Bootstrapping Black Holes

Henry Lin, Stanford University

February 5, 2025

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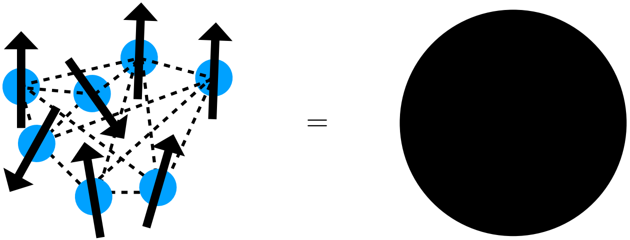

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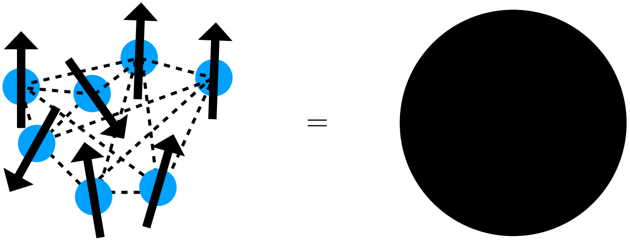

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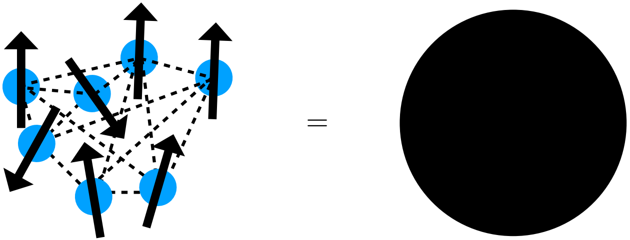

It would be nice to solve the quantum system. This means, e.g., computing correlators like $\frac{1}{Z}\,\mathrm{tr}\big(e^{-\beta H}\mathcal{O}\big)$ on the LHS.

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So why haven't we solved these black holes yet?

Why bootstrap?

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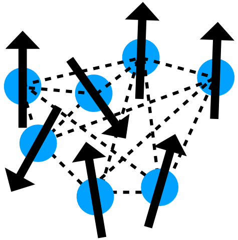
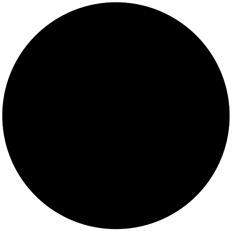
Review gauge/gravity duality for maximally supersymmetric Yang Mills (SYM) theory in $p + 1$ dimensions, for $p < 3$.
[Itzhaki, Maldacena, Sonneschein, Yankielowicz '00]

$\text{SYM}_{p+1} =$  $=$ 

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In slightly different regimes, terminology:

- $p = 0$: BFSS
- $p = -1$: IKKT
- $p = 1$: matrix string theory

- Review of non-conformal holography/black holes
- Matrix bootstrap for simple models
- Matrix bootstrap for BFSS, future directions

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$$\boxed{\text{Decoupling limit: } E^2(\alpha') \to 0, \quad E^{p-3}\, g^2_{\mathrm{YM}} = \text{fixed}}$$

Note that for $p < 3$ this implies that $g_s \to 0$.

Black brane solution

In the 't Hooft limit, with $\tilde{\lambda} \gg 1$, Type II supergravity.

$$I = \frac{1}{(2\pi)^7(\alpha')^4} \int \mathrm{d}^{10}x\sqrt{g}\left[e^{-2\phi}(R + 4(\nabla\phi)^2) - \frac{1}{2(p+2)!}F_{p+2}^2\right]$$

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This has the famous black brane solution:

$$\begin{aligned}
\mathrm{d}s^2 &= f^{-1/2}[h(r)\mathrm{d}t^2 + \mathrm{d}x_p^2] + f^{1/2}\left[h^{-1}(r)\mathrm{d}r^2 + r^2\mathrm{d}\Omega_{8-p}^2\right], \\
e^{-2\phi} &= g_\mathrm{s}^{-2} f^{(p-3)/2}, \quad A_{0\cdots p} = f^{-1}, \\
f &= 1 + \frac{d_0 g_\mathrm{YM}^2 N}{(\alpha')^2 (r/\alpha')^{7-p}}, \quad d_p = 2^{7-2p}\pi^{\frac{9-3p}{2}}\Gamma\left(\tfrac{1}{2}(7-p)\right), \\
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In the decoupling limit, delete 1.

This solution is somewhat similar to the AdS black brane $\times S_{8-p}$

$$\frac{\mathrm{d}s^2}{\alpha'} = \left(\frac{z}{R_{\mathrm{AdS}}}\right)^{\frac{3-p}{5-p}} \left[R_{\mathrm{AdS}}^2 \left(\frac{h(z)\,\mathrm{d}\tau^2 + h^{-1}(z)\mathrm{d}z^2 + \mathrm{d}x_p^2}{z^2}\right) + \mathrm{d}\Omega_{8-p}^2\right],$$

$$h = 1 - \frac{z^d}{z_0^d}, \quad d = 1 + \frac{9-p}{5-p}, \quad R_{\mathrm{AdS}} = \frac{2}{5-p},$$

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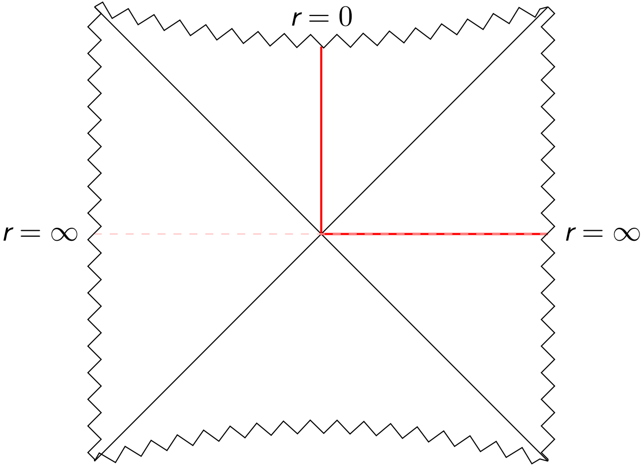
Exercise: work out the change of coordinates and check that finite τ, z is consistent with the decoupling limit. Work out z_0 as a function of temperature.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Some features for $p = 0$:

$$\frac{\mathrm{d}s^2}{\alpha'} = \left(\frac{z}{R_{\mathrm{AdS}}}\right)^{\frac{3}{5}} \left[R_{\mathrm{AdS}}^2 \left(\frac{h(z)\,\mathrm{d}\tau^2 + h^{-1}(z)\mathrm{d}z^2}{z^2}\right) + \mathrm{d}\Omega_8^2\right],$$

