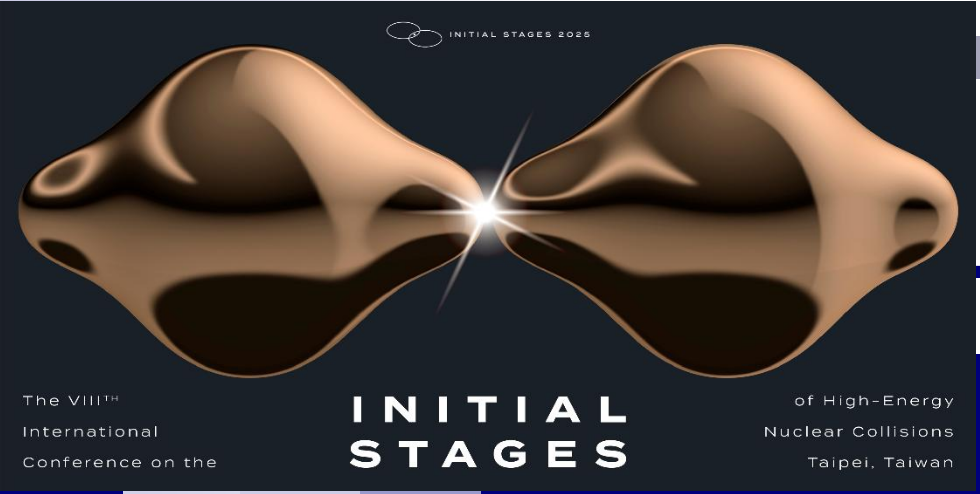

Initial-State-Driven Spin and Polarization Effects

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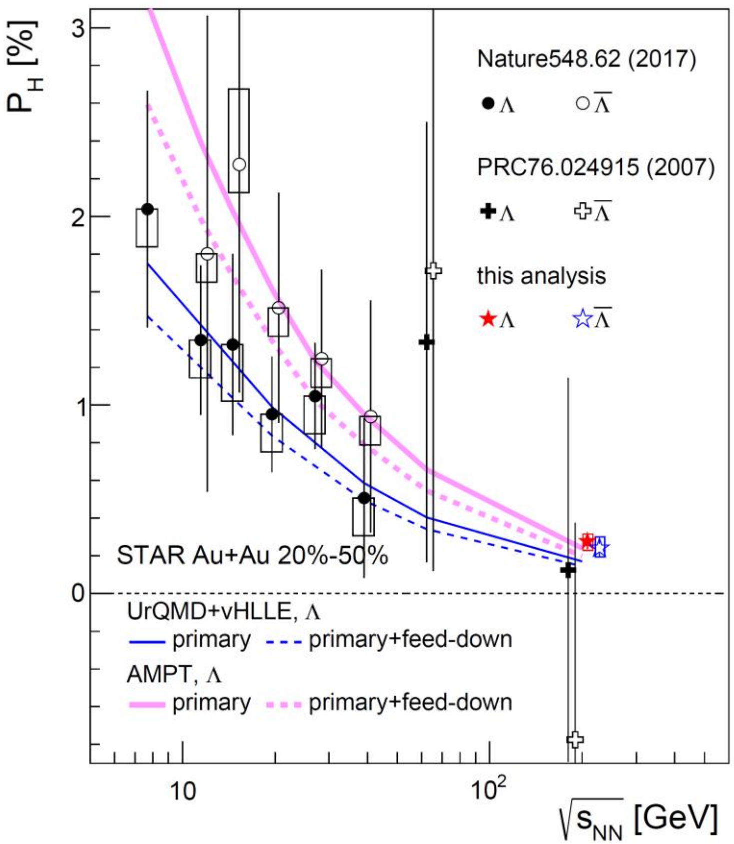
(The 8th International Conference on the Initial Stages in High-Energy Nuclear Collisions, Sep. 9th, 2025)

NSTC 國家科學及技術委員會
National Science and Technology Council

Global Λ polarization in HIC

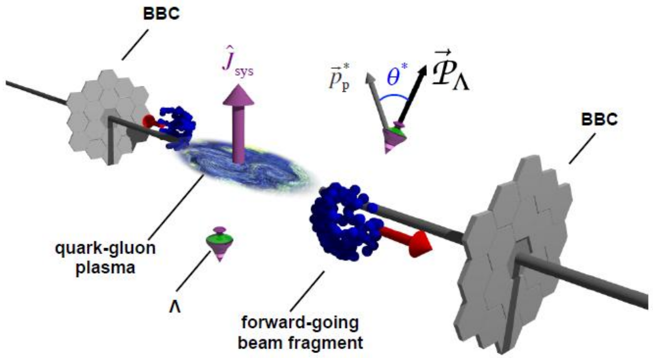
- The measurement of global polarization of Λ hyperons revealed the spin-orbit interaction & strong vorticity in heavy ion collisions. Z.-T. Liang & X.-N. Wang, PRL. 94, 102301 (2005)

(relativistic Barnett effect)



L. Adamczyk et al. (STAR), Nature 548, 62 (2017)

❖ Self-analyzing via the weak decay : $\Lambda \to p + \pi^-$



❖ Successfully described by the modified Cooper–Frye formula in global equilibrium : F. Becattini, et al., Ann. Phys. 338, 32 (2013) R. Fang, et al., PRC 94, 024904 (2016)

$$\mathcal{P}^\mu = \frac{\int d\Sigma \cdot p f_p^{(0)} (1 - f_p^{(0)}) \epsilon^{\mu\nu\rho\sigma} q_\nu \omega_{\rho\sigma}}{8 M_\Lambda \int d\Sigma \cdot p f_p^{(0)}},$$

$$\omega_{\rho\sigma} = \frac{1}{2}\left(\partial_\rho \left(\frac{u_\sigma}{T}\right) - \partial_\sigma \left(\frac{u_\rho}{T}\right)\right).$$ thermal vorticity

❖ Global pol. from (average) kinetic vorticity :

$$P_{\Lambda(\bar{\Lambda})} \approx \int_p \mathcal{P}^{-y}(p) \simeq \frac{\omega}{2T} \Longrightarrow \omega = \frac{1}{2}|\nabla \times u| \sim 10^{22} s^{-1}$$

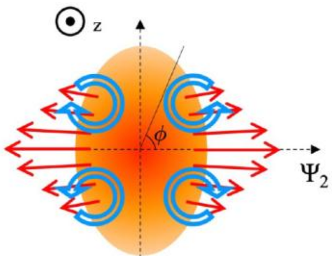
F. Becattini et al., PRC95, 054902 (2017)

Longitudinal (local) polarization

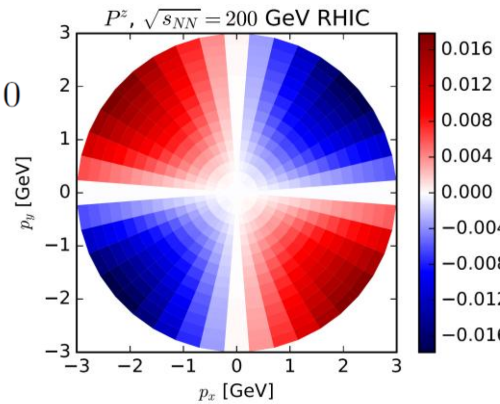
- Longitudinal polarization along the beam direction in HIC :

F. Becattini, I. Karpenko, PRL 120, 012302 (2018)

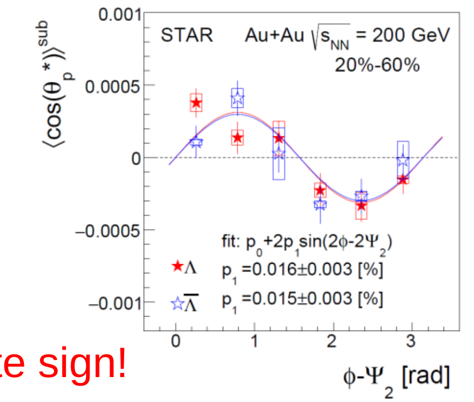
J. Adam et al. (STAR), PRL. 123, 132301 (2019)



$P_{2,z} < 0$



V.S.



