Parton structures in deep inelastic scattering with quantum computing

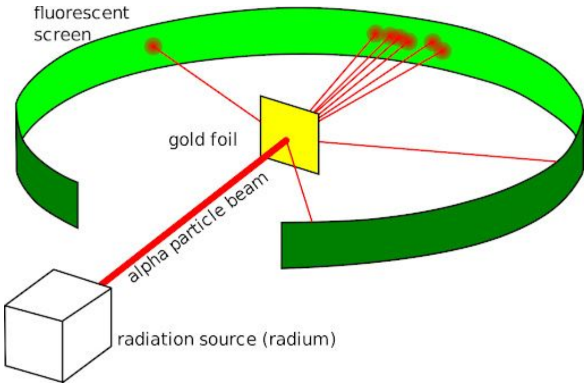
Wenyang Qian (IGFAE)

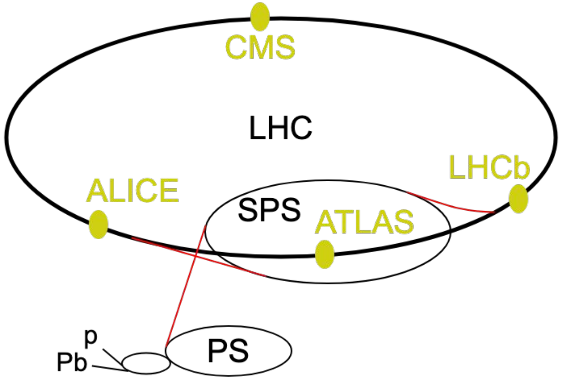
In collaboration with Zhongbo, Peter, Noah, based on 2501.09738

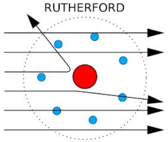
Sept 9, Initial Stages 2025, Taipei

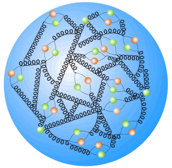
Parton structures

Primary goal in modern physics is to understand matter, from Rutherford to Large Hadron Colliders.



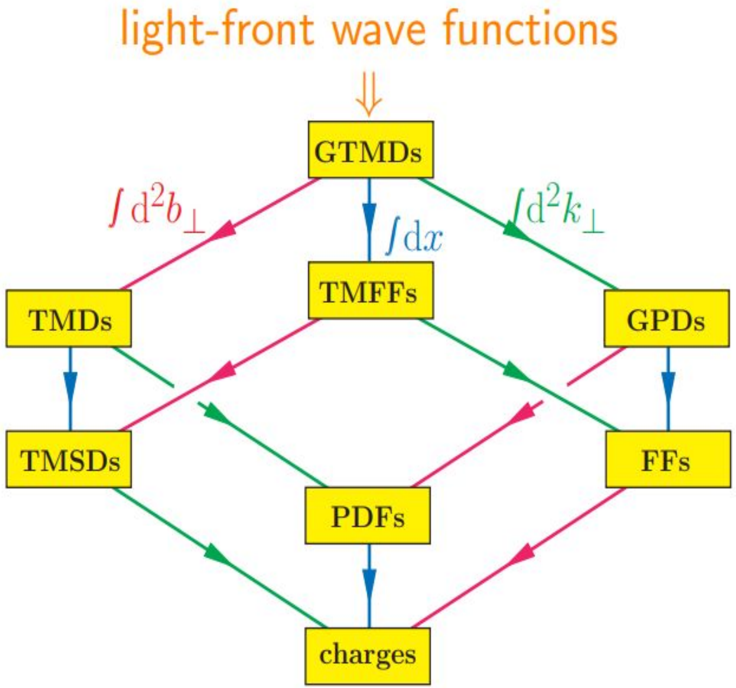


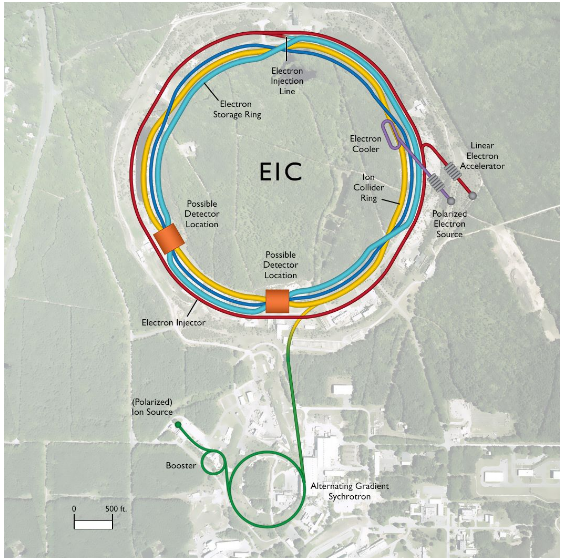




Parton structures

Multi-dimensional view of the parton structures, TMDs, GPDs, FFs, PDFs ...





