Dynamics of Heavy Quarks in Quark-Gluon Plasma

towards a systematic characterization

Initial Stages 2025: 8th International Conference on the Initial Stages of High-Energy Nuclear Collisions, Taipei

Sep 12, 2025

UC SANTA BARBARA
Kavli Institute for Theoretical Physics

Bruno Scheihing-Hitschfeld

Heavy Quarks in Quark-Gluon Plasma

Transport models: Fokker-Planck equation / Langevin stochastic dynamics

Langevin equation:

$$\frac{dp_i}{dt} = \zeta_i$$

ζ_i: stochastic force

The stochastic force encodes properties of the fluid: we can study the motion of the heavy particle to study the fluid.

Heavy Quarks in Quark-Gluon Plasma

Transport models: Fokker-Planck equation / Langevin stochastic dynamics

Langevin equation:

$$\frac{dp_i}{dt} = \zeta_i$$

ζ_i: stochastic force

The stochastic force encodes properties of the fluid: we can study the motion of the heavy particle to study the fluid.

Heavy Quarks in Quark-Gluon Plasma

Transport models: Fokker-Planck equation / Langevin stochastic dynamics

Langevin equation:

$$\frac{dp_i}{dt} = \zeta_i$$

ζ_i: stochastic force

The stochastic force encodes properties of the fluid: we can study the motion of the heavy particle to study the fluid.

Heavy Quarks in Quark-Gluon Plasma

Transport models: Fokker-Planck equation / Langevin stochastic dynamics

Langevin equation:

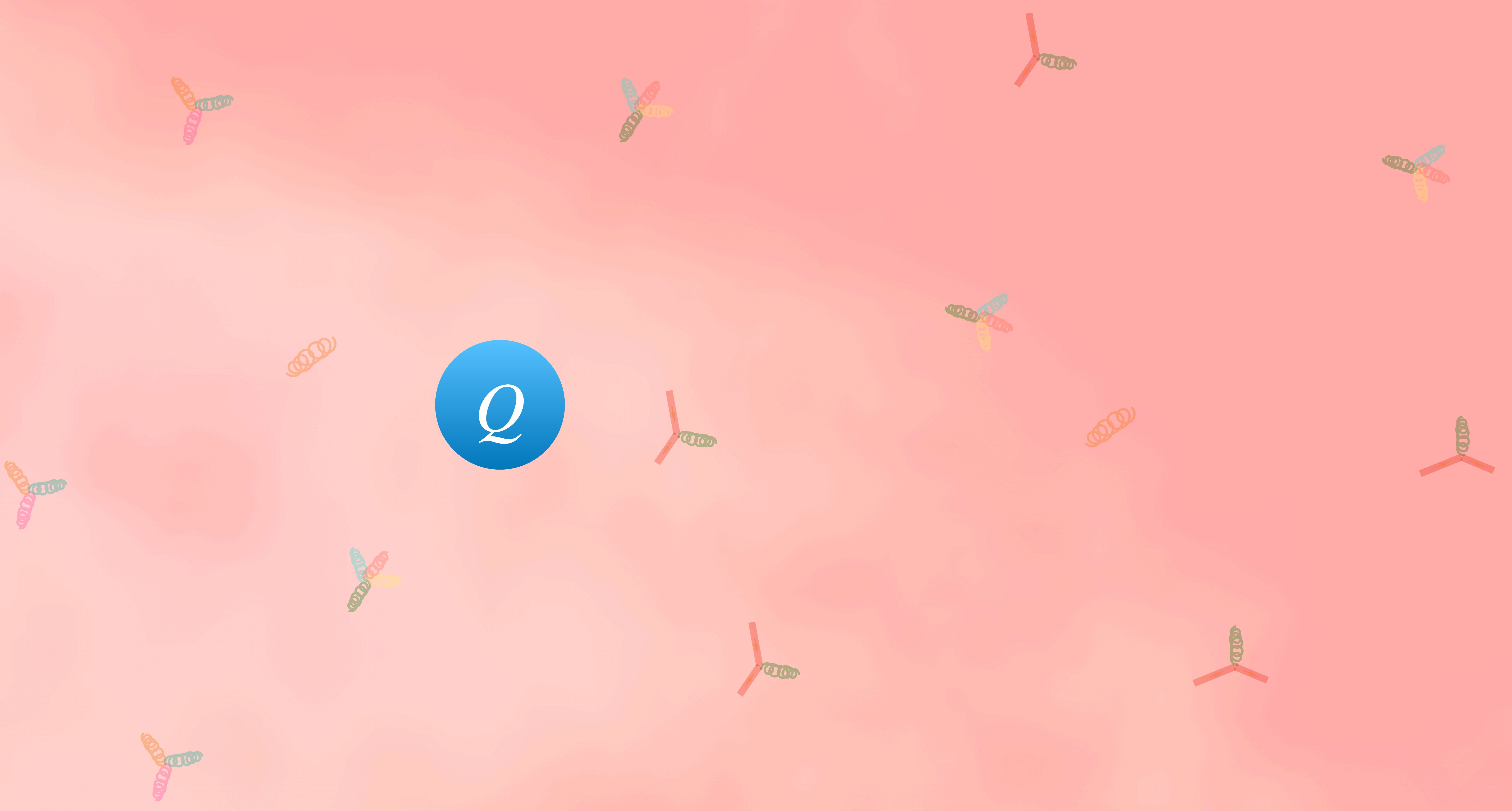
$$\frac{dp_i}{dt} = \zeta_i$$

ζ_i: stochastic force

The stochastic force encodes properties of the fluid: we can study the motion of the heavy particle to study the fluid.

Heavy Quarks in Quark-Gluon Plasma

Transport models: Fokker-Planck equation / Langevin stochastic dynamics



Langevin equation:

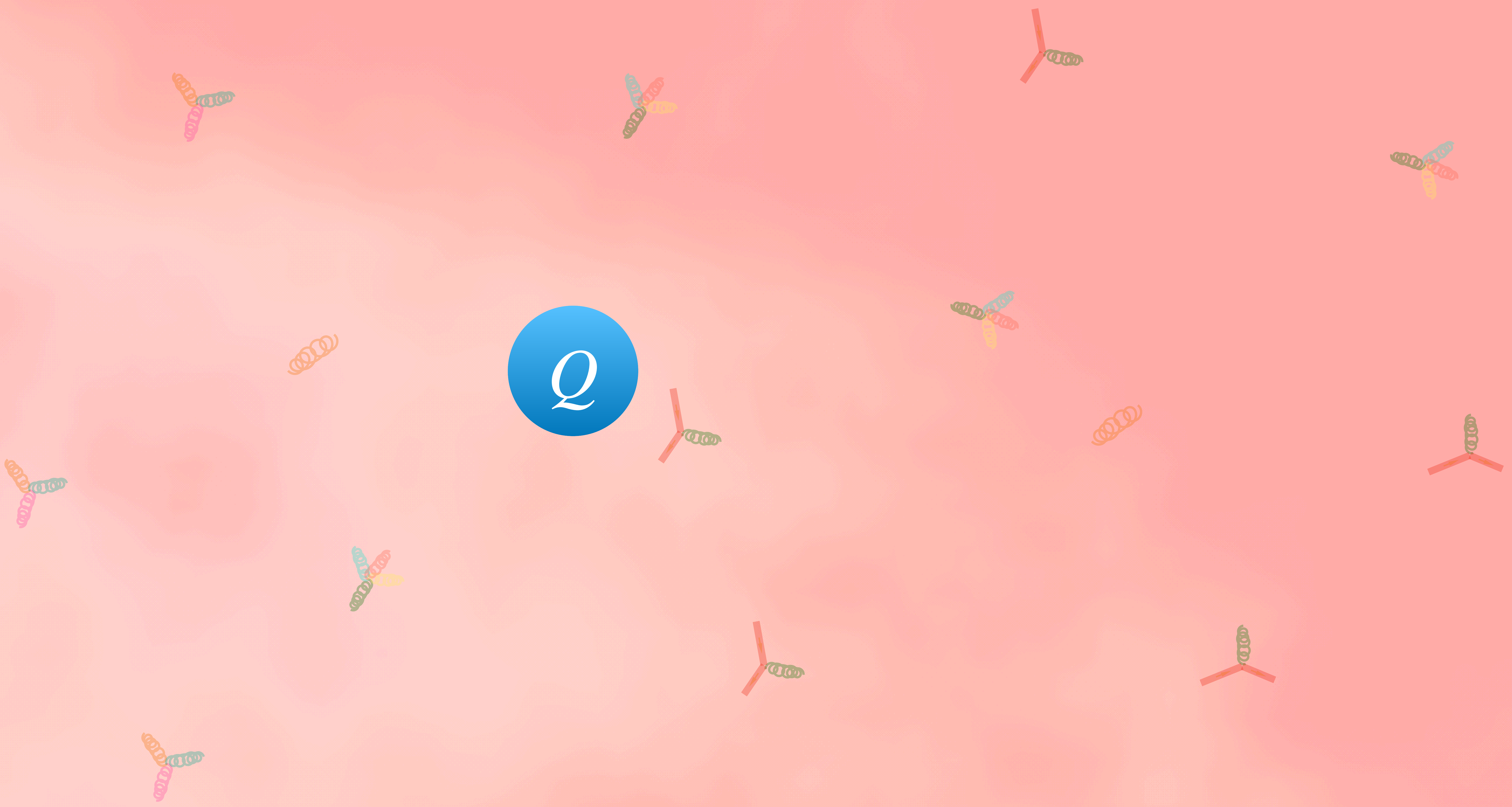
$$\frac{dp_i}{dt} = \zeta_i$$

ζ_i: stochastic force

The stochastic force encodes properties of the fluid: we can study the motion of the heavy particle to study the fluid.

