Electron Cooling for a Muon Collider

Del Larson

Particle Beam Lasers, Inc.

Outline

- Overview of the Problem
- Review of Electron Cooling
- The Coulomb Analogy
- Electron Cooling of Muons – Simplified Examples
- Stability Problems in High Current Low Energy Electron Beams
- A Possible Solution to the Stability Problems
- Summary Comments

The Problem

- Muons are Produced with High Emittance (~ 1 cm)
- Beams Ideally Would Have Low Emittance (<~ 1 micron)
- Desired Physics Output Needs < ~30 micron Emittance or Better
- Cooling is Needed!
- And Cooling must be Fast; Muon lifetime is 2.2 μsec

Electron Cooling

- Electron Cooling Works Best at High Electron Currents
- Electron Cooling Works Best at Low Energy
- Yet High Current, Low Energy Electron Beams Are Often Unstable. So Not Considered. We'll Look at Stability Later, but first we'd see what we'd Like to Achieve.

Specific Opportunities to Apply Cooling

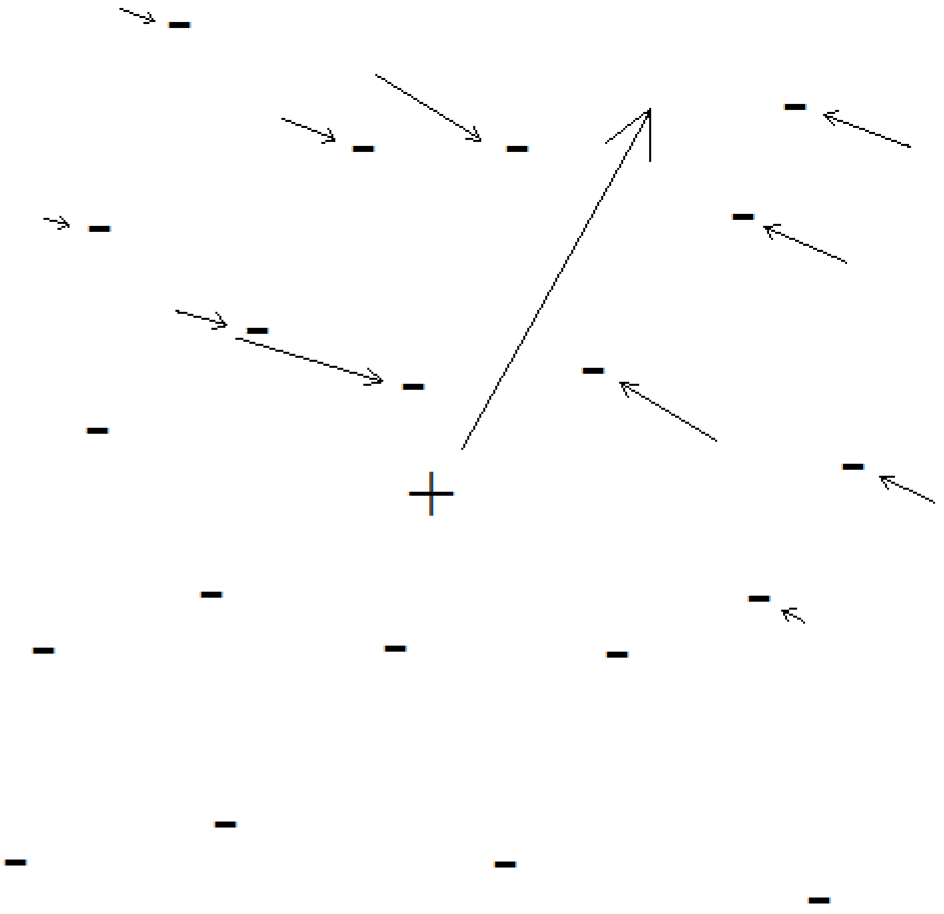
- Presently 6D Ionization Cooling is Proposed to Take the Emittance from ~1 cm to ~300 microns
- A ‘Final’ Cooler Cools From ~300 microns to ~25 microns
- Ideally, a Further (Post-Final?) Cooling to ~0.3 microns Would be Very Beneficial

Potential Problems

- The Problem: Ionization Cooling Is Not Yet Fully Demonstrated and May Face Technical Hurdles, especially in the Final Ionization Cooler
- We Should Explore All Alternatives!