$P_{2,z} > 0$

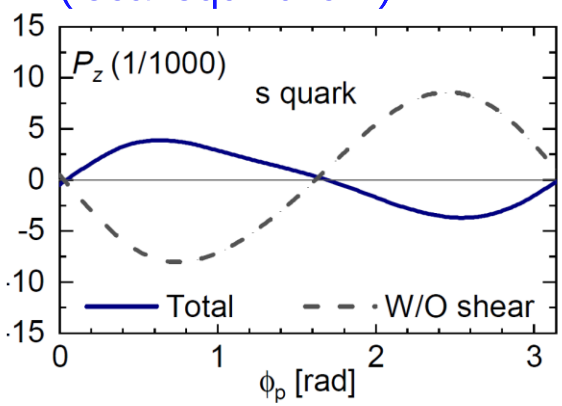
$$P_{2,z} = \langle P_z \sin 2(\phi - \Psi_2) \rangle$$
$$\approx \int_0^{2\pi} d(\phi-\Psi_2) \frac{P_z \sin 2(\phi-\Psi_2)}{2\pi}$$

opposite sign!

- Thermal-shear corrections : $\mathcal{P}^\mu = \dfrac{\int d\Sigma\cdot p f_p^{(0)}(1-f_p^{(0)})\epsilon^{\mu\nu\rho\sigma}q_\nu\left[\omega_{\rho\sigma} - u_\rho\pi_{\sigma\lambda}p^\lambda/(Tp\cdot u)\right]}{8M_\Lambda\int d\Sigma\cdot p f_p^{(0)}}$

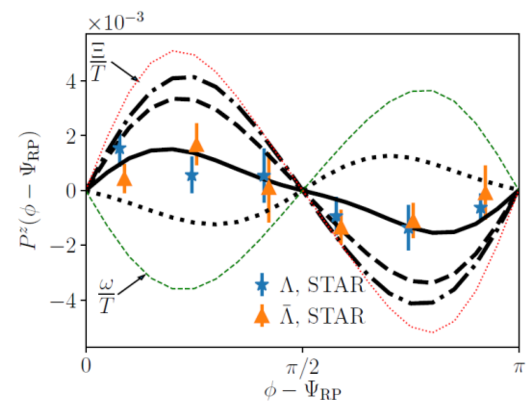
(local equilibrium)

Y. Hidaka, S. Pu, DY, PRD 97, 016004 (2018)
S. Liu, Y. Yin, JHEP 07, 188 (2021)
F. Becattini, et al., PLB 820,136519 (2021)



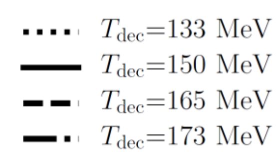
strange-memory scenario : $M_\Lambda \to m_s$

B. Fu et al., PRL 127, 142301 (2021)



isothermal approx. : no T gradient correction

F. Becattini et al., PRL. 127, 272302 (2021)



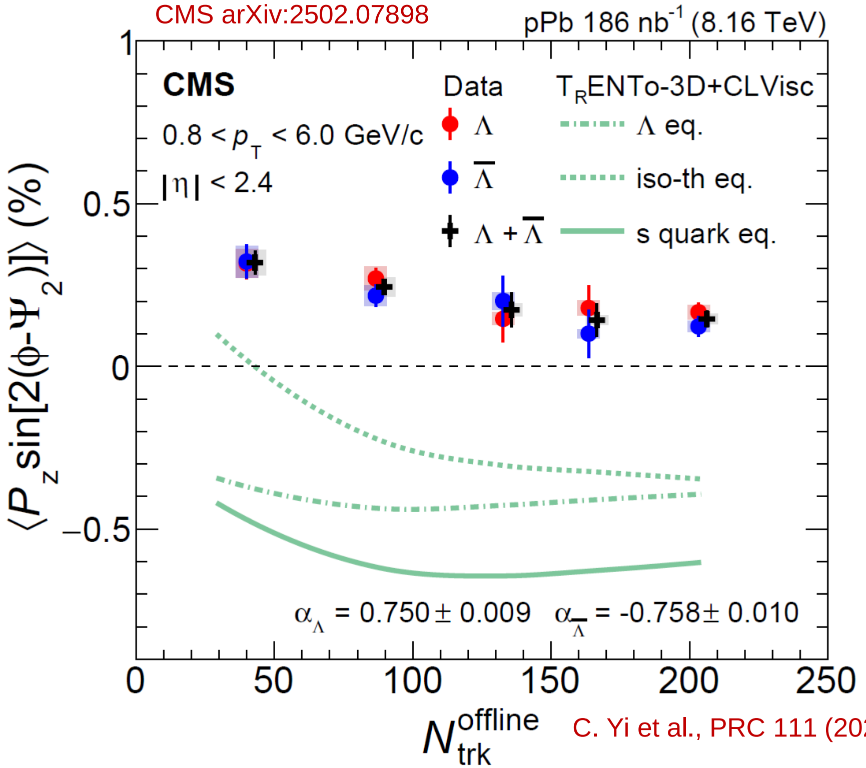
- The overall sign depends on adopted approximations.

see also C. Yi et al., PRC 104. 064901 (2021), W. Florkowski et al., PRC 105, 064901(2022)

Small-collision systems

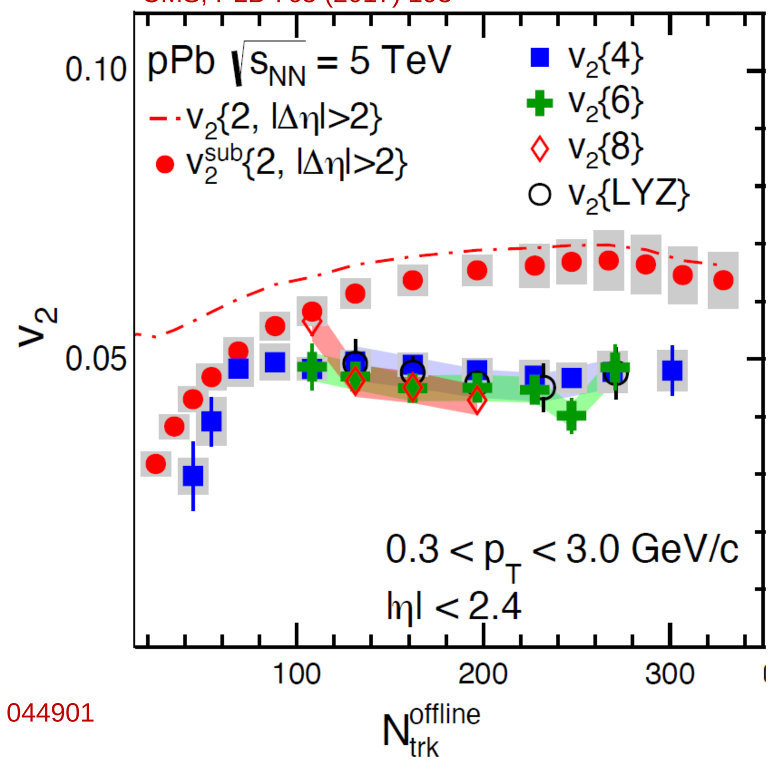
- Longitudinal polarization in high-multiplicity pA collisions :

CMS arXiv:2502.07898



C. Yi et al., PRC 111 (2025) 044901

CMS, PLB 765 (2017) 193



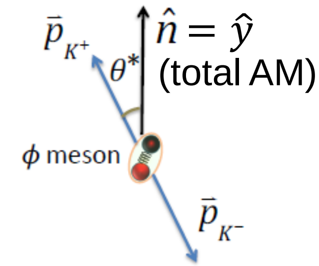
- Thermal vorticity + thermal shear as the final-state effect fails to describe the measurements
- An opposite trend w.r.t the flow : initial-state or non-equilibrium effects?

