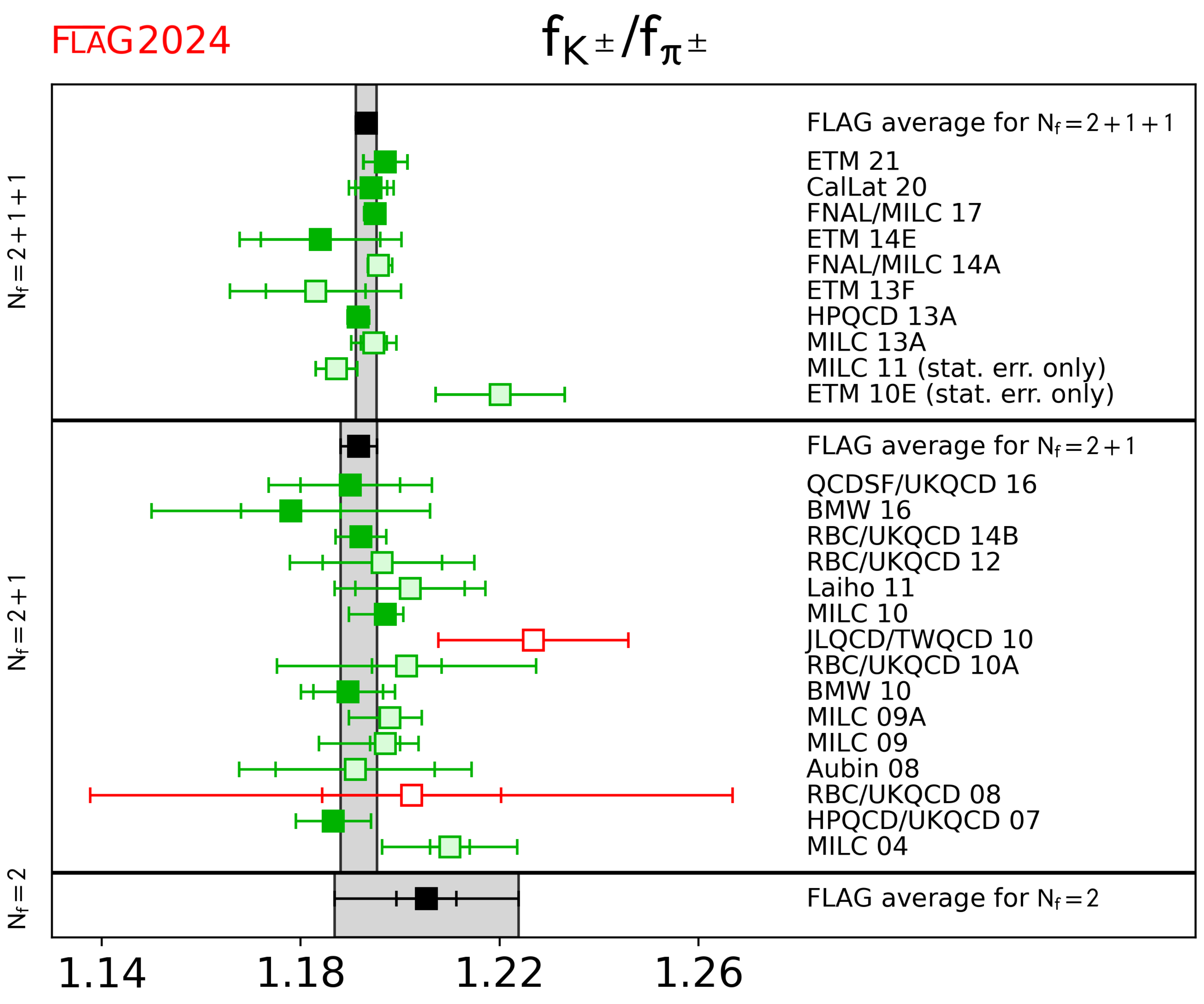
Different **tensions** in the V_{us}-V_{ud} plane:

$$|V_u|^2 - 1 = 2.8\sigma$$

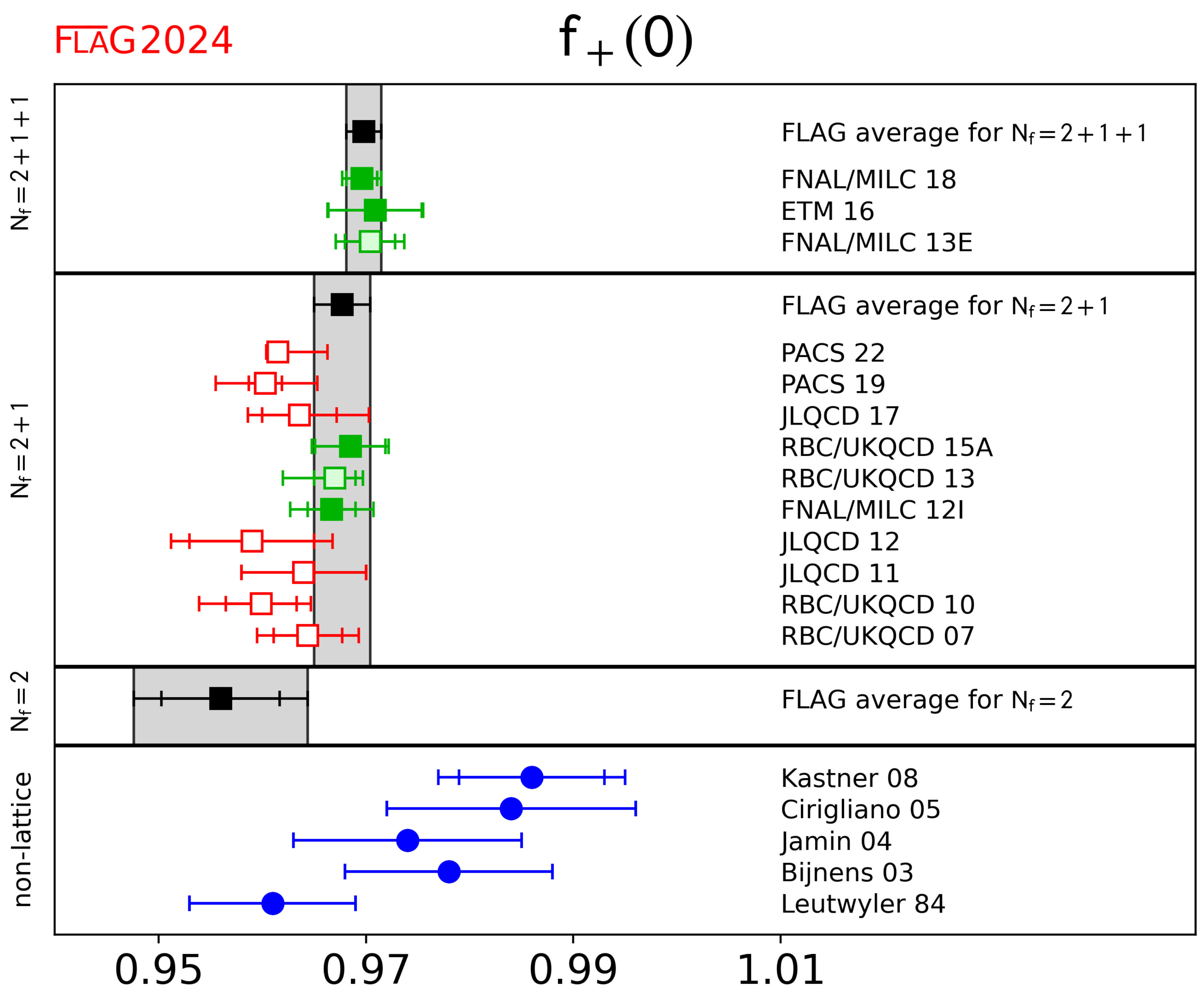
$$|V_u|^2 - 1 = 3.1\sigma \qquad |V_u|^2 - 1 = 1.7\sigma$$

Experimental and **theoretical** control of these quantities is of crucial importance to solve the issue

Lattice QCD inputs



$$f_{K^\pm}/f_{\pi^\pm} = 1.1934\,(19)$$

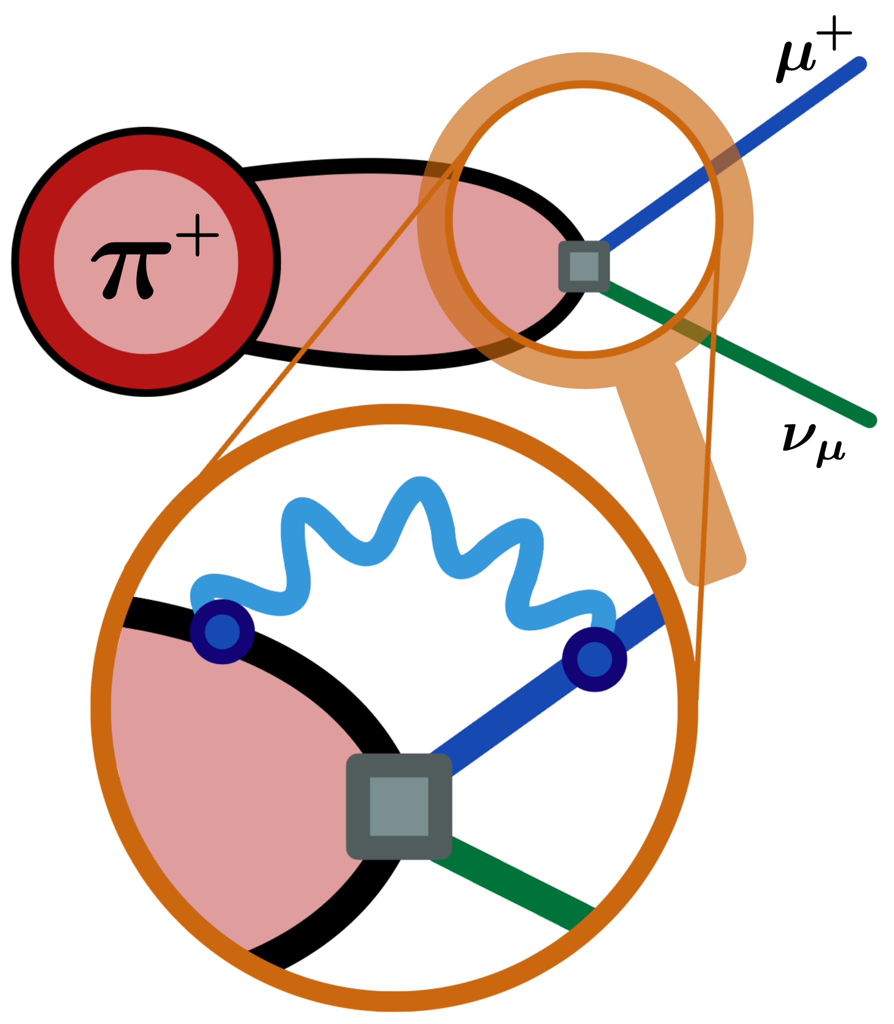


$$f_+^{K\pi}(0) = 0.9698\,(17)$$

FLAG 2024 Flavour Lattice Averaging Group

f_K/f_π and $f_+^{K\pi}(0)$ determined from lattice QCD with sub percent precision!

QED and isospin-breaking effects



Current level of precision requires the inclusion of isospin-breaking corrections due to

- strong effects $\quad [m_u - m_d]_{\mathrm{QCD}} \neq 0$
- electromagnetic effects $\quad \alpha \neq 0$

$\sim \mathcal{O}(1\%)$

$$\frac{\Gamma(K \to \ell\nu_\ell)}{\Gamma(\pi \to \ell\nu_\ell)} \propto \frac{|\boldsymbol{V_{us}}|^2}{|\boldsymbol{V_{ud}}|^2} \left(\frac{f_K}{f_\pi}\right)^2 (1 + \delta R_{K\pi}) \qquad \Gamma(K \to \pi\ell\nu_\ell) \propto |\boldsymbol{V_{us}}|^2\, |f_+^{K\pi}(0)|^2\, \mathcal{I}^\ell_{K\pi}\, (1 + \delta R^\ell_{K\pi})$$

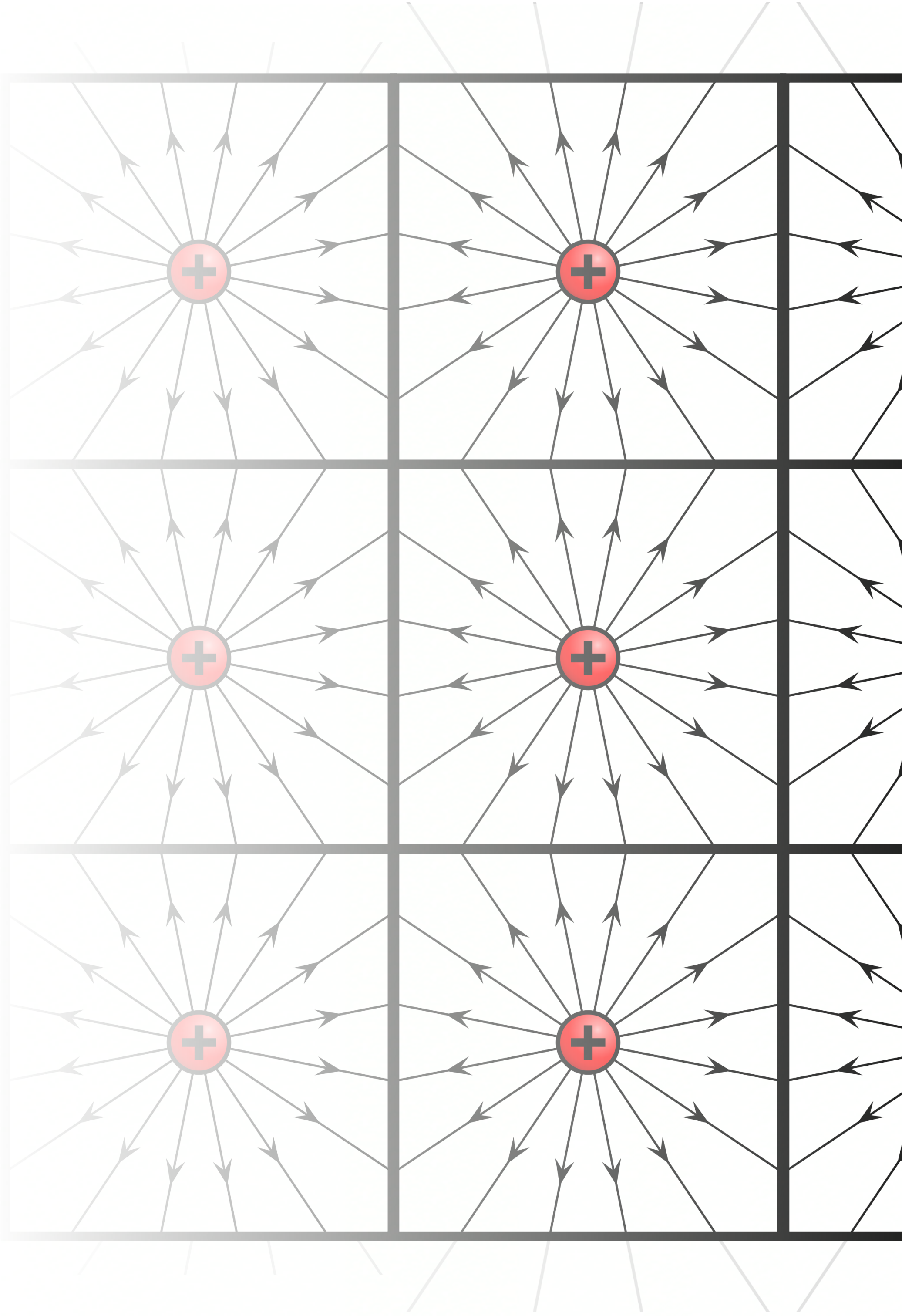
- results currently quoted in the PDG come from χPT — V.Cirigliano & H.Neufeld, PLB **700** (2011)
- fully **non-perturbative** (structure dependent) quantities
- first-principle **lattice calculations** are possible!

Lattice QCD + QED

A conceptual challenge: how to define QED in a finite periodic box?

- need to circumvent **Gauss' law:** no charged states in a periodic box
- **finite-volume effects** can be sizeable and power-like
- logarithmic **infrared divergences** arise when studying decays

Problems well studied. Different lattice QED formulations proposed and used.