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- Sphere shrinks near boundary $z = 0$. When $z \sim 1$ curvature scale is of order $\sim \ell_s$.
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- Exercise: compute the proper distance from the boundary to the horizon. Use this to estimate the thermal 2-pt function of a massive stringy mode $e^{-m\ell}$ and its β dependence.

Extrapolation to strong coupling

- $p = 0$ view D0s as gravitons in 11d $\Rightarrow$ boosted Schwarzschild black hole (homogeneous in the 11th dimension) $\Rightarrow$ BFSS conjecture
- $p = 2$, view the D2 branes as M2 branes, $\mathrm{AdS}_4 \times S_7$ ABJM
- $p = 1$, S-duality relates D1 solution to F1s $\Rightarrow$ matrix string ($(R^8)^N/S_N$ CFT)
- $p = -1$ ...?

Relation to AdS

Fluctuations of the dilaton $\phi = \phi_{\text{sol}} + \chi$

$$I \propto \int \mathrm{d}^{10}x\sqrt{g}e^{-2\phi_{\text{sol}}}(\nabla\chi)^2$$
$$= \int \mathrm{d}^{8-p}\Omega\, \mathrm{d}^{d-1}\vec{x}\,\mathrm{d}z\,\mathrm{d}\tau\, \sqrt{g_{\text{AdS}}}\left[(\nabla_{\text{AdS}}\chi)^2 + m_k^2\chi^2\right]$$

Using $m_k = k(k+7-p) \Rightarrow$ fields in AdS_{d+1}:

$$\langle\mathcal{O}_\phi(x)\mathcal{O}_\phi(0)\rangle \sim \frac{1}{|x|^{2(\Delta-d-p-1)}}, \quad \Delta = R_{\text{AdS}}(k+2)+2.$$

This applies to SUGRA modes.

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$$I_{DBI} \sim \int \mathrm{d}^{p+1}x\, e^{-\phi(x,X)} F^2(x) + \cdots$$

Boundary operators schematically of the form

$$S_{\mathsf{SYM}} \to S_{\mathsf{SYM}} + \mathcal{N} \sum_j \frac{1}{k!} \int \mathrm{d}^{p+1}x\, \partial_{I_1} \cdots \partial_{I_k} \phi \, \mathrm{Tr}\Big(F^2_{\mu\nu} X^{(I_1} \cdots X^{I_k)} \Big).$$

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This is a super-descendant of the $1/2$ BPS operator:

$$\mathrm{Tr}\Big(F^2_{\mu\nu} X^{(I_1} \cdots X^{I_k)} \Big) \sim QQQQ\, \mathrm{Tr}\, X^{(I_1} \cdots X^{I_{k+2})}$$

We learn that the dimensions of the $\frac{1}{2}$ BPS operator:

$$\Delta = R_{\mathrm{AdS}}(k+2) + 2 \Rightarrow \Delta_{\frac{1}{2}\mathsf{BPS}} = R_{\mathrm{AdS}} k$$

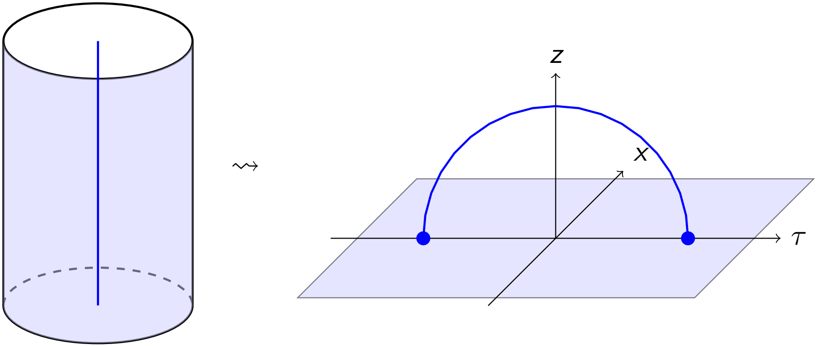
Giant gravitons

As a fun aside [WIP w/ Gauri Batra], we can also reproduce the relation

$$\Delta_{\frac{1}{2}\text{BPS}} = R_{\text{AdS}} k$$

For values of k that are very large $k \sim N$ by considering a classical solution where a giant graviton $D(6-p)$ brane that couples to magnetically-dual RR flux.

Giant gravitons



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$$S = N^2 \left(c_0 (T/\lambda)^{9/5} + c_1 (T/\lambda)^{18/5} + \cdots \right) + N^0 b_0 T^{-3/5}$$

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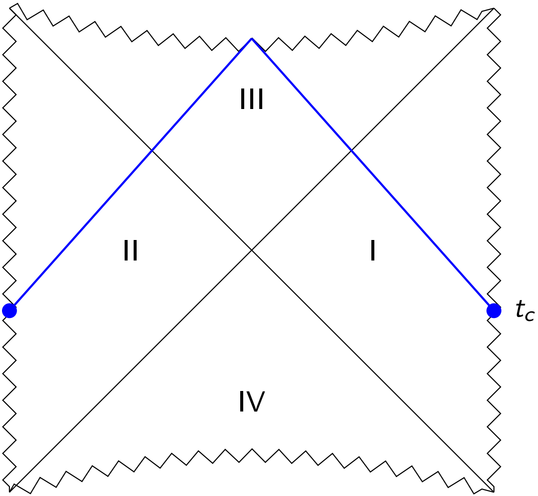
5. exercise: debug the following wrong argument. In the decoupling limit $E^2 \alpha' \to 0$, so we should be able to neglect all α' corrections to SUGRA.


