

Perturbative calculations

- Perturbative calculations = fixed-order expansion in the coupling constant, or more refined expansions that include terms to all orders
- Perturbative calculations are possible because the coupling is small at high energy
- In QCD (or in a generic QFT) the coupling depends on the energy (renormalization scale)
- So changing scale the result changes. By how much? What does this dependence mean?
- **Let's consider some examples**

Leading order n-jet cross-section

- Consider the cross-section to produce n jets. The leading-order result at scale μ result will be

$$\sigma_{\text{njets}}^{\text{LO}}(\mu) = \alpha_s(\mu)^n A(p_i, \epsilon_i, \dots)$$

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So the change of scale is a NLO effect ($\propto \alpha_s$), but this becomes more important when the number of jets increases ($\propto n$)

- Notice that at leading order (LO) the normalization is not under control:

$$\frac{\sigma_{\text{njets}}^{\text{LO}}(\mu)}{\sigma_{\text{njets}}^{\text{LO}}(\mu')} = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu')} \right)^n$$

NLO n-jet cross-section

Now consider an n-jet cross-section at NLO. At scale μ the result reads

$$\sigma_{\text{njets}}^{\text{NLO}}(\mu) = \alpha_s(\mu)^n A(p_i, \epsilon_i, \dots) + \alpha_s(\mu)^{n+1} \left(B(p_i, \epsilon_i, \dots) - nb_0 \ln \frac{\mu^2}{Q_0^2} \right) + \dots$$

- So the NLO result compensates the LO scale dependence. The residual dependence is NNLO.
- Scale dependence and normalization start being under control only at NLO, since a **compensation mechanism** kicks in
- Notice also that a good scale choice automatically resums large logarithms to all orders, while **a bad one spuriously introduces large logs and ruins the perturbative expansion**
- Scale variation is conventionally used to estimate the **theory uncertainty**, but the validity of this procedure should not be overrated (see later)

Leading order calculations

Get *any* LO cross-section from the Lagrangian

1. draw all Feynman diagrams
2. put in the explicit Feynman rules and get the amplitude
3. do some algebra, simplifications
4. square the amplitude
5. integrate over phase space + flux factor + sum/average over outgoing/
incoming states

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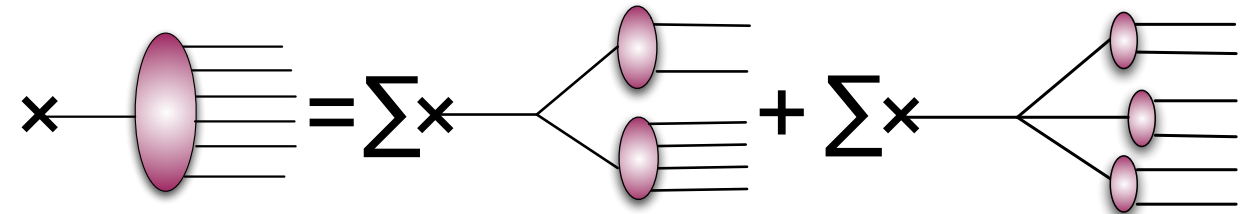
Bottlenecks

- a) number of Feynman diagrams diverges factorially
- b) algebra becomes more cumbersome with more particles

But given enough computer power everything can be computed at LO

Techniques beyond Feynman diagrams

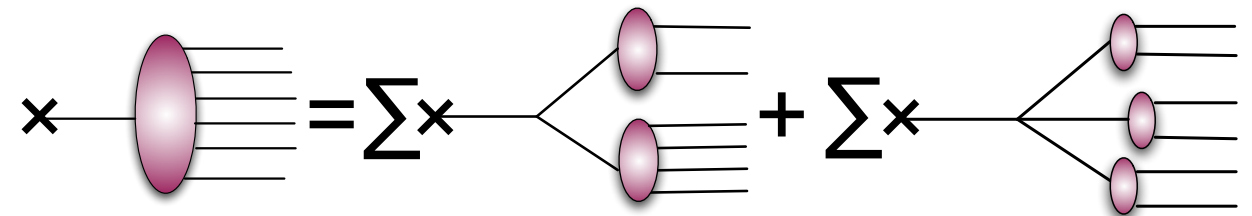
✓ Berends-Giele relations: compute helicity amplitudes **recursively** using off-shell currents



Berends, Giele '88

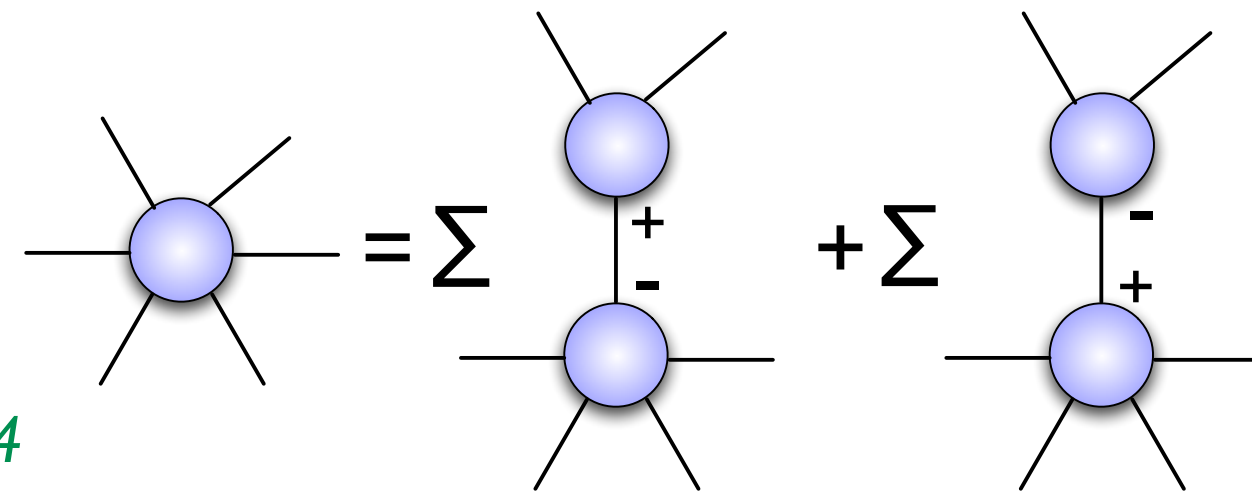
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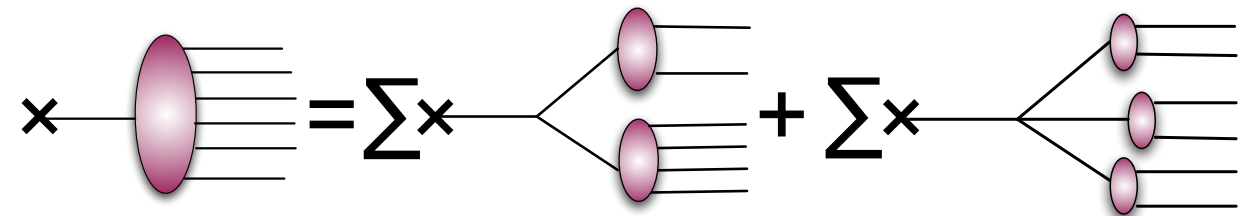
✓ BCF relations: compute helicity amplitudes via on-shell **recursions** (use complex momentum shifts)



Britto, Cachazo, Feng '04

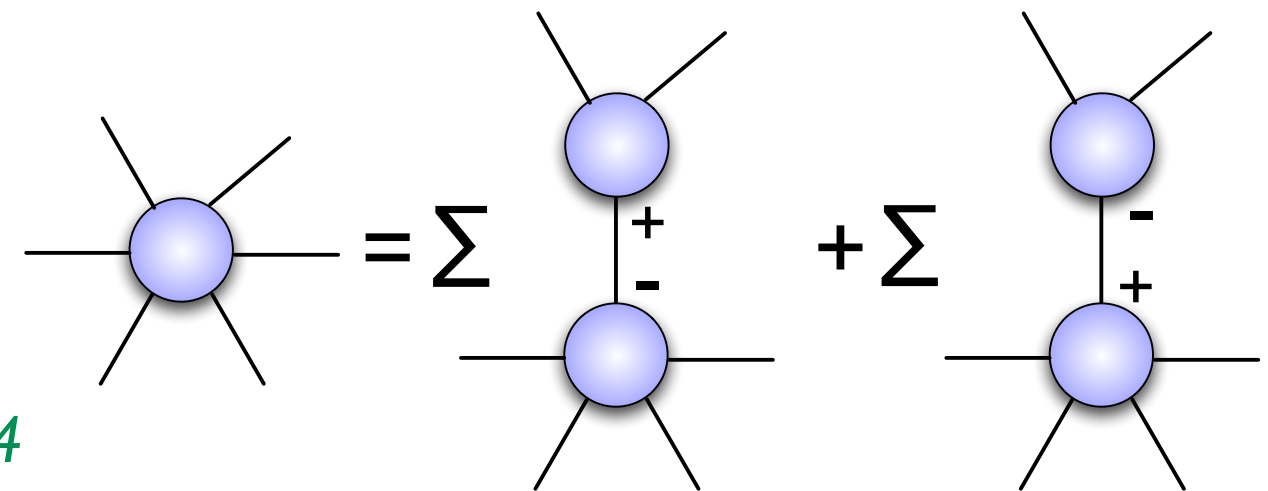
Techniques beyond Feynman diagrams

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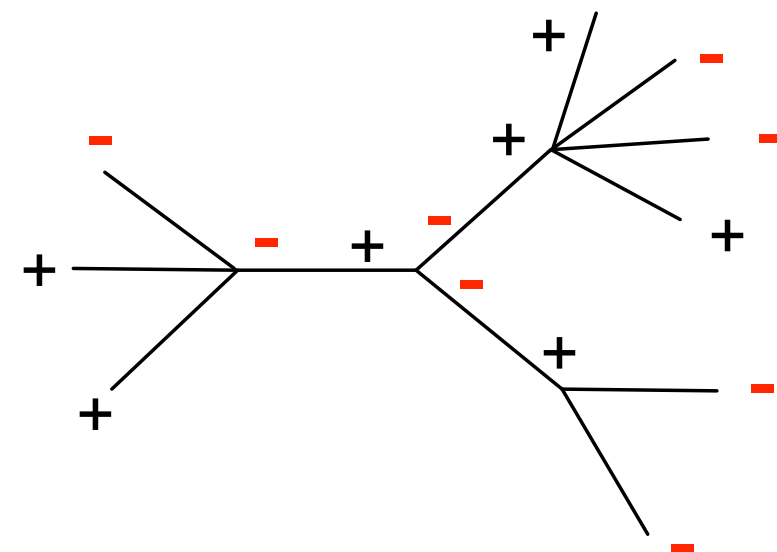
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- ✓ BCF relations: compute helicity amplitudes via on-shell **recursions** (use complex momentum shifts)



Britto, Cachazo, Feng '04

- ✓ CSW relations: compute helicity amplitudes by **sewing together** MHV amplitudes [- - + + ... +]



Cachazo, Svrcek, Witten '04

Matrix element generators

Fully automated calculation of leading-order cross-sections:

- ▶ generation of tree-level matrix elements
 - Feynman diagrams [CompHEP/CalcHEP, Madgraph/Madevent, HELAS, Sherpa, ...]
 - Helicity amplitudes + off-shell Berends-Giele recursion [ALPHA/ALPGEN, Helac, Vecbos]
- ▶ phase space integration
- ▶ interface to parton showers (see later)

These codes are currently used extensively in many analysis of LHC data

Benefits and drawbacks of LO

Benefits of LO:

- fastest option; often the only one
- test quickly new ideas with fully exclusive description (New Physics)
- many working, well-tested approaches
- highly automated, crucial to explore new ground, but no precision

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Drawbacks of LO:

- large scale dependences, reflecting large theory uncertainty
- no control on normalization
- poor control on shapes
- poor modeling of jets

Example: $W+4$ jet cross-section $\propto \alpha_s(Q)^4$

Vary $\alpha_s(Q)$ by $\pm 10\%$ via change of $Q \Rightarrow$ cross-section varies by $\pm 40\%$

Next-to-leading order

Benefits of next-to-leading order (NLO)

- reduce dependence on **unphysical scales (renormalization/factorization)**
- establish **normalization** and **shape** of cross-sections
- small scale dependence at LO can be very misleading (see later), small dependence at NLO robust sign that **PT is under control**
- large NLO correction or large scale dependence at NLO robust sign that neglected **other higher order** are important
- through loop effects get **indirect information** about sectors not directly accessible

We'll look at a few concrete examples in few minutes

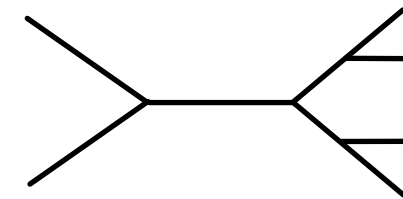
Ingredients at NLO

A full N-particle NLO calculation requires:

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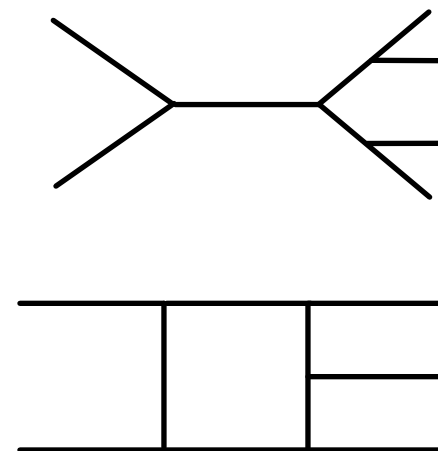
- tree graph rates with $N+1$ partons
→ soft/collinear divergences



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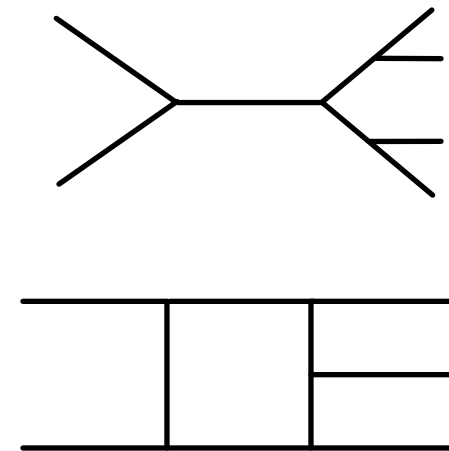
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- virtual correction to N-leg process
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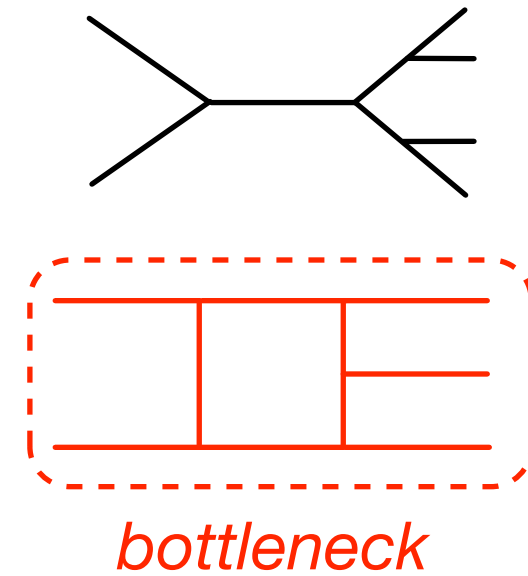
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Approaches to virtual (loop) part of NLO

Two complementary approaches:

- ▶ **Numerical/traditional Feynman diagram methods:**
use robust computational methods [integration by parts, reduction techniques...], then let the computer do the work for you

Bottleneck:

factorial growth, 2 → 4 doable, very difficult to go beyond

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▶ Analytical approaches:

improve understanding of field theory [e.g. unitarity, onshell methods, OPP, recursion relations, twistor methods, ...]

Bottleneck:

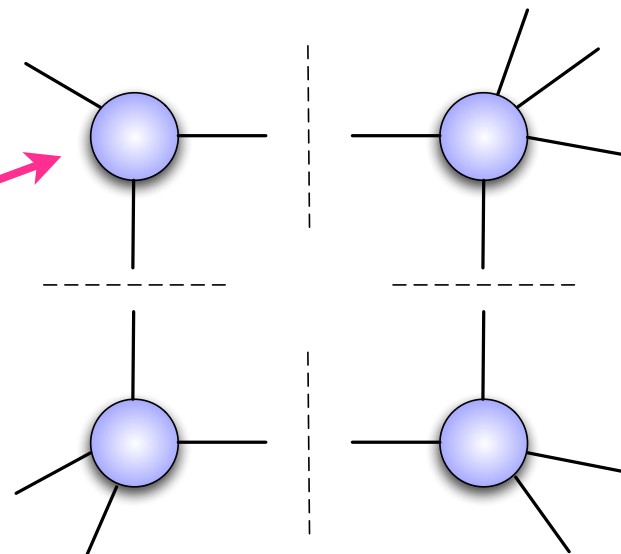
still lack of complete automation, fermions in general more difficult

Two breakthrough ideas

Aim: NLO loop integral without doing the integration

1) “... we show how to use generalized unitarity to read off the (box) coefficients. The generalized cuts we use are quadrupole cuts ...”

NB: non-zero
because cut gives
complex momenta



Britto, Cachazo, Feng '04

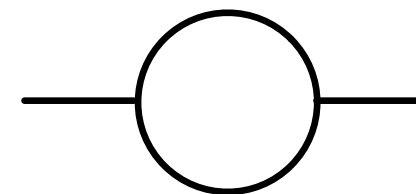
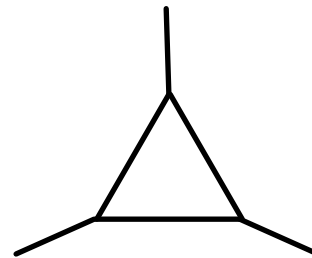
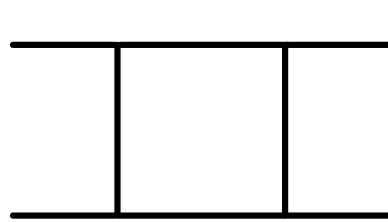
Quadrupole cuts: 4 on-shell conditions on 4 dimensional loop momentum) freezes the integration. But **rational part** of the amplitude, coming from $D=4-2\epsilon$ not 4, computed separately

Two breakthrough ideas

Aim: NLO loop integral without doing the integration

2) *The OPP method: “We show how to extract the coefficients of 4-, 3-, 2- and 1-point one-loop scalar integrals ...”*

$$\mathcal{A}_N = \sum_{[i_1|i_4]} \left(d_{i_1 i_2 i_3 i_4} I_{i_1 i_2 i_3 i_4}^{(D)} \right) + \sum_{[i_1|i_3]} \left(c_{i_1 i_2 i_3} I_{i_1 i_2 i_3}^{(D)} \right) + \sum_{[i_1|i_2]} \left(b_{i_1 i_2} I_{i_1 i_2}^{(D)} \right)$$



Ossola, Pittau, Papadopolous '06

Coefficients can be determined by solving system of equations: no loops, no twistors, just algebra!

Status of NLO in 2005

Table 42: The LHC “priority” wishlist for which a NLO computation seems now feasible.

process ($V \in \{Z, W, \gamma\}$)	relevant for
1. $pp \rightarrow V V \text{ jet}$	$t\bar{t}H$, new physics
2. $pp \rightarrow t\bar{t} b\bar{b}$	$t\bar{t}H$
3. $pp \rightarrow t\bar{t} + 2 \text{ jets}$	$t\bar{t}H$
4. $pp \rightarrow V V b\bar{b}$	VBF $\rightarrow H \rightarrow VV$, $t\bar{t}H$, new physics
5. $pp \rightarrow V V + 2 \text{ jets}$	VBF $\rightarrow H \rightarrow VV$
6. $pp \rightarrow V + 3 \text{ jets}$	various new physics signatures
7. $pp \rightarrow V V V$	SUSY trilepton

The QCD, EW & Higgs Working group report [hep-ph/0604120](https://arxiv.org/abs/hep-ph/0604120)

The 2007 update

Process ($V \in \{Z, W, \gamma\}$)	Comments
Calculations completed since Les Houches 2005	
1. $pp \rightarrow VV\text{jet}$	$WW\text{jet}$ completed by Dittmaier/Kallweit/Uwer [3]; Campbell/Ellis/Zanderighi [4] and Binoth/Karg/Kauer/Sanguinetti (in progress) NLO QCD to the gg channel completed by Campbell/Ellis/Zanderighi [5]; NLO QCD+EW to the VBF channel completed by Ciccolini/Denner/Dittmaier [6, 7] ZZZ completed by Lazopoulos/Melnikov/Petriello [8] and WWZ by Hankele/Zeppenfeld [9]
2. $pp \rightarrow \text{Higgs}+2\text{jets}$	
3. $pp \rightarrow VVV$	
Calculations remaining from Les Houches 2005	
4. $pp \rightarrow t\bar{t}b\bar{b}$	relevant for $t\bar{t}H$ relevant for $t\bar{t}H$ relevant for $\text{VBF} \rightarrow H \rightarrow VV, t\bar{t}H$ relevant for $\text{VBF} \rightarrow H \rightarrow VV$ VBF contributions calculated by (Bozzi/)Jäger/Oleari/Zeppenfeld [10–12] various new physics signatures
5. $pp \rightarrow t\bar{t}+2\text{jets}$	
6. $pp \rightarrow VVb\bar{b},$	
7. $pp \rightarrow VV+2\text{jets}$	
8. $pp \rightarrow V+3\text{jets}$	
NLO calculations added to list in 2007	
9. $pp \rightarrow b\bar{b}b\bar{b}$	Higgs and new physics signatures
Calculations beyond NLO added in 2007	
10. $gg \rightarrow W^*W^* \mathcal{O}(\alpha^2\alpha_s^3)$	backgrounds to Higgs normalization of a benchmark process Higgs couplings and SM benchmark
11. NNLO $pp \rightarrow t\bar{t}$	
12. NNLO to VBF and $Z/\gamma+\text{jet}$	
Calculations including electroweak effects	
13. NNLO QCD+NLO EW for W/Z	precision calculation of a SM benchmark

} with Feynman diagrams

} with Feynman diagrams or unitarity/onshell methods

The NLO multi-leg Working group report 0803.0494

Table 1: The updated experimenter's wishlist for LHC processes

Status of NLO today

Status of NLO:

- $2 \rightarrow 2$: all known (or easy) in SM and beyond
- $2 \rightarrow 3$: essentially all SM processes known
[but: often do not include decays, codes private]
- $2 \rightarrow 4$: a number of calculations performed in the last 1- or 2 years
[$W/Z+3$ jets, $WW+2$ jets, $WWbb$, $tt+2$ jets, $ttbb$, $bbbb$].
Calculations done using different techniques
- $2 \rightarrow 5$: dominant corrections for only two processes [$W/Z+4$ jets]

Top-pair production

The top quark plays a unique role in the SM

It is much heavier than all other quarks, therefore

- top quark mass crucial for EW precision tests
- strong coupling to scalars (see later)
- prominent decay product in many BSM models
- window to new physics ?

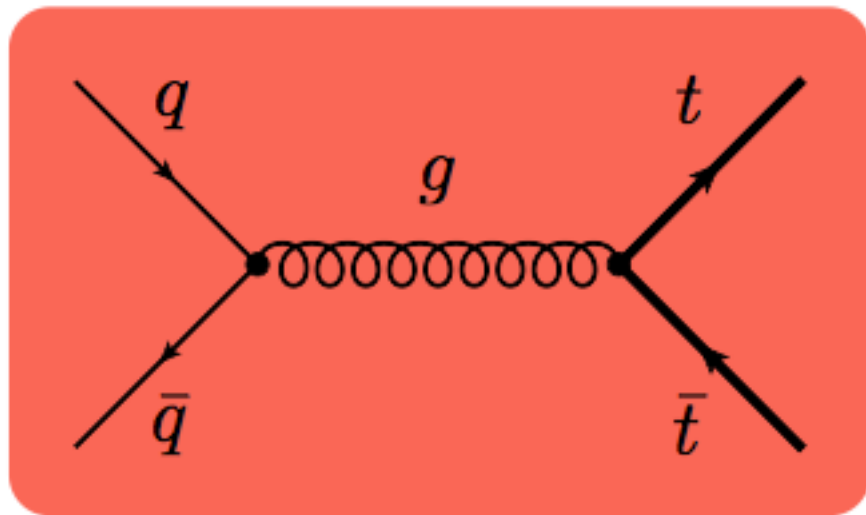
From a QCD point of view

- top lifetime $\sim 5 \cdot 10^{-25}$ s (dominant decay mode is to Wb)
- typical time scale for hadron formation $\sim 3 \cdot 10^{-24}$ s

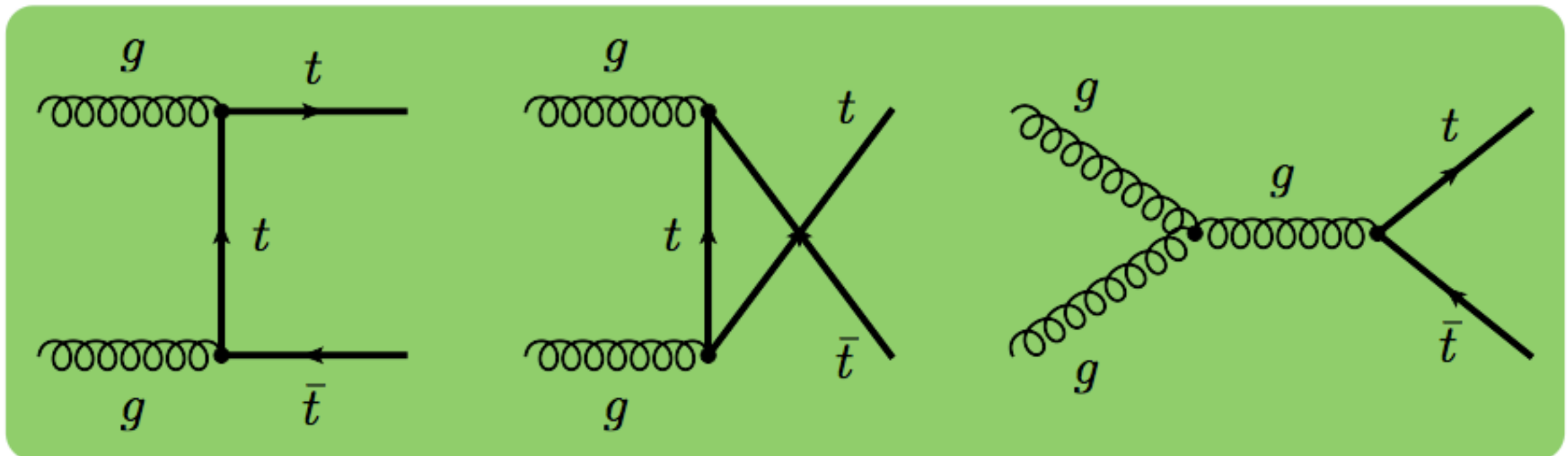
The top quark is the only one that decays before forming a bound state

Top-pair production

Basic production mechanisms: initiated from quarks or gluons



*What is the dominant production mechanism, at the Tevatron/LHC?
[And why?]*



Top-pair production: Tevatron

Running the program MCFM gives

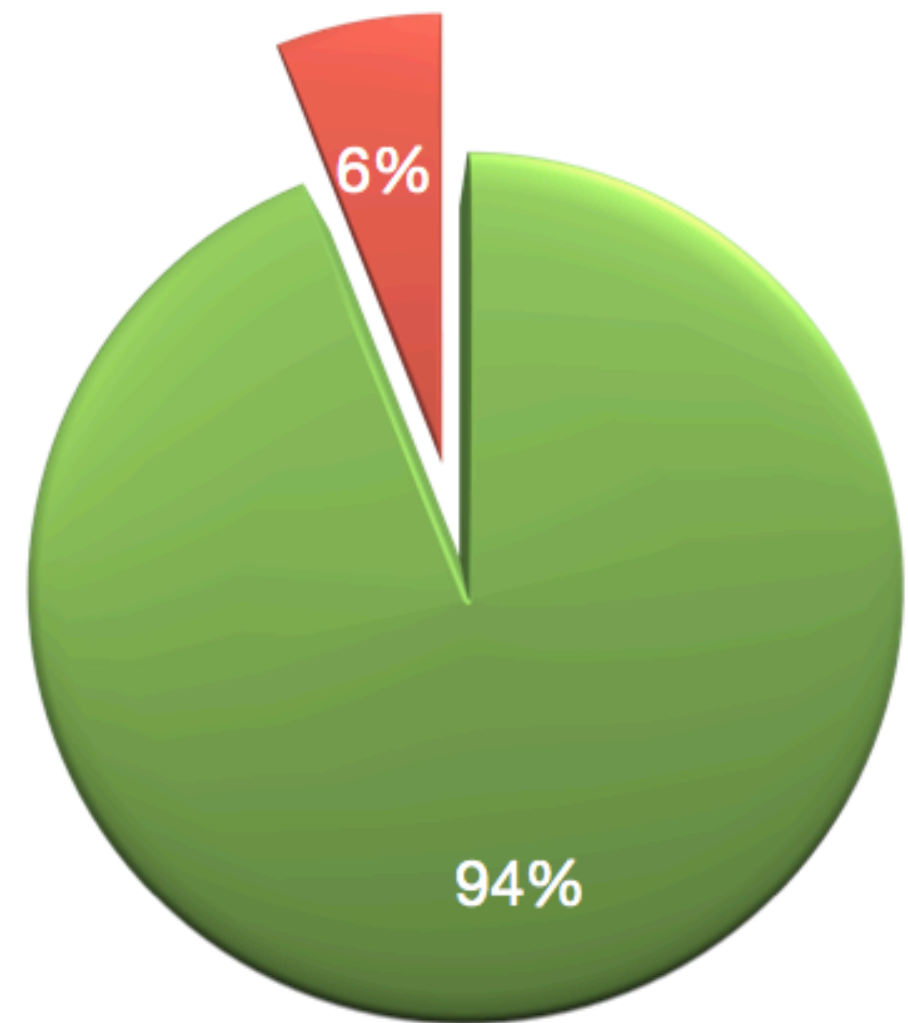
Value of final lord integral is 9334.461 +/- 3.530 fb

Total number of shots : 200000
 Total no. failing cuts : 0
 Number failing jet cuts : 0
 Number failing process cuts : 0

Jet efficiency : 100.00%
 Cut efficiency : 100.00%
 Total efficiency : 100.00%

Contribution from parton sub-processes:

GG	563.36203	6.04%
GQ	0.00000	0.00%
QGB	0.00000	0.00%
QG	0.00000	0.00%
QBG	0.00000	0.00%
QQ	0.00000	0.00%
QBQB	0.00000	0.00%
QQB	8723.36136	93.45%
QBQ	47.73759	0.51%



● $q\bar{q}$ ● gg

Top-pair production: pp @ 1.96 TeV

Running the program MCFM gives

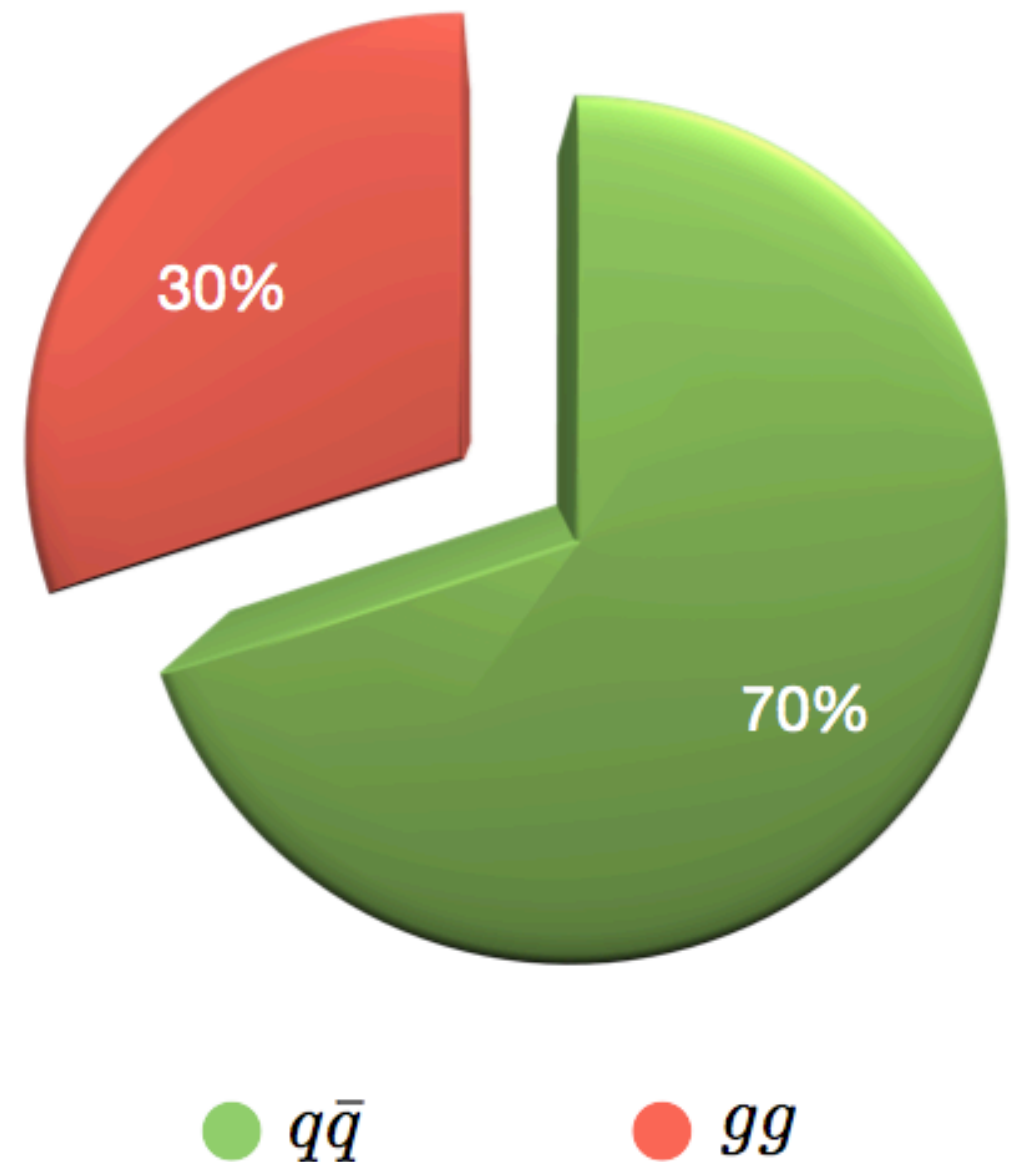
Value of final lord integral is 1889.320 +/- 0.723 fb

Total number of shots : 200000
Total no. failing cuts : 0
Number failing jet cuts : 0
Number failing process cuts : 0

Jet efficiency : 100.00%
Cut efficiency : 100.00%
Total efficiency : 100.00%

Contribution from parton sub-processes:

GG	563.26857	29.81%
GQ	0.00000	0.00%
QGB	0.00000	0.00%
QG	0.00000	0.00%
QBG	0.00000	0.00%
QQ	0.00000	0.00%
QBQB	0.00000	0.00%
QQB	662.81972	35.08%
QBQ	663.23143	35.10%



Top-pair production: LHC

Running the program MCFM gives

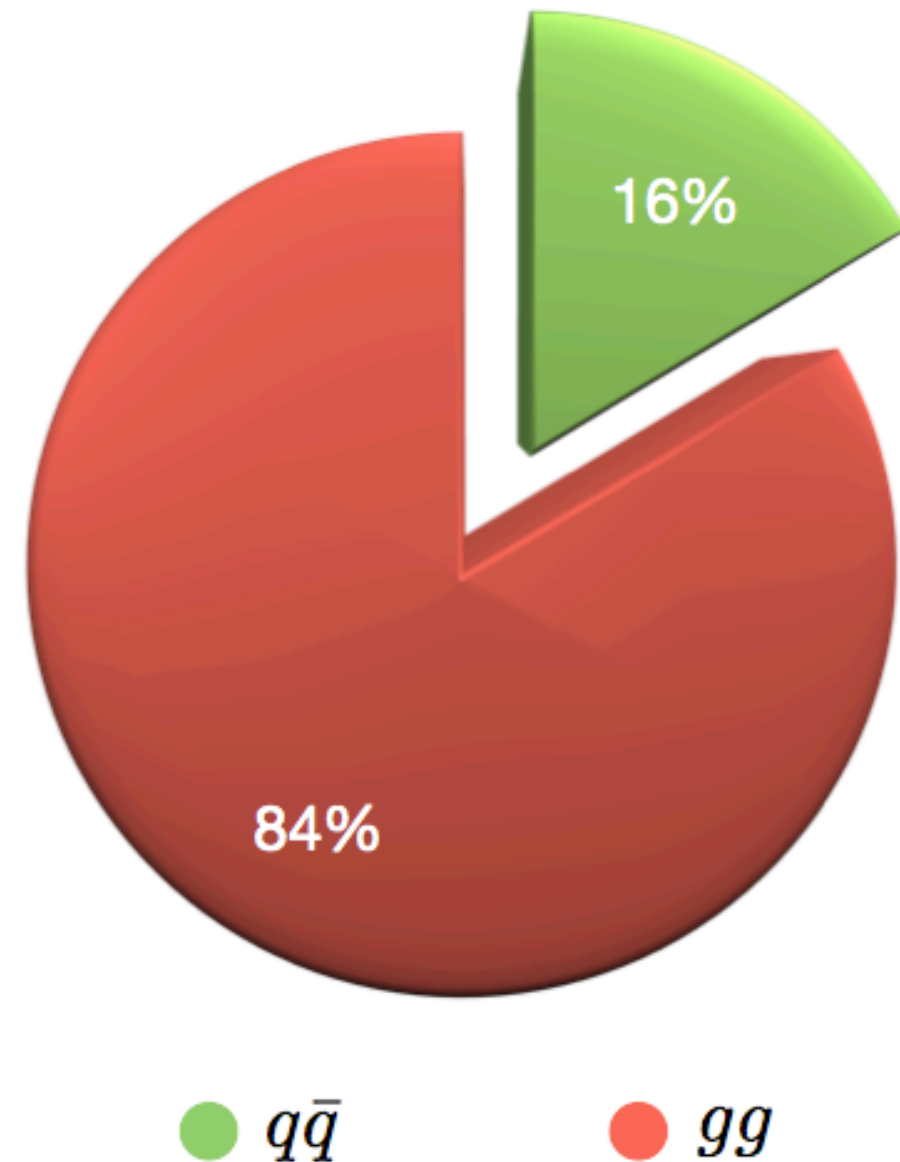
Value of final lord integral is 373635.066 +/- 148.259 fb

Total number of shots : 200000
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Number failing jet cuts : 0
Number failing process cuts : 0

Jet efficiency : 100.00%
Cut efficiency : 100.00%
Total efficiency : 100.00%

Contribution from parton sub-processes:

GG	312453.03253	83.63%
GQ	0.00000	0.00%
GQB	0.00000	0.00%
QG	0.00000	0.00%
QBG	0.00000	0.00%
QQ	0.00000	0.00%
QBQB	0.00000	0.00%
QQB	30598.98764	8.19%
QBQ	30583.04606	8.19%



Top-asymmetry

At the Tevatron, one interesting top measurement is its **asymmetry**

$$A_{fb} = \frac{N_{\text{top}}(\eta > 0) - N_{\text{top}}(\eta < 0)}{N_{\text{top}}(\eta > 0) + N_{\text{top}}(\eta < 0)}$$