Brookhaven

Parton Distribution Function

PDF ***f(x)*** is the probability of finding parton in a hadron carrying ***x*** fraction of the momentum of the constituent.

They are key to high energy and nuclear physics

- Understand innate structure of QCD bound states (EIC, EicC, JLab…)
- Provide essential baseline for Hard Probes in heavy ion collisions (RHIC, LHC, …)

PDFs are **not directly** observed in experiments, but through QCD **factorization**

Collins, Soper, Sterman, 0409313

PDFs are **universal, process-independent** (DIS, Drell-Yan, …) => global fit using world's data

[See Aleksander Kusina's review talk on Tues]

Theoretical methods to compute PDF

Formally, the PDF is defined as real-time correlators of quark and gluon fields

$$f_{q/h}(x) = \int_{-\infty}^{\infty} \frac{\mathrm{d}t}{4\pi} e^{-itx\vec{n}\cdot\vec{P}} \langle h(\vec{P})| \bar{\psi}(t\vec{n}) W(t\vec{n} \leftarrow \vec{0}) \vec{n} \cdot \vec{\gamma} \psi(\vec{0}) |h(\vec{P})\rangle$$

There are many direct and indirect approaches:

- Lattice QCD methods (traditional and new: quasi-PDF, pseudo-PDF, …)
- Light-front Hamiltonian (DLCQ, BLFQ, …)
- Dyson-Schwinger equations (extract from BS wavefunctions)

Ji, 1305.1539,
Radyushkin, 1705.01488
Ma & Qiu, 1709.03018

Brodsky, Pauli, Pinsky, 9705477

…

Direct, complete calculation remains complicated → Quantum computing may emerge as **new tool** to compute PDF directly.

Why bother with **QC** and why **now**?

Theoretical

- Real-time correlator are prevented by a sign problem with Euclidean lattice

- Hilbert space dimensionality grows exponentially with particles quickly surpassing classical resources

- Classically hard, real-time evolution are most natural on quantum computers

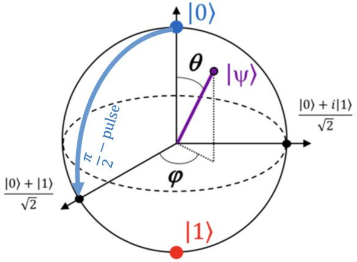
Practical

- New tools like quantum computing will become accessible like any other tools (i.e., ML)

- Though *ideal* QC not available now, we can use **QIS tools** to investigate physical probs of interests

Quantum computing

Quantum bit (qubit)



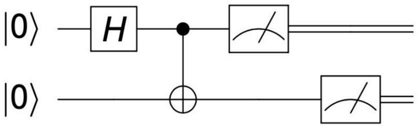
$$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix},\ |1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$$