Spin alignment of vector mesons

- Angular dep. of the decay particle w.r.t the spin quantization axis :

$$\frac{dN}{d\cos\theta^*} \propto [1-\rho_{00}+\cos^2\theta^*(3\rho_{00}-1)]$$

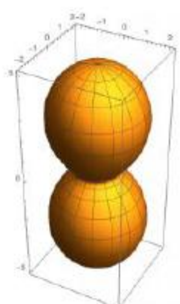
$$\rho_{00} = \frac{1-\langle \mathcal{P}_q^y \mathcal{P}_{\bar{q}}^y \rangle}{3+\langle \mathcal{P}_q^y \mathcal{P}_{\bar{q}}^y \rangle}$$



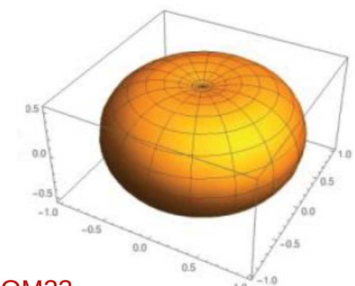
Z.-T. Liang and X.-N. Wang, PLB 629, 20 (2005)

$\rho_{00} \neq 1/3$: spin correlation

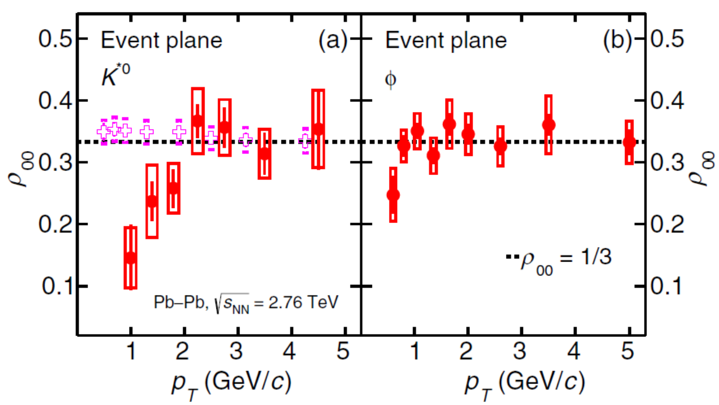
$\rho_{00} > 1/3$:



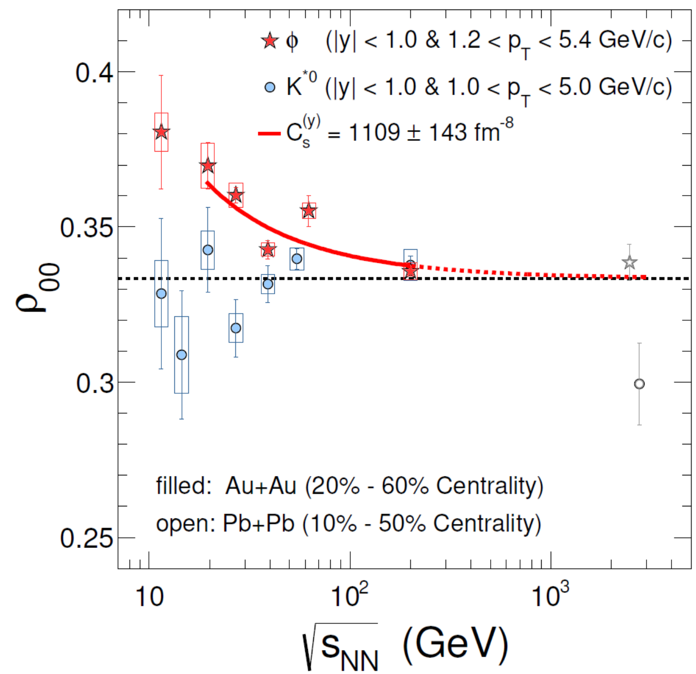
$\rho_{00} < 1/3$:



B. Xi, QM23



S. Acharya et al. (ALICE), PRL.125, 012301 (2020)



M.S. Abdallah et al. (STAR), Nature 614 (2023) 7947, 244-248,

Spin polarization beyond subatomic swirls?

- Spin alignment puzzle : the deviation of ρ_{00} from 1/3 is unexpectedly large

  e.g. $\rho_{00} \approx \frac{1}{3} - \left(\frac{\omega}{T}\right)^2, \frac{\omega}{T} \sim 0.1\%$ at LHC energy. (from Λ pol.~ s quark pol.)

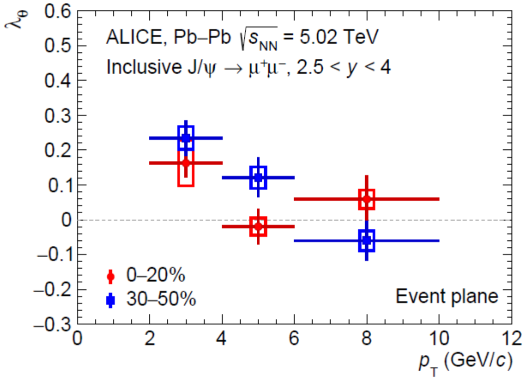
- Flavor & collision energy dep. :

	ϕ	K^{*0}
ALICE	$\rho_{00} < 1/3$ ($p_T \leq 1$ GeV)	$\rho_{00} < 1/3$
STAR	$\rho_{00} > 1/3$	$\rho_{00} \approx 1/3$

- Spin alignment for J/ψ :

S. Acharya et al. (ALICE), PRL 131,042303 (2023)

$$\lambda_\theta = \frac{1 - 3\rho_{00}}{1 + \rho_{00}} > 0 \Rightarrow \rho_{00} < \frac{1}{3}$$



- Other sources for the spin correlation (alignment) beyond vorticity?

- ❑ Small spin pol. & large spin correlation imply that the source may be fluctuating.
- ❑ Electromagnetic fields can polarize the spin. How about gluon fields in QCD matter? $\mathcal{P} \propto \boldsymbol{B} - \boldsymbol{u} \times \boldsymbol{E}$

magnetic pol. spin Hall effect with momentum anisotropy

An intuitive picture

- An intuitive construction of color-singlet spin observables :

❖ Building blocks for quark transport :

$$\boldsymbol{F}^a = g(\boldsymbol{E}^a + \boldsymbol{u} \times \boldsymbol{B}^a), \quad \boldsymbol{\mathcal{P}}^a = g(\boldsymbol{B}^a - \boldsymbol{u} \times \boldsymbol{E}^a).$$

chromo-Lorentz force   chromo-magnetic polarization & spin Hall effect

❖ Parity-even correlators only :

$$\langle E^i(x)E^j(x)\rangle = \delta^{ij}\langle E^i(x)E^i(x)\rangle, \quad \langle B^i(x)B^j(x)\rangle = \delta^{ij}\langle B^i(x)B^i(x)\rangle, \quad \langle E^i(x)B^j(x)\rangle = 0.$$

➢ spin alignment (without flow) : $\delta\rho_{00} = \rho_{00} - \frac{1}{3} \sim \langle \boldsymbol{\mathcal{P}}^a \cdot \boldsymbol{\mathcal{P}}^a \rangle \sim \langle \boldsymbol{B}^a \cdot \boldsymbol{B}^a \rangle$

p – even scalar

➢ spin polarization of Λ (strange-equilibrium scenario) :

$$\boldsymbol{\mathcal{P}} \sim \langle (\boldsymbol{p} \cdot \boldsymbol{F}^a)\boldsymbol{\mathcal{P}}^a \rangle$$

flow-induced polarization

$$\sim (\boldsymbol{p} \times \boldsymbol{u})(\langle \boldsymbol{B}^a \cdot \boldsymbol{B}^a \rangle + \langle \boldsymbol{E}^a \cdot \boldsymbol{E}^a \rangle)$$

p – even axial-vector

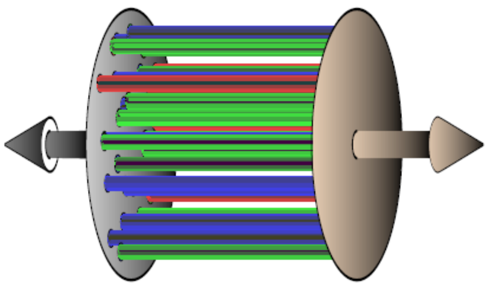
- Such effects as non-equilibrium spin transport can be systematically derived from the quantum kinetic theory & Wigner functions.

