Formation of the Quark-Gluon Plasma

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Contents

Initial stages from point of view of hydrodynamic model for high-energy heavy-ion collisions

First part : Quark-Gluon Plasma from Hydrodynamic Model

- One of successful dynamical models
- Hydrodynamic model basics and related topics

Second part : Initial conditions of hydrodynamic model

- Phenomenological approach
  - Glauber model, TRENTO, Core-Corona model
- Beyond phenomenological approach
  - Parton cascade model, Hydrodynamization, Effective kinetic theory

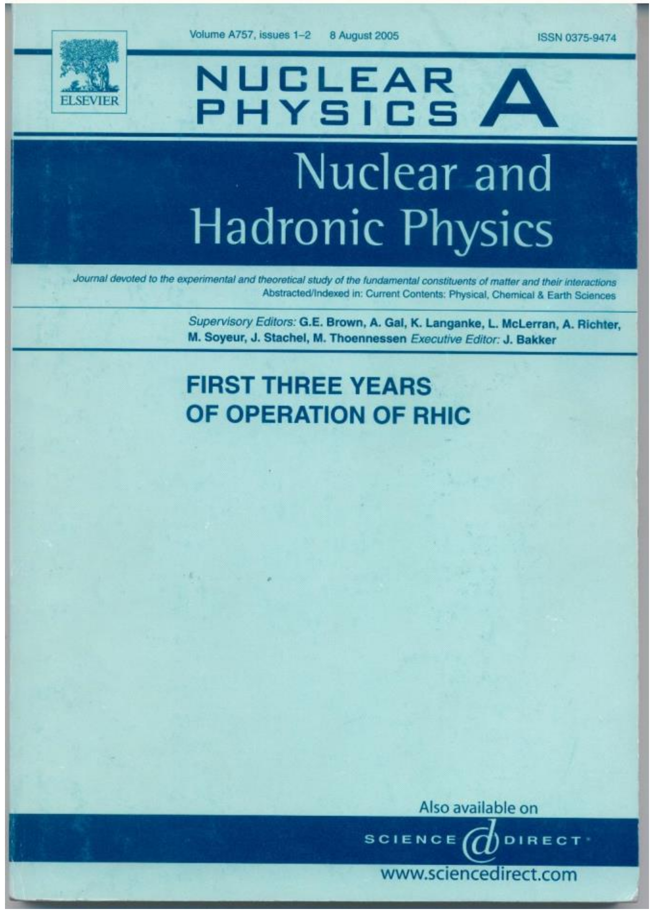
HYDRODYNAMIC MODEL

QGP Production @ RHIC

- White papers : First three years of operation of RHIC

  BRAHMS, PHOBOS, STAR, PHENIX

Nuclear Physics A757(2005)



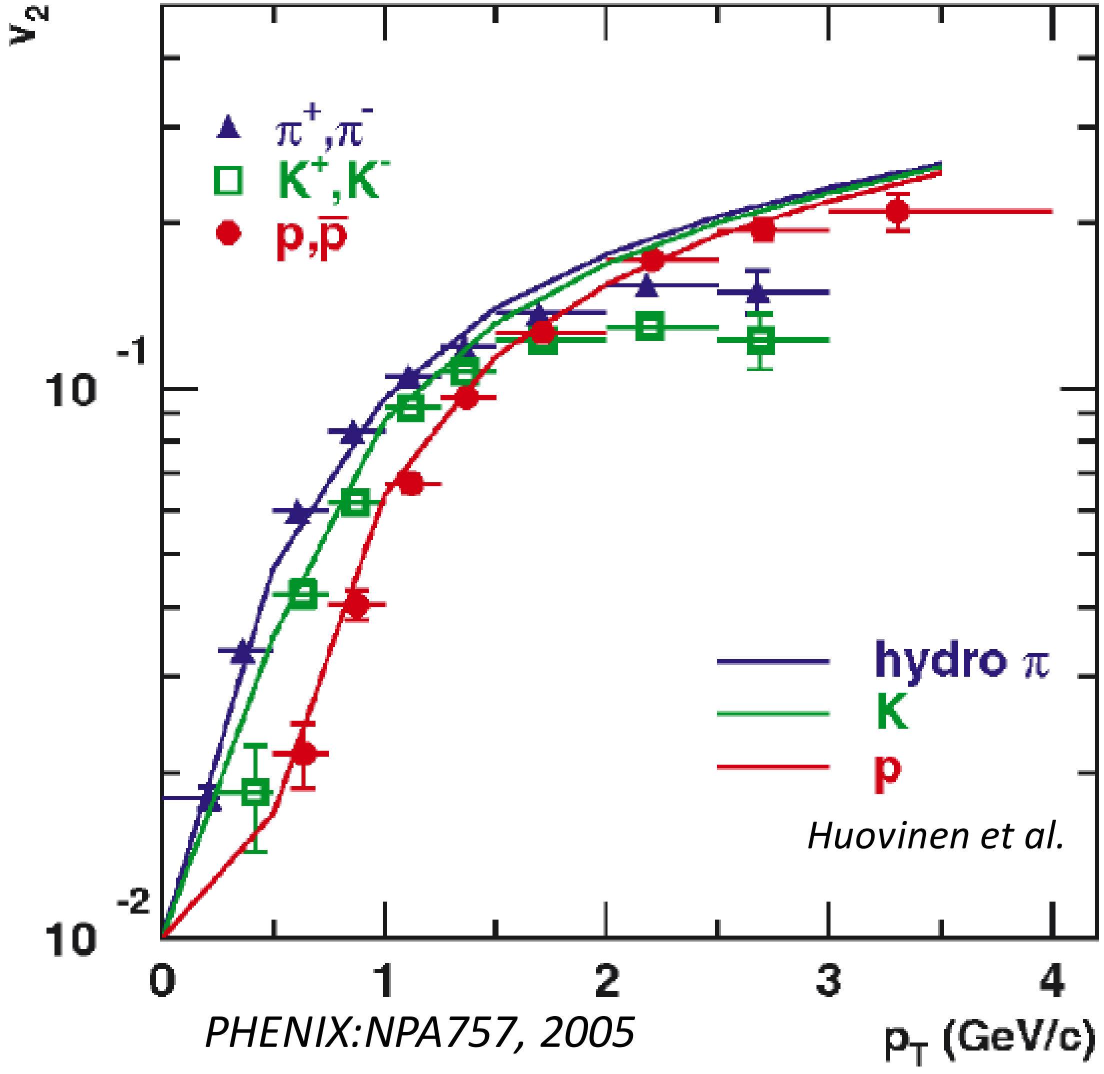
Key

- Relativistic Hydrodynamic Models
- Recombination Models
- Jet Quenching
- Color Glass Condensate

Hydrodynamic picture was widely accepted.

Formation of Quark-Gluon Plasma

- **Strong** elliptic flow



PHENIX:NPA757, 2005

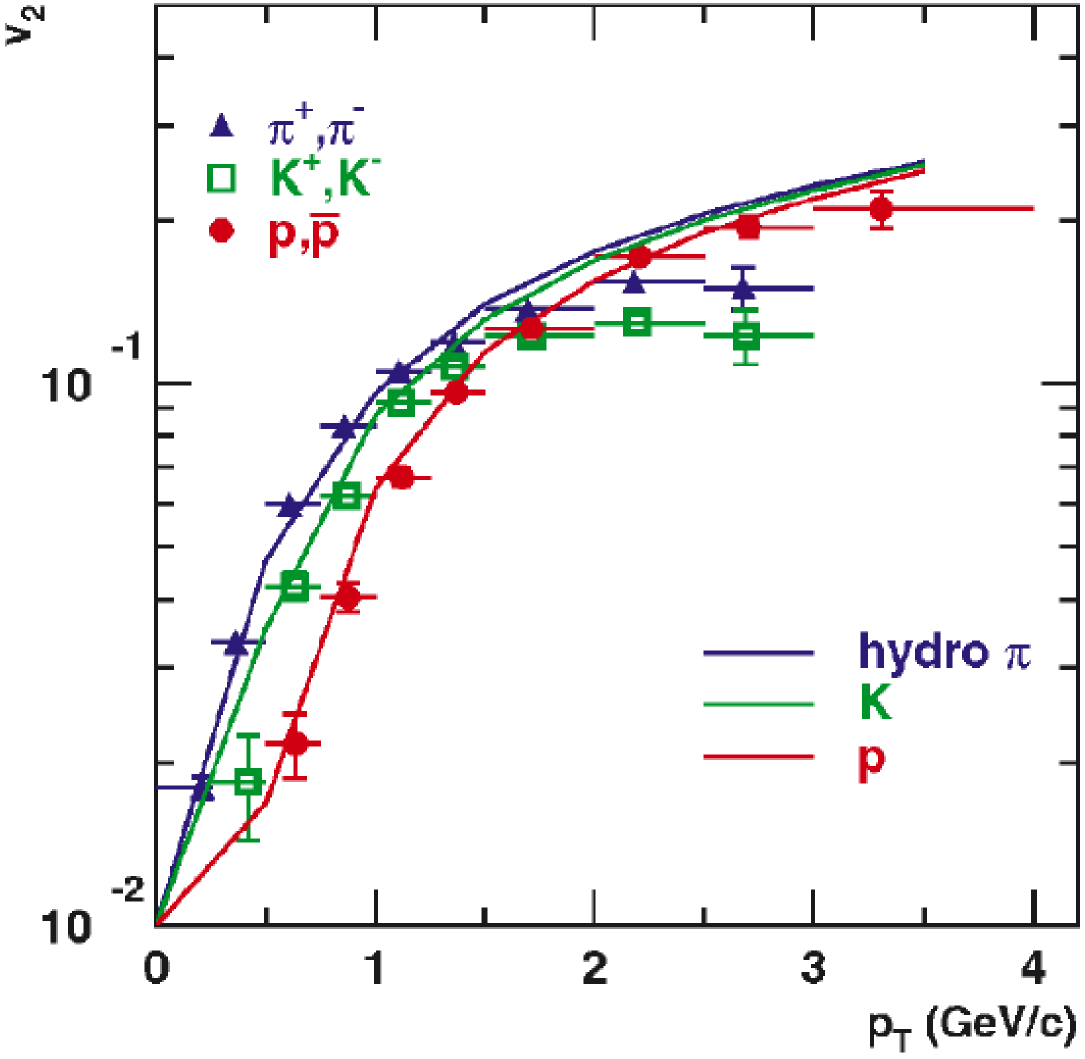
Key issues:

•Perfect match to experimental data at low P_T

•mass dependence

Formation of Quark-Gluon Plasma

- **Strong** elliptic flow



Key issues:

•Particle mass dependence of collectivity

•Early thermalization

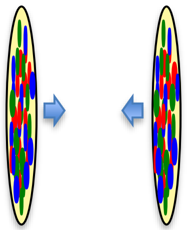
initial time of hydrodynamic models

τ_0~ 0.6 fm

Hirano, PRC 65, 011901(R)2001

Hydrodynamic Model

collisions



Experimental data

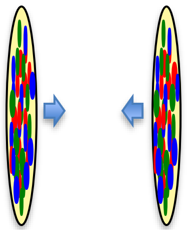
Hadrons
Leptons
Photons

Hydrodynamic Model

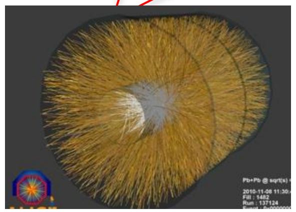
collisions

Model

Experimental data



QGP
Production?