RM123 approach:

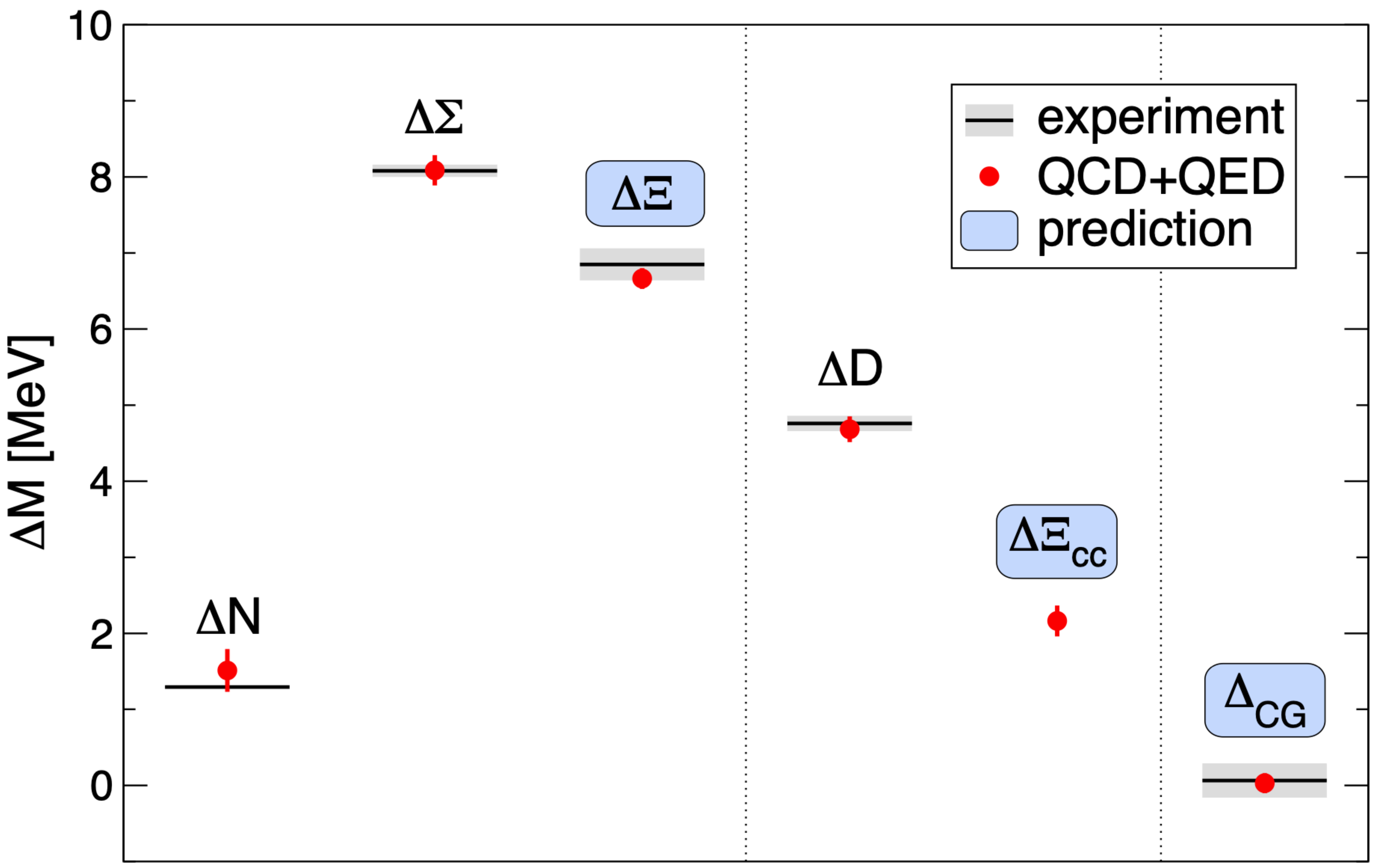
G.M.de Divitiis et al. [RM123], PRD 87 (2013)

$$\boxed{\langle \mathcal{O} \rangle = \int \mathcal{D}\Phi \, \mathcal{O} \, e^{-S_{\mathrm{iso}} - \Delta S} = \langle \mathcal{O} \rangle_{\mathrm{iso}} + \langle \Delta S \, \mathcal{O} \rangle_{\mathrm{iso}} + \ldots}$$

$$\text{“iso”} = \begin{cases} m_{\mathrm{u}} = m_{\mathrm{d}} \\ \alpha_{\mathrm{em}} = 0 \end{cases}$$

Hadron mass splitting

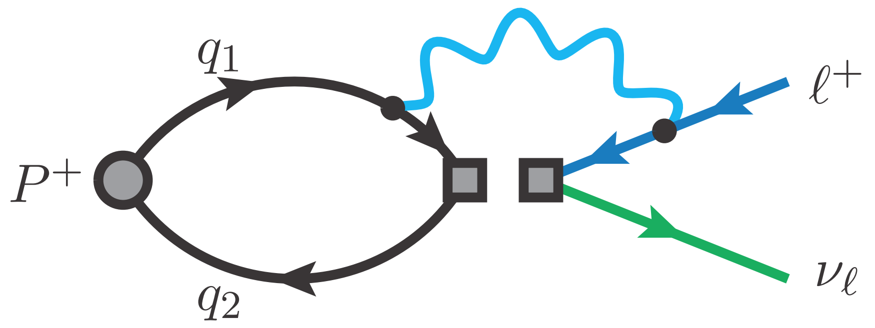
Pioneering work on **neutron-proton** mass difference BMW Collaboration, Science 347 (2015)



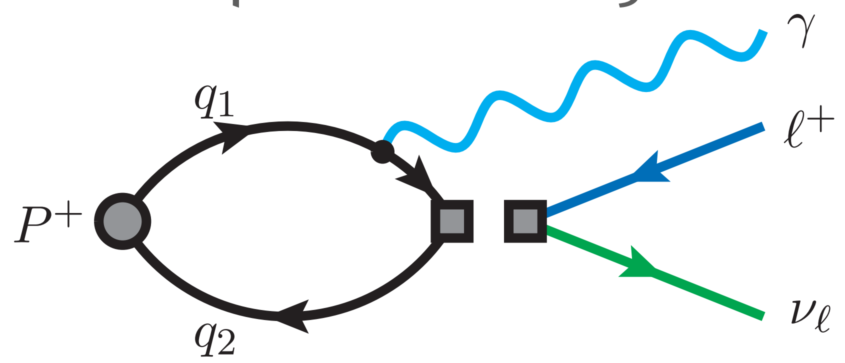
Pion & **kaon** mass splittings now computed by different collaborations —› u & d quark masses

Weak decays — some recent works

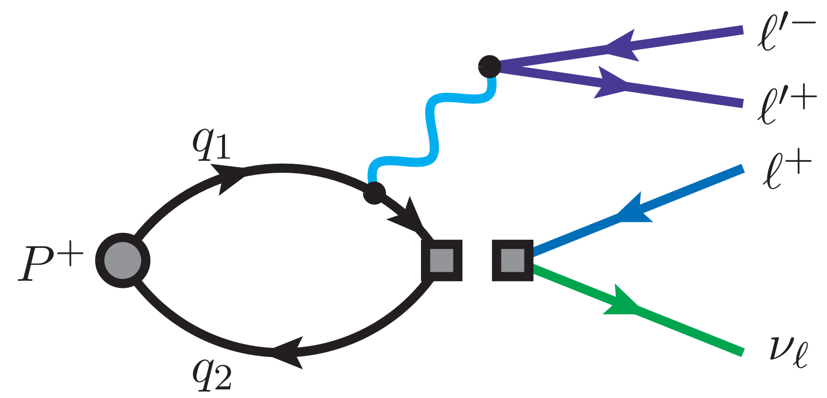
leptonic decays



real photon emission
leptonic decays



virtual photon emission
leptonic decays



N.Carrasco et al., PRD 91 (2015)
V.Lubicz et al., PRD 95 (2017)
N.Tantalo et al., [1612.00199v2]
D.Giusti et al., PRL 120 (2018)
MDC et al., PRD 100 (2019)
MDC et al., PRD 105 (2022)
P.Boyle, MDC et al., JHEP 02 (2023)
N.Christ et al., [2304.08026]

G.M.de Divitiis et al., [1908.10160]
C.Kane et al., [1907.00279 & 2110.13196]
R.Frezzotti et al., PRD 103 (2021)
A.Desiderio et al., PRD 102 (2021)
D.Giusti et al., [2302.01298]
R.Frezzotti et al., [2306.05904]

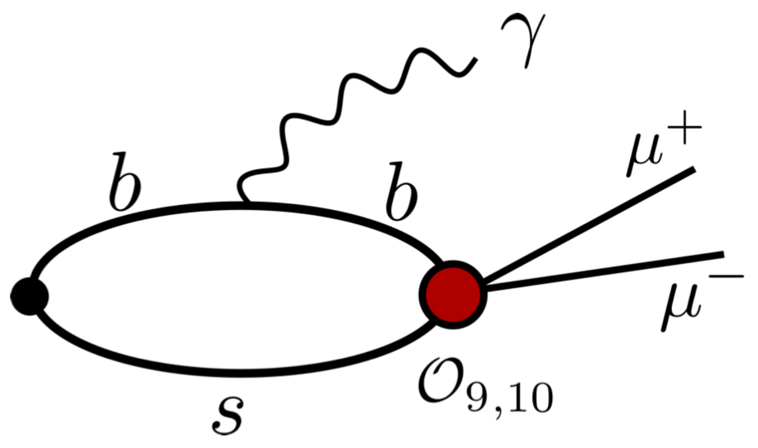
G.Gagliardi et al., Phys. Rev. D 105 (2022)
R.Frezzotti et al., [2306.07228]

R.Frezzotti et al., [2402.03262]

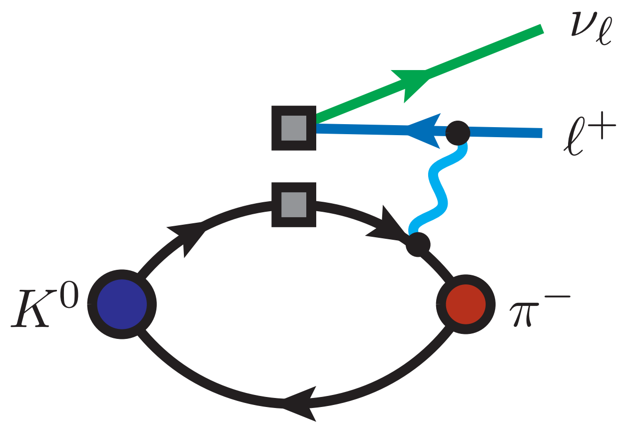
C.Sachrajda et al., [1910.07342]
N.Christ et al., PRD 108 (2023)
N.Christ et al., [2402.08915]

R.Abbott et al., PRD 102 (2020)
Z.Bai et al., PRL 115 (2015)
N.Christ et al., PRD 106 (2022)
N.Christ & X.Feng, EPJ Web Conf. 175 (2018)
Y.Cai & Z.Davoudi, [1812.11015]

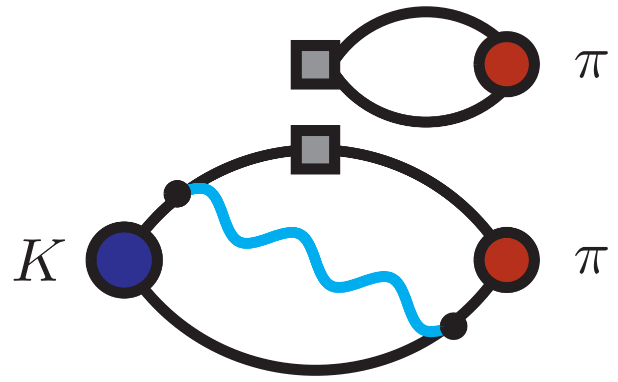
rare FCNC decays



semileptonic decays



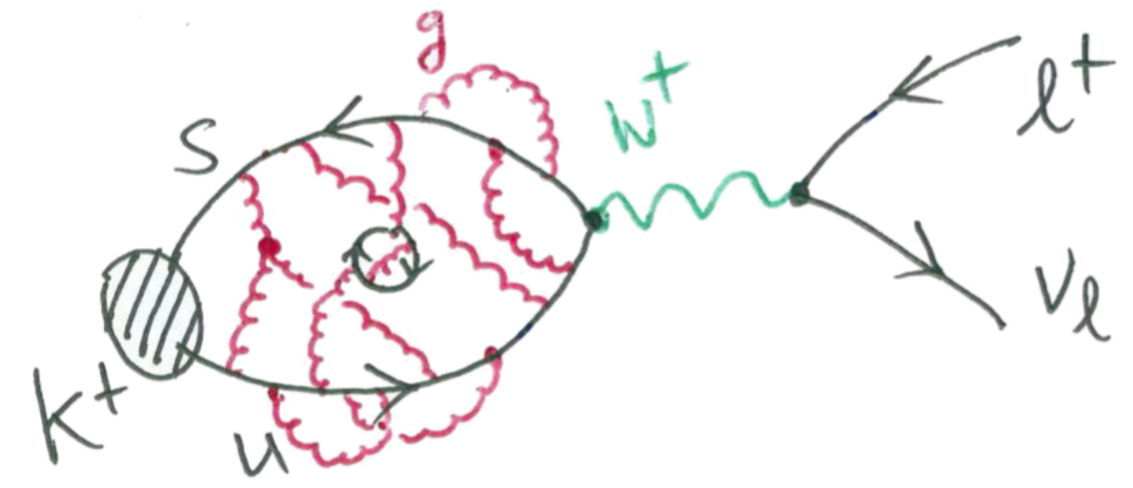
hadronic decays



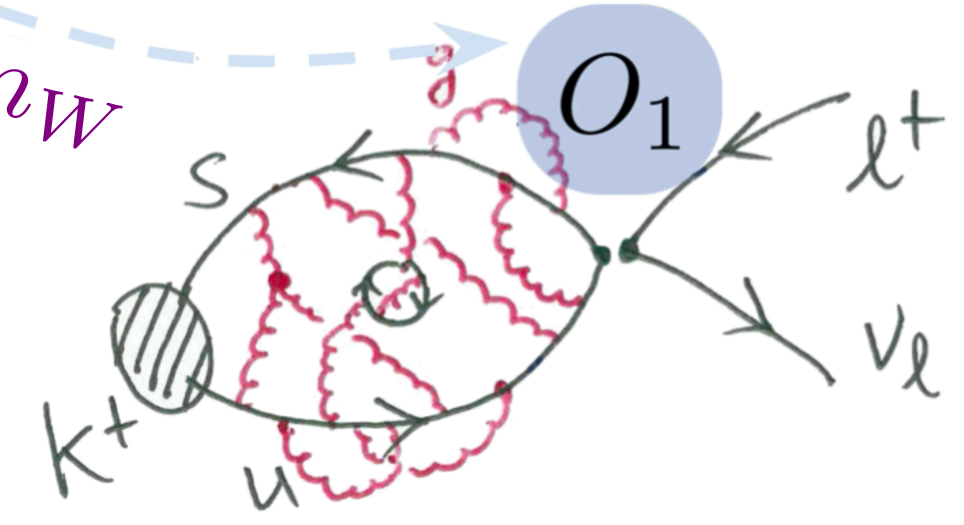
Leptonic decays of pseudoscalar mesons

Can be studied in an **effective Fermi theory** with the W-boson integrated out and the local interaction described by

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V^*_{q_1 q_2} \left[\bar{q}_2\, \gamma_\mu (1-\gamma_5)\, q_1\right] \left[\bar{\nu}_\ell\, \gamma^\mu (1-\gamma_5)\, \ell\right]$$

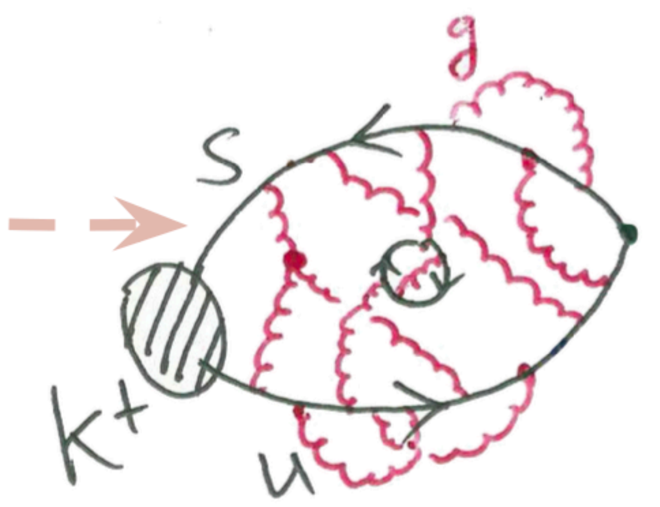


$1/a \ll m_W$



In the **PDG convention**, the tree-level decay rate takes the form

$$\Gamma_P^{\rm tree} = \frac{G_F^2}{8\pi} m_\ell^2 \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2 m_P \left[f_{P,0}\right]^2$$



with the non-perturbative dynamic encoded in the **decay constant**

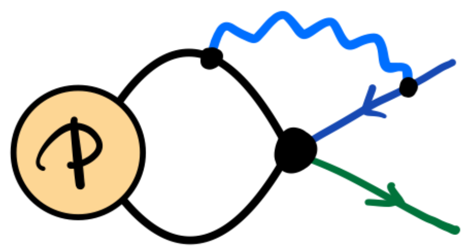
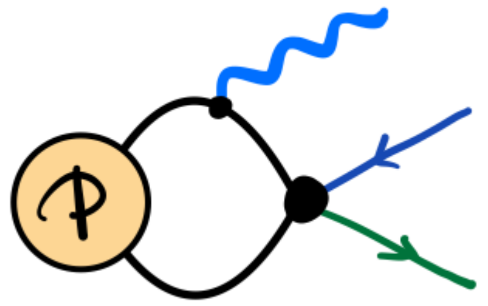
$$\mathcal{Z}_0 \langle 0 | \bar{q}_2\, \gamma_0 \gamma_5\, q_1 | P, \mathbf{0}\rangle^{(0)} = i\, m_{P,0}\, f_{P,0}$$

Leptonic decay rate at $\mathcal{O}(\alpha)$

- The **decay constant** $f_{P,0}$ becomes an ambiguous and unphysical quantity

- **IR divergences** appear in intermediate steps of the calculation

F. Bloch & A. Nordsieck, PR 52 (1937) 54

$$\Gamma(P_{\ell 2}) = \lim_{\Lambda_{\mathrm{IR}}\to 0} \Big\{$$  $$+$$  $$\Big\}$$

IR finite — IR divergent — IR divergent

- **UV divergences**: need to include QED corrections to the renormalization of the weak Hamiltonian

$$\mathcal{H}_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}} V^*_{q_1 q_2} \left(1 + \frac{\alpha_{\mathrm{em}}}{\pi} \ln\left(\frac{M_Z}{M_W}\right)\right) \underbrace{[\, \bar{q}_2\, \gamma_\mu (1-\gamma_5)\, q_1 \,]\, [\, \bar{\nu}_\ell\, \gamma^\mu (1-\gamma_5)\, \ell \,]}_{O_1^{\text{W-reg}}(M_W)}$$

A.Sirlin, NPB **196** (1982)

E.Braaten & C.S.Li, PRD **42** (1990)