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$

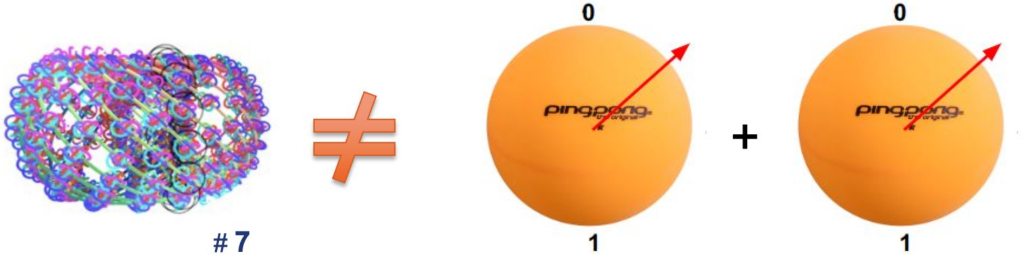
Quantum gates are unitary operators

Probabilistic nature



Entanglement

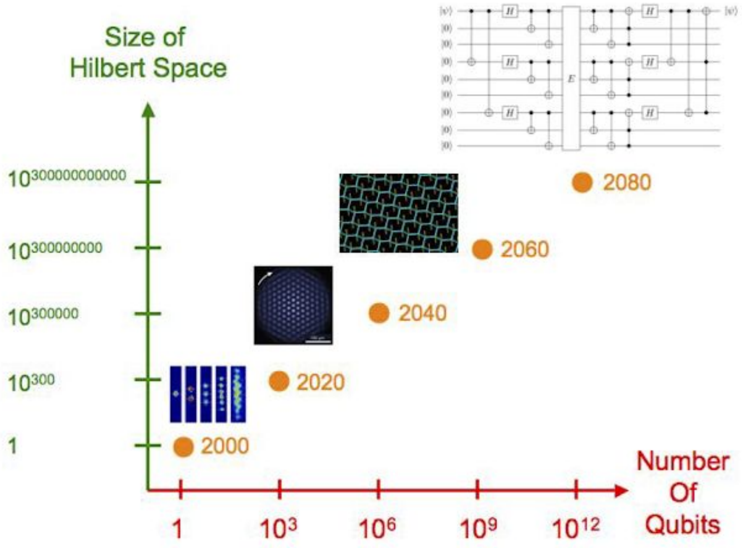
d.o.f. 2^{n+1}-1 ≫ 3n



Quantum computing = using quantum mechanics to solve problems…

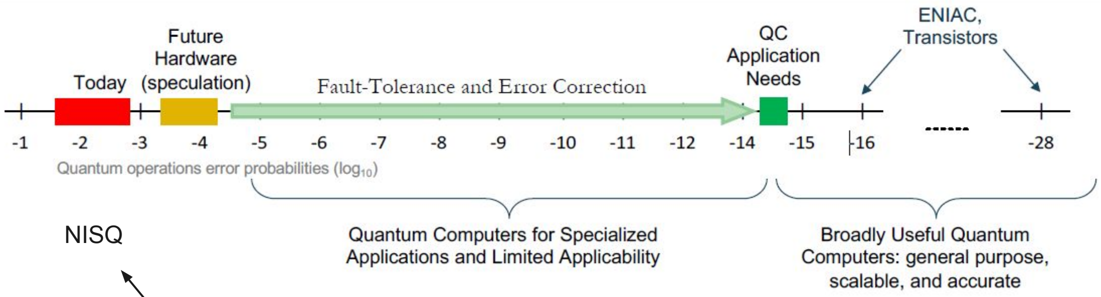
Review paper of QC to HEP
Bauer et al, 2204.03381
Meglio et al, 2307.03236

Scaling law vs. reality

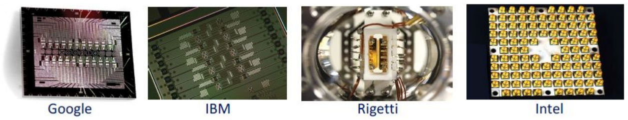


Optimist's prediction:

Neven's law = Double-exponential scaling



NISQ



State-of-the-art:

- Noisy intermediate-scale quantum (NISQ)
- Analog quantum computers (not universal)

Practical path nowadays = tensor network (TN)

Basic idea: Efficient local representation of Hilbert space to approximate the true Hilbert space

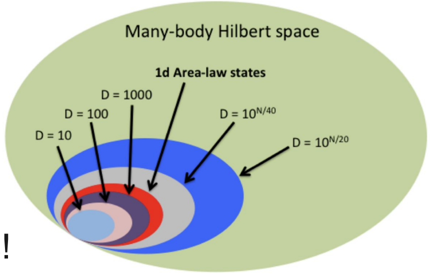
In 1D: Matrix Product States (MPS) $$|\psi\rangle = \sum_{s_1 s_2, \dots, s_N} A_1^{s_1} A_2^{s_2} \cdots A_N^{s_N} |s_1 s_2 \cdots s_N\rangle$$

Seen as a parameterized ansatz between mean-field and full problem size (bond dimension)

- Good for finding ground states
- Evolution obeys area laws of entanglement entropy

Hastings, 0705.2024

$S \sim \log D$



Just “diagramical” way to perform quantum computation instead of circuits!

Computation of PDF with QIS

All are in 1+1
still far from full QCD

2019 Theoretical framework on computing PDF with quantum computing

Thirring model, exact diag

Lamm, Lawrence, Yamauchi, 1908.10439

2021 Calculation of PDF with implementation using quantum circuits

NJL model, QuSpin

Li et al, 2106.03865
later extends to LCDA and FF

2024 Calculation of quasi-PDF with boost mesons

Schwinger model, exact diag

Grieninger, Ikeda, Zahed, 2404.05112

2024 Calculation of PDF with dynamical gauge fields in QED

Schwinger model, tensor network

Banuls et al, 2409.16996 & 2504.07508
[See Manuel Schneider's poster]

2025 Uniform computation of PDF & LCDA in large systems

NJL model, tensor network

This work, 2501.09738

2025 First calculation of PDF on real quantum devices

Schwinger model, IBM quantum hardware

Chen, Chen, Meher, 2506.16829

Recipe to compute PDF

Basic idea:

1. Map the lattice Hamiltonian with staggered fermions onto quantum circuit
2. Solve for the hadron state using quantum algorithm
3. Apply local field operators and evolve the state in real time
4. Extract matrix elements at different time and perform Fourier transform

$$f_{q/h}(x) = \int_{-\infty}^{\infty} \frac{\mathrm{d}t}{4\pi} e^{-itx\vec{n}\cdot\vec{P}