Hadrons
Leptons
Photons

Multiple particle production

➡ Hydrodynamic picture

Landau
Bjorken

Success of hydrodynamic model at RHIC
One of important phenomenological models

Relativistic Hydrodynamic Model

- Relativistic hydrodynamic equation

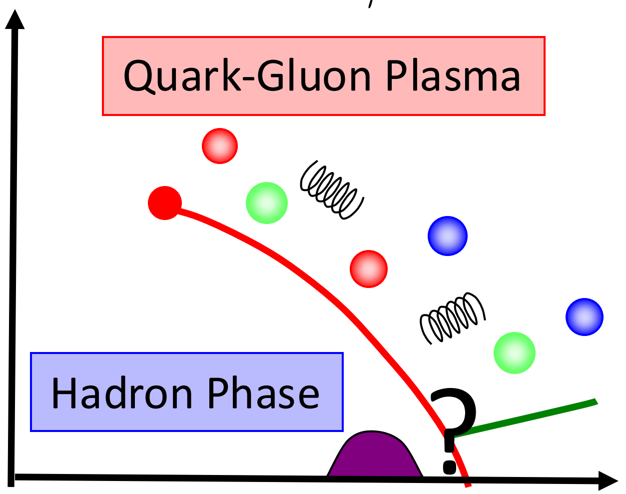
$$\partial_\mu T^{\mu\nu} = 0$$

Energy momentum tensor

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}$$

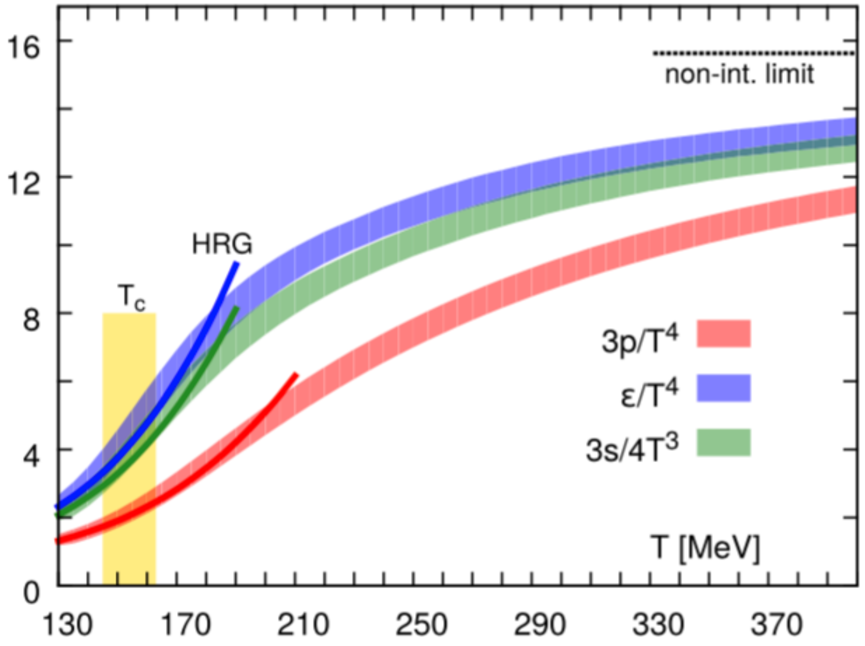
ϵ : energy density p : pressure u^μ : four velocity $\frac{1}{\gamma}(1, \vec{v})$

- One more equation : equation of states
  Close relation to QCD phase diagram and QCD phase transition

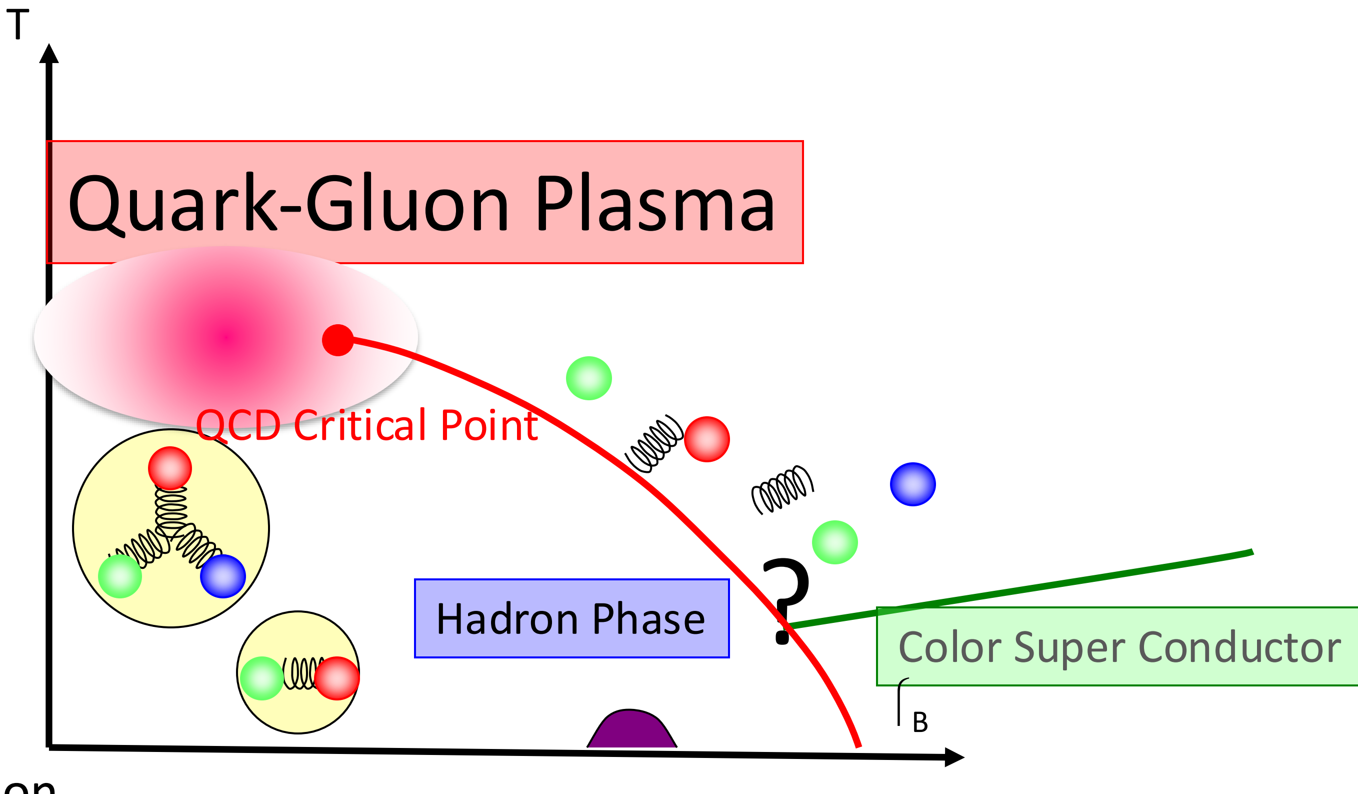


Equation of State

- Equation of State
  - Lattice QCD *HotQCD,PRD90,094503(2014)*



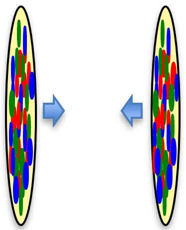
(2+1) flavor, Highly improved staggered quark action
Nt=6,8,10,12,Ns=4Nt → continuum limit
Parametrization of EoS $T_c \sim 155$ MeV



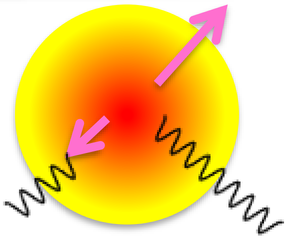
finite μ: sign problem

Hydrodynamics

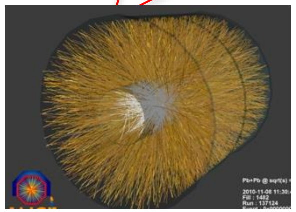
collisions



hydrodynamics



Experimental data



Hydrodynamics

QCD phase diagram
EoS: lattice QCD

Hadrons
Leptons
Photons

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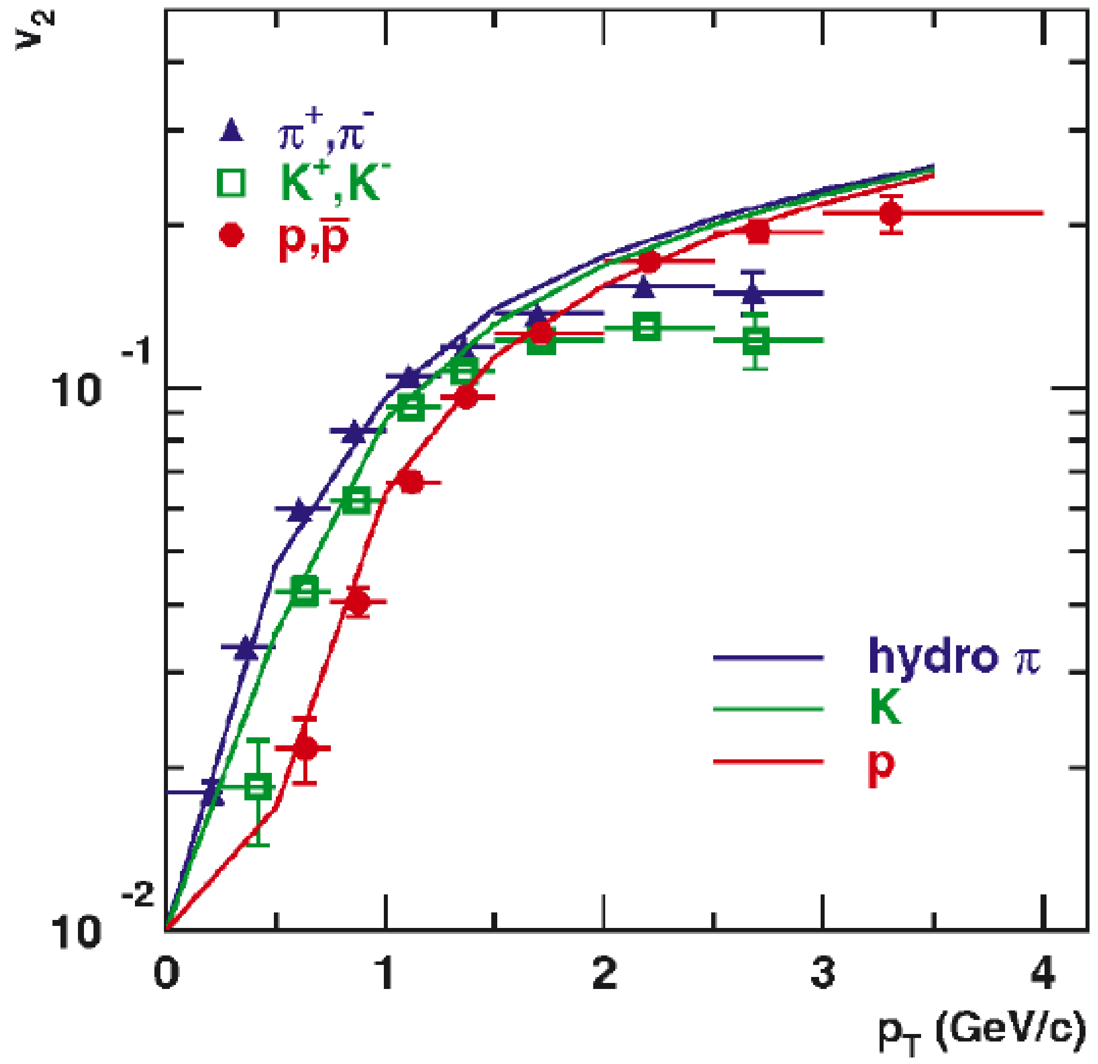
Formation of Quark-Gluon Plasma