Heavy Quarks in Quark-Gluon Plasma

Transport models: Fokker-Planck equation / Langevin stochastic dynamics



Langevin equation:

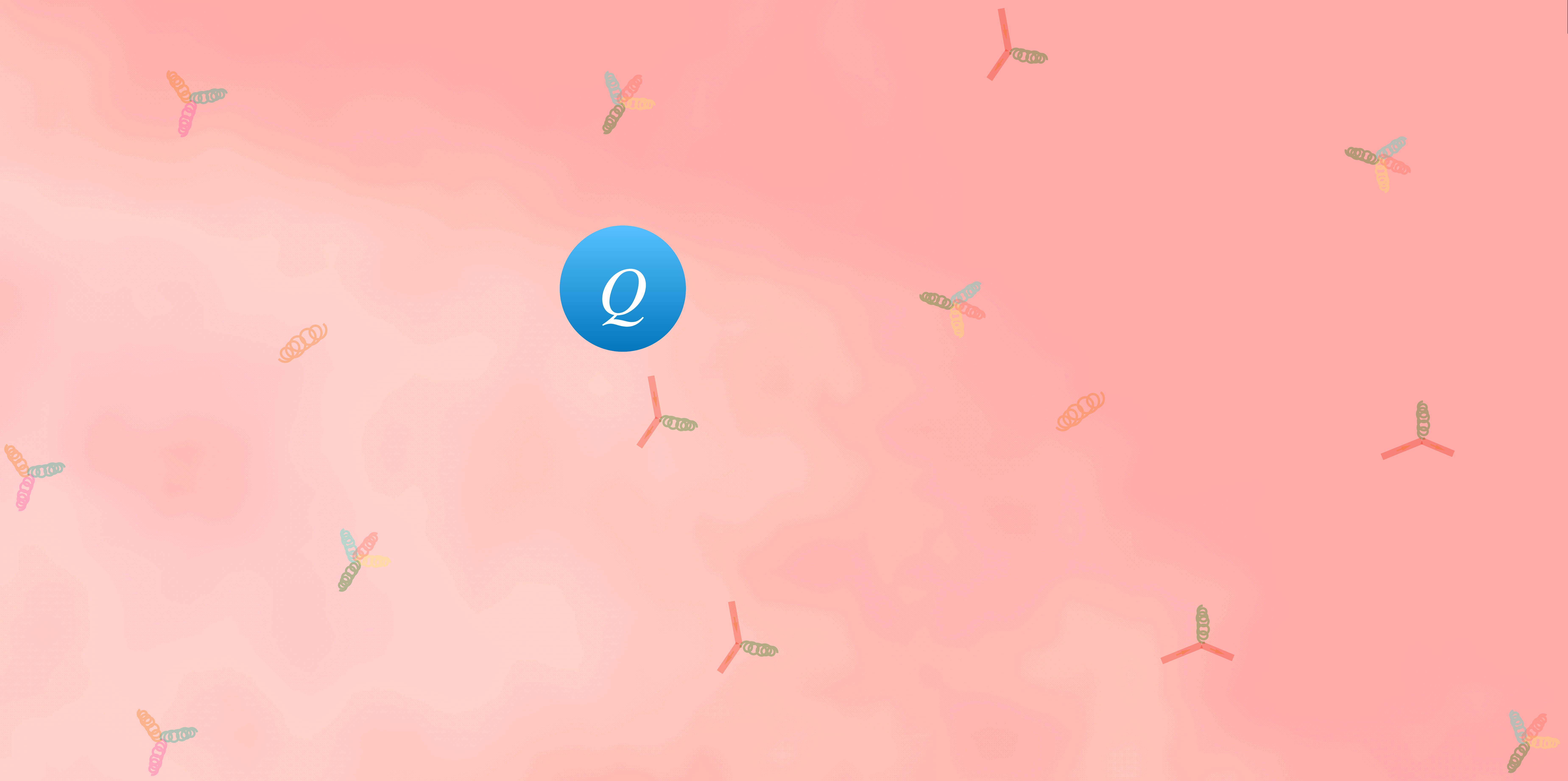
$$\frac{dp_i}{dt} = \zeta_i$$

ζ_i: stochastic force

The stochastic force encodes properties of the fluid: we can study the motion of the heavy particle to study the fluid.

Heavy Quarks in Quark-Gluon Plasma

Transport models: Fokker-Planck equation / Langevin stochastic dynamics



Langevin equation:

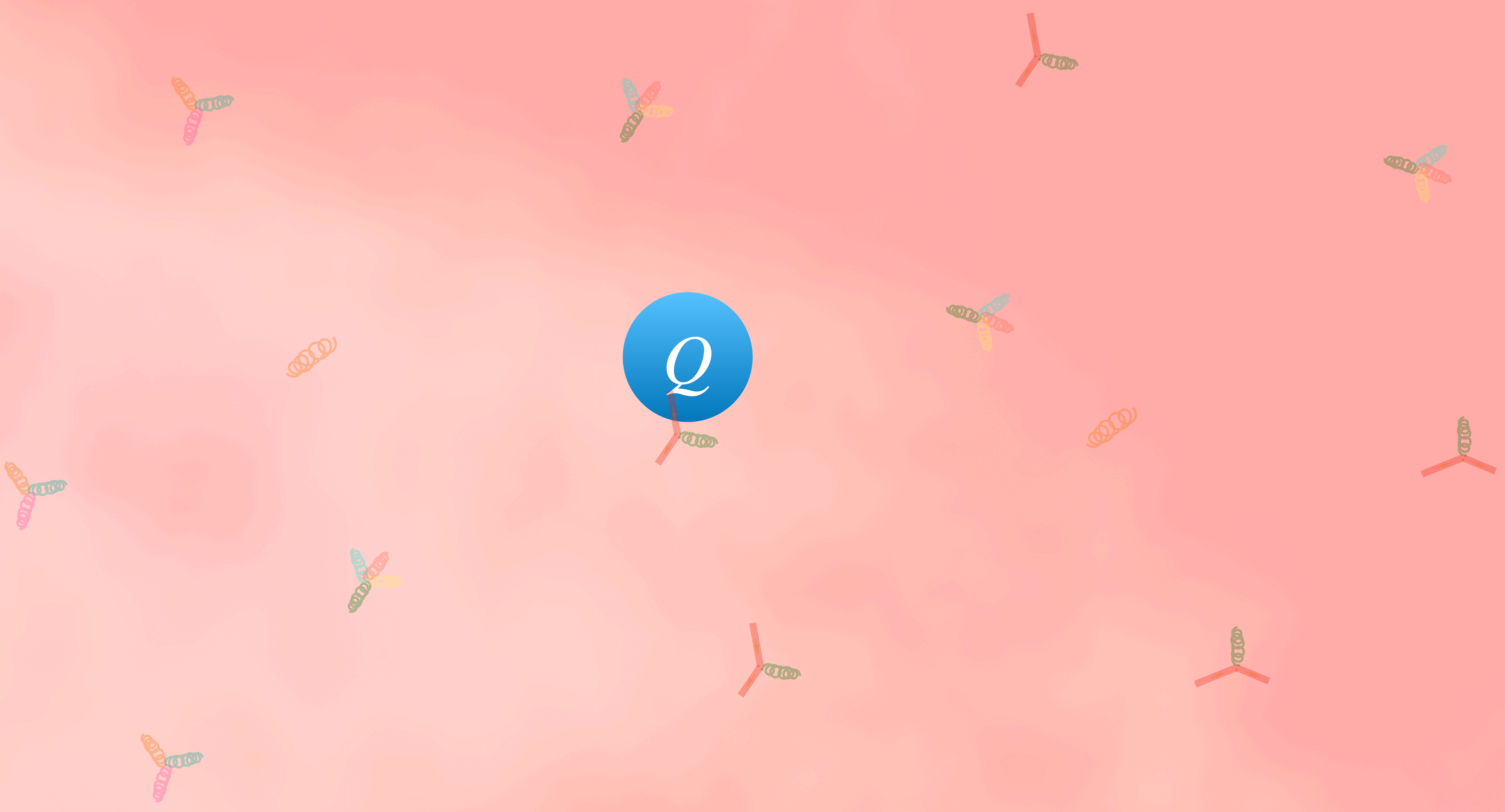
$$\frac{dp_i}{dt} = \zeta_i$$

ζ_i: stochastic force

The stochastic force encodes properties of the fluid: we can study the motion of the heavy particle to study the fluid.

Heavy Quarks in Quark-Gluon Plasma

Transport models: Fokker-Planck equation / Langevin stochastic dynamics



Langevin equation:

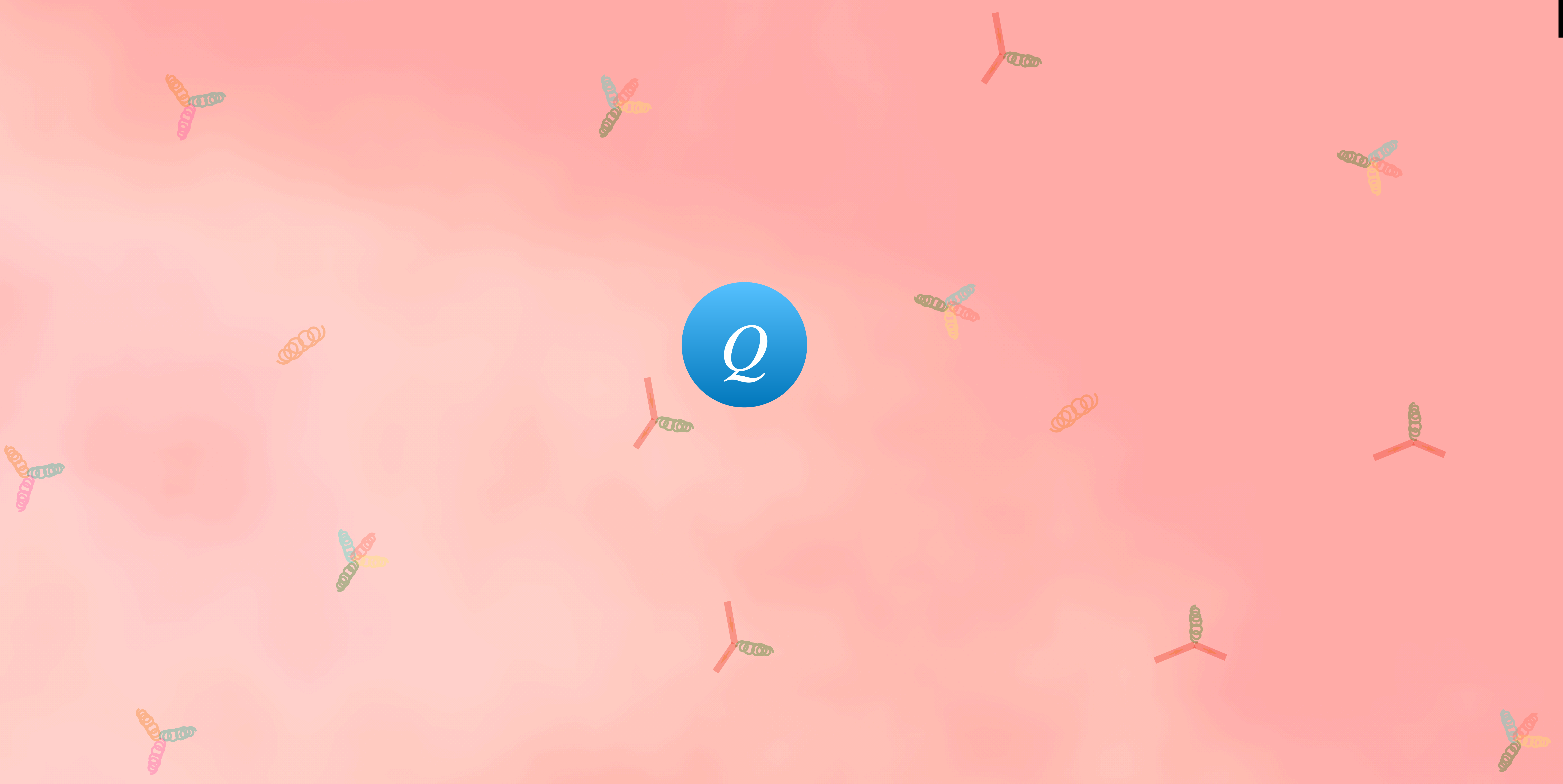
$$\frac{dp_i}{dt} = \zeta_i$$

ζ_i: stochastic force

The stochastic force encodes properties of the fluid: we can study the motion of the heavy particle to study the fluid.