Review : Y. Hidaka S. Pu, Q, Wang, DY, PPNP 127, 103989 (2022)

DY, JHEP 06, 140 (2022)
B. Müller, DY, PRD 105, L011901 (2022)

Color-field induced spin alignment

- Initial-state gluons may form the glasma phase characterized by predominantly longitudinal chromo-electromagnetic fields from CGC.



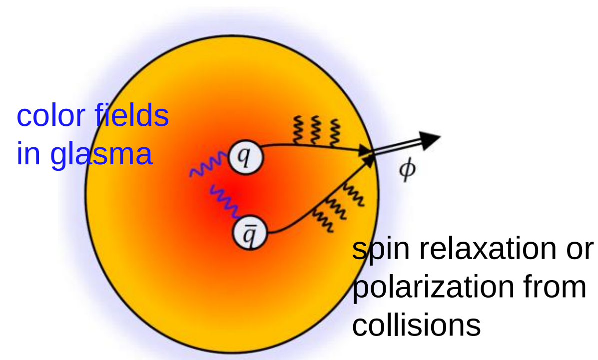
(at top RHIC & LHC energies)

Reviews : F. Gelis et al., Ann.Rev.Nucl.Part.Sci.60:463-489,2010
J. Berges et al., Rev. Mod. Phys. 93 (2021) 3, 035003

- Assuming the early quark production in glasma

N. Tanji, J. Berges, PRD, 97, 034013 (2018)

❖ Why glasma fields for spin alignment?
(1) intrinsic saturation scale $Q_s \gg \omega$
(2) fluctuating (no effect on global Λ pol.)
(3) intrinsic anisotropy (need not be along $\hat{n}$)



Updated coalescence model :

$$\rho_{00}(q) \approx \frac{1 - \mathrm{Tr}_c \langle \hat{\mathcal{P}}^y_q(\boldsymbol{q}/2) \hat{\mathcal{P}}^y_{\bar{q}}(\boldsymbol{q}/2)\rangle_{\boldsymbol{q}=0}}{3 - \sum_{i=x,y,z} \mathrm{Tr}_c \langle \hat{\mathcal{P}}^i_q(\boldsymbol{q}/2) \hat{\mathcal{P}}^i_{\bar{q}}(\boldsymbol{q}/2)\rangle_{\boldsymbol{q}=0}}$$

glasma effect :

$$\mathrm{Tr}_c \langle \hat{\mathcal{P}}^z_q(\boldsymbol{q}/2) \hat{\mathcal{P}}^z_{\bar{q}}(\boldsymbol{q}/2)\rangle_{\boldsymbol{q}=0} < 0$$

$$\delta\rho_{00} = \rho_{00} - \frac{1}{3} \propto -\langle B^{az}(x) B^{az}(x')\rangle e^{-2\Delta t/\tau_R} < 0$$

(mesons at rest)

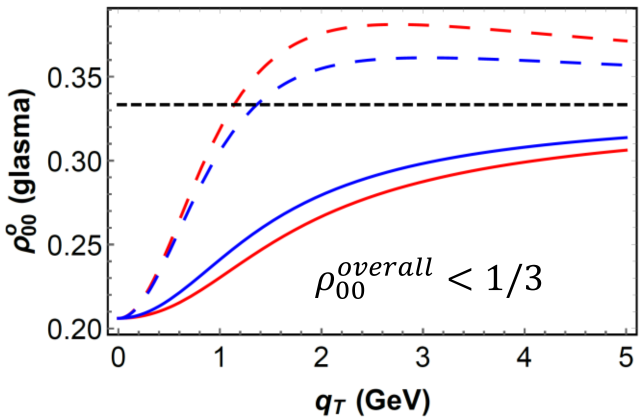
glasma effect — relaxation in QGP

A. Kumar, B. Müller, DY, PRD 107, 076025 (2023)

Transverse spin alignment spectra

- Out-of-plane spin alignment (ϕ mesons) : DY, PRD 110, 056005 (2025)

❖ Initial-state effect (glasma) :



$v^y = 0$

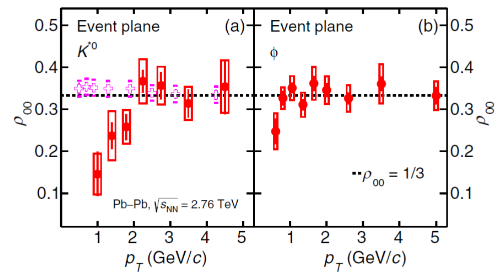
$v^x = v^y$

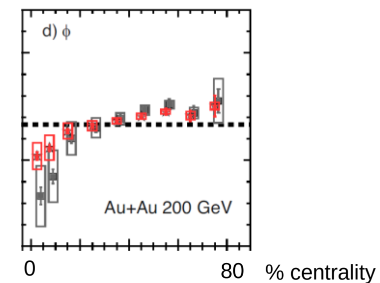
$y_q = 1$

$y_q = 0$

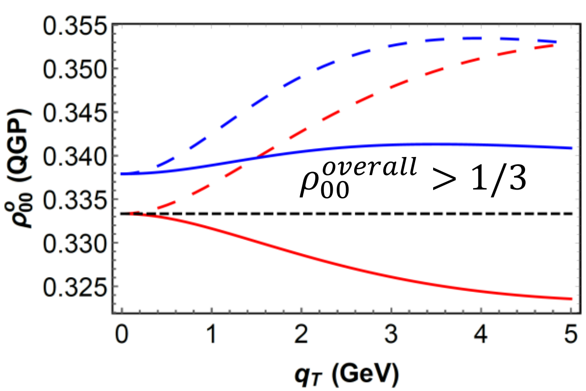
weighting needed :

$$\langle \rho_{00}(q_T) \rangle = \frac{\int d\phi_q dy_q [\rho_{00}(\boldsymbol{q})\mathcal{N}]}{\int \phi_q dy_q \mathcal{N}}$$

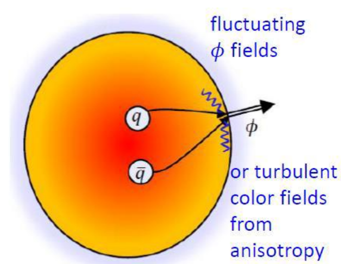




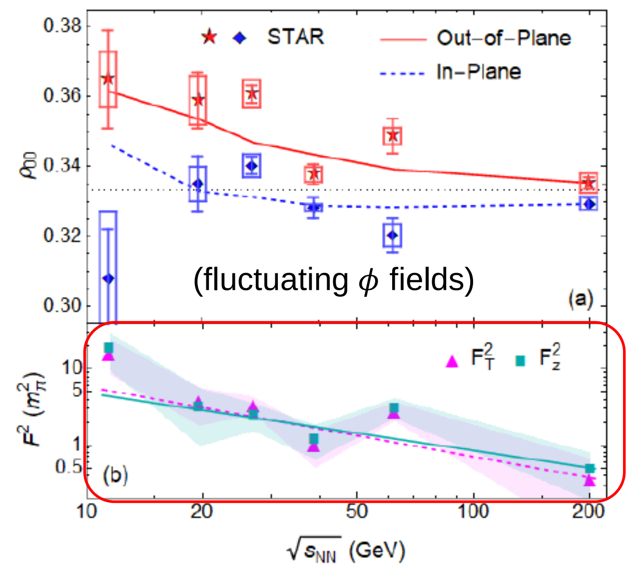
❖ Final-state effect (QGP) : strong-force fields



(isotropic color fields from QGP : qualitative)



X.-L. Sheng et al., PRD 109, 036004, (2024)
PRL 131, 042304 (2023)
B. Müller, DY, PRD 105, L011901 (2022)
DY, JHEP 06, 140 (2022)

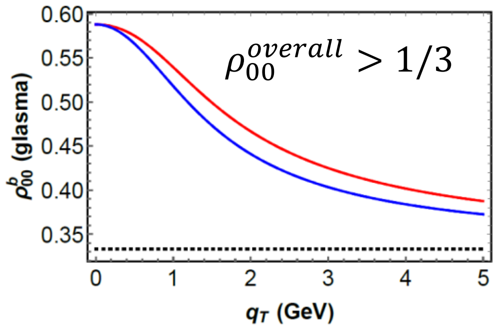


isotropic, competition with glasma effect at high energies?