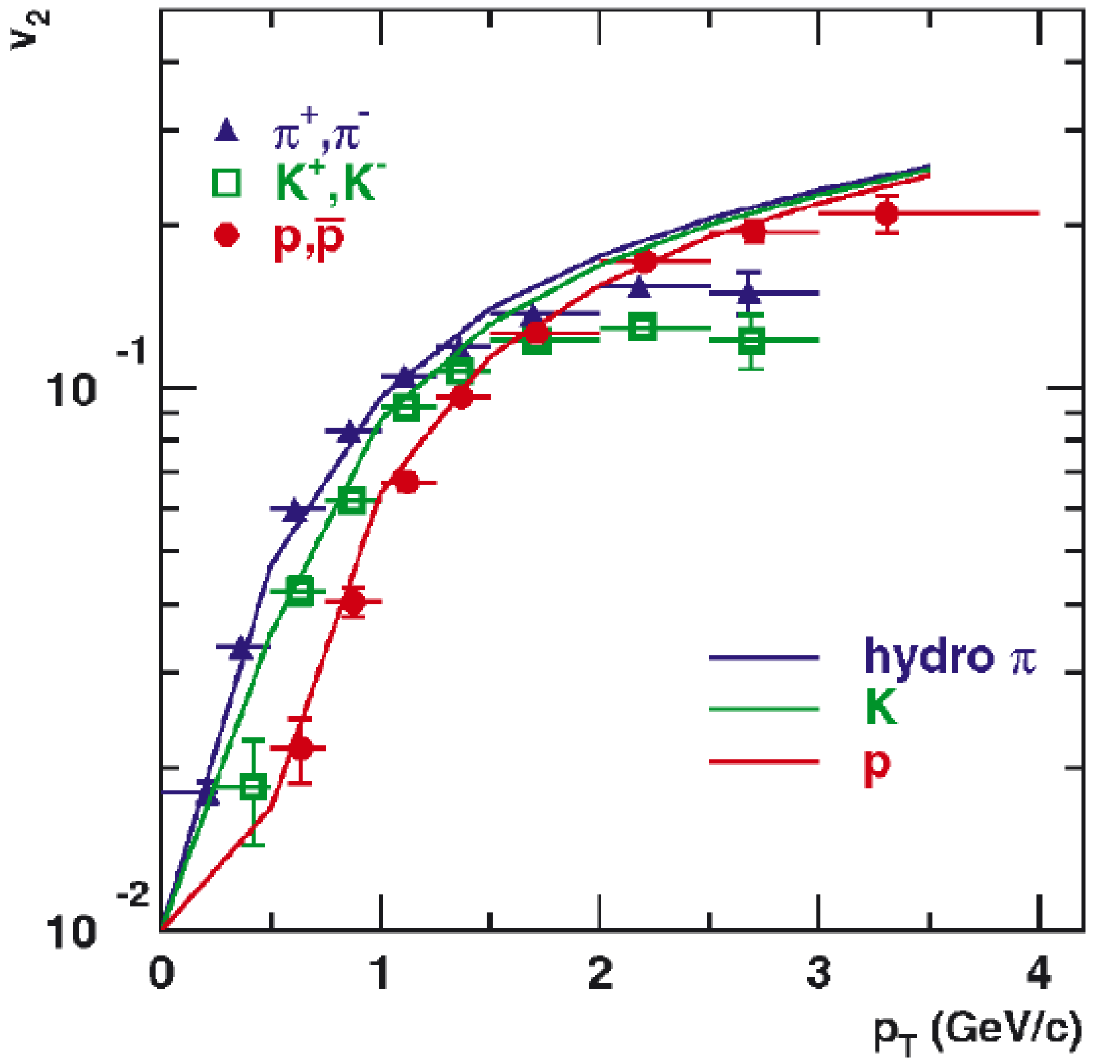
- **Strong** elliptic flow



This result gives us idea of formation of QGP behaves like fluid.

Formation of Quark-Gluon Plasma

- **Strong** elliptic flow



This result gives us idea of formation of QGP behaves like fluid.

Caveat !

- Ideal hydrodynamics
- Minimum bias
- mid rapidity, (2+1) d
- Equation of state : first phase transition
- Without chemical equilibrium
- No final state interactions

➡ Relativistic viscous hydrodynamics

Relativistic Viscous Hydrodynamics

- Relativistic viscous hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0$$

Energy Momentum tensor

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} + \Delta T^{\mu\nu}$$

$$\Delta T^{\mu\nu} = q^\mu u^\nu + q^\nu u^\mu + \pi^{\mu\nu}$$

$$\partial_\mu N^\mu = 0$$

$$N^\mu = nu^\mu + \nu^\mu$$ Net charge current

Four velocity

Eckart frame: $u_E^\mu \equiv \frac{N^\mu}{\sqrt{N \cdot N}}$

$$\nu^\mu = 0$$

Landau frame: $u_L^\mu \equiv \frac{T^\mu_\nu u_L^\nu}{\sqrt{u_L^\alpha T_\alpha^\beta T_{\beta\gamma} u_L^\gamma}}$

$$q^\mu = 0$$

Relativistic Viscous Hydrodynamics

- Relativistic Hydrodynamic Equation

$$\partial_\mu T^{\mu\nu} = 0$$

Energy Momentum Tensor

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu} + \Delta T^{\mu\nu}$$

$$\Delta T^{\mu\nu} = q^\mu u^\nu + q^\nu u^\mu + \pi^{\mu\nu}$$

$$\partial_\mu N^\mu = 0$$

$$N^\mu = nu^\mu + \nu^\mu$$ Net charge current

q^μ $\pi^{\mu\nu}$ ν^μ

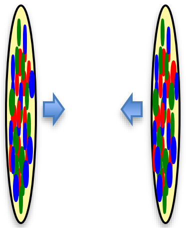
various approaches for fixing the parameters
Ex.
Phenomenology
From kinetic theory

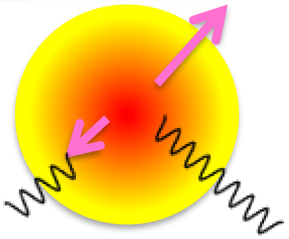
Viscous Effect

collisions

hydrodynamics

Experimental data





Hydrodynamics

QCD phase diagram

EoS: lattice QCD

Transport coefficients

Hadrons
Leptons
Photons

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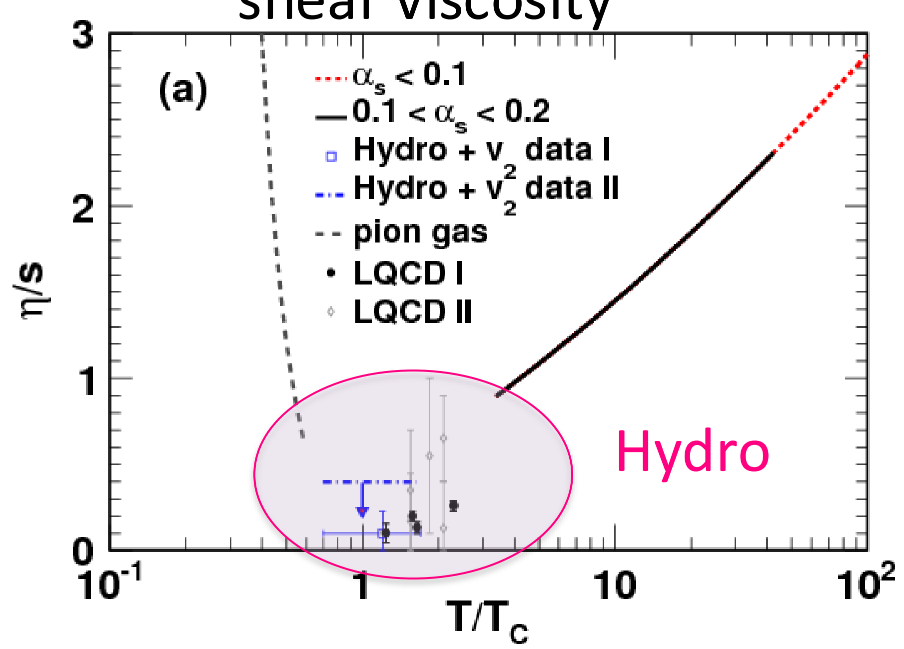
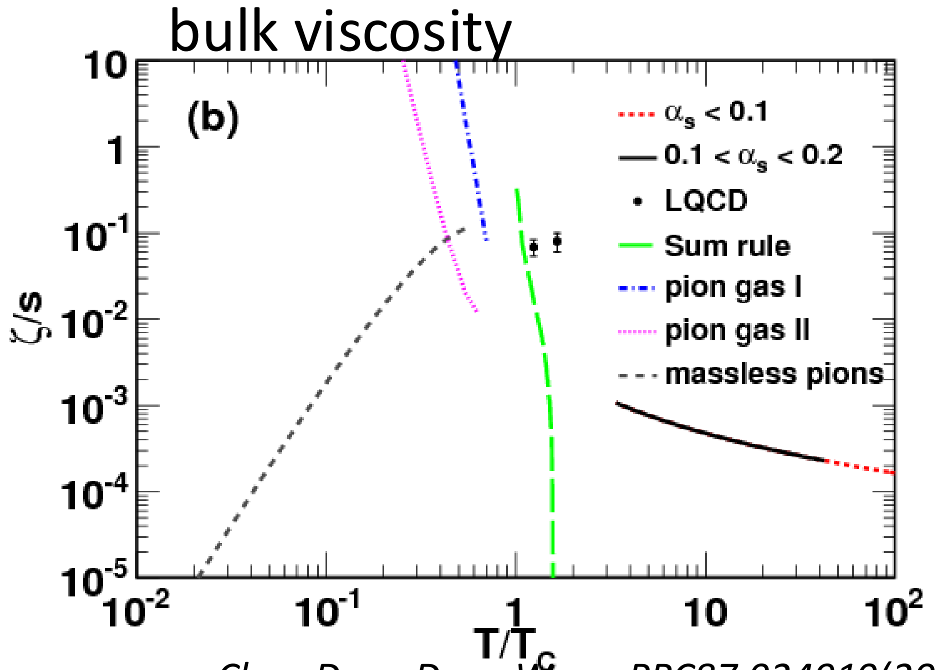
Temperature dependent Transport Coefficients

- Status for transport coefficients from theories





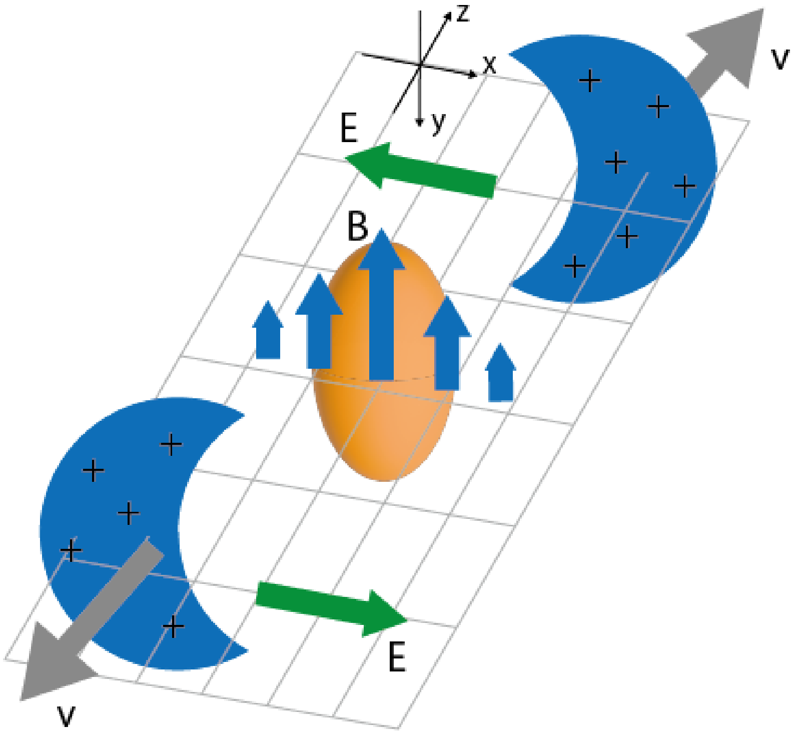
Chen,Deng,Dong,Wang,PRC87,024910(2013)

- Shear viscosity takes the minimum around T_c.
  Cf. $\eta/s=1/4\pi$ AdS/CFT
- Bulk viscosity
  Temperature dependence is unclear.