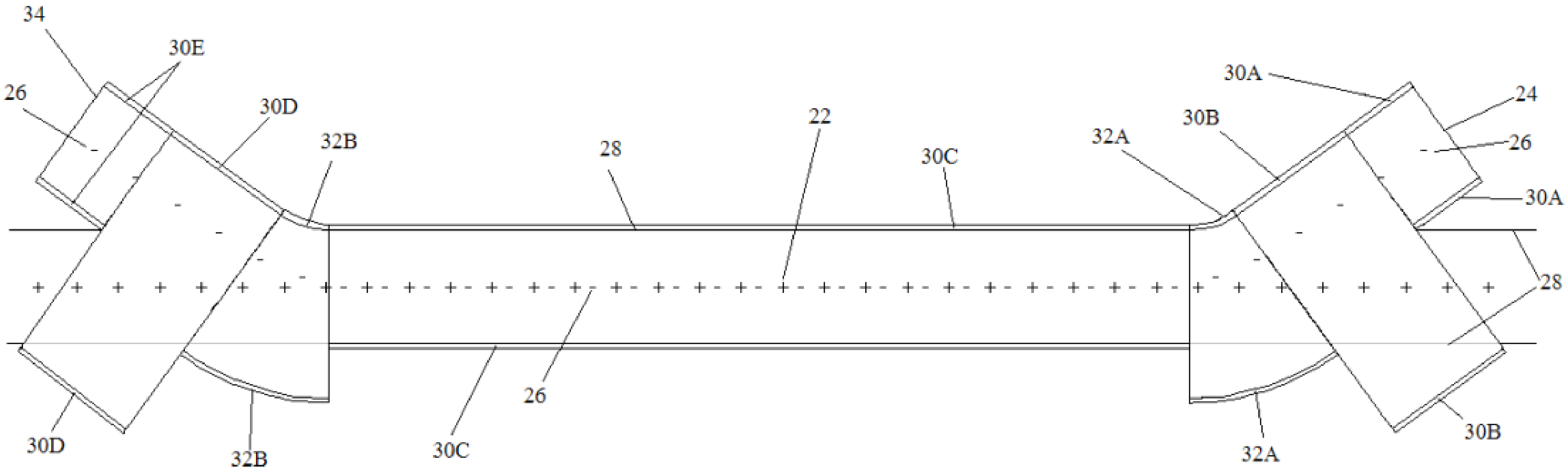
What Is Electron Cooling?

- Spitzer, 1956: Warm Ions Come to Equilibrium with Cooler Electrons in a Plasma



What Is Electron Cooling?

- Budker, 1966: Electron Beam is Simply a Moving Electron Plasma. Superimpose electron beam on ion beam to cool:



Electron Cooling Formulas

- $dE/dx = [\omega_p^2 z^2 e^2/v^2] \ln(\Lambda v/\omega_p b_{min})$ (Found in Textbooks) (Eq. 1)
- Now $\ln(\Lambda v/\omega_p b_{min}) \sim 10$; $\omega_p^2 = 4\pi n e^2/m = 4\pi n c^2 r_e$; and $r_i = e^2/m_i c^2$ Leaves:
- $dE/dx = [40\pi n c^4 r_e r_i m_i/v^2]$ (Eq. 2)
- With $p = m_i v$, $dE/dx = v dp/dx = dp/dt = m_i dv/dt$ we get to:
- $dv/dt = -[40\pi n c^4 r_e r_i/v^2]$
- We must integrate over the velocity distribution $g_e(\mathbf{v_e})$:
- $d\mathbf{v}/dt = -40\pi n c^4 r_e r_i \iiint [\mathbf{u}\, g_e(\mathbf{v_e})/(\mathbf{v}-\mathbf{v_e})^2] d\mathbf{v_e}$
  - $\mathbf{u}$ is a unit vector in the direction of the relative velocity, $\mathbf{u} = (\mathbf{v}-\mathbf{v_e})/|\mathbf{v}-\mathbf{v_e}|$.
- Electron density is given by $n = I/\pi a^2 e \beta c$ leaving
- $d\mathbf{v}/dt = -[40 I c^3 r_e r_i/a^2 e\beta] \iiint [\mathbf{u}\, g_e(\mathbf{v_e})/(\mathbf{v}-\mathbf{v_e})^2] d\mathbf{v_e}$ (Eq. 3)