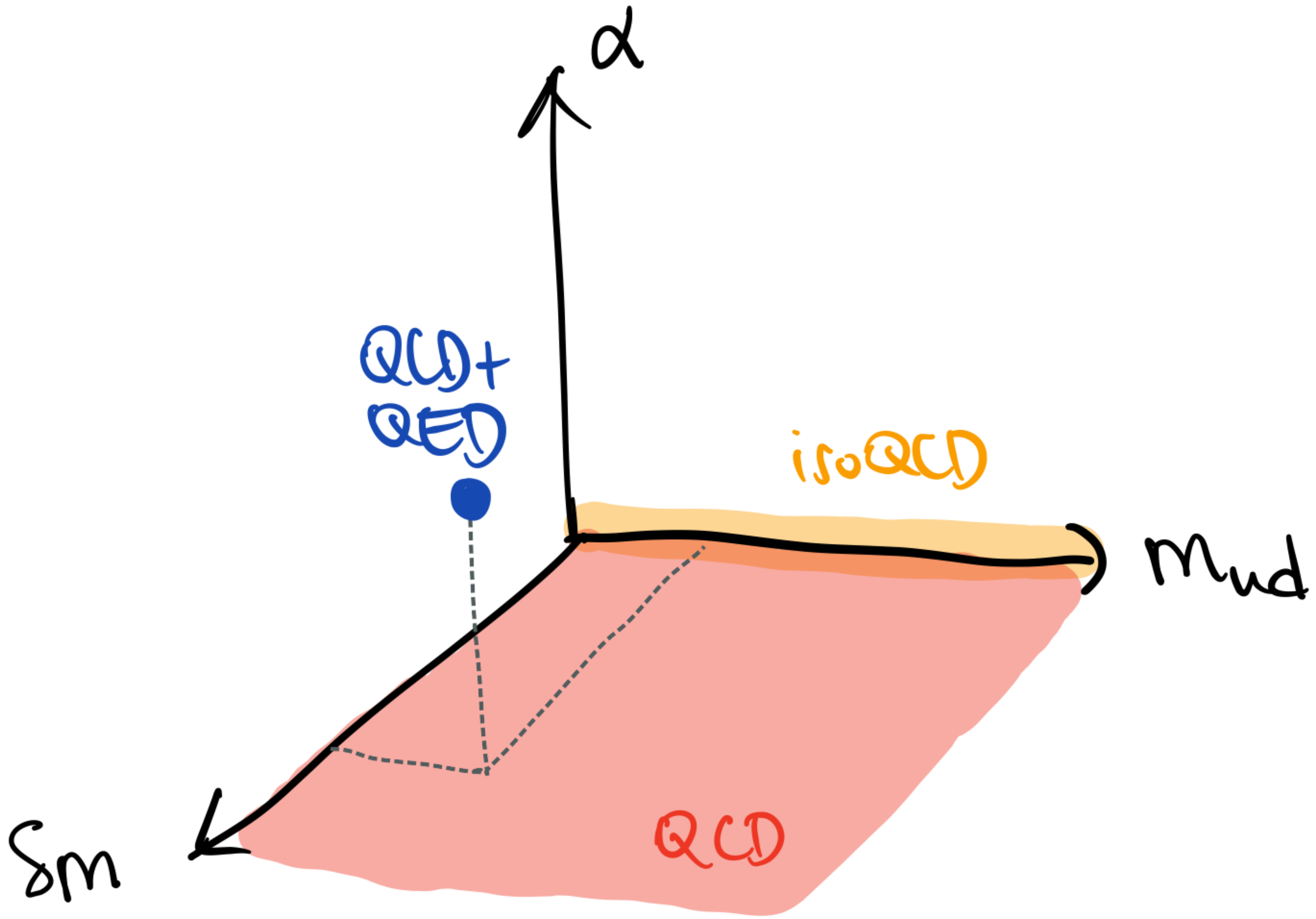
$$O_1^{\text{W-reg}}(M_W) = Z^{\text{W-S}}\left(\frac{M_W}{\mu}, \alpha_s(\mu), \alpha_{\mathrm{em}}\right) O_1^{\mathrm{S}}(\mu)$$

- perturbative @ 2 loops in QCD+QED
- non-perturbative in lattice QCD+QED

MDC et al., PRD **100** (2019)

Leptonic decay rate at $\mathcal{O}(\alpha)$

Defining the isospin symmetric world



- The full QCD+QED theory is unambiguously defined after **matching** a set of observables to the real world

$$\left[\frac{\hat{\mathrm{M}}_j}{\hat{\Lambda}}\right]^2(g, e^\phi, \hat{\mathbf{m}}^\phi) = \left(\frac{\mathrm{M}_j^\phi}{\Lambda^\phi}\right)^2 \longrightarrow \hat{\mathbf{m}}^\phi(g)$$

$$j = 1, \dots, N_f$$

- The definition of QCD or isoQCD requires a prescription, i.e. some renormalization conditions to **fix the bare parameters** of the action

$$\boldsymbol{\sigma}^{\mathrm{QCD}} = (g^{\mathrm{QCD}}, 0, \hat{\mathbf{m}}^{\mathrm{QCD}}) \qquad \hat{\mathbf{m}}^{\mathrm{QCD}} = (\hat{m}_{ud}^{\mathrm{QCD}}, \delta\hat{m}^{\mathrm{QCD}}, \hat{m}_s^{\mathrm{QCD}}, \dots)$$

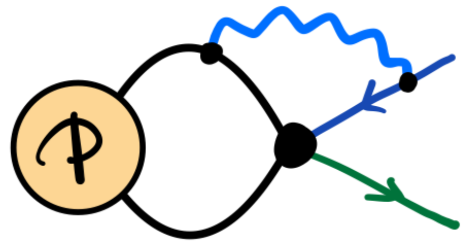
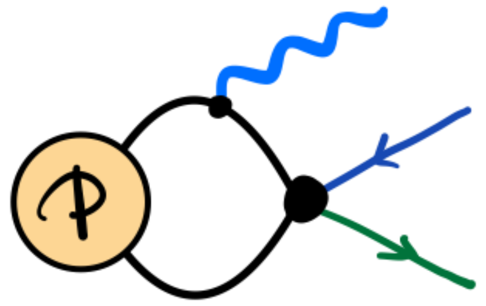
$$\boldsymbol{\sigma}^{(0)} = (g^{(0)}, 0, \hat{\mathbf{m}}^{(0)}) \qquad \hat{\mathbf{m}}^{(0)} = (\hat{m}_{ud}^{(0)}, 0, \hat{m}_s^{(0)}, \dots)$$

FLAG 2024 now includes a discussion on this topic where a reference scheme is proposed.

Leptonic decay rate at $\mathcal{O}(\alpha)$

The RM123+Soton approach

F. Bloch & A. Nordsieck, PR **52** (1937)
N. Carrasco et al., PRD **91** (2015)
D. Giusti et al., PRL **120** (2018)
MDC et al., PRD **100** (2019)
P.Boyle, MDC et al., JHEP **02** (2023)

$\Gamma(P_{\ell 2}) = \lim\limits_{\Lambda_{\mathrm{IR}}\to 0} \Big\{$  $+$  $\Big\}$

IR finite — IR divergent — IR divergent

Leptonic decay rate at $\mathcal{O}(\alpha)$

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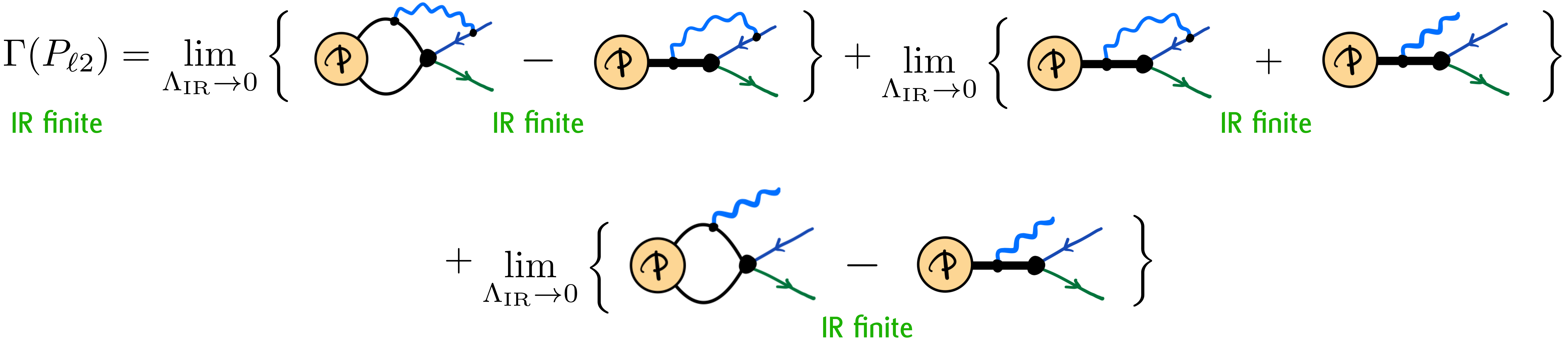

Leptonic decay rate at $\mathcal{O}(\alpha)$

The RM123+Soton approach

F. Bloch & A. Nordsieck, PR **52** (1937)
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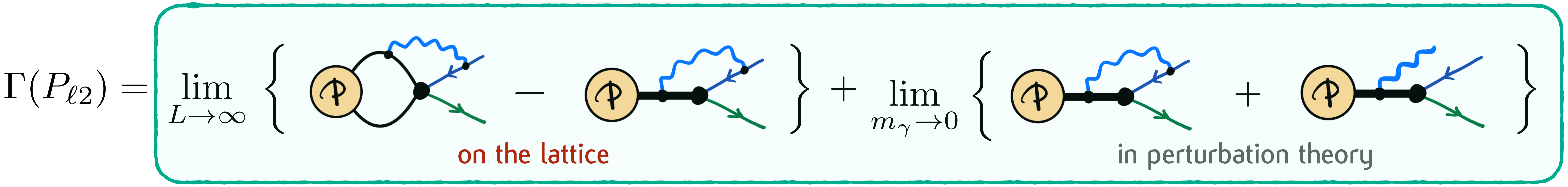
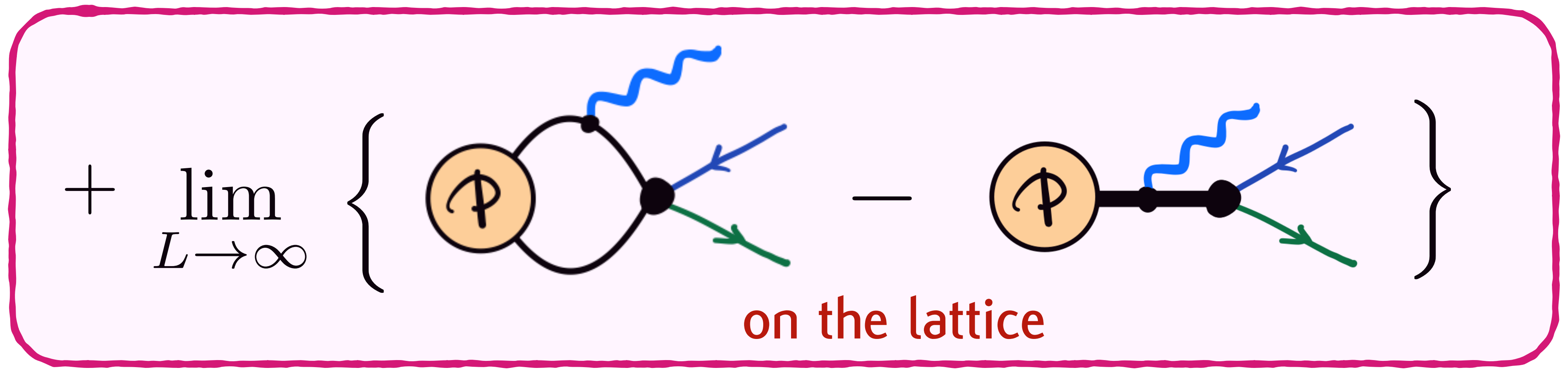
MDC et al., PRD **100** (2019)
P.Boyle, MDC et al., JHEP **02** (2023)





enough for $K_{\mu 2}$ and $\pi_{\mu 2}$

leading **finite-volume scaling** well studied

V.Lubicz et al., PRD **95** (2017) N.Tantalo et al., [1612.00199v2]
MDC et al., PRD **105** (2022)

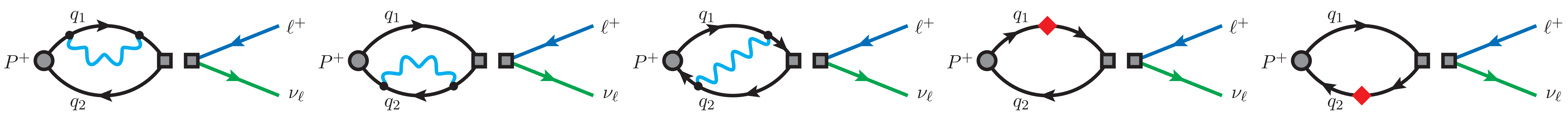
relevant for K_{e2} and π_{e2}
& decays of heavier mesons

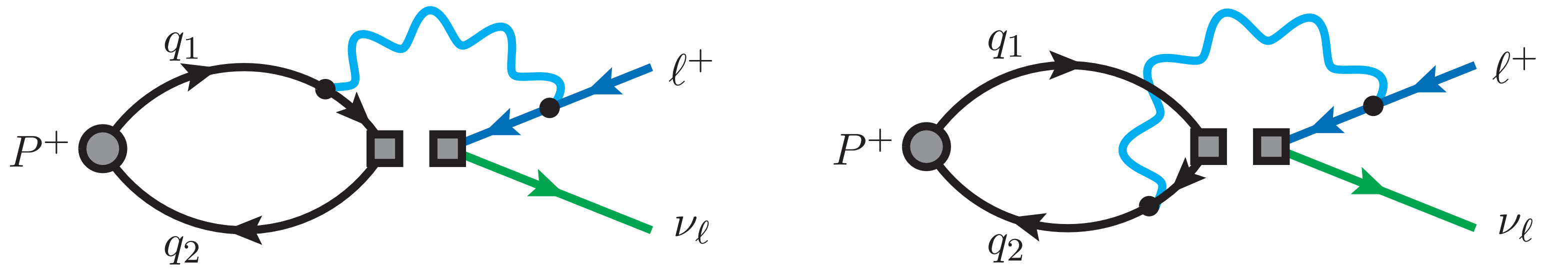
G.M. de Divitiis et al., [1908.10160]
R. Frezzotti et al., PRD **103** (2021)
A. Desiderio et al., PRD **102** (2021)
C. Kane et al., [1907.00279 & 2110.13196]
D. Giusti et al., [2302.01298]
R.Frezzotti et al., [2306.05904]

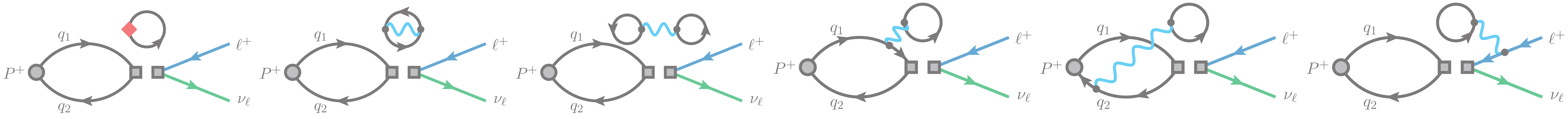
IB corrections to the decay amplitude

G.M.de Divitiis et al. [RM123], PRD **87** (2013)

RM123 perturbative method: expand lattice path-integral around isosymmetric point $\alpha = m_{\mathrm{u}} - m_{\mathrm{d}} = 0$