Observables for future measurements

- Longitudinal spin alignment along the beam direction : DY, PRD 110, 056005 (2025)

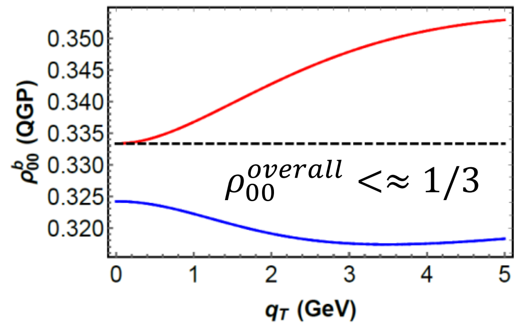
initial-state



v.s.

$y_q = 1$

$y_q = 0$



final state

(qualitative)

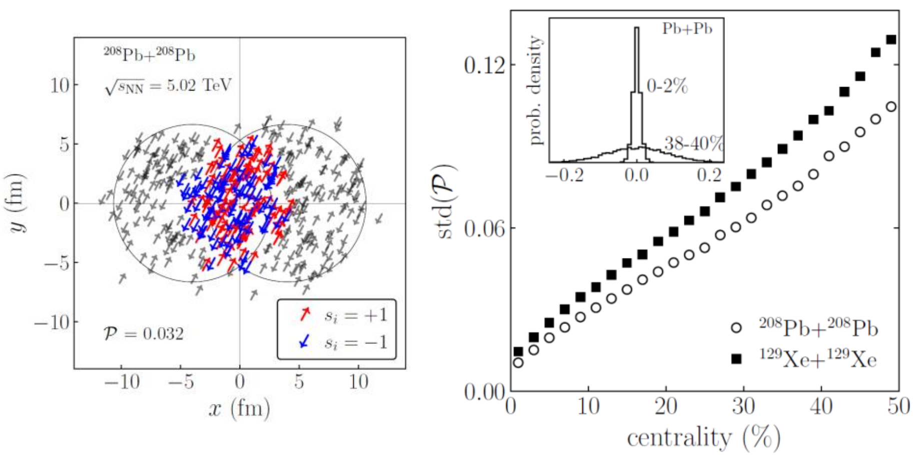
- Spin-spin correlation of hyperons : $C_{12}^{\mu\nu}(p_1, p_2) = \frac{1}{N_{\text{event}}} \sum_{\text{event}} \frac{\int d\Sigma(x_1)\cdot p_1 \int d\Sigma(x_2)\cdot p_2 \, P_1^{\mu} P_2^{\nu} f_1 f_2}{\left[\int d\Sigma(x_1)\cdot p_1 f_1\right]\left[\int d\Sigma(x_2)\cdot p_2 f_2\right]}$

D. Shen, Ji. Chen, A. Tang, arXiv: 2407.21291

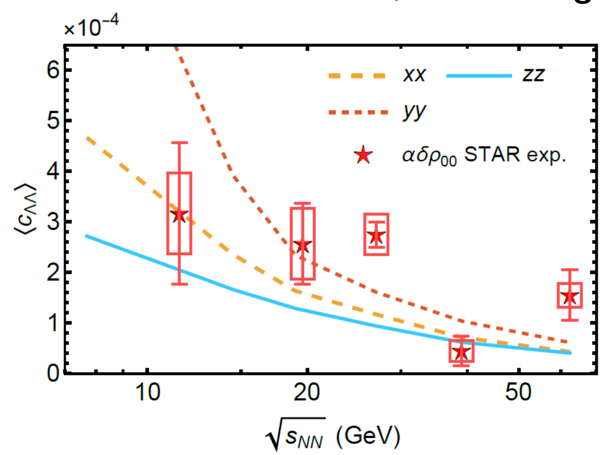
J.-p. Lv et al., PRD 109, 114003 (2024)

initial-state : nucleon spin fluct., glasma fields?

final-state : meson fields, thermal-gluon fields?



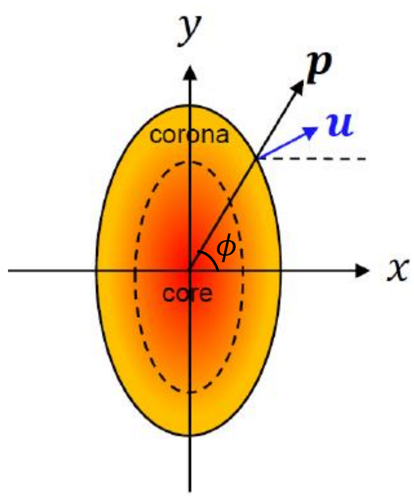
G. Giacalone, E. Speranza, arXiv: 2502.13102



X.-L. Sheng et al., arXiv:2508.03496

Longitudinal Λ polarization (focused on pA)

- Longitudinal polarization from the corona of glasma (early hadronization) :



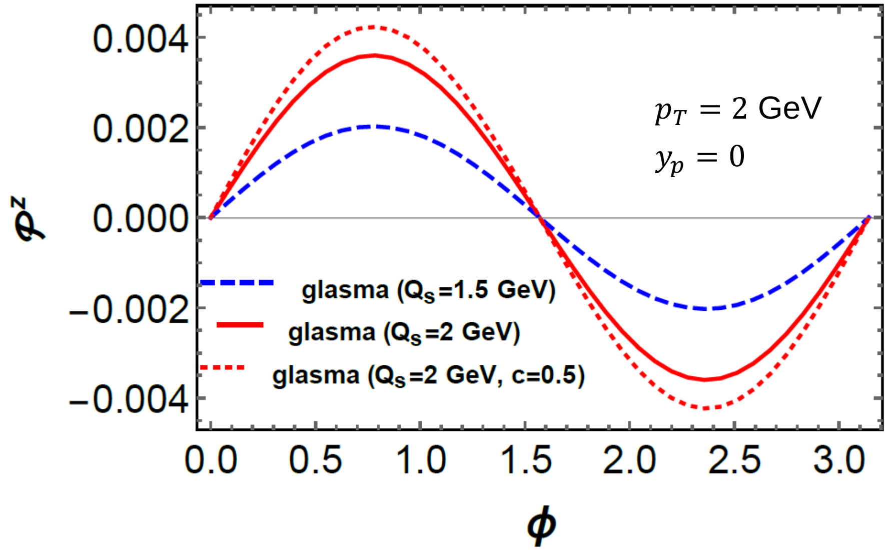
weak initial anisotropic flow from pressure gradient

traveling time in corona

$\boldsymbol{p} \times \boldsymbol{u}$ structure

$$\mathcal{P}^z \approx \frac{\hbar(N_c^2-1)Q_s^2\Delta t}{16M_\Lambda N_c^2\epsilon_{\boldsymbol{p}}\int d\Sigma\cdot p f_V^{\rm s}}\int d\Sigma\cdot p(p^x u^y - p^y u^x) \times f_V^{\rm s}(1-f_V^{\rm s})\left((1-2f_V^{\rm s})u^0 + Q_s/\epsilon_{\boldsymbol{p}}\right), \qquad f_V^{\rm s} = \frac{1}{e^{p\cdot u/Q_s}+c}.$$