The Coulomb Analogy

- Recall Eq. 3 from the Previous Slide:
- $d\mathbf{v}/dt = -[40Ic^3 r_e r_i/a^2 e\beta] \iiint [\mathbf{u}\ g_e(\mathbf{v_e})/(\mathbf{v}\text{-}\mathbf{v_e})^2]d\mathbf{v_e}$ (Eq. 3)
- Now also Recall the Expression for the Coulomb Force:
- $\delta F_{\mathrm{Coulomb}} = [K_2/(\mathbf{r} - \mathbf{r_p})^2]\rho(\mathbf{r_p})d\mathbf{r_p}$
- Cooling dependence on v same as Coulomb dependence on r
- For a Sphere of Uniform Density we Know:
  - Coulomb Force Increases Linearly With Radius Inside of Sphere
  - Coulomb Force Decreases as $1/r^2$ Outside of Sphere
- By Analogy (the Coulomb Analogy) We Get:
- $d\mathbf{v}/dt = -[40Ic^3 r_e r_i/a^2 e\beta][\mathbf{u}/\mathbf{v}^2] = -K_{\mathrm{cool}}[\mathbf{u}/\mathbf{v}^2]\ (v > v_{\mathrm{emax}})$ (Eq. 4)
- $d\mathbf{v}/dt = -K_{\mathrm{in}}\mathbf{v}\ (v < v_{\mathrm{emax}})$ (Eq. 5)

The Cooling Time and Cooler Length

- Recall the last two Expressions from the Previous Slide:
- $d\mathbf{v}/dt = –[40Ic^3r_er_i/a^2e\beta][\mathbf{u}/\mathbf{v}^2] = –K_{cool}[\mathbf{u}/\mathbf{v}^2]$ ($v > v_{emax}$) (Eq. 4)
- $d\mathbf{v}/dt = –K_{in}\mathbf{v}$ ($v < v_{emax}$) (Eq. 5)
- At $v = v_{emax}$ the expressions must be equal, so we get $K_{in} = K_{cool}/v_{emax}{}^3$.
- For $v < v_{emax}$, $v = v_0\exp(–K_{in}t)$. Defining $\theta_{emax} = v_{emax}/\beta c$, and setting Constants:

- $T_{cool} = [(1.044\text{x}10^7\ C/cm^2)\theta_{emax}{}^3\beta^4] / (I/a^2)$ (e-drop time, $v < v_{emax}$) (Eq. 6)

- We see we want small θ_{emax}, small β and large I/a^2 for FAST COOLING.
- The invariant is the emittance, $\varepsilon = \pi\theta r$. We can set θ by varying r!
- We'll set starting $v = v_{emax}$ as the one sigma thermal velocity of the electrons.
- EXAMPLE: We'll use 20 A/cm^2 (commercially available now) and $\beta = 0.02$.
- Adiabatically expand electron beam radius 7X to reduce θ_{emax} leaving ~0.4 A/cm^2
- For a 10X emittance reduction, cooler is ~5 m long and cool time ~0.84 μsec.

Adiabatic Expansion and Scaling Comments

- Recall Eq. 6, $T_{cool} = [(1.044\text{x}10^7\ C/cm^2)\theta_{emax}{}^3\beta^4] / (I/a^2)$
- We’ve set starting $v = v_{emax}$ as the one sigma thermal velocity of the electrons.
- Since Cooling Time is Proportional to $\theta_{emax}{}^3$ we want θ_{emax} SMALL.
- To make θ_{emax} small, Adiabatic Beam Expansion is Proposed.
- Adiabatic Beam Expansion decreases θ_{emax} but also Increases a.
- Due to Scaling, Adiabatic Beam Expansion is a Linear Cooling Time Improvement.

- Also, Note that $\theta = \theta_N/\beta$. Therefore, Cooling Really Scales as $\theta_{eNmax}{}^3\beta / (I/a^2)$
- Desire is for Low θ, Low β and High I/a^2
- However, Some Increase in β May be Desired
  - Scattering off of Trapped Ions Leads to Electron Beam Emittance Growth
  - Higher β May Assist in Stability