Bayesian analysis: Duke, JETSCAPE, Trajectum

Electromagnetic Fields in Heavy Ion Collisions

- Strong Electromagnetic field
  - Au + Au ($\sqrt{s_{NN}} = 200$ GeV) : 10^{14} T ~10 m_π^2
  - Pb + Pb ($\sqrt{s_{NN}} = 2.76$ TeV) : 10^{15} T



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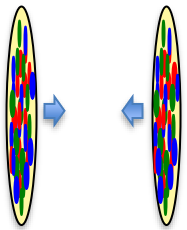
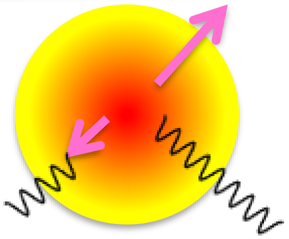
Electromagnetic Fields

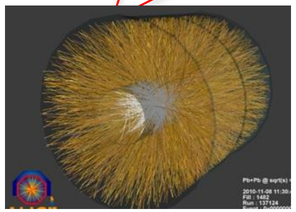
collisions

hydrodynamics

Experimental data







Hydrodynamics

QCD phase diagram

EoS: lattice QCD

Electromagnetic fields

Magnetohydrodynamics

Hadrons
Leptons
Photons

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Magnetohydrodynamics

- Magnetohydrodynamics

  - Conservation law

$$\nabla_\mu T^{\mu\nu} = 0$$

Energy momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_m + T^{\mu\nu}_f$$

Maxwell eq.

$$\nabla_\mu F^{\mu\nu} = -J^\nu$$

matter

$$T^{\mu\nu}_m = (e+p)u^\mu u^\nu + pg^{\mu\nu}$$

EM field

$$T^{\mu\nu}_f = F^{\mu\lambda}F^\nu_\lambda - \frac{1}{4}g^{\mu\nu}F^{\lambda\kappa}F_{\lambda\kappa};$$

$$\nabla_\mu T^{\mu\nu}_f = J_\mu F^{\mu\nu}$$

$$\nabla_\mu T^{\mu\nu}_m = -J_\mu F^{\mu\nu}$$

  - Ohm's law *Blackman and Field, PRL71,3481(1993)*

$$J^\mu = \sigma F^{\mu\nu}u^\nu + qu^\mu$$

Electrical conductivity

$$q = -J^\mu u_\mu$$

Magnetohydrodynamics

- Magnetohydrodynamics
  - Conservation law

$$\nabla_\mu T^{\mu\nu} = 0$$

Energy momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_m + T^{\mu\nu}_f$$

Maxwell eq.

$$\nabla_\mu F^{\mu\nu} = -J^\nu$$

matter

$$T^{\mu\nu}_m = (e+p)u^\mu u^\nu + pg^{\mu\nu}$$

+viscosity, *Hattori..*

EM field

$$T^{\mu\nu}_f = F^{\mu\lambda}F^\nu_\lambda - \frac{1}{4}g^{\mu\nu}F^{\lambda\kappa}F_{\lambda\kappa};$$

$$\nabla_\mu T^{\mu\nu}_f = J_\mu F^{\mu\nu}$$

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  - Ohm's law *Blackman and Field, PRL71,3481(1993)*

$$J^\mu = \sigma F^{\mu\nu}u^\nu + qu^\mu$$

Electrical conductivity

$$q = -J^\mu u_\mu$$

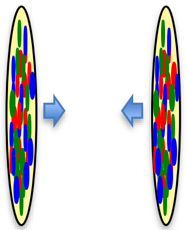
Charge diffusion, *Dash et al., PRD107,056003(2023)*

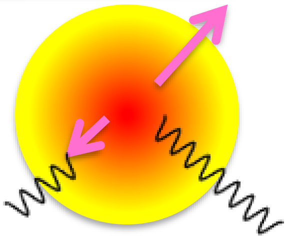
Electromagnetic Fields

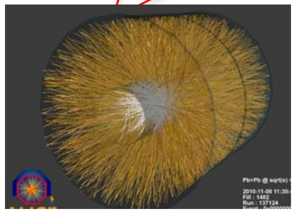
Experimental data

collisions

hydrodynamics







Hydrodynamics

QCD phase diagram

EoS: lattice QCD

Electromagnetic fields

Hadrons
Leptons
Photons

Understanding of QGP property

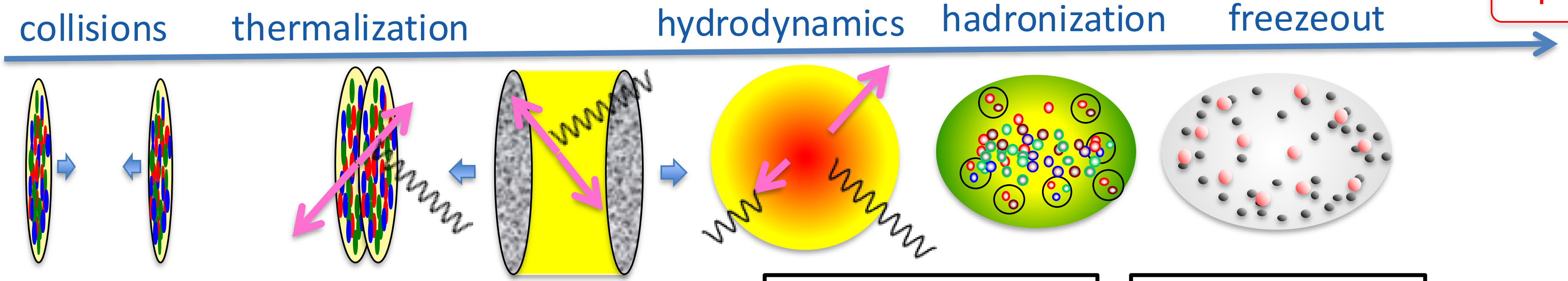
Understanding of QGP Property

Hydrodynamics has close relation to the QGP bulk property

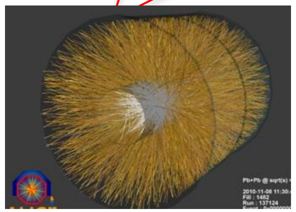
Physical property	Related Observables	Quantitative analysis
Charge conductivity	Charge dependent flow, EM probes..	Just started
Shear viscosity	Azumithal anisotoropy v_n	○
Bulk viscosity	P_{T} distributions	○
Diffusion coefficient	Jet energy loss	○

Space-Time Evolution of HIC

- Hadronization



Experimental data



Hydrodynamics

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EoS: lattice QCD

Electromagnetic fields

Hadronization

Particlization

Final state interactions

UrQMD, JAM, SMASH..

Recombination Model

Key ingredient of QGP formation

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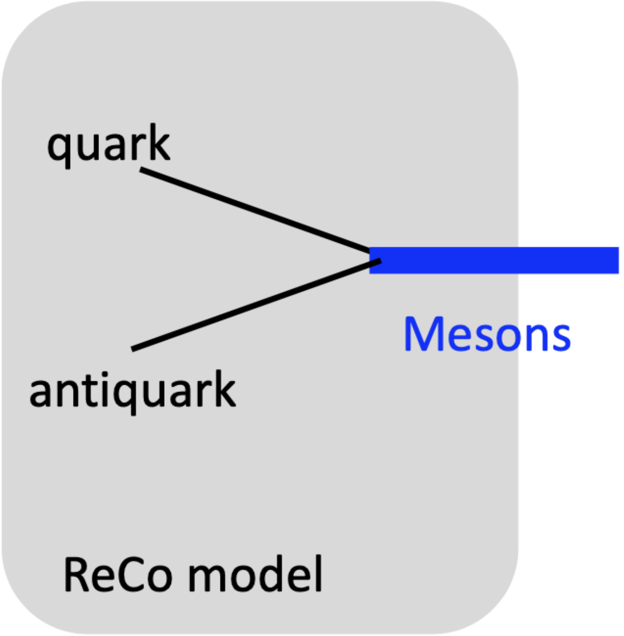
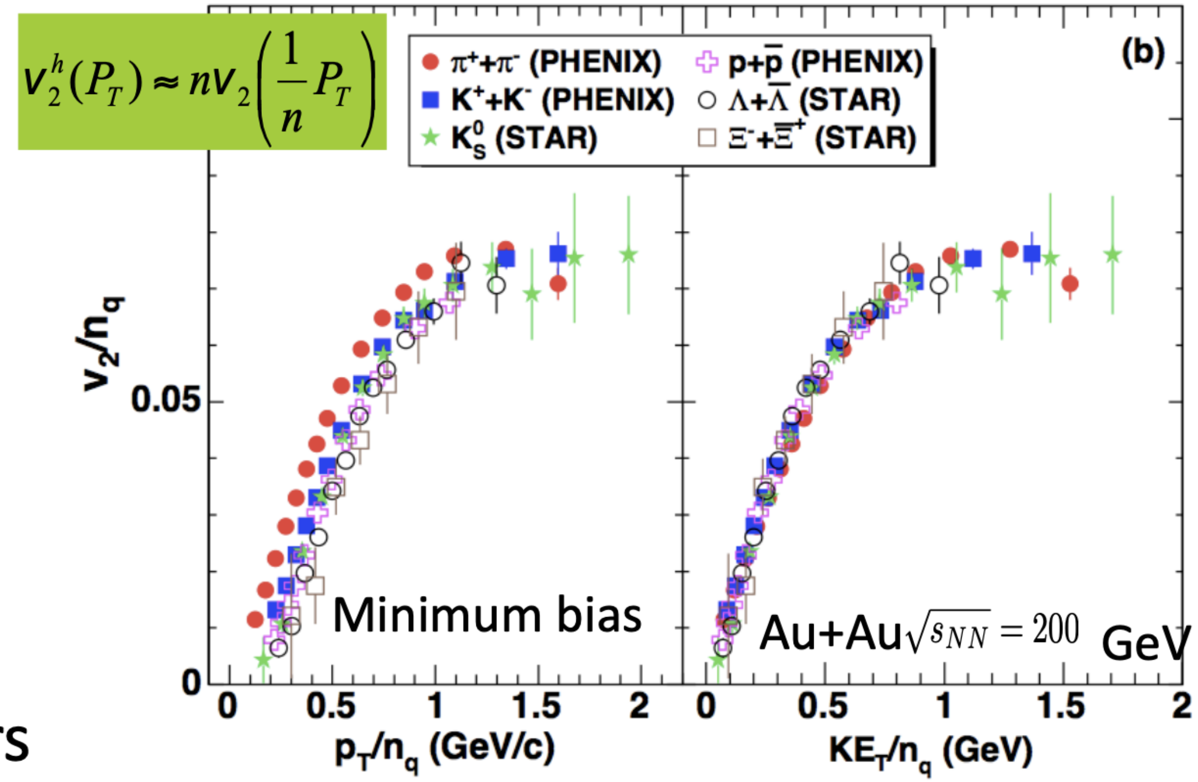
Recombination Model

Fries, Mueller, CN and Bass, PRC68(2003)

- One of successful models



Ex: Quark number scaring



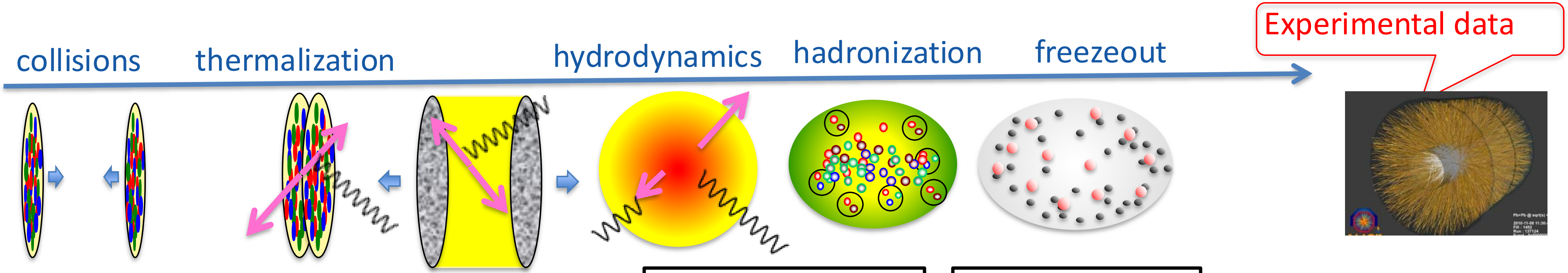
PHENIX,nucl-ex:0608033

- Baryon/Meson ratios
- Nuclear modification factors
- Quark number scaling in elliptic flow

➡ Radiative recombination model

Space-Time Evolution of HIC

•



Initial Condition

Hydrodynamics

QCD phase diagram

EoS: lattice QCD

Electromagnetic fields

$$\partial_\mu T^{\mu\nu} = 0$$

Hadronization

Particlization

Final state interactions

Recombination Model

Key ingredient of QGP formation

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INITIAL CONDITIONS
OF HYDRODYNAMIC MODEL

Initial Conditions

Assumption: Thermalization is achieved in a short time ~O(1) fm.