(the overall sign is related to the freeze-out hypersurface)

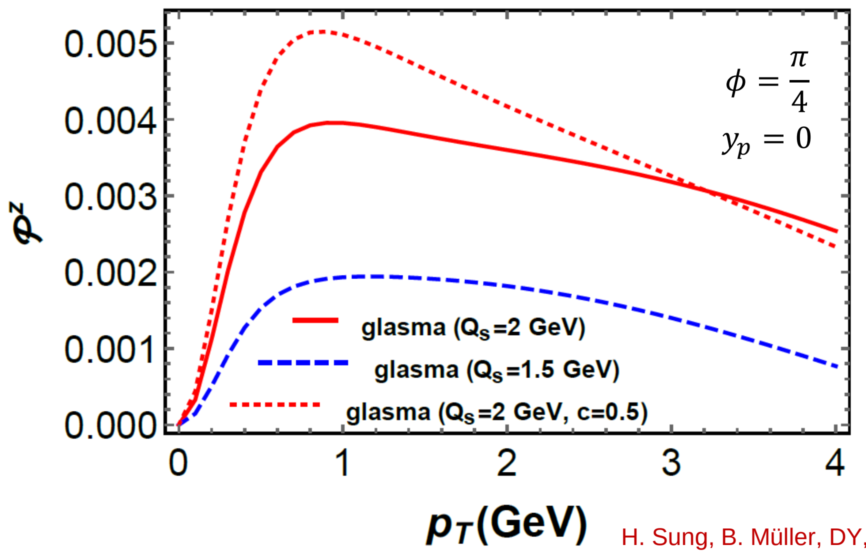


H. Sung, B. Müller, DY, arXiv: 2507.23210

✓ consistent sign & comparable order of magnitude with observations

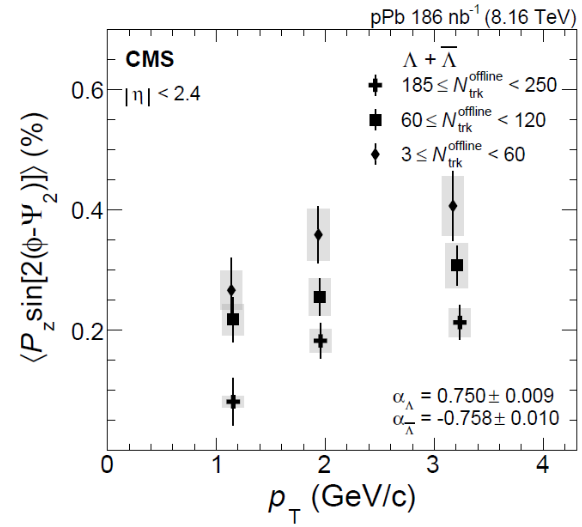
P_T & system-size dependences

- Transverse-momentum dep. :



sign flipping when $|\boldsymbol{p}\cdot\boldsymbol{u}| \sim Q_s$

H. Sung, B. Müller, DY, arXiv: 2507.23210



CMS arXiv:2502.07898

- Competing effects from soft thermal gluons in QGP (with an opposite sign) + thermal vorticity/shear effects as the core contribution.

For smaller systems & high pT For larger systems & low pT

❖ Full pol. : $$\mathcal{P}^z = \frac{N_{\rm GL}\mathcal{P}^z_{\rm GL} + N_{\rm QGP}\mathcal{P}^z_{\rm QGP}}{N_{\rm GL} + N_{\rm QGP}}$$ ⇒ monotonic increase with pT ?

Conclusions & outlook

- Conclusions :

- ✓ Initial-state effects such as glasma fields could play a significant role on local spin polarization and spin alignment, while there exist competing final-state effects.
- ✓ The size and energy dependence of collision systems may be helpful to disentangle the competing effects.
- ✓ New observables like longitudinal spin alignment & spin-spin correlations of hyperons will be also useful.

- Outlook :

- ➢ More sophisticated modeling & simulations are needed.

  e.g., more accurate estimation on spin relaxation in QGP

- ➢ (Collective) spin transport for heavy quarks : more sensitive to the initial-state effects.

Thank you!

Axial kinetic theory

- Axial kinetic theory : scalar/axial-vector kinetic eqs. (SKE/AKE)

K. Hattori, Y. Hidaka, DY, PRD 100, 096011 (2019)
DY, K. Hattori, Y. Hidaka, JHEP 20, 070 (2020)
Z. Wang, X. Guo, P. Zhuang, Eur. Phys. J. C 81, 799 (2021)

➢ SKE : $p\cdot\Delta f_V = \mathcal{C}[f_V], \quad \Delta_\mu = \partial_\mu + eF_{\nu\mu}\partial_p^\nu.$

standard Vlasov eq. — EM fields ($eF_{\nu\mu}$)

➢ AKE : $p\cdot\Delta\tilde{a}^\mu + eF^{\nu\mu}\tilde{a}_\nu - \frac{e}{2}\hbar\epsilon^{\mu\nu\rho\sigma}p_\rho(\partial_\sigma F_{\beta\nu})\partial_p^\beta f_V = \hat{L}^{\mu\nu}\tilde{a}_\nu + \hbar\hat{H}^{\mu\nu}\partial_\nu f_V.$

($\tilde{a}^\mu(p,x)$: effective spin four vector)
($\hbar$: gradient correction in phase space)

(entangled f_V & $\tilde{a}^\mu$) — spin relaxation ($\hat{L}^{\mu\nu}\tilde{a}_\nu$) — dynamical spin pol. from spin-orbit int. ($\hbar\hat{H}^{\mu\nu}\partial_\nu f_V$)

➢ Axial Wigner functions :

$$\mathcal{A}^\mu(\boldsymbol{p},x) = \frac{1}{2\epsilon_{\boldsymbol{p}}}\left[\tilde{a}^\mu - \frac{\hbar}{2}\tilde{F}^{\mu\nu}\left(\partial_{p\nu}f_V - \frac{\epsilon_{\boldsymbol{p}}}{2}\partial_{p_\perp\nu}(f_V/\epsilon_{\boldsymbol{p}})\right)\right]_{p_0=\epsilon_{\boldsymbol{p}}=\sqrt{|\boldsymbol{p}|^2+m^2}}.$$

dynamical (w/ memory effect) — non-dynamical (w/o memory effect)

$$\Rightarrow \quad \mathcal{P}^\mu(\boldsymbol{p}) = \frac{\int d\Sigma\cdot p\,\mathcal{A}^\mu(\boldsymbol{p},x)}{2m\int d\Sigma\cdot p\,(2\epsilon_{\boldsymbol{p}})^{-1}f_V(\boldsymbol{p},x)}.$$

- Relaxation-time approx. & weak coupling :

$$p\cdot\partial\tilde{a}^\mu - \frac{e}{2}\hbar\epsilon^{\mu\nu\rho\sigma}p_\rho(\partial_\sigma F_{\beta\nu})\partial_p^\beta f_V = -\frac{p_0\delta\tilde{a}^\mu}{\tau_{\rm R}}, \quad \delta\tilde{a}^\mu = \tilde{a}^\mu - \tilde{a}^\mu_{\rm eq}.$$

$$\Rightarrow \quad \delta\tilde{a}^\mu(\boldsymbol{p},x) = \frac{\hbar e}{2}\int_{t_{\rm i}}^{t_{\rm f}} dx_0' e^{-(x_0-x_0')/\tau_{\rm R}}\epsilon^{\mu\nu\rho\sigma}\hat{p}_\rho\left(\partial_{x'\sigma}F_{\beta\nu}(x')\right)\partial_p^\beta f_V(p,x')$$