Scenarios

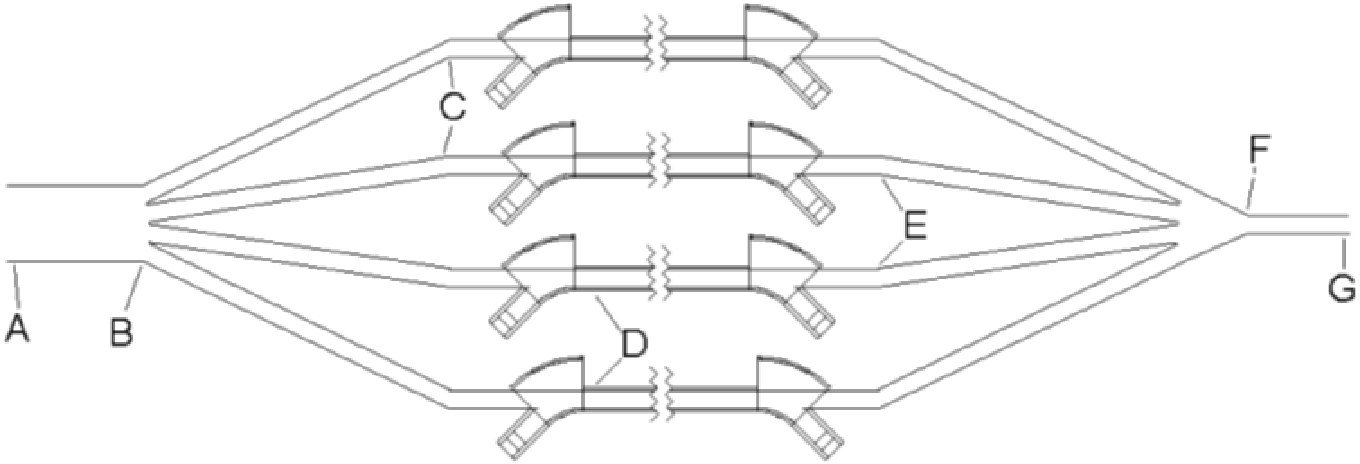
- Can Have Stages of Cooling, Focusing the Beams to Set θ before each stage.

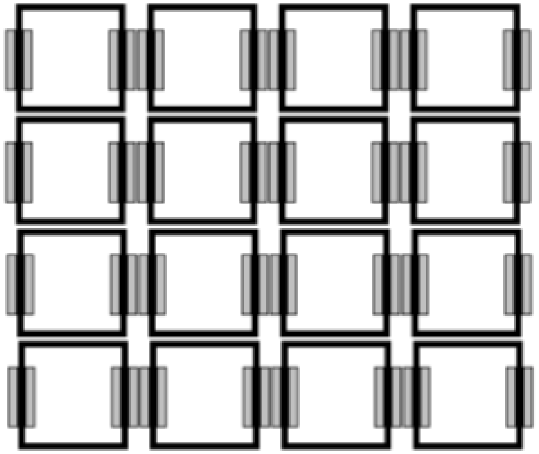
Initial ε_n	Final ε_n	Beam Radius	Total Electron Current	Number of Beamlets	Electron Current per Beamlet
3 π μm	0.3π μm	3.36 cm	14.5 A	1	14.5 A
30 π μm	3 π μm	33.6 cm	1.45 kA	1	1.45 kA
300 π μm	30 π μm	3.36 m	145 kA	100	1.45 kA
3 π mm	300 π μm	33.6 m	14.5 MA	10,000	1.45 kA

- Each Stage has ~5 m long cooling straight & 0.84 microsecond cool time
- The bigger the starting emittance, the more electron current we need
- Other Parameters Possible, this is just an Example; Optimization Needed
- For Beamlets, we’ll Operate in Parallel; See Next Slide