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At $O(\alpha_s^3)$ the asymmetry is non-zero, an **NLO calculation** gives

$$A_{fb}^{\text{NLO}} = 0.050 \pm 0.015$$

Kuehn et al. '99

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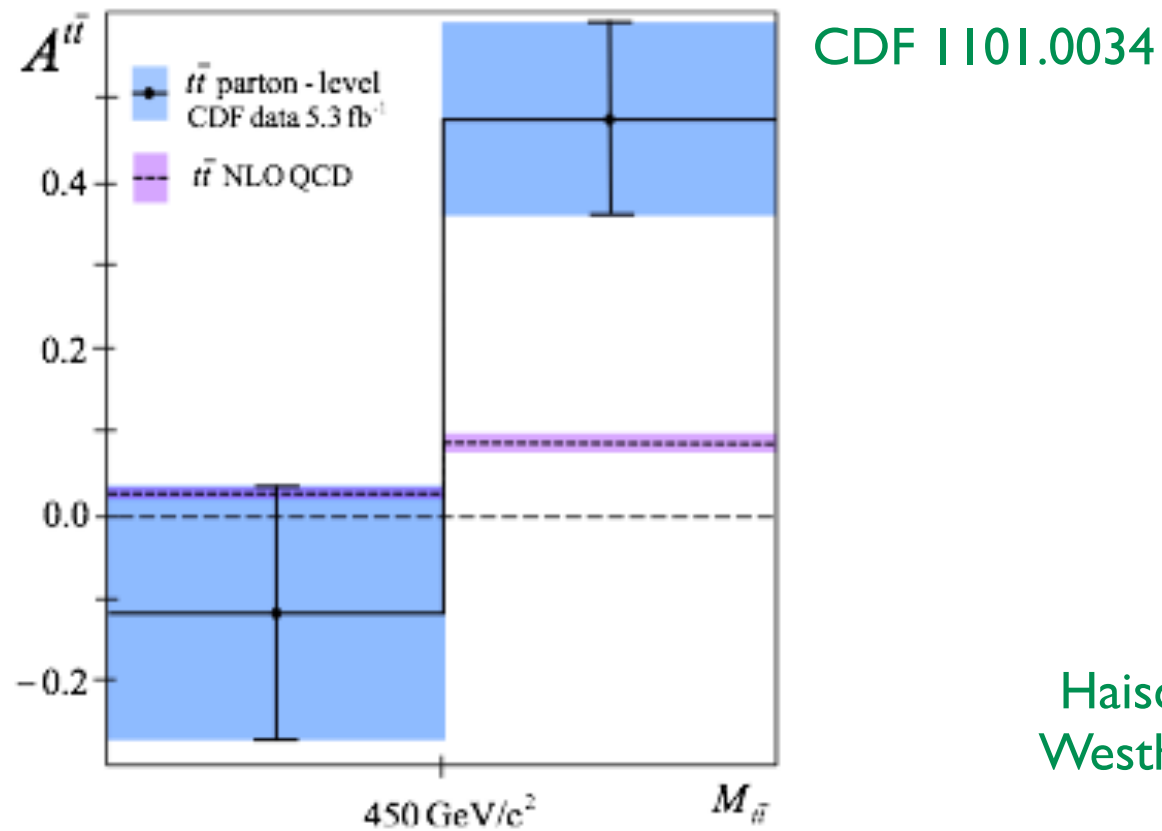
Kuehn et al. '99

But **CDF & D0 measurements** give

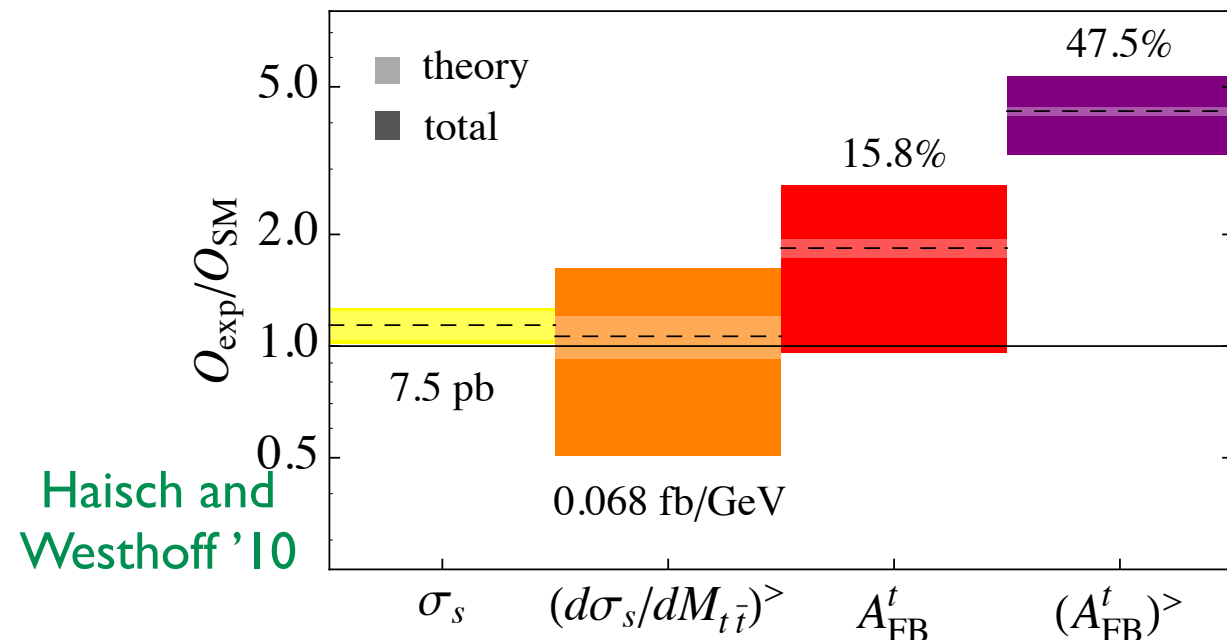
$$A_{fb}^{\text{exp.}} = 0.193 \pm 0.065 (\text{stat.}) \pm 0.024 (\text{syst.})$$

⇒ more than 2-sigma deviation from NLO

Top-asymmetry: high mass region



Tension between symmetric and asymmetric cross-section



2.7 σ / 4.2 σ away from the NLO+NNLL theory. Seen both by CDF and D0, CDF effect enhanced at large $M_{t\bar{t}}$, also in dilepton channel

Asymmetry is 0 at LO, but theoretical arguments and partial higher orders suggest that NLO is robust under higher-order corrections

Almeida et al. 0805.1885; Melnikov and Schulze 1004.3284; Ahrens et al. 1106.6051, ...

Various new models try to explain data, but difficult to preserve good agreement with symmetric cross-section, like-sign top decays, ...

Top at the LHC

Large Yukawa coupling and prominent decay product in many New-Physics models. The place where new physics will show up?

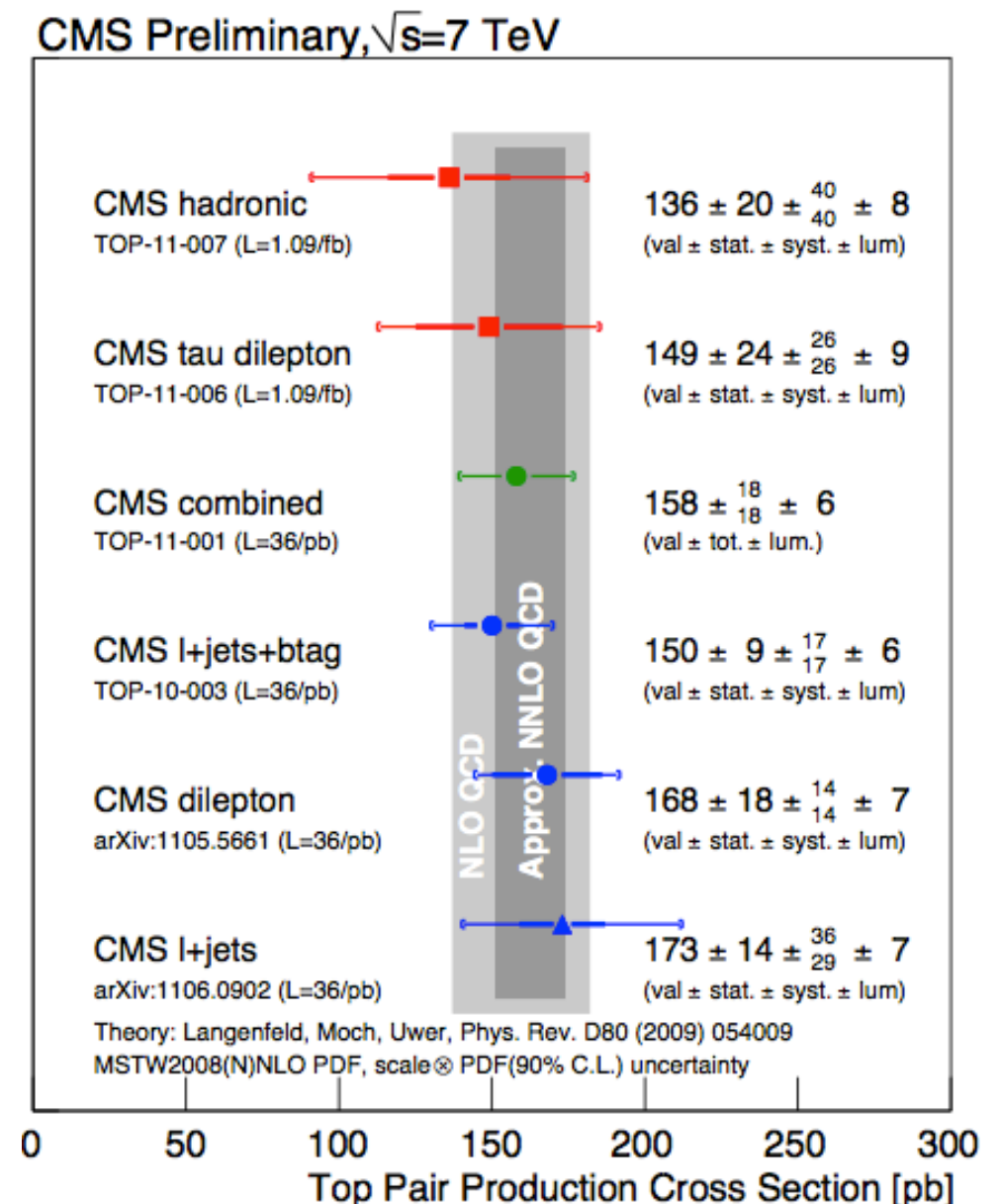
Good agreement between LHC data and NLO (and approx. NNLO) QCD

The frontier of NNLO

[...]

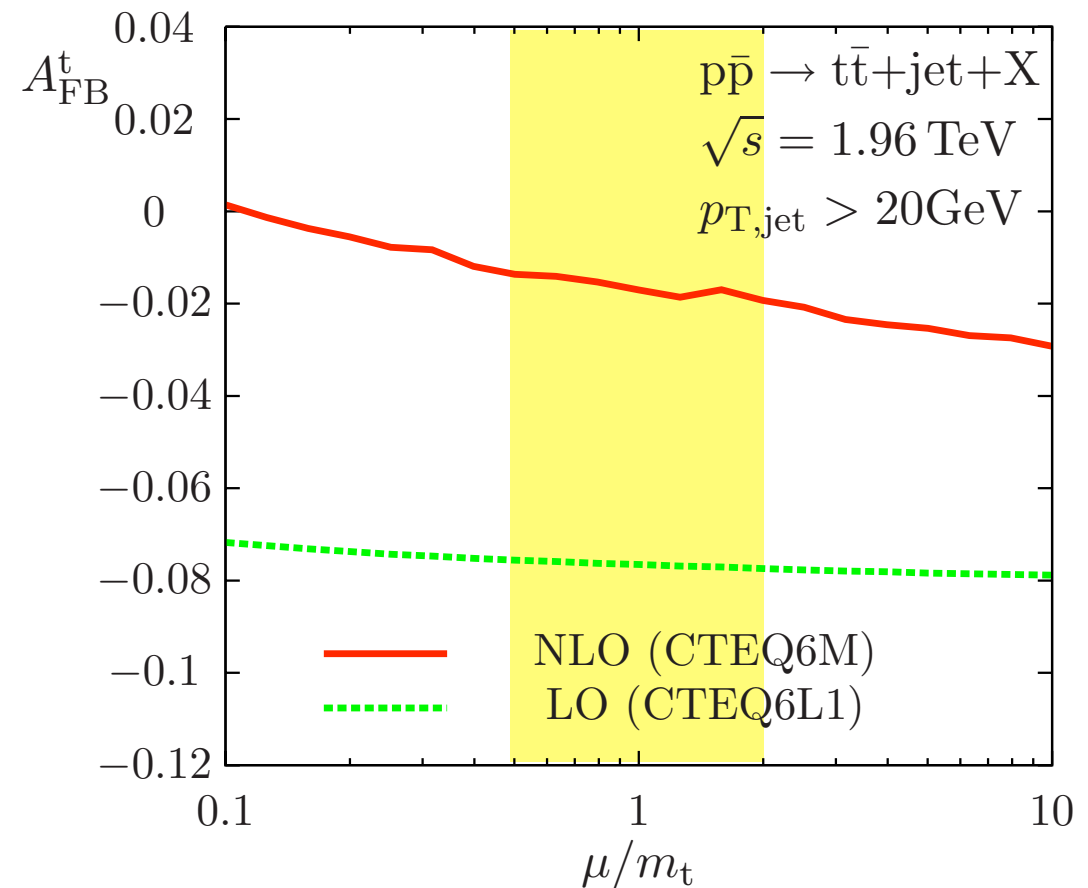
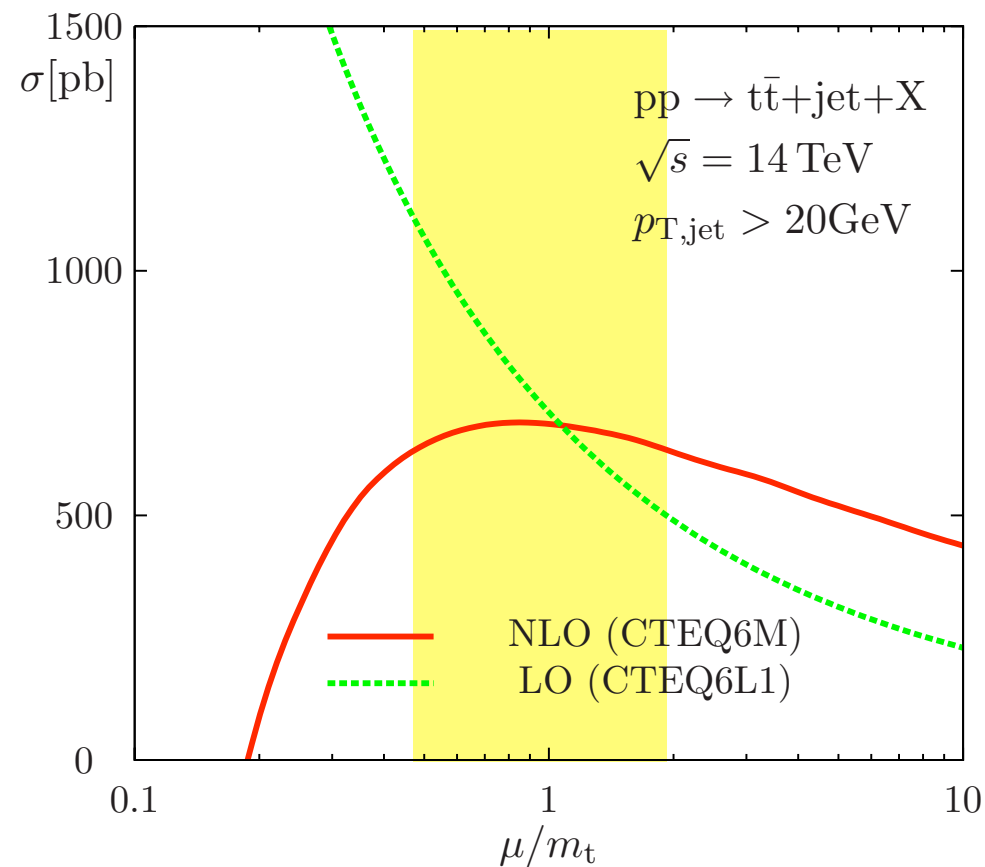
Motivation for NNLO

- constrain gluon PDF
- top mass from cross-section
- top FB asymmetry



$t\bar{t} + 1 \text{ jet}$

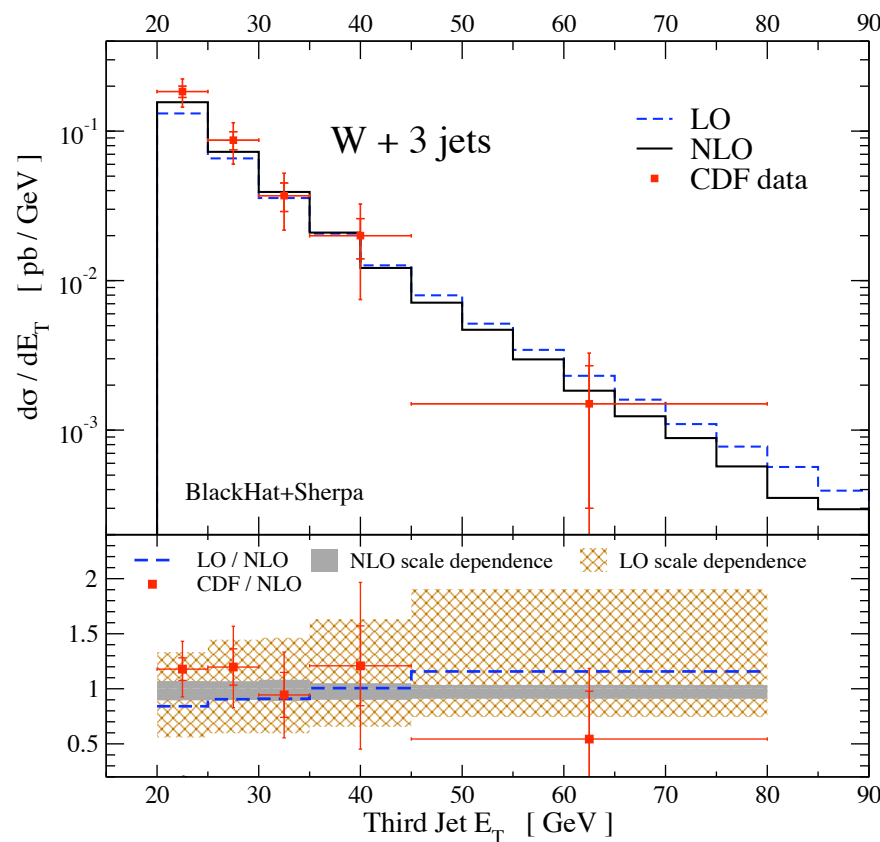
Dittmaier, Kallweit, Uwer '07-'08



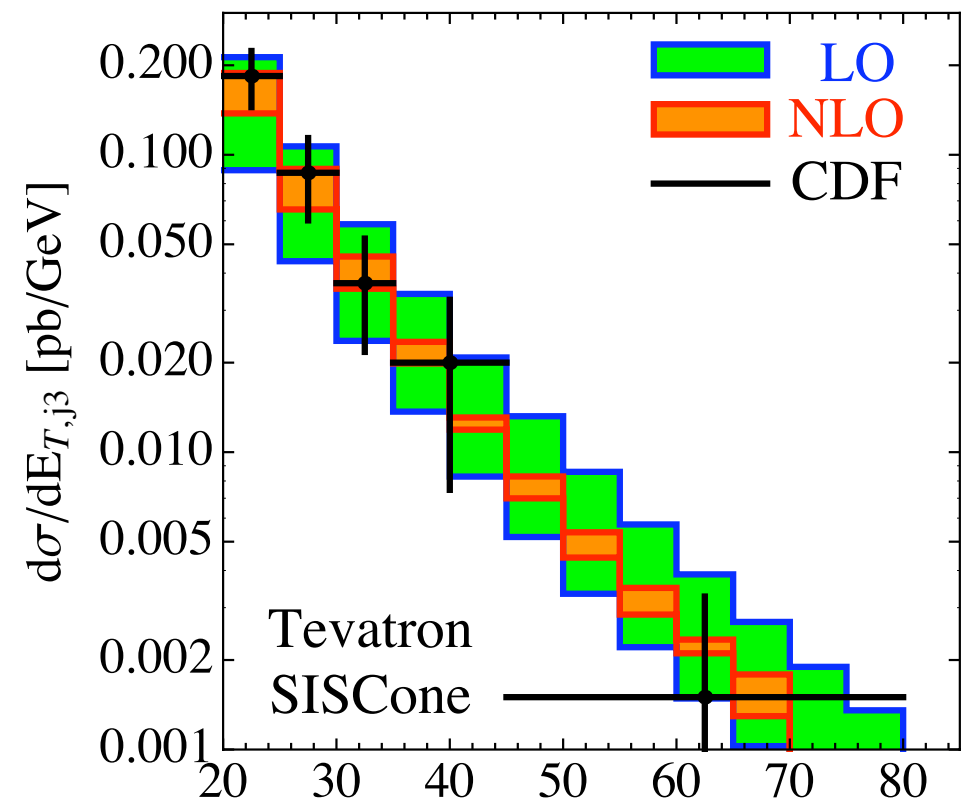
- ▶ improved stability of NLO result [\[but no decays\]](#)
- ▶ forward-backward asymmetry at the Tevatron compatible with zero
- ▶ essential ingredient of NNLO $t\bar{t}$ production (hot topic)

W + 3jets

Measured at the Tevatron + of primary importance at the LHC:
background to **model-independent new physics searches using jets + MET**



Berger et al. '09



$E_{T,j3}$ Ellis et al. '09

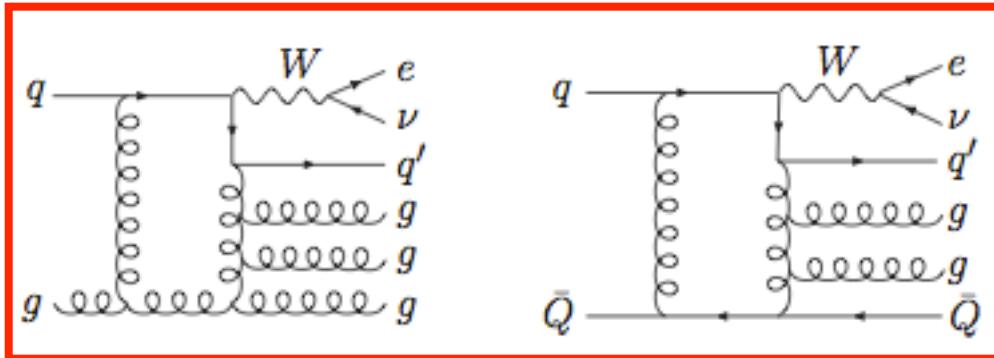
☺ Small $K=1.0-1.1$, reduced uncertainty: **50% (LO) → 10% (NLO)**

☺ First applications of new techniques to **2 → 4** LHC processes

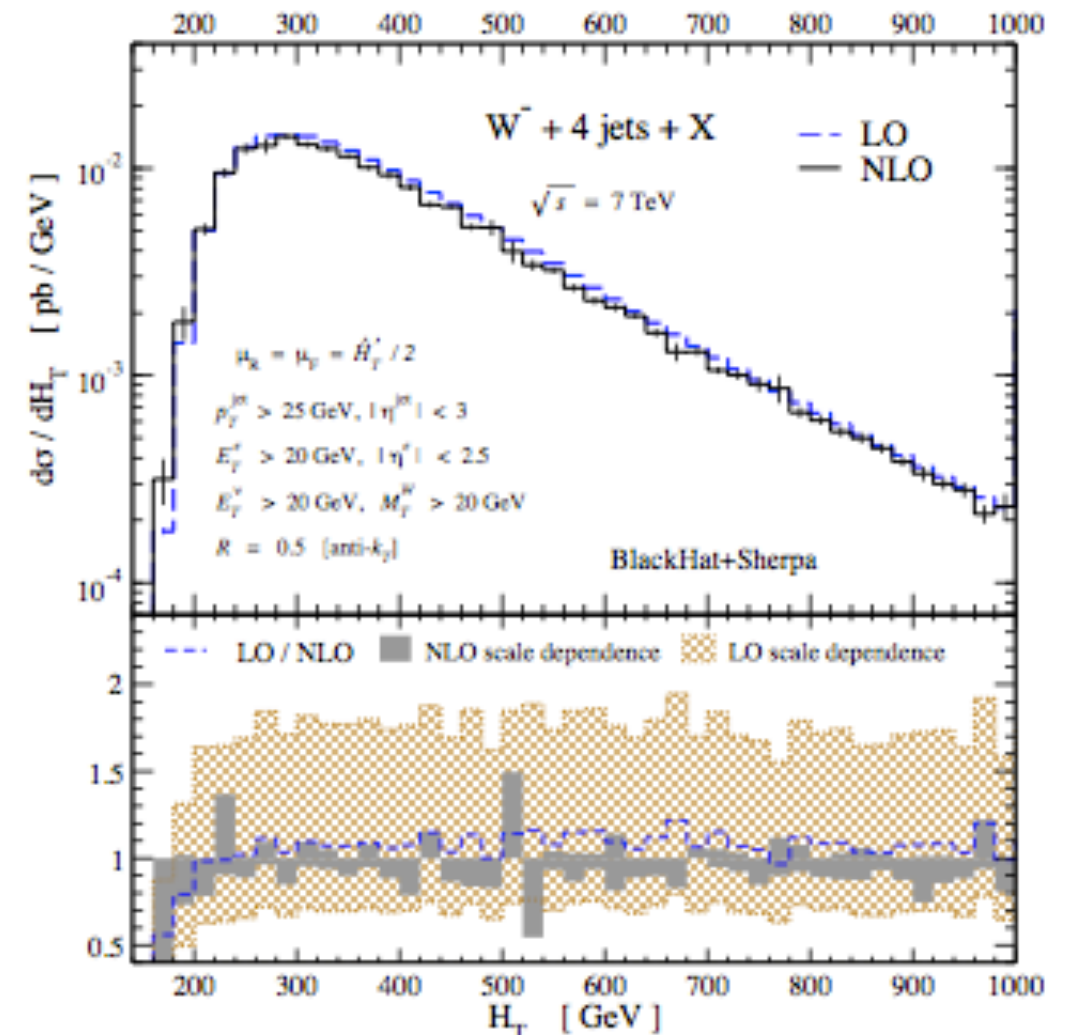
W + 4 jets at NLO

Sample diagrams*

Berger et al. '10



- first pp \rightarrow 5
- expected reduction of theoretical uncertainties
- key to top physics analyses: main background to tt in semi-leptonic channel



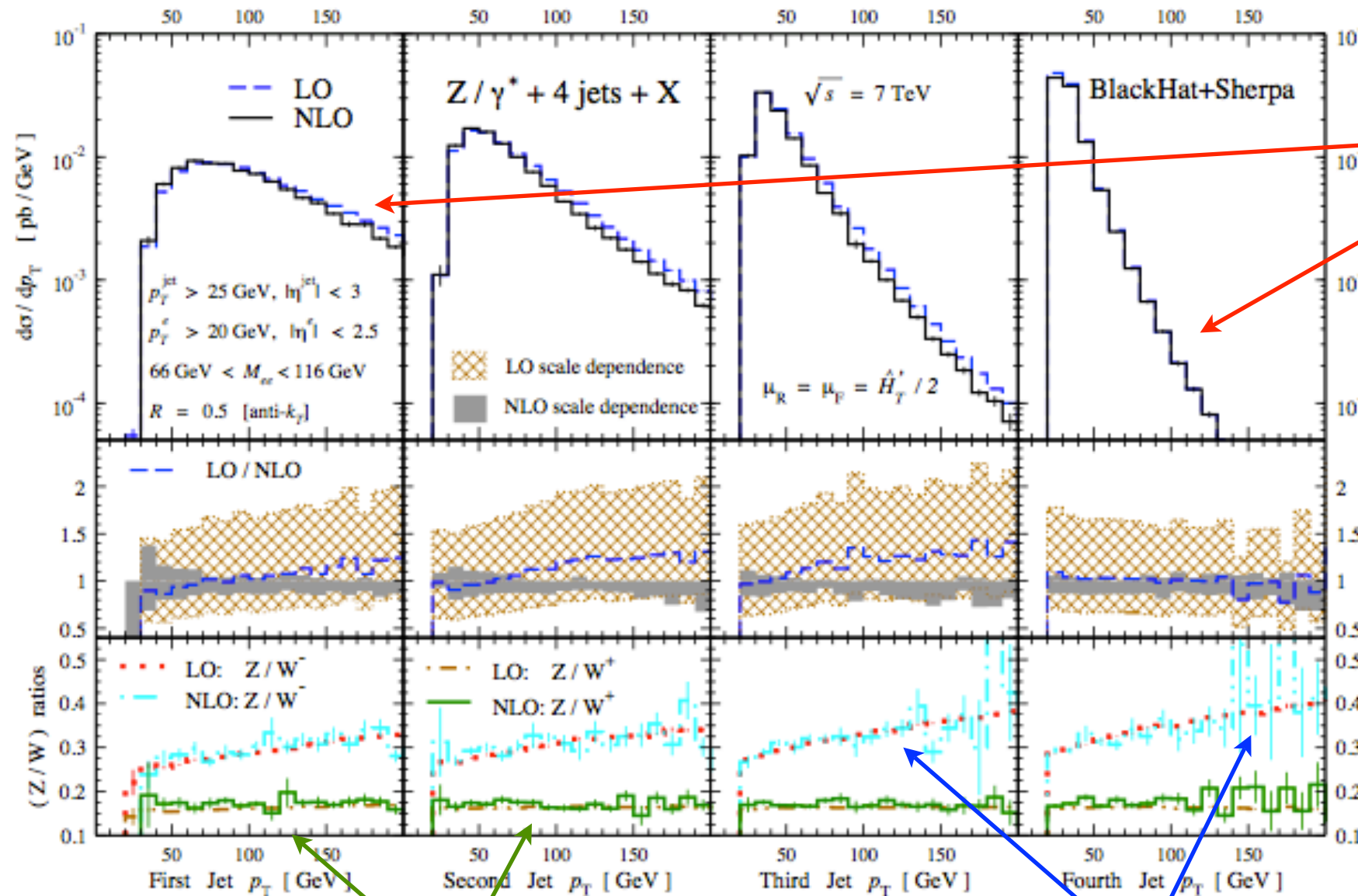
$$H_T = \sum_j p_{T,j} + p_{T,e} + p_{T,miss}$$

*Leading color calculation (OK to within 3% for lower multiplicities); missing W + 6q channels (also very small)

Z + 4 jets at NLO

4 jets + MET: important background to SUSY searches

Ita et al.'11



additional jets steeper

LO/NLO not always flat

ratios: excellent PT control

Z/W⁺: flat u(x)/u(x)

Z/W⁻: u(x)/d(x) enhancement

General NLO features?

Process	Typical scales		Tevatron K -factor			LHC K -factor		
	μ_0	μ_1	$\mathcal{K}(\mu_0)$	$\mathcal{K}(\mu_1)$	$\mathcal{K}'(\mu_0)$	$\mathcal{K}(\mu_0)$	$\mathcal{K}(\mu_1)$	$\mathcal{K}'(\mu_0)$
W	m_W	$2m_W$	1.33	1.31	1.21	1.15	1.05	1.15
$W+1\text{jet}$	m_W	p_T^{jet}	1.42	1.20	1.43	1.21	1.32	1.42
$W+2\text{jets}$	m_W	p_T^{jet}	1.16	0.91	1.29	0.89	0.88	1.10
$WW+\text{jet}$	m_W	$2m_W$	1.19	1.37	1.26	1.33	1.40	1.42
$t\bar{t}$	m_t	$2m_t$	1.08	1.31	1.24	1.40	1.59	1.48
$t\bar{t}+1\text{jet}$	m_t	$2m_t$	1.13	1.43	1.37	0.97	1.29	1.10
$b\bar{b}$	m_b	$2m_b$	1.20	1.21	2.10	0.98	0.84	2.51
Higgs	m_H	p_T^{jet}	2.33	–	2.33	1.72	–	2.32
Higgs via VBF	m_H	p_T^{jet}	1.07	0.97	1.07	1.23	1.34	1.09
Higgs+1jet	m_H	p_T^{jet}	2.02	–	2.13	1.47	–	1.90
Higgs+2jets	m_H	p_T^{jet}	–	–	–	1.15	–	–

$$\mathcal{K} = \frac{NLO}{LO}$$

[NLO report 0803.0494]

General features:

- ▶ color annihilation, gluon dominated \Rightarrow large K factors ?
- ▶ extra legs in the final state \Rightarrow smaller K -factors ?

But be careful, only full calculations can really tell!

NNLO: when is NLO not good enough?

 when **NLO corrections are large** (NLO correction \sim LO)

This may happen when

- process involve very different scales \rightarrow large logarithms of ratio of scales appear
- new channels open up at NLO (at NLO they are effectively LO)
- master example: Higgs production

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 when **high precision is needed** to match small experimental error

- W/Z hadro-production, heavy-quark hadro-production, α_s from event shapes in e^+e^- ...

NNLO: when is NLO not good enough?

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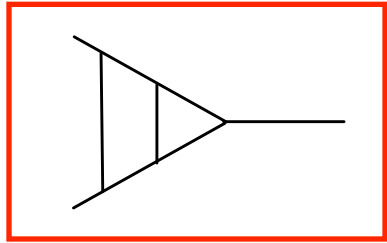
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🎤 when **high precision is needed** to match small experimental error

- W/Z hadro-production, heavy-quark hadro-production, α_s from event shapes in e^+e^- ...