III
II
I
t_c
IV

End of the gravity review
$\Rightarrow$ Boundary methods for analyzing low dimensional SYM

Refs for bootstrap:
[Anderson & Kruczenski, 1612.08140],
[HL, 2002.08387], [Kazakov & Zheng, 2108.04830]
[Han, Harnoll, Kruthoff, 2004.10212]
[Fawzi, Fawzi, Scalet, 2311.18706] [Cho, Gabai, Sandor, Yin, 2410.04262]
[HL, 2302.04416] [HL & Zheng, 2410.14647]

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- metastability: some problems ill-defined at finite N

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  Number of correlators of fixed length grows $\sim D^L$.

$$\langle \mathrm{Tr}\, ABABBBA \rangle \,, \langle \mathrm{Tr}\, AAABBBB \rangle \,, \cdots$$

1-Matrix model

Probability distribution over $N \times N$ Hermitian matrix M_{ij}:

$$p(M) = \frac{1}{\mathcal{Z}} e^{-N^2 \operatorname{tr} V(M)}, \quad V(M) = \frac{1}{2}M^2 + \frac{g}{4}M^4$$

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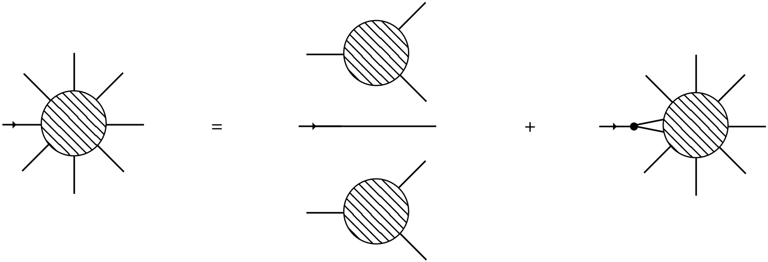
Goal: compute moments $\langle \operatorname{tr} M^k \rangle$ as a function of g.

$$\langle \operatorname{tr} M^2 \rangle = \lim_{N \to \infty} \frac{1}{\mathcal{Z}} \int \mathrm{d}M \, e^{-N^2 \operatorname{tr} V(M)} \operatorname{tr} M^2$$

Bootstrapping matrices

1. Guess the value of some simple correlator, e.g. $\left\langle \operatorname{tr} M^2 \right\rangle$
2. Feed it through the loop eqns to generate more correlators
3. Demand that $\left\langle \operatorname{tr} \mathcal{O}^\dagger \mathcal{O} \right\rangle \geq 0$.

Loop (Schwinger-Dyson) equations



- relates lower-pt correlators to higher-pt correlators
- uses large N factorization ('t Hooft)

$$\langle \mathrm{tr}\, M^k \rangle = \sum_{\ell=0}^{k-1} \langle \mathrm{tr}\, M^\ell \rangle \langle \mathrm{tr}\, M^{k-\ell-2} \rangle + g \langle \mathrm{tr}\, M^{k+2} \rangle$$

Positivity

Naive algorithm: starting with some guess for $\left\langle \mathrm{tr}\, M^2 \right\rangle$, generate moments $\left\langle \mathrm{tr}\, M^4 \right\rangle, \left\langle \mathrm{tr}\, M^6 \right\rangle, \left\langle \mathrm{tr}\, M^8 \right\rangle, \cdots$.

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Positivity

More systematically, we can consider a general polynomial in the matrix M:

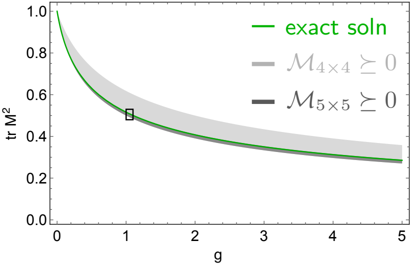
$$\mathcal{O} = \sum \alpha_k M^k \Rightarrow \operatorname{tr} \mathcal{O}^\dagger \mathcal{O} \geq 0.$$

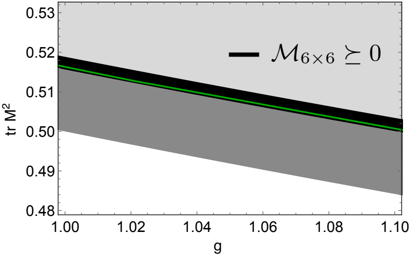
This implies that $\alpha_i^* \mathcal{M}_{ij} \alpha_j \geq 0$ for all coefficients α, where we have assembled all the correlators into a big matrix $\mathcal{M}_{ij} = \left\langle \operatorname{tr} M^{i+j} \right\rangle$:

$$\mathcal{M} = \begin{pmatrix} 1 & \langle \operatorname{tr} M \rangle & \left\langle \operatorname{tr} M^2 \right\rangle \\ \langle \operatorname{tr} M \rangle & \left\langle \operatorname{tr} M^2 \right\rangle & \left\langle \operatorname{tr} M^3 \right\rangle \\ \left\langle \operatorname{tr} M^2 \right\rangle & \left\langle \operatorname{tr} M^3 \right\rangle & \left\langle \operatorname{tr} M^4 \right\rangle \end{pmatrix} \succeq 0$$

Here $\mathcal{M}_{ij} = \left\langle \operatorname{tr} M^{i+j} \right\rangle$.

Review of the matrix bootstrap

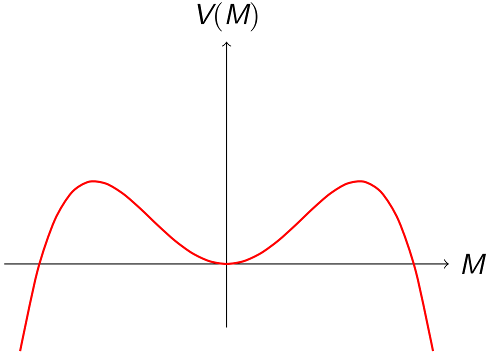




As the size of $\mathcal{M}$ increases, rapid convergence to the exact solution.