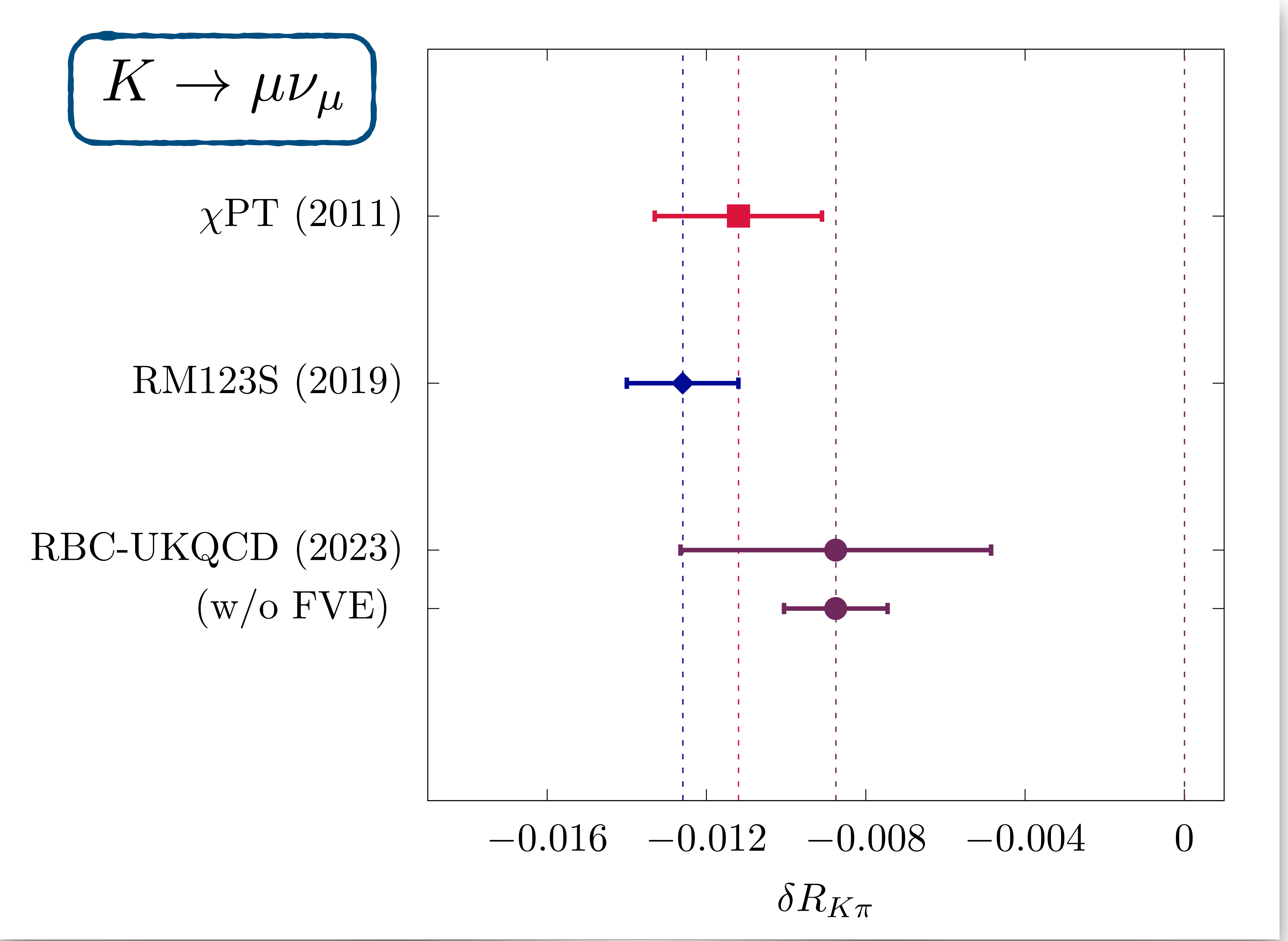


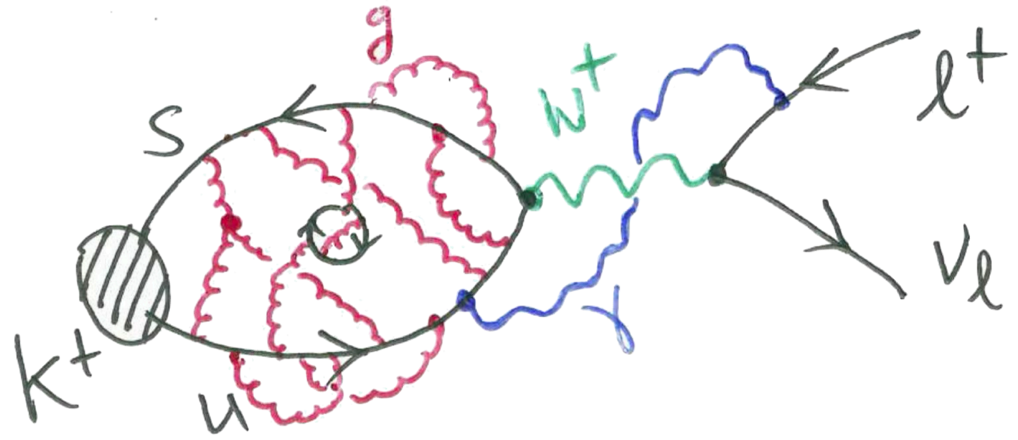
Current calculations have been performed in the **electro-quenched approximation (sea quarks electrically neutral).**
Work is in progress to compute the remaining diagrams.

T.Harris et al., PoS LATTICE 2022 (2023) 013

Results for $\delta R_{K\pi}$

- ■ $\delta R_{K\pi} = -0.0112\,(21)$
- ◆ $\delta R_{K\pi} = -0.0126\,(14)$
- ● $\delta R_{K\pi} = -0.0086\,(13)(39)_{\text{vol.}}$





V.Cirigliano et al., PLB **700** (2011)
MDC et al., PRD **100** (2019)
P.Boyle, MDC et al., JHEP **02** (2023)

$$\frac{\Gamma(K \to \ell\nu_\ell)}{\Gamma(\pi \to \ell\nu_\ell)} \propto \frac{|V_{us}|^2}{|V_{ud}|^2} \left(\frac{f_K}{f_\pi}\right)^2 (1 + \delta R_{K\pi})$$

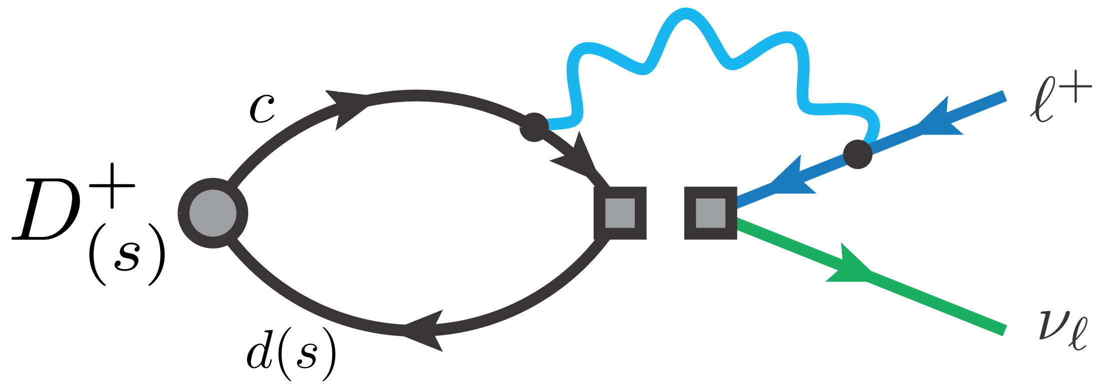
- Good evidence that $\delta R_{K\pi}$ **can be computed from first principles non-perturbatively on the lattice!**
- **RBC-UKQCD error** dominated by a large systematic uncertainty related to **finite-volume effects** (!)
  Work in progress to improve the result.
- **Errors on** $|V_{us}|/|V_{ud}|$ from **theoretical** inputs could become comparable with those from **experiments**

Some comments on light mesons...

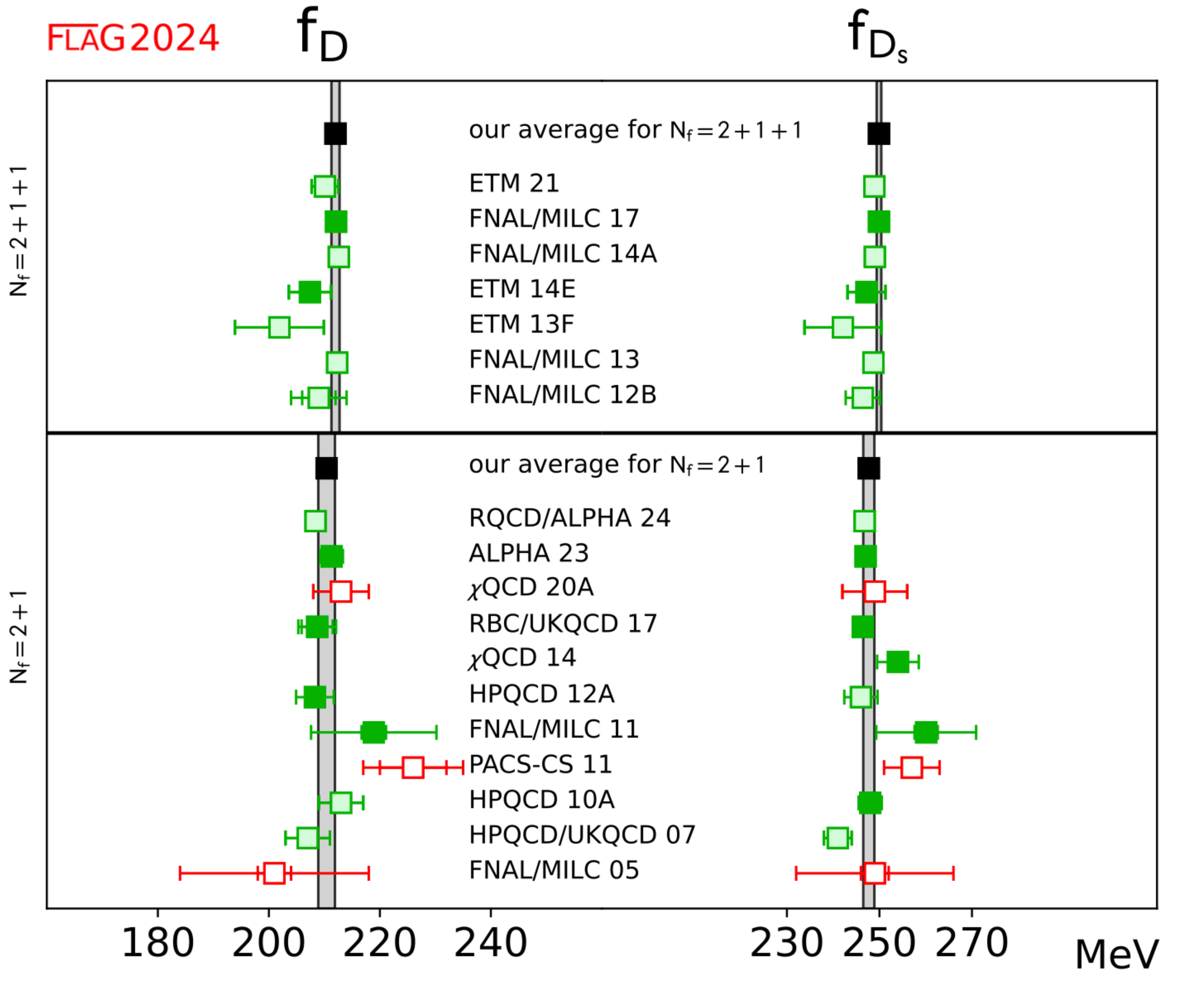
- **Theory uncertainties** on K/π leptonic decays **can become comparable** with experimental ones
- Current kaon experiments: **NA62** (CERN) & **KOTO** (J-PARC)
- Proposal for high intensity kaon program at **CERN** (**HIKE**) was not approved [March '24], so the future of kaon measurements is a bit unclear...
- New analyses of old datasets possible?

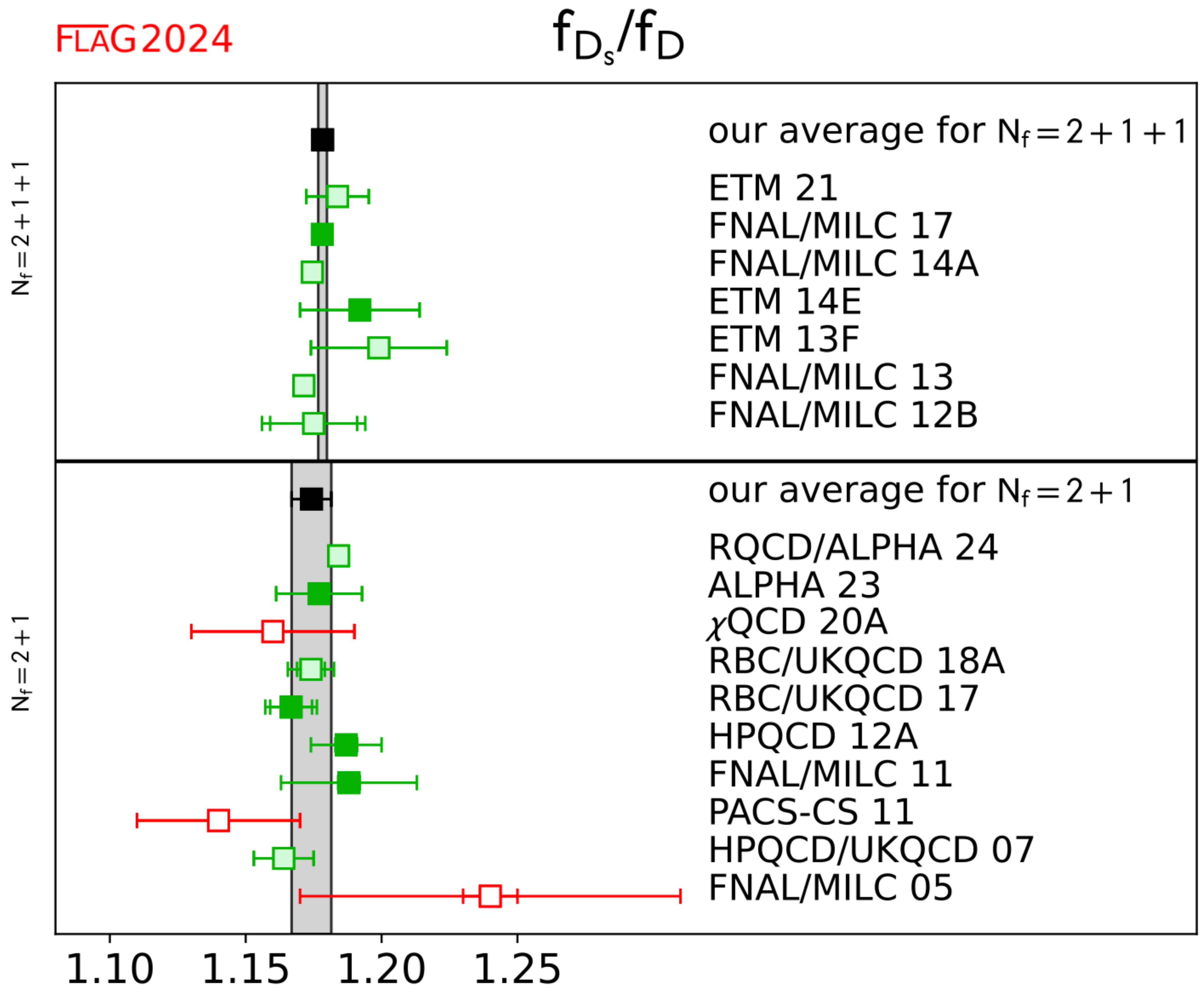
... and heavy ones

- Lattice QCD+QED calculations of **D and Ds mesons decays** are under study
- **B-meson decays** require very fine lattices, but exploratory work on **radiative decay** $B^+ \to \mu^+ \bar{\nu} \gamma$ is ongoing