🎤 when **a reliable error estimate is needed**



Collider processes known at NNLO

Collider processes known at NNLO today:

(a) Drell-Yan (Z,W)

(b) Higgs, also associated HV

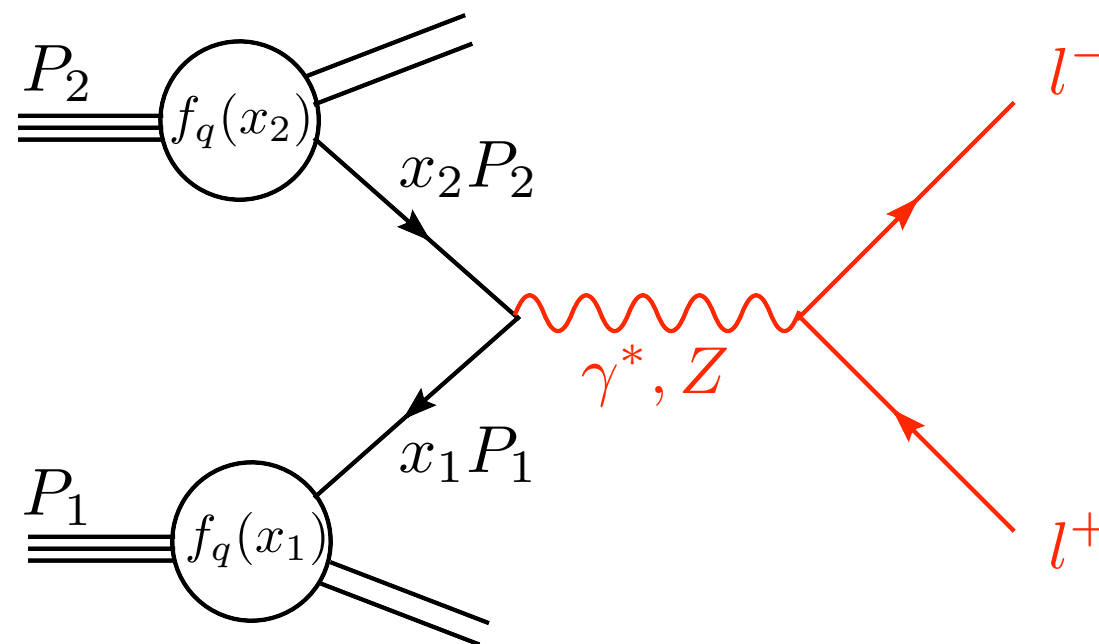
(c) 3-jets in e^+e^-

Drell-Yan processes

Drell-Yan processes: Z/W production ($W \rightarrow l\nu$, $Z \rightarrow l^+l^-$)

Very clean, golden-processes in QCD because

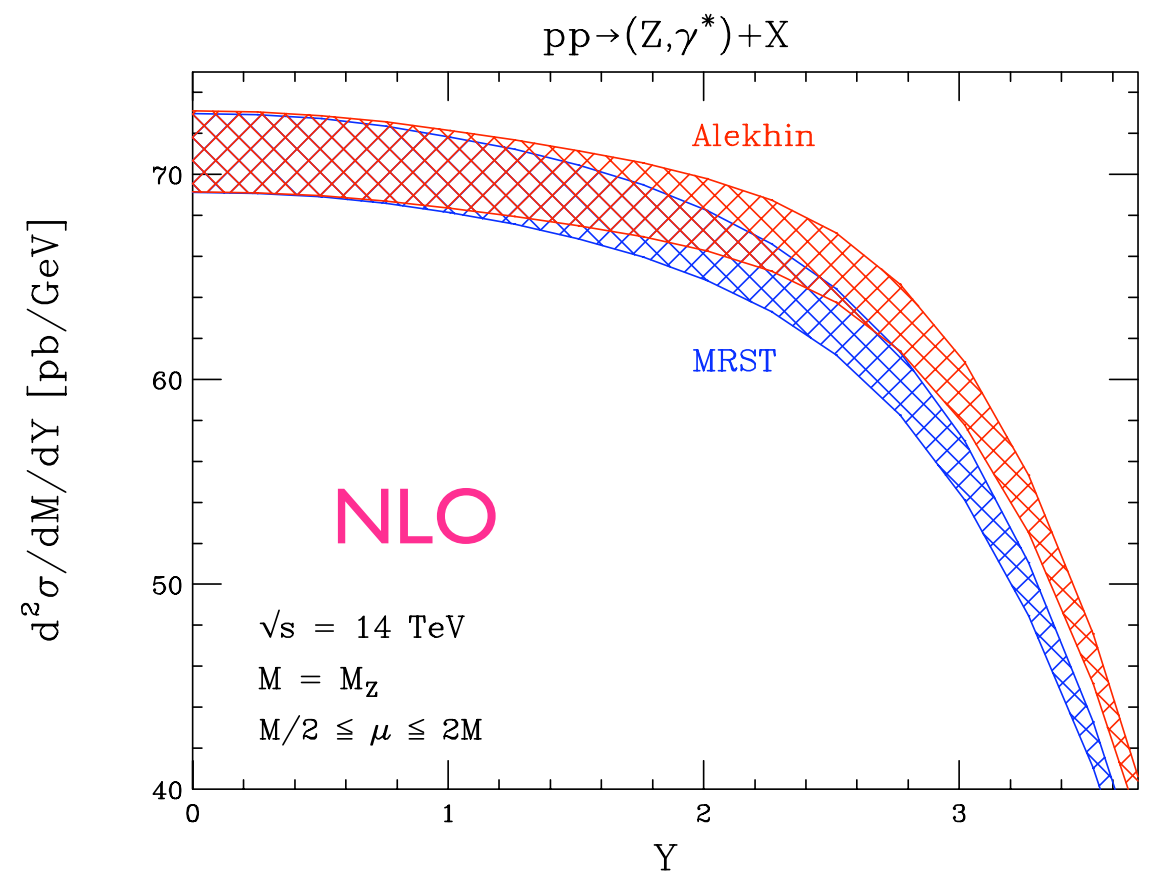
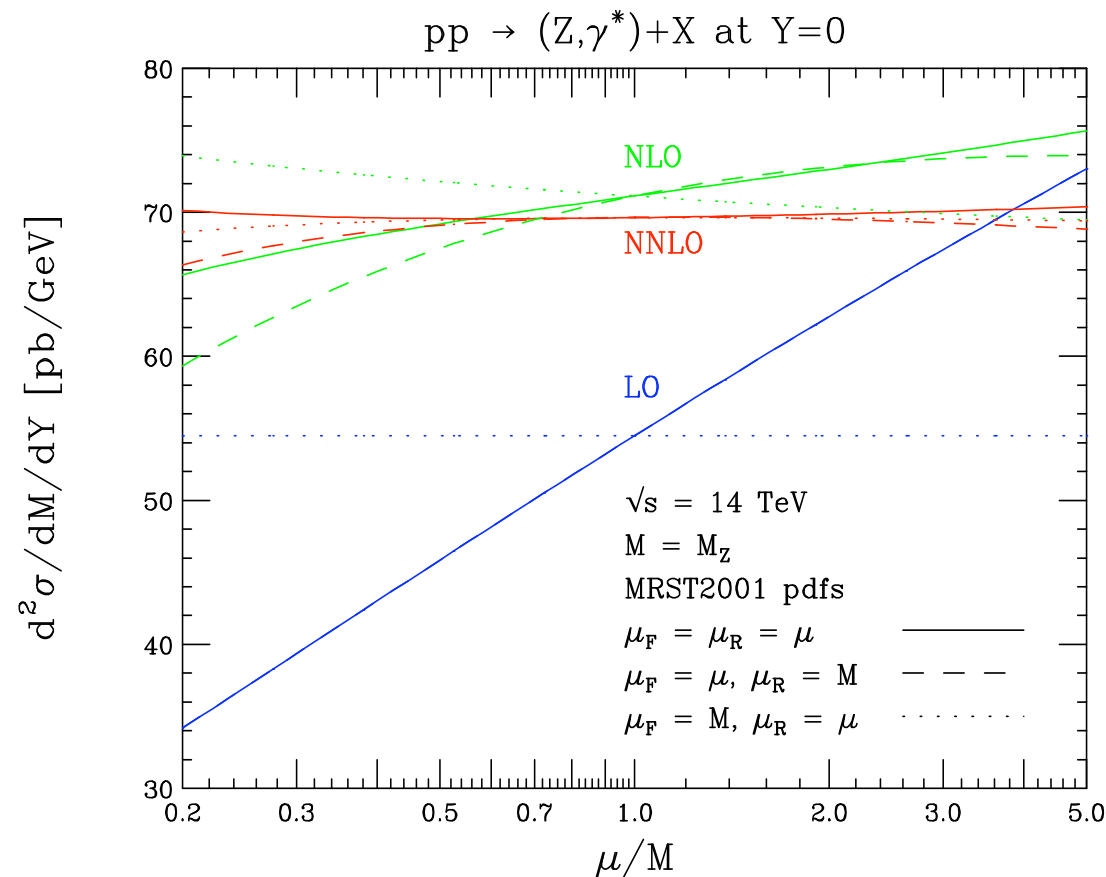
- ✓ dominated by quarks in the initial state
 - ✓ no gluons or quarks in the final state (QCD corrections small)
 - ✓ leptons easier experimentally (clear signature)
- ⇒ as clean as it gets at a hadron collider



Drell-Yan processes

- most important and precise test of the SM at the LHC
- best known process at the LHC: spin-correlations, finite-width effects, γ -Z interference, fully differential in lepton momenta

Scale stability and sensitivity to PDFs

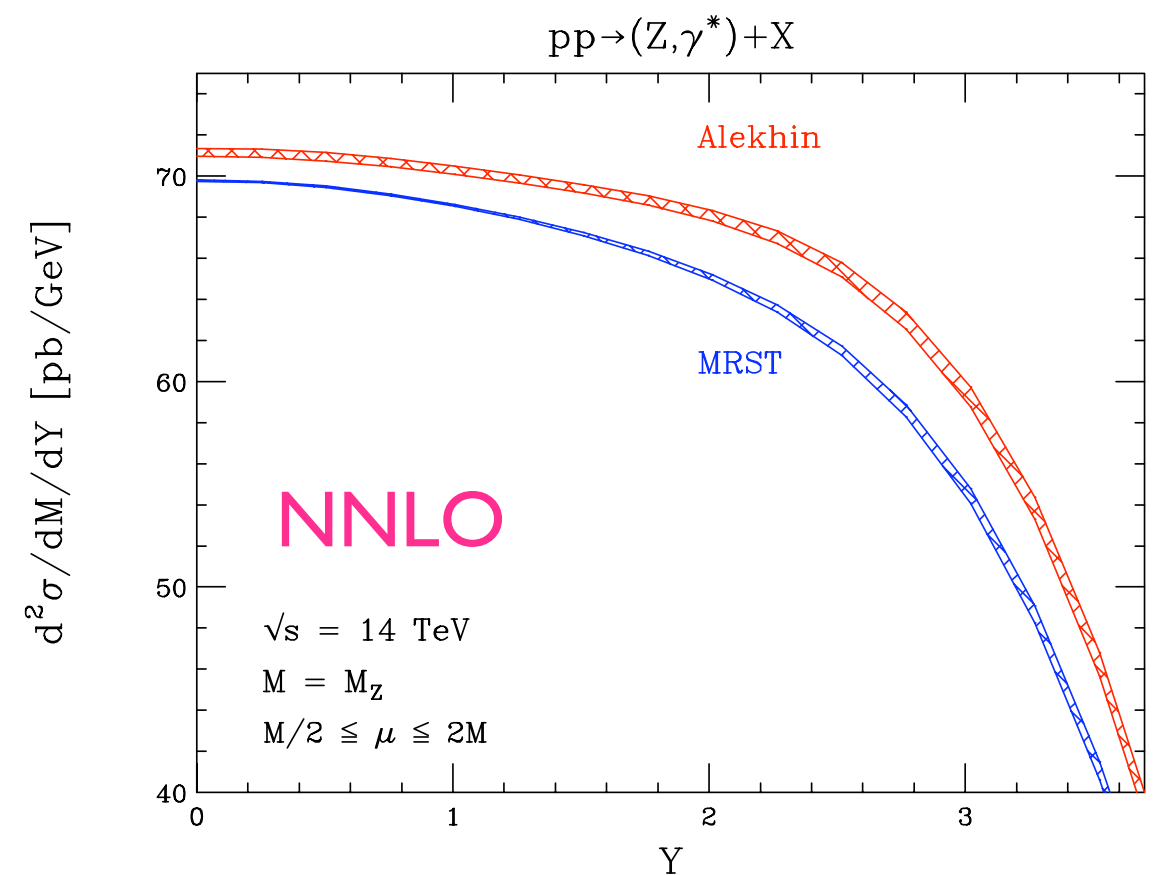
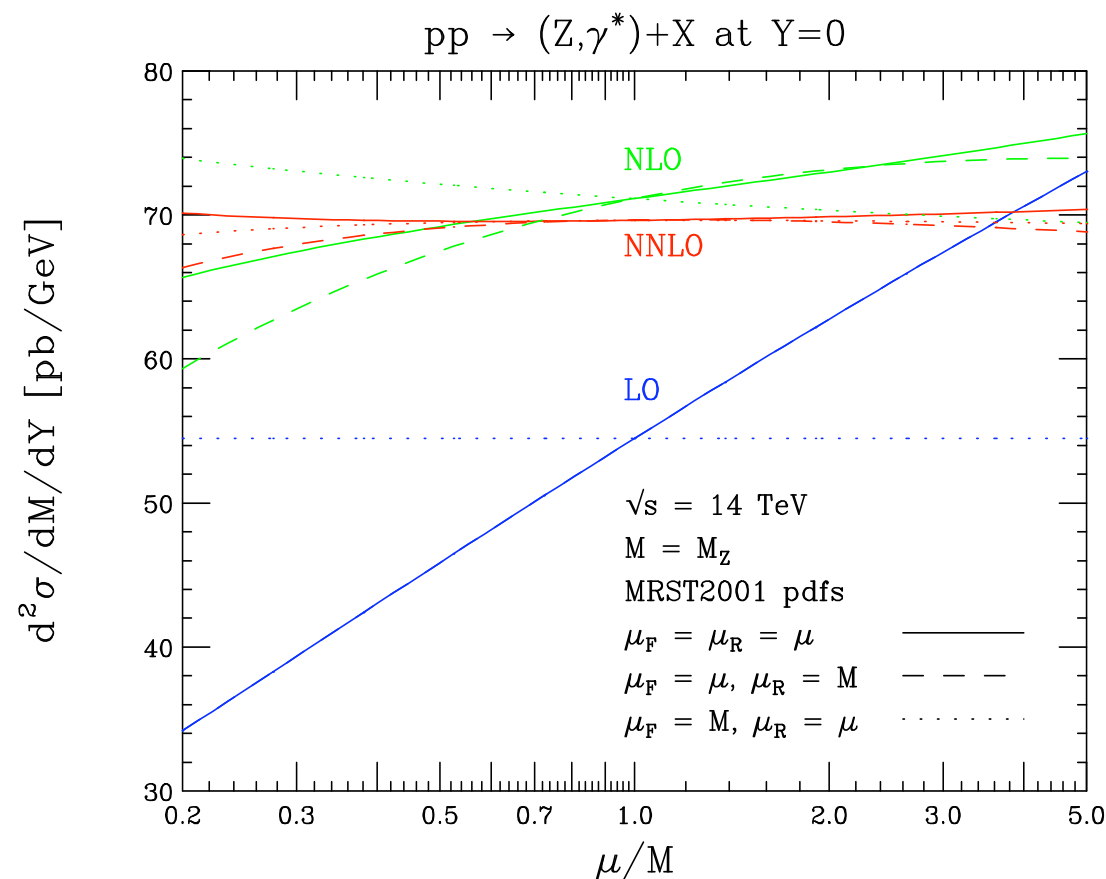


Anastasiou, Dixon, Melnikov, Petriello '03, '05; Melnikov, Petriello '06

Drell-Yan processes

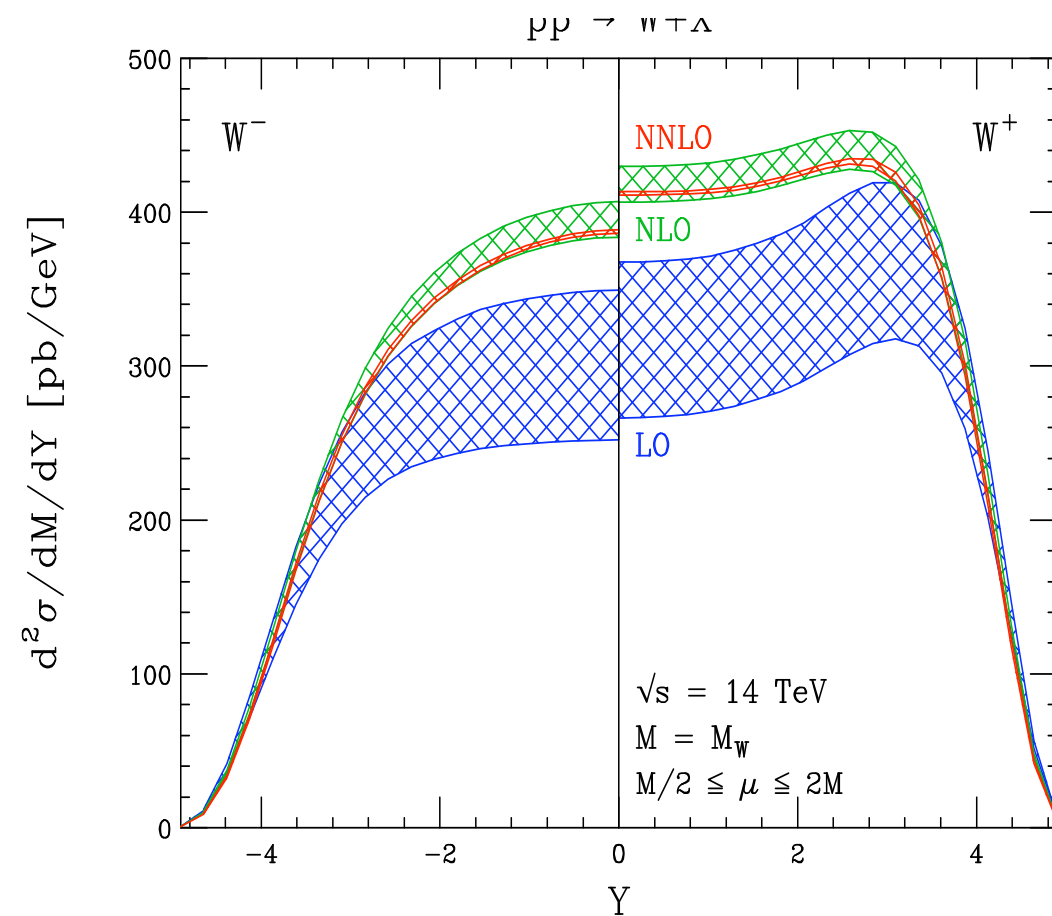
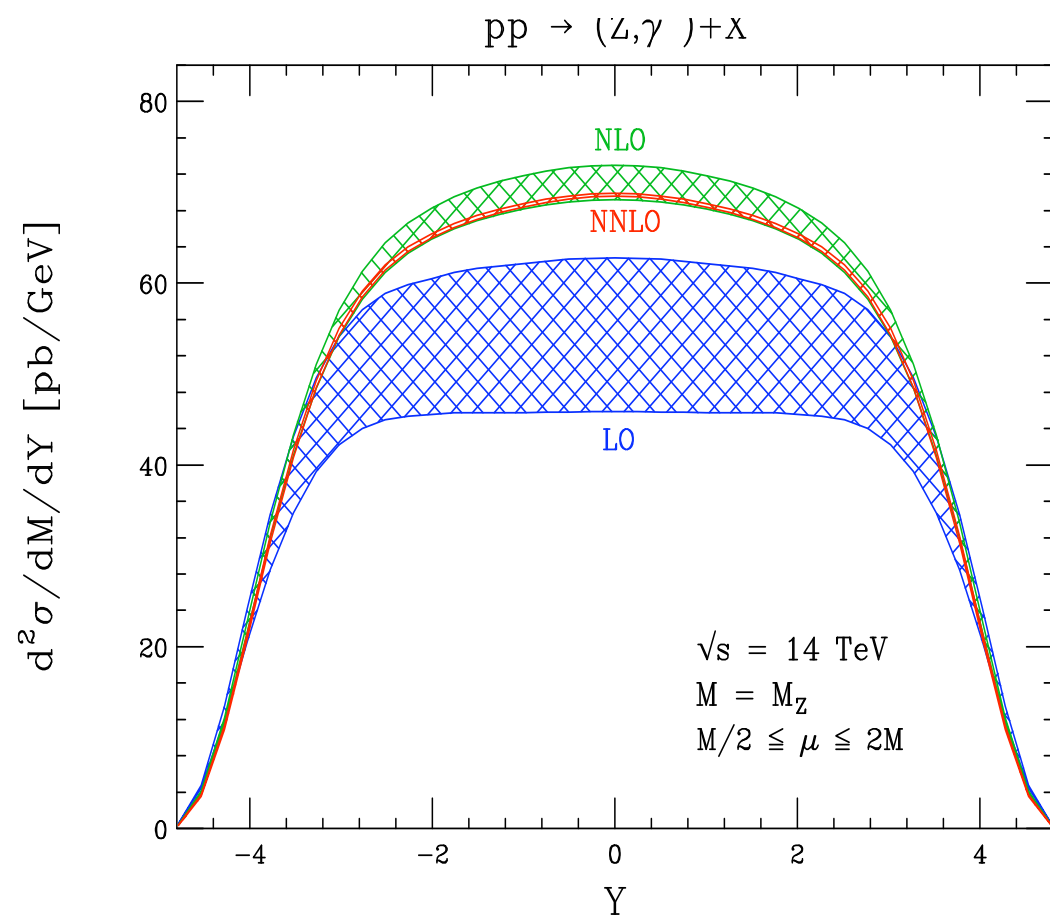
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Scale stability and sensitivity to PDFs



Anastasiou, Dixon, Melnikov, Petriello '03, '05; Melnikov, Petriello '06

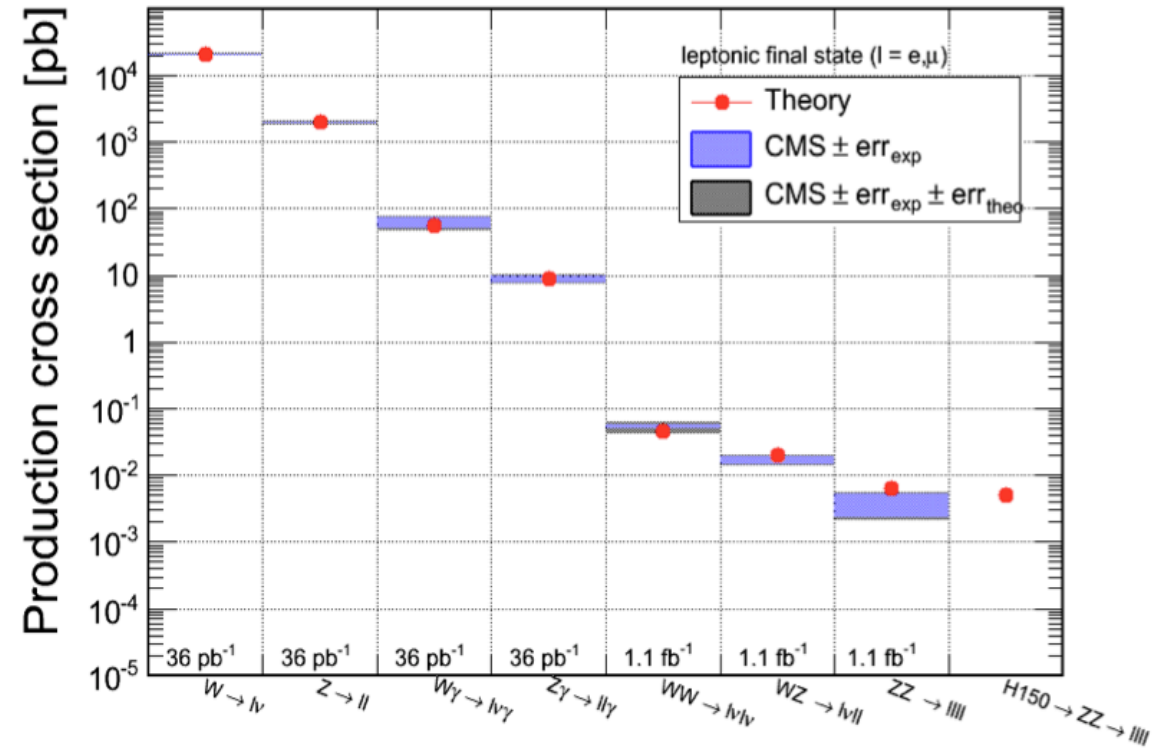
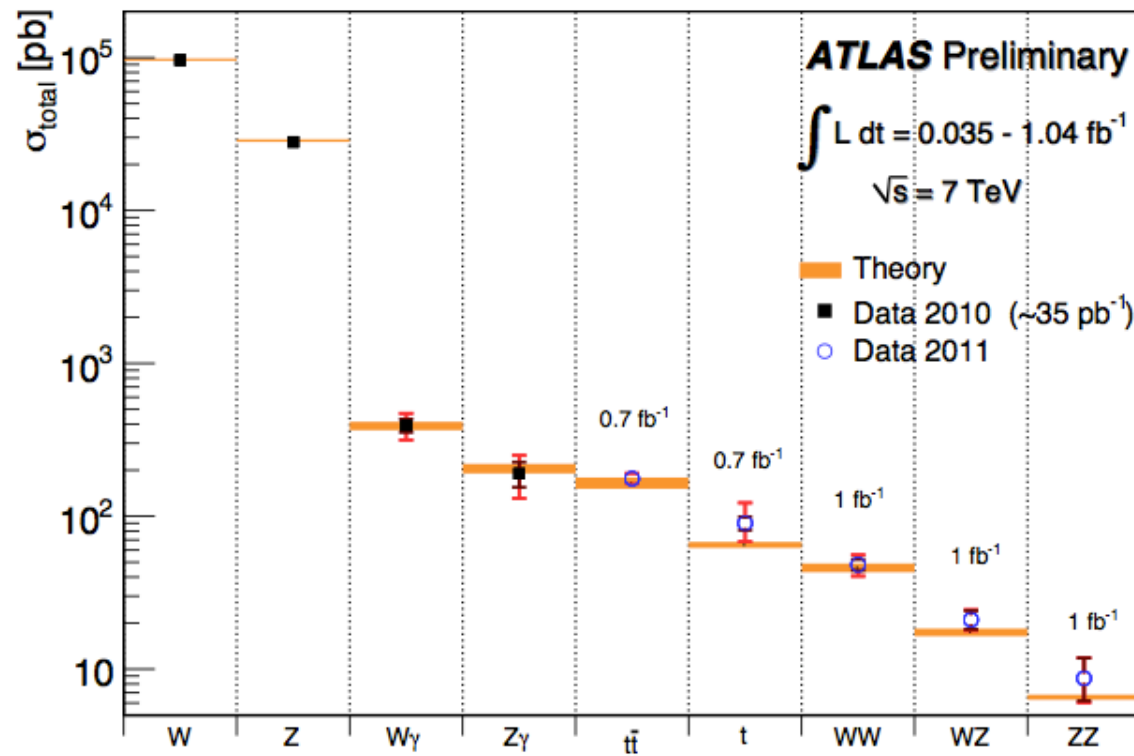
Drell-Yan: rapidity distributions



Anastasiou, Dixon, Melnikov, Petriello '03, '05; Melnikov, Petriello '06

☛ *LHC: perturbative accuracy of the order of 1%. This is absolutely unique!*

NNLO vs LHC data



$$\sigma(W) \cdot B(W \rightarrow e\nu) \sim 10 \text{ nb} \qquad \sigma(Z) \cdot B(Z \rightarrow e^+e^-) \sim 1 \text{ nb}$$

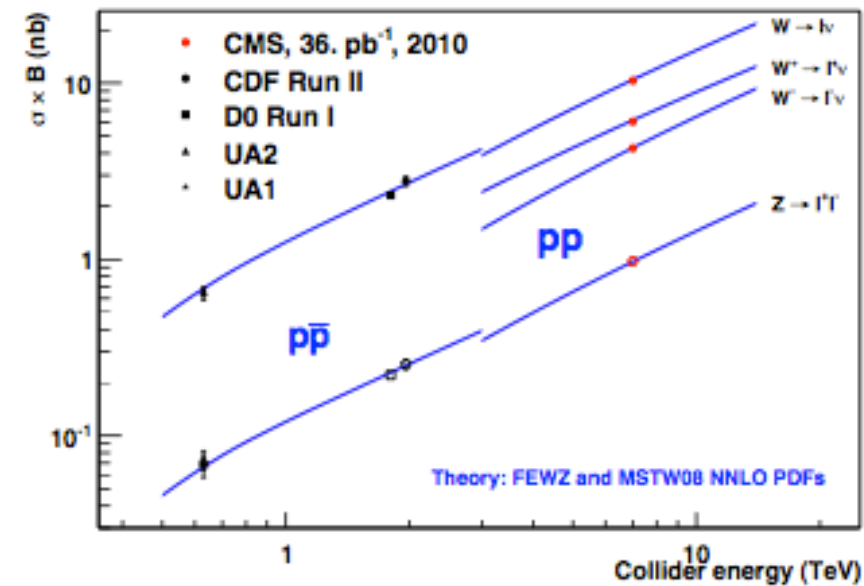
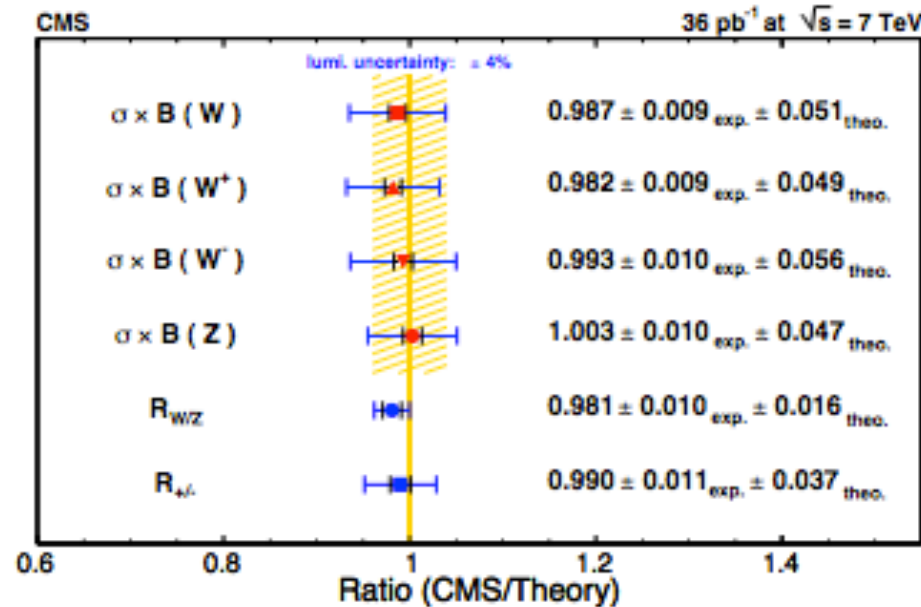
$$\sigma(WW) \cdot B(W \rightarrow l\nu)^2 \sim 100 \text{ fb} \qquad \sigma(ZZ) \cdot B(W \rightarrow l^+l^-)^2 \sim 10 \text{ fb}$$

E. g. per 1 fb^{-1} :

- $O(10^6)$ W and $O(10^5)$ Z events per experiment and lepton channel
- $O(100)$ WW and $O(10)$ ZZ per experiment including all lepton channels

NNLO vs LHC data

Impressive agreement between experiment and NNLO theory



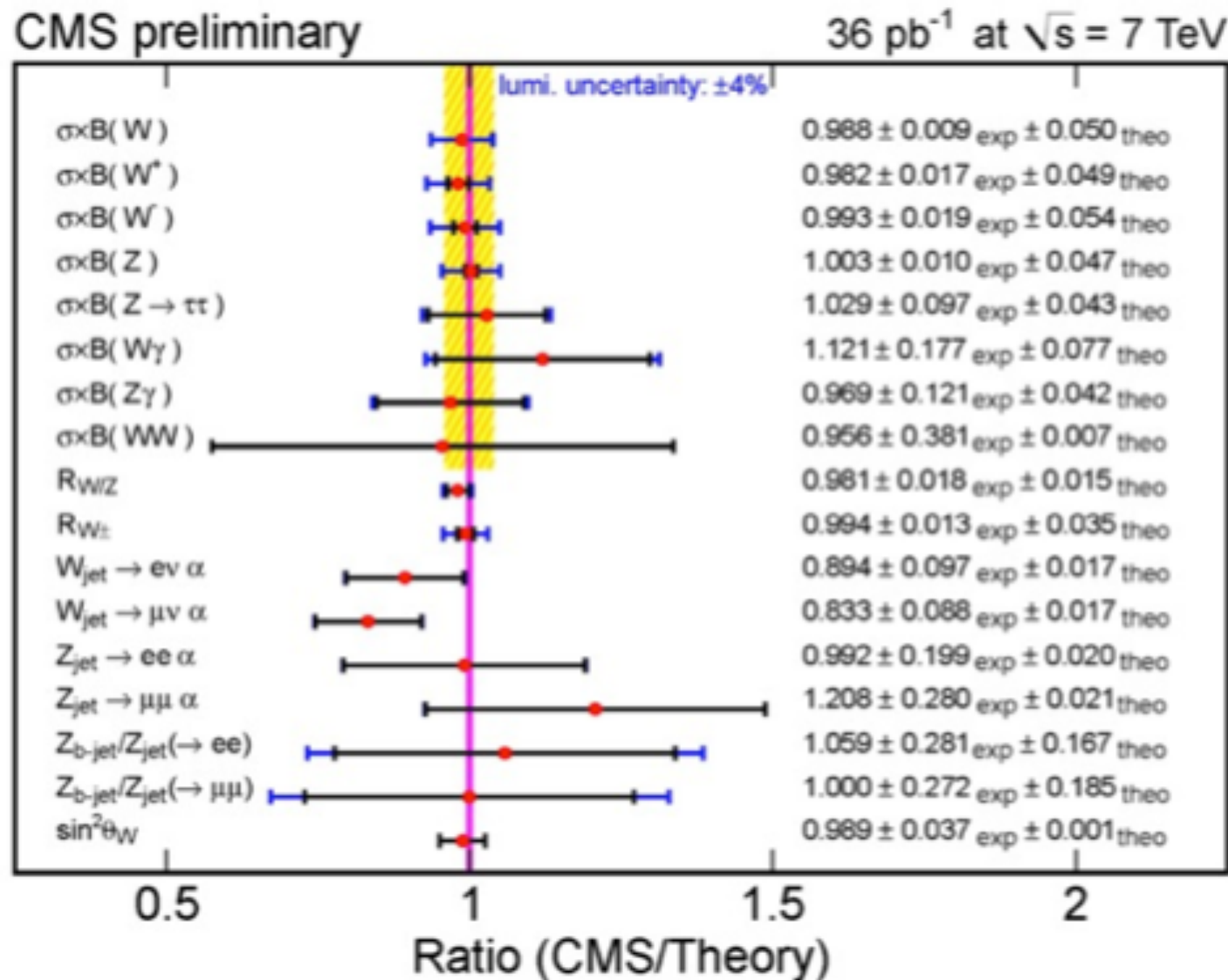
Quantity	Ratio (CMS/Theory)	Lumi. uncert. (4%)
$\sigma \times BF(W^\pm)$	0.987 ± 0.009 (ex) ± 0.051 (th) [± 0.051 (tot)]	0.039
$\sigma \times BF(W^+)$	0.982 ± 0.009 (ex) ± 0.049 (th) [± 0.050 (tot)]	0.039
$\sigma \times BF(W^-)$	0.993 ± 0.010 (ex) ± 0.056 (th) [± 0.057 (tot)]	0.040
$\sigma \times BF(Z)$	1.003 ± 0.010 (ex) ± 0.047 (th) [± 0.048 (tot)]	0.040
$\sigma \times BF(W)/\sigma \times BF(Z)$	0.981 ± 0.010 (ex) ± 0.016 (th) [± 0.019 (tot)]	—
$\sigma \times BF(W^+)/\sigma \times BF(W^-)$	0.990 ± 0.011 (ex) ± 0.037 (th) [± 0.039 (tot)]	—

Theory error completely dominated by PDFs

CMS PAS EWK-10-005, similar results from ATLAS not shown here

NNLO vs LHC data

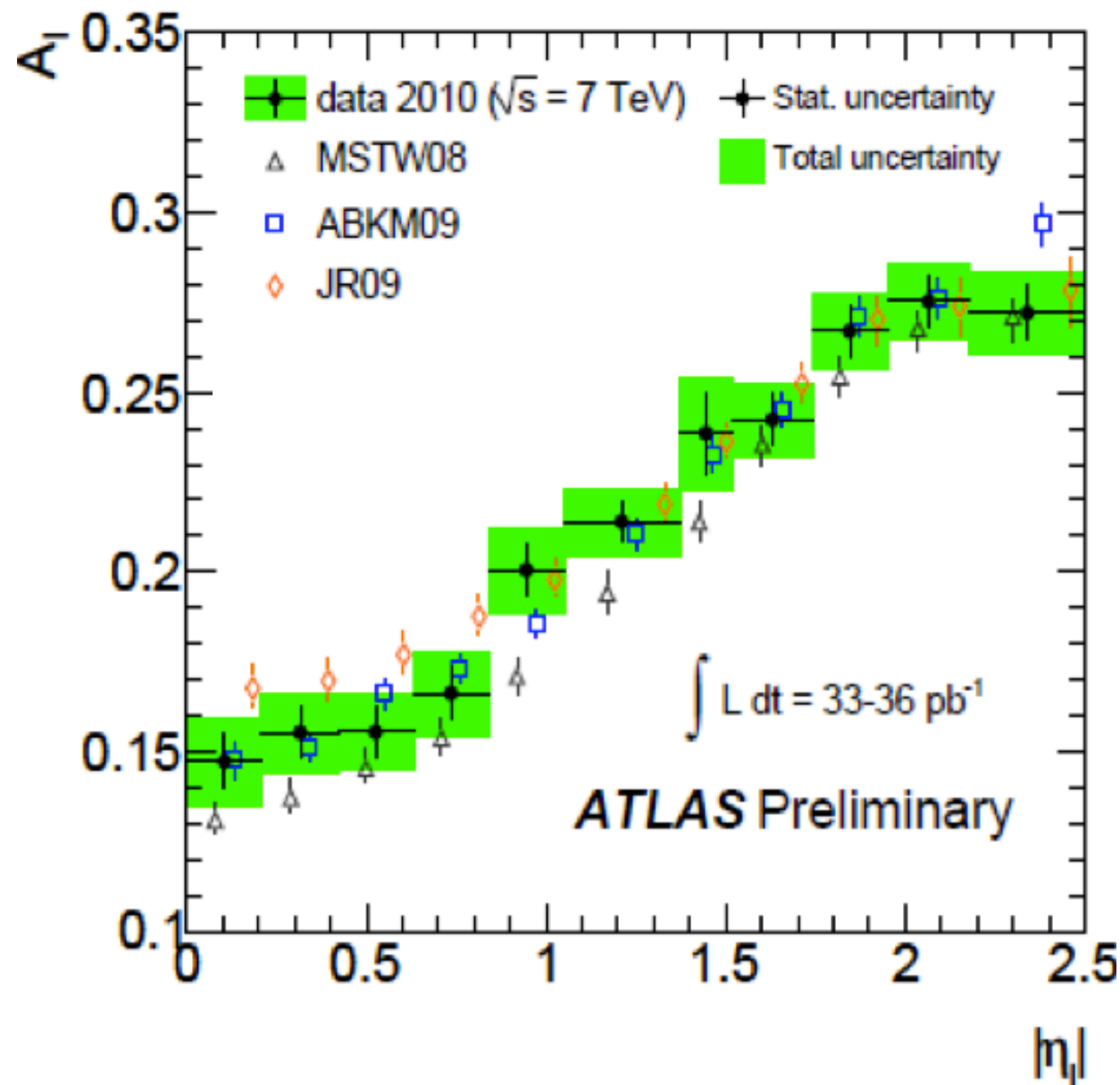
Spectacular experimental achievements in very little time!



- remarkable agreement with theory
- precise measurement of W/Z properties (also notice measurement of $\sin^2\theta_W$)
- achieved control and precision already allows improvements on PDFs

Charge asymmetry

Natural extension of the inclusive cross-section is the $R_W = W^+/W^-$ ratio. Study R_W as a function of kinematics variables, e.g. charge asymmetry as a function of lepton rapidity

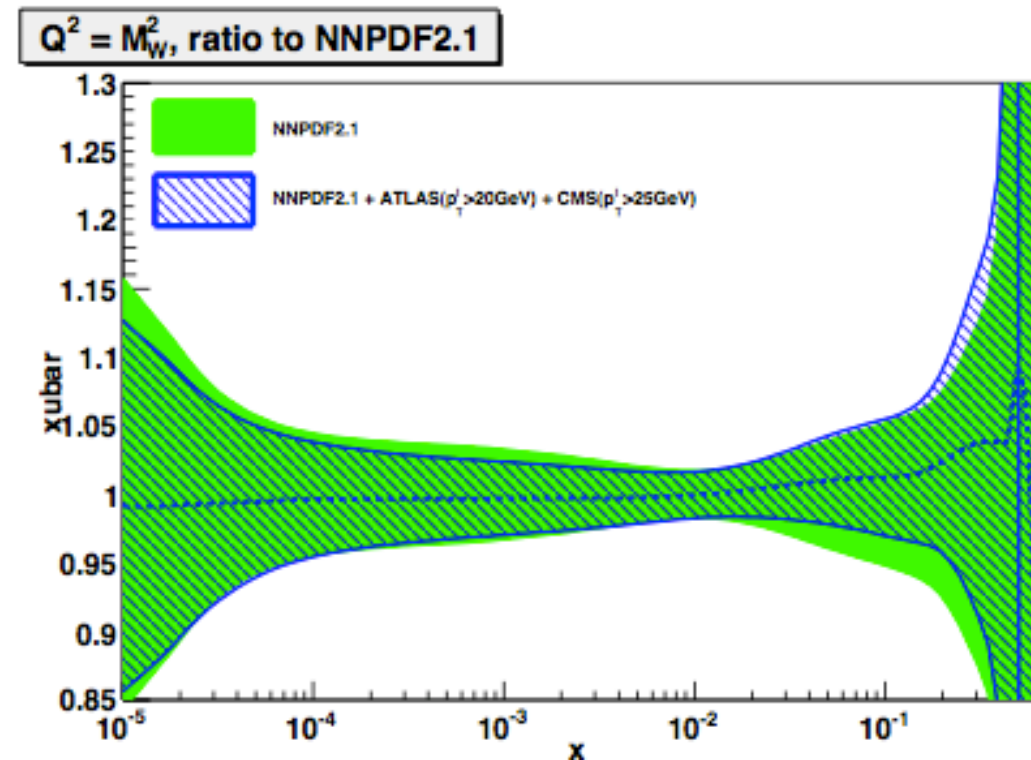
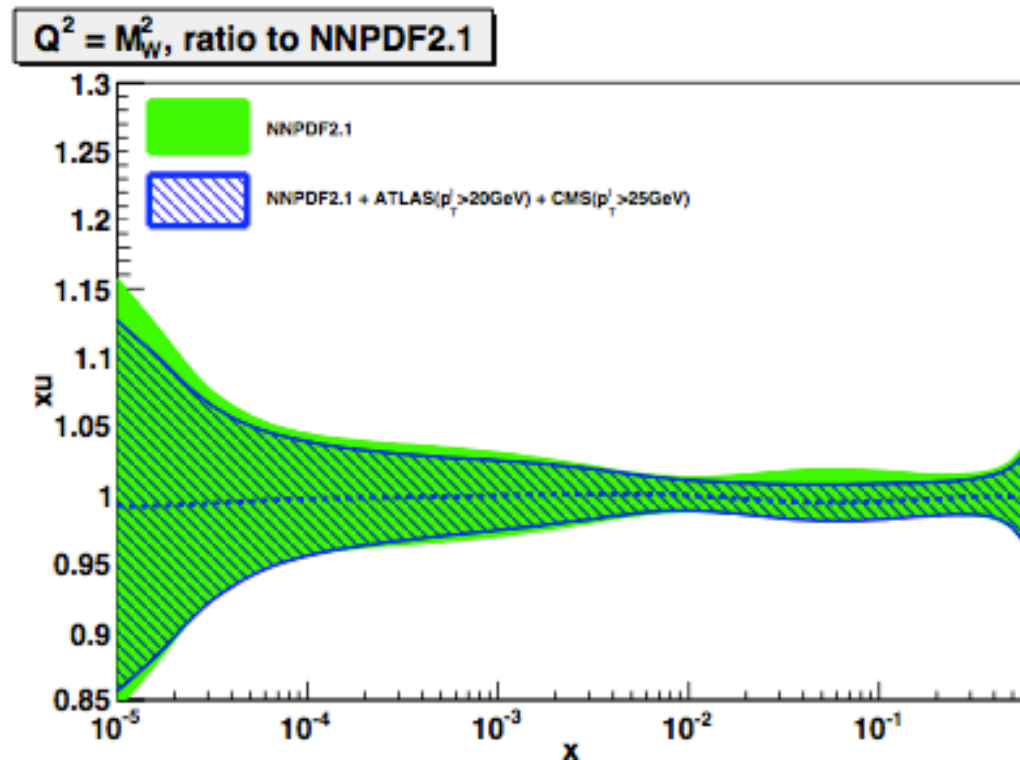


$$A(\eta) = \frac{R_W(\eta) - 1}{R_W(\eta) + 1}$$

- measurement very sensitive to PDFs since many uncertainties cancel in ratios
- good agreement with various PDFs but very sensitive to shape details
- similar results by CMS

Charge asymmetry

Effect of ATLAS and CMS lepton charge asymmetry on NNPDF global fit



Reduction of uncertainty of the order of 10-30% in the range $x=10^{-3}-10^{-1}$
Similar results for d-quark and other sea distributions

NNPDF 1108.1758

NB:

LHCb data at larger rapidities probe larger and smaller values of x that are less constraint, they will have a larger impact than ATLAS/CMS soon

Higgs

Besides Drell-Yan, we know the inclusive Higgs production cross-section at NNLO. But before discussing results, a short theory introduction is in order....