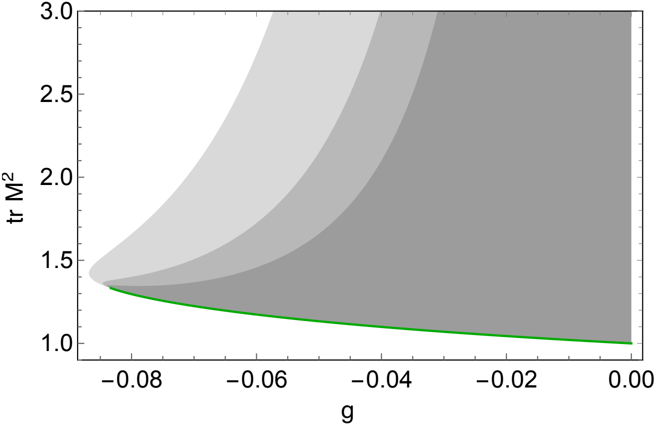
Metastability

To address the issue of metastability, consider $g < 0$. The potential is unbounded from below:



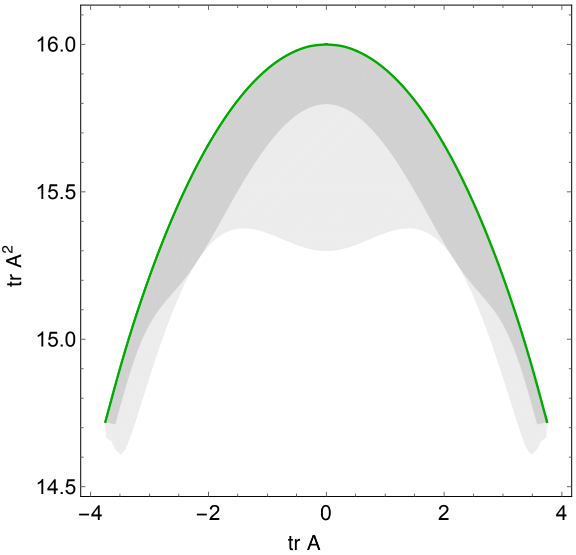
In the large N limit, tunneling is suppressed.

Metastability



For $-g_* < g < 0$ the model still makes sense at $N = \infty$

Metastability $V = -\frac{1}{2}A^2 + \frac{1}{4}gA^4$



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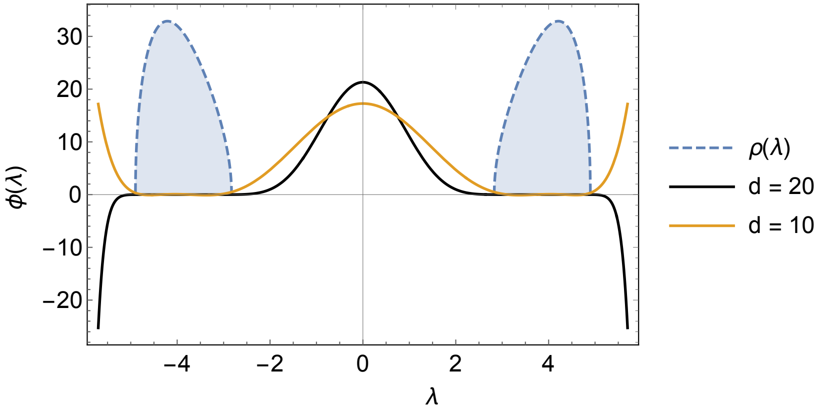

	$d = 0$ (stat mech)	$d = 1$
solvable	1-matrix integral	1-matrix quantum mech $c = 1$ matrix model
unsolvable	**multi-matrix integral**	D0-brane quantum mech BFSS matrix theory

Multi-matrix integrals

Main challenge: exponentially many correlators for a given length L, e.g., for $L = 7$:

$$\langle \mathrm{Tr}\, ABABBBA \rangle, \langle \mathrm{Tr}\, BBBABAB \rangle, \cdots$$

Also more loop equations and more positivity constraints:

$$\mathcal{M} = \begin{pmatrix} 1 & \mathrm{Tr}\, A & \mathrm{Tr}\, B & \cdots \\ \mathrm{Tr}\, A & \mathrm{Tr}\, A^2 & \mathrm{Tr}\, AB & \\ \mathrm{Tr}\, B & \mathrm{Tr}\, BA & \mathrm{Tr}\, B^2 & \\ \vdots & & & \ddots \end{pmatrix}$$

Despite these challenges, the bootstrap gives strong results for multi-matrix integrals [HL '20].

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$$\mathcal{Z} = \int \mathrm{d}A \, \mathrm{d}B \, e^{-N^2 \operatorname{tr} V(A,B)}$$

$$V(X, Y) = -\frac{1}{2}[A, B]^2 + v(A) + v(B),$$

$$v(X) = \frac{1}{2}X^2 + \frac{1}{4}X^4$$

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Using non-linear relaxation, one can convert it to a standard semi-definite programming problem [Kazakov & Zheng '22].

$$0.4217836 \leq \left\langle \operatorname{tr} A^2 \right\rangle \leq 0.4217847$$

$$0.3333413 \leq \left\langle \operatorname{tr} A^4 \right\rangle \leq 0.3333421$$

~ 6 decimal digits on a laptop!

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1-matrix QM

N^2 non-relativistic particles arranged in a matrix.

$$\mathrm{i}[X_{ij}, P_{kl}] = \delta_{il}\delta_{jk}.$$

Hamiltonian:

$$H = N\left(\frac{1}{2}\,\mathrm{Tr}\,P^2 + \frac{m^2}{2}\,\mathrm{Tr}\,X^2 + \frac{g}{4}\,\mathrm{Tr}\,X^4\right).$$

U(N) gauge constraint:

$$J_{ik} = \mathrm{i}(X_{ij}P_{jk} - P_{ij}X_{jk}) + N\delta_{ik} = 0$$

[for a review, see Klebanov hep-th/9108019] [Brezin, Itzykson, Parisi, Zuber, Douglas, Klebanov, Kutasov, Maldacena, Martinec, Takayangi, Toumbas, Verlinde, $\cdots$]

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known as $c = 1$ or $\hat{c} = 1$ matrix model[1].