Charmed QCD decay constants



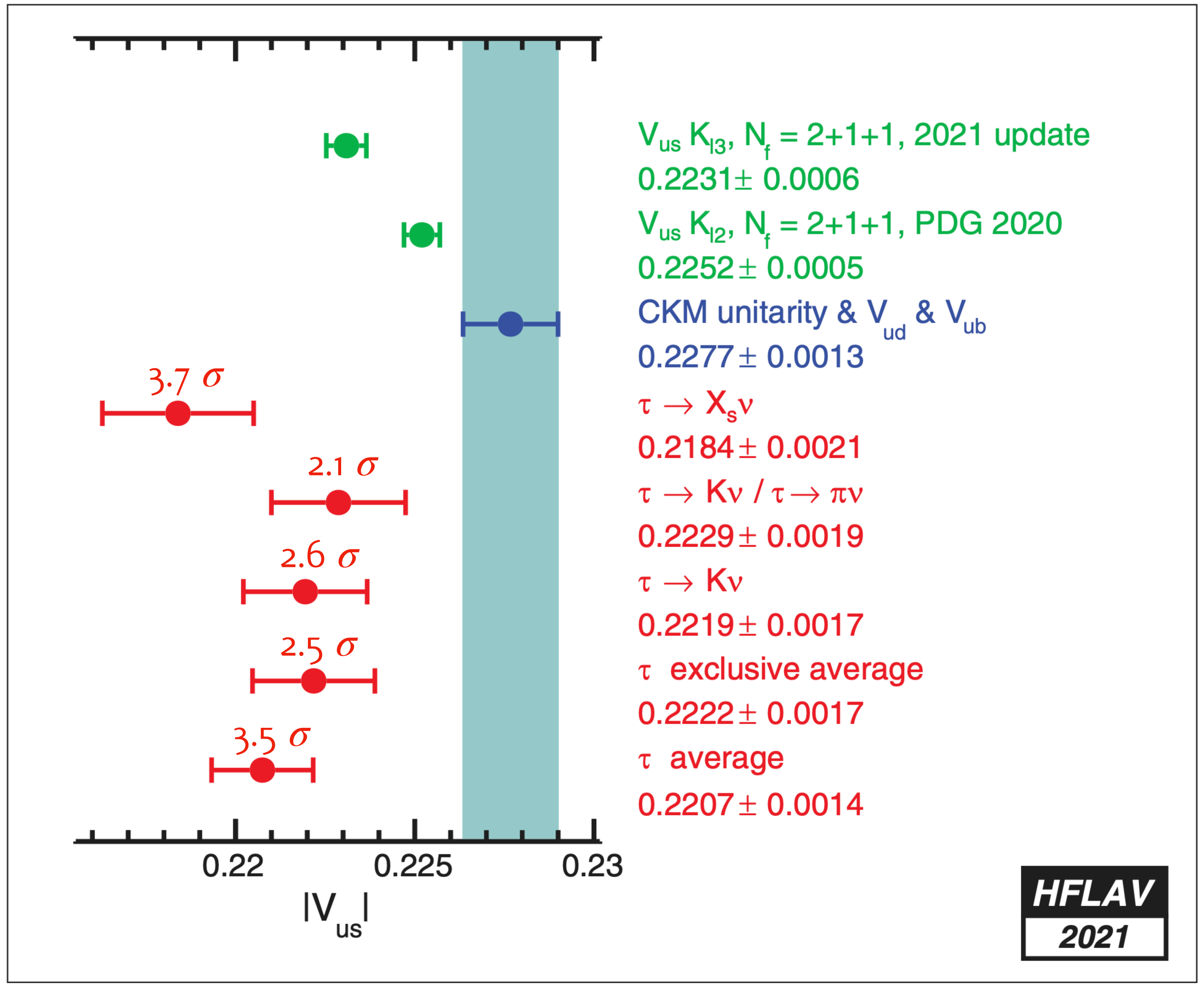


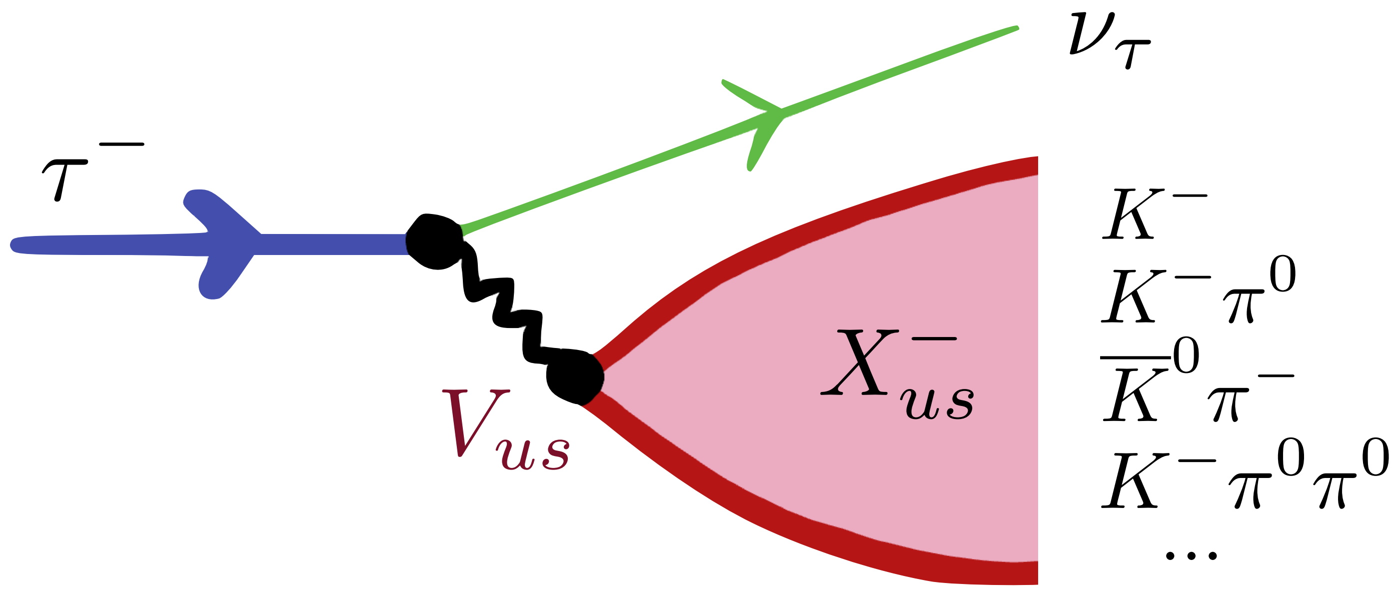
$f_D = 212.0(0.7)$ MeV $f_{D_s} = 249.9(0.5)$ MeV

$f_{D_s}/f_D = 1.1783(0.0016)$

Inclusive hadronic τ decays

Alternative determinations of $|V_{us}|$ can be obtained from inclusive **hadronic τ decays**





- And yet another **puzzle**: lower value of $|V_{us}|_{\tau-\mathrm{incl.}}$
- Inclusive $\tau \to X_{us}\nu_\tau$ result in HFLAV plot obtained using truncated operator product expansion (OPE)
- Exclusive channels give results larger than $|V_{us}|_{\tau-\mathrm{incl.}}$ but smaller than that obtained imposing CKM unitarity

Inclusive hadronic τ decays

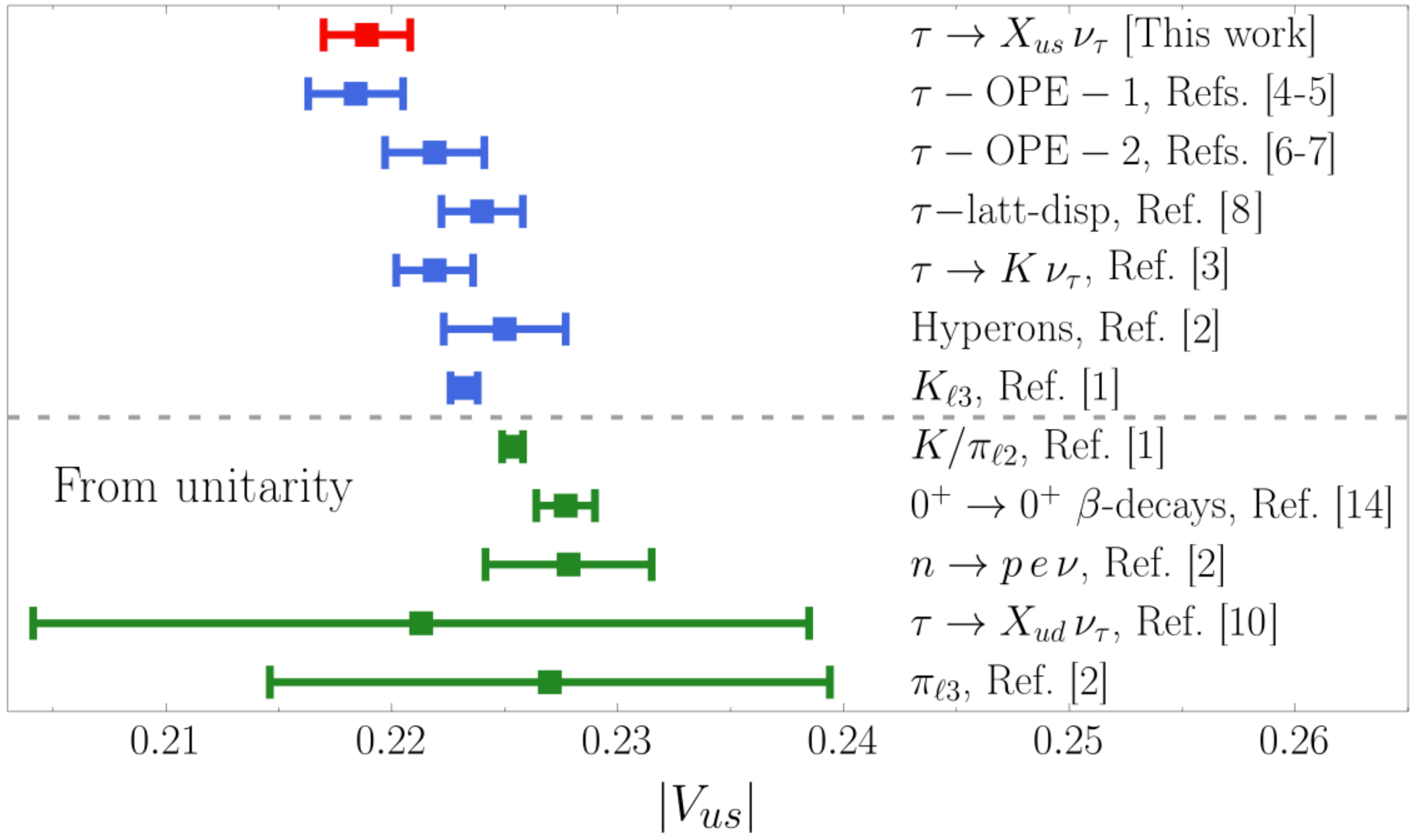
A.Evangelista et al. (ETMC), PRD **108** (2023)
C.Alexandrou et al. (ETMC), PRL **132** (2024)

Recent calculation obtains **inclusive decay rate** using **smeared spectral densities** reconstructed from finite-volume **Euclidean lattice correlators** M.Hansen, A.Lupo & N.Tantalo, PRD **99** (2019)

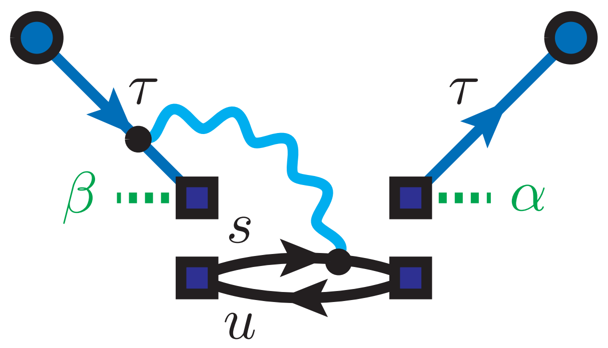
$$\rho(\omega) = \langle \tau^- | H_{\mathrm{w}}^{us} \, (2\pi)\delta(\mathbb{H} - \omega) \, H_{\mathrm{w}}^{us} | \tau^- \rangle$$

$$\hat{\rho}_L(E,\epsilon) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \Delta_\epsilon(E,\omega) \, \rho_L(\omega) = \sum_{t=0}^{T} g_t(E,\epsilon) \, C_L(t)$$

$$\Rightarrow \quad \Gamma(\tau \to X_{us}\nu_\tau) = \lim_{\epsilon\to 0} \lim_{L\to\infty} \frac{\hat{\rho}_L(m_\tau,\epsilon)}{2m_\tau}$$



› **Next step:** inclusion of **QED** and **strong isospin-breaking effects**



CP violation in neutral kaons

flavour eigenstates

$$\mathrm{i}\frac{\partial}{\partial t}\begin{pmatrix} K^0(t) \\ \overline{K}^0(t) \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)\begin{pmatrix} K^0(t) \\ \overline{K}^0(t) \end{pmatrix}$$

weak eigenstates

$$|K_{L,S}\rangle = \frac{1}{\sqrt{1+|\bar{\epsilon}|^2}}\left(\bar{\epsilon}\,|K_\pm\rangle + |K_\mp\rangle\right)$$

CP eigenstates

$$|K_\pm\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle \pm |\overline{K}^0\rangle\right)$$

Neutral mesons can mix because the **flavour eigenstates** are different from the **weak eigenstates**

- "Indirect" **CP violation** in the **mixing**

$$|\epsilon| = (2.228 \pm 0.011) \times 10^{-3}$$

PDG, PTET **2022** (2022)

- "Direct" **CP violation** in the **decay**

$$\eta_{+-} = \frac{\langle \pi^+\pi^-|\mathcal{H}_\mathrm{w}|K_L\rangle}{\langle \pi^+\pi^-|\mathcal{H}_\mathrm{w}|K_S\rangle} = \epsilon + \epsilon'$$

$$\eta_{00} = \frac{\langle \pi^0\pi^0|\mathcal{H}_\mathrm{w}|K_L\rangle}{\langle \pi^0\pi^0|\mathcal{H}_\mathrm{w}|K_S\rangle} = \epsilon - 2\epsilon'$$

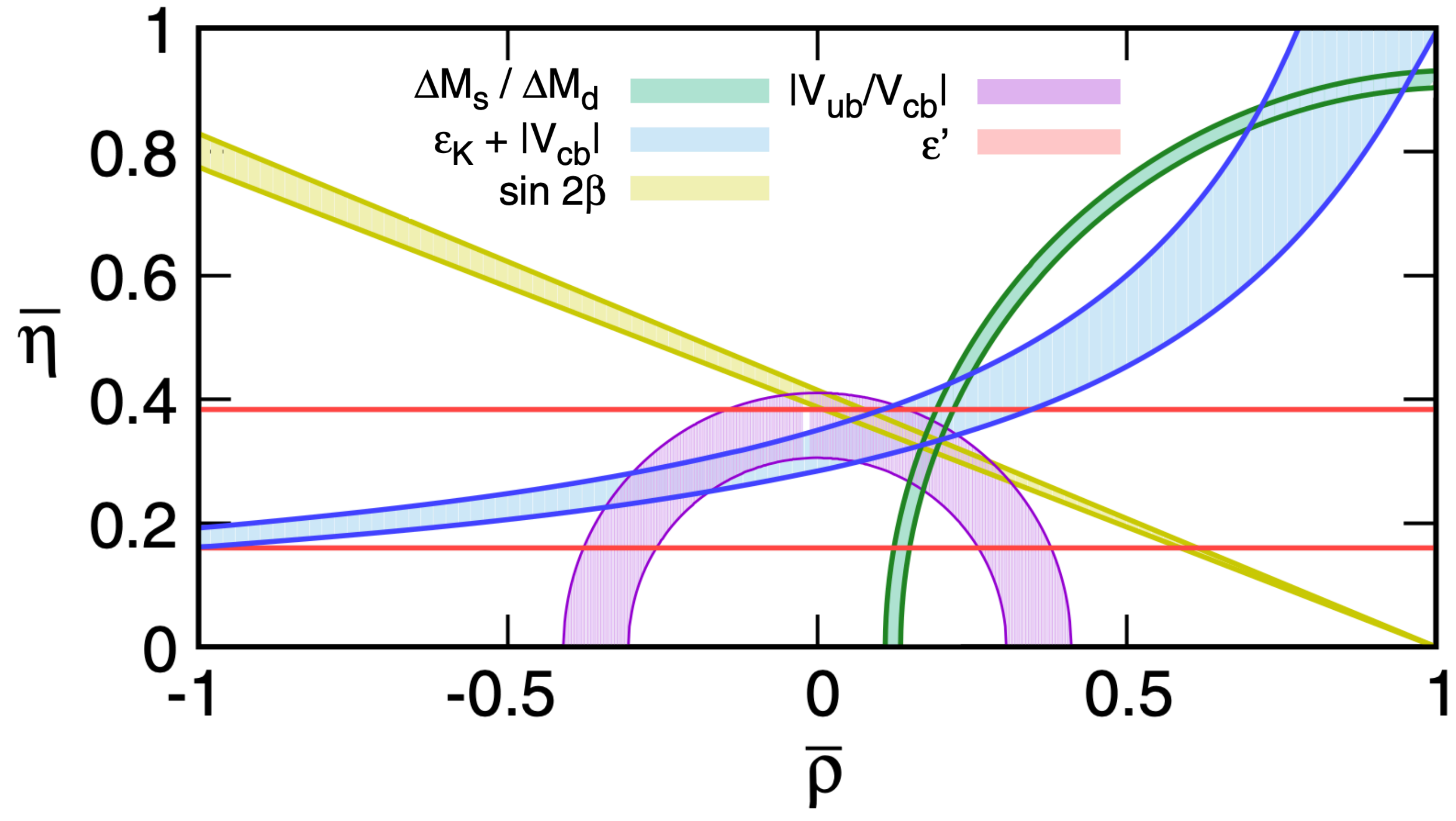
$$\mathrm{Re}(\epsilon'/\epsilon) = (1.66 \pm 0.23) \times 10^{-3}$$

PDG, PTET **2022** (2022)

CP violation in neutral kaons

lattice: $\mathrm{Re}(\epsilon'/\epsilon) = 21.7\,(2.6)_{\mathrm{stat.}}(8.0)_{\mathrm{sys.}} \times 10^{-4}$

experiments: $\mathrm{Re}(\epsilon'/\epsilon) = 16.6\,(2.3) \times 10^{-4}$

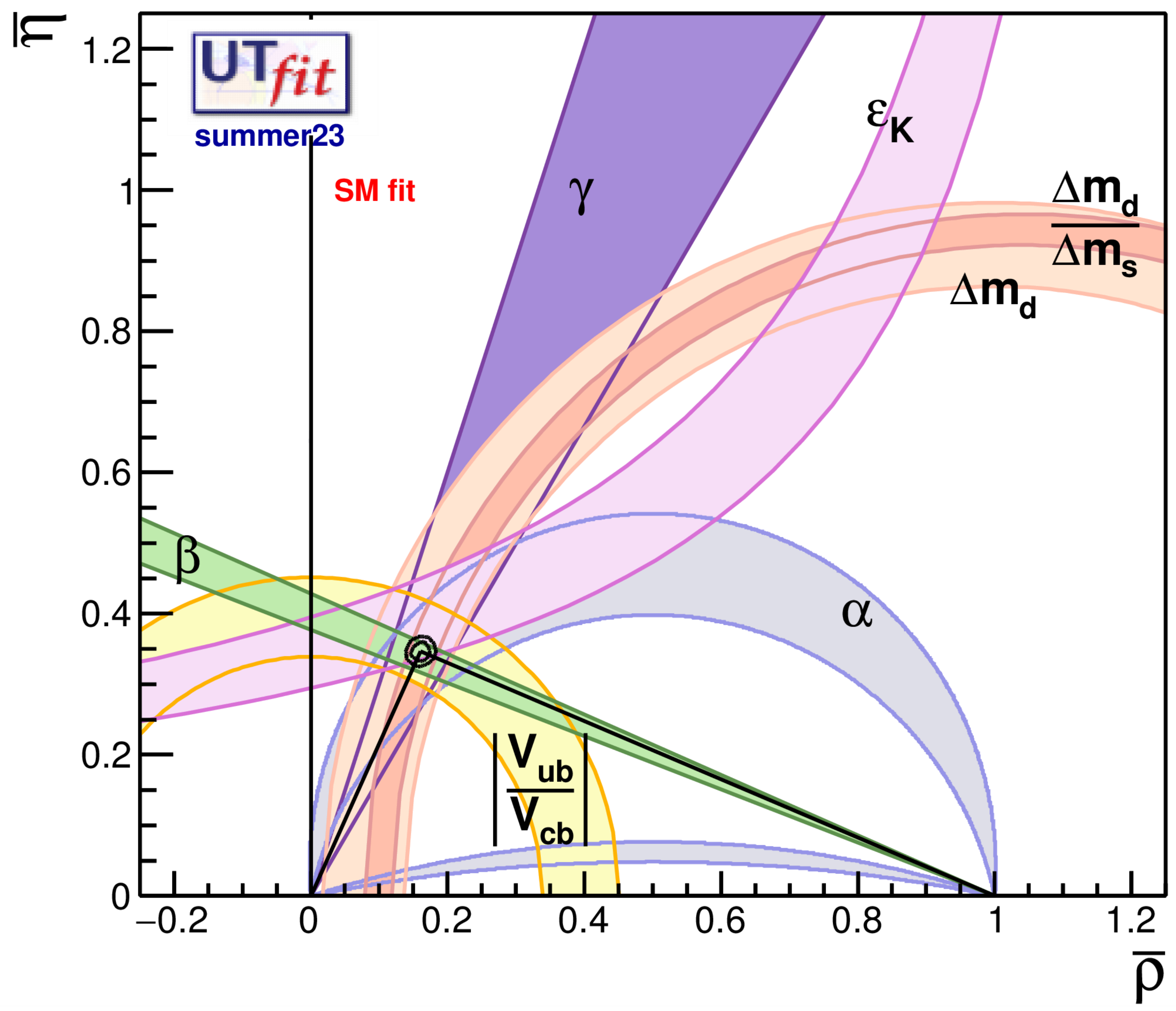


R.Abbott et al., PRD **102** (2020)

- $\mathrm{Re}(\epsilon'/\epsilon)$ currently at **40% precision** (RBC-UKQCD). Significant improvements expected in next couple of years -> QED?