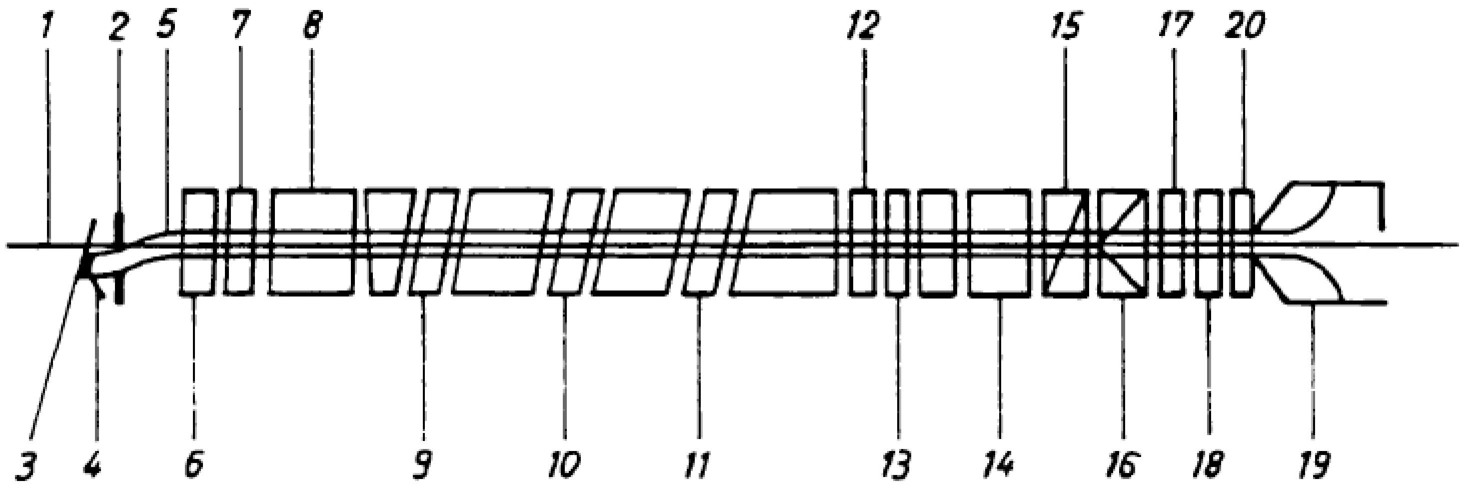
Beamlet Formation

- To Form Beamlets, Use Window Frame Magnet Septa
- Then Steer into Individual Coolers
- Then Steer and Coalesce Into a Single Beam





But What about Low Energy Electron Beam Instabilty?



- Low Energy Beam Instability was Studied in great detail in the “Solenoid Model”.
  - Schematic of the Novosibirsk Solenoid Model is in the figure above.
- Many measurements were done that agreed with electron/ion plasma theory.
  - Impressively Thorough Experimentation and Analysis. Ion Neutralization Region Begins at Electrostatic Mirror between 6 and 7 and ends prior to the Collector.
- Oscillations remained Stable below a Threshold Current but Grew Above that Threshold. Appeared to Indicate a Basic Instability of Low Energy Electron Beams.

Low Energy Electron Beam Bursian Limit

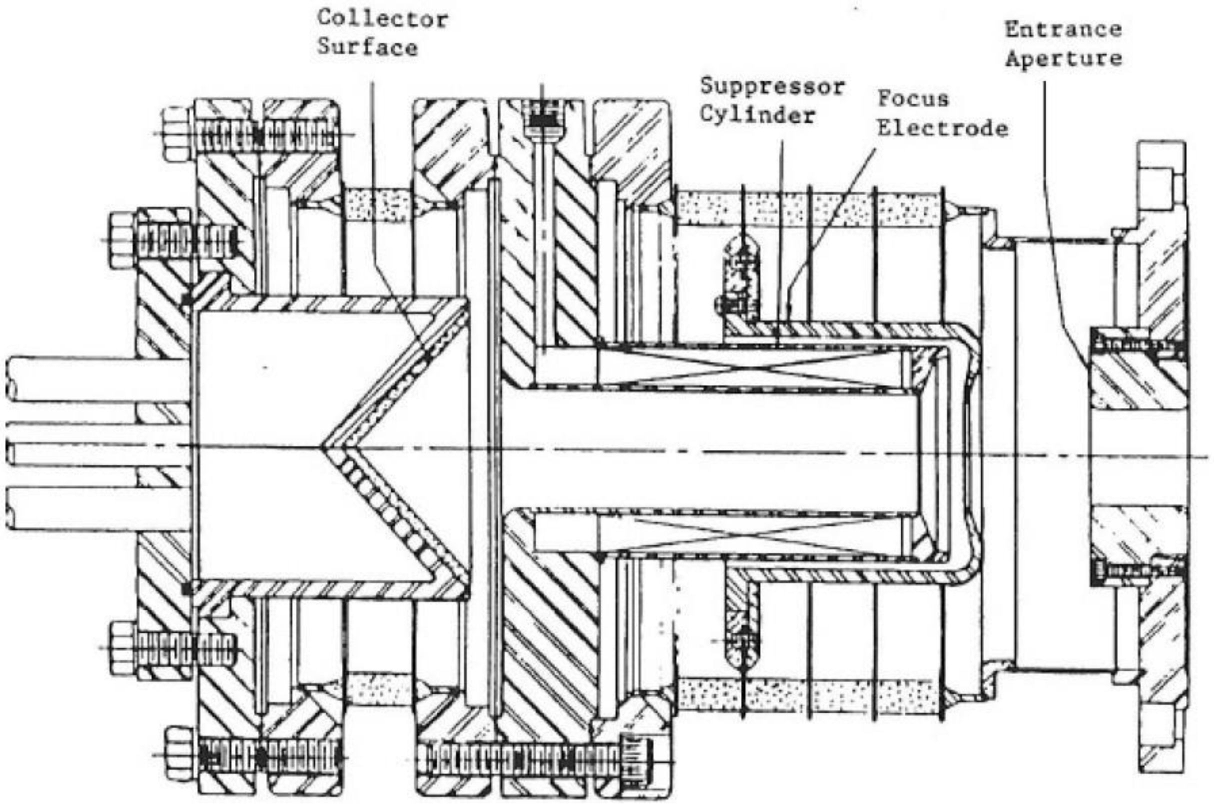
- Several Studies Have Shown Instability in Low Energy Electron Beams
- A Formula by Bursian Describes the Instability Limit for non-Neutralized Beams:

$$I_B = (25.4\mathrm{x}10^{-6}\mathrm{x}V_0^{3/2})/[1+2\ln(R/a)]$$

- The Above Equation comes from Simple Space Charge Voltage Depression. Too Large, and electrons slow, further increasing the charge, resulting in instability.
- For $\beta = 0.02$, $V_0 = 102$ V and with R/a = 1.5, the Bursian Limit is 14.6 mA!
- But we want 1.45 kA to do the cooling! (100,000 X more than Bursian Limit.)
- Pierce showed that Quasi-neutralized Beams get about 5 times more.
- But Even with Pierce, We’re very far away from What we Want.
- So Electron Cooling Hasn’t Been Considered an Option for Muon Cooling.
- However, There is Some Evidence of Stable Low Energy Beams!

Electron Beam Collectors Surpass Bursian Limit

-

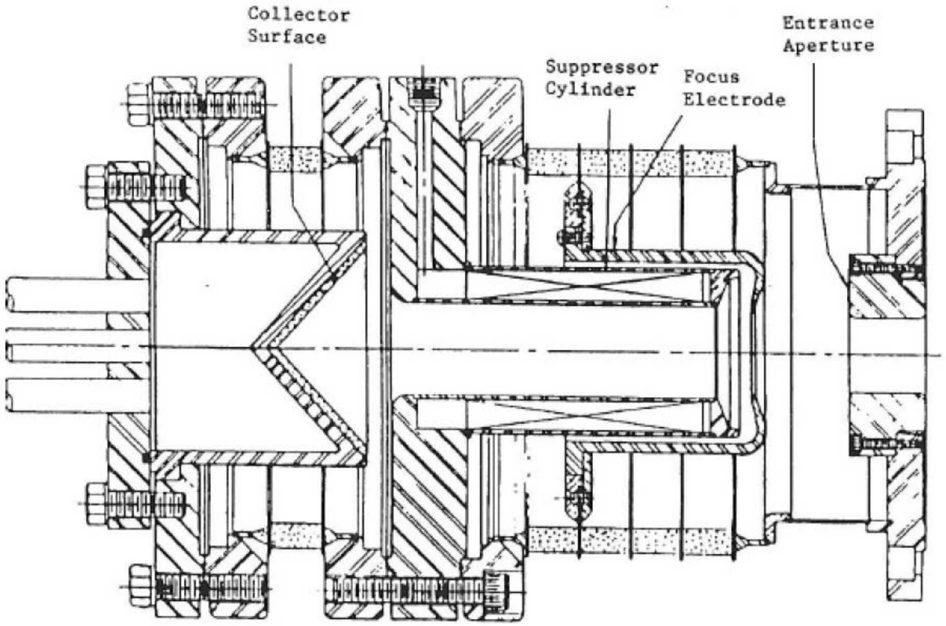


- For β = 0.028, V_0 = 200 V and with R/a = 1.5, the Bursian Limit is 37.5 mA.
- But we saw stable operation at 500 mA! Other collectors did even better.
- Note that no one looked for a max current. Collectors just met the design needs. Operations were not suffiently studied, and V_0 = 200 V is only a recollection.

Why Was Solenoid Model Experiment Unstable?