Spontaneous Symmetry Breaking (SSB)

In the first lecture we saw that a mass term $m^2 A_\mu A^\mu$ violates gauge invariance. How can one generate mass terms for W/Z?

Solution: add to the Lagrangian of a spin one field **a complex scalar field**

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + |D_\mu \Phi|^2 - V(\Phi) \quad V(\Phi) = \mu^2 |\Phi|^2 + \lambda |\Phi|^4$$

For $\mu^2 > 0$: unique minimum at $\Phi = 0 \Rightarrow M_A = 0$ and $M_\phi = \mu$ (QED)

Reverse sign of μ^2 in V: $V(\Phi) = -\mu^2 |\Phi|^2 + \lambda |\Phi|^4$

Minimum of the potential at $\Phi = \sqrt{\frac{\mu^2}{2\lambda}} \equiv \frac{v}{\sqrt{2}}$

Expand Φ around minimum $\Phi = \frac{1}{\sqrt{2}}(v + H + i\chi)$

SSB mechanism

The Lagrangian becomes

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_{\mu}H\partial^{\mu}H - \frac{1}{2}\partial_{\mu}\chi\partial^{\mu}\chi + \frac{1}{2}e^2v^2A_{\mu}A^{\mu} \\ + \frac{1}{2}e^2A_{\mu}A^{\mu}(H^2 + \chi^2) + \dots + V(\Phi)$$

We now have

- a massive scalar H with cubic&quartic interactions
- a photon of mass $M_A = ev$
- massless scalar field χ (Goldstone boson)

The field χ can be reabsorbed into a redefinition of A and via a gauge transformation (unitary gauge)

$$A_{\mu} \rightarrow A_{\mu} - \frac{1}{ev}\partial_{\mu}\chi \quad \Phi \rightarrow e^{-i\frac{\chi}{v}}\Phi$$

Degrees of freedom **before** and **after** symmetry breaking:

- before: 1 massless gauge boson (2 dof) + 1 complex field (2 dof)
- after: 1 massive gauge boson (3 dof) + 1 real field (1 dof)

Higgs boson in the SM

Consider a Higgs kinetic term

$$|D_\mu \Phi|^2$$

With the covariant derivative of the $SU(2) \otimes U(1)$ gauge theory

$$D_\mu = \partial_\mu + i \frac{g_w}{2} \sigma^i W_\mu^i + i \frac{g'_W}{2} B_\mu$$

Expanding Φ around the vacuum expectation value

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H \end{pmatrix}$$

Leads to

$$|D_\mu \Phi|^2 = \frac{1}{2} (\partial_\mu H)^2 + \frac{g_W^2 v^2}{4} W^{+\mu} W_\mu^- + \frac{v^2}{8} (g_W W_\mu^0 - g'_W B_\mu) + \dots$$

Higgs boson in the SM

So one gets **three massive bosons** W^\pm and Z with

$$Z_\mu = \frac{1}{\sqrt{g_W^2 + g'_W{}^2}} (g_W W_\mu^0 - g'_W B_\mu)$$

with masses

$$M_{W^\pm} = \frac{1}{2} g_W v \quad M_Z = \frac{1}{2} \sqrt{g_W^2 + g'^2} v$$

and a **massless photon** (orthogonal to the Z)

$$A_\mu = \frac{1}{\sqrt{g_W^2 + g'^2}} (g_W W_\mu^0 + g'_W B_\mu)$$

Higgs boson in the SM

It is customary to introduce the weak mixing angle

$$\sin \theta_W = \frac{g'_W}{\sqrt{g_W^2 + g'_W{}^2}} \quad \cos \theta_W = \frac{M_W}{M_Z}$$

The Higgs vev can then be expressed through the Fermi constant G_F , which is known precisely from μ decays

$$\frac{G_F}{\sqrt{2}} = \left(\frac{g_W}{2\sqrt{2}} \right)^2 \frac{1}{M_W^2} \quad \Rightarrow \quad v = \sqrt{\frac{1}{\sqrt{2}G_F}} \approx 246.22 \text{ GeV}$$

Similarly, fermion masses are generated through **Yukawa interactions**

Fermion masses and interactions

Consider the electron

$$\mathcal{L}_e = -G_e \bar{e}_L^i \Phi_i e_R + \text{h.c.}$$

In the unitarity gauge this becomes

$$\mathcal{L}_e = -\frac{G_e}{2} \begin{pmatrix} \bar{\nu}_L \\ \bar{e}_L \end{pmatrix}^T \begin{pmatrix} 0 \\ v + H \end{pmatrix} e_R + \text{h.c.}$$

So this gives rise to a mass term and an interaction term

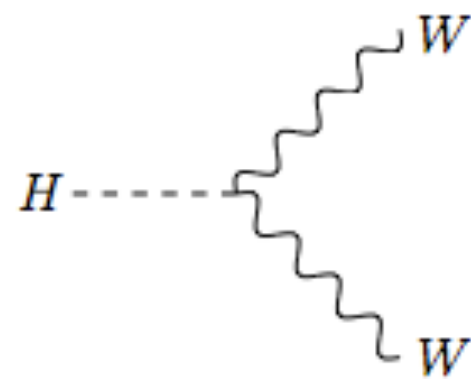
$$\mathcal{L}_e = \frac{G_e v}{2} \bar{e}e - \frac{G_e v}{2} \bar{e}He$$

We read off the **electron mass** and the **Yukawa coupling**

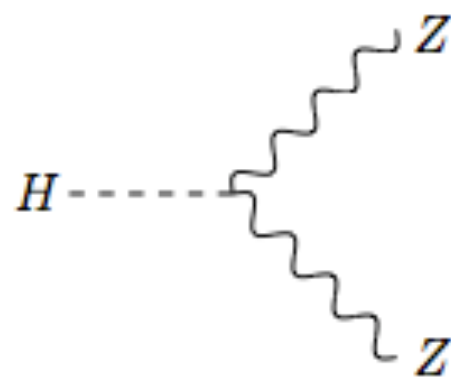
Quark masses are generated in a similar way through Yukawa interactions

Couplings to the SM Higgs boson

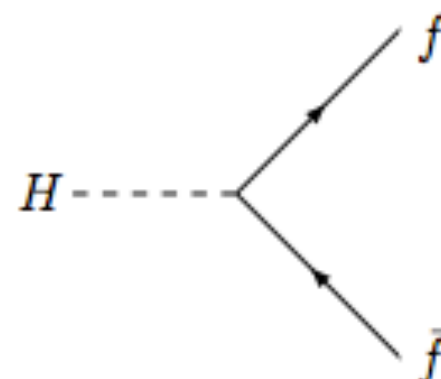
Three-point couplings to Higgs boson:



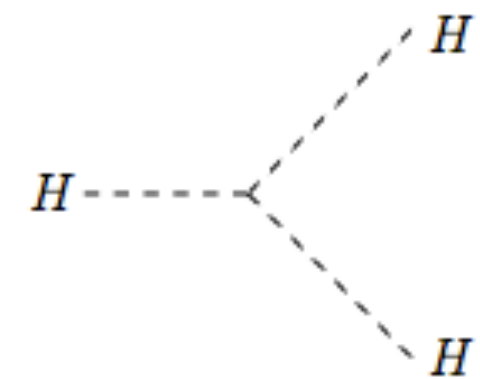
$$= ig_W M_W g_{\mu\nu}$$



$$= i \frac{g_W}{\cos^2 \theta_W} M_W g_{\mu\nu}$$

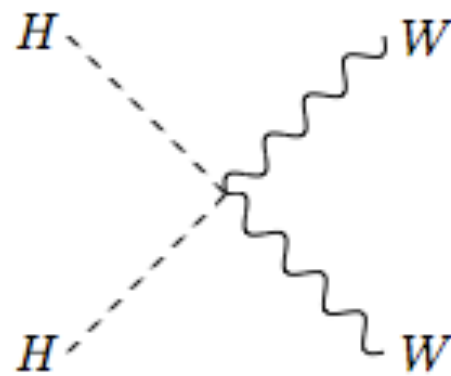


$$= -\frac{ig_W m_f}{2M_W}$$

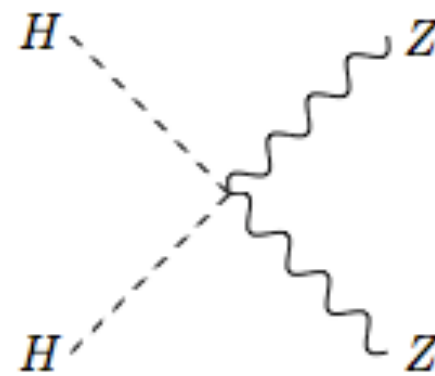


$$= -\frac{3ig_W M_H^2}{2M_W}$$

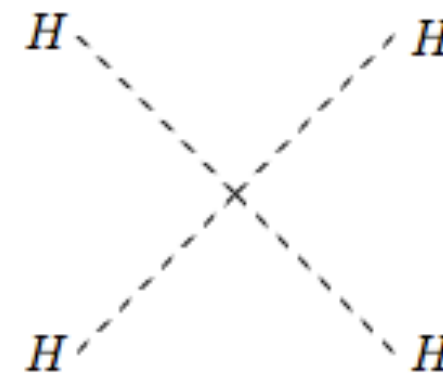
Four-point couplings to Higgs boson:



$$= \frac{1}{2} ig_W^2 g_{\mu\nu}$$



$$= \frac{ig_W^2}{2 \cos^2 \theta_W} g_{\mu\nu}$$



$$= -\frac{3ig_W M_H^2}{4M_W^2}$$

Couplings to the SM Higgs boson

The SM Higgs boson mechanism is **testable** at the LHC since given the Higgs mass, all couplings to the Higgs are known

$$g_{ffH} \propto m_f/v \quad (\text{fermions})$$

$$g_{VVH} \propto M_V^2/v \quad (\text{gauge bosons})$$

Therefore the Higgs properties (production modes, decay modes and branching ratios, and lifetime) are fully determined by its mass

Extended Higgs models have a more complicated structure

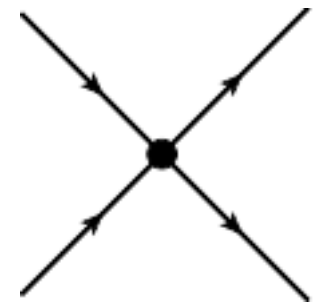
Unitarity violation of Fermi model

Consider muon decay in the effective Fermi four-fermion interaction model

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} [\bar{\nu}_\mu \gamma^\lambda (1 - \gamma_5) \mu] [\bar{e} \gamma^\lambda (1 - \gamma_5) \nu_e]$$

with $G_F \approx 1.17 \text{ GeV}^{-2}$ (Fermi coupling)

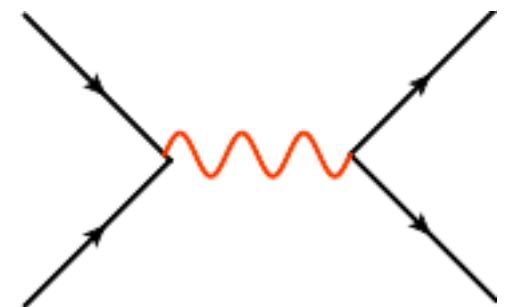
$$\mathcal{M}[\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e] \sim \frac{G_F}{2\sqrt{2}\pi} s$$



Cross section for $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ at high energies **violates unitarity!**

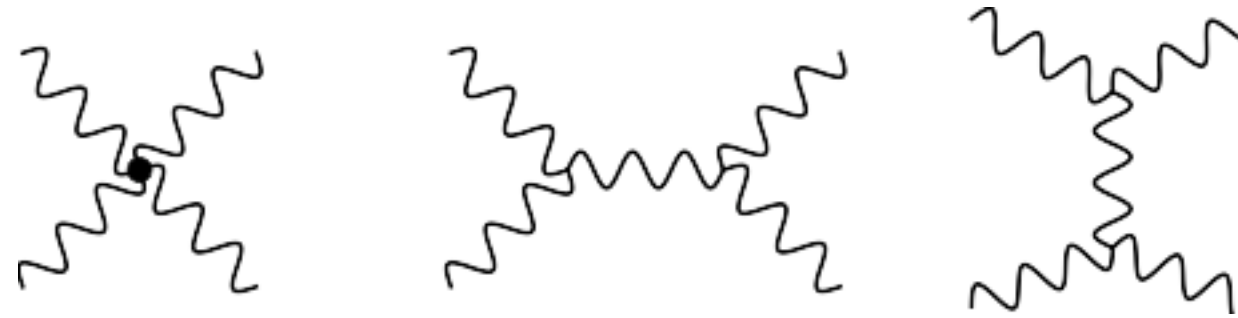
Solution: interaction mediated by heavy vector boson

$$\mathcal{M}[\mu^- \rightarrow e^- \nu_\mu \bar{\nu}_e] \sim \frac{G_F}{2\sqrt{2}\pi} \frac{M_W^2 s}{M_W^2 - s}$$



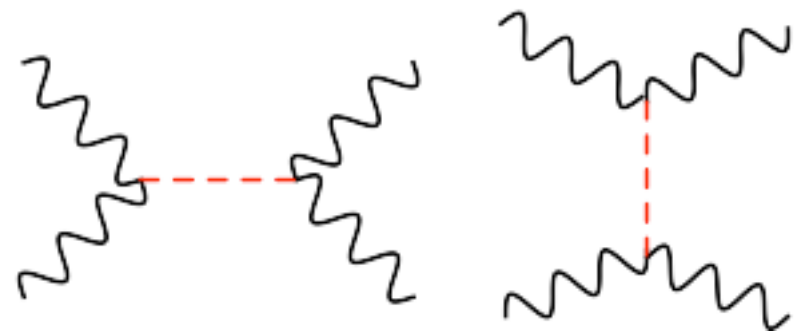
Higgs boson in WW scattering

Similarly, consider WW scattering in the SM **without a Higgs boson**



$$\mathcal{M}[W_L W_L \rightarrow W_L W_L] \propto s$$

With a Higgs boson

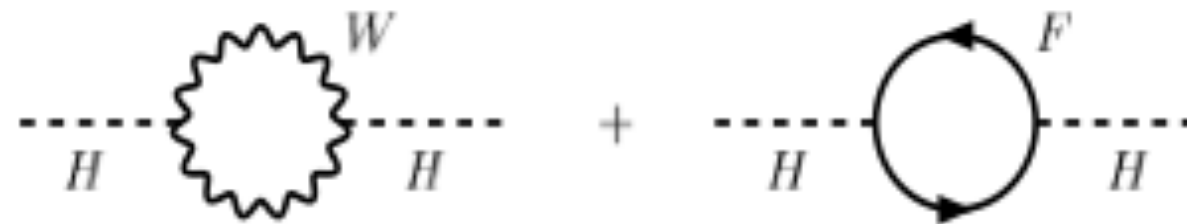


$$\mathcal{M}[W_L W_L \rightarrow W_L W_L] \rightarrow \frac{G_F M_H^2}{4\sqrt{2}\pi}$$

Crucial properties $g_{WWH} \propto M_W$ and $M_H \lesssim 1\text{TeV}$

Hierarchy problem: why is $M_H \ll M_{\text{Planck}}$

Quantum corrections to the Higgs mass have quadratic UV divergences




$$\delta M_H^2 \sim \frac{\alpha}{\pi} (\Lambda^2 + m_F^2)$$

The cutoff Λ represents the scale up to which the SM is valid. We need $\Lambda \sim 1 \text{ TeV}$ to avoid unnaturally large corrections.

Most popular BSM models with a solution to the hierarchy problem:

- Supersymmetry, Extra dimensions, Dynamical symmetry breaking ...

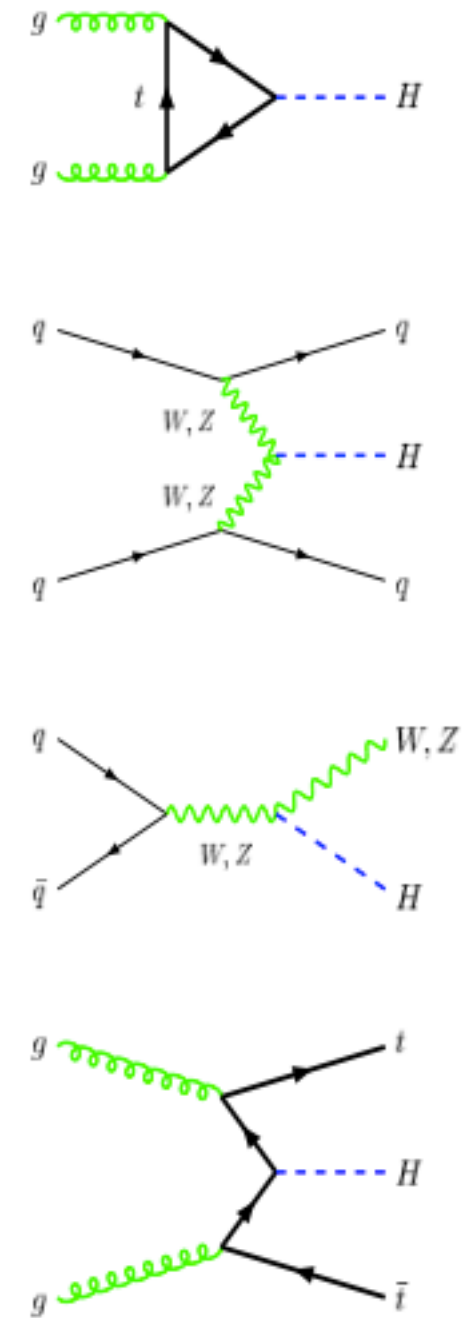
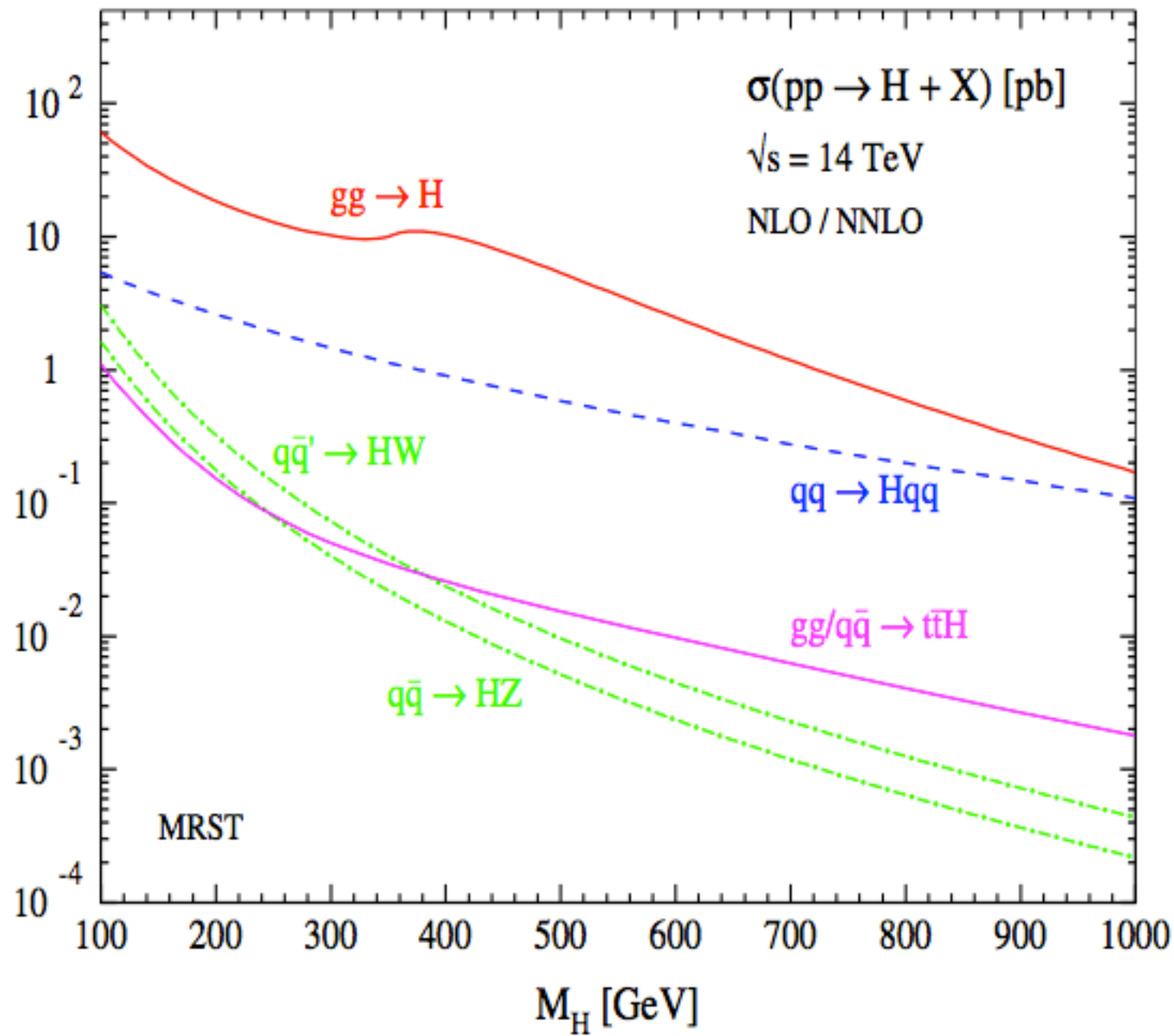
e.g. in Supersymmetry



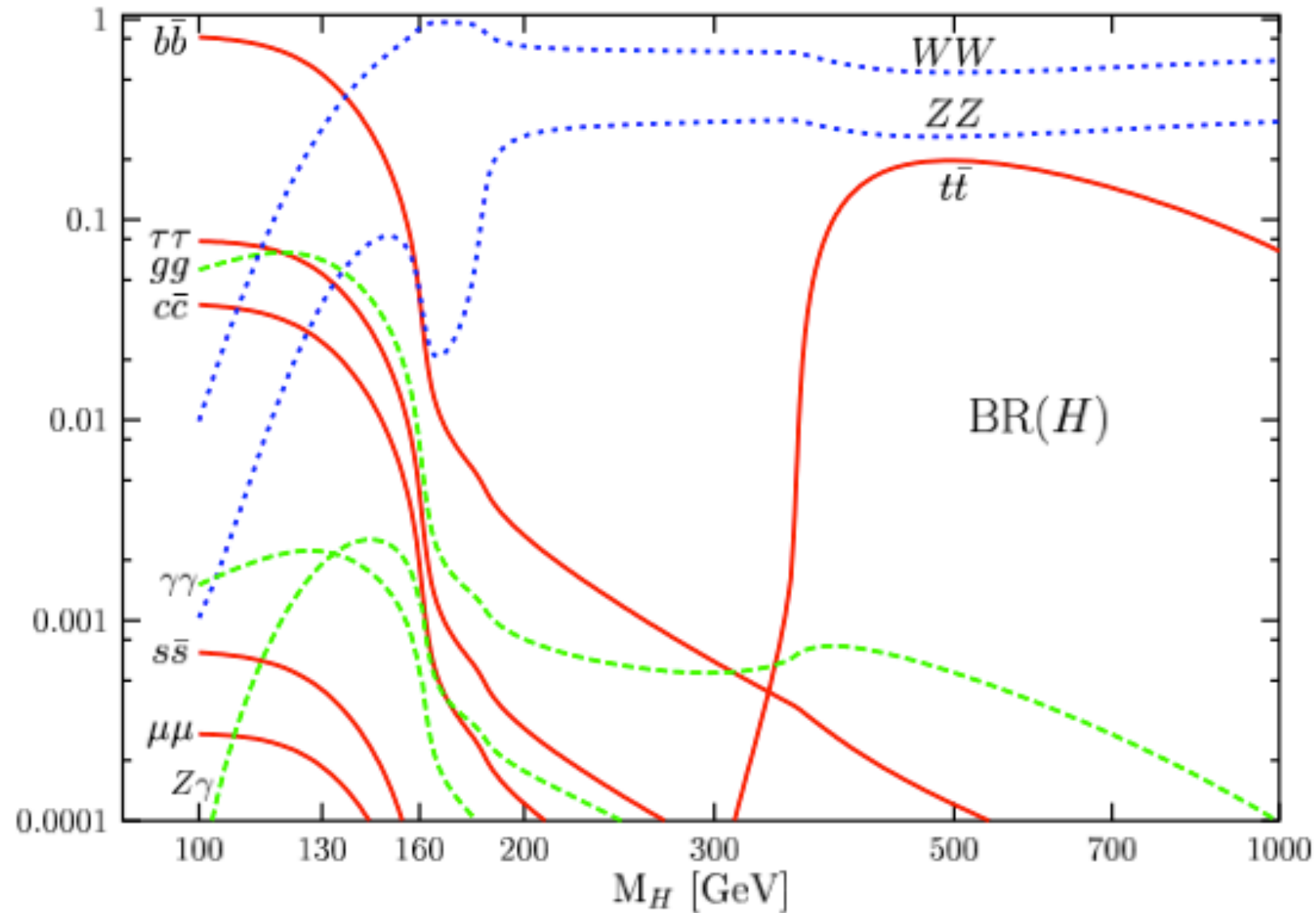
$$\delta M_H^2 \sim \frac{\alpha}{\pi} (-\Lambda^2 + \tilde{m}_F^2)$$

no fine tuning if $\tilde{m} \lesssim O(1) \text{ TeV}$

SM Higgs production at the LHC

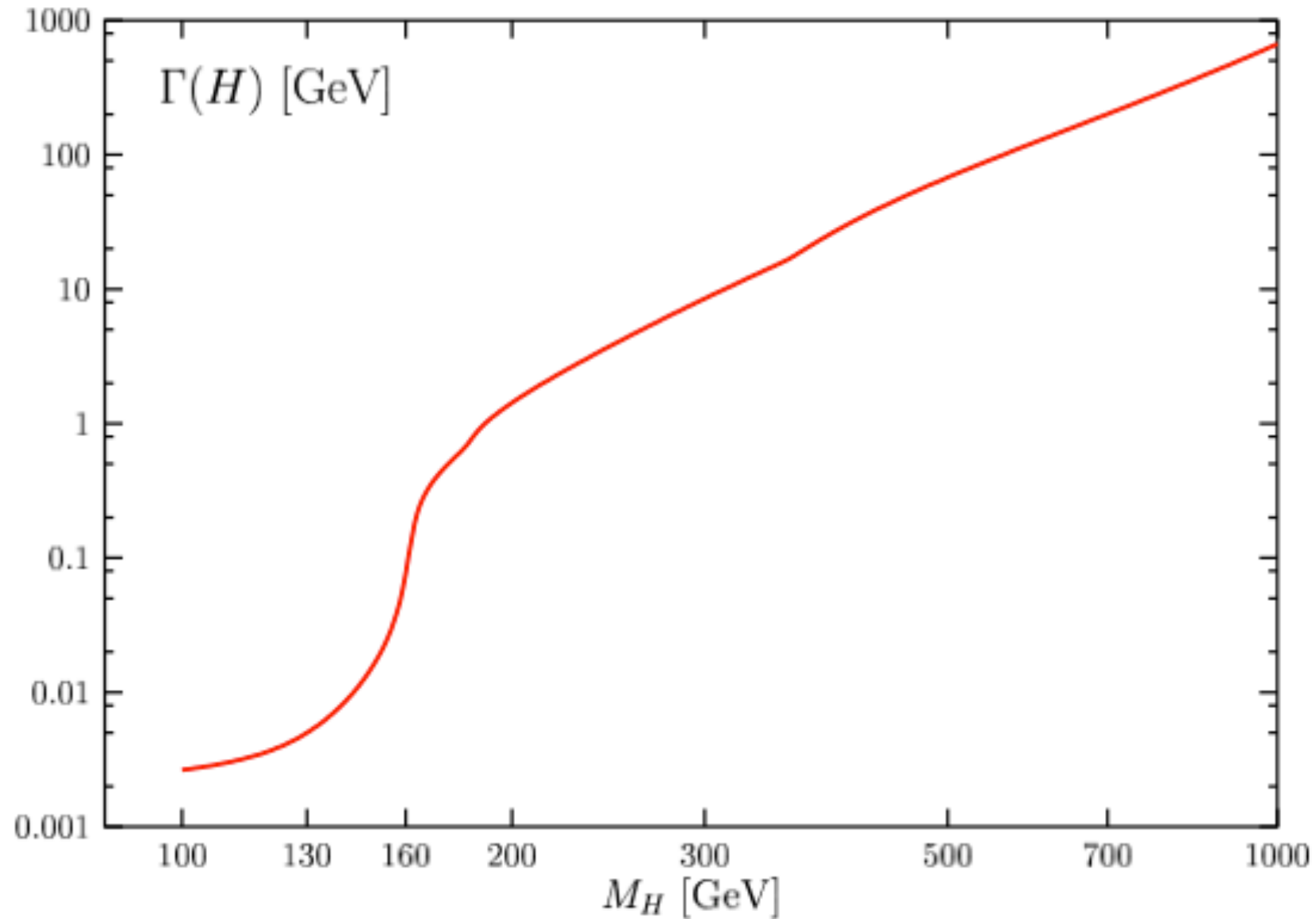


SM Higgs decay modes and branching ratios



Dominant decay into - WW/ZZ for $M_H > 130$ GeV
- bb for $M_H < 130$ GeV (but difficult background, while $\gamma\gamma$ is very small but much cleaner)

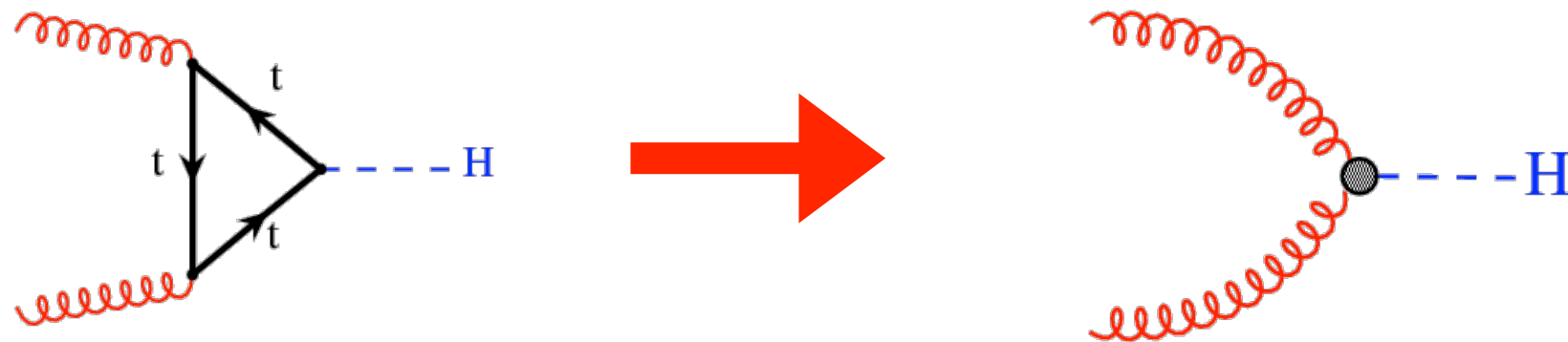
SM Higgs total width



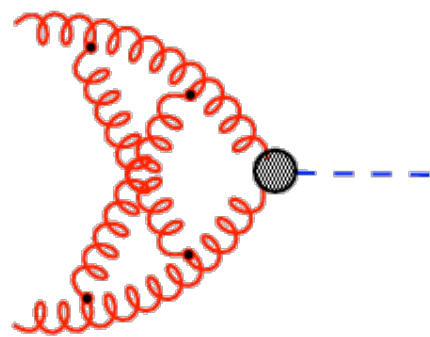
Heavy Higgs ($M_H > 500$ GeV) has a width comparable to its mass. Unclear how to represent a Higgs propagator. Unclear also how legitimate it is to think of the Higgs as particle

Inclusive NNLO Higgs ggf production

Inclusive Higgs production via gluon-gluon fusion in the large m_t -limit:



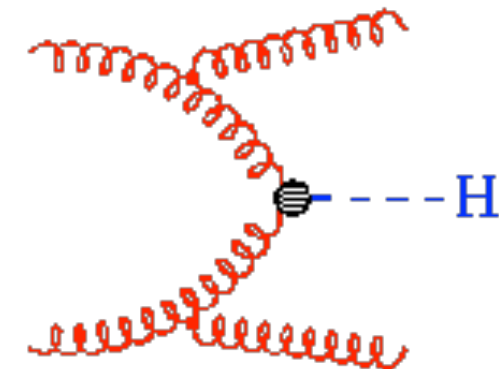
NNLO corrections known since few years now:



virtual-virtual

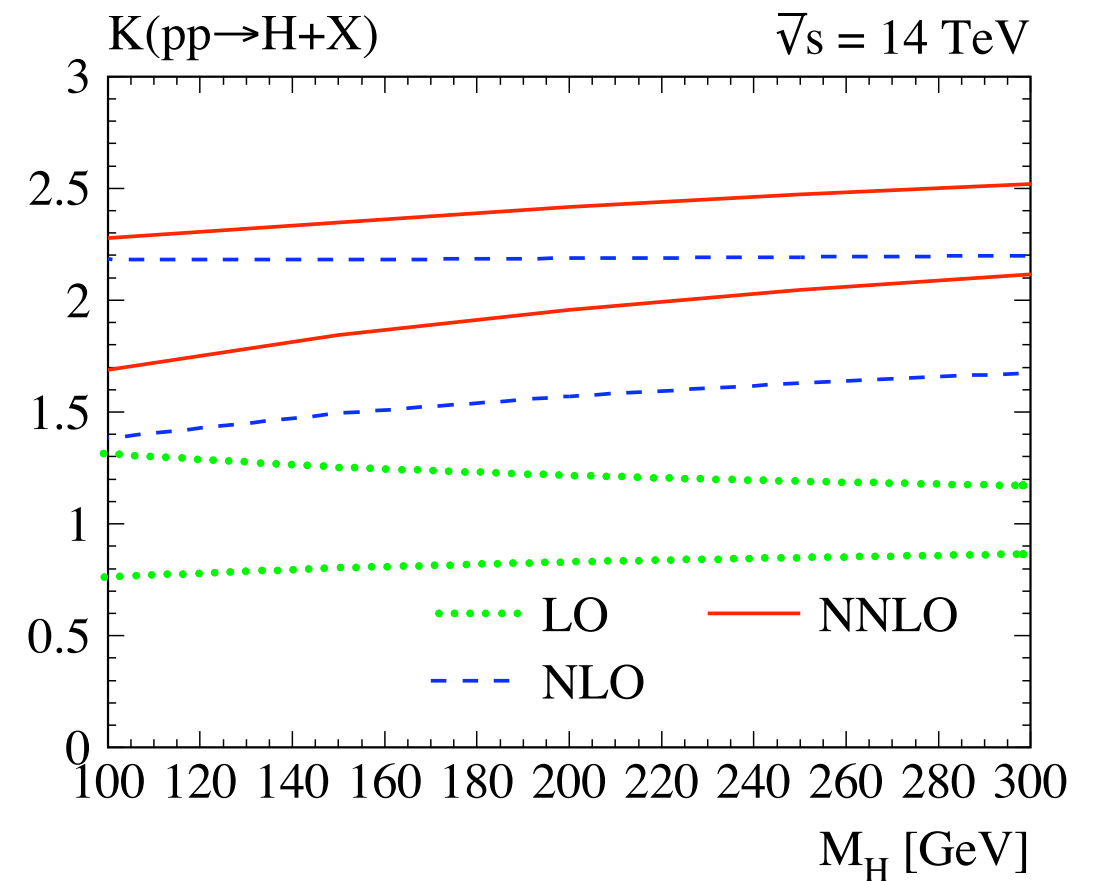
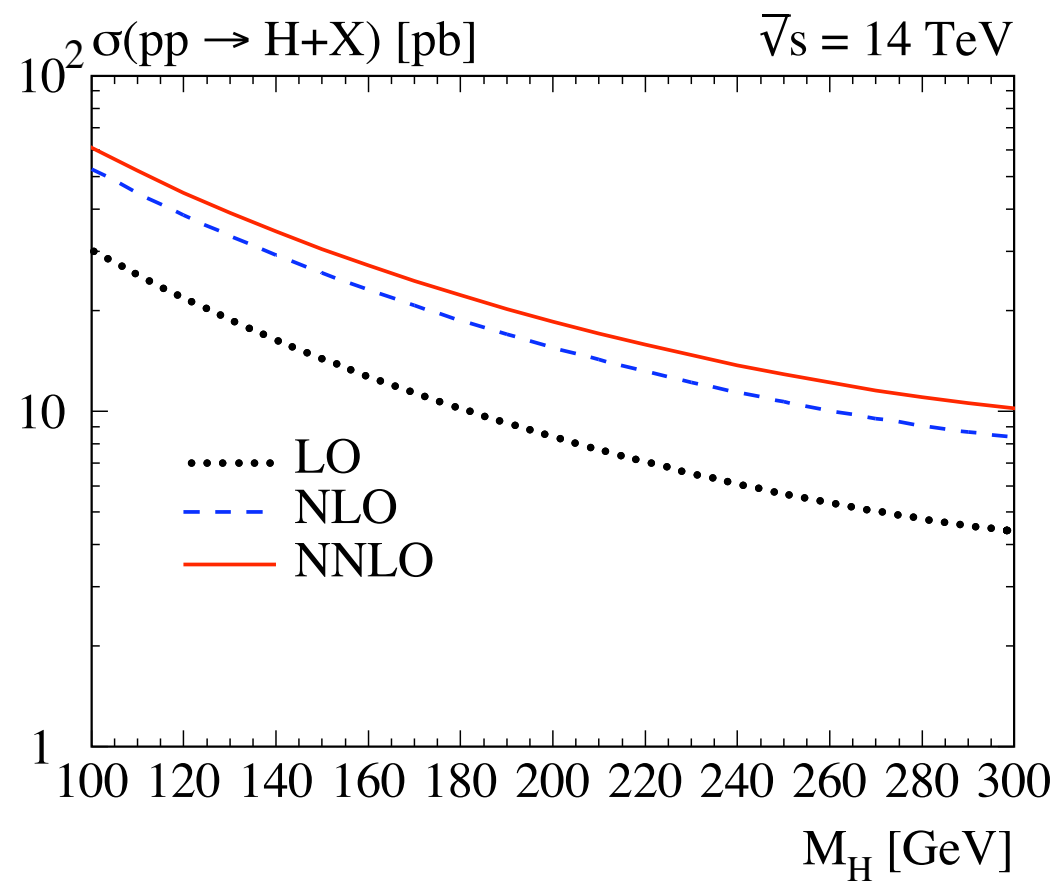


real-virtual



real-real

Inclusive NNLO Higgs ggf production



Kilgore, Harlander '02
Anastasiou, Melnikov '02

Further improvement on $gg \rightarrow H$

The urge to understand EW symmetry breaking led to most advanced theoretical predictions, for instance, we know the main $gg \rightarrow H$ production mechanism in the SM including

