Hard Pion Chiral Perturbation Theory

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Chiral Perturbation Theory

- Chiral Perturbation Theory is an expansion in the mass and momenta
- Consider the vector form factor defined by:

$$\langle \pi^i(p_2)|\frac{1}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right)|\pi^j(p_1)\rangle = i\epsilon^{i3j}(p_{1\mu} - p_{2\mu})F_V(s)$$

- ChPT is a effective theory which alows us to compute

$$F_V(s) = 1 + c_1\frac{M_\pi^2}{16\pi^2 F_\pi^2} + c_2\frac{s}{16\pi^2 F_\pi^2} + c_L\frac{M_\pi^2}{16\pi^2 F_\pi^2} ln\frac{M_\pi^2}{\mu^2} + \mathcal{O}(p^4)$$

- $$M_\pi \sim 140\ MeV \qquad 4\pi F_\pi \sim 1\ GeV$$

Hard Pion Chiral Perturbation Theory

- Power counting breaks down if momenta gets large, but
- Observation/claim:
  the leading chiral logarithm can be predicted nevertheless
  [Flynn and Sachrajda 09; Bijnens, Celis and Jemos 09-11]
- H$\pi\chi PT$ *states for our Formfactor* :

$$F_V(s) = \bar{F}_V(s) \cdot (1 + \alpha L) + \mathcal{O}(M_\pi^2)$$

where L is the chiral log defined by:

$$L = \frac{M_\pi^2}{16\pi^2 F_\pi^2} ln \frac{M_\pi^2}{\mu^2} \ and \ \alpha = -1$$

Factorization

- We found a proof of this factorization for the elastic part of the formfactor
- proof based on a dispersive representation

$$F_V(s) = 1 + \frac{s}{\pi} \int_{4M_\pi^2}^{\infty} ds' \frac{\mathrm{Im} F_V(s')}{s'(s'-s)} \qquad (1)$$

- the optical theorem states:

$$\mathrm{Im} F_V(s) = \sigma(s) F_V(s) t^*(s) \qquad \sigma(s) = \sqrt{1 - \frac{4M_\pi^2}{s}}$$

- where the t_l^I are the $\pi\pi$ partial waves. An explicit callculation yields that they do not contain any chiral log in the limit s$\gg M_\pi^2$

Factorization

- Chiral logs are only produced at order p^2
- Apply the power counting of ChPT to the dispersive equation
- induction $\longrightarrow$ *factorization*

Applications

- Consider the decay $K \to \pi + \mathrm{l} + \nu_{\mathrm{l}}$
- The experimental rate is given by

$$\Gamma_{K\to\pi l\nu_l} = C_K^2 \frac{G_f^2 m_K^5}{192\pi^3} \, I \, S_{EW}(1+2\Delta_{SU(2)}+2\Delta_{EM})|V_{us}|^2|f^+(0)|^2$$

- this experiment is used to determine V_{us}
- The formfactor must be determined from the theory
- perturbation in α_{strong} not possible
  $\to$ lattice calculations combined with χPT do the extrapolation towards the physical masses are used
- our factorization theorem improves the extrapolation