- Parametrization: smoothed distribution -> fluctuating initial condition
  - Glauber Model
    - Number of binary nucleon-nucleon collisions

$$N_{\mathrm{coll}}(b) = \sigma_{NN}^{inel} \int d^2s\, T_A(s)\, T_B(|\vec{b} - \vec{s}|)$$

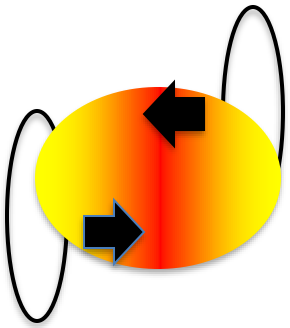
    - Number of participant nucleons

$$N_{\mathrm{part}}(b) = \int d^2s\, T_A(s) \left[1 - \left(1 - \sigma_{NN}^{inel} T_B(|\vec{b} - \vec{s}|)\right)^A\right] + (A \leftrightarrow B)$$

    - Superposition of them

$$N(b) = \alpha N_{\mathrm{coll}}(b) + (1-\alpha) N_{\mathrm{part}}(b)$$

  - TRENTO



σ_{NN}^{inel} : Inelastic NN cross section

$T_A(s)$ Nuclear thickness function

ρ_A : Woods-Saxon distribution

TRENTO

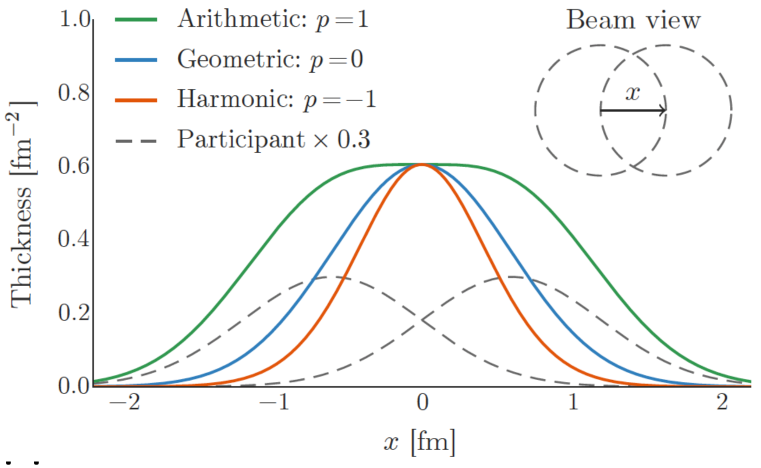
Moreland, Bernhard, Bass, PRC 92 (2015) 1, 011901
Weiyao et al TRENTO-3d

- Phenomenological model can mimic the shape of initial conditions

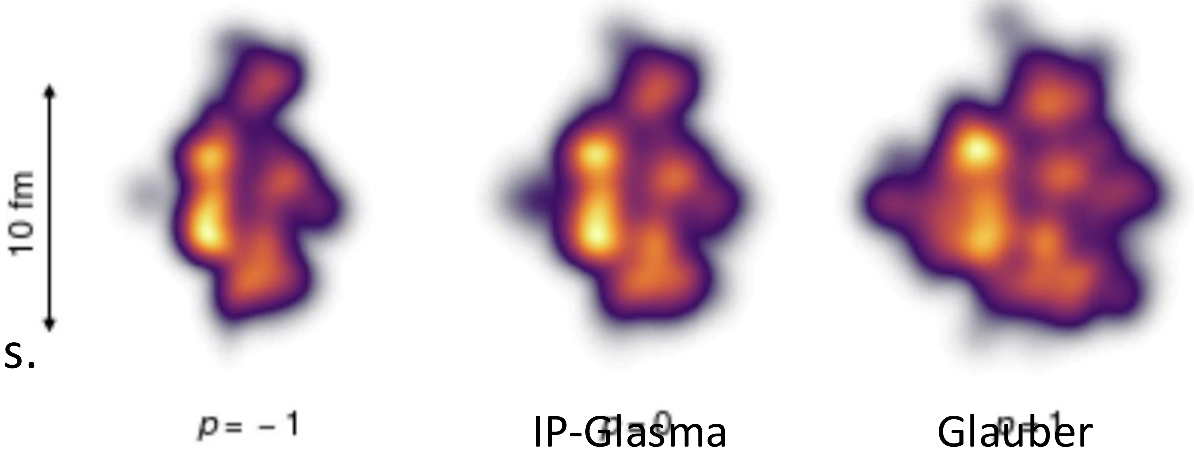
$$T_R(p; T_A, T_B) \equiv \left(\frac{T_A^p + T_B^p}{2} \right)^{1/p}$$

p: parameter interpolates different physical mechanism

$$T_R = \begin{cases} \max(T_A, T_B) & p \to +\infty, \\ (T_A + T_B)/2 & p = +1, \quad \text{(arithmetic)} \\ \sqrt{T_A T_B} & p = 0, \quad \text{(geometric)} \\ 2\,T_A T_B/(T_A + T_B) & p = -1, \quad \text{(harmonic)} \\ \min(T_A, T_B) & p \to -\infty. \end{cases}$$



Pb+Pb collisions



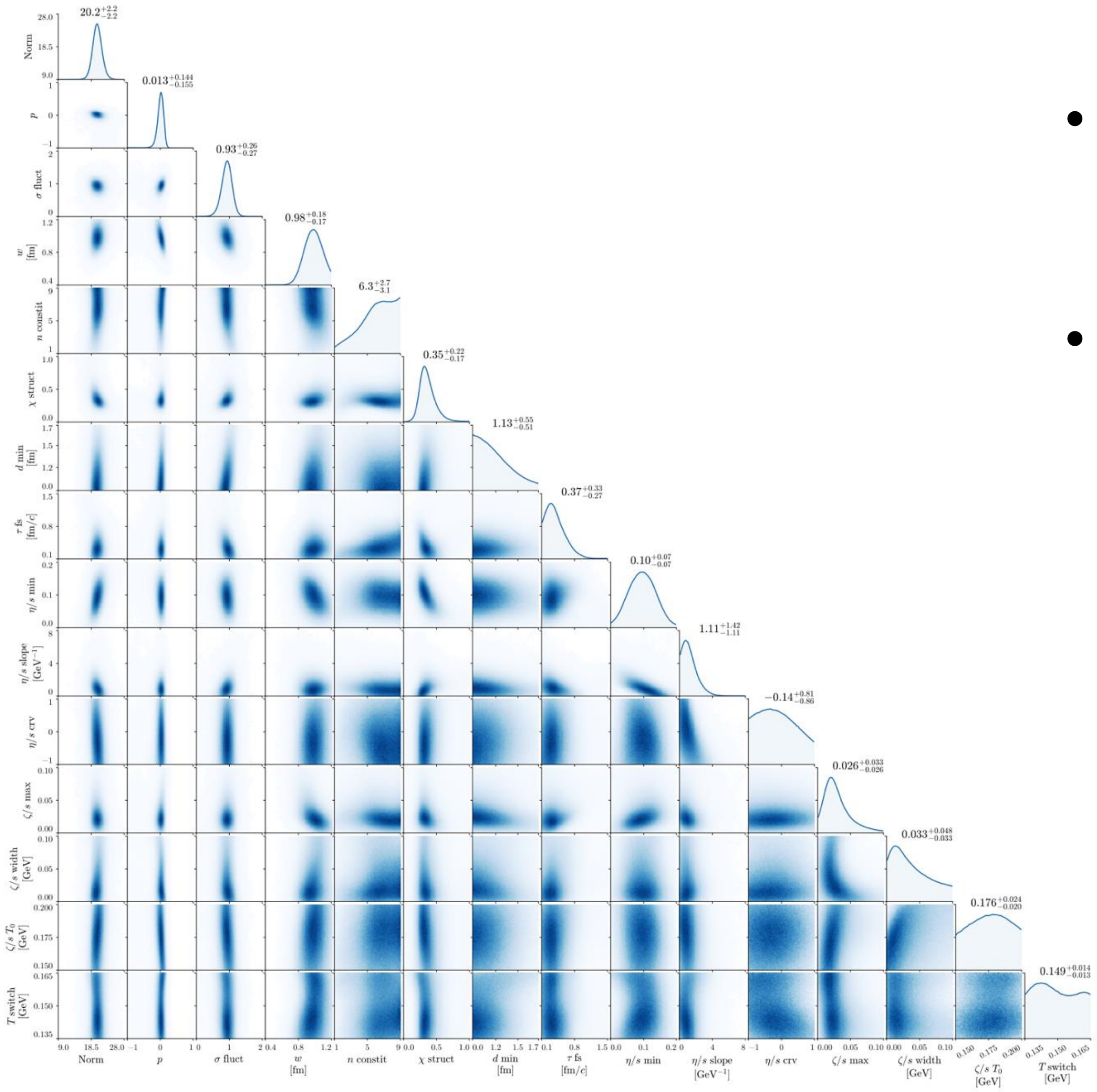
IP-Glasma

Glauber

This initial condition works well with Bayesian analysis.

https://github.com/keweiyao/trento3d/tree/subnucleon-3d

Bayesian Analysis



Moreland et al, PRC101,024911(2020)

- Hydrodynamic Model
  ex. TRENTO + VISH2+1 + UrQMD
  fifteen parameters
- Experimental data （LHC)
  p+Pb, Pb+Pb $\sqrt{s_{NN}} = 5.02$ TeV

Pb-Pb $\sqrt{s_{\rm NN}} = 5.02$ TeV
Charged-particle multiplicity $dN_{\rm ch}/d\eta$, $\lvert\eta\rvert < 0.5$ [72]
Two-particle flow cumulants $v_n\{2\}$ for $n = 2, 3, 4$, $\lvert\eta\rvert < 0.8$, charged-particles, $\lvert\Delta\eta\rvert > 1$, $0.2 < p_T < 5.0$ GeV [74]

p-Pb $\sqrt{s_{\rm NN}} = 5.02$ TeV
Charged-particle multiplicity $dN_{\rm ch}/d\eta$, $\lvert\eta\rvert < 1.4$ [73]
Two-particle flow cumulants $v_n\{2\}$ for $n = 2, 3$, $\lvert\eta\rvert < 2.4$, charged-particles, $\lvert\Delta\eta\rvert > 2$, $0.3 < p_T < 3.0$ GeV [75]
Charged-particle mean p_T, $0.15 < p_T < 10$ GeV, $\lvert\eta\rvert < 0.3$ [76]

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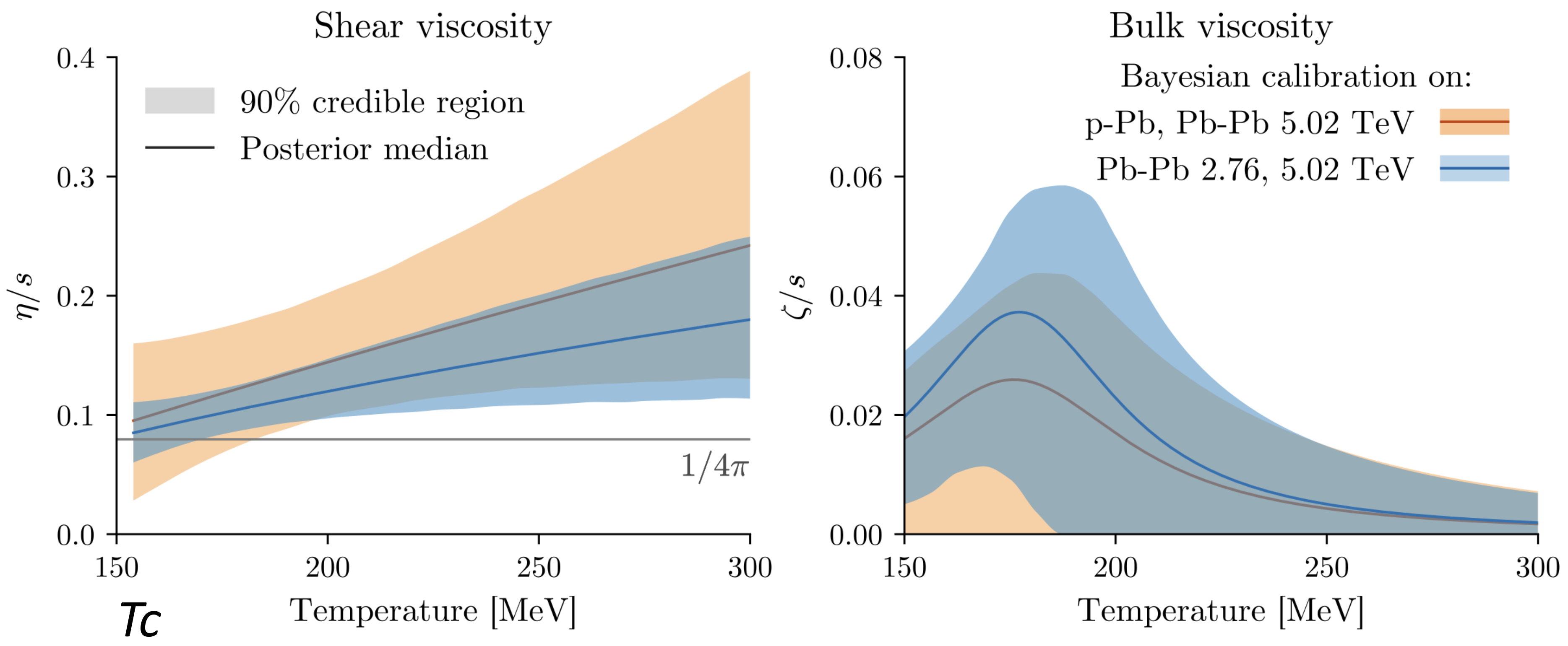
Temperature Dependence of Vicosity

Moreland et al, PRC101,024911(2020)



- Shear viscosity increase with T.
- Temperature dependence of bulk viscosity is not clear.