- NLO with exact top and bottom loop Djouadi, Graudenz, Spira, Zerwas '93,'95
- NNLO in large m_t limit Ravindran, Smith, van Neerven '03; Kilgore and Harlander '02
Anastasiou, Melnikov '02
- electroweak corrections Actis, Passarino, Sturm, Uccirati '08
- mixed QCD - EW corrections Anastasiou, Boughezal, Petriello '09
- resummation and/or N³LO soft Catani, De Florian, Grazzini, Nason '03; Moch and Vogt '05;
Laenen, Magnea '06; Ahrens, Becher, Neubert, Yang '08
- fully exclusive decays to $\gamma\gamma, WW \rightarrow l^+l^- \nu\nu$ and $ZZ \rightarrow 4l$ Catani and Grazzini '08
Anastasiou, Melnikov Petriello '05; Anastasiou, Dissertori, Stoeckli '07
- also exclusive NNLO $VH(\rightarrow bb)$ Ferrera, Grazzini, Tramontano '11

Further improvement on $gg \rightarrow H$

*So, how well do we know this process?
What is the theory error on it ?*

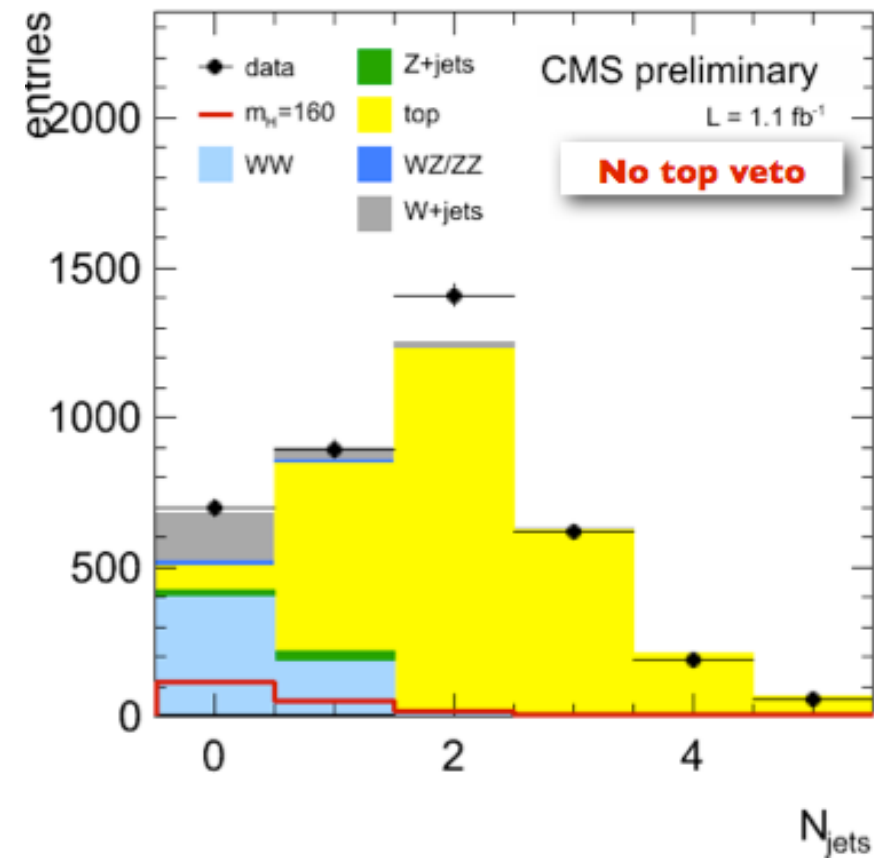
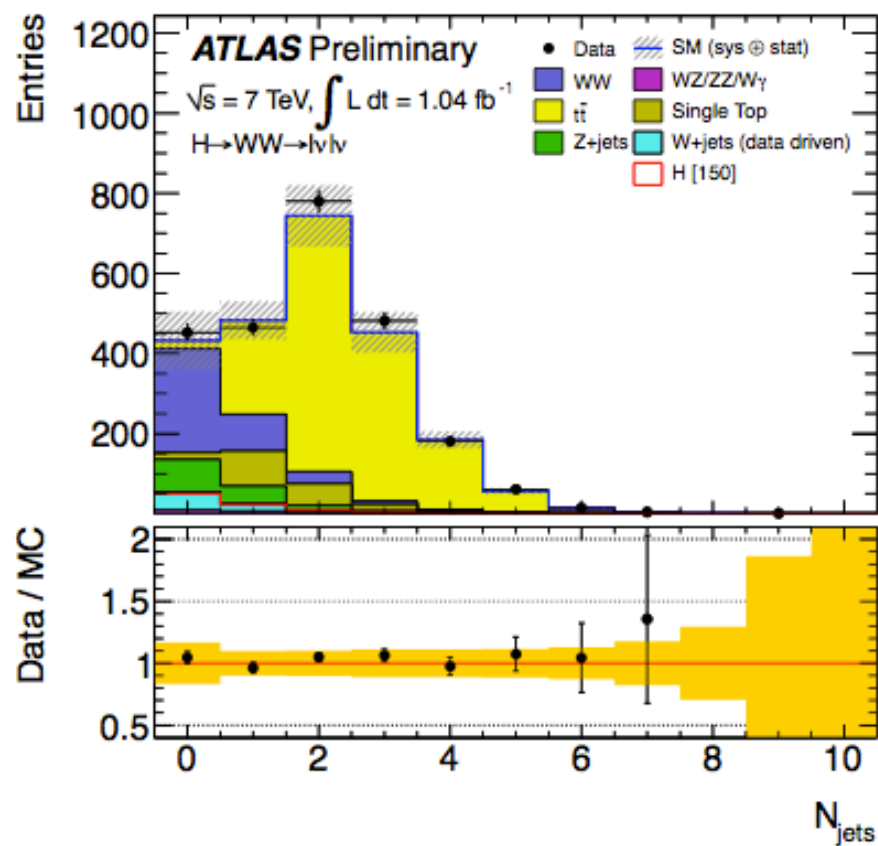
You'll find quoted errors ranging from 10% to 40%

Assigning a theoretical error very important to claim exclusion/excess, and for measurements of couplings. Yet, even for the main Higgs production channel there are still controversies. **I will illustrate here one of them.**

Many issues, discussions, recommendations can be found in the **Handbook of LHC cross-sections (Vol I and II)** [1101.0593](#) and [1201.3084](#)

Jet veto

Need jet veto to kill large top background, ideally $p_T^{\text{veto}} \approx 25$ GeV



Higgs production studied in 0-, 1-, 2-jet bin separately to maximize sensitivity

Tevatron $\rightarrow \frac{\Delta\sigma_{\text{tot}}}{\sigma_{\text{tot}}} = 66.5\%_{-9\%}^{+5\%} + 28.6\%_{-22\%}^{+24\%} + 4.9\%_{-41\%}^{+78\%} = [-14.3\%; +14.0\%]$

NB: $\mu_R = \mu_F$

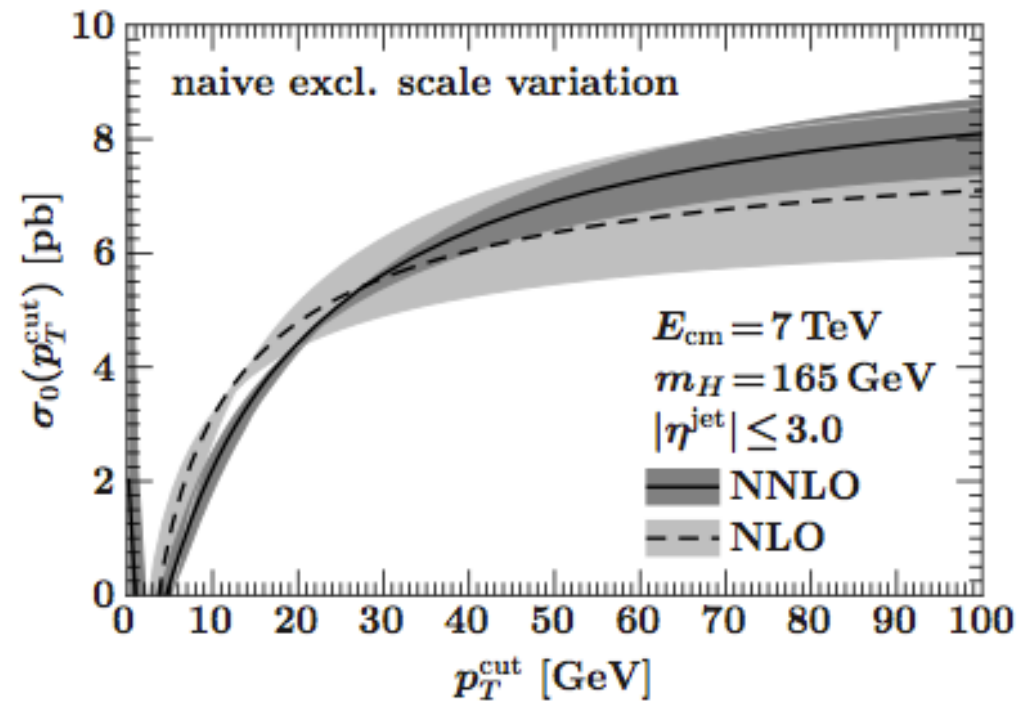
0-jet

1-jet

≥ 2 -jets

Anastasiou et al. 0905.3529

Jet veto uncertainties



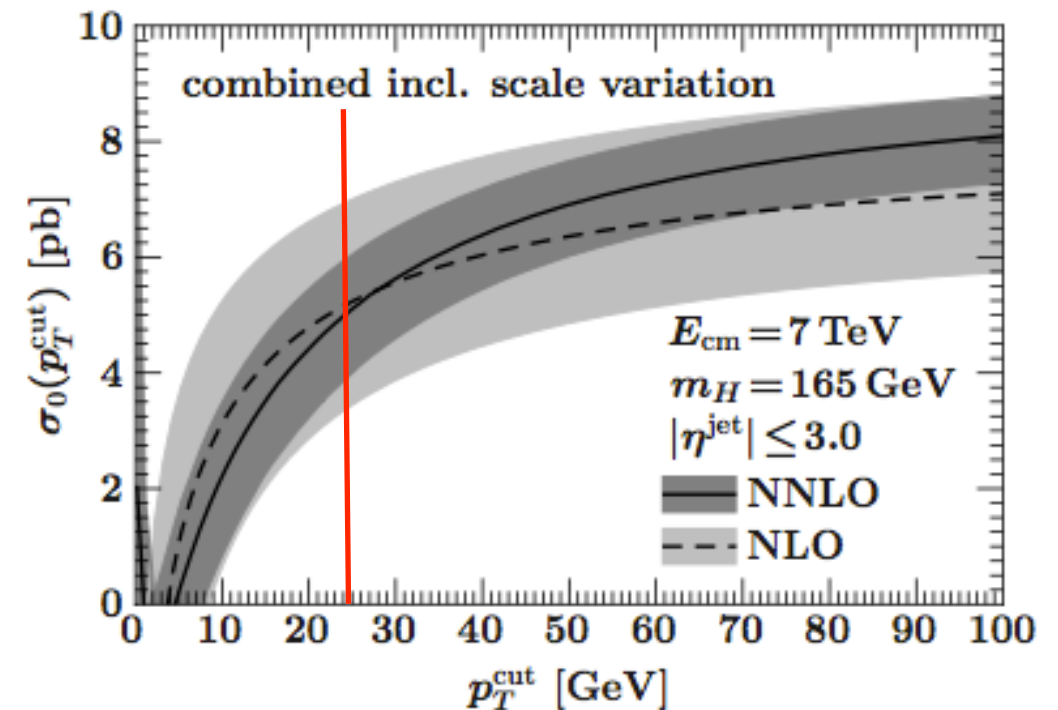
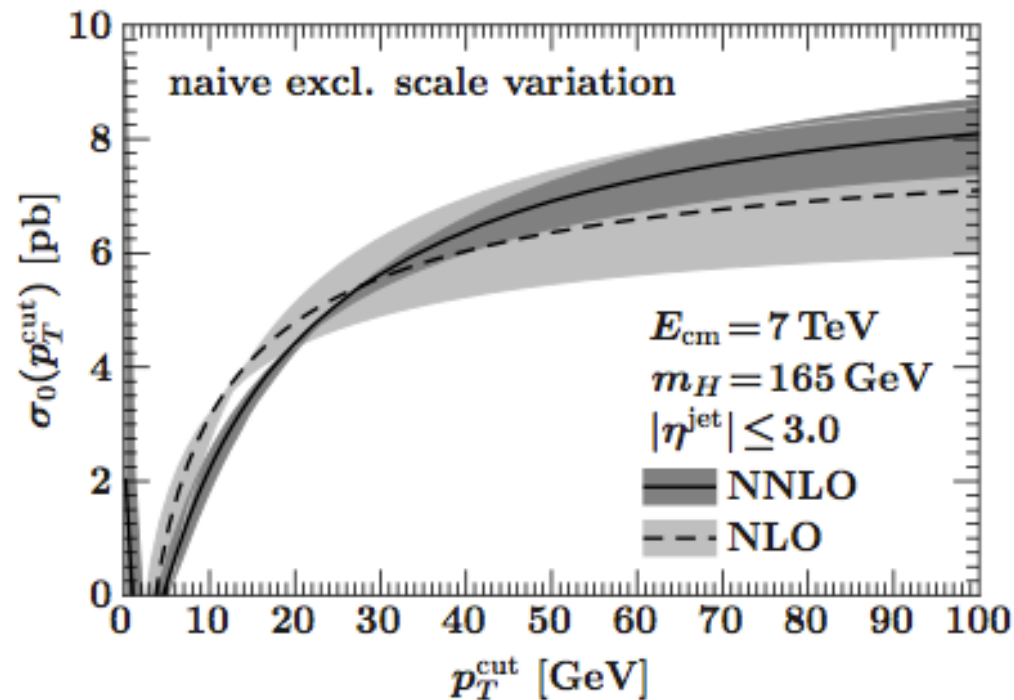
- with p_T^{veto} much smaller error
- large positive correction (K-factor) and large negative logarithms

$$-\frac{2C_A\alpha_s}{\pi} \ln^2 \frac{M_H}{p_T^{\text{veto}}}$$

Scale variation alone underestimates uncertainties?

Jet veto uncertainties

Stewart and Tackman '11



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Scale variation alone underestimates uncertainties?

- full correlations between jet bins

$$\sigma_{0 \text{ jets}} = \sigma_{\text{tot}} - \sigma_{\geq 1 \text{ jet}}$$

$$\Delta^2 \sigma_{0 \text{ jets}} = \Delta^2 \sigma_{\text{tot}} + \Delta^2 \sigma_{\geq 1 \text{ jet}}$$

large K (pointing to σ_{tot}) and *large logarithms* (pointing to $\sigma_{\geq 1 \text{ jet}}$)

Uncertainties overestimated?

Higgs searches: current status

After 5fb^{-1} of data in 2011

CMS excludes (95CL) the region $127\text{ GeV} < M_H < 600\text{ GeV}$
while the expected exclusion is $117\text{ GeV} < M_H < 543\text{ GeV}$
small window left for a light H $114.4\text{ GeV} < M_H < 127\text{ GeV}$

ATLAS has restricted the allowed ranges at 95CL to
 $115.5 < M_H < 131\text{ GeV}$ or $127 < M_H < 251\text{ GeV}$ or $M_H > 468\text{ GeV}$

More data and a combination of the results is needed to come to a conclusion

2012 is the decisive year

Other NNLO on the horizon



Single-jet production

- constrain gluon PDF
- matrix elements known for some time
- subtraction in progress



Top pair production

- needed for more precise m_t determination
- possibly for further constraining PDFs
- top asymmetry



Vector boson pair production

- NLO corrections are large
- study gauge structure of SM (triple gauge couplings)
- most important and irreducible background for Higgs production in intermediate mass region

Recap of higher orders

Leading order

- everything can be computed in principle today (practical edge: 8 particles in the final state), many public codes
- techniques: standard Feynman diagrams or recursive methods (Berends-Giele, BCF, CSW, ...)

Next-to-leading order




- current frontier $2 \rightarrow 5$ in the final state
- many new, promising techniques

Next-to-next-to-leading order

- very few $2 \rightarrow 1$ processes available (Higgs, Drell-Yan)
- expect $2 \rightarrow 2$ calculations soon

Next

Next will focus on

-  parton showers and Monte Carlo methods
-  matching of parton showers and fixed order calculations
-  jets

Parton shower & Monte Carlo methods

- 📌 today at the frontier of NLO calculations are **processes with 4 or 5 particles in the final state**. Difficult to expect much more in the coming years. However, typical LHC processes have much larger multiplicity

Parton shower & Monte Carlo methods

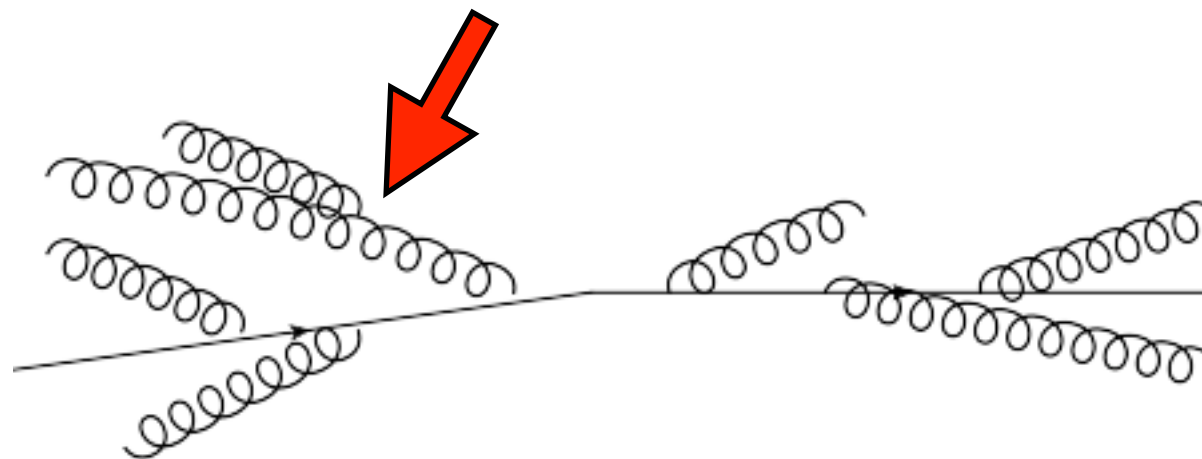
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Parton shower & Monte Carlo methods

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- we have also seen that **large logarithms can spoil the convergence of PT, NLO results become unreliable**
- instead, one can **seek for an approximate result such that soft and collinear enhanced terms are taken into account to all orders**
- this leads to a **'parton shower' picture**, which is implemented in computer simulations, usually called **Monte Carlo programs or event generators**



Angular ordering

When a soft gluon is radiated from a $(p_i p_j)$ dipole one gets a universal eikonal factor

$$\omega_{ij} = \frac{p_i p_j}{p_{ik} p_{jk}} = \frac{1 - v_i v_j \cos \theta_{ij}}{\omega_k^2 (1 - v_i \cos \theta_{ik}) (1 - v_j \cos \theta_{jk})}$$

Massless emitting lines $v_i = v_j = 1$, then

$$\omega_{ij} = \omega_{ij}^{[i]} + \omega_{ij}^{[j]} \qquad \omega_{ij}^{[i]} = \frac{1}{2} \left(\omega_{ij} + \frac{1}{1 - \cos \theta_{ik}} - \frac{1}{1 - \cos \theta_{jk}} \right)$$

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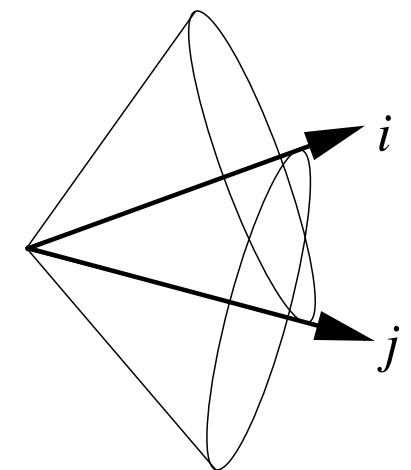
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Angular ordering

$$\int_0^{2\pi} \frac{d\phi}{2\pi} \omega_{ij}^{[i]} = \begin{cases} \frac{1}{\omega_k^2 (1 - \cos \theta_{ik})} & \theta_{ik} < \theta_{ij} \\ 0 & \theta_{ik} > \theta_{ij} \end{cases}$$



Proof: see e.g. QCD and collider physics, Ellis, Stirling, Webber

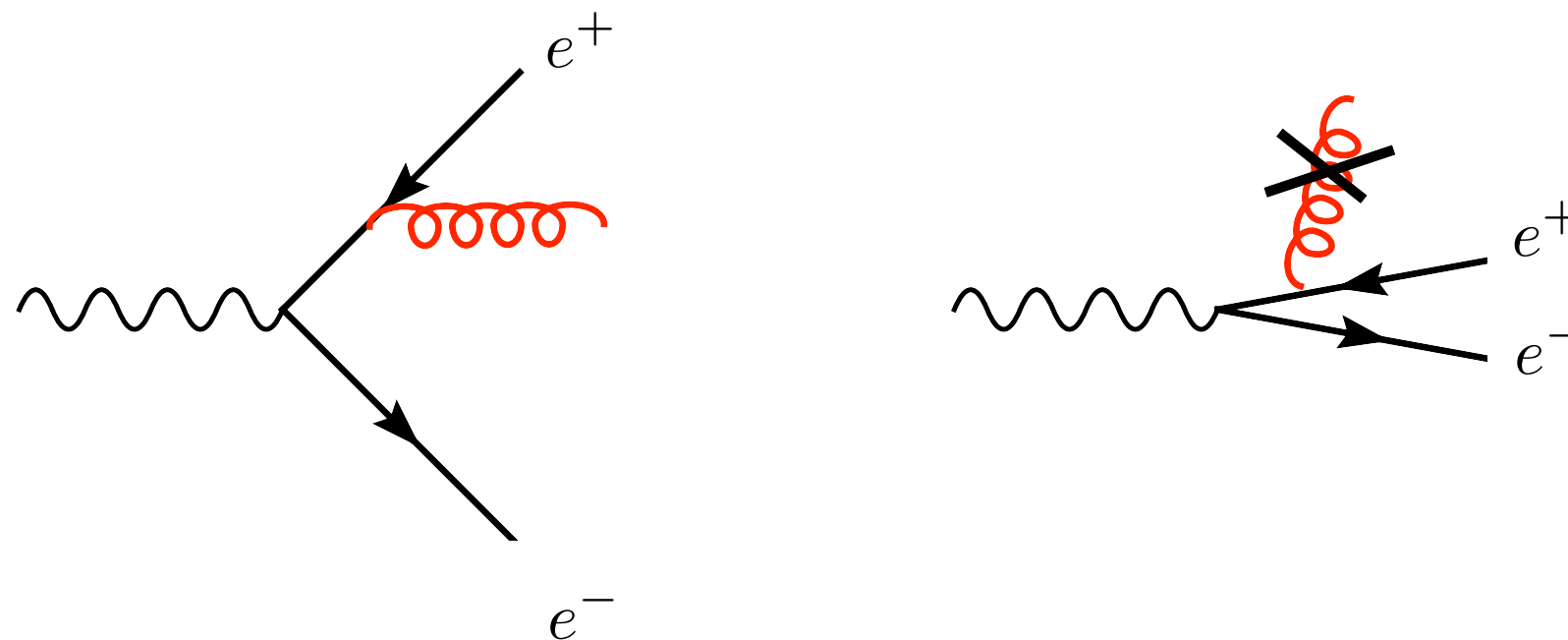
Angular ordering & coherence

A. O. is a manifestation of coherence of radiation in gauge theories

In QED

suppression of soft bremsstrahlung from an e^+e^- pair (Chudakov effect)

At large angles the e^+e^- pair is seen coherently as a system without total charge \Rightarrow radiation is suppressed



Herwig uses the angle as an evolution variable, therefore has coherence built in. Other parton showers force angular ordering in the evolution

Parton showers (PS) at the LHC

[Ariadne, Pythia, Herwig, Isajet, ...]

Standard parton shower programs

- hard ($2 \rightarrow 2$) scattering
- parton shower (in the soft-collinear approximation)
- hadronization model + underlying event model (UE)

PS differ in the ordering variable of the shower, e.g. angle Herwig, transverse momentum Ariadne and Pythia (new), virtuality Pythia (old), in UE model, in the hadronization model

Every LHC analysis will make use of one or more PS simulation for

- the signal and/or the background
- underlying event / non-perturbative corrections
- pile-up
- efficiency studies / detector response

An example with Herwig

Select the initial state, e.g. pp collision at 14 TeV

```
---INITIAL STATE---
```

IHEP	ID	IDPDG	IST	MO1	MO2	DA1	DA2	P-X	P-Y	P-Z	ENERGY	MASS
1	P	2212	101	0	0	0	0	0.00	0.00	7000.0	7000.0	0.94
2	P	2212	102	0	0	0	0	0.00	0.00	-7000.0	7000.0	0.94
3	CHF	0	103	1	2	0	0	0.00	0.00	0.0	14000.0	14000.0

An example with Herwig

Select the hard process of interest, e.g. Z^+ jet production

---HARD SUBPROCESS---

IHEP	ID	IDPDG	IST	MO1	MO2	DA1	DA2	P-X	P-Y	P-Z	ENERGY	MASS
4	UQK	2	121	6	8	9	5	0.00	0.00	590.8	590.8	0.32
5	GLUON	21	122	6	4	17	8	0.00	0.00	-232.1	232.1	0.75
6	HARD	0	120	4	5	7	8	0.40	-9.40	358.7	823.0	740.63
7	Z0/GAMA*	23	123	6	7	22	7	-261.59	-217.31	329.3	481.6	88.56
8	UQK	2	124	6	5	23	4	261.59	217.31	29.4	341.3	0.32

An example with Herwig

Then Herwig dresses the process for you, both with initial state and final state shower

---PARTON SHOWERS---

IHEP	ID	IDPDG	IST	M01	M02	DA1	DA2	P-X	P-Y	P-Z	ENERGY	MASS
9	UQPK	94	141	4	6	11	16	2.64	-9.83	592.2	590.2	-49.07
10	CONE	0	100	4	5	0	0	-0.27	0.96	0.1	1.0	0.00
11	GLUON	21	2	9	12	32	33	-1.02	3.59	5.6	6.7	0.75-
12	GLUON	21	2	9	13	34	35	0.25	1.46	3.6	4.0	0.75-
13	GLUON	21	2	9	14	36	37	-0.87	1.62	4.7	5.1	0.75-
14	GLUON	21	2	9	15	38	39	-0.81	4.17	3611.7	3611.7	0.75-
15	GLUON	21	2	9	16	40	41	-0.19	-1.01	1727.7	1727.7	0.75-
16	UD	2101	2	9	25	42	41	0.00	0.00	1054.6	1054.6	0.32-
17	GLUON	94	142	5	6	19	21	-2.23	0.44	-233.5	232.8	-18.36
18	CONE	0	100	5	8	0	0	0.77	0.64	0.2	1.0	0.00
19	GLUON	21	2	17	20	43	44	1.60	0.58	-2.1	2.8	0.75
20	UD	2101	2	17	21	45	44	0.00	0.00	-2687.6	2687.6	0.32
21	UQPK	2	2	17	32	46	45	0.63	-1.02	-4076.9	4076.9	0.32
22	Z0/GAMA*	23	195	7	22	251	252	-257.66	-219.68	324.8	477.5	88.56
23	UQPK	94	144	8	6	25	31	258.06	210.29	33.9	345.5	86.10
24	CONE	0	100	8	5	0	0	0.21	0.17	-1.0	1.0	0.00
25	UQPK	2	2	23	26	47	42	26.82	24.33	23.7	43.3	0.32
26	GLUON	21	2	23	27	48	49	8.50	8.18	6.0	13.3	0.75
27	GLUON	21	2	23	28	50	51	73.27	61.24	12.0	96.2	0.75
28	GLUON	21	2	23	29	52	53	73.66	58.54	-6.3	94.3	0.75
29	GLUON	21	2	23	30	54	55	67.58	52.13	-7.3	85.7	0.75
30	GLUON	21	2	23	31	56	57	6.98	4.60	2.3	8.7	0.75
31	GLUON	21	2	23	43	58	59	1.24	1.26	3.6	4.1	0.75

Add hadronization + UE then perform your desired physics study

Accuracy of Monte Carlos

Formally, Monte Carlos are Leading Logarithmic (LL) showers

- because they don't include any higher order corrections to the $1 \rightarrow 2$ splitting
- because they don't have any $1 \rightarrow 3$ splittings
-

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- they have energy conservation (NLO effect) implemented
- they have coherence
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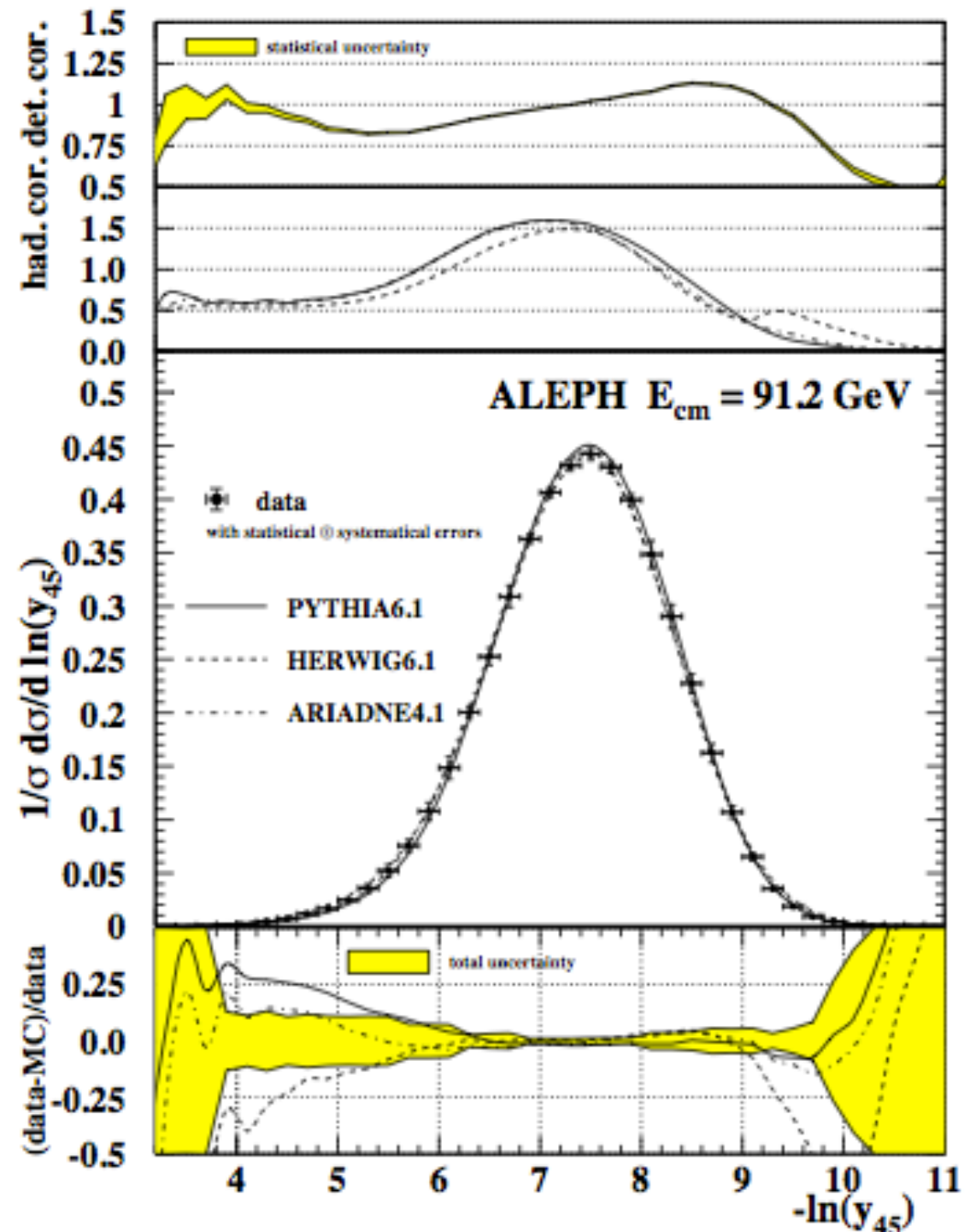
So, despite not guaranteeing any formal accuracy, they fare better than LL calculations. *The problem is that we don't know the uncertainty. Often comparison between different PS is the only way to estimate the uncertainty*

Parton shower vs data

Example:

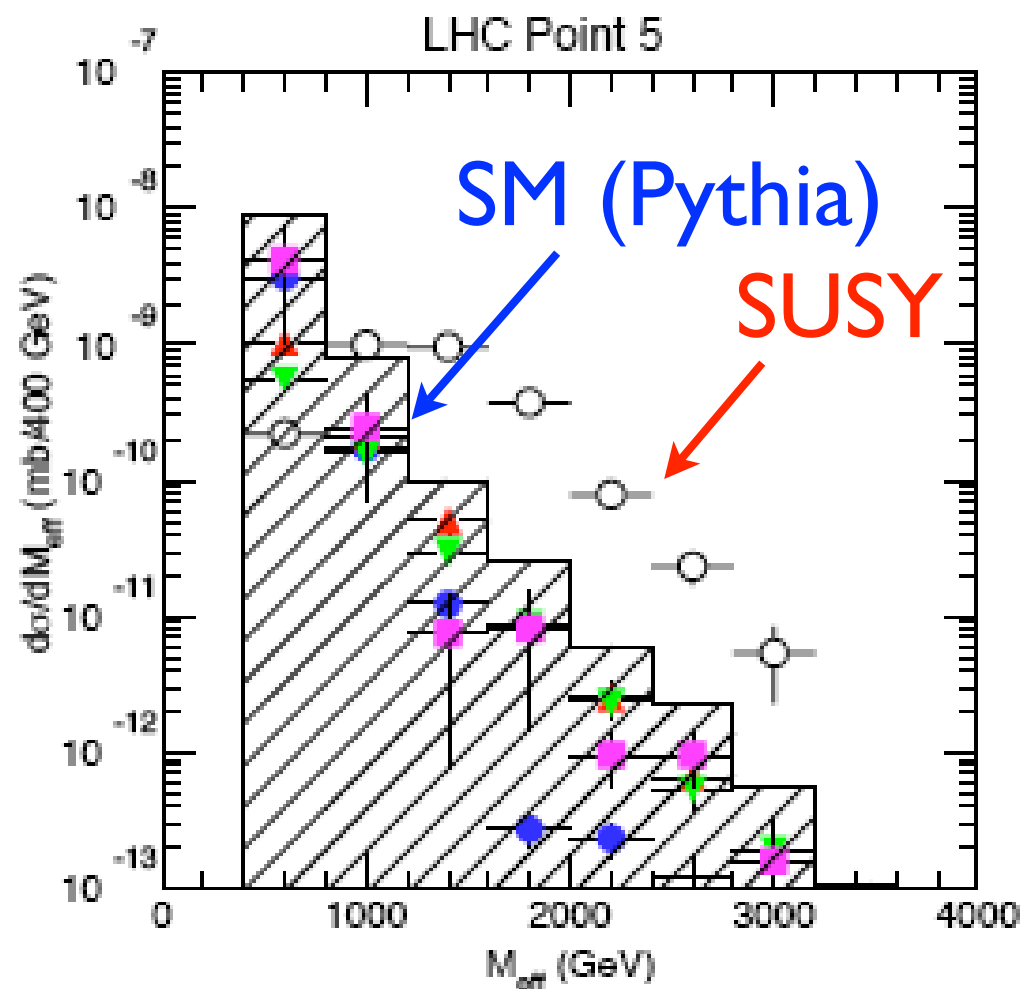
five-jet resolution parameter y_{45}

- Agreement over 3 orders of magnitudes for a variable that describes a multi-jet final state
- Surprising since MCs rely on the soft-collinear approximation + a model for hadronization
- Note however that MCs have been tuned to LEP data



Accuracy of parton showers

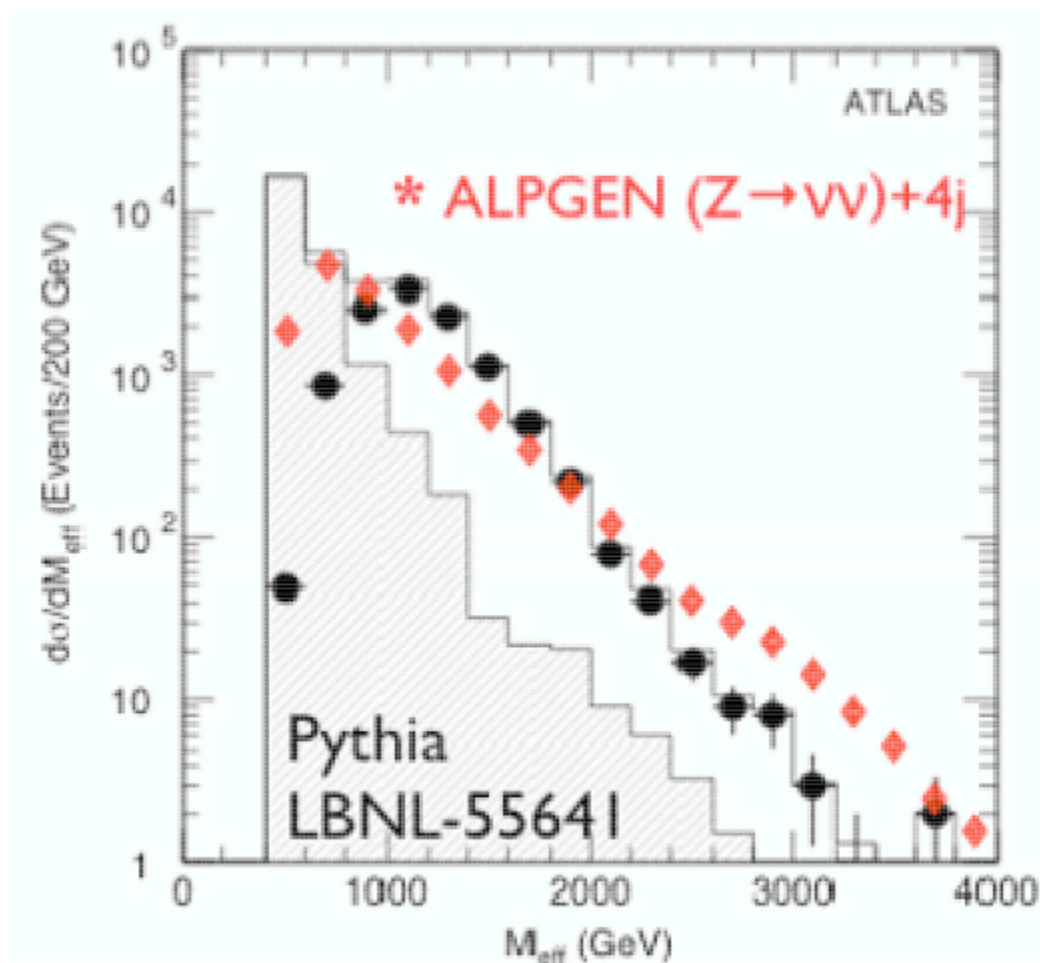
M_{eff} = total transverse energy in the event



- SUSY: position of the peak determined by the mass spectrum
- Pure PS predict steeply falling SM background
- With matrix element calculation: SM and SUSY comparable size and shape
- In this example: SUSY search much more difficult than originally thought

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Lesson to take away

- PS fail to describe hard radiation and it is difficult to understand the uncertainty of their predictions
- techniques and public code (AlpGen, Sherpa, Madgraph, ...) exist to match matrix element calculations with Monte Carlos

NLO + parton shower

Even better than LO matrix element + shower is NLO + shower.