Heavy Quarks in Quark-Gluon Plasma

Transport models: Fokker-Planck equation / Langevin stochastic dynamics



Langevin equation:

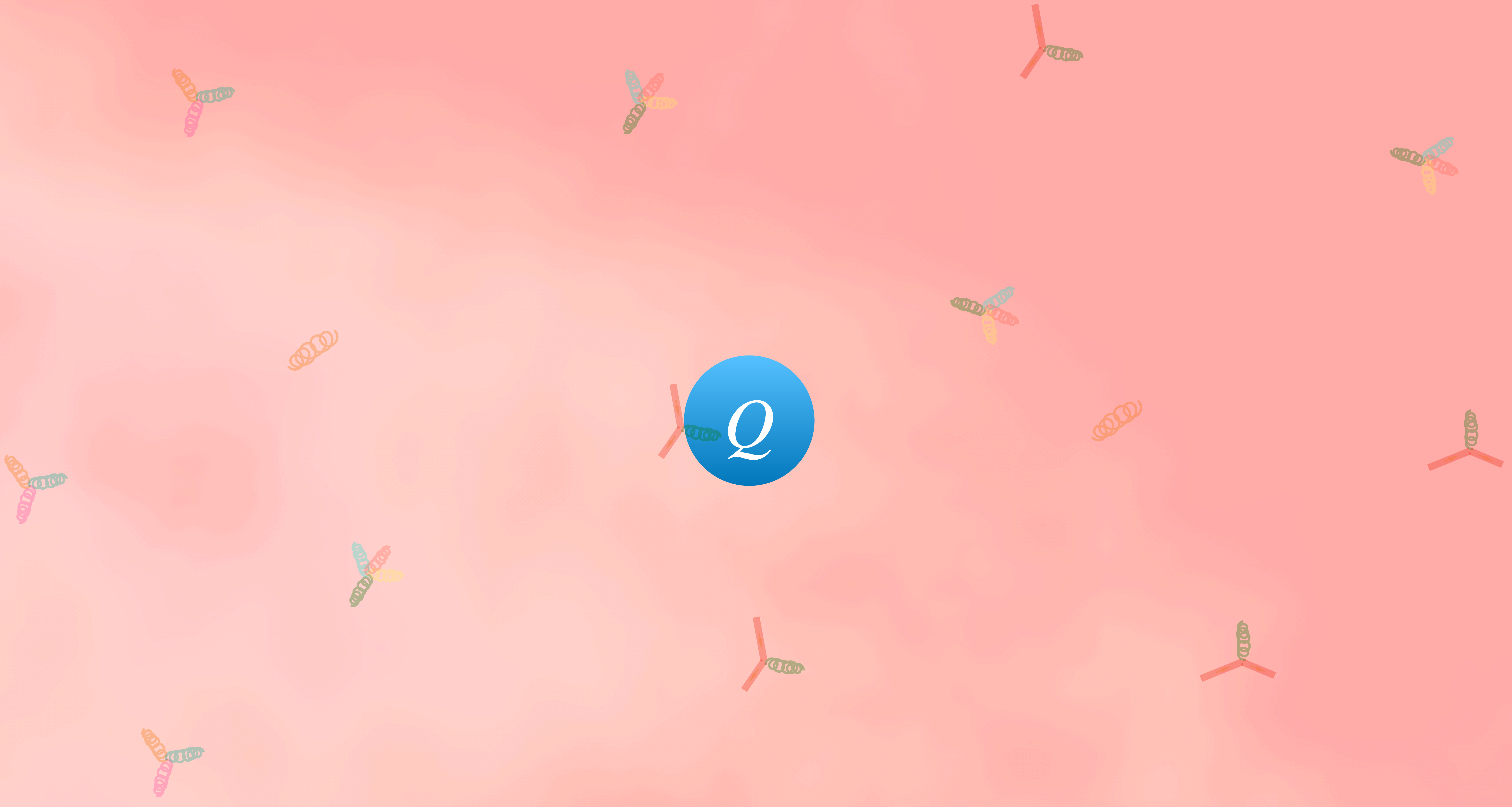
$$\frac{dp_i}{dt} = \zeta_i$$

ζ_i: stochastic force

The stochastic force encodes properties of the fluid: we can study the motion of the heavy particle to study the fluid.

Heavy Quarks in Quark-Gluon Plasma

Transport models: Fokker-Planck equation / Langevin stochastic dynamics



Langevin equation:

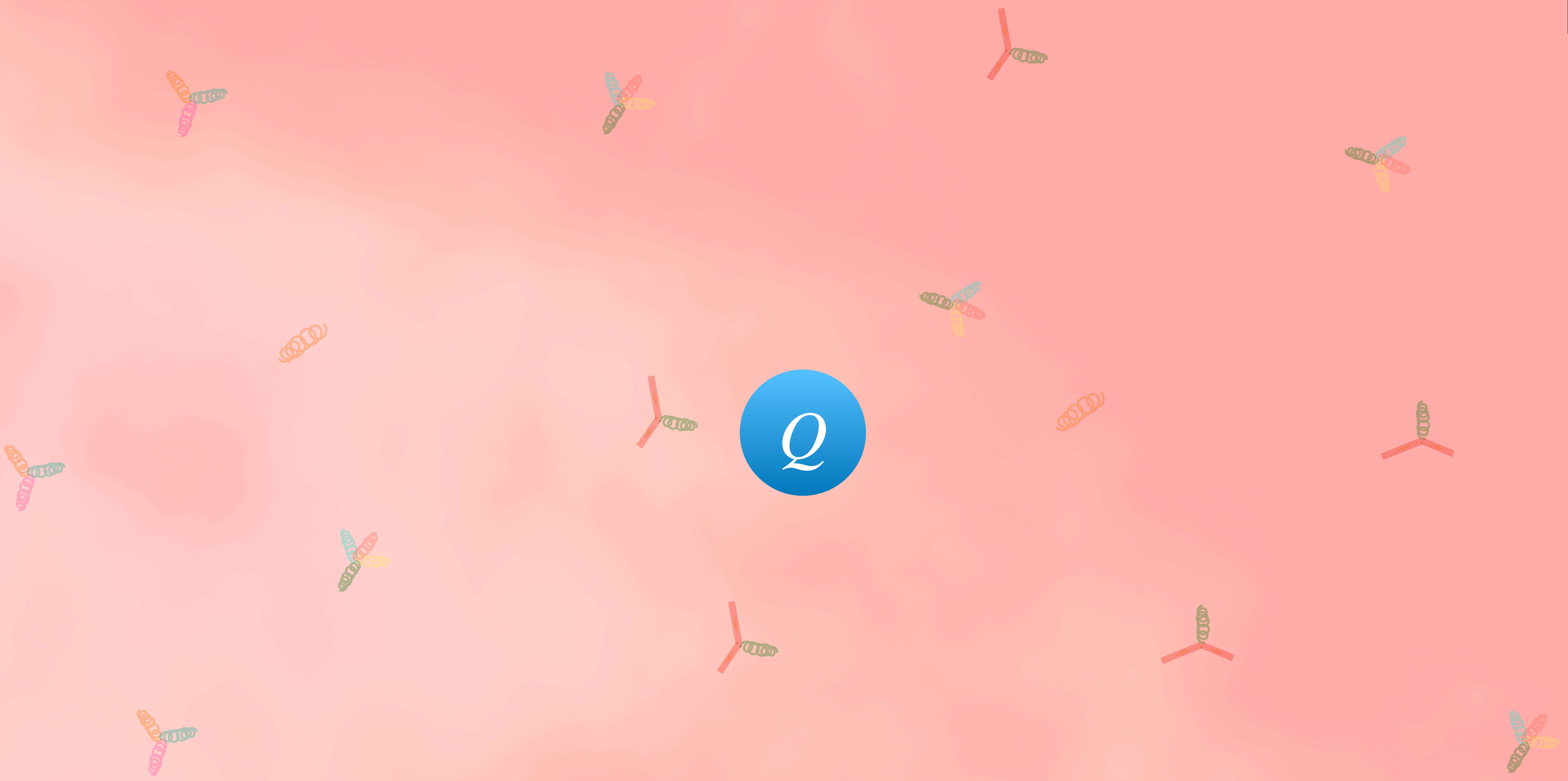
$$\frac{dp_i}{dt} = \zeta_i$$

ζ_i: stochastic force

The stochastic force encodes properties of the fluid: we can study the motion of the heavy particle to study the fluid.

Heavy Quarks in Quark-Gluon Plasma

Transport models: Fokker-Planck equation / Langevin stochastic dynamics



Langevin equation:

$$\frac{dp_i}{dt} = \zeta_i$$

ζ_i: stochastic force

The stochastic force encodes properties of the fluid: we can study the motion of the heavy particle to study the fluid.

Heavy Quarks in Quark-Gluon Plasma

Transport models: Fokker-Planck equation / Langevin stochastic dynamics

Langevin equation:

$$\frac{dp_i}{dt} = \zeta_i$$

ζ_i: stochastic force

The stochastic force encodes properties of the fluid: we can study the motion of the heavy particle to study the fluid.