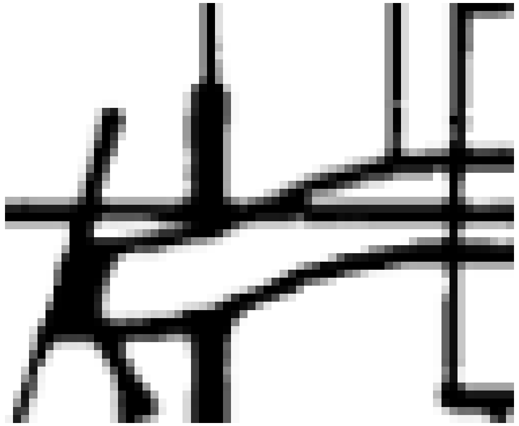
- In the Solenoid Model Experiment, a Drift Region Exists Before the Ion-Trap Region
- In the Solenoid Model Experiment, a Bursian Instability Will Exist in the Region Before the Ion-Trap Region. The Electron Gun Works to Sweep Ions Back to Cathode. This Leaves a Non-Neutralized Drift Region Prior to the Ion-Trap Region. In that Non-Neutralized Drift Region the Electron Beam is Susceptible to the Bursian Limit.
- However, A.V. Burov reports that the Threshold Current was even Lower than the Bursian Limit, and Instability of Space Charge Oscillations was Observed. A Two Stream Plasma Instability was Suspected.
- But the Beam Only Survives for $t \sim 2.5$ m/v, and with $v/c \sim 0.041$; $t \sim 0.2$ μs. Not a lot of time for Instability. Also, a Feedback Mechanism is Required.
- Burov et al., Speculate Feedback comes from Collector Secondary Electrons and also there are Electrons Freed from Gas. (The Beam Itself May not be the Entity Responsible for a Two Stream Instability.)

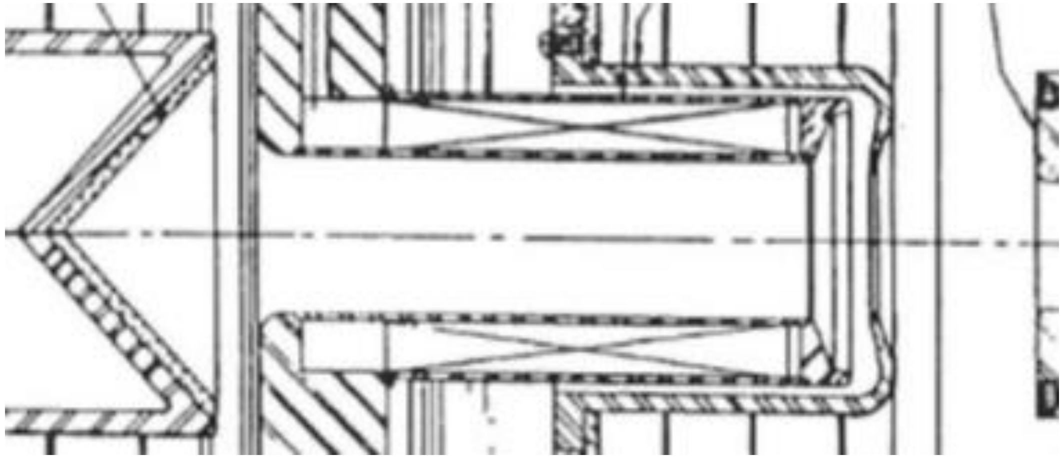
How Can Electron Collector Beams be Stable?



- Within the Electron Collector Drift, Ions are Trapped in All Three Dimensions
  - Collector Fully Neutralizes Space Charge, Allowing Stable Operation
- Outside of the Neutralized Electron Collector Drift Region, There is No Other Drift Region. Beams Rapidly Get to Higher β.
- The Electric Fields will also Clear Secondary Electrons.

Review





- (Left) Solenoid Model System has Non-Neutral Drift Region Prior to Ion-Trap Region.
  - In the Drift Prior to the Ion-Trap Region, the Cathode/Anode Electric Field Extracts Ions Out.
  - Hence, the Drift Region Prior to the Ion-Trap Region is Non-neutral and Bursian Unstable.
  - Secondary and Gas Stripped Electrons may Make the Instability Worse.
- (Right) Collector has Internal Neutralized Drift Region
  - Electric fields on Either Side work to Trap, Not Extract, Neutralizing Ions Inside the Drift Region.
  - Hence, the Drift Region is Neutralized and Bursian Instability Does not Manifest.
  - Secondary Electrons Are Also Swept Out by the Adjacent Electric Fields.