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[1] in the double scaling limit

Review of the quantum mechanical bootstrap

1. Replace loop eqns with $O' = [O, H]$. In energy eigenstates $\langle E| O' |E\rangle = \langle E|O|E\rangle E - E \langle E|O|E\rangle = 0$.

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   example: $0 = \langle [\operatorname{tr} XP, H] \rangle = -\operatorname{tr} P^2 + \operatorname{tr} X^2 + g \operatorname{tr} X^4$

2. Positivity of measure replaced w/ Hilbert space positivity (fermions ☺)

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2. Positivity of measure replaced w/ Hilbert space positivity (fermions ☺)
$$\langle E|\, \mathrm{tr}\, \mathcal{O}^\dagger \mathcal{O}\, |E\rangle \geq 0 \Rightarrow \mathcal{M}_{ij} = \langle E|\, \mathrm{tr}\, \mathcal{O}_i^\dagger \mathcal{O}_j\, |E\rangle \geq 0$$
3. Optional: ground state bootstrap positivity:
$$\langle O^\dagger [H, O]\rangle_{\text{gs}} = \langle O^\dagger H O\rangle_{\text{gs}} - E_{\text{gs}} \langle O^\dagger O\rangle_{\text{gs}} \geq 0$$

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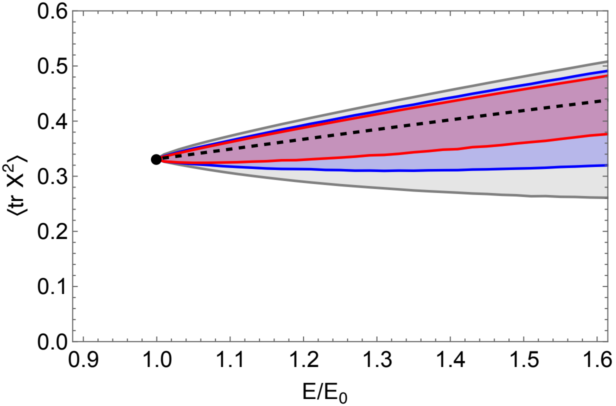
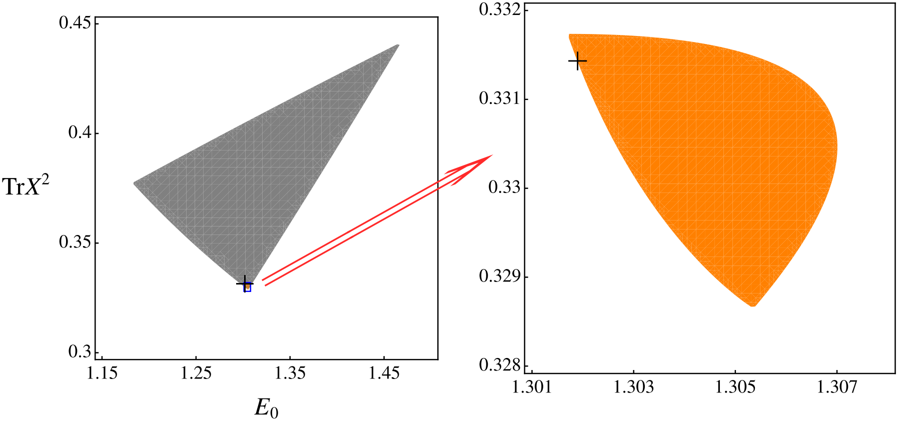
Finite energy bootstrap



Dashed line is the exact solution for $g = 1$.

[WIP w/ Zechuan Zheng; see also Han, Hartnoll, Kruthoff '20]

Ground state bootstrap



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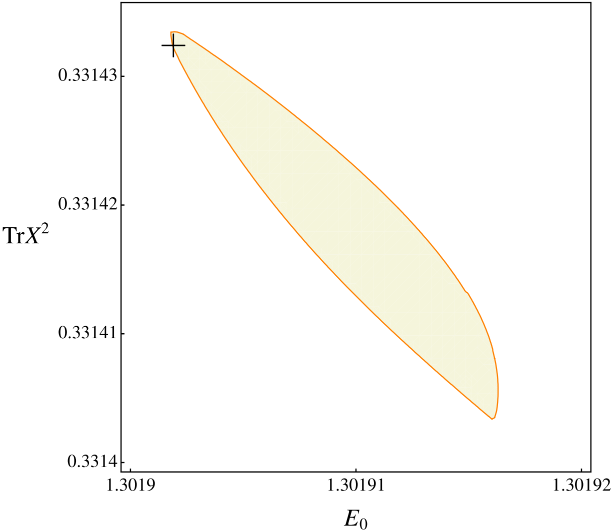

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~ 6 digit precision on a laptop.

D0-brane quantum mechanics

9 bosonic matrices and 16 fermionic matrices. Transform as a vector and spinor of $SO(9)$.

$$H = \frac{1}{2}\mathrm{Tr}\left(P_I^2 - \frac{1}{2}\left[X_I, X_J\right]^2 - \psi_\alpha \gamma^I_{\alpha\beta}\left[X_I, \psi_\beta\right]\right)$$

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$\mathcal{N} = 16$ SUSY:

$$\begin{aligned} Q_\alpha &= \mathrm{Tr}\, P_I \gamma^I_{\alpha\beta}\psi_\beta - \frac{\mathrm{i}}{2}\,\mathrm{Tr}\left[X^I, X^J\right]\gamma^{IJ}_{\alpha\beta}\psi_\beta, \\ \{Q_\alpha, Q_\beta\} &= 2\delta_{\alpha\beta} H + 2\gamma^I_{\alpha\beta}\,\mathrm{Tr}\, X^I C \end{aligned}$$

$C_{ij} =$ generator of SU(N).