This combines the best features: correct rates (NLO) and hadron-level description of events (PS)

Difficult because need to avoid double counting

Two working examples:

▶ MC@NLO

Frixione&Webber '02 and later refs.

▶ POWHEG (POWHEG-BOX)

Nason '04 and later refs.

Processes implemented:

- W/Z boson production
- WW, WZ, ZZ production
- inclusive Higgs production
- heavy quark production
- V + 1 jet
- single-top
- dijets
- Wbb
- W^+W^+ + dijets ...
- ...

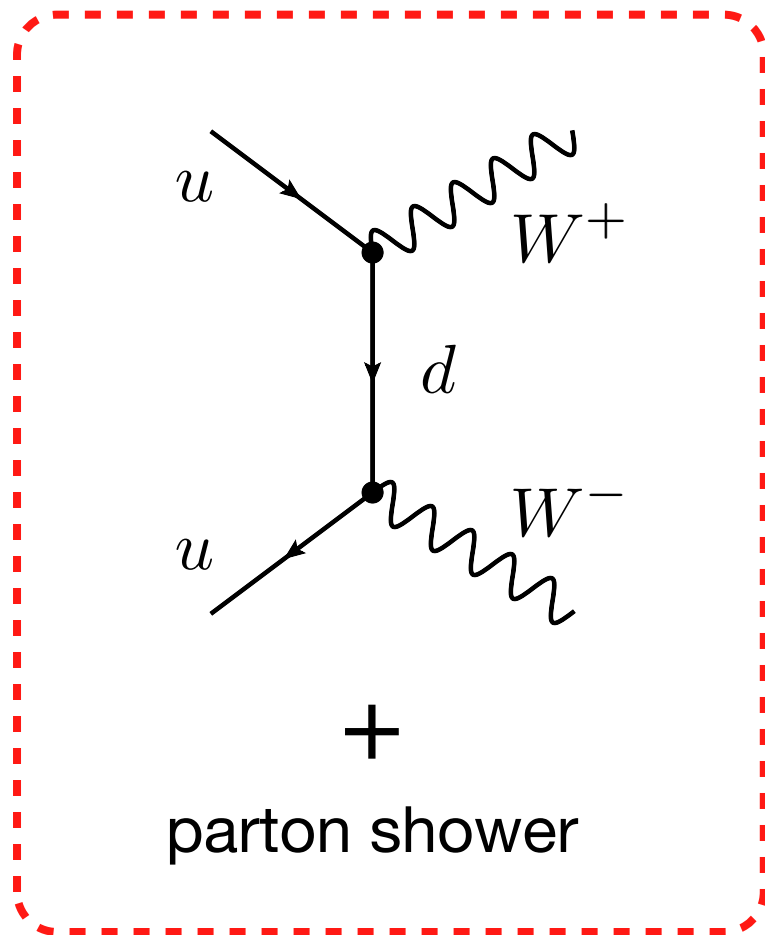
MC@NLO

IPROC	IV	IL ₁	IL ₂	Spin	Process
-1350-IL				✓	$H_1 H_2 \rightarrow (Z/\gamma^* \rightarrow) l_{\text{IL}} \bar{l}_{\text{IL}} + X$
-1360-IL				✓	$H_1 H_2 \rightarrow (Z \rightarrow) l_{\text{IL}} \bar{l}_{\text{IL}} + X$
-1370-IL				✓	$H_1 H_2 \rightarrow (\gamma^* \rightarrow) l_{\text{IL}} \bar{l}_{\text{IL}} + X$
-1460-IL				✓	$H_1 H_2 \rightarrow (W^+ \rightarrow) l_{\text{IL}}^+ \nu_{\text{IL}} + X$
-1470-IL				✓	$H_1 H_2 \rightarrow (W^- \rightarrow) l_{\text{IL}}^- \bar{\nu}_{\text{IL}} + X$
-1396				×	$H_1 H_2 \rightarrow \gamma^* (\rightarrow \sum_i f_i \bar{f}_i) + X$
-1397				×	$H_1 H_2 \rightarrow Z^0 + X$
-1497				×	$H_1 H_2 \rightarrow W^+ + X$
-1498				×	$H_1 H_2 \rightarrow W^- + X$
-1600-ID					$H_1 H_2 \rightarrow H^0 + X$
-1705					$H_1 H_2 \rightarrow b\bar{b} + X$
-1706		7	7	×	$H_1 H_2 \rightarrow t\bar{t} + X$
-2000-IC		7		×	$H_1 H_2 \rightarrow t/\bar{t} + X$
-2001-IC		7		×	$H_1 H_2 \rightarrow \bar{t} + X$
-2004-IC		7		×	$H_1 H_2 \rightarrow t + X$
-2030		7	7	×	$H_1 H_2 \rightarrow tW^-/\bar{t}W^+ + X$
-2031		7	7	×	$H_1 H_2 \rightarrow \bar{t}W^+ + X$
-2034		7	7	×	$H_1 H_2 \rightarrow tW^- + X$
-2600-ID	1	7		×	$H_1 H_2 \rightarrow H^0 W^+ + X$
-2600-ID	1	i		✓	$H_1 H_2 \rightarrow H^0 (W^+ \rightarrow) l_i^+ \nu_i + X$
-2600-ID	-1	7		×	$H_1 H_2 \rightarrow H^0 W^- + X$
-2600-ID	-1	i		✓	$H_1 H_2 \rightarrow H^0 (W^- \rightarrow) l_i^- \bar{\nu}_i + X$
-2700-ID	0	7		×	$H_1 H_2 \rightarrow H^0 Z + X$
-2700-ID	0	i		✓	$H_1 H_2 \rightarrow H^0 (Z \rightarrow) l_i \bar{l}_i + X$
-2850		7	7	×	$H_1 H_2 \rightarrow W^+ W^- + X$
-2860		7	7	×	$H_1 H_2 \rightarrow Z^0 Z^0 + X$
-2870		7	7	×	$H_1 H_2 \rightarrow W^+ Z^0 + X$
-2880		7	7	×	$H_1 H_2 \rightarrow W^- Z^0 + X$

- ▶ $H_{1,2}$ denote nucleon and antinucleon
- ▶ “Spin” indicates whether spin correlations in vector boson fusion or top decays are included (✓), neglected (×) or absent (void entry)
- ▶ The values of IV, IL, IL₁, and IL₂ control the identities of vector bosons and leptons

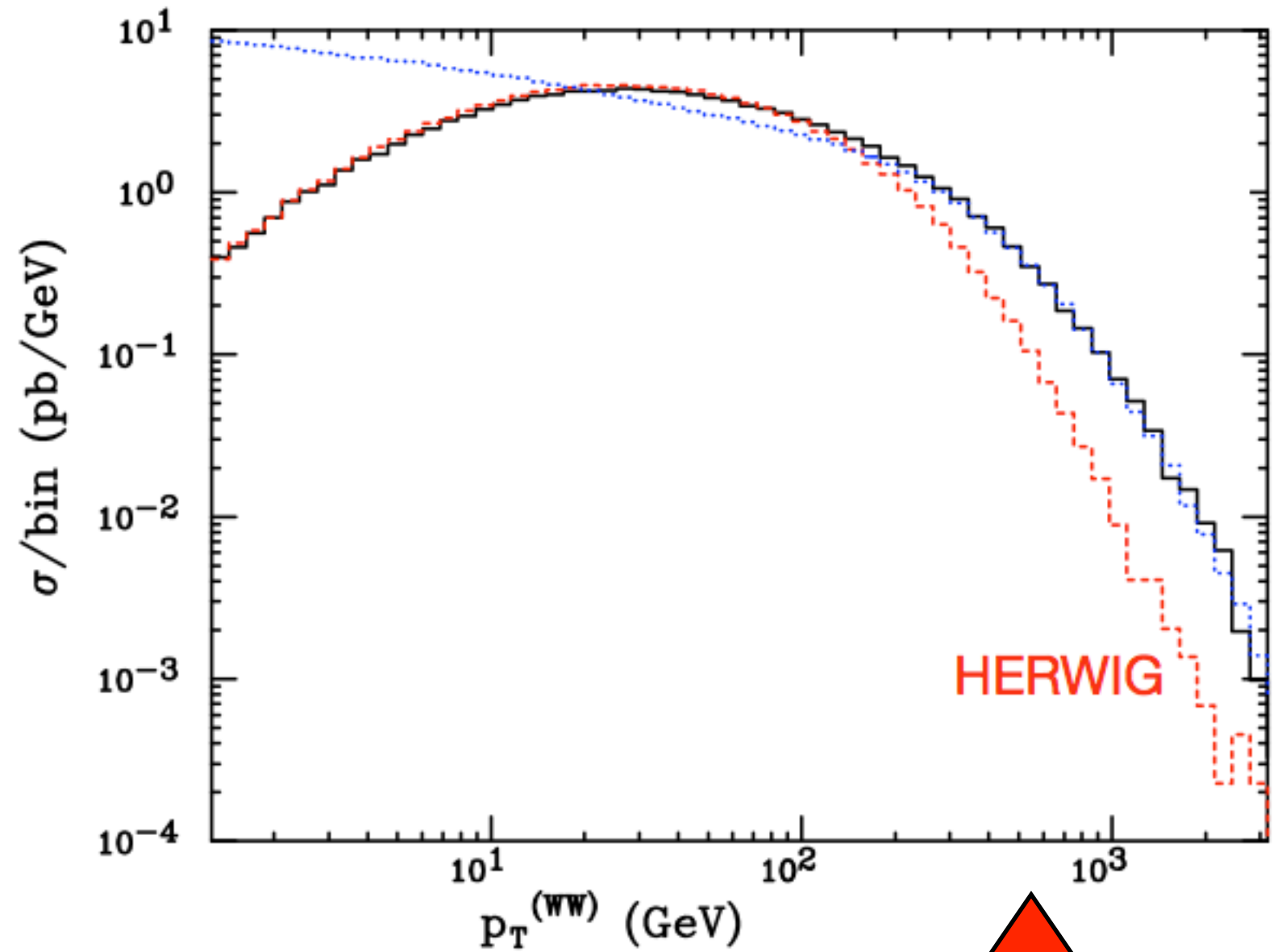
IPROC	IV	IL ₁	IL ₂	Spin	Process
-1706		i	j	✓	$H_1 H_2 \rightarrow (t \rightarrow) b_k f_i f'_i (\bar{t} \rightarrow) \bar{b}_l f_j f'_j + X$
-2000-IC		i		✓	$H_1 H_2 \rightarrow (t \rightarrow) b_k f_i f'_i / (\bar{t} \rightarrow) \bar{b}_k f_i f'_i + X$
-2001-IC		i		✓	$H_1 H_2 \rightarrow (\bar{t} \rightarrow) \bar{b}_k f_i f'_i + X$
-2004-IC		i		✓	$H_1 H_2 \rightarrow (t \rightarrow) b_k f_i f'_i + X$
-2030		i	j	✓	$H_1 H_2 \rightarrow (t \rightarrow) b_k f_i f'_i (W^- \rightarrow) f_j f'_j / (\bar{t} \rightarrow) \bar{b}_k f_i f'_i (W^+ \rightarrow) f_j f'_j + X$
-2031		i	j	✓	$H_1 H_2 \rightarrow (\bar{t} \rightarrow) \bar{b}_k f_i f'_i (W^+ \rightarrow) f_j f'_j + X$
-2034		i	j	✓	$H_1 H_2 \rightarrow (t \rightarrow) b_k f_i f'_i (W^- \rightarrow) f_j f'_j + X$
-2850		i	j	✓	$H_1 H_2 \rightarrow (W^+ \rightarrow) l_i^+ \nu_i (W^- \rightarrow) l_j^- \bar{\nu}_j + X$

MC@NLO: W^+W^- production (LHC)



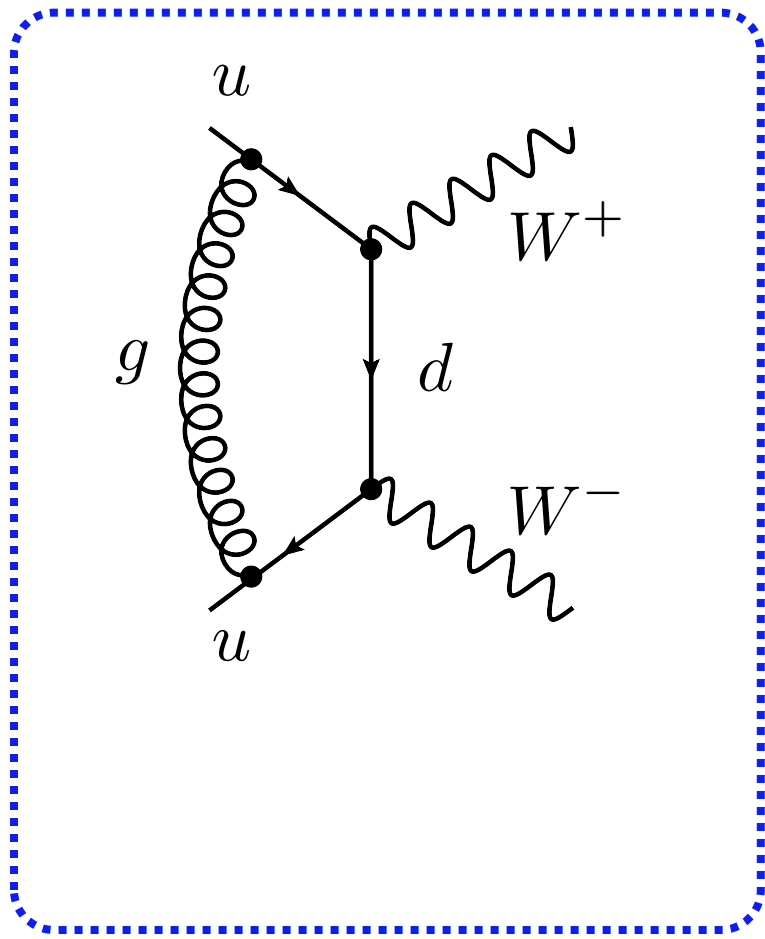
parton shower

HERWIG

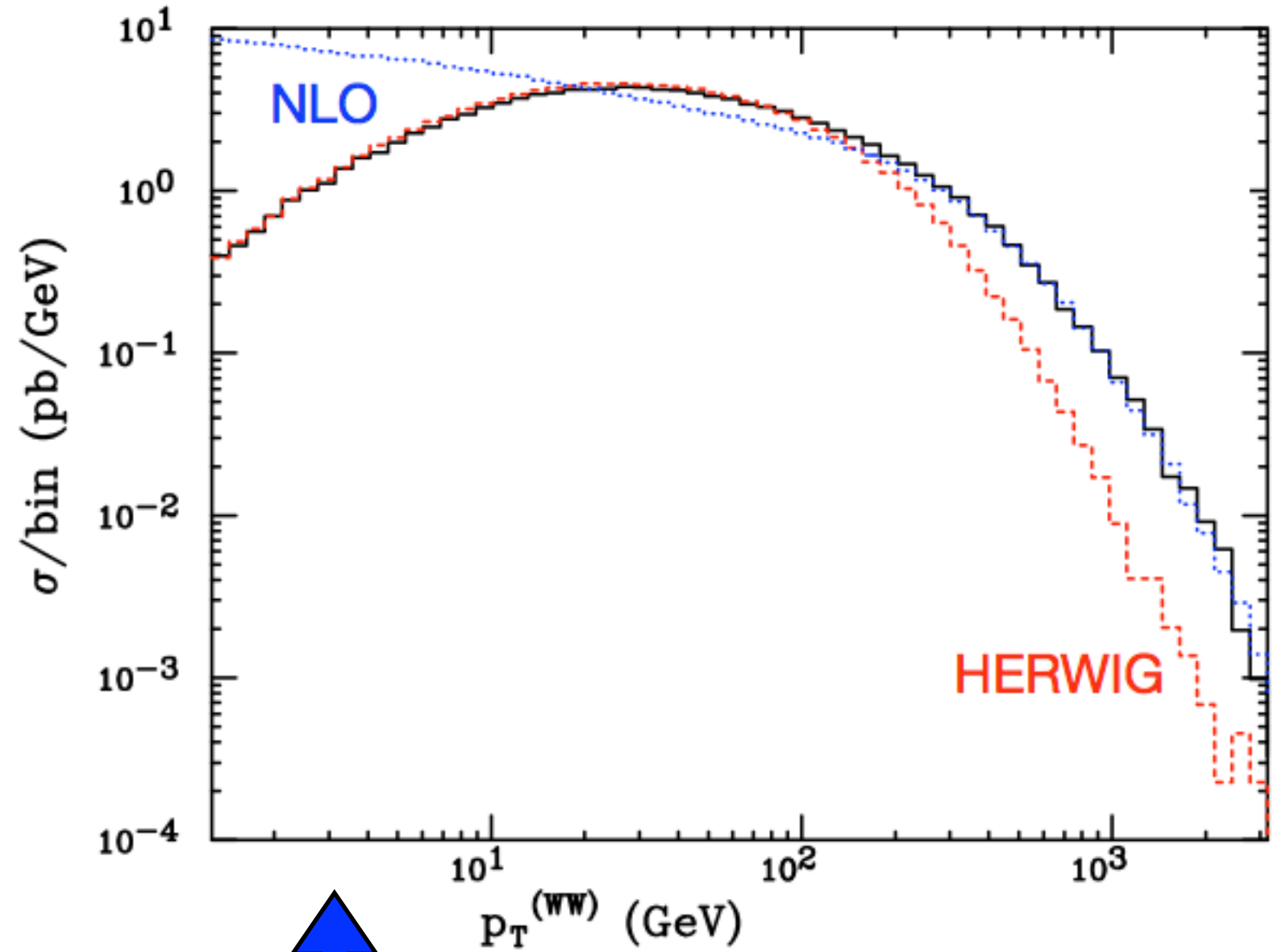


Herwig too soft in the high- p_t region

MC@NLO: W^+W^- production (LHC)

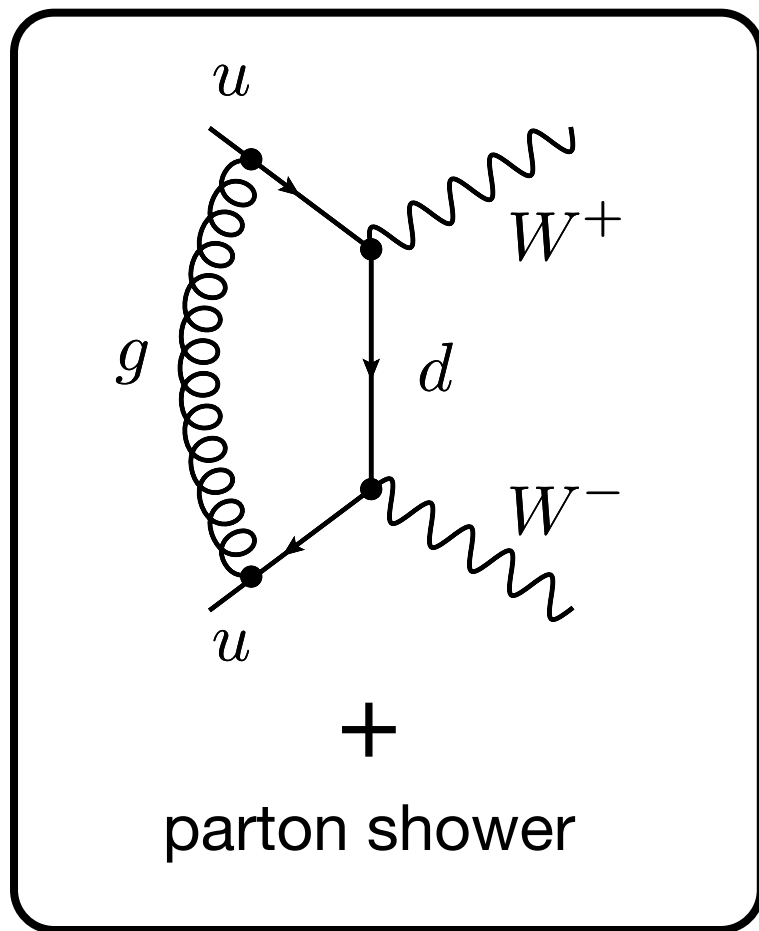


NLO

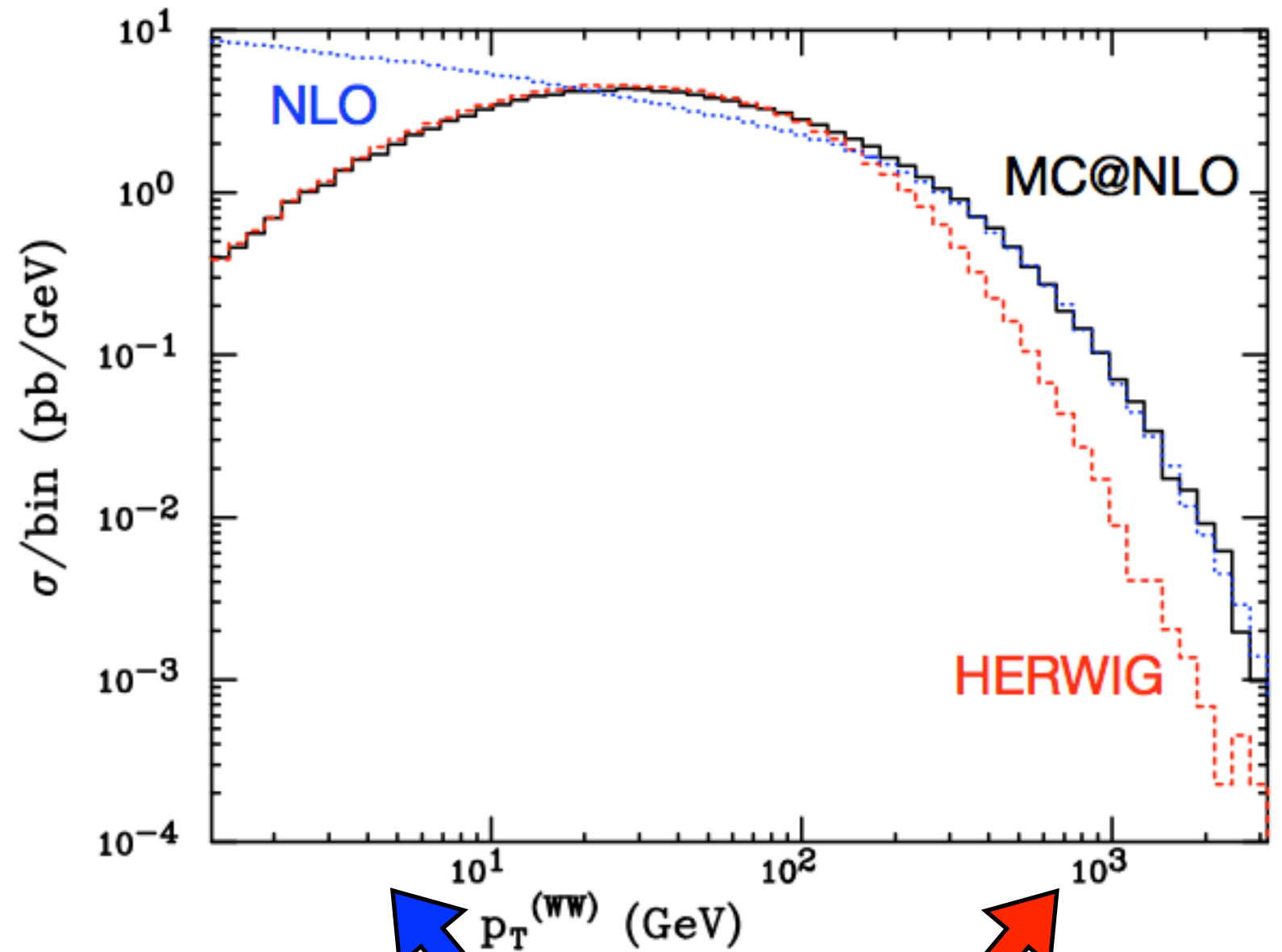


NLO divergent
in the soft region

MC@NLO: W^+W^- production (LHC)



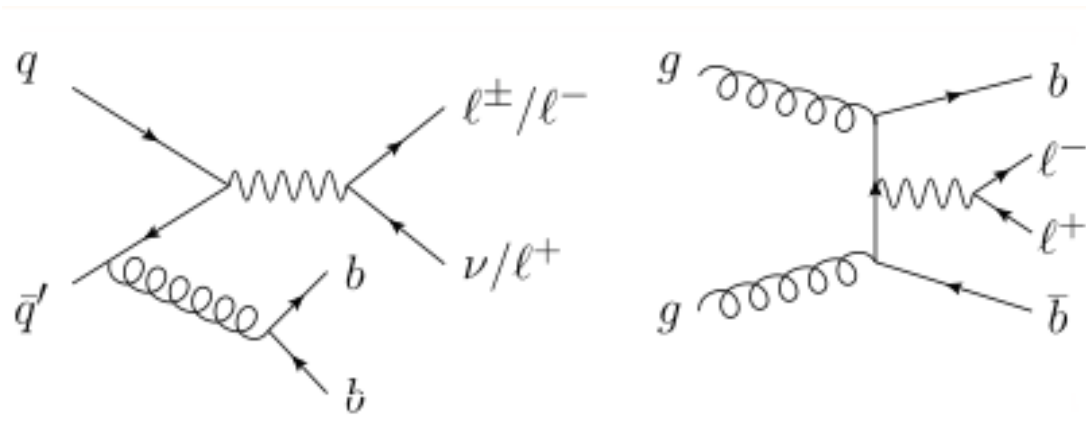
MC@NLO



MC@NLO correctly interpolates
between the two regimes

Wbb/Zbb in MC@NLO

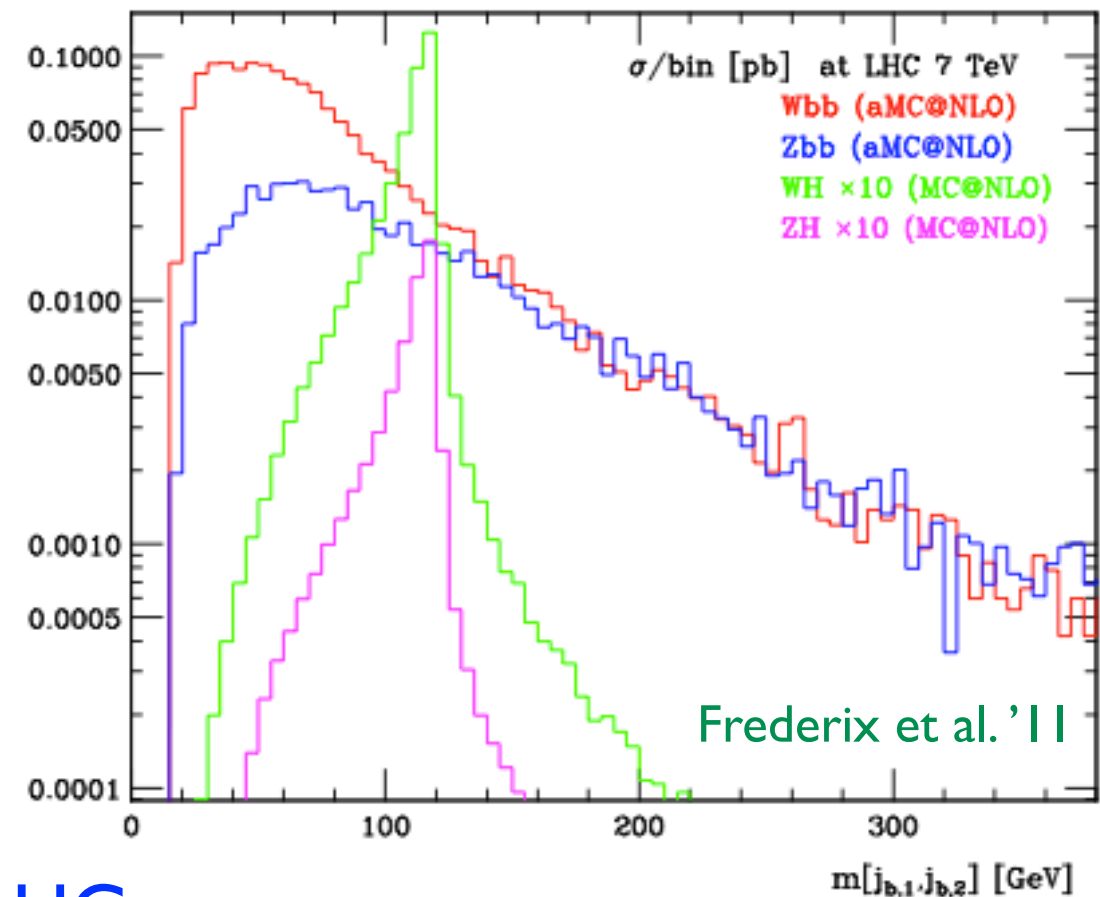
Irreducible background to $pp \rightarrow HW$ and $pp \rightarrow HZ$, with $H \rightarrow bb$



LO: gg channel present only for Zbb. Most differences Wbb vs Zbb due to this

	Cross section (pb)					
	Tevatron $\sqrt{s} = 1.96$ TeV			LHC $\sqrt{s} = 7$ TeV		
	LO	NLO	K factor	LO	NLO	K factor
$\ell\nu b\bar{b}$	4.63	8.04	1.74	19.4	38.9	2.01
$\ell^+\ell^-b\bar{b}$	0.860	1.509	1.75	9.66	16.1	1.67

Example: signal & background with the same accuracy



Wbb/Zbb: ≈ 5 ≈ 2
Reason: gg enhancement in Zbb at the LHC



Jets: five years ago



Cones are IR unsafe!

The Cone is too rigid!

IR unsafety affects jet cross-sections by less than 1%, so don't need to care!

kt collects too much soft radiation!



Cones have a well-defined circular area!

Jet area not well defined in kt: U.E. and pile-up subtraction too difficult!

What about dark towers??

After all, if $D=1.35 R$ Cone and kt are practically the same thing....

Where do jets enter?

Essentially everywhere at colliders!

Jets are an essential tool for a variety of studies:

- 🎧 top reconstruction
- 🎧 mass measurements
- 🎧 most Higgs and NP searches
- 🎧 general tool to attribute structure to an event
- 🎧 instrumental for QCD studies, e.g. inclusive-jet measurements
⇒ important input for PDF determinations

Jets

Jets provide a way of projecting away the multiparticle dynamics of an event \Rightarrow leave a simple quasi-partonic picture of the hard scattering

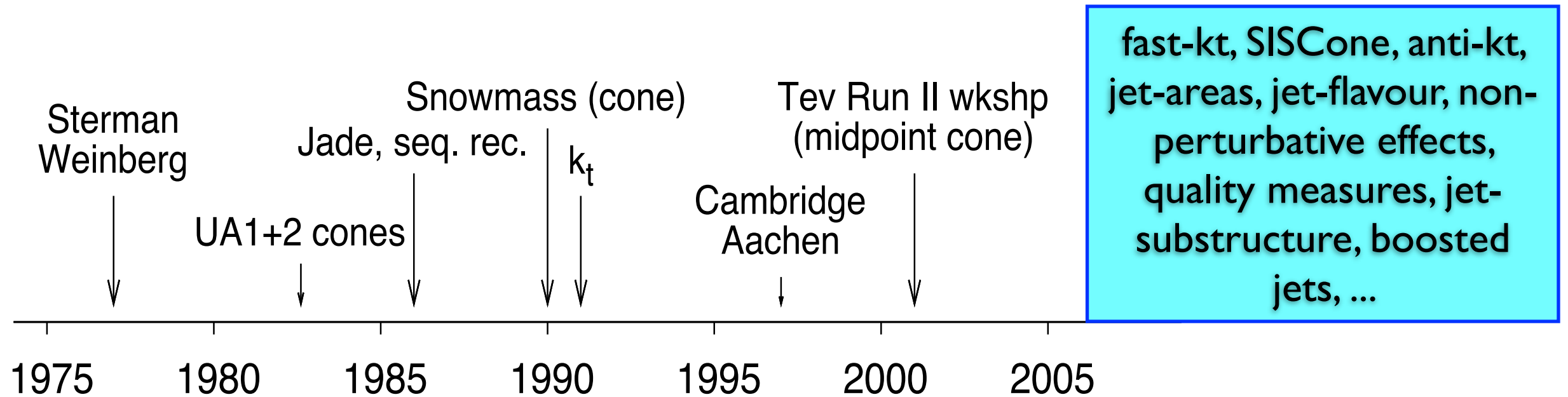
The projection is fundamentally ambiguous \Rightarrow jet physics is a rich subject



Ambiguities:

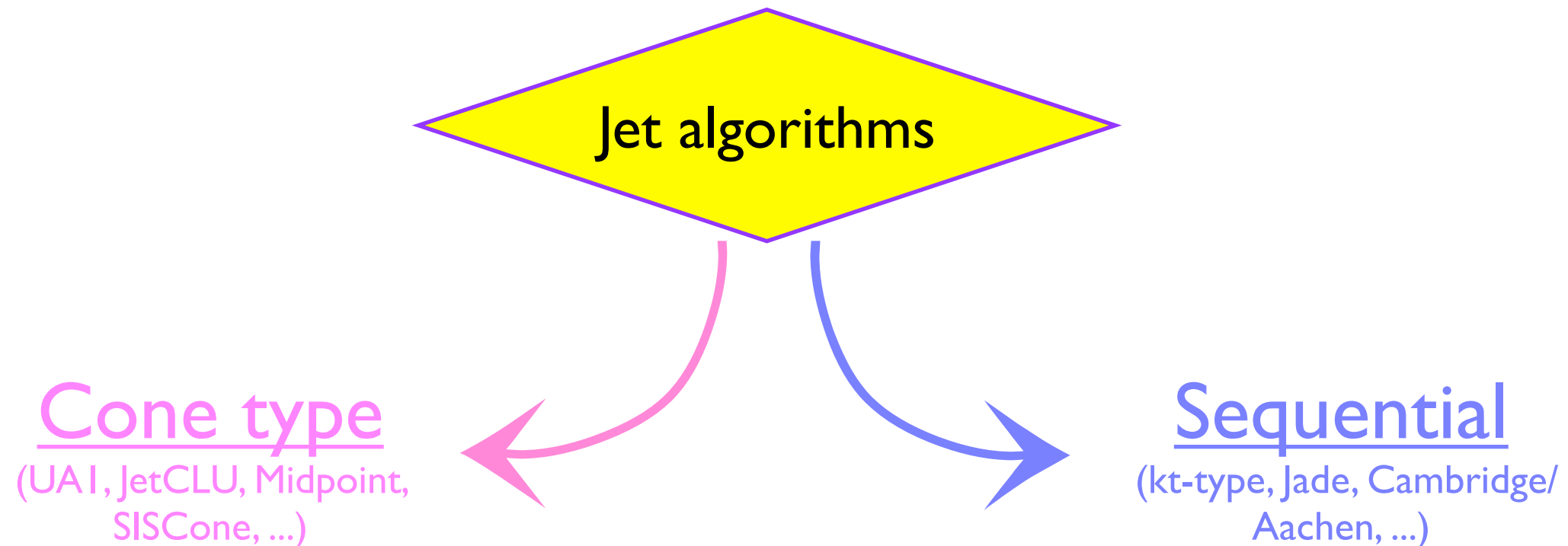
- 1) Which particles should belong to a same jet?
- 2) How does recombine the particle momenta to give the jet-momentum?

Jet developments



Two broad classes of jet algorithms

Today many extensions of the original Stermann-Weinberg jets.
Modern jet-algorithms divided into two broad classes



top down approach:

cluster particles according to distance in **coordinate-space**

Idea: put cones along dominant direction of energy flow

bottom up approach: cluster particles according to distance in **momentum-space**
Idea: undo branchings occurred in the perturbative evolution

Jet requirements

Snowmass accord

FERMILAB-Conf-90/249-E
[E-741/CDF]

Toward a Standardization of Jet Definitions

Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross section at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronization.