Heavy Quarks in Quark-Gluon Plasma

Transport models: Fokker-Planck equation / Langevin stochastic dynamics

Langevin equation:

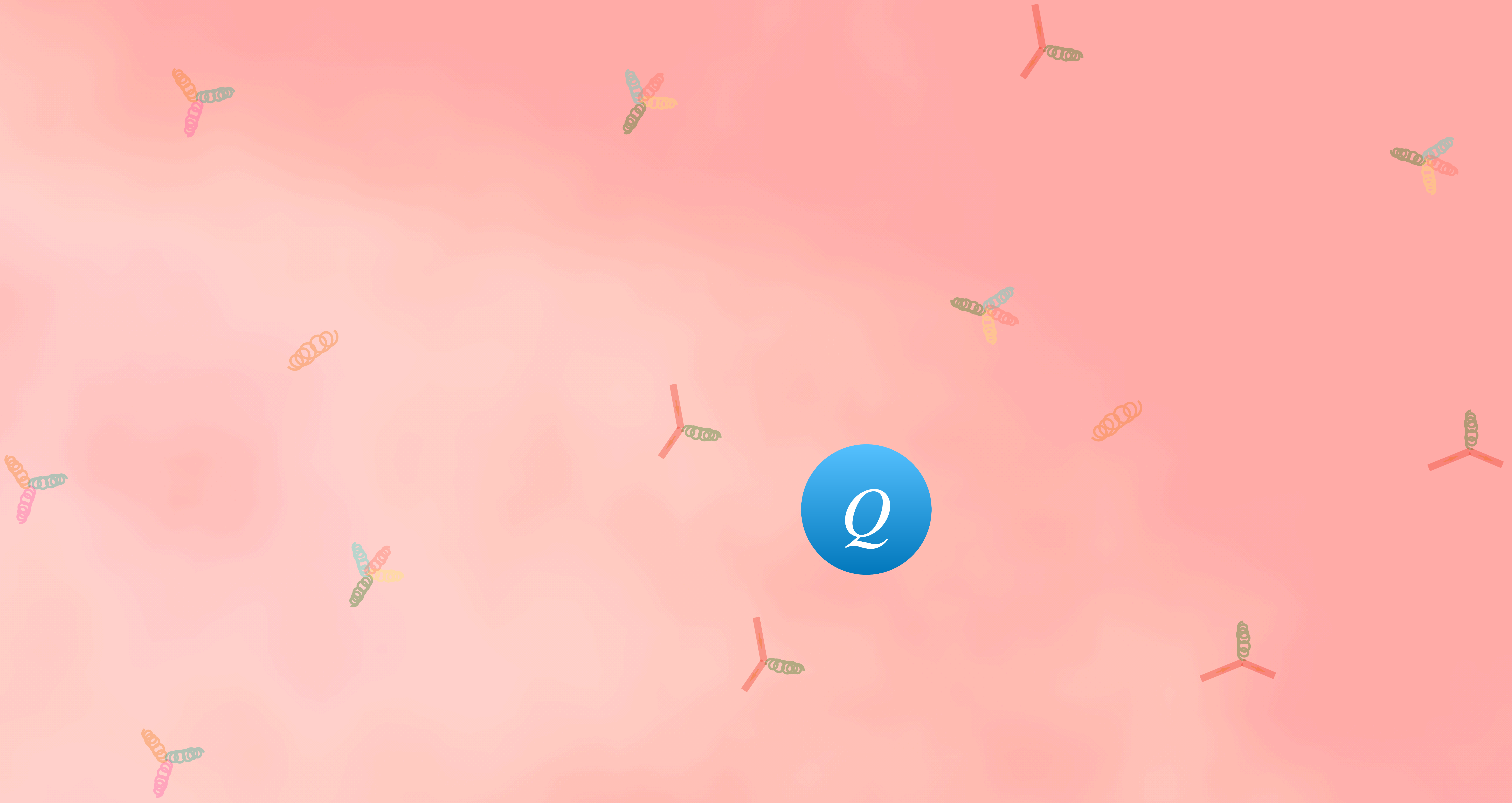
$$\frac{dp_i}{dt} = \zeta_i$$

ζ_i: stochastic force

The stochastic force encodes properties of the fluid: we can study the motion of the heavy particle to study the fluid.

Heavy Quarks in Quark-Gluon Plasma

Transport models: Fokker-Planck equation / Langevin stochastic dynamics



Langevin equation:

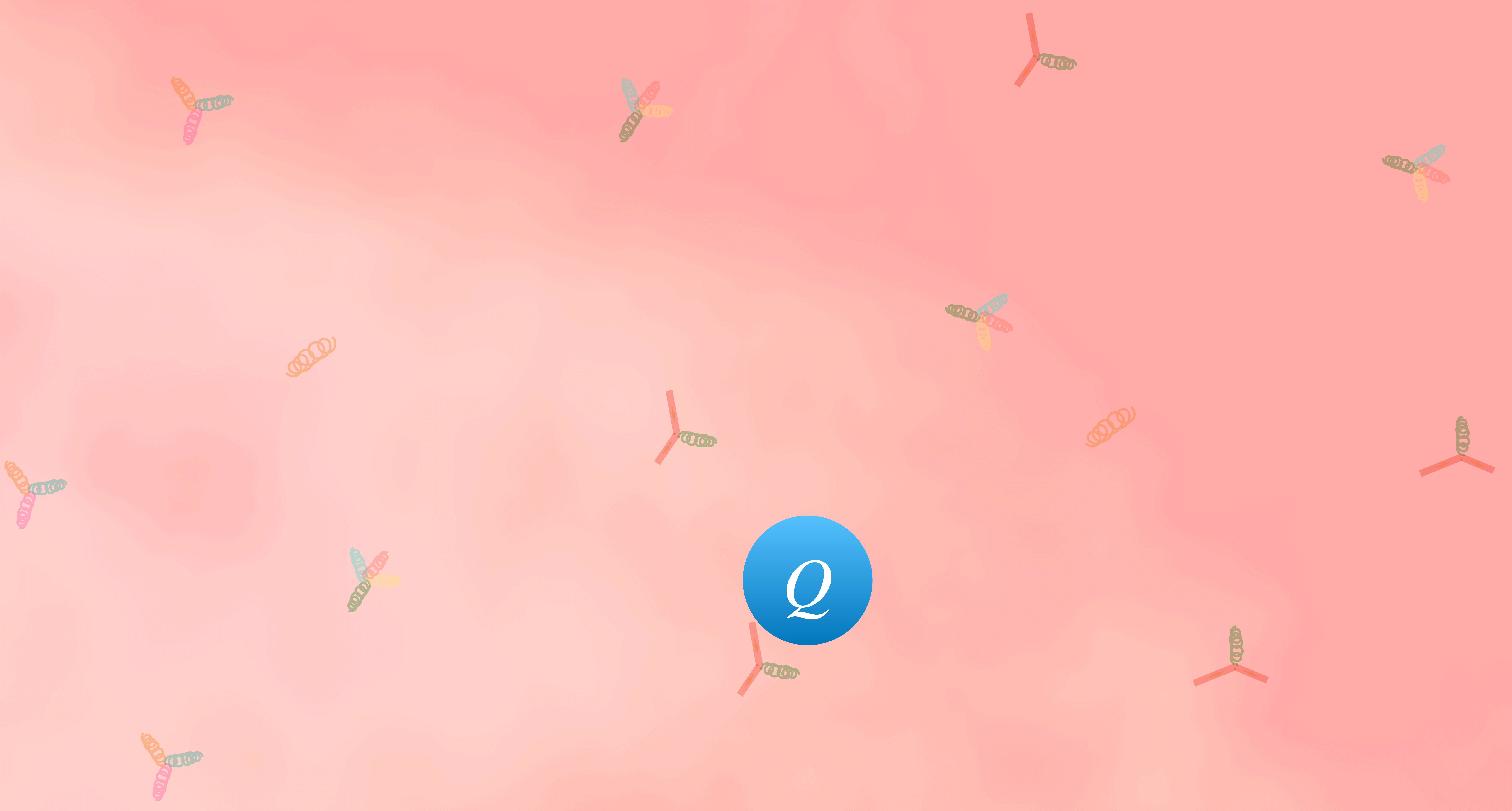
$$\frac{dp_i}{dt} = \zeta_i$$

ζ_i: stochastic force

The stochastic force encodes properties of the fluid: we can study the motion of the heavy particle to study the fluid.

Heavy Quarks in Quark-Gluon Plasma

Transport models: Fokker-Planck equation / Langevin stochastic dynamics



Langevin equation:

$$\frac{dp_i}{dt} = \zeta_i$$

ζ_i: stochastic force

The stochastic force encodes properties of the fluid: we can study the motion of the heavy particle to study the fluid.

Heavy Quarks in Quark-Gluon Plasma

Transport models: Fokker-Planck equation / Langevin stochastic dynamics

Common model: Gaussian ζ_i. Then, equilibration $\Longleftrightarrow$ Einstein relation $\nu\kappa_L = 2TF_D$ between drag F_D and diffusion κ_L

Langevin equation:

$$\frac{dp_i}{dt} = \zeta_i$$

ζ_i: stochastic force

The stochastic force encodes properties of the fluid: we can study the motion of the heavy particle to study the fluid.

Q

Heavy Quarks in Quark-Gluon Plasma

Transport models: Fokker-Planck equation / Langevin stochastic dynamics

Common model: Gaussian ζ_i. Then, equilibration $\Longleftrightarrow$ Einstein relation $v\kappa_L = 2TF_D$ between drag F_D and diffusion κ_L

Problem: explicit QFT calculations show that for $v > 0$

$$v\kappa_L \neq 2TF_D$$

Observed at weak coupling beyond leading log: Moore & Teaney hep-ph/0412346
and at strong coupling: Gubser hep-th/0612143

Langevin equation:

$$\frac{dp_i}{dt} = \zeta_i$$

ζ_i: stochastic force

The stochastic force encodes properties of the fluid: we can study the motion of the heavy particle to study the fluid.

Heavy Quarks in Quark-Gluon Plasma

Transport models: Fokker-Planck equation / Langevin stochastic dynamics

Langevin equation:

$$\frac{dp_i}{dt} = \zeta_i$$

ζ_i: stochastic force

Can we derive ζ_i from QFT?

Q

The stochastic force encodes properties of the fluid: we can study the motion of the heavy particle to study the fluid.


$T > 0$
$|\mathbf{p}\rangle$
$M \gg T$
Q


$T > 0$
$|\mathbf{p}\rangle$
$|\mathbf{p} - \mathbf{k}\rangle$
Propagate for a time t
$M \gg T$
Q
Q

$T > 0$
$|\mathbf{p}\rangle$
$|\mathbf{p} - \mathbf{k}\rangle$
Propagate for a time t
$M \gg T$
Q
Q
What is the probability distribution $P(\mathbf{k})$?