$\langle \overline{K}^0 | \mathcal{H}_{\mathrm{w}} | K^0 \rangle_{\mathrm{SD}}$ P.A.Boyle et al. PRD 110 (2024) (dominates)

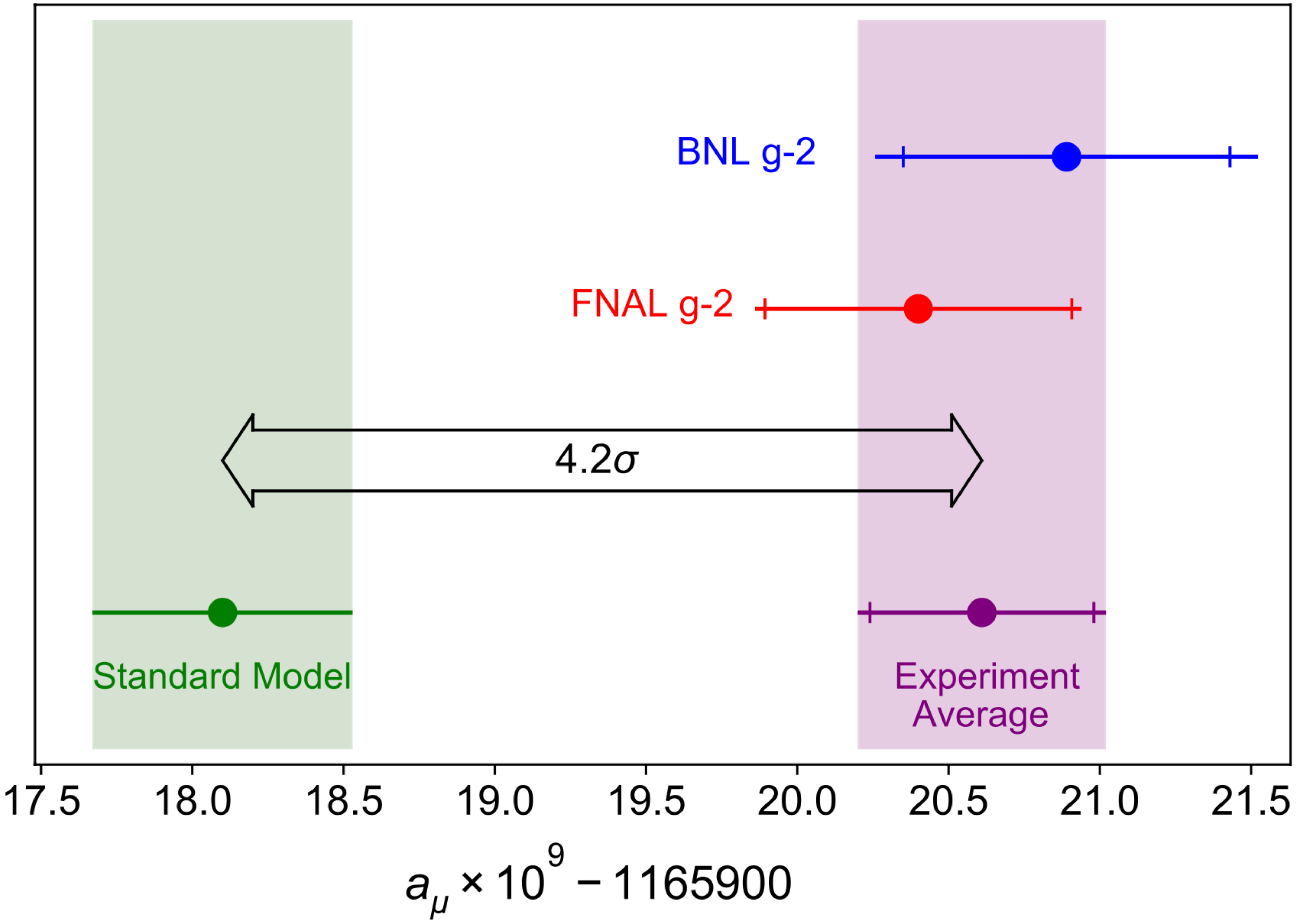
$\langle \overline{K}^0 | \mathcal{H}_{\mathrm{w}} | K^0 \rangle_{\mathrm{LD}}$ Z.Bai et al. PRD 109 (2024)

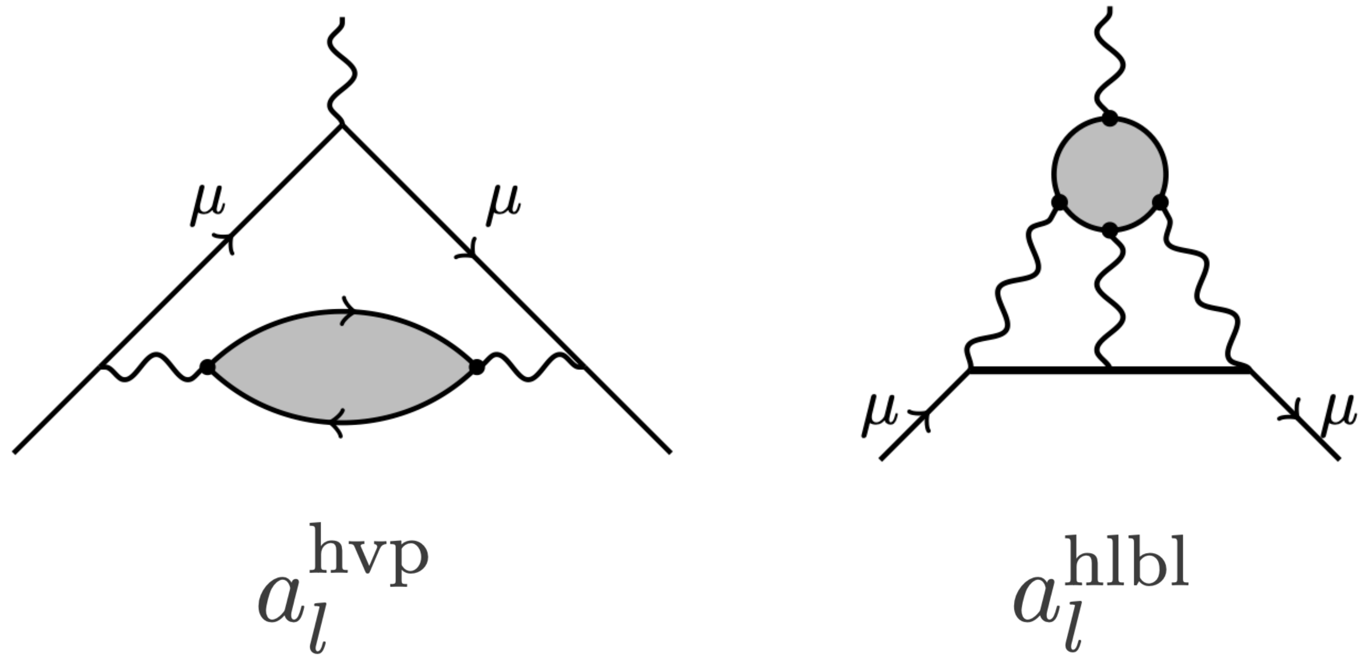


- $(\epsilon_K)_{\mathrm{LD}}$ also at **40% precision** (RBC-UKQCD). Errors of ~10% can be achieved on the long term
- Lattice inputs to $(\epsilon_K)_{\mathrm{SD}}$ can be computed with high precision, but **overall uncertainty is dominated by** $|V_{cb}|$
- $|V_{cb}|$**-puzzle** affects the SM prediction for $|\epsilon_K|$

The muon anomalous magnetic moment

The picture in 2020-2021





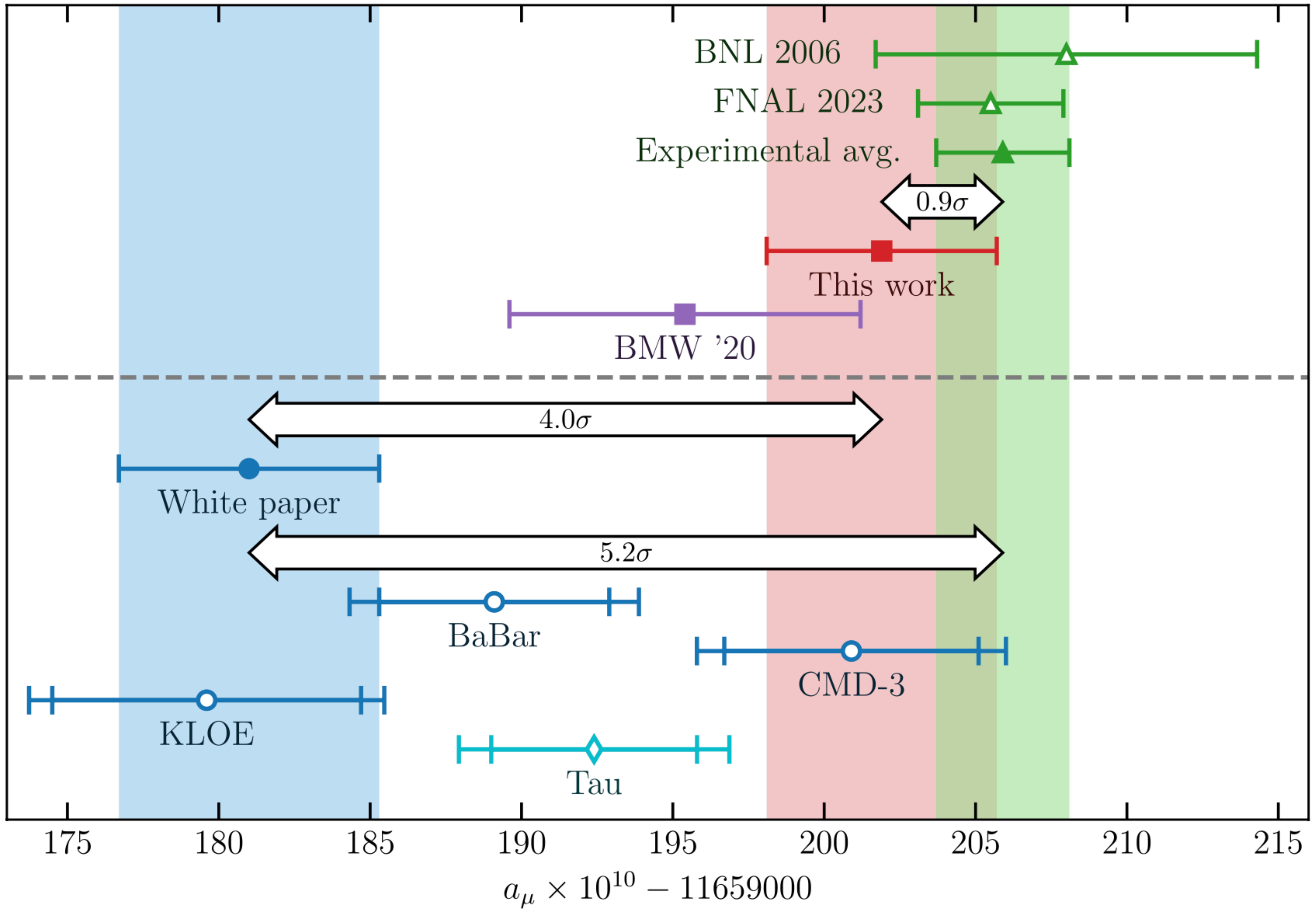
$$a_l^{\mathrm{SM}} = a_l^{\mathrm{QED}} + a_l^{\mathrm{EW}} + a_l^{\mathrm{had}}$$

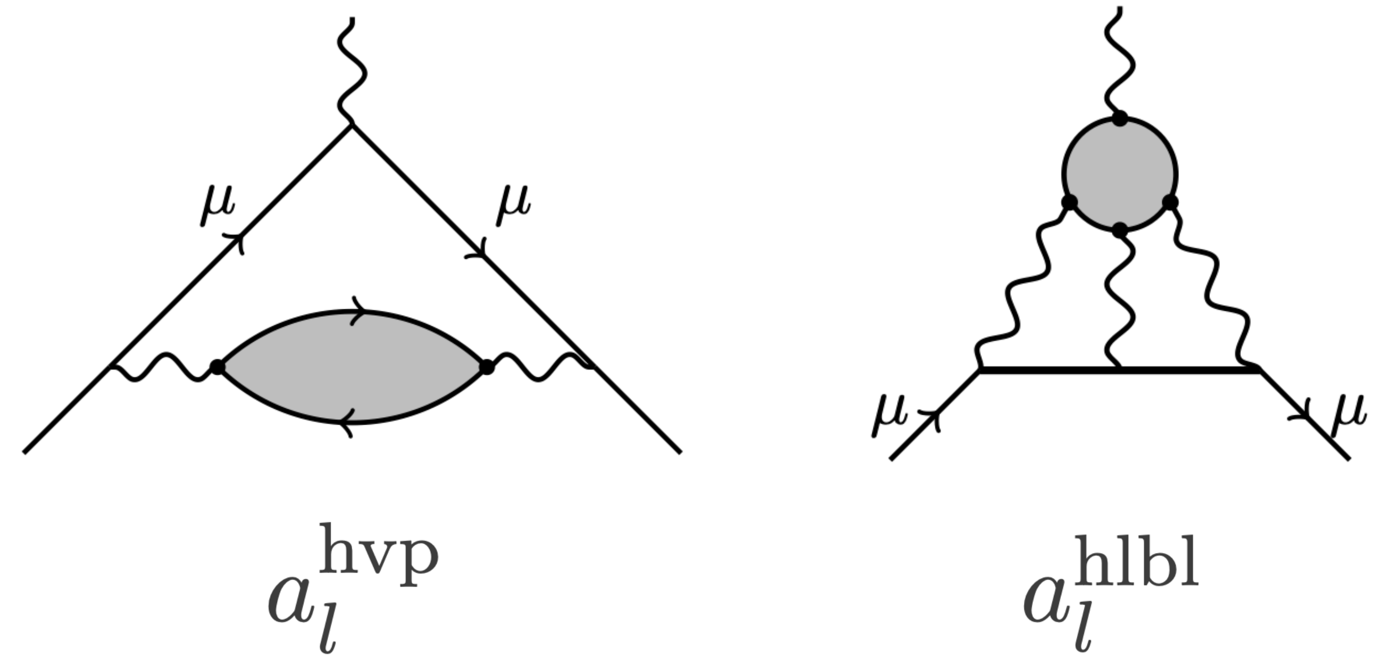
Standard Model prediction based on the **White Paper** of the Muon g-2 Theory Initiative

Aoyama et al., 2006.04822

The muon anomalous magnetic moment

The picture in 2024





$$a_l^{\mathrm{SM}} = a_l^{\mathrm{QED}} + a_l^{\mathrm{EW}} + a_l^{\mathrm{had}}$$

Standard Model prediction based on the **White Paper** of the Muon g-2 Theory Initiative