Axial kinetic theory with color fields

- Incorporation of background color fields into Wigner functions and kinetic equations.
- Color decomposition : $O = O^s I + O^a t^a$

U. W. Heinz, Phys. Rev. Lett. 51, 351 (1983)
H. T. Elze, M. Gyulassy, D. Vasak, Nucl. Phys. B276, 706 (1986).

e.g., $\mathcal{A}^{\mu}(\boldsymbol{p},x) = \mathcal{A}^{s\mu}(\boldsymbol{p},x)I + \mathcal{A}^{a\mu}(\boldsymbol{p},x)\,t^a, \quad f_V(\boldsymbol{p},x) = f^{s}_{V}(\boldsymbol{p},x)I + f^{a}_{V}(\boldsymbol{p},x)\,t^a,$

$\tilde{a}^{\mu}(\boldsymbol{p},x) = \tilde{a}^{s\mu}(\boldsymbol{p},x)I + \tilde{a}^{a\mu}(\boldsymbol{p},x)\,t^a.$

- Kinetic equations : DY, JHEP 06, 140 (2022)

SKEs : $p^{\rho}\left(\partial_{\rho} f^{s}_{V} + \frac{g}{2N_c} F^{a}_{\nu\rho}\partial^{\nu}_{p} f^{a}_{V}\right) = \mathcal{C}_{s}, \quad p^{\rho}\left(\partial_{\rho} f^{a}_{V} + g F^{a}_{\nu\rho}\partial^{\nu}_{p} f^{s}_{V} + \frac{d^{bca}}{2} g F^{b}_{\nu\rho}\partial^{\nu}_{p} f^{c}_{V}\right) = \mathcal{C}^{a}_{o},$

diffusion — dynamical spin polarization

AKEs : $p^{\rho}\partial_{\rho}\tilde{a}^{s\mu} + \frac{g}{2N_c}\left(p^{\rho}F^{a}_{\nu\rho}\partial^{\nu}_{p}\tilde{a}^{a\mu} + F^{a\nu\mu}\tilde{a}^{a}_{\nu}\right) - \frac{\hbar}{4N_c}\epsilon^{\mu\nu\rho\sigma}p_{\rho}(\partial_{\sigma} g F^{a}_{\beta\nu})\partial^{\beta}_{p} f^{a}_{V} = \mathcal{C}^{\mu}_{s},$

$$p^{\rho}\partial_{\rho}\tilde{a}^{a\mu} + g\left(p^{\rho}F^{a}_{\nu\rho}\partial^{\nu}_{p}\tilde{a}^{s\mu} + F^{a\nu\mu}\tilde{a}^{s}_{\nu}\right) + \frac{d^{bca}}{2} g\left(p^{\rho}F^{b}_{\nu\rho}\partial^{\nu}_{p}\tilde{a}^{c\mu} + F^{b\nu\mu}\tilde{a}^{c}_{\nu}\right) - \frac{\hbar}{2}\epsilon^{\mu\nu\rho\sigma}p_{\rho}(\partial_{\sigma} g F^{a}_{\beta\nu})\partial^{\beta}_{p} f^{s}_{V} = \mathcal{C}^{a\mu}_{o}.$$

Axial Wigner functions :

$$\mathcal{A}^{s\mu}(\boldsymbol{p},x) = \frac{1}{2\epsilon_{\boldsymbol{p}}}\left[\tilde{a}^{s\mu} - \frac{\hbar}{4N_c}\tilde{F}^{a\mu\nu}\left(\partial_{p\nu} f^{a}_{V} - \frac{\epsilon_{\boldsymbol{p}}}{2}\partial_{p_{\perp}\nu}(f^{a}_{V}/\epsilon_{\boldsymbol{p}})\right)\right]_{p_0=\epsilon_{\boldsymbol{p}}},$$

$$\mathcal{A}^{a\mu}(\boldsymbol{p},x) = \frac{1}{2\epsilon_{\boldsymbol{p}}}\left[\tilde{a}^{a\mu} - \frac{\hbar}{2}\tilde{F}^{a\mu\nu}\left(\partial_{p\nu} f^{s}_{V} - \frac{\epsilon_{\boldsymbol{p}}}{2}\partial_{p_{\perp}\nu}(f^{s}_{V}/\epsilon_{\boldsymbol{p}})\right)\right]_{p_0=\epsilon_{\boldsymbol{p}}}.$$

dynamical (w/ memory effect) — non-dynamical (w/o memory effect)

Spin polarization:

$$\mathcal{P}^{\mu}(\boldsymbol{p}) = \frac{\int d\Sigma\cdot p\,\mathrm{Tr_c}\mathcal{A}^{\mu}(\boldsymbol{p},x)}{2m\int d\Sigma\cdot p\,(2\epsilon_{\boldsymbol{p}})^{-1} f^{s}_{V}(\boldsymbol{p},x)} = \frac{\int d\Sigma\cdot p\,\mathcal{A}^{s\mu}(\boldsymbol{p},x)}{2m\int d\Sigma\cdot p\,(2\epsilon_{\boldsymbol{p}})^{-1} f^{s}_{V}(\boldsymbol{p},x)}.$$

Spin alignment from glasma

- Generalized AKT with color fields : $\tilde{a}^{\mu}(\boldsymbol{p},x) = \tilde{a}^{s\mu}(\boldsymbol{p},x)I + \boxed{\tilde{a}^{a\mu}(\boldsymbol{p},x)}\, t^{a}.$

  DY, JHEP 06, 140 (2022)
  B. Müller, DY, PRD 105, L011901 (2022)

  (more dominant in the perturbative approach)

- Dynamical spin polarization from glasma fields : A. Kumar, B. Müller, DY, PRD 108, 016020 (2023)

  suppressed

$$\tilde{a}^{a\mu}(\boldsymbol{p},x) \approx -\frac{\hbar g}{2} e^{-(t_{\rm f}-t_{\rm i})/\tau_{\rm R}^{\rm o}} \left(B^{a\mu}(t_{\rm i})\partial_{\epsilon_{\boldsymbol{p}}} f_{\rm V}^{\rm s}(\epsilon_{\boldsymbol{p}},t_{\rm i}) - B^{a\mu}(t_{\rm f})\partial_{\epsilon_{\boldsymbol{p}}} f_{\rm V}^{\rm s}(\epsilon_{\boldsymbol{p}},t_{\rm f})\right)$$

- Correlation of initial color magnetic fields : $g^2\langle B^{az}(x)B^{az}(x)\rangle_{x_0=t_{\rm i}} \sim \frac{Q_s^4(N_c^2-1)}{2N_c}$

  K. J. Golec-Biernat and M. Wusthoff, Phys. Rev. D 59, 014017 (1998)
  P. Guerrero-Rodr´ıguez and T. Lappi, Phys. Rev. D 104, 014011 (2021)

- Initial quark distribution function : $f_{\rm V}^{\rm s}(\epsilon_{\boldsymbol{p}},t_{\rm i}) = 1/\left(e^{\epsilon_{\boldsymbol{p}}/Q_s}+1\right)$

❖ spin correlation : $\langle \mathcal{P}_q^z \mathcal{P}_{\bar{q}}^z\rangle \sim \frac{Q_s^2}{m_q m_{\bar{q}}} e^{-2(t_{\rm f}-t_{\rm i})/\tau_{\rm R}^{\rm o}}$

❖ Order-of-magnitude estimation (for ϕ) : $\rho_{00} \sim \frac{1}{3 + 10\, e^{-2(t_{\rm f}-t_{\rm i})/\tau_{\rm R}^{\rm o}}} < \frac{1}{3}$

$Q_s \approx 1 \sim 2\ \mathrm{GeV}$

glasma effect   relaxation effect

❖ Heavy-quark approx. : $\tau_{\rm R}^{\rm o} \approx \left(\frac{g^2 C_2(F) m_D^2 T}{6\pi m^2}\ln g\right)^{-1} \approx 5\ \mathrm{fm/c} \Rightarrow \rho_{00} \approx 0.24$

M. Hongo et al., JHEP 08, 263 (2022)

(model dependent)

Spin correlations from color fields

- Spin density matrix can be directly related to Wigner functions of the coalesced quark and antiquark through the quark-meson interaction.