$T > 0$

$M \gg T$

What is the probability distribution $P(\mathbf{k})$?

$$P(\mathbf{k}) = P(\mathbf{k}; \mathbf{p}, T, t, \ldots)$$

for more on this, see Rajagopal, **BSH**, Wiedemann [arXiv:2501.06289, arXiv:2504.21139]

What to calculate

at leading order in T/M

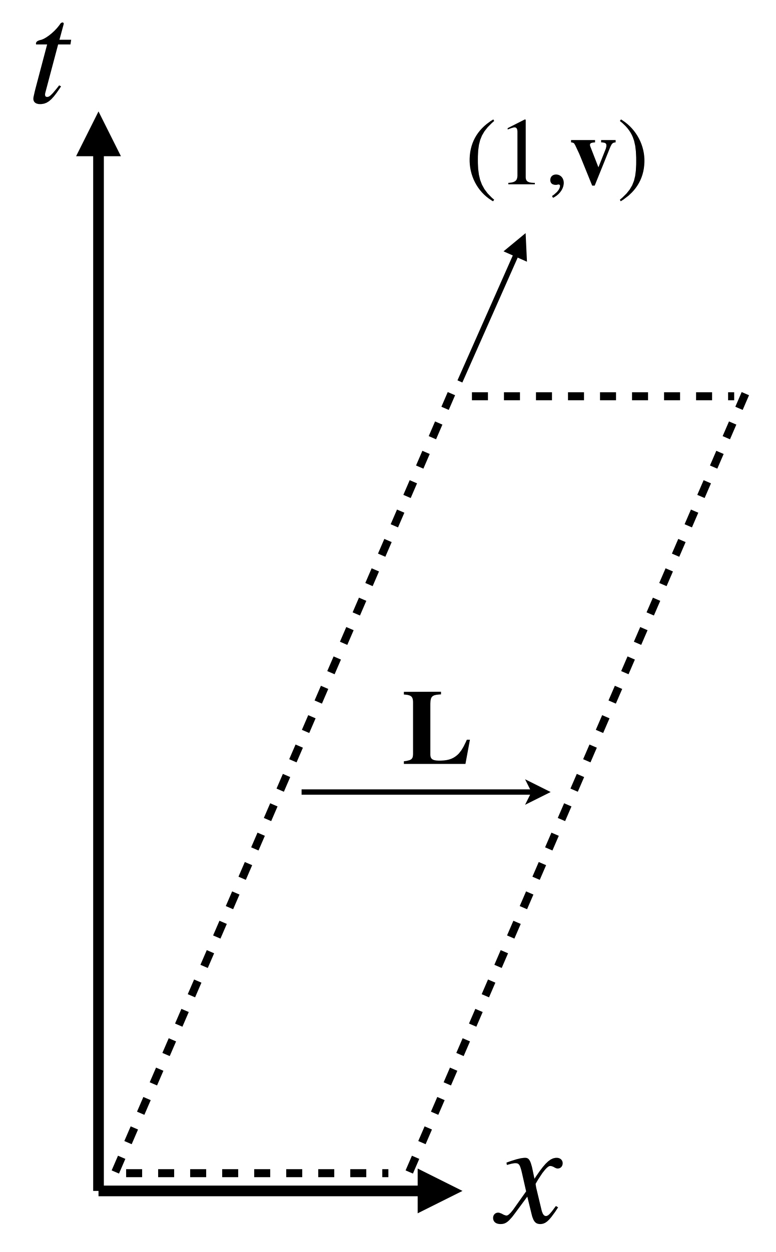
$$\mathscr{L}_{\rm HQET} = \bar{Q}_v i v^\mu D_\mu Q_v + \mathcal{O}(1/M)$$

$$\mathscr{L} = \mathscr{L}_{\rm HQET} + \mathscr{L}_{\rm light}$$

- Heavy quark effective theory (HQET) provides the answer:

$$\langle \mathbf{p} - \mathbf{k} |_{\rm out} | \mathbf{p} \rangle_{\rm in} = \int d^3x \, e^{i\mathbf{x}\cdot\mathbf{k}} \, W_{[x_f, x_i]}$$

where W is a Wilson line. (I have omitted color indices)



- The momentum change probability is given by

$$P(\mathbf{k}; \mathbf{v}(\mathbf{p})) \propto \frac{\mathrm{Tr}\{ \, |\langle \mathbf{p} - \mathbf{k} |_{\rm out} | \mathbf{p} \rangle_{\rm in}|^2 e^{-\beta H} \}}{\mathrm{Tr}\{ e^{-\beta H} \}} \propto \int d^3L \, e^{-i\mathbf{k}\cdot\mathbf{L}} \, \langle W[\mathscr{C}] \rangle_T(\mathbf{L}) \, ,$$

where $W[\mathscr{C}]$ is a Wilson loop characterized by a velocity $\mathbf{v}$ and “width” $\mathbf{L}$.

*in a homogenous system

for more on this, see Rajagopal, **BSH**, Wiedemann [arXiv:2501.06289, arXiv:2504.21139]

Dynamics

in terms of the HQ phase space distribution $\mathscr{P}(\mathbf{p})$

- The momentum distribution of heavy quarks in medium* evolves following

$$\mathscr{P}(\mathbf{p}, t+\Delta t) = \int d^3\mathbf{k}\, P(\mathbf{k}; \mathbf{v}(\mathbf{p}+\mathbf{k}), \Delta t)\, \mathscr{P}(\mathbf{p}+\mathbf{k}, t) \implies \partial_t \mathscr{P} = -T\, K(\partial_{\mathbf{p}}, \mathbf{p})\mathscr{P}$$

where $K(\mathbf{x}, \mathbf{p}) = -\frac{1}{tT} \log \left[\langle W[\mathscr{C}] \rangle_T \big(\mathbf{L} = -i\mathbf{x}; \mathbf{v} = \mathbf{v}(\mathbf{p}) \big) \right]$.

- Therefore, HQ transport $\Longleftrightarrow$ characterizing K.
  - New result 2504.21139: in any QFT,

$$K(\mathbf{x}; \mathbf{p}) = K(-\mathbf{x} - \mathbf{v}(\mathbf{p})/T; \mathbf{p}) \implies \text{Equilibration is guaranteed!}$$

*in a homogenous system

for more on this, see Rajagopal, **BSH**, Wiedemann [arXiv:2501.06289, arXiv:2504.21139]

Dynamics

in terms of the HQ phase space distribution $\mathscr{P}(\mathbf{p})$

- The momentum distribution of heavy quarks in medium* evolves following

$$\mathscr{P}(\mathbf{p}, t+\Delta t) = \int d^3\mathbf{k}\, P(\mathbf{k}; \mathbf{v}(\mathbf{p}+\mathbf{k}), \Delta t)\, \mathscr{P}(\mathbf{p}+\mathbf{k}, t) \implies \partial_t \mathscr{P} = -T K(\partial_{\mathbf{p}}, \mathbf{p})\mathscr{P}$$

where $K(\mathbf{x}, \mathbf{p}) = -\frac{1}{tT} \log \left[\langle W[\mathscr{C}] \rangle_T \big(\mathbf{L} = -i\mathbf{x}; \mathbf{v} = \mathbf{v}(\mathbf{p}) \big) \right]$.

- Therefore, HQ transport $\iff$ characterizing K.
  - New result 2504.21139: in any QFT,

Non-Gaussian generalization of Einstein relation at $|\mathbf{v}| > 0$

$$K(\mathbf{x}; \mathbf{p}) = K(-\mathbf{x} - \mathbf{v}(\mathbf{p})/T; \mathbf{p}) \implies \text{Equilibration is guaranteed!}$$

- The equilibration condition

$$K(\mathbf{x};\mathbf{p}) = K(-\mathbf{x} - \mathbf{v}(\mathbf{p})/T;\mathbf{p})$$

is in fact equivalent to

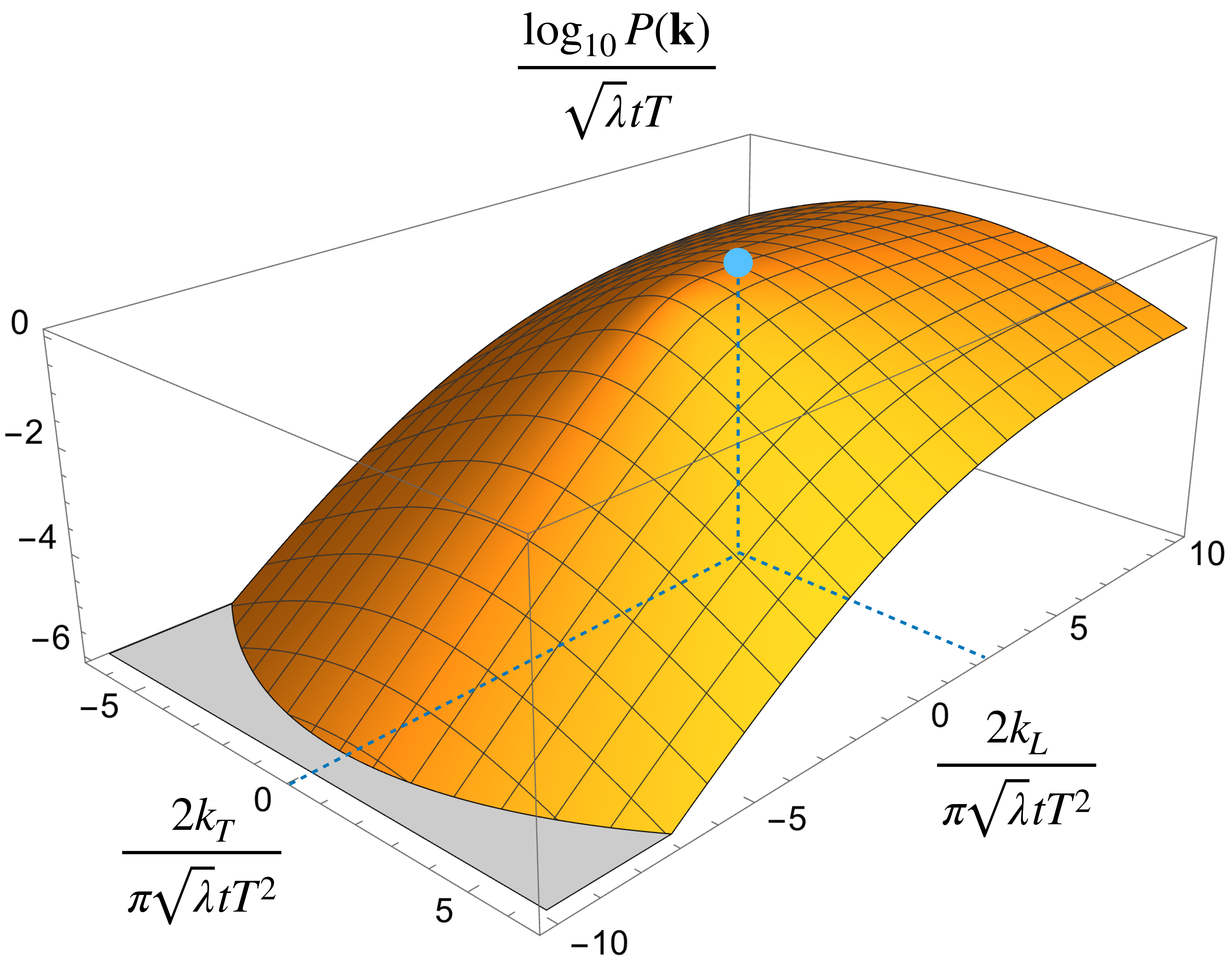
$$P(\mathbf{k};\mathbf{v})\ \exp\left[-\frac{\mathbf{v}\cdot\mathbf{k}}{2T}\right]$$

$$= P(-\mathbf{k};\mathbf{v})\ \exp\left[\frac{\mathbf{v}\cdot\mathbf{k}}{2T}\right],$$

a universal symmetry property of $P(\mathbf{k};\mathbf{v})$.