- Standard analysis
- Results are limited in the used framework.
  →model construction

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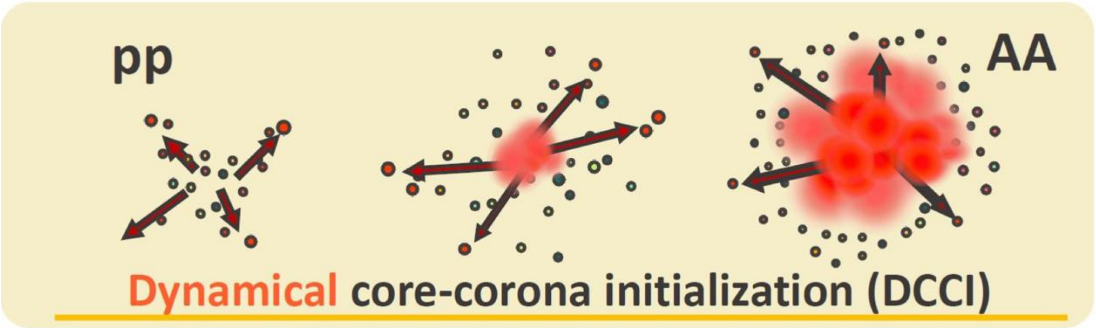
Dynamical Core-Corona Initialization

Kanakubo, Hirano, Tachubana, PRC 101, 024912 (2020)

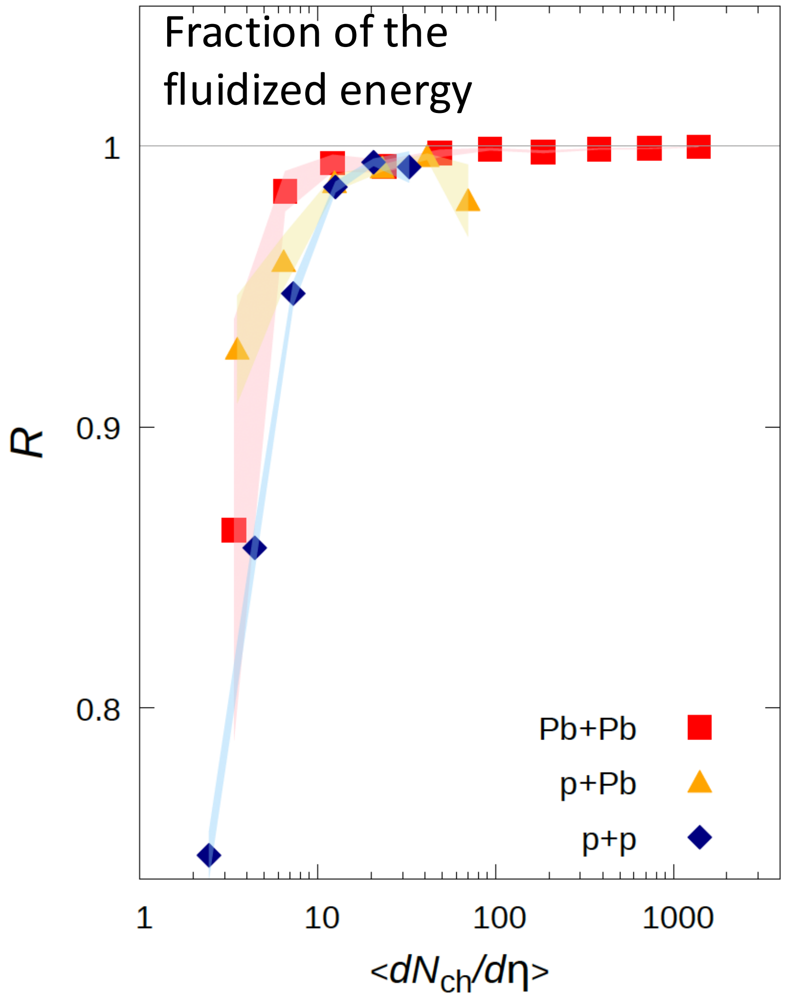
- The model dynamically describes the process where initially produced partons deposit energy and momentum into QGP fluids.

$$\partial_\mu T^{\mu\nu}_{\mathrm{fluid}}(x) = J^\nu(x)$$

Source term from partons



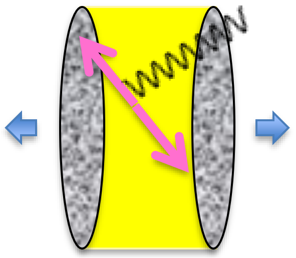
https://www.u.tsukuba.ac.jp/~esumi.shinichi.gn/workshop20210924/T03_kanakubo.pdf



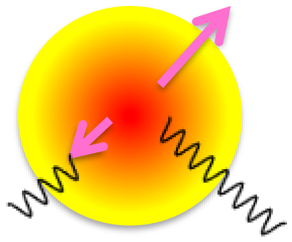
- Core(fluids)-corona(non-equilibriated partons) is determined dynamically based on the initial parton density.

Beyond the Phenomenological Approach

Thermalization/Hydrodynamization in a short time after the collisions.



Strong longitudinal flow
Out-of equilibrium



Hydrodymamics
equilibrium

- Kinetic thermalization
  - The momentum distributions of quarks and gluons become isotropic and approach thermal (Boltzmann/Fermi/Bose) distributions.
- Chemical thermalization
  - The particle abundances (gluon density, q, q-bar) reach their equilibrium values at a given temperature.
- Hydrodynamization
  - Even before full equilibrium, the system can be effectively described by relativistic viscous hydrodynamics.

Parton Cascade Model

- VNI/PCM, BAMPS..
- Boltzmann equations

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}_1}{E_1}\frac{\partial}{\partial \mathbf{r}}\right) f_1(\mathbf{r}, \mathbf{p}_1, t) = \mathcal{C}_{22} + \mathcal{C}_{23} + \cdots ,$$

Collision term

The system is not thermalized in a short time.

- Ex. BAMPS: Three-body interactions (2 ↔ 3) *Xu, Greiner, PRCC71 (2005) 064901*
  - Three-body interactions (2 ↔ 3) are crucial for gluon thermalization.
  - They accelerate both chemical and kinetic equilibration.
  - Thermalization occurs at ~1 fm/c; full chemical equilibrium at ~3 fm/c.
  - Nearly isotropic angular distribution of three-body processes aids fast equilibration.
  - Two-body interactions alone lead to slower thermalization and cannot explain collective flow.

Recent update

Abdi’s talk, Septmber 10

Hydrodynamization

The stage in which the far-from-equilibrium quark–gluon matter becomes well described by relativistic hydrodynamics, even before full local thermal equilibrium is reached.

Key References:

- Kurkela, Zhu, Isotropization and Hydrodynamization in Weakly Coupled Heavy-Ion Collisions, PRL115, 182301 (2015)
- Heller, Holography, Hydrodynamization and Heavy-Ion Collisions, Acta Phys. Polon. B 47, 2581 (2016)
- A. Soloviev, Hydrodynamic Attractors in Heavy Ion Collisions: A Review, EPJ.C 82, 319 (2022)

Isotropization and Hydrodynamization

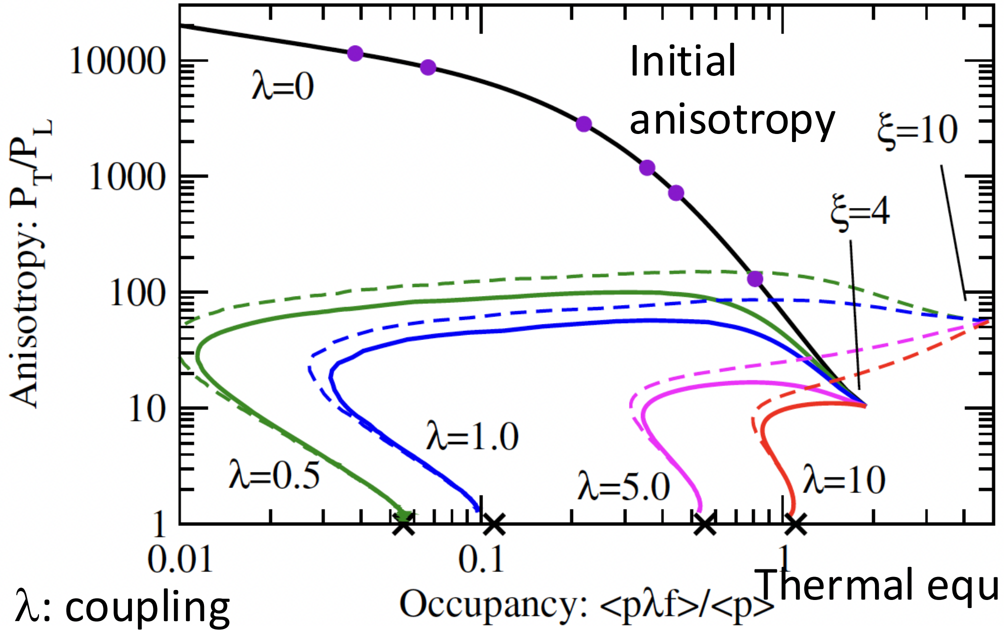
Kurkela, Y. Zhu, PRL115, 182301 (2015)

- the early stages of weakly coupled heavy-ion collisions
- (2+1)-dimensional effective kinetic theory (EKT)

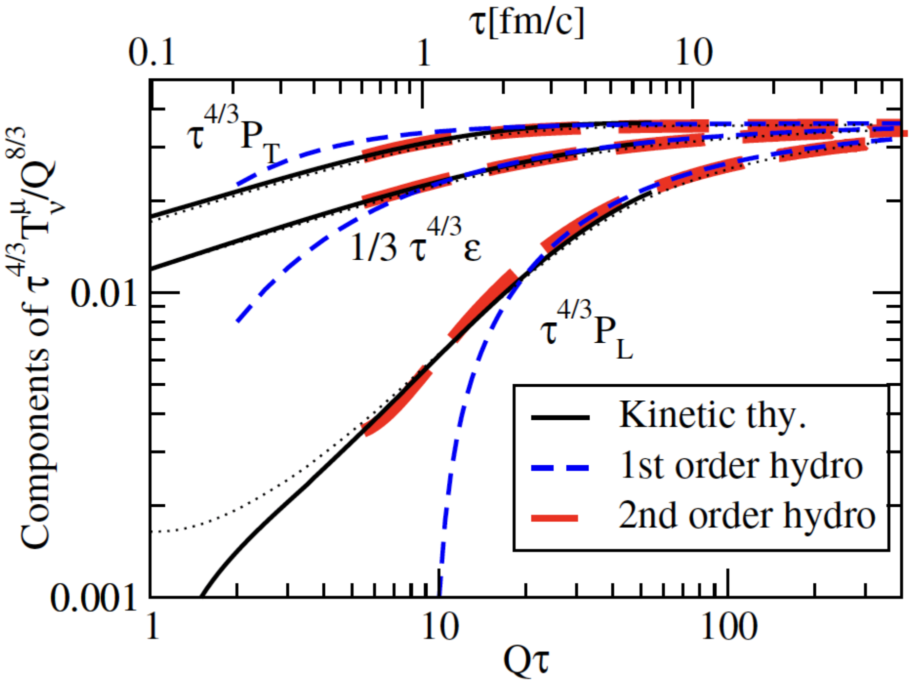
$$-\frac{df_{\mathbf{p}}}{d\tau} = \mathcal{C}_{1\leftrightarrow 2}[f_{\mathbf{p}}] + \mathcal{C}_{2\leftrightarrow 2}[f_{\mathbf{p}}] + \mathcal{C}_{\exp}[f_{\mathbf{p}}].$$