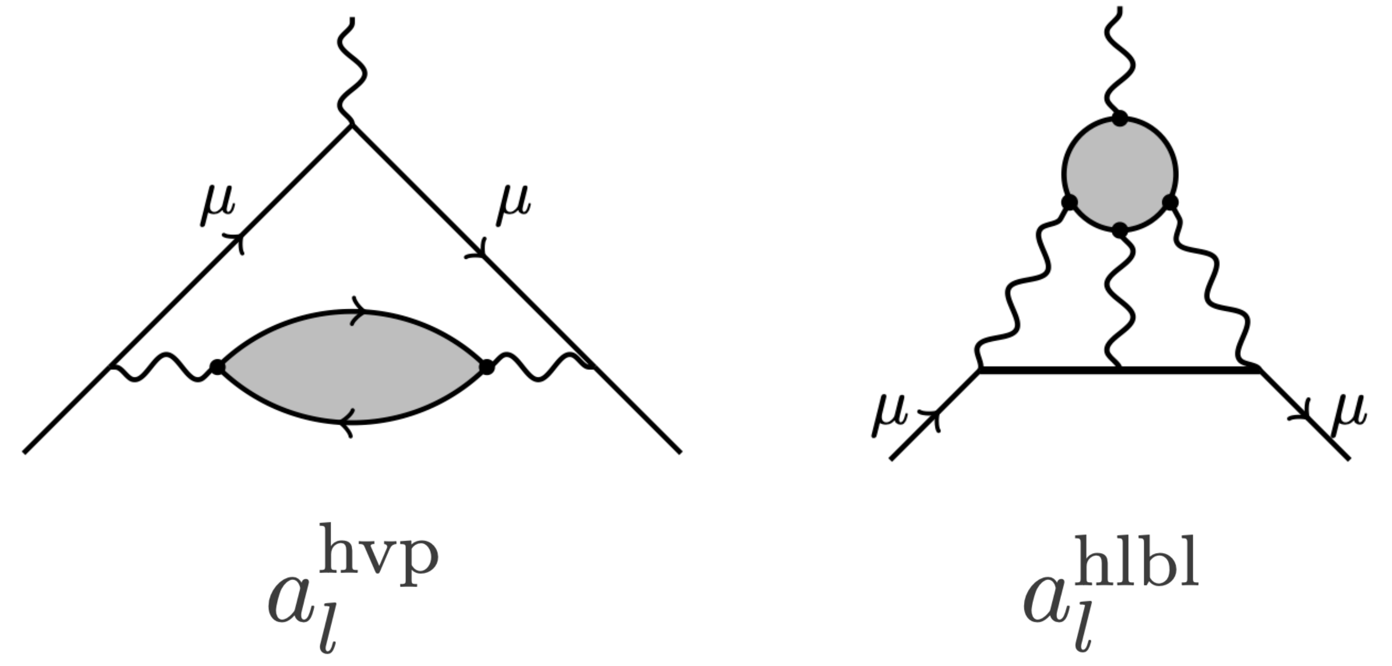
Aoyama et al., 2006.04822

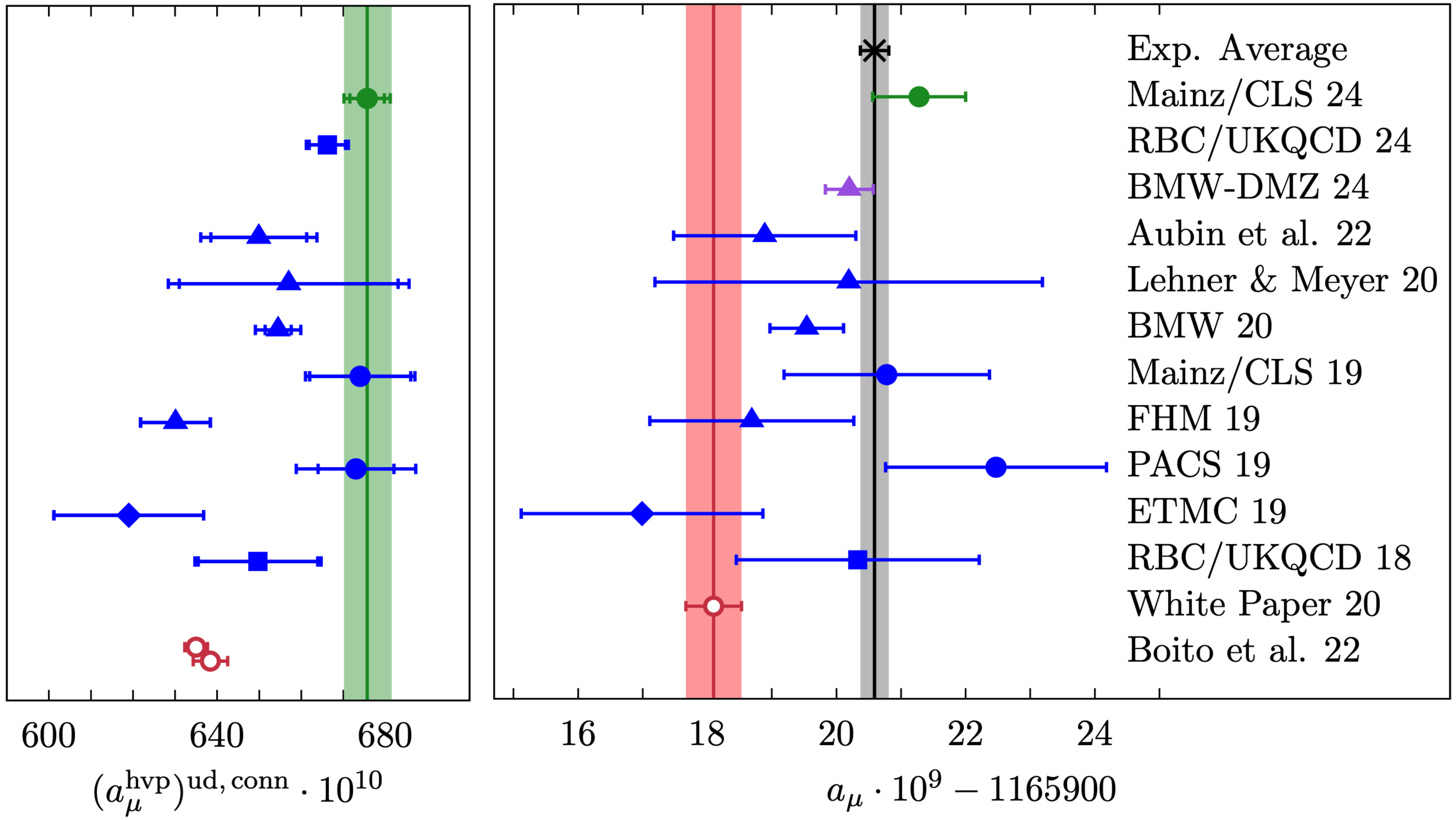
The situation is now more complicated...

Boccaletti et al. [BMW], 2407.10913

The muon anomalous magnetic moment

The picture in 2024





D.Djukanovic et al. [Mainz/CLS], 2411.07969

Conclusions

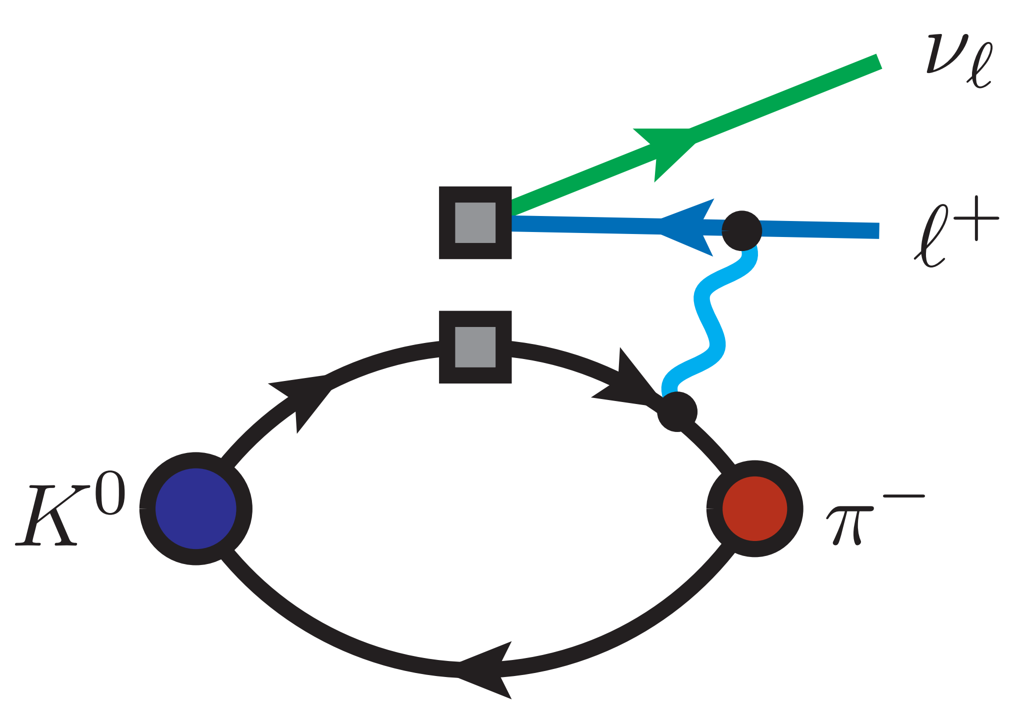
- **Flavour physics** offers unique opportunities for **indirect searches** of New Physics
- **Lattice QCD** is at a mature stage on many flavour observables and is now in the **precision era**:
  towards physical pion masses, QED & isospin-breaking effects, physical b quarks, ...
- Progress expected on **various flavour observables** in next years
  Maybe some of the tensions will be clarified?
- **Developments of new techniques** & **algorithmic advances** make it possible to study **long-distance observables** considered before unaccessible
- The **muon g-2** effort teaches us that the lattice calculations can get **precise impactful results.**
  What will be the next "g-2"?

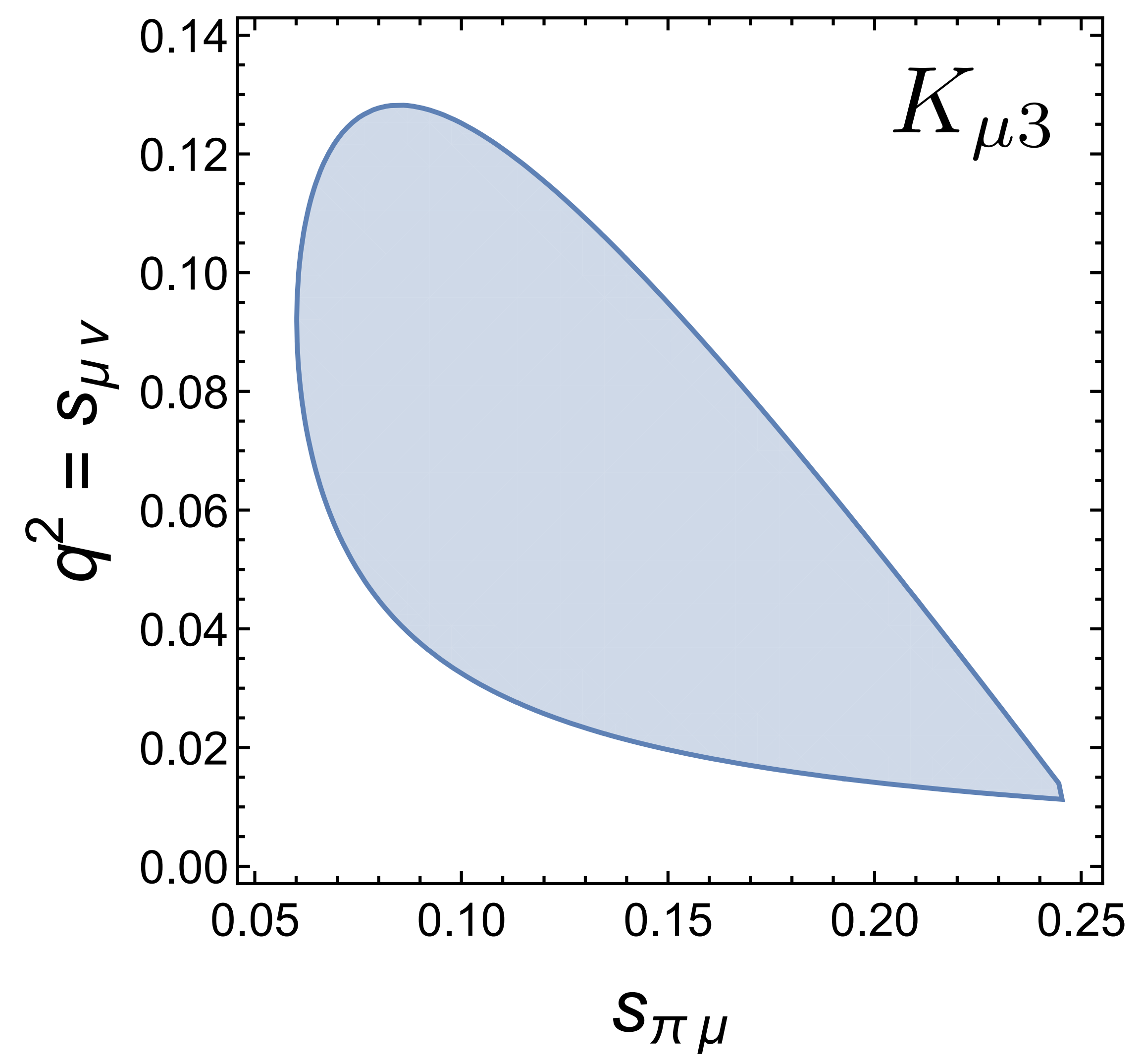
Thank you

This work has received funding from the European Union's Horizon Europe research and innovation programme under the Marie Sklodowska-Curie grant agreement No 101108006

Backup slides

Semileptonic kaon decays





To go beyond current precision, we need to include **isospin-breaking effects** computed from first principles lattice QCD+QED.

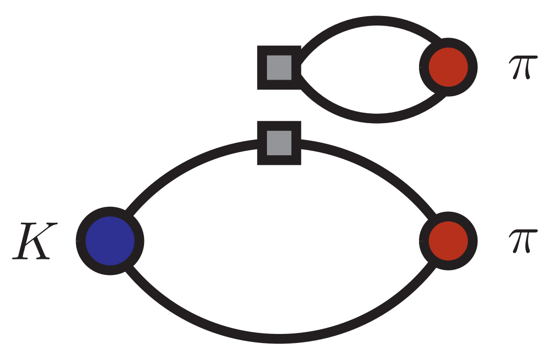
Significant additional **difficulties** compared to leptonic decays:

- integration over **three-body phase-space**
- problems of **analytical continuation** from Euclidean to Minkowski when intermediate on shell π-ℓ states are lighter than external ones

Proper **finite-volume QED formalism is still missing**, but solutions are under study by different groups.

Recent **QED$_\infty$ proposal**: N.Christ et al., PRD 108 (2023) & PoS LATTICE2023 (2024) 266

Direct CP violation in $K \to \pi\pi$



If **isospin-symmetry** is conserved, then the CP violation parameters can be expressed as

$$\frac{\epsilon'}{\epsilon} = \frac{i e^{i(\delta_2 - \delta_0)}}{\sqrt{2}\epsilon} \frac{\mathrm{Re}(A_2)}{\mathrm{Re}(A_0)} \left[\frac{\mathrm{Im}(A_2)}{\mathrm{Re}(A_2)} - \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)} \right]$$

$$A_I = \langle (\pi\pi)_I | H_W^{\Delta S=1} | K \rangle$$

$\delta_I = \pi\pi$ scattering phase shifts

($I =$ isospin)

1. **RBC-UKQCD** performed **first calculation** of ϵ' in 2015 — Z.Bai et al., PRL **115** (2015)
2. **Improved result** in 2020: 3.5x more statistics + improved systematics — R.Abbott et al., PRD **102** (2020)

lattice: $\mathrm{Re}(\epsilon'/\epsilon) = 21.7\,(2.6)_{\mathrm{stat.}}(8.0)_{\mathrm{sys.}} \times 10^{-4}$

experiments: $\mathrm{Re}(\epsilon'/\epsilon) = 16.6\,(2.3) \times 10^{-4}$

Direct CP violation in $K \to \pi\pi$

Systematic error budget

(from C.Kelly @Lattice2023)

- (~12%) Perturbation theory in Wilson coeffs to match 3f – 4f weak EFT at m_c
  - Improve with 4f calculation (active charm) : computationally infeasible?
  - Non-perturbative calculation of matching matrix : investigation underway

    [M.Tomii, PoS LATTICE2018 (2019) 216]

- (~23%) Lack of EM+isospin-breaking contributions in lattice calculation
  - Lattice measurement of these effects extremely challenging but approach is being formulated. *[Phys.Rev.D* 106 (2022) 1, 014508] *[Christ, PoS* LATTICE2021 (2022) 312]

    V.Cirigliano et al., JHEP **02** (2020)

- (~12%) Use of single lattice spacing to compute I=0 amplitude
  - Repeat calculation with multiple, finer lattice spacings: **my current focus**

- Intense work by RBC-UKQCD to reduce ~12% error due single lattice spacing (C.Kelly @Lattice2024) + parallel ongoing project using different computational approach (M.Tomii @Lattice2024)

- IB correction will soon become relevant! (but very tricky to compute on the lattice) Usually O(1%), but the "$\Delta I = 1/2$ rule" can give a ~20x enhancement in ϵ'/ϵ.