Quantum mechanical bootstrap

I. Constraints

II. Positivity

- **Dynamics**:

$$\langle\Omega|\left\{Q_\alpha, O_\alpha\right\}|\Omega\rangle = 0.$$

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example:

$O_\alpha \propto \gamma^{IJ}_{\alpha\beta} \operatorname{tr} \psi_\beta X^I P^J \Rightarrow -2\mathrm{i} \left\langle \operatorname{tr}\left[X^I, X^J\right] X^I P^J \right\rangle = \left\langle \operatorname{tr} \psi_\alpha \psi_\alpha X^I X^I \right\rangle.$

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- **Hierarchy:**

  $\langle \mathrm{tr}\, W \rangle$ where W a word made of X, ψ, P.
  level: $\ell(X) = 1, \ell(\psi) = \frac{3}{2}, \ell(P) = 2$.

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  - example:

$$\left\langle \mathrm{tr}\, X^{I_1} X^{I_2} X^{I_3} P^{I_4} X^{I_5} X^{I_6} \right\rangle = \left\langle \mathrm{tr}\, X^{I_2} X^{I_3} P^{I_4} X^{I_5} X^{I_6} X^{I_1} \right\rangle + \mathrm{i} \left\langle \mathrm{tr}\, X^{I_2} X^{I_3} \right\rangle \left\langle \mathrm{tr}\, X^{I_5} X^{I_6} \right\rangle \delta^{I_1 I_4}$$

SO(9) group theory

Only SO(9)-invariant operators have non-zero vevs.

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Trivial example: $\langle \mathrm{tr}(X^J X^J) \rangle \geq 0$. Derived by observing that it is the sum of squares of X^J (a non-invariant operator).

Kinematic constraints

Less trivial example:

$$\mathcal{M}_{\alpha\beta\gamma\eta} = \left\langle \mathrm{tr}\left(\psi^{\alpha}\psi^{\beta}\psi^{\delta}\psi^{\eta} \right) \right\rangle$$

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Viewed as a matrix in the $\{\alpha, \beta\}$ and $\{\gamma, \eta\}$ indices, positivity requires $\mathcal{M} \succeq 0$.

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Using the "addition of SO(9) angular momentum" rules:

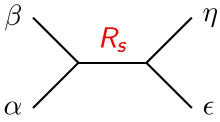
$$(\mathbf{16})^4 = (\mathbf{16}\times\mathbf{16})^2 = (\mathbf{1}+\mathbf{9}+\mathbf{36}+\mathbf{84}+\mathbf{128})^2$$
$$= 5(\mathbf{1}) + \text{non-singlets}$$

Thus group theory determines this $16^4 = 65536$ to just 5 unknowns.

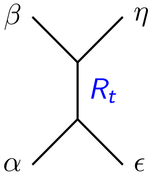
$$\mathcal{M}_{\alpha\beta\gamma\eta} = \delta_{\alpha\beta}\delta_{\eta\epsilon}a_{\mathbf{1}} + \gamma^{I}_{\alpha\beta}\gamma^{I}_{\eta\epsilon}a_{\mathbf{9}} + \gamma^{IJ}_{\alpha\beta}\gamma^{IJ}_{\eta\epsilon}a_{\mathbf{36}} + \gamma^{IJK}_{\alpha\beta}\gamma^{IJK}_{\eta\epsilon}a_{\mathbf{84}}$$
$$+ \gamma^{IJKL}_{\alpha\beta}\gamma^{IJKL}_{\eta\epsilon}a_{\mathbf{128}}$$

Cyclicity and the fermion anti-commutation relations cuts this further to just 2 unknowns.

Expand *s*-channel block in terms of *t*-channel blocks:



$$= \sum_{R_t} \mathbb{F}_{R_s,R_t}\begin{bmatrix} \mathbf{16} & \mathbf{16} \\ \mathbf{16} & \mathbf{16} \end{bmatrix}$$



$\Rightarrow$ 6j symbol. At higher levels, need higher-pt crossing kernels.

Kinematics determined $\mathcal{M}_{\alpha\beta\gamma\eta}$ in terms of 2 unknowns. We still need to impose positivity of a large matrix $\mathcal{M}_{\alpha\beta,\gamma\eta}$. By decomposing $\psi_\alpha\psi_\beta$ into irreps, one can easily diagonalize $\mathcal{M}$.

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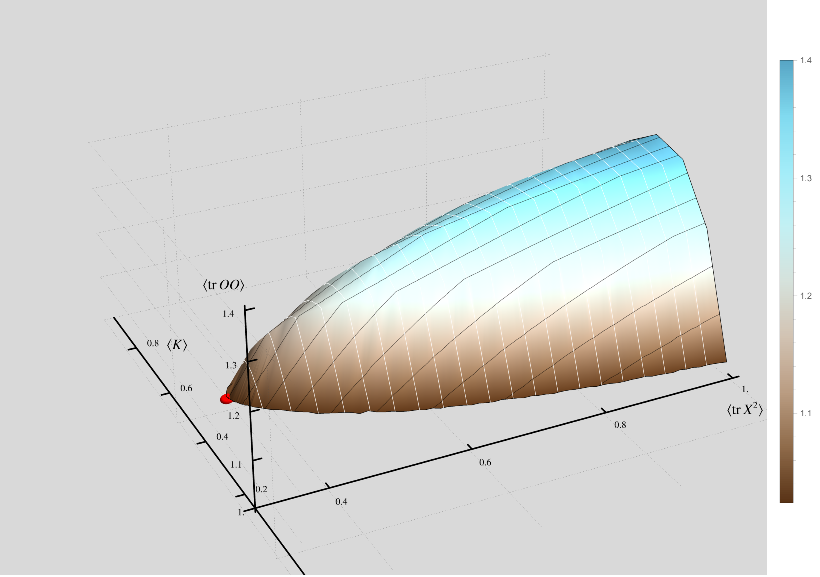
The upshot is that by leveraging the symmetries of the model, the D0-brane bootstrap is practical. ☺

level	total variables	free variables
4	11	3
5	38	4
6	140	11
7	569	18
8	2528	59
9	12077	149