Inclusive k_t /Durham-algorithm

Catani et. al '92-'93; Ellis and Soper '93

Inclusive algorithm:

I. For any pair of final state particles i, j define the distance

$$d_{ij} = \frac{\Delta y_{ij}^2 + \Delta \phi_{ij}^2}{R^2} \min\{k_{ti}^2, k_{tj}^2\}$$

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2. For each particle i define a distance with respect to the beam

$$d_{iB} = k_{ti}^2$$

Inclusive k_t/R /Durham-algorithm

Catani et. al '92-'93; Ellis and Soper '93

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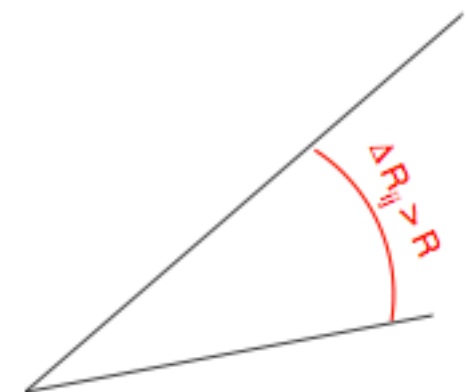
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3. Find the smallest distance. If it is a d_{ij} recombine i and j into a new particle (\Rightarrow recombination scheme); if it is d_{iB} declare i to be a jet and remove it from the list of particles

NB: if $\Delta R_{ij}^2 \equiv \Delta y_{ij}^2 + \Delta \phi_{ij}^2 < R^2$ then partons (ij) are always recombined, so **R sets the minimal interjet angle**



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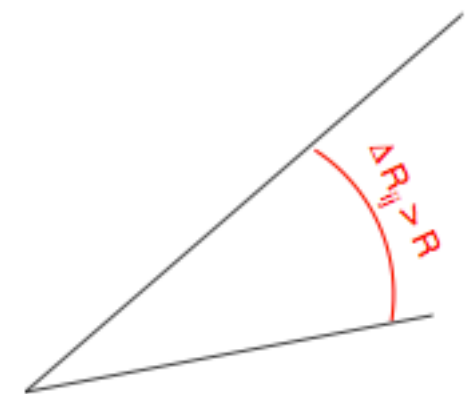
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4. repeat the procedure until no particles are left

Exclusive k_t /Durham-algorithm

Inclusive algorithm gives a variable number of jets per event, according to the specific event topology

Exclusive k_t /Durham-algorithm

Inclusive algorithm gives a variable number of jets per event, according to the specific event topology

Exclusive version: run the inclusive algorithm but stop when either

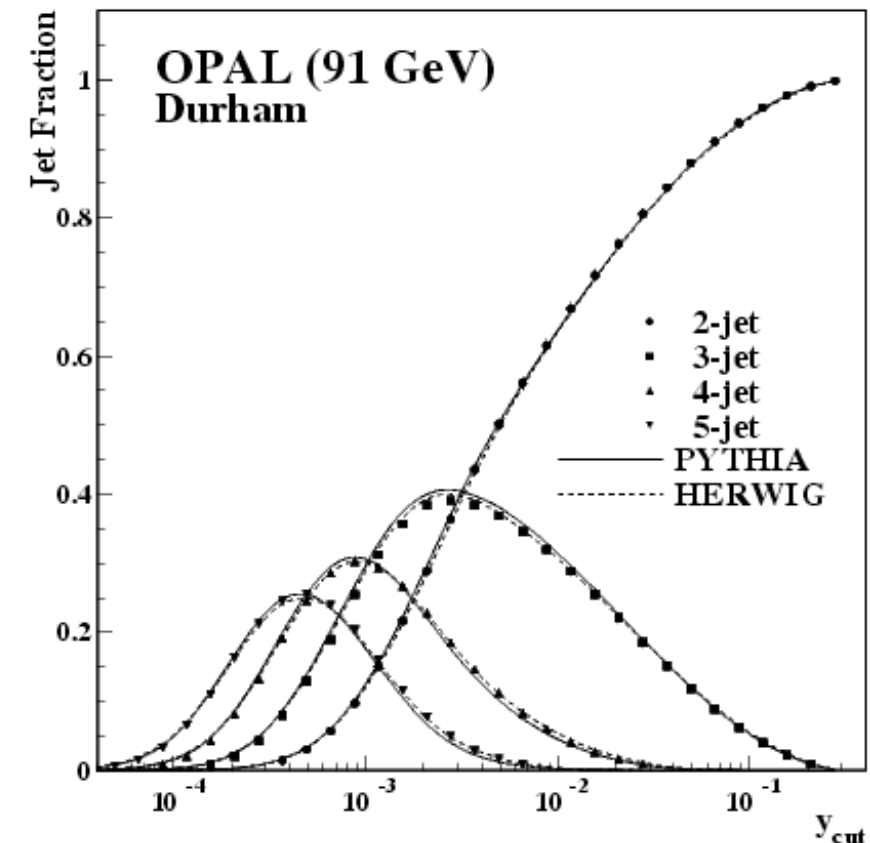
- all $d_{ij}, d_{iB} > d_{\text{cut}}$ or
- when reaching the desired number of jets n

k_t /Durham-algorithm in e^+e^-

k_t originally designed in e^+e^- , most widely used algorithm in e^+e^- (LEP)

$$y_{ij} = 2 \min\{E_i^2, E_j^2\} (1 - \cos \theta_{ij}^2)$$

- can classify events using $y_{23}, y_{34}, y_{45}, y_{56} \dots$
- resolution parameter related to minimum transverse momentum between jets



k_t /Durham-algorithm in e^+e^-

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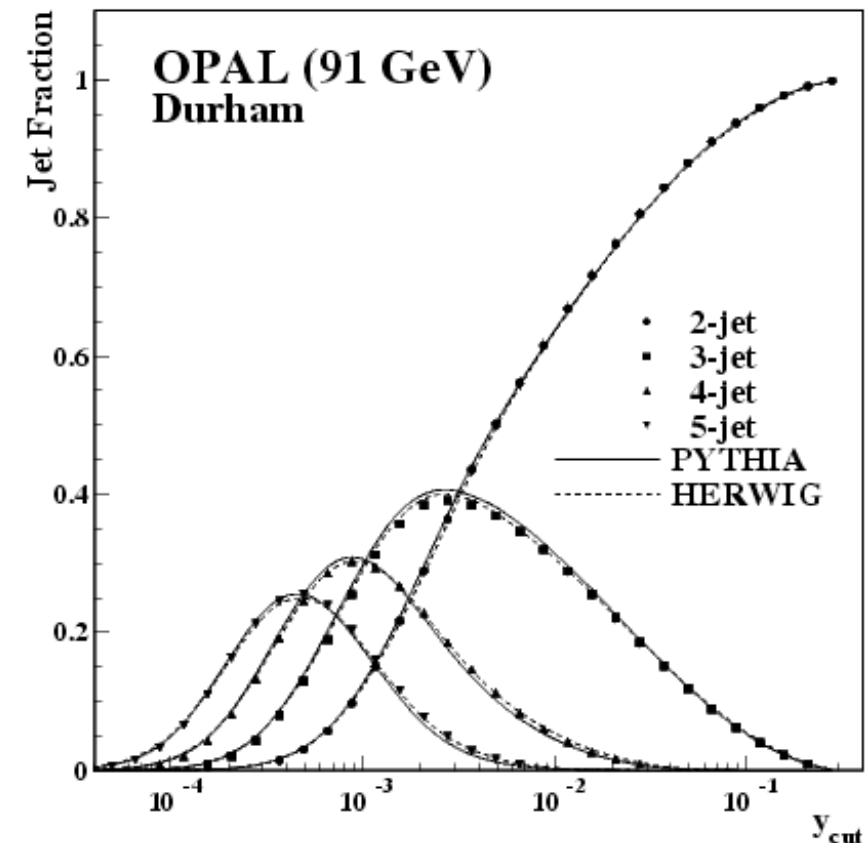
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- can classify events using $y_{23}, y_{34}, y_{45}, y_{56} \dots$
- resolution parameter related to minimum transverse momentum between jets

Satisfies fundamental requirements:

1. **Collinear safe:** collinear particles recombine early on
2. **IR-safe:** soft particles do not influence the clustering sequence

\Rightarrow *collinear + IR safety important: it means that cross-sections can be computed at higher order in pQCD (no divergences)!*



The CA and the anti- k_t algorithm

The Cambridge/Aachen: sequential algorithm like k_t , but uses only angular properties to define the distance parameters

$$d_{ij} = \frac{\Delta R_{ij}^2}{R^2} \quad d_{iB} = 1 \quad \Delta R_{ij}^2 = (\phi_i - \phi_j)^2 + (y_i - y_j)^2$$

Dotshitzer et. al '97; Wobisch and Wengler '99

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Cacciari, Salam, Soyez '08

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Cacciari, Salam, Soyez '08

anti- k_t is the default algorithm for ATLAS and CMS

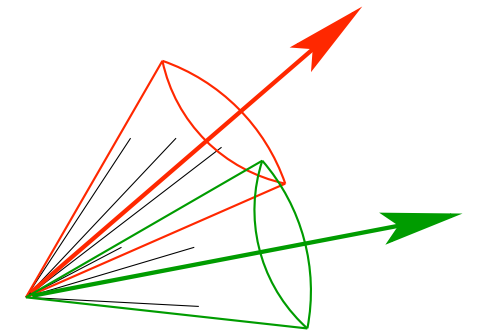
unfortunately with different default R 0.4 & 0.6 [ATLAS] 0.5 & 0.7 [CMS]

First time only IR-safe algorithms are used systematically at a collider!

Cone algorithms

I. A particle i at rapidity and azimuthal angle $(y_i, \Phi_i) \in \text{cone } C$ iff

$$\sqrt{(y_i - y_C)^2 + (\phi_i - \phi_C)^2} \leq R_{\text{cone}}$$



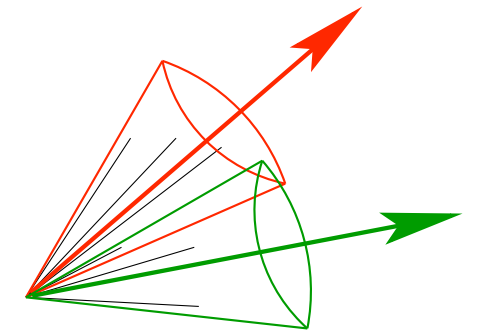
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$$\bar{y}_C \equiv \frac{\sum_{i \in C} y_i \cdot p_{T,i}}{\sum_{i \in C} p_{T,i}} \quad \bar{\phi}_C \equiv \frac{\sum_{i \in C} \phi_i \cdot p_{T,i}}{\sum_{i \in C} p_{T,i}}$$



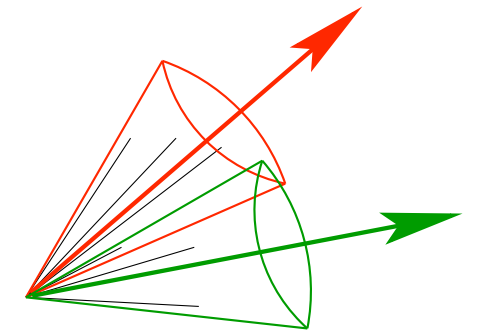
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3. If weighted and geometrical averages coincide $(y_C, \phi_C) = (\bar{y}_C, \bar{\phi}_C)$
a stable cone (\Rightarrow jet) is found, otherwise set $(y_C, \phi_C) = (\bar{y}_C, \bar{\phi}_C)$ & iterate

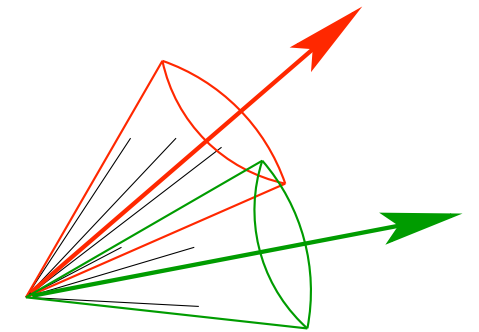
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4. Stable cones can overlap. Run a split-merge on overlapping jets: merge jets if they share more than an energy fraction f , else split them and assign the shared particles to the cone whose axis they are closer to.

Remark: too small f (<0.5) creates high jets, not recommended

Cone algorithms

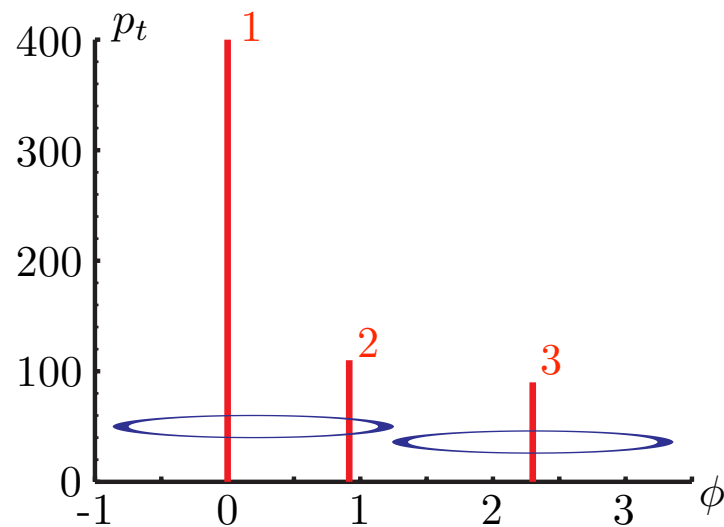
- The question is where does one start looking for stable cone?
- The direction of these trial cones are called **seeds**
- Ideally, place seeds everywhere, so as not to miss any stable cone
- Practically, this is unfeasible. Speed of recombination grows fast with the number of seeds. So place only some seeds, e.g. at the (y, Φ) -location of particles.

Cone algorithms

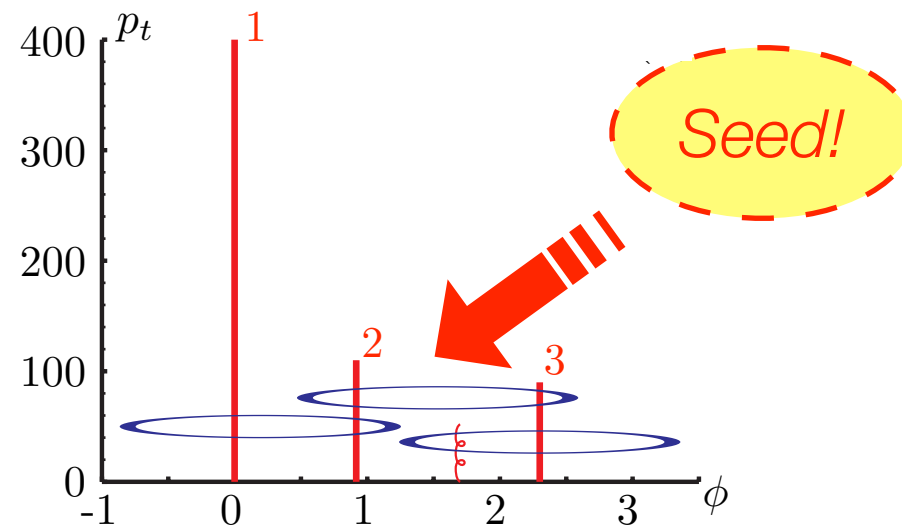
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Seeds make cone algorithms IR unsafe

Jets: IR unsafety of cones



3 hard \Rightarrow 2 stable cones



3 hard + 1 soft \Rightarrow 3 stable cones

Soft emission changes the hard jets \Rightarrow algorithm is IR unsafe

Midpoint algorithm: take as seed position of emissions **and midpoint between two emissions** (postpones the IR safety problem)

Seedless cones

Solution:

use a seedless algorithm, i.e. consider all possible combinations of particles as candidate cones, so find all stable cones [\Rightarrow jets]

Blazey '00

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The problem:

clustering time growth as $N2^N$. So for an event with **100 particles need 10^{17} ys to cluster the event** \Rightarrow prohibitive beyond PT (N=4,5)

Seedless cones

Solution:

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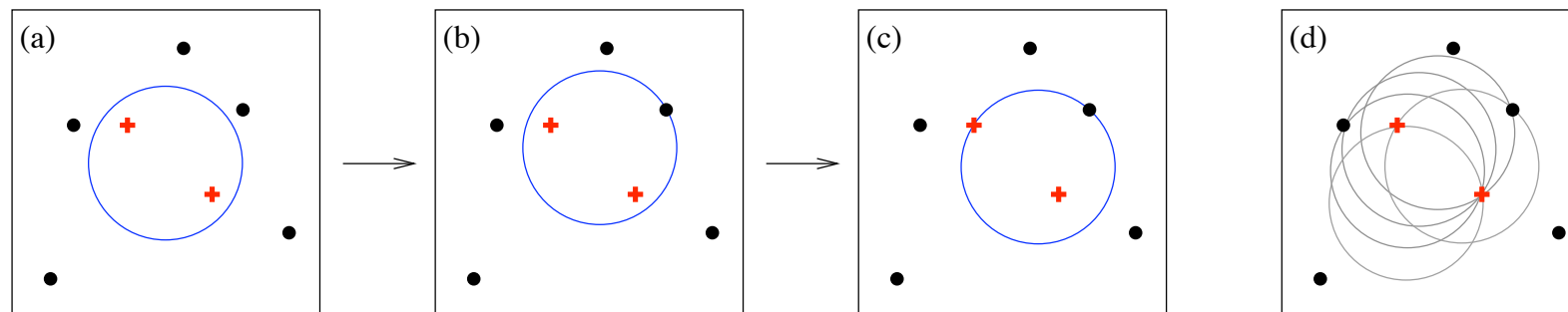
Blazey '00

The problem:

clustering time growth as N^2^N . So for an event with **100 particles need 10^{17} ys to cluster the event** \Rightarrow prohibitive beyond PT ($N=4,5$)

Better solution:

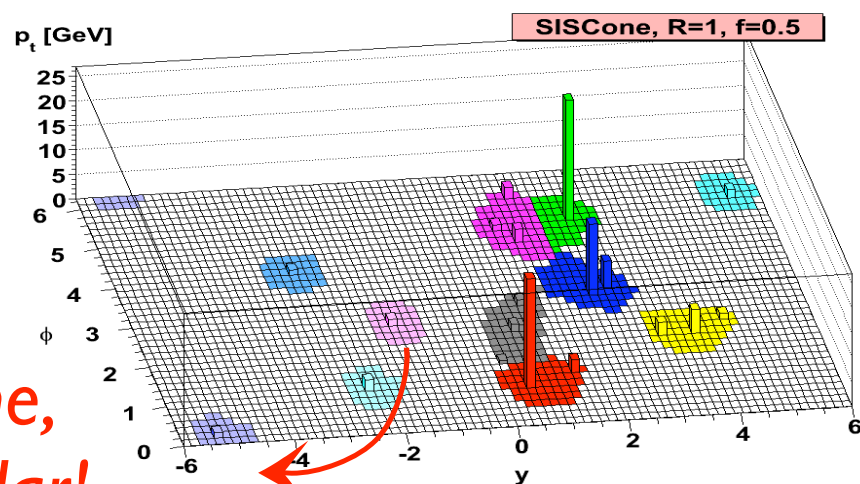
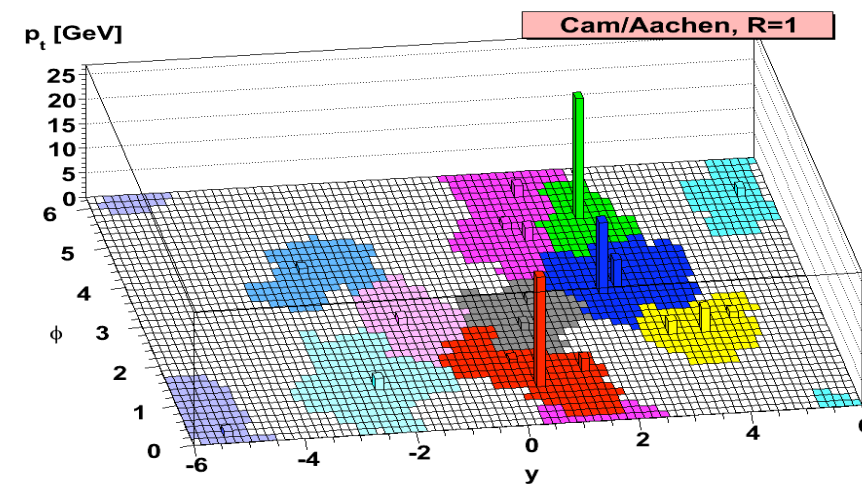
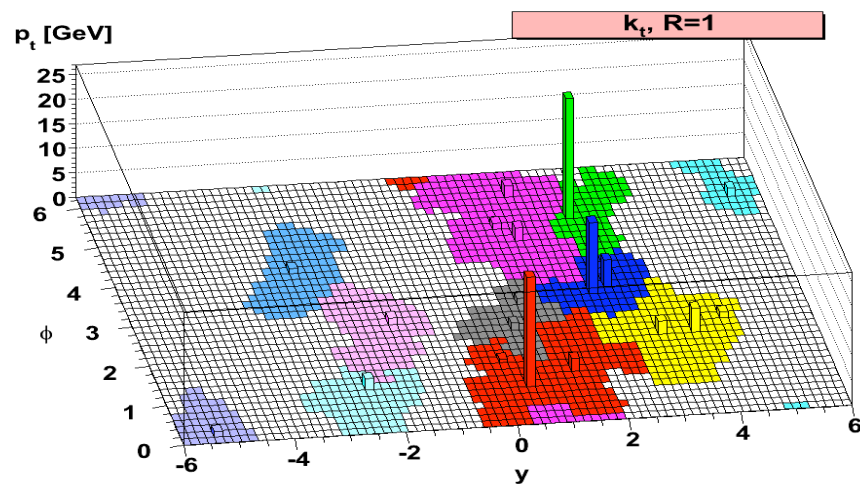
SISCone recasts the problem as a computational geometry problem, the identification of all distinct circular enclosures for points in 2D and finds a solution to that \Rightarrow **N^2 In N time IR safe algorithm**



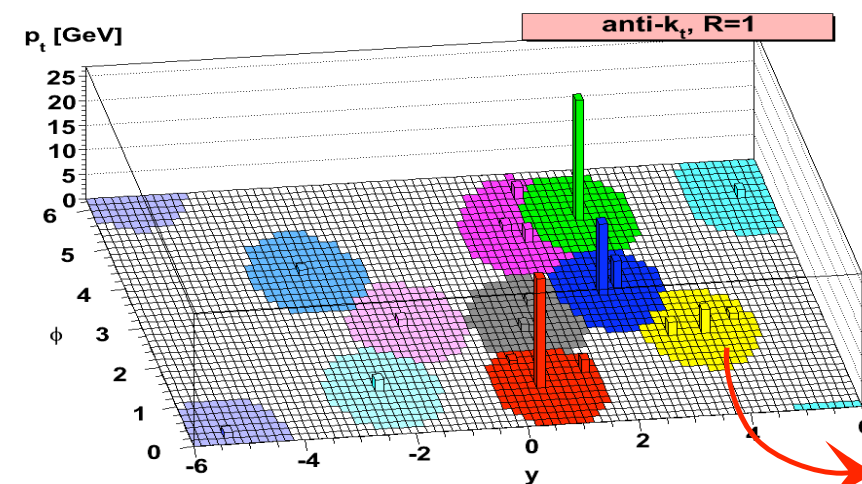
Salam, Soyez '07

Jet area

Given an IR safe, fast jet-algorithm, can define the jet area A as follows:
fill the event with an infinite number of infinitely soft emissions uniformly distributed in η - ϕ and make A proportional to the # of emissions clustered in the jet



*NB: cone,
not circular!*



*NB: new
anti-kt*

What jet areas are good for

jet-area \equiv catching area of the jet when adding soft emissions

\Rightarrow use the jet area to formulate a **simple area based subtraction** of pile-up events

1. cluster particle with an IR safe jet algorithm
2. from **all jets** (most are pile-up ones) in the event define the median

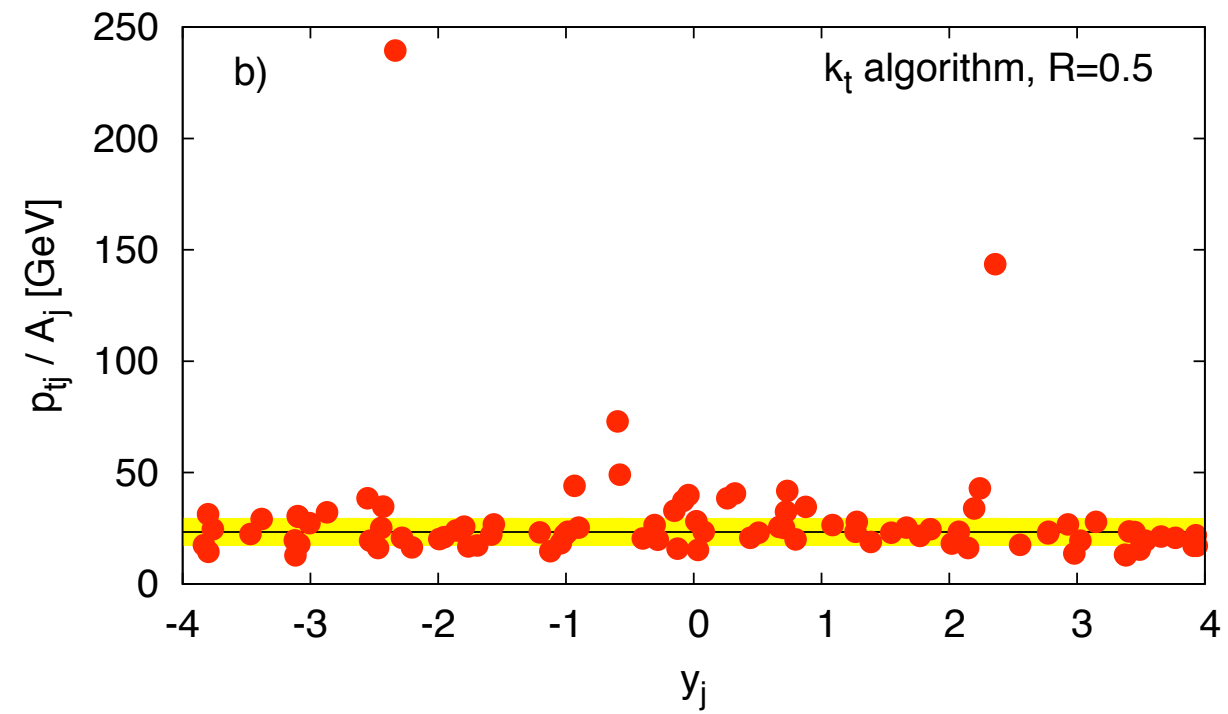
$$\rho = \frac{p_{t,j}}{A_j}$$

3. the median gives the typical p_t/A_j for a given event
4. use the median to subtract off dynamically the soft part of the soft events

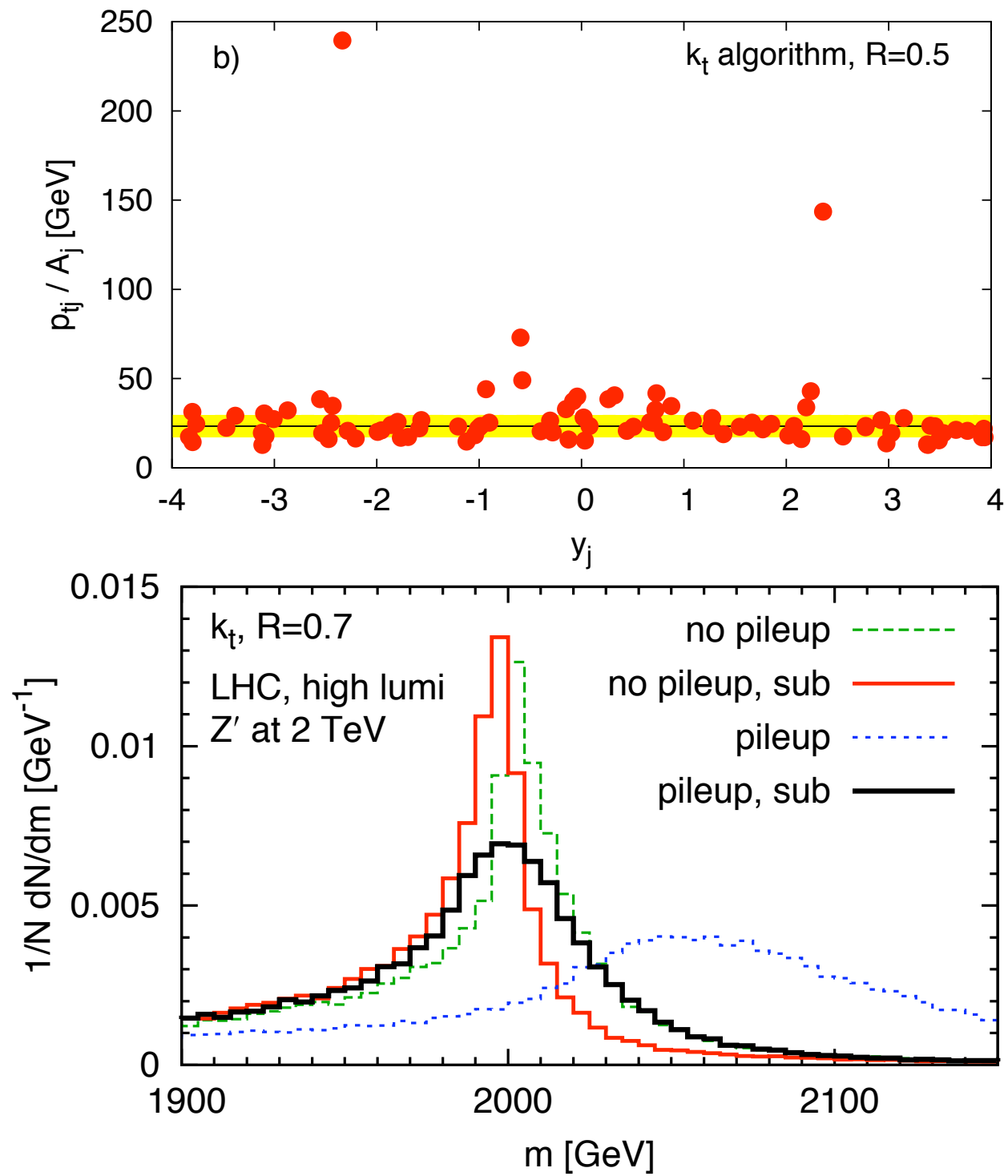
$$p_j^{\text{sub}} = p_j - A_j \rho$$

Pileup = generic p-p interaction (hard, soft, single-diffractive, ...) overlapping with hard scattering

Sample 2 TeV mass reconstruction



Sample 2 TeV mass reconstruction



Cacciari et al. '07

Quality measures of jets

Suppose you are searching for a heavy state ($H \rightarrow gg, Z' \rightarrow qq, \dots$)

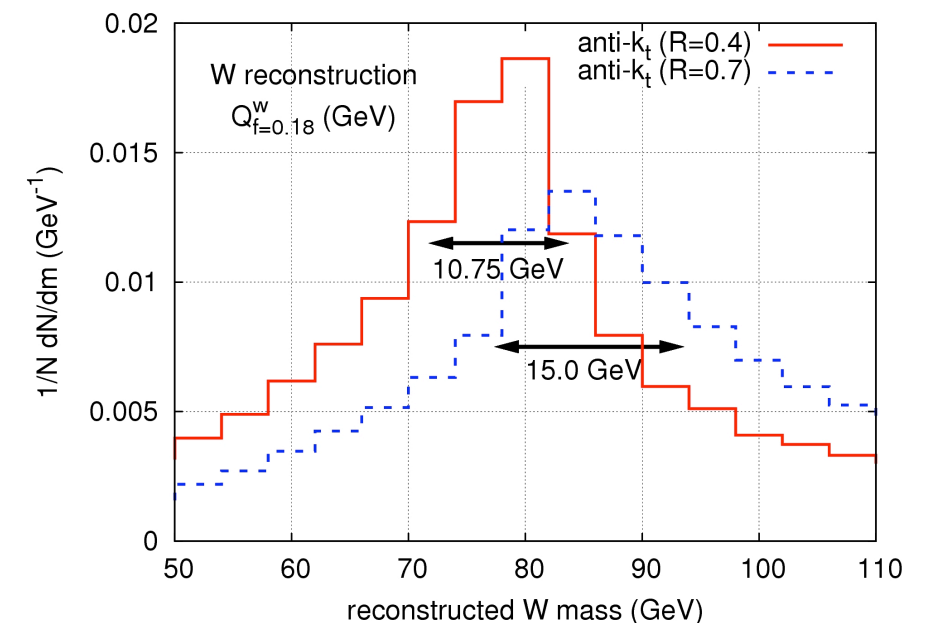
The object is reconstructed through its decay products

\Rightarrow Which jet algorithm (JA) is best? Does the choice of R matter?

Define: $Q_f^w(JA, R) \equiv$ width of the smallest mass window that contains a fraction f of the generated massive objects

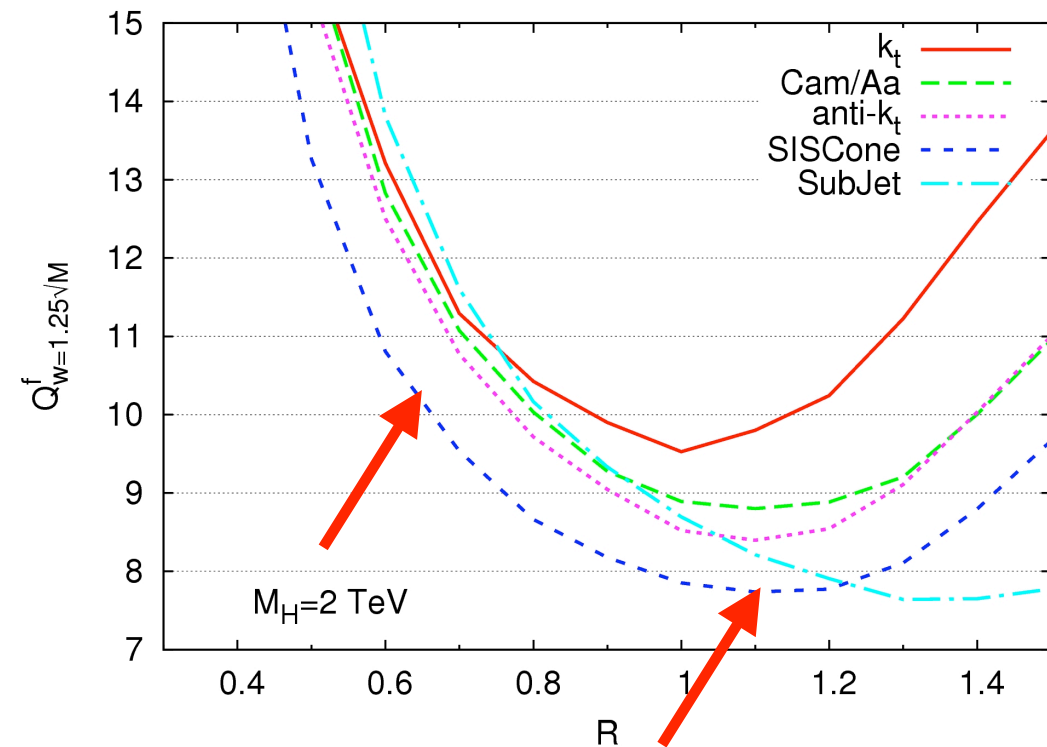
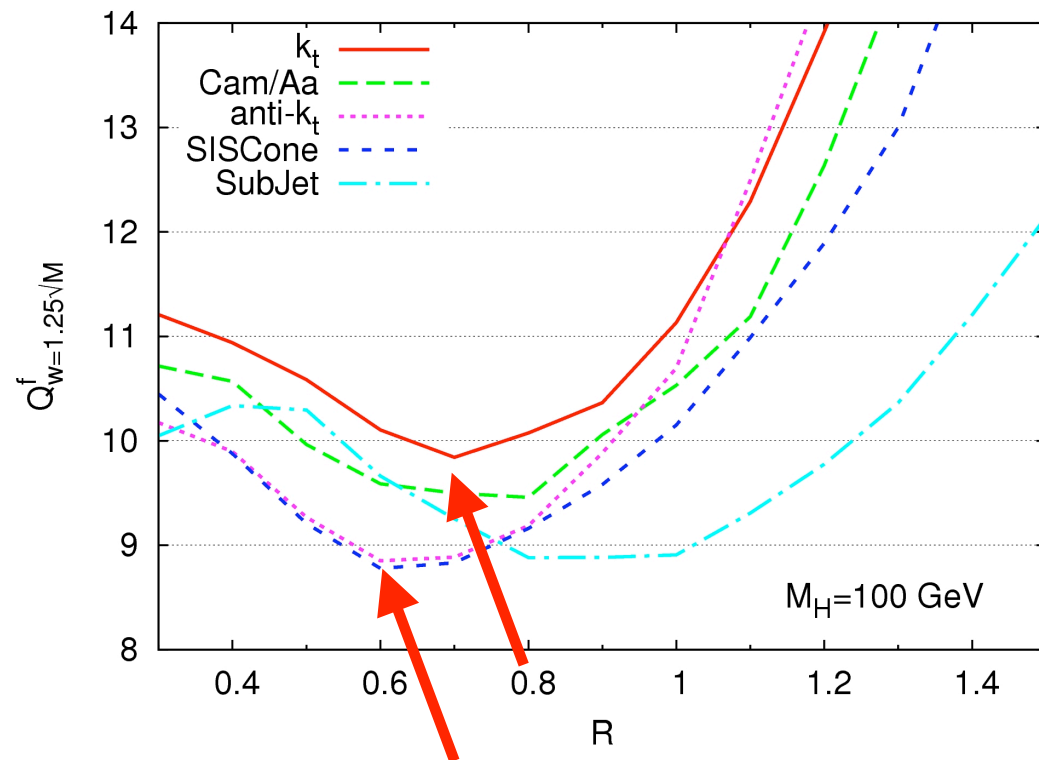
- good algo \Leftrightarrow small $Q_f^w(JA, R)$
- ratios of $Q_f^w(JA, R)$: mapped to ratios of effective luminosity (with same S/\sqrt{B})

$$\mathcal{L}_2 = \rho \mathcal{L} \mathcal{L}_1 \quad \rho \mathcal{L} = \frac{Q_z^f(JA_2, R_2)}{Q_z^f(JA_1, R_1)}$$



Quality measures: sample results

NB: Here “fake Higgs” = narrow resonance decaying to gluons

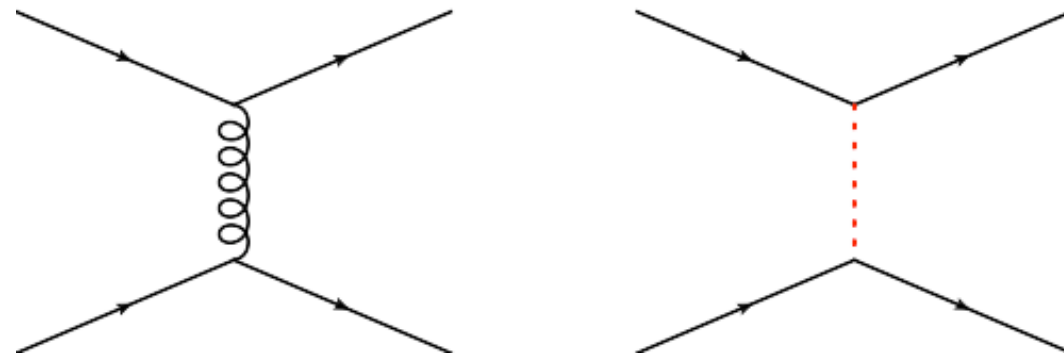


- ▶ At 100GeV: use a Tevatron standard algo (k_t , $R=0.7$) instead of best choice (SIScone, $R=0.6$) \Rightarrow lose $\rho_{\mathcal{L}} = 0.8$ in effective luminosity
- ▶ At 2 TeV: use $M_Z=100$ GeV Tevatron best choice instead SIScone, $R=1.1$ \Rightarrow lose $\rho_{\mathcal{L}} = 0.6$ in effective luminosity

A good choice of jet-algorithm can make the difference
 Bad choice of jet-algorithm \Leftrightarrow loose in discrimination power

Jets and New Physics searches

New Physics can modify the scattering of quarks and gluons, e.g. through the exchange of a heavy object



At energies much smaller than M , the details of the new particles exchanged can not be resolved. The effect can be simulated by adding new terms to the QCD Lagrangian, typically dimension 6 operators