Theoretical Electron Cooling Program

1. Improve Electron Cooling Analysis and Design.
    1. A real distribution is not a simple uniform sphere in velocity space: a Gaussian has a significant distribution outside of the 1σ boundary used in our simple example. This is a LOSS compared to the simple example.
    2. Optimize starting velocity: We should start at $v > 1\sigma$ since the cooling force is stronger in the near-outside region than in the Interior of the Spherical Distribution. This GAIN regains much of what is lost in 1.1; possibly more.
    3. Any electron beam space charge depression will increase the longitudinal velocity spread, which would be a LOSS compared to the simple example.
    4. However, the longitudinal spread is smaller than the transverse, a GAIN.
    5. Cathodes with 100 A/cm^2 are possible. This is a 5X GAIN.
    6. A guess is that 1.1 through 1.4 roughly Cancel each other, and Cooling will be 5X better than the earlier Simple Estimate via 1.5.
    7. We Need a Numerical Integration rather than the Simple Model to obtain a More Accurate and Detailed Evaluation of the Cooling.
2. Determine the Cost Effectiveness (both Construction Cost and Operational Costs) to See Where Electron Cooling Makes Sense.

Experimental Electron Cooling Program

1. Build a Test Bed System with a 15 cm Drift. Investigate to Find the Maximum Stable Current at Low Energy. (Design for 100 A.)
2. Extend the Test Bed System to a 1 m Drift. Investigate Stability Limit.
3. Add Toroidal Merging and Separation for a Cooler Prototype. Investigate Stability Limit.
4. Add Adiabatic Expansion. Investigate Stability Limit.
5. If Toroidal Inclusive System is only Stable at a Lower Current or at Smaller Lengths than Desired, Arrive at a Design using Many More Such Coolers (in series and in parallel) to achieve Electron Cooling for Muons. Evaluate Cost Effectiveness.

Note – Much of the Theoretical and Experimental Work is the Subject of a Pending SBIR Proposal to the US DOE. (Of course, any support would be appreciated.)

Ramifications of a Final Electron Cooler

- Electron Collectors Proven to Operate Well Beyond Bursian Limit
- We've Specified a 5 meter long Drift with 1.45 kA as one Example
- Collectors already Operate at 15 cm Drifts with Ampere Intensity
- If we Can Achieve the Needed Electron Beam Technology, Muon Beams with Less than 1 micron Emittances are Possible
- With a 100 X emittance reduction, 10 X less Muons Are Needed
  - Same Physics Results would be Obtained; but 10 X Less Background
  - 10 X Less of a Radiation Problem
  - 10 X Less Beam Loading on the RF in the Ionization Cooling Channels
  - A Less Intense Proton Driver is Needed
- Smaller Good Field Aperture Requirement for Accelerator Magnets
- The Advantages Would Likely Result in a Significantly Cheaper Collider (Maybe 3X?)

Conclusion

- Electron Cooling of Muons May be Possible.
- Key to the Success of Electron Cooling for Muons is Demonstration of Stable, High Current, Low Energy, Electron Beams.
- Such Beams Have Been Proven to Exist in Electron Collectors, Although the Drift Lengths are Shorter, and Currents less, than What we Desire.
- If the Desired Electron Beam Technology Can be Demonstrated, a Muon Collider will be Significantly Cheaper to Build than Presently Envisioned.
- If the Desired Electron Beam Technology Can be Demonstrated, a Muon Collider May be Realizable Quicker than Presently Envisioned.
- We Should Begin Efforts to Demonstrate the Desired Electron Beam Technology.
- We Should Include Electron Cooling in a New Full System Design.

For Discussion: Scenarios With a 100 A/cm^2 Source

- Can Have Stages of Cooling, Focusing the Beams to Set θ before each stage.

Initial ε_n	Final ε_n	Beam Radius	Total Electron Current	Number of Beamlets	Electron Current per Beamlet
3 π μm	0.3π μm	3.36 cm	14.5 A	1	14.5 A
30 π μm	3 π μm	33.6 cm	1.45 kA	1	1.45 kA
300 π μm	30 π μm	3.36 m	145 kA	100	1.45 kA
3 π mm	300 π μm	33.6 m	14.5 MA	10,000	1.45 kA

- Each Stage has ~1 m Long Cooling Straight & 0.17 Microsecond Cool Time.
- The Larger the Starting Emittance, the More Electron Current We Need.
- Other Parameters Possible, this is just Example 2; Optimization is Needed.
- For Beamlets, we’ll Operate in Parallel and Perhaps in Series.
- May want 100 A Beamlets; Leading to 16X More Coolers Where Needed.