Splitting scattering longitudinal expansion

- Robustness of hydrodynamic description even in the presence of large anisotropies



λ: coupling



C. NONAKA

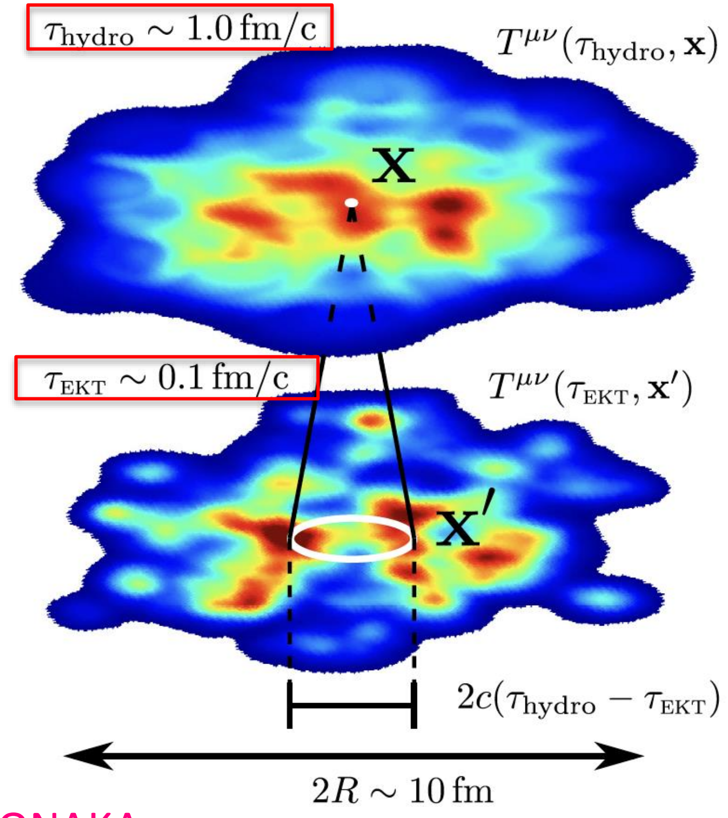
From Initial Stage to Hydrodynamics

Kurkela et.al, PRL122, 122302 (2019)

- KØMPØST: Linearized kinetic theory propagator for heavy-ion collisions

  Bridges early non-equilibrium dynamics to hydrodynamic evolution

Evolution of transverse energy profile



KØMPØST's linear response framework

Causal Contributions: The energy-momentum tensor at each point x receives causal contributions from the local average background and linearized energy and momentum perturbations propagated from τ_{EKT} to τ_{hydro}.

Causal Circle: The black cone and white circle in the figure represent the causal past for point x, which is used to calculate the energy density.

From Initial Stage to Hydrodynamics

Kurkela et.al, PRL122, 122302 (2019)

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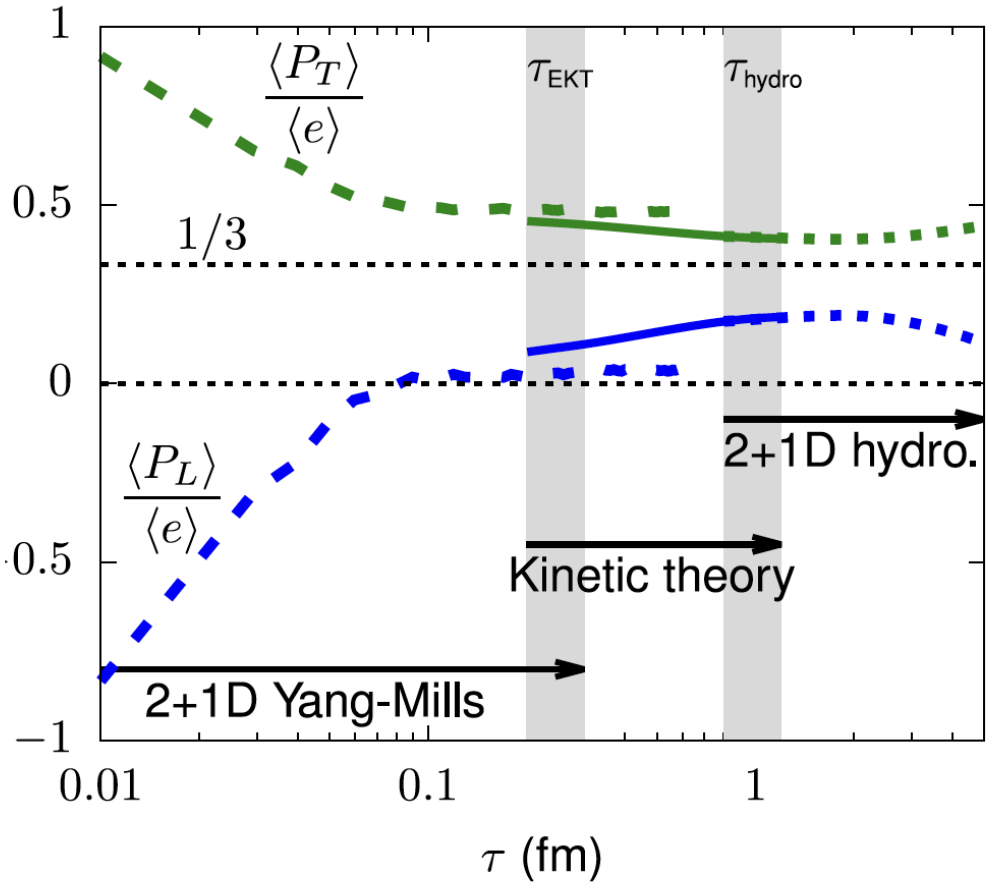


2+1D Yang-Mills Evolution (IP-Glasma Model):
Initially, the longitudinal pressure $\langle P_L \rangle$ is negative.
As the system evolves, the classical fields lose coherence,
and $\langle P_L \rangle$ approaches zero.
Kinetic Theory (KØMPØST): During the kinetic phase ($\tau_{EKT} \rightarrow \tau_{hydro}$),
the energy-momentum tensor begins to equilibrate.
The pressures gradually approach their equilibrium values,
with $\langle P_T \rangle$ approaching 1/3 of the energy density, consistent
with a locally equilibrated fluid of massless particles.
Relativistic Viscous Hydrodynamics: After τ_{hydro},
the system transitions to hydrodynamic evolution,
where the pressures are further refined by viscous corrections.

Initial Electromagnetic Fields

- Liénard–Wiechert potentials: Skokov et al., IJMPA 24, 5925 (2009).
- MC Glauber + decay models: Deng, Huang, PRC 85, 044907 (2012).
- IP-Glasma + Maxwell equations: Inghirami et al., EPJC 76, 659 (2016).

Initial Electromagnetic Fields

Tuchin, Phys.Rev.C88,024911(2013)

■Asymptotic solution of Maxwell eq.

➢Electromagnetic field made by point charge moving in the longitudinal axis

- Proton distribution in nucleus : uniform sphere
- Constant charge conductivity (σ =0.023 fm^{-1})

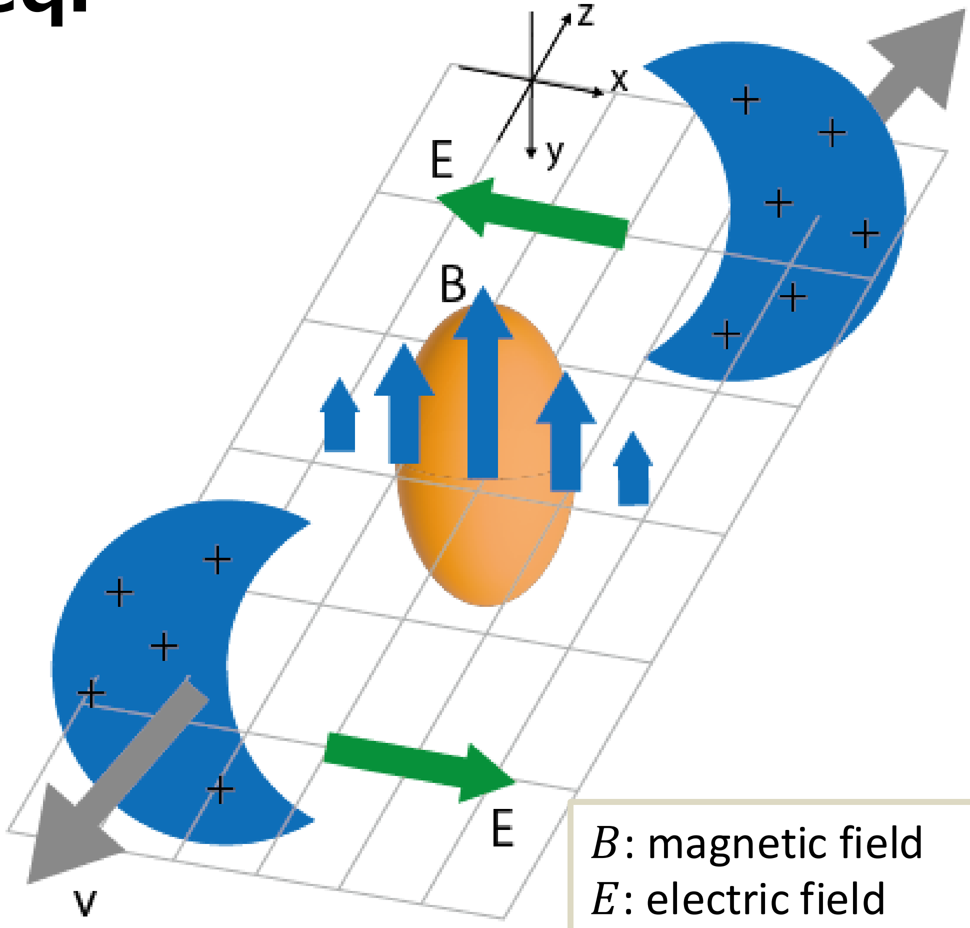
$$\nabla \cdot \boldsymbol{B} = 0\,, \quad \nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}\,,$$

$$\nabla \cdot \boldsymbol{D} = e\delta(z - vt)\delta(\boldsymbol{b})\,,$$

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} + \sigma \boldsymbol{E} + ev\hat{z}\delta(z - vt)\delta(\boldsymbol{b})$$

Integration of the asymptotic solutions over

the charge distribution inside of nucleus



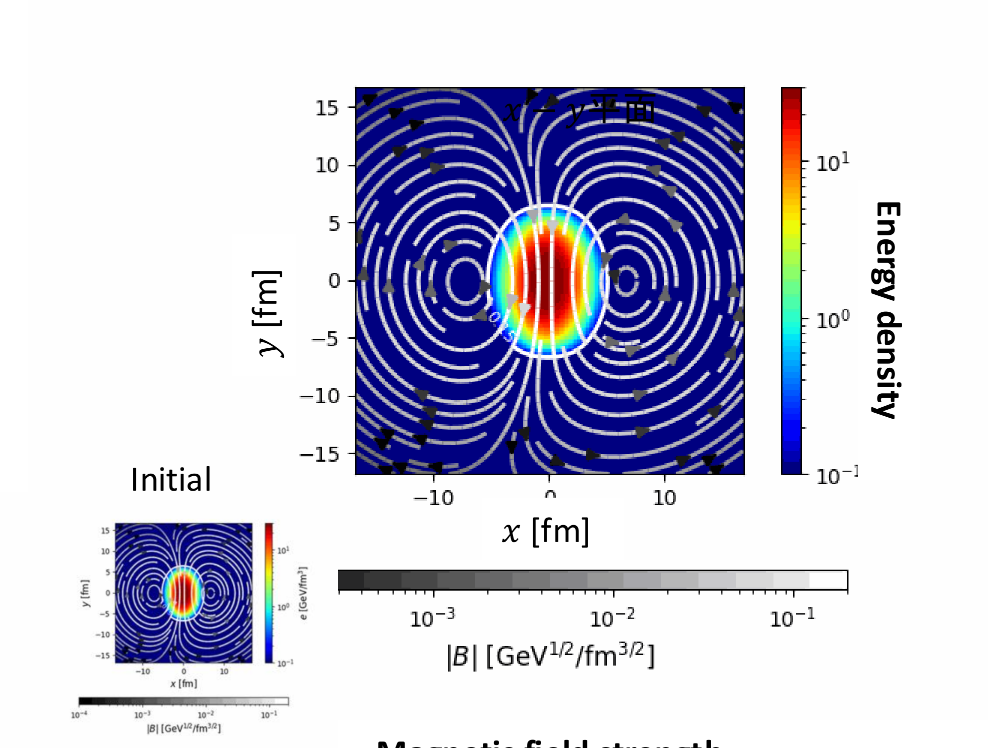
B: magnetic field
E: electric field
v: velocity of heavy ion