# of single trace SO(9) singlets, before and after modding out by the EOM/kinematic constraints.


⟨tr OO⟩
⟨K⟩
⟨tr X^2⟩
1.4
1.3
1.2
1.1
1.
0.8
0.6
0.4
0.2
0.4
0.6
0.8
1.
1.4
1.3
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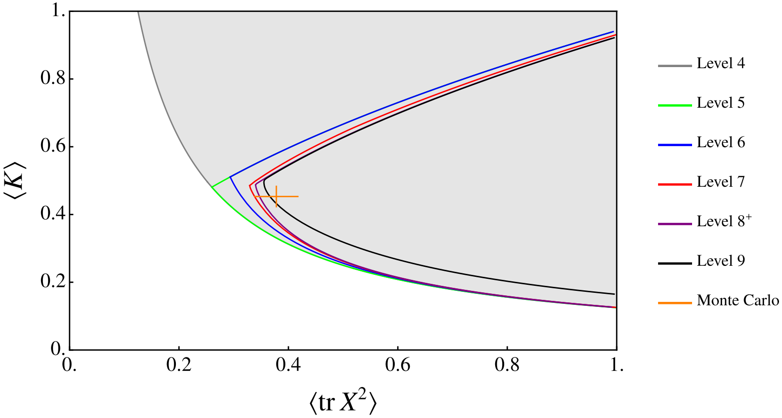
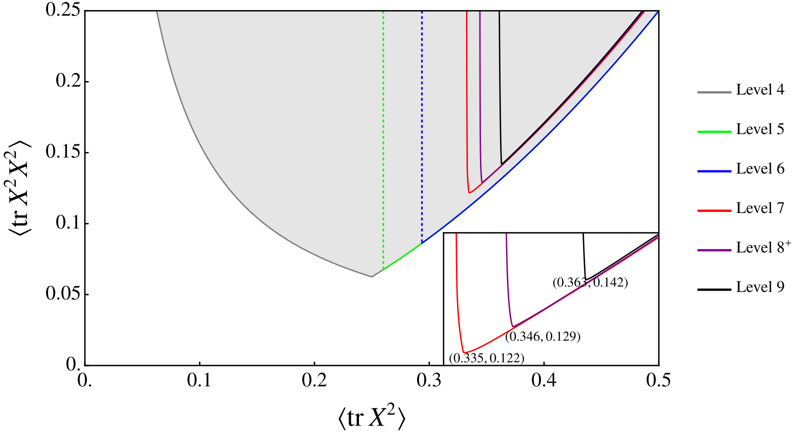
1.1



Cross + is the Monte Carlo result* of [Berkowitz *et al.*'16].



The lower bound on $\langle \mathrm{tr}\, X^2 X^2 \rangle$ was derived (up to some factors) in [Polchinski '99]. It can also be improved to finite energy [HL '23].

	method	$\langle \mathrm{tr}\, X^2 \rangle$
	Monte Carlo [Pateloudis *et al.*'22]	$\approx 0.37 \pm 0.05$
	primitive bootstrap [HL '23]	≥ 0.1875
	bootstrap level 6	≥ 0.294
	bootstrap level 7	≥ 0.331
	bootstrap level 8^+	≥ 0.3401
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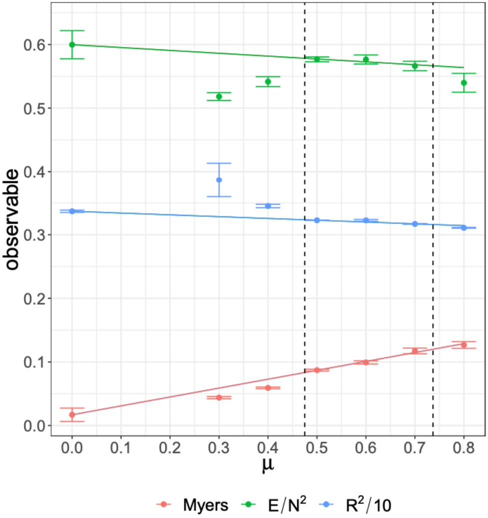
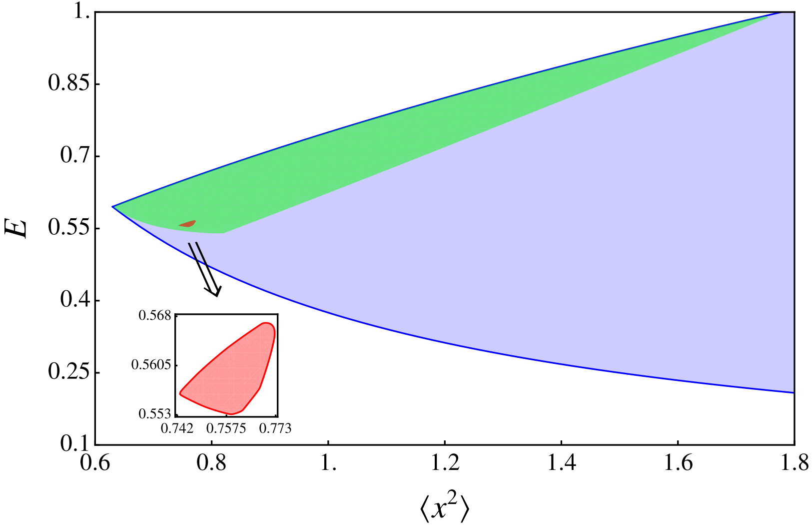
$\sim 90\%$ of the MC value with just level 7:
19 variable SDP, ~ 170 EoMs, matrices of size $\lesssim 20 \times 20$.

Metastability in Monte Carlo



Monte Carlo results [Pateloudis *et al.* '22]

Toy supermembrane problem



In a simpler toy problem, we see a similar-looking peninsula at low levels, but an island at higher levels.

I have presented some evidence that the bootstrap could yield precision data on some correlators like $\langle \mathrm{tr}\, X^2 \rangle$.