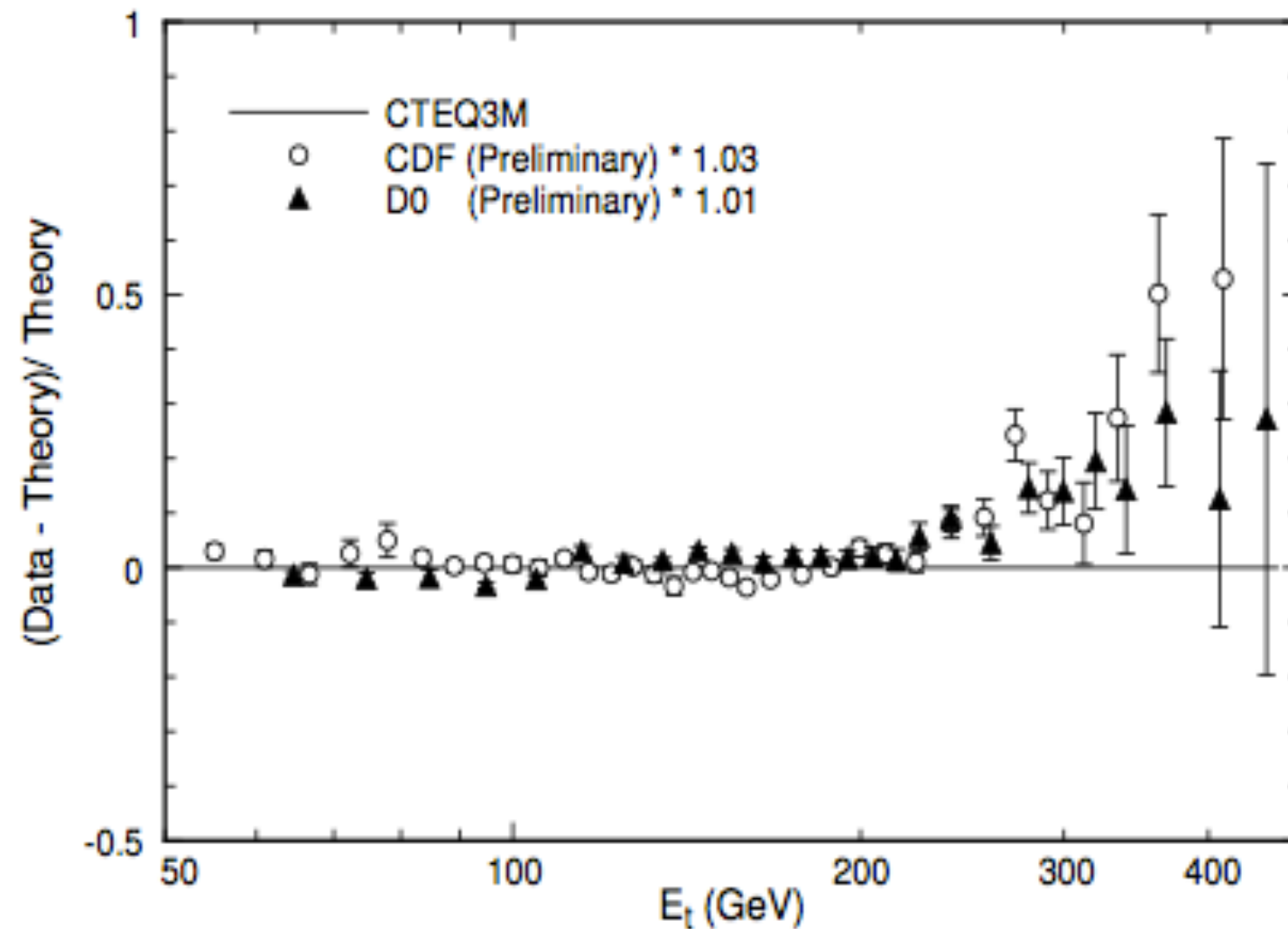
$$\Delta\mathcal{L} = \frac{\tilde{g}^2}{M^2} \bar{\psi}\gamma^\mu\psi\bar{\psi}\gamma_\mu\psi$$

Then one expects a correction to the transverse energy cross-section of the form

$$\sim \tilde{g}^2 E_T^2 / M^2$$

Jets and New Physics searches

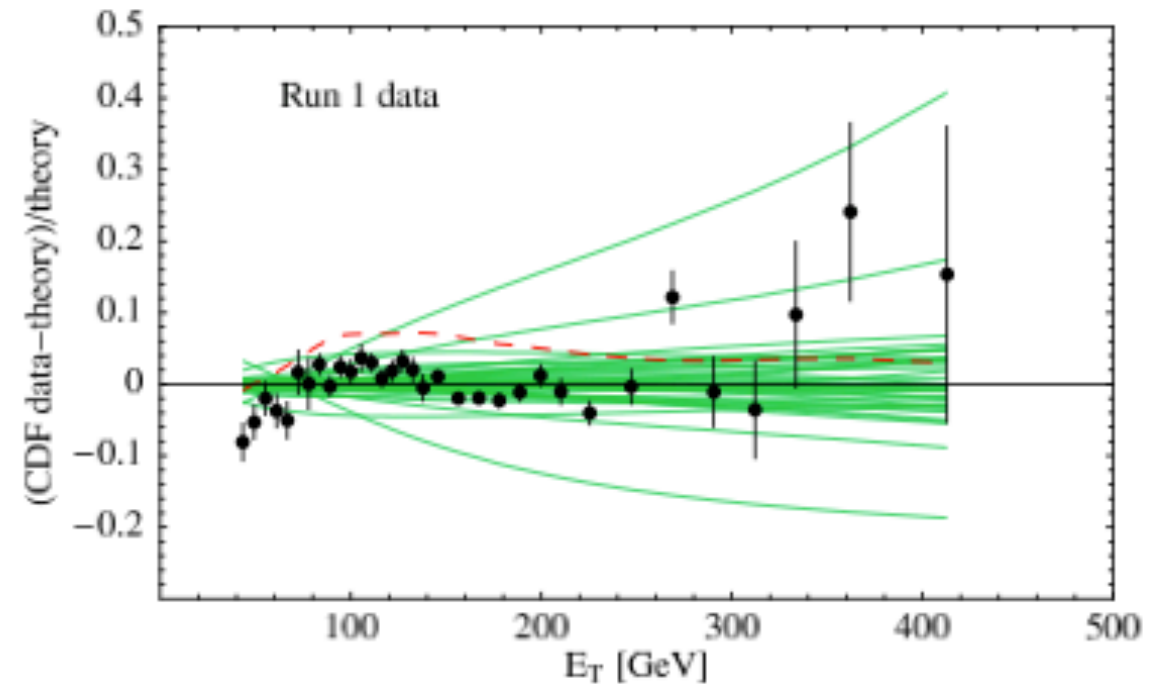
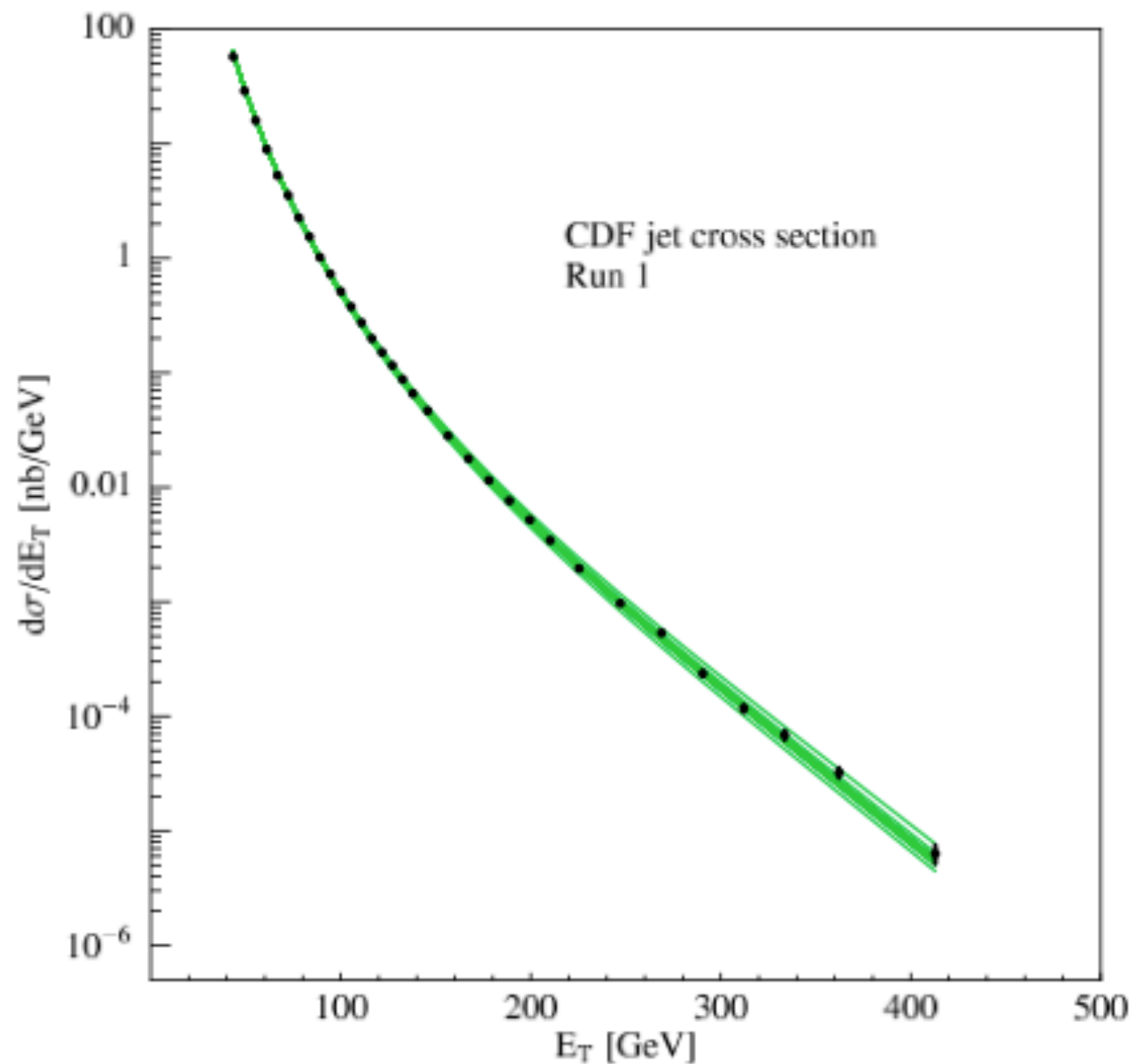
An example: NLO QCD vs Tevatron data (1996)



New Physics ? No! Poor modeling of gluon PDF at large x .

Jets and New Physics searches

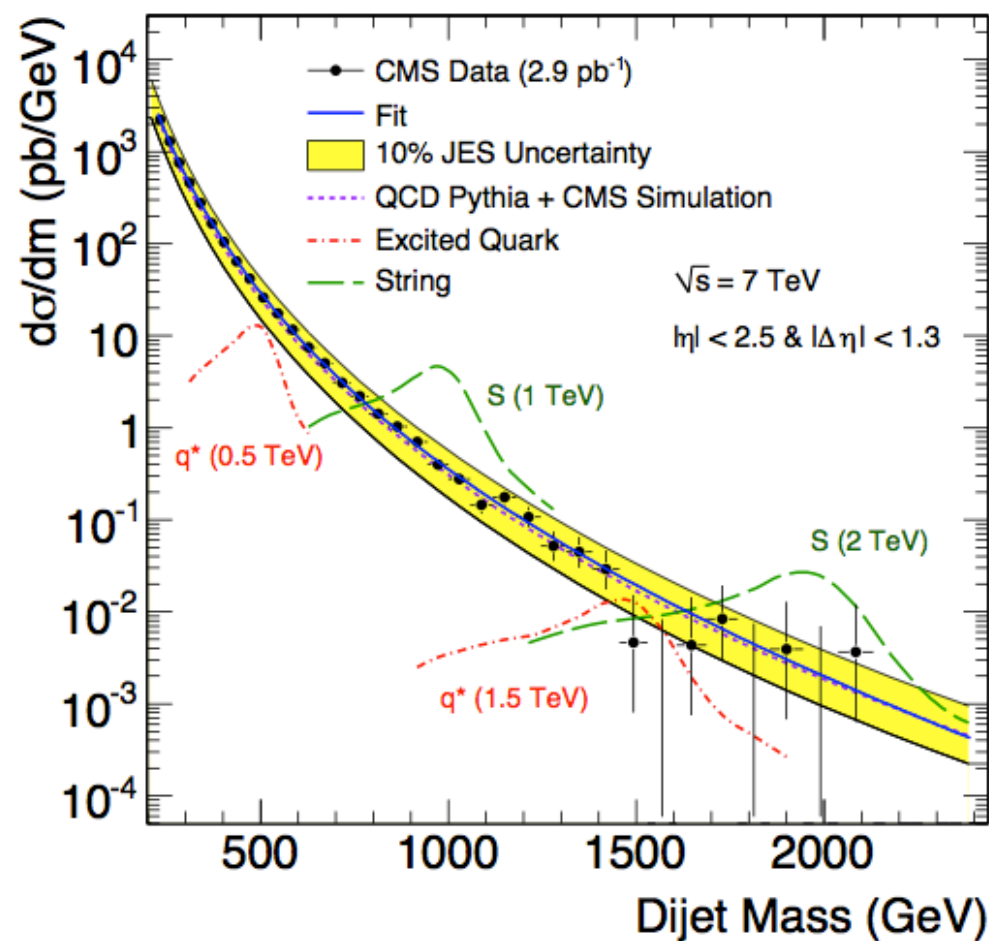
With better treatment and inclusion of uncertainties on gluon PDFs



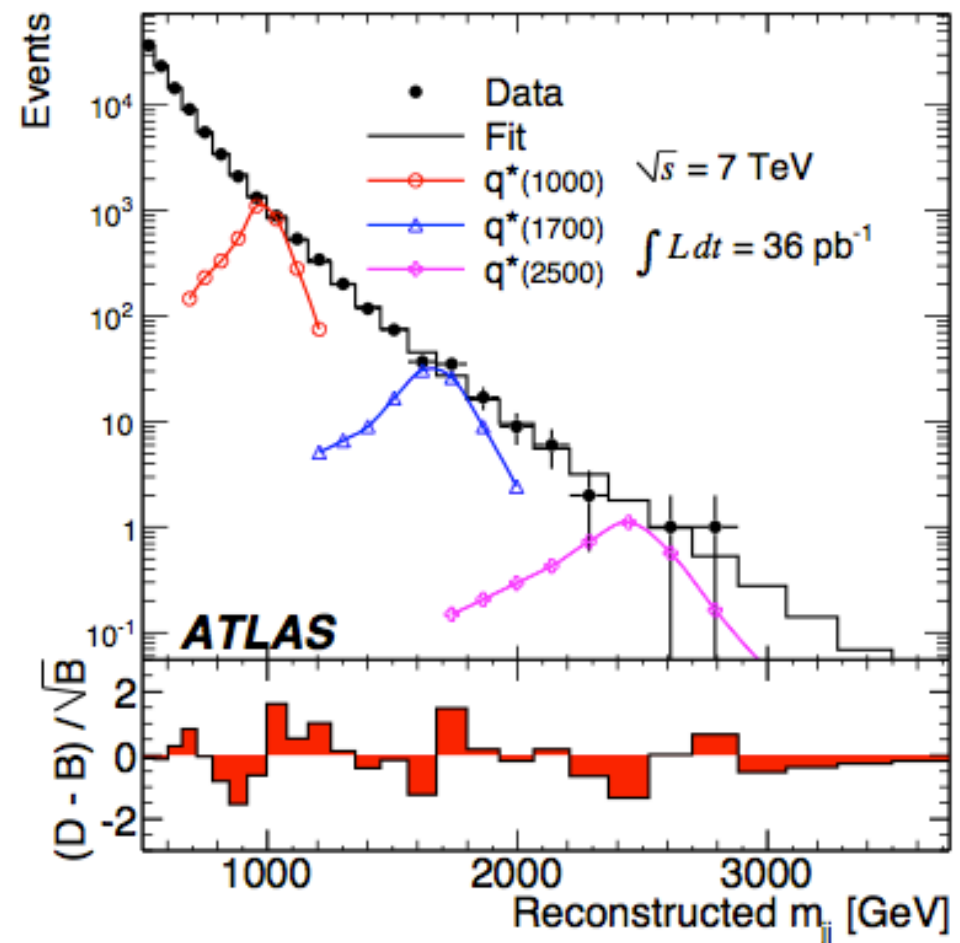
Lots of care is needed in data interpretation, especially when PDF are probed in regions with none or little data

Jets today at the LHC

So far, at the LHC jets could probe the highest energy scales $\sim 4 \text{ TeV}$
 [Tevatron $\sim 1 \text{ TeV}$]

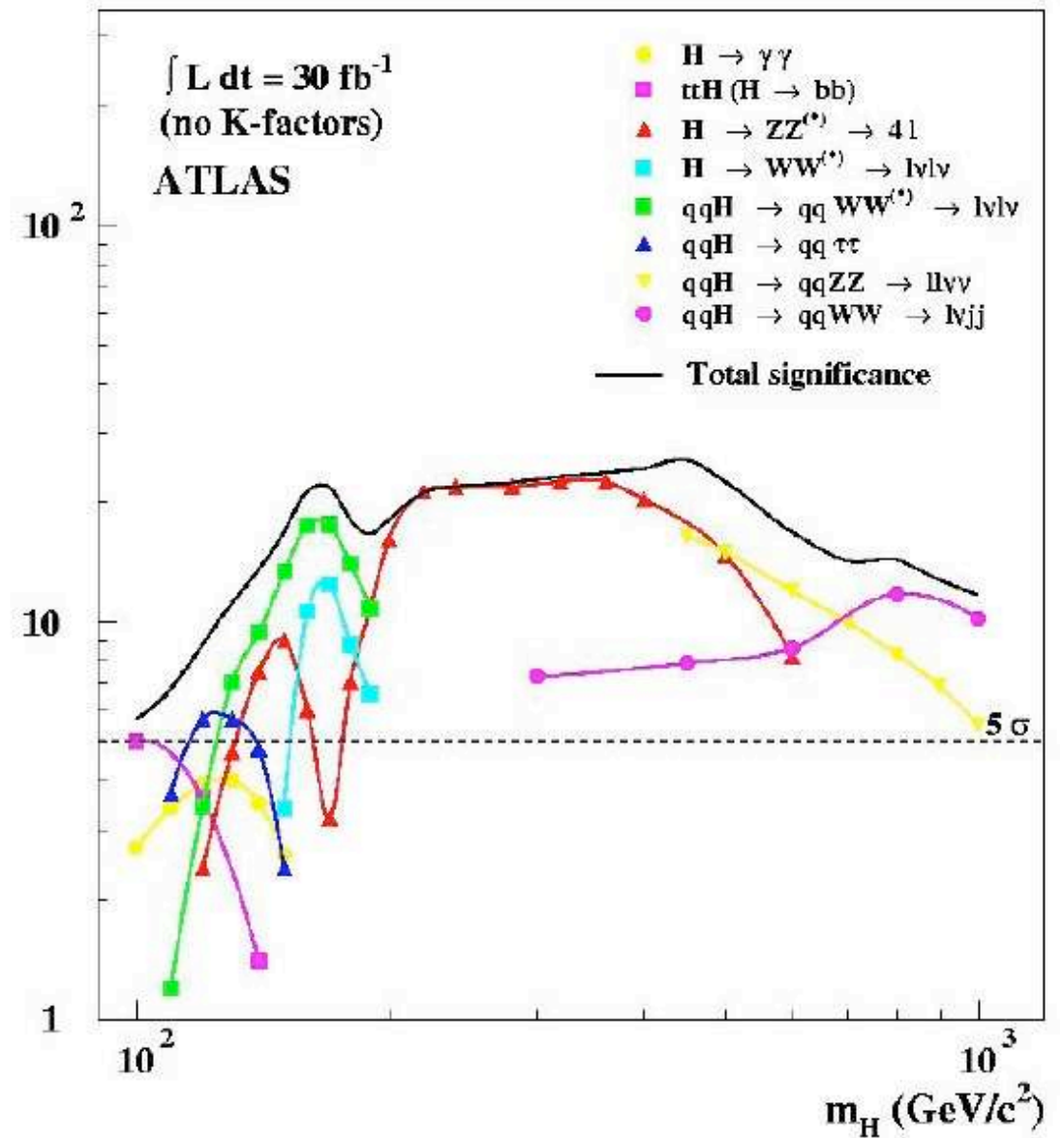
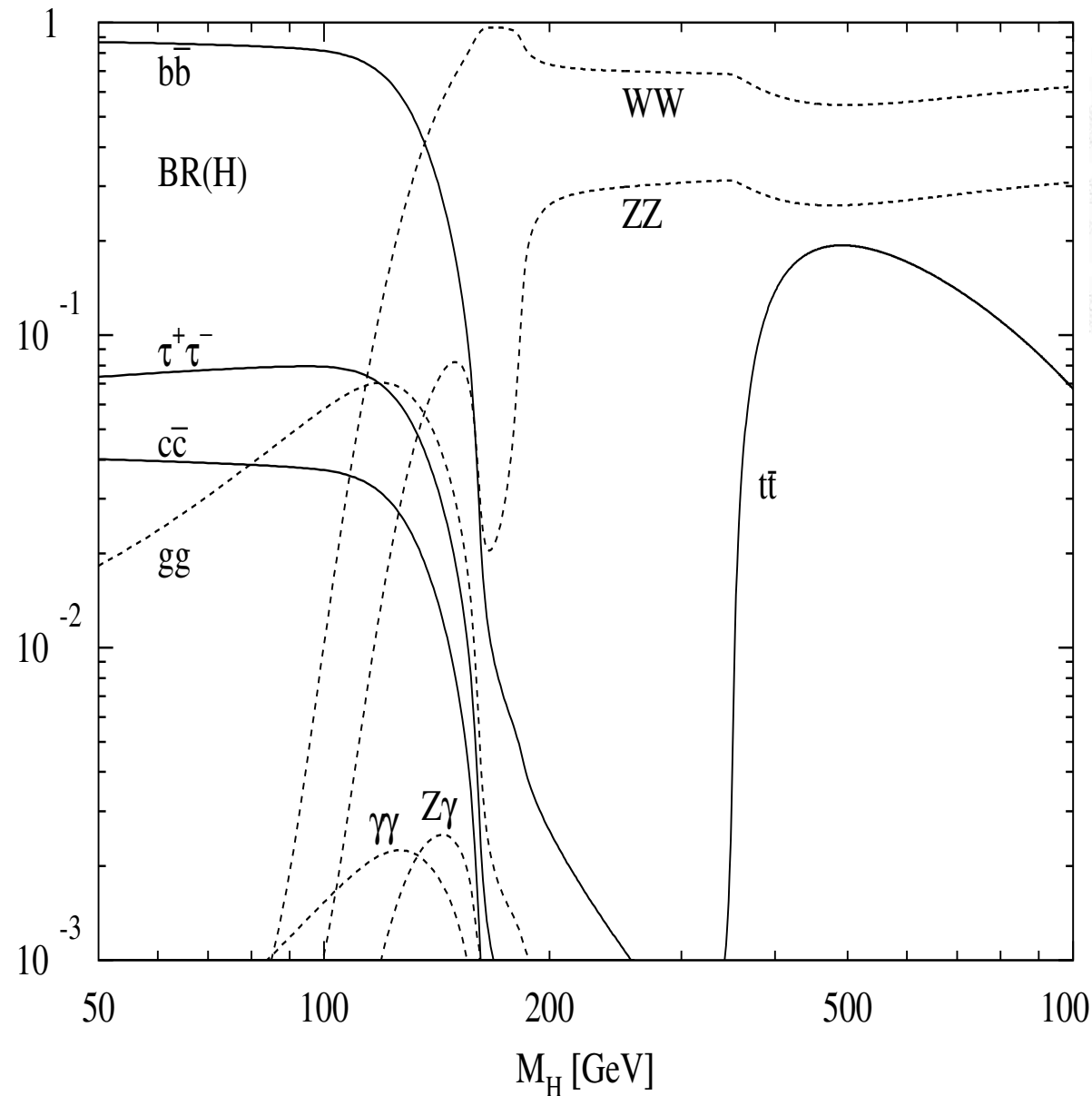


CMS PRL 105 (2010)



ATLAS New J. Phys 13 (2011)

Z/W+ H ($\rightarrow bb$) rescued ?



\Rightarrow **Light Higgs hard:** Higgs mainly produced in association with Z/W, decay $H \rightarrow bb$ is dominant, but overwhelmed by QCD backgrounds

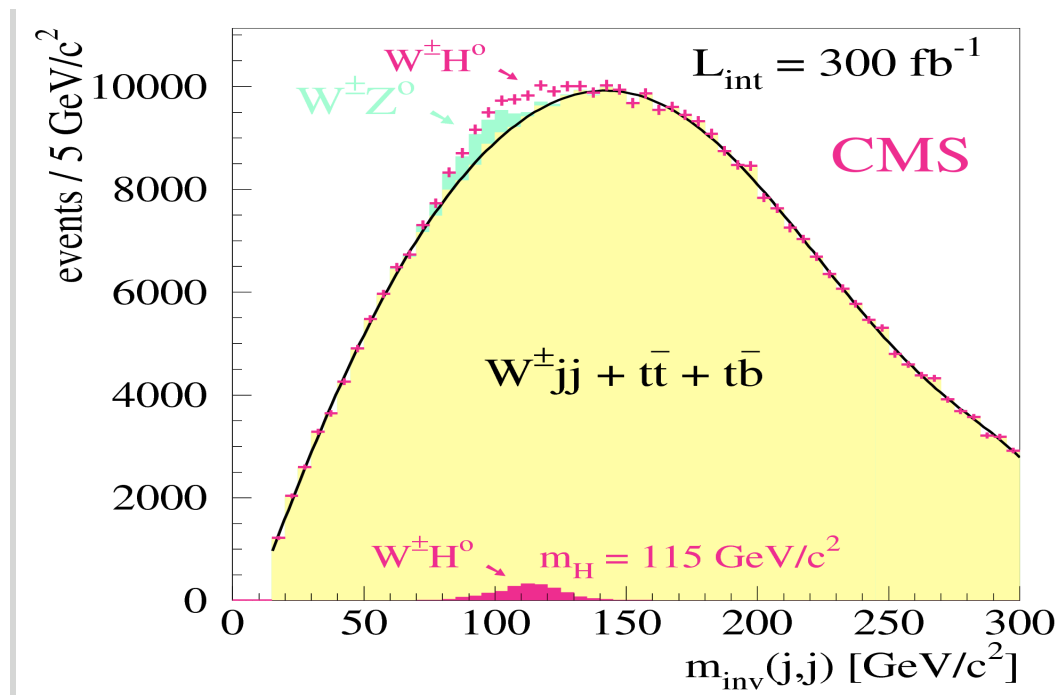
Z/W+ H ($\rightarrow bb$) rescued ?

Recall why searching for $pp \rightarrow WH(bb)$ is hard:

$$\sigma(pp \rightarrow WH(bb)) \sim \text{few pb} \quad \sigma(pp \rightarrow Wbb) \sim \text{few pb}$$

$$\sigma(pp \rightarrow tt) \sim 800\text{pb} \quad \sigma(pp \rightarrow Wjj) \sim \text{few } 10^4\text{pb} \quad \sigma(pp \rightarrow bb) \sim 400\text{pb}$$

\Rightarrow signal extraction very difficult



Conclusion [ATLAS TDR]:

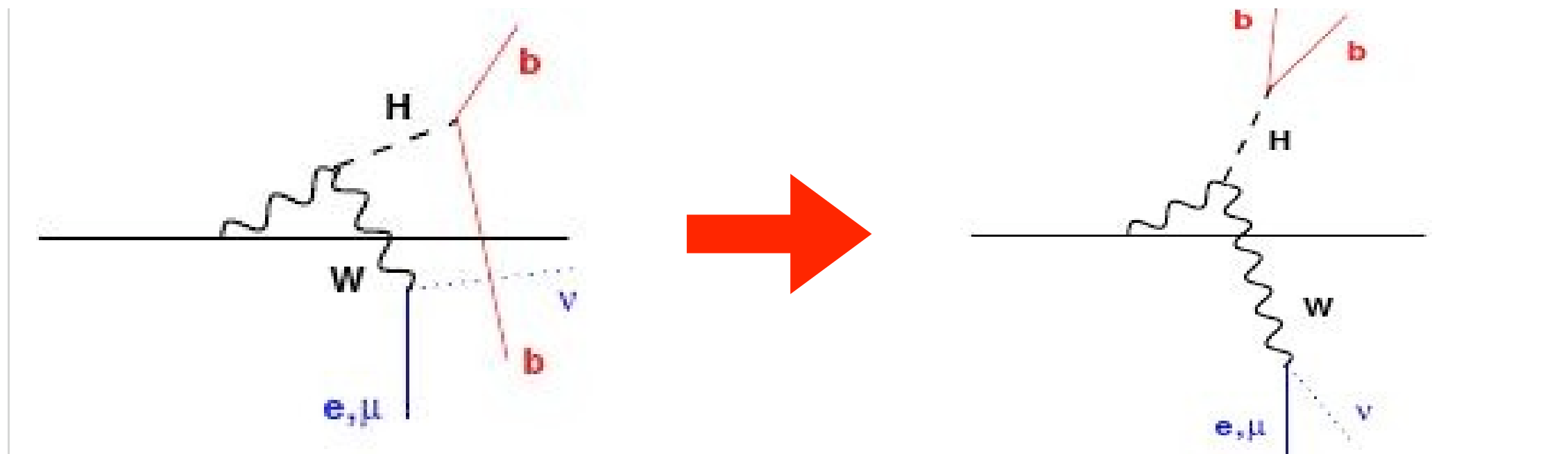
The extraction of a signal from $H \rightarrow bb$ decays in the WH channel will be very difficult at the LHC even under the most optimistic assumptions [...]

Z/W+ H (\rightarrow bb) rescued ?

But ingenious suggestions open up to window of opportunity

Central idea: require high- p_T W and Higgs boson in the event

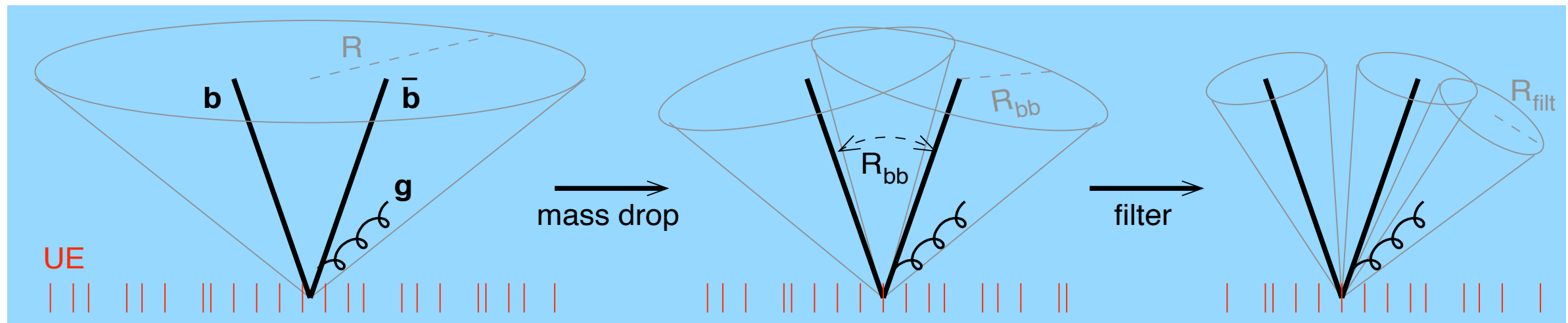
- leads to back-to-back events where two b-quarks are contained within the same jet
- high p_T reduces the signal but reduces the background much more
- improve acceptance and kinematic resolution



Z/W+ H ($\rightarrow bb$) rescued ?

Then use a jet-algorithm geared to exploit the specific pattern of $H \rightarrow bb$ vs $g \rightarrow gg, q \rightarrow gg$

- QCD partons prefer soft emissions (hard \rightarrow hard + soft)
- Higgs decay prefers symmetric splitting
- try to beat down contamination from underlying event
- try to capture most of the perturbative QCD radiation



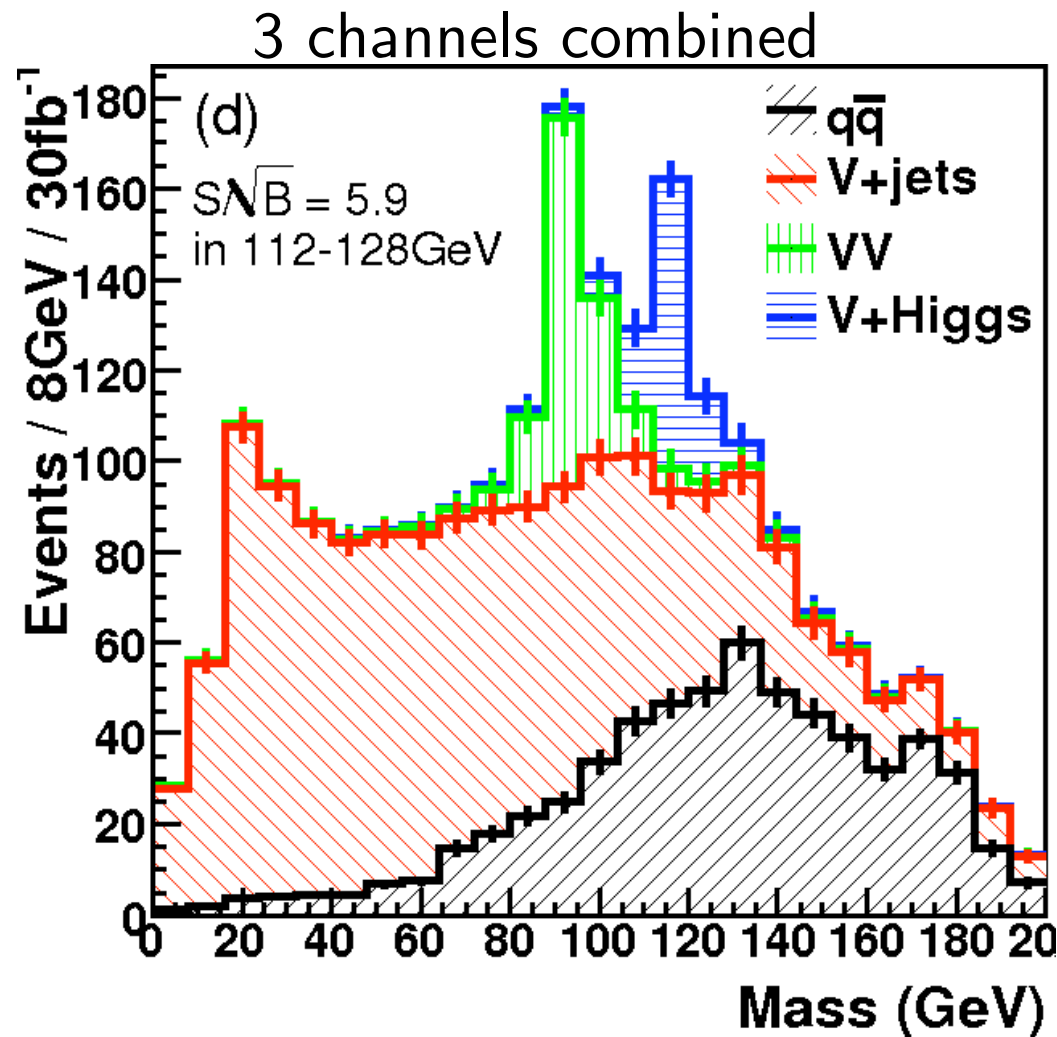
1. **cluster** the event with e.g. CA algo and large-ish R

2. undo last recomb: **large mass drop** + symmetric + b tags

3. **filter** away the UE: take only the 3 hardest sub-jets

Z/W+ H (\rightarrow bb) rescued ?

Mass of the three hardest sub-jets:



- ▶ with common & channel specific cuts:
 $p_{tV}, p_{tH} > 200\text{GeV}$, ...
- ▶ real/fake b-tag rate: 0.7/0.01
- ▶ NB: very neat peak for WZ (Z \rightarrow bb)
Important for calibration

Butterworth, Davison, Rubin, Salam '08

5.9 σ at 30 fb⁻¹: VH with H \rightarrow bb recovered as one of the best discovery channels for light Higgs ?

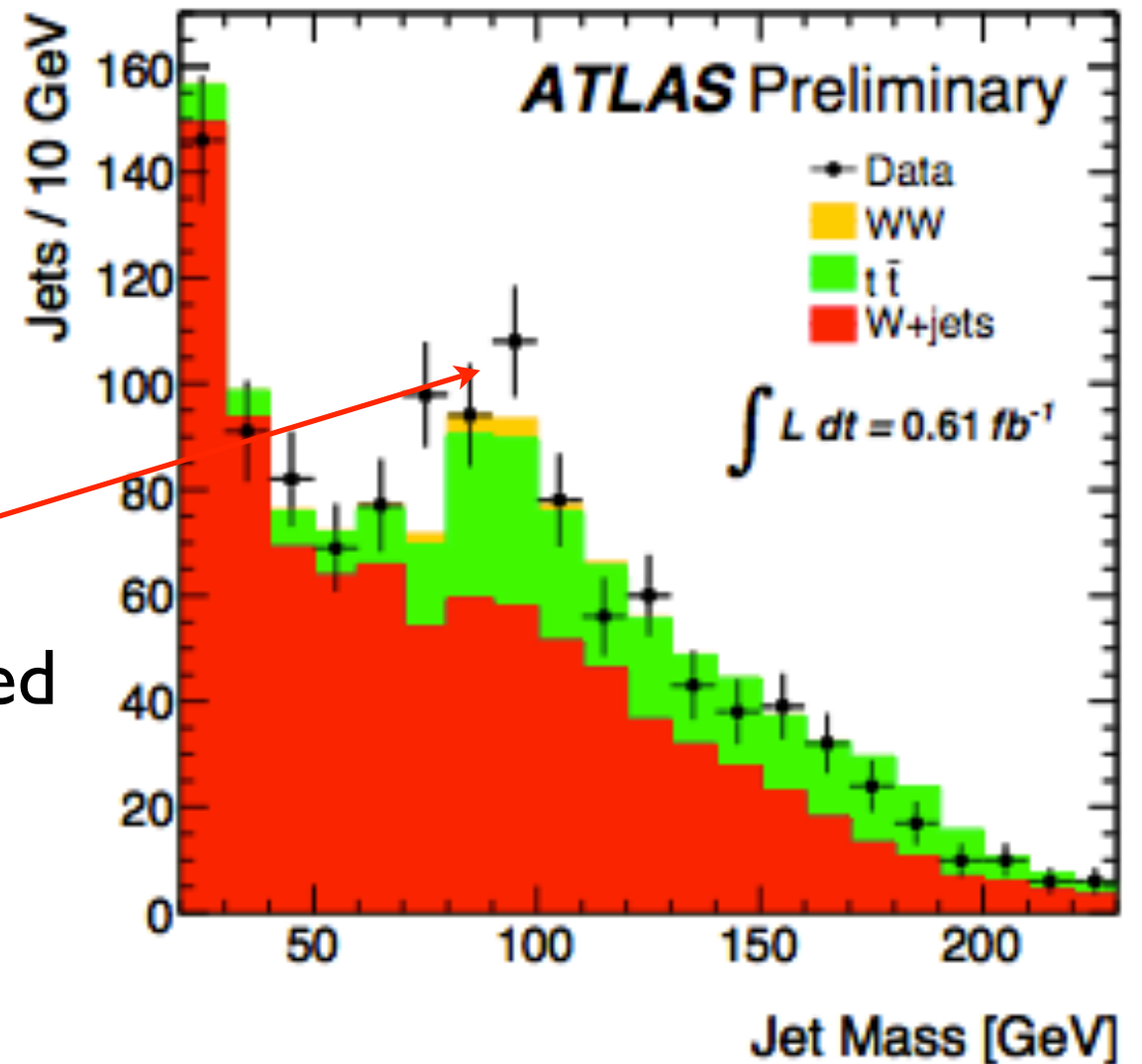
Z/W+ H (\rightarrow bb) rescued ?

These very recent techniques already in use at the LHC!

Presented at EPS 2011

Example relevant for WH(\rightarrow bb):
single jet hadronic mass in W+lj

Z peak evident. **Very promising**
Expect many new results with boosted
techniques at higher statistics soon



Recap on jets

- **Two major jet classes:** sequential (k_t , CA, ...) and cones (UAI, midpoint, ...)
- Jet algorithm is fully specified by: **clustering + recombination + split merge or removal procedure + all parameters**
- **Standard cones based on seeds are IR unsafe**
- **SISCone is new IR safe** cone algorithm (no seeds) and **anti- k_t** a new sequential algorithm
- Using IR-unsafe jets you **can not use perturbative QCD calculations**
- With IR-safe jets: sophisticated studies e.g. **jet-area for pile-up subtraction**
- Not all algorithms fare the same for BSM/Higgs searches: **quality measures**
- Recent applications using **boosted techniques and jet substructure** (Higgs example)