- Note: $P(\mathbf{k};\mathbf{v})$ can be highly non-Gaussian!

$$P(\mathbf{k};\mathbf{v}) = \int \frac{d^3L}{(2\pi)^3}\, e^{-i\mathbf{k}\cdot\mathbf{L}}\, \langle W[\mathscr{C}]\rangle_T(\mathbf{L})$$



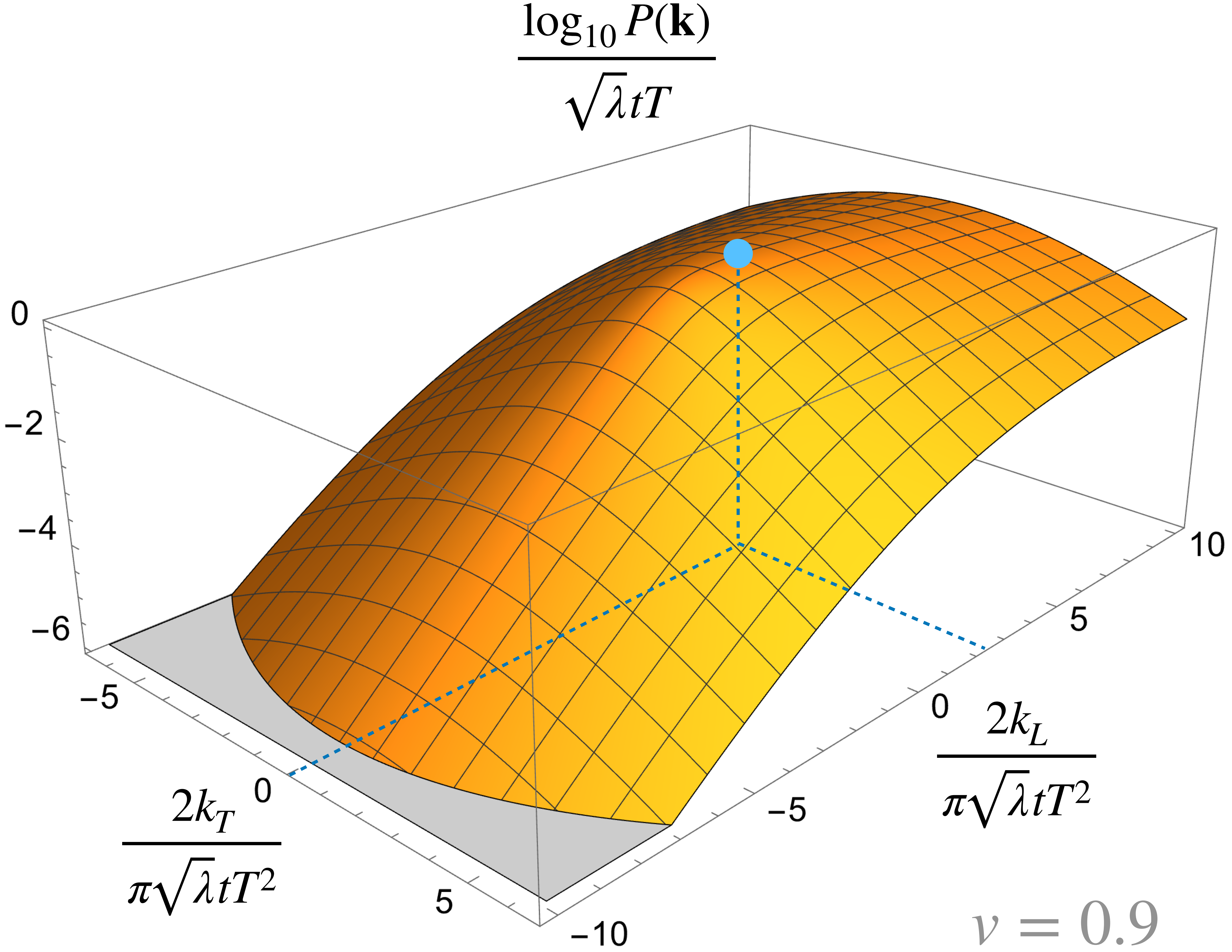
Example $P(\mathbf{k};\mathbf{v})$

$$P(\mathbf{k};\mathbf{v}) = \int \frac{d^3L}{(2\pi)^3}\, e^{-i\mathbf{k}\cdot\mathbf{L}} \langle W[\mathscr{C}]\rangle_T(\mathbf{L})$$

$\mathcal{N} = 4$ Super Yang-Mills

A tractable quantum field theory

- Non-perturbative calculations of K, or equivalently, of $P(\mathbf{k};\mathbf{v})$, are sorely needed in order to have intuition for how to interpret QCD dynamics.
- To date, we have one example where this has been done at strong coupling: $\mathcal{N} = 4$ Yang-Mills plasma [arXiv:2501.06289].
- It would be very nice to calculate these objects in QCD. Can get input from perturbative and lattice methods.



$P(\mathbf{k};\mathbf{v})$ in $\mathcal{N} = 4$ SYM

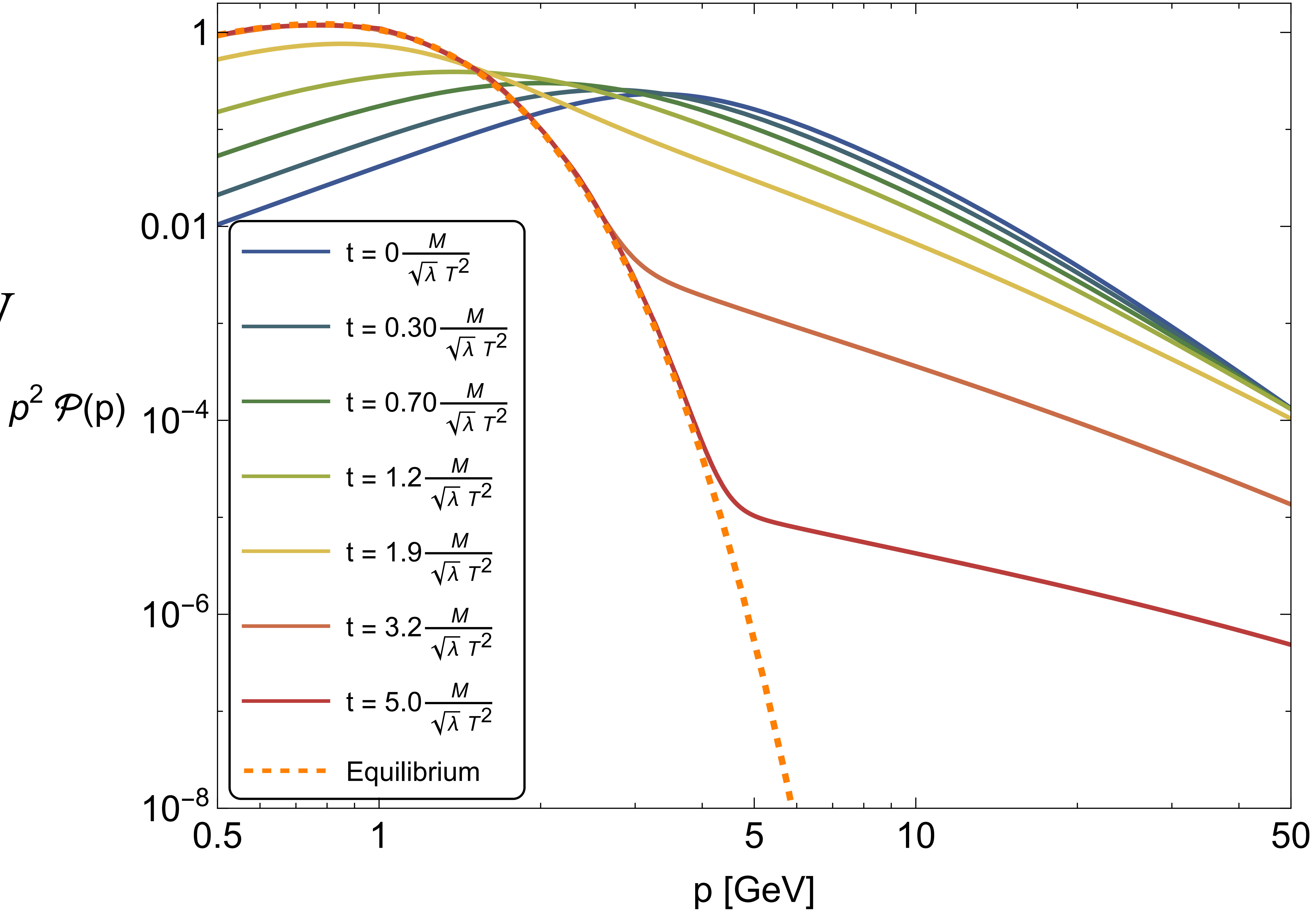
*Spherically symmetric

Static brick of strongly coupled plasma, with analytic expressions for K !

Heavy quark dynamics

$\mathcal{N} = 4$, $\lambda \gg 1$, FONLL-like initial condition*, $M = m_c$, LO in T/M

- Using
$$M/T = \frac{1.27\,\mathrm{GeV}}{200\,\mathrm{MeV}} \approx 6.4$$
- With $\lambda \sim 10$, $M = 1.27\,\mathrm{GeV}$ and $T = 200\,\mathrm{MeV}$, the timescale is
$$\frac{M}{\sqrt{\lambda}T^2} \sim 2\,\mathrm{fm}/c\,.$$
- Within a few fm, equilibration at low p, but still memory of initial spectrum at high p.