❖ Kinetic theory of vector mesons :

$$q\cdot\partial f^{\phi}_{\lambda}=\epsilon^{*}_{\mu}(\lambda,\boldsymbol{q})\epsilon_{\nu}(\lambda,\boldsymbol{q})\left[\mathcal{C}^{\mu\nu}_{\rm coal}(q,x)(1+f^{\phi}_{\lambda})-\mathcal{C}^{\mu\nu}_{\rm diss}(q,x)f^{\phi}_{\lambda}\right]\approx\boxed{\epsilon^{*}_{\mu}(\lambda,\boldsymbol{q})\epsilon_{\nu}(\lambda,\boldsymbol{q})\mathcal{C}^{\mu\nu}_{\rm coal}(q,x)}$$

quark-meson int. :

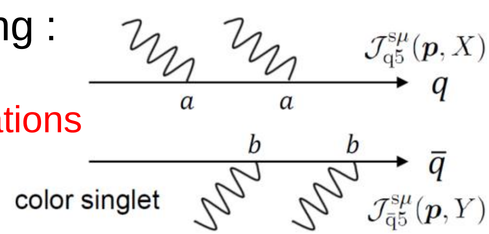
$$\mathcal{L}_{\rm int}=g_{\phi}\bar{\psi}\gamma^{\mu}V_{\mu}\psi$$

$$\rho_{00}(q)=\frac{\int d\Sigma_X\cdot q f^{\phi}_{0}(q,X)}{\int d\Sigma_X\cdot q\big(f^{\phi}_{0}(q,X)+f^{\phi}_{+1}(q,X)+f^{\phi}_{-1}(q,X)\big)}$$
$$=\frac{1-{\rm Tr}_{\rm c}\langle\hat{\mathcal{P}}^{y}_{q}(\boldsymbol{q}/2)\hat{\mathcal{P}}^{y}_{\bar{q}}(\boldsymbol{q}/2)\rangle_{\boldsymbol{q}=0}}{3-\sum_{i=x,y,z}{\rm Tr}_{\rm c}\langle\hat{\mathcal{P}}^{i}_{q}(\boldsymbol{q}/2)\hat{\mathcal{P}}^{i}_{\bar{q}}(\boldsymbol{q}/2)\rangle_{\boldsymbol{q}=0}}$$

❖ Spin correlations in the non-relativistic limit :

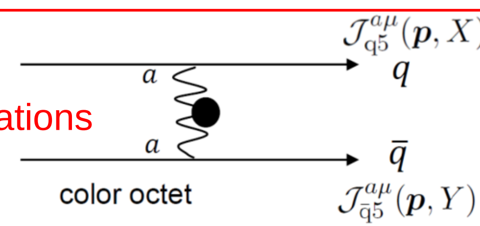
$${\rm Tr}_{\rm c}\langle\hat{\mathcal{P}}^{i}_{q}(\boldsymbol{p})\hat{\mathcal{P}}^{i}_{\bar{q}}(\boldsymbol{p})\rangle\approx\frac{4\int d\Sigma_X\cdot p\big(\langle\mathcal{A}^{si}_{q}(\boldsymbol{p},X)\mathcal{A}^{si}_{\bar{q}}(\boldsymbol{p},X)\rangle+\langle\mathcal{A}^{ai}_{q}(\boldsymbol{p},X)\mathcal{A}^{ai}_{\bar{q}}(\boldsymbol{p},X)\rangle/(2N_c)\big)}{\int d\Sigma_X\cdot p f^{\rm s}_{{\rm V}q}(\boldsymbol{p},X)f^{\rm s}_{{\rm V}\bar{q}}(\boldsymbol{p},X)}$$

➢ Weak coupling :



4-field correlations $\propto g^4$

2-field correlations $\propto g^2$



A. Kumar, B. Müller, DY, PRD 107, 076025 (2023)

A. Kumar, B. Müller, DY, PRD 108, 016020 (2023)

Color fields from the glasma

- Solving linearized Yang-Mills eqs. : $[D_\mu, F^{\mu\nu}] = J^\nu$

P. Guerrero-Rodrguez, T. Lappi, PRD 104, 014011 (2021)

- Color-field correlators in the glasma :

evolution in time

e.g.
$$\langle E_T^{ai}(X')E_T^{aj}(X'')\rangle = -\bar{N}_c \epsilon^{in}\epsilon^{jm} \int_{\perp;q,u}^{X'} \int_{\perp;l,v}^{X''} \Omega_-(u_\perp, v_\perp)\frac{q^n l^m}{ql} \times J_1(qX_0')J_1(lX_0''),$$

$$\langle B_T^{ai}(X')B_T^{aj}(X'')\rangle = -\bar{N}_c \epsilon^{in}\epsilon^{jm} \int_{\perp;q,u}^{X'} \int_{\perp;l,v}^{X''} \Omega_+(u_\perp, v_\perp)\frac{q^n l^m}{ql} \times J_1(qX_0')J_1(lX_0''),$$

$$\bar{N}_c \equiv \frac{1}{2} g^2 N_c (N_c^2 - 1),$$

$$\Omega_\mp(u_\perp, v_\perp) = [G_1(u_\perp, v_\perp)G_2(u_\perp, v_\perp) \mp h_1(u_\perp, v_\perp)h_2(u_\perp, v_\perp)],$$

$$\int_{\perp;q,u}^{X'} \equiv \int \frac{d^2 q_\perp}{(2\pi)^2} \int d^2 u_\perp e^{iq_\perp (X'-u)_\perp}.$$

unpolarized & linearly polarized
Gluon distribution functions

- Golec-Biernat Wusthoff (GBW) distribution : K. J. Golec-Biernat and M. Wustho, PRD 59, 014017 (1998)

$$\Omega_\pm(u_\perp, v_\perp) = \Omega(u_\perp, v_\perp) = \frac{Q_s^4}{g^4 N_c^2} \left(\frac{1 - e^{-Q_s^2 |u_\perp - v_\perp|^2/4}}{Q_s^2 |u_\perp - v_\perp|^2/4} \right)^2$$

More on spin polarization from color fields

- Isochronous freeze-out in 3+1 D : $\tilde{\tau} \equiv \sqrt{\tau^2 - x^2 - y^2}$

$$d\Sigma \cdot p = dr d\Phi d\eta\, r\sqrt{1-\epsilon^2}\left(G_1 \cosh(y_p - \eta) + G_2\right), \qquad G_1 = \sqrt{(m^2 + p_T^2)}\tau_f, \quad G_2 = -(xp^x + yp^y)$$

- Manifestation of $sin2\phi$: when $|\boldsymbol{p} \cdot \boldsymbol{u}| \ll Q_s$

$$\int d\Sigma \cdot p(p^x u^y - p^y u^x) \xrightarrow{G_2 \text{ dominates}} \int d\Sigma \cdot p u^{[y} p^{x]} u^0 \approx u_T p_T^2 \pi \sin 2\phi \int \frac{dr d\eta r^3 \tau \delta}{r_m^2}$$

$$u^\mu = \frac{1}{N_u}(t, x\sqrt{1+\delta}, y\sqrt{1-\delta}, z)$$

- QGP case : when $|\boldsymbol{p} \cdot \boldsymbol{u}| \gtrsim T$ ⟹ G_1 dominates

H. Sung, B. Müller, DY, arXiv: 2507.23210

single freeze-out thermal model at $\sqrt{s_{NN}} = 130$ GeV