Indirect CP violation in kaon mixing

$$\boxed{\epsilon = e^{i\phi_\epsilon}\sin(\phi_\epsilon)\left(\frac{-\mathrm{Im}M_{\bar{0}0}}{\Delta M_K} + \frac{\mathrm{Im}(A_0)}{\mathrm{Re}(A_0)}\right)}$$

$$\tan(\phi_\epsilon) = \frac{\Delta M_K}{\Delta\Gamma_K/2}$$

$$\Delta\Gamma_K = \Gamma_{K_S} - \Gamma_{K_L}$$

$$\Delta M_K = M_{K_L} - M_{K_S}$$

The quantity $M_{\bar{0}0}$ splits into a short and long distance parts

$$M_{\bar{0}0} = \langle\overline{K}^0|\mathcal{H}_\mathrm{w}|K^0\rangle = \langle\overline{K}^0|\mathcal{H}_\mathrm{w}|K^0\rangle_\mathrm{SD} + \langle\overline{K}^0|\mathcal{H}_\mathrm{w}|K^0\rangle_\mathrm{LD}$$

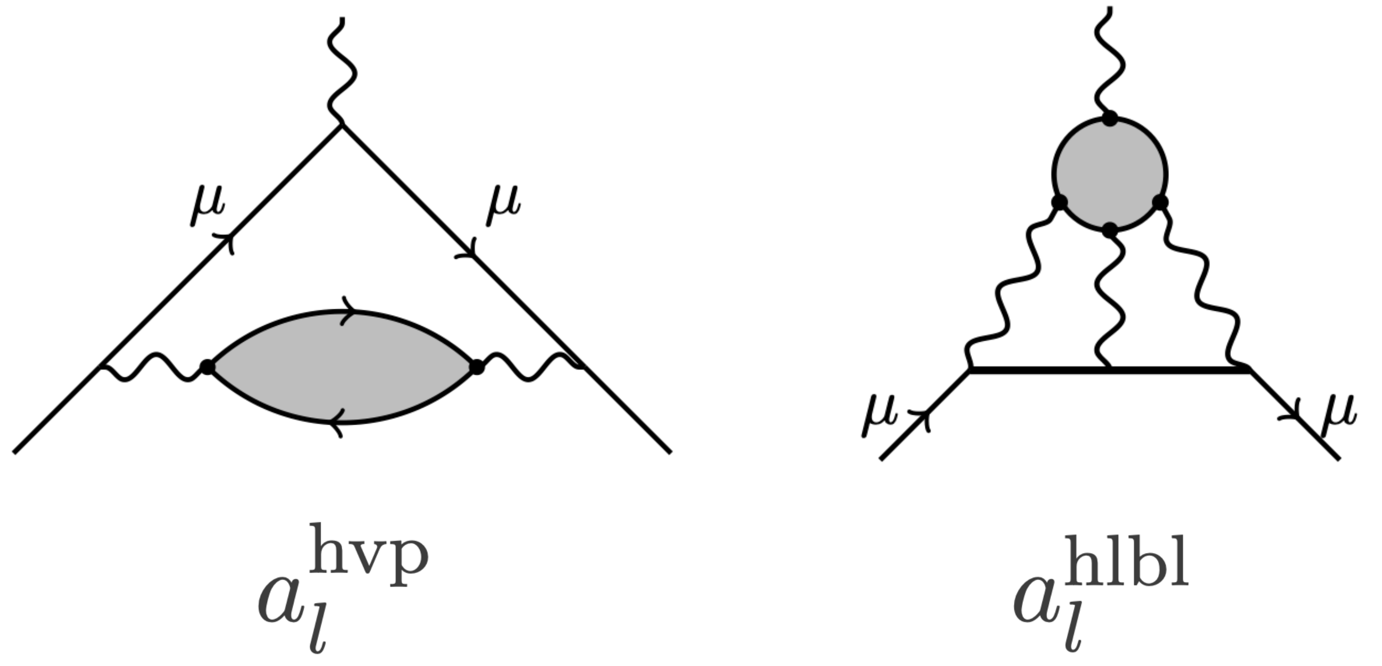
$$= \langle\overline{K}^0|\mathcal{H}_\mathrm{w}^{\Delta\mathrm{S}=2}|K^0\rangle + \mathcal{P}\sum_n \frac{\langle\overline{K}^0|\mathcal{H}_\mathrm{w}^{\Delta\mathrm{S}=1}|n\rangle\langle n|\mathcal{H}_\mathrm{w}^{\Delta\mathrm{S}=1}|K^0\rangle}{M_K - E_n}$$

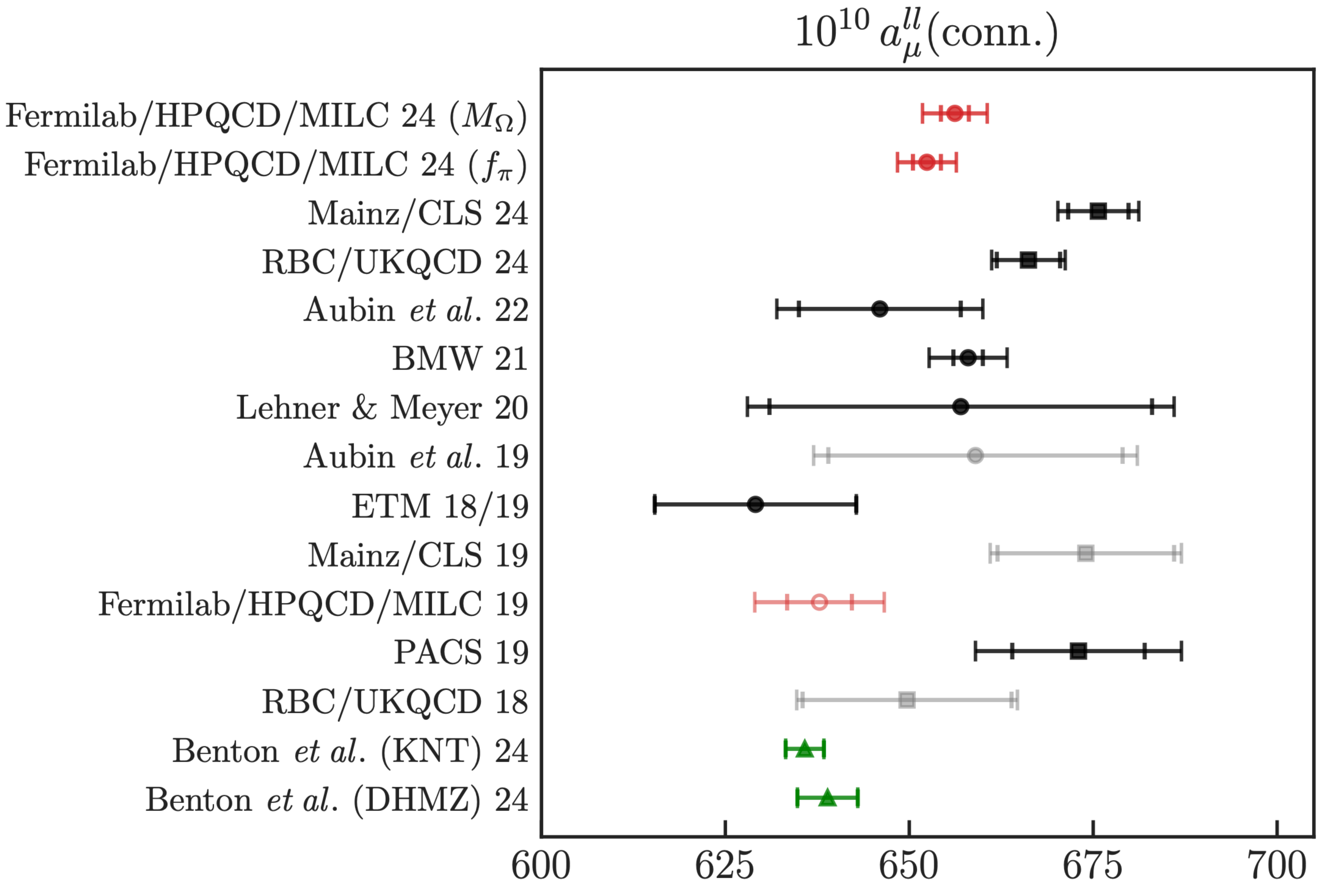
On the **lattice**, we can compute both:

- $\langle\overline{K}^0|\mathcal{H}_\mathrm{w}|K^0\rangle_\mathrm{SD}$ e.g. P.A.Boyle et al. PRD 110 (2024) ⟵ dominating contribution
- $\langle\overline{K}^0|\mathcal{H}_\mathrm{w}|K^0\rangle_\mathrm{LD}$ Z.Bai et al. PRD 109 (2024)

The muon anomalous magnetic moment

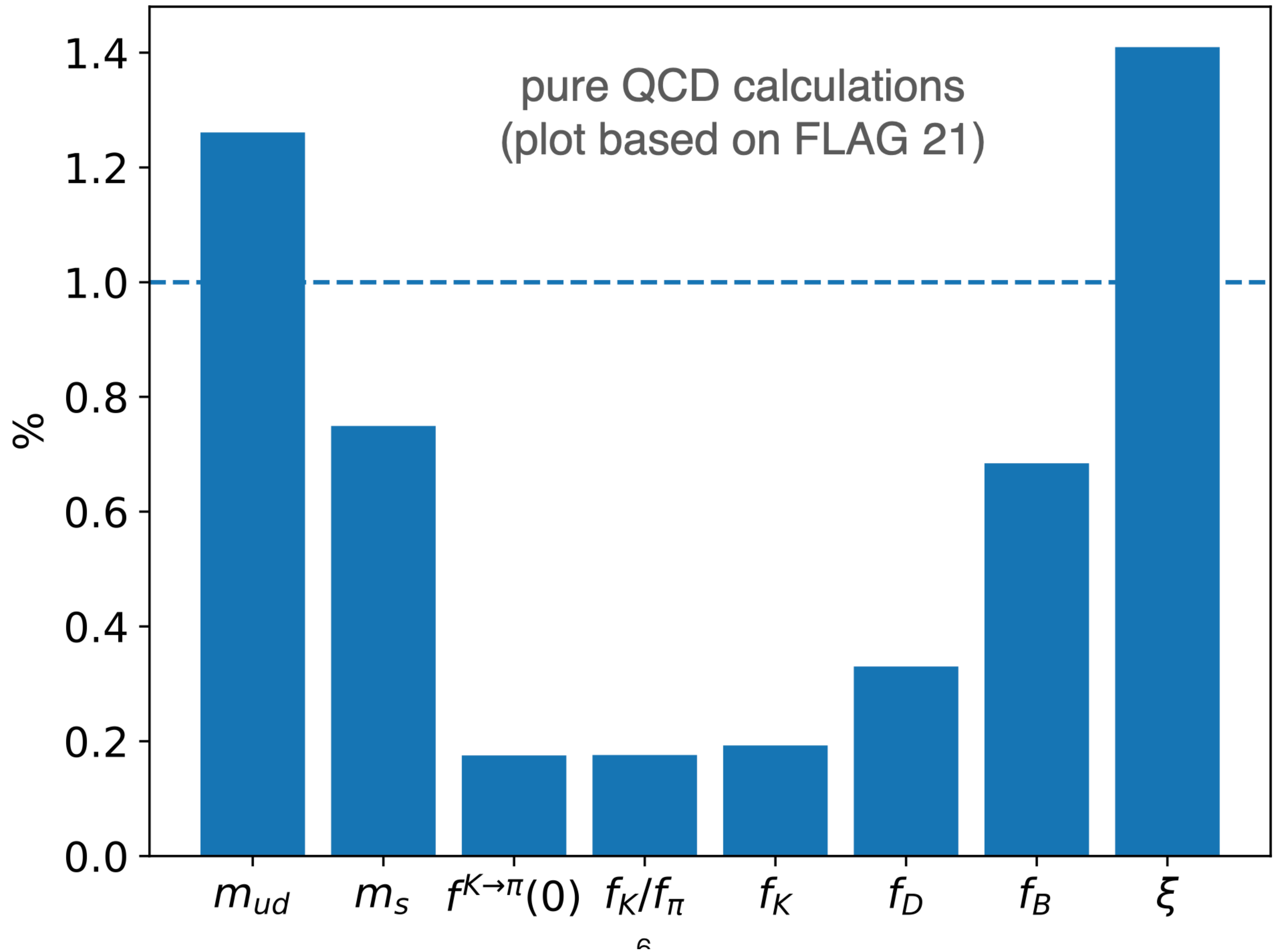
The picture in 2024





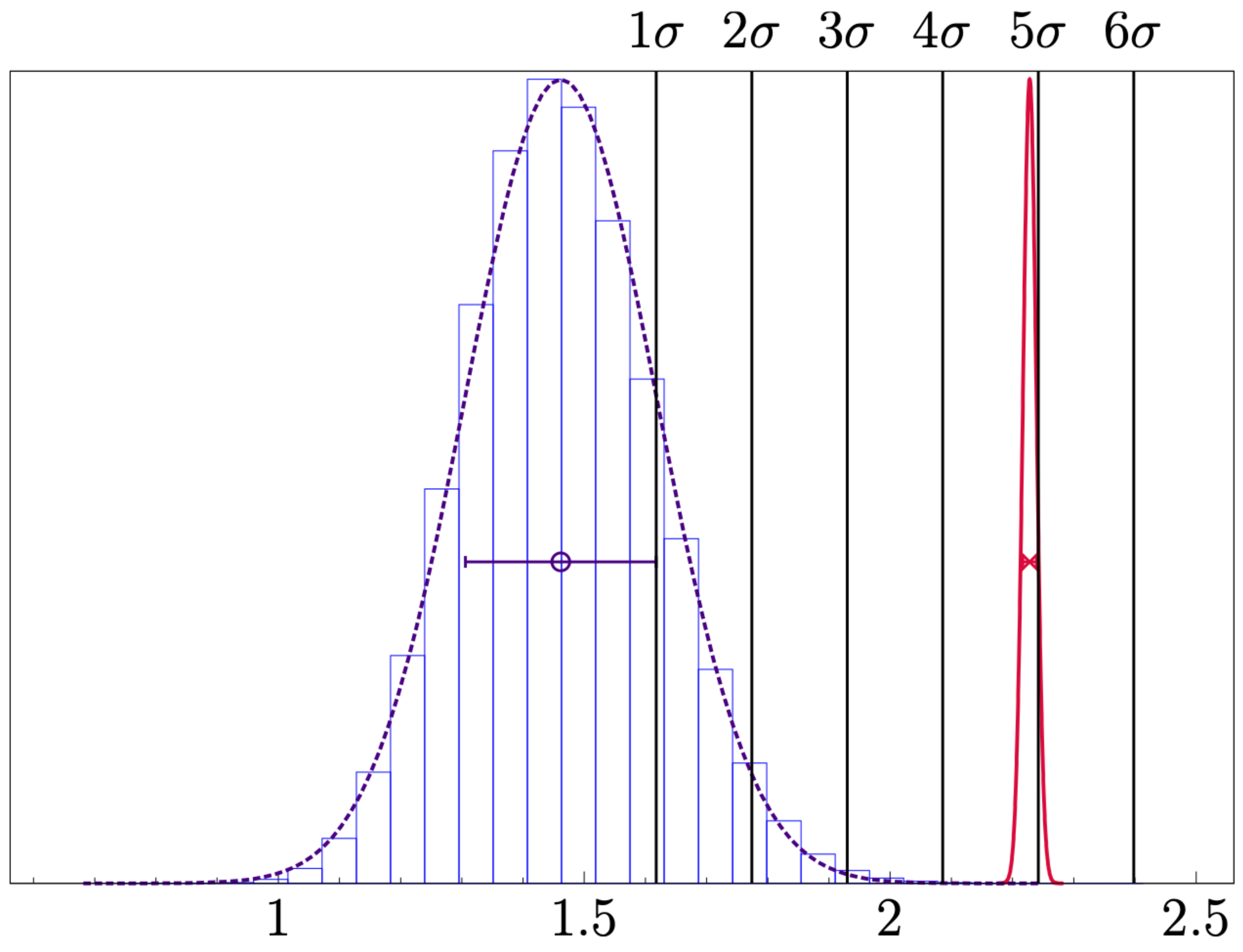
A.Bazavov et al. [Fermilab/HPQCD/MILC], 2412.18491

Current accuracy on some QCD quantities

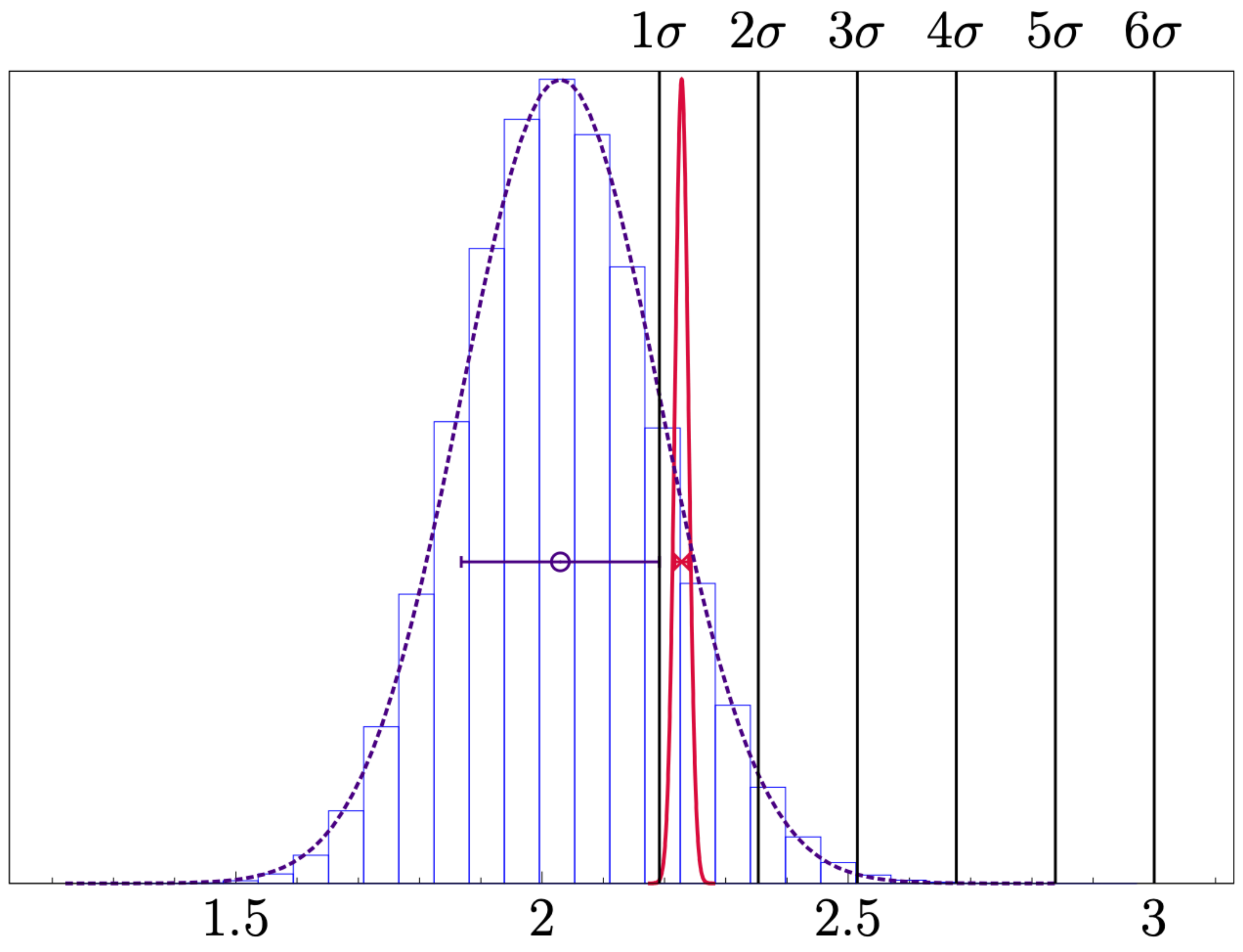


plot by A.Jüttner

ϵ_K and $|V_{cb}|$ puzzle



(a) Exclusive $|V_{cb}|$ (FNAL/MILC-22, BGL)



(b) Inclusive $|V_{cb}|$ (HFLAV-23, 1S scheme)