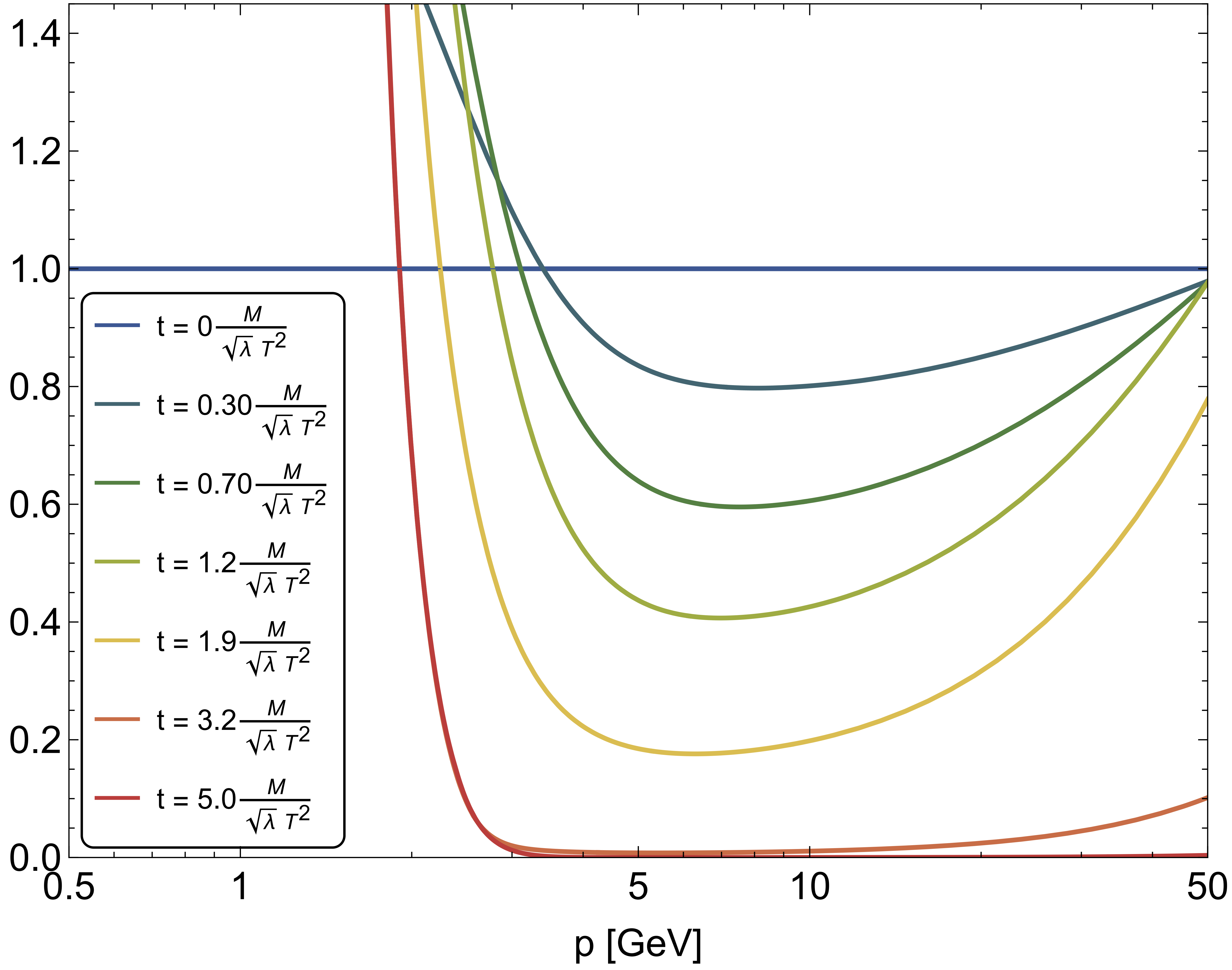


Static brick of strongly coupled plasma, with analytic expressions for K !

Medium modifications

comparing the evolved distribution with the initial spectrum

- Ratio of the number of heavy quarks with momentum p at time t to itself at the initial time.
- Minimum between $5 - 10\,\mathrm{GeV}$.
- "Bare" R_{AA} in a static brick $\frac{\mathcal{P}(p,\tau)}{\mathcal{P}(p,0)}$
    - No hadronization
    - No cold nuclear matter effects
    - No hydrodynamic medium