A high precision measurement of these and/or related correlators could give the *leading* corrections to the semiclassical black hole background. [A nice candidate is $\langle O_{\mathsf{BPS}} \rangle \sim T^{\Delta+\delta}$.]

In principle, we could use this to constrain unknown $O(\alpha'^3)$ corrections to the IIa effective action. [See Hanada, Berkowitz, Pateloudis, ... for similar discussions involving BH thermodynamics. Similar in spirit to the CFT bootstrap program by e.g. Binder, Chester, Pufu, Wang, ...]

Finite temperature generalization

Somewhat surprising reformulation of KMS condition:

$$\langle [ ] \rangle \tag{1}$$

Future directions

- Islands?
- Constraints on the bound state?
- Finite energy/temperature
- Large N lattice systems, especially those with sign problems?
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Thanks!

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In the strongly coupled regime $\lambda\beta^3 \gg 1$, dual to a metastable black hole in Type IIA [Klebanov & Tsetlyin '96, Itzhaki, Maldacena, Sonneschein, Yankielowicz '20]:

$$\frac{\mathrm{d}s^2}{\alpha'} = -f(r) r_c^2 \, \mathrm{d}t^2 + \frac{\mathrm{d}r^2}{f(r) r_c^2} + \left(\frac{r}{r_c}\right)^{-3/2} \mathrm{d}\Omega_8^2$$

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S_8 *shrinks* with r. At $r \sim \lambda^{1/3} \Rightarrow$ string scale curvature.

Suppose that one day we have high precision measurements of 1-pt functions like $\langle \mathrm{tr}\, X^n \rangle$. What can we learn?

The semiclassical BH geometry and its stringy corrections

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The semiclassical BH geometry and its stringy corrections

In principle, this includes properties that are currently inaccessible by worldsheet methods.

[See Hanada *et al.*, Berkowitz *et al.*, Pateloudis, *et al.*for similar discussions involving the BH thermodynamics. In principle using the Fawzi, Fawzi, & Scalet one can bootstrap the thermodynamics.]

In principle, a theory of quantum gravity should predict the higher-derivative corrections to Einstein gravity, e.g.,

$$\mathcal{L} \sim R + \#\alpha'^3 R^4 + \#\alpha'^3 R^3 F^2 + \cdots .$$

For charged black holes (with Ramond-Ramond gauge fields), the *leading* correction is unknown.

A precision measurement of certain correlators will give us information about these corrections. Similar program in the CFT bootstrap; e.g., [Binder, Chester, Pufu, Wang '19]

A clear target is the only SO(9) singlet field in this background χ.

χ has scaling dimension $\Delta = 28/5$ [Sekino & Yoneya '00, Biggs & Maldacena '23]. The leading α'^3 correction breaks the scaling symmetry and gives rise to a non-trivial 1-pt function:

$$S_{\text{eff}} \supset \frac{(\alpha')^3}{G_N} \int \sqrt{g} e^{-2\phi} \chi \left(\#_1 R^4 + \#_2 e^{2\phi} R^3 F^2 + \cdots + \right)$$

$$\langle \mathcal{O}_\chi \rangle \propto T^{\Delta+\delta} = T^{28/5}$$

On the matrix side, the operator $\mathcal{O}_\chi$ is a known level 8 operator [Van Raamsdonk and Taylor '98] :

$$\mathcal{O}_\chi \sim \mathrm{Tr}\, P^I P^I P^J P^J + \mathrm{Tr}[X_I, X_J][X_J, X_K] P^K P^I + \cdots + \text{fermions}$$

χ is also expected to contribute to a generic SO(9) singlet due to operator mixing, e.g.,

$$\left\langle \mathrm{tr}\, X^2 \right\rangle \sim \#_1 + \#_H T^{14/5} + \#_{H'} T^{23/5} + \#_\chi T^{28/5} + \cdots$$

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1. solvable matrix models can also be solved by bootstrap
2. for "unsolvable" models like BFSS, bootstrap yields non-trivial bounds. Old results from the matrix side [Polchinski '99] can be reformulated and improved as a bootstrap result.
3. In principle, we could learn about stringy black holes using the bootstrap. We are in the process of putting this into practice.

Future directions I.

Bootstrapping the thermal entropy, e.g.,

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Recent progress [Fawzi, Fawzi & Scalet '23] in inputting the KMS condition into the bootstrap (in the Hamiltonian approach). Uses a non-linear relaxation of the relative entropy.

Can be applied to large N matrix quantum mechanics [Cho, Sandor, & Yin, WIP]

Future directions II.

	$d=0$	$d=1$	$d \geq 2$
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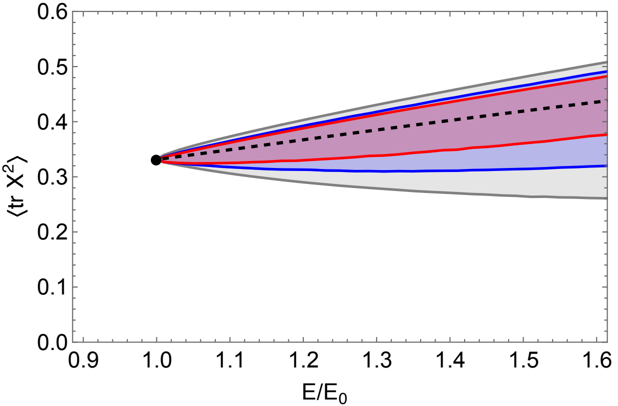
	multi-matrix integral	D0-brane BFSS	large N Yang Mills large N QCD

Already some interesting progress...

[Anderson & Kruczenski '16] [Kazakov & Zheng '22] [Kazakov & Zheng, '24]

Many other strongly-coupled lattice systems seem possible...

Finite energy bootstrap



Dashed line is the exact solution for $g = 1$.

[WIP w/ Zechuan Zheng; see also Han, Hartnoll, Kruthoff '20]

(note: we are considering high energies $E \sim N^2$ even though for the 1-matrix model there are only N eigenvalues).