Space-time Evolution

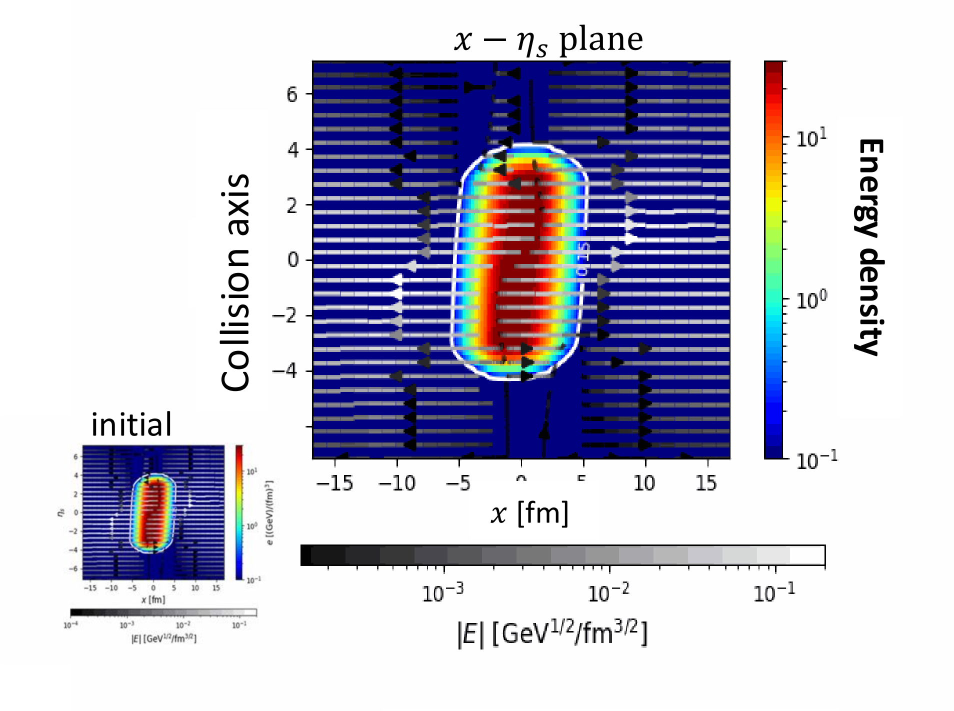
Nakamura, Miyoshi, CN and Takahashi, PRC 107, no.1, 014901 (2023)

Au+Au collision system

First calculation in HIC with RRMHD code



Magnetic field strength



Electric field strength

Analysis of Heavy Ion Collisions

Initial Electromagnetic Fields

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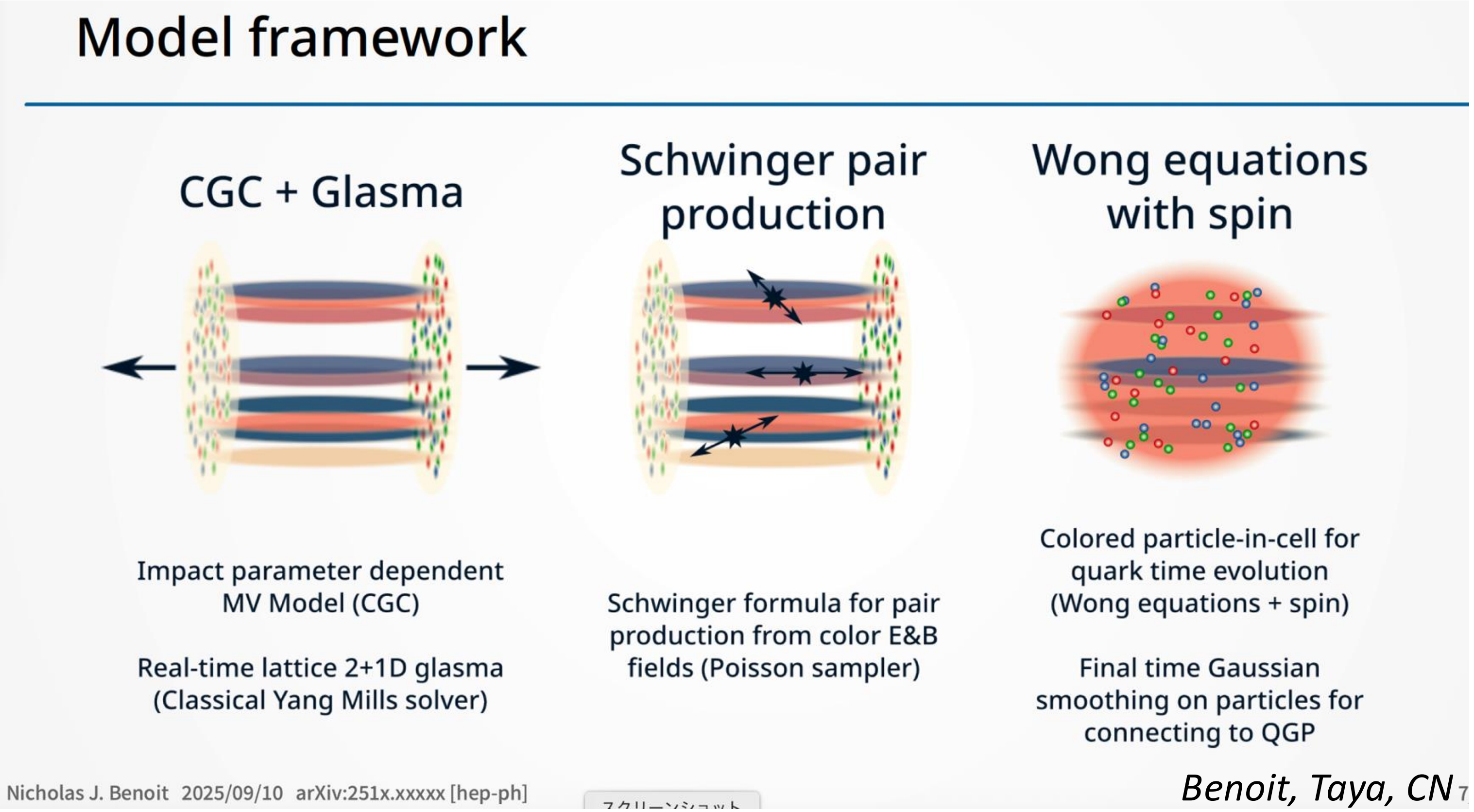
- IP-Glasma + Maxwell equations: Inghirami et al., EPJC 76, 659 (2016).



Benoit’s talk

September 10

Initial Electromagnetic Fields

- Liénard–Wiechert potentials: Skokov et al., IJMPA 24, 5925 (2009).
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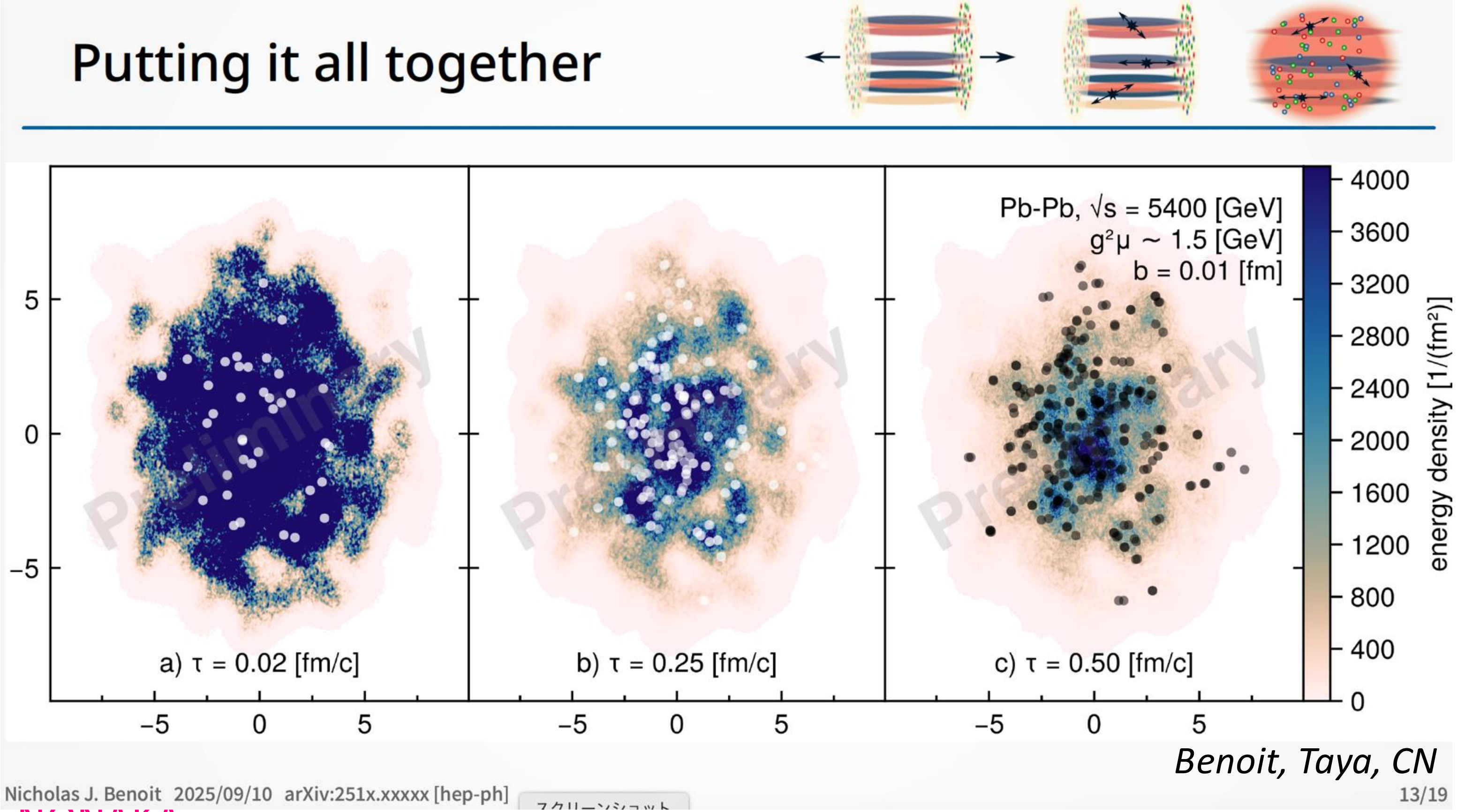
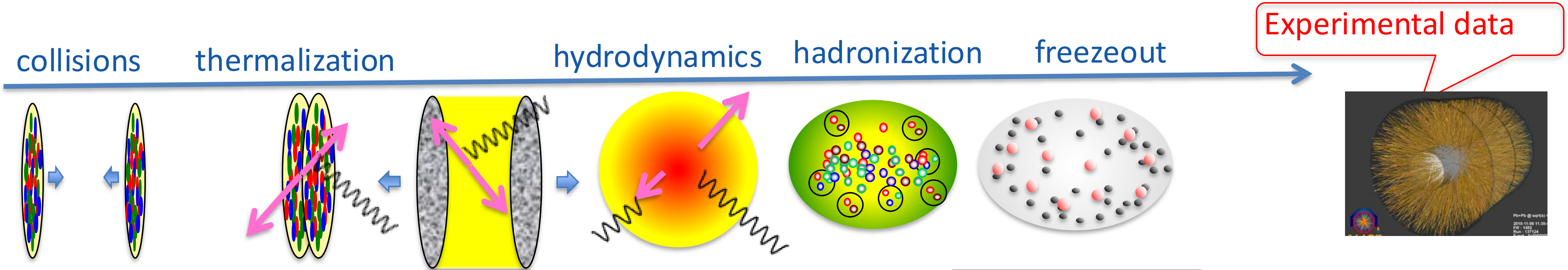


Benoit's talk

September 10

Summary



Initial Condition

Hydrodynamics

Hadronization

Particlization

Final state interactions

Recombination Model

Hydrodynamic model gives us information of QGP bulk property, however still uncertainty exists in its initial condition.

- Much about the initial conditions is becoming clear
- Various models are still under development