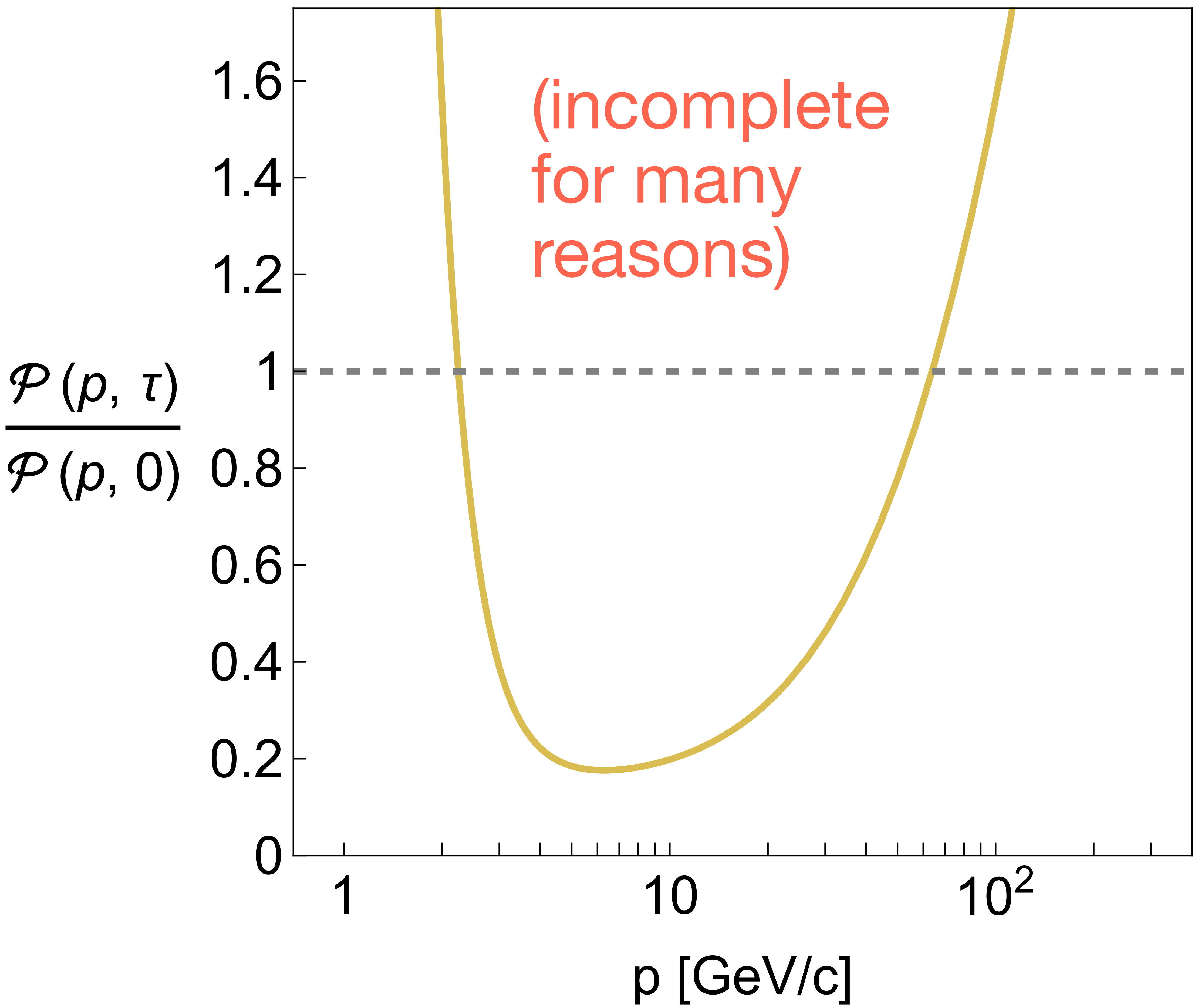
and still…

Looking forward

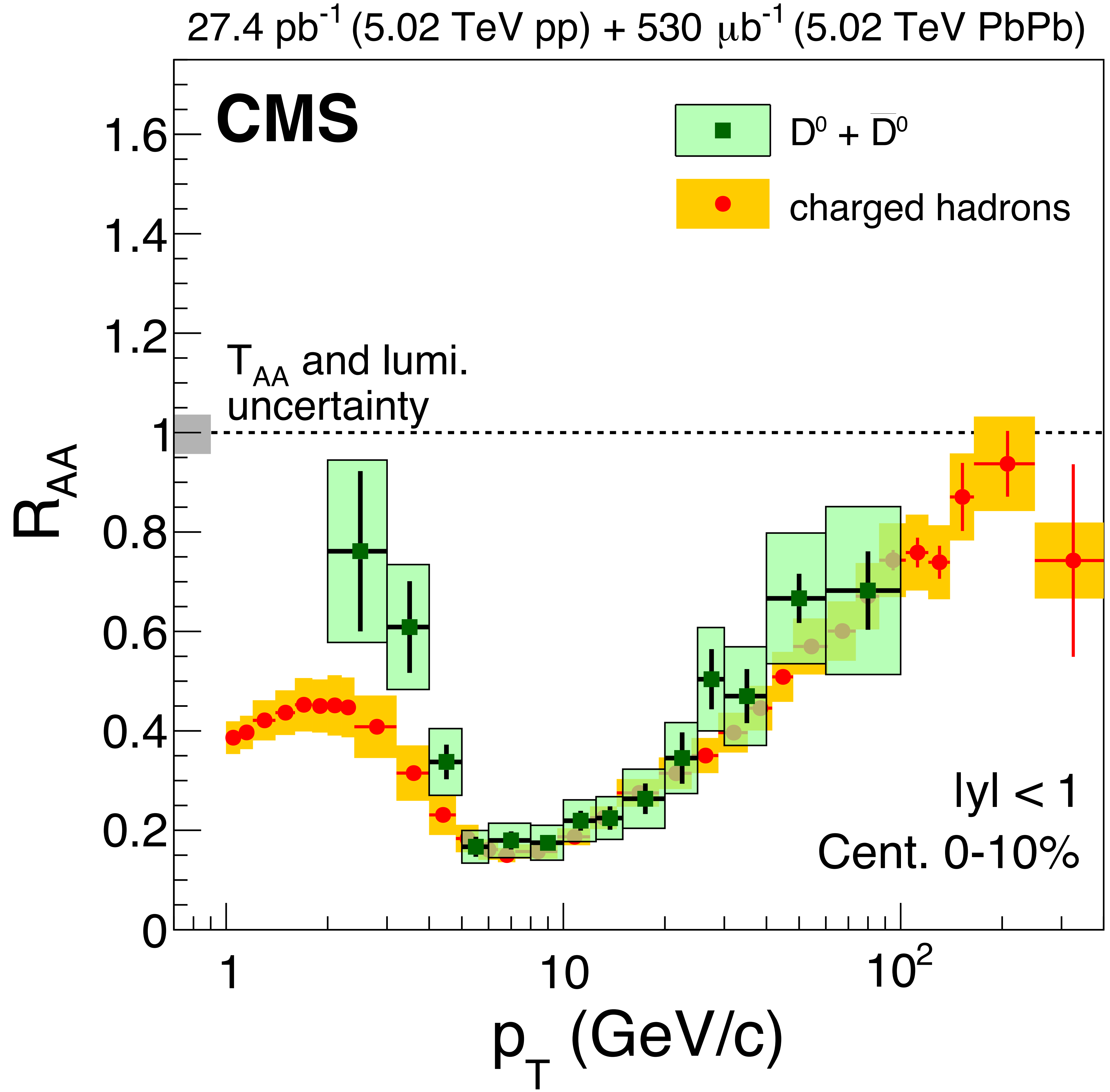
this might be very interesting

Static brick of strongly coupled plasma

$\mathcal{N} = 4$ SYM calculation at $\tau \approx 4\,\mathrm{fm}/c$



CMS *Phys.Lett.B* 782 (2018) 474-496

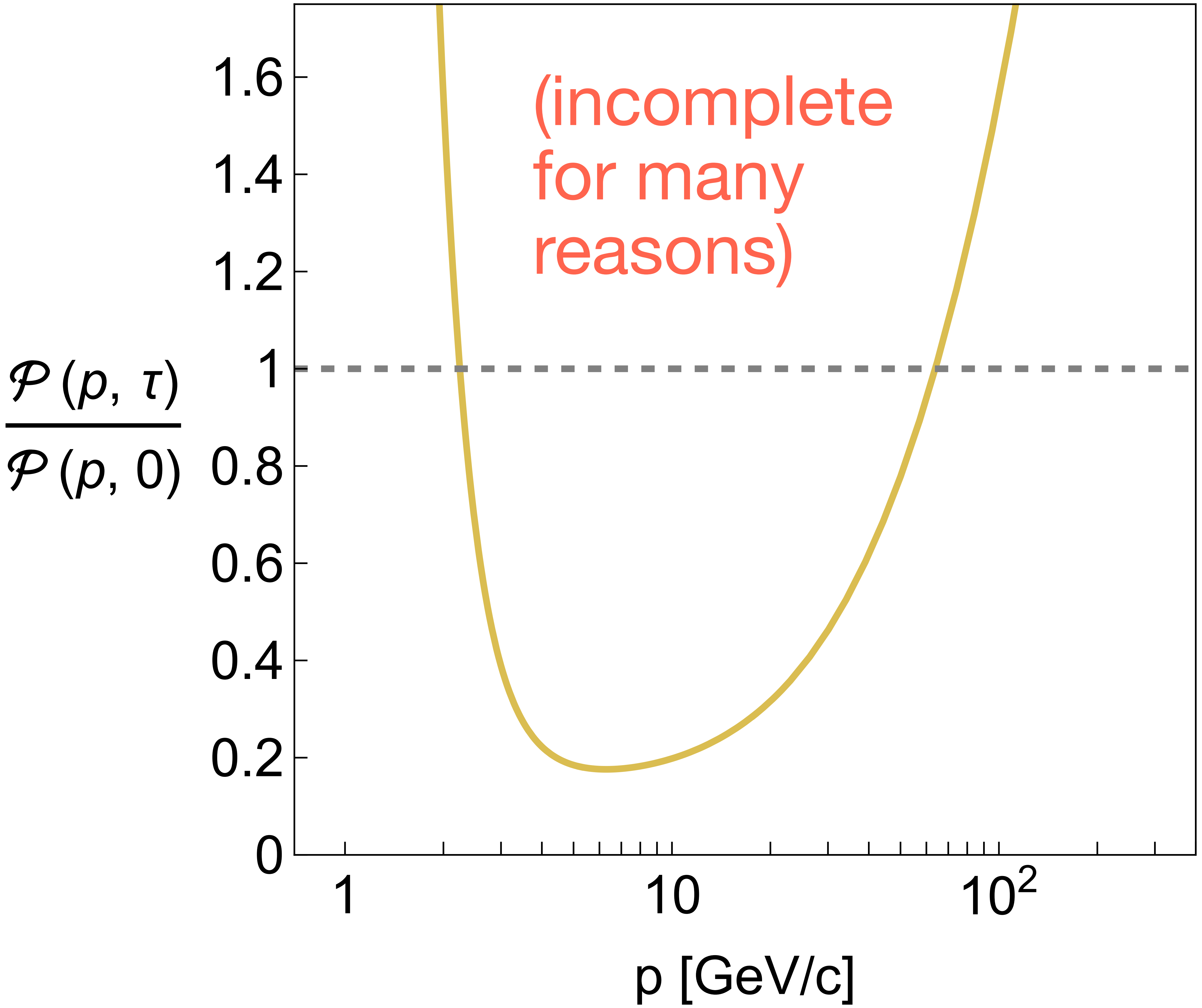


Looking forward

this might be very interesting

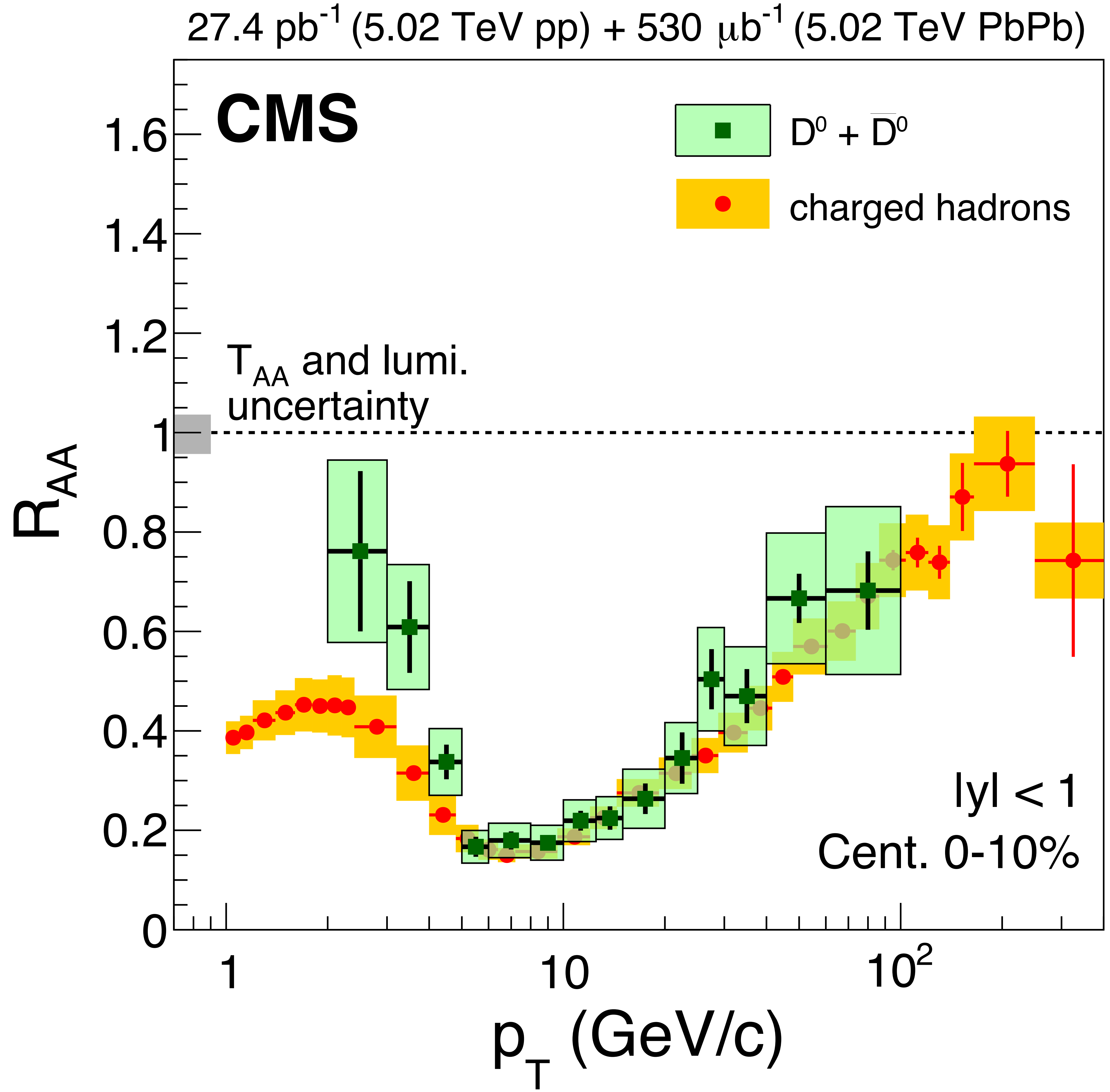
Static brick of strongly coupled plasma

$\mathcal{N} = 4$ SYM calculation at $\tau \approx 4\,\mathrm{fm}/c$



Thank you for your attention!

CMS *Phys.Lett.B* 782 (2018) 474-496



Extra slides

Analytic expressions for P

in $\mathcal{N} = 4$ SYM

- The (not normalized) momentum change probability is given by

$$P(\mathbf{k}) = \exp\left[-\sqrt{\lambda}Tt\,\tilde{S}_{\text{tot}}\left(\frac{2\mathbf{k}}{\pi\sqrt{\lambda}T^2t}\right)\right],$$

where

$$\tilde{S}_{\text{tot}}(\mathbf{C}) = \sqrt{1+\mathbf{C}^2}\frac{\pi}{32}\frac{(z_+^4 - z_-^4)^2}{z_-^5(1-z_-^4)}F_1\left(\frac{3}{2},\frac{5}{4},1;3;-\frac{z_+^4 - z_-^4}{z_-^4},\frac{z_+^4 - z_-^4}{1-z_-^4}\right) + \frac{\pi}{2}|vC_3|\,\theta(-vC_3)\,,$$

$$z_\pm^4 = \frac{1 + C_\perp^2 + (1-v^2)(1+C_3^2)}{2(1+C_\perp^2+C_3^2)} \pm \frac{\sqrt{C_\perp^4 + 2C_\perp^2(v^2 + C_3^2(1-v^2)) + (v^2 - (1-v^2)C_3^2)^2}}{2(1+C_\perp^2+C_3^2)}.$$

with $C_3 = \hat{\mathbf{v}}\cdot\mathbf{C}$, $C_\perp^2 = \mathbf{C}^2 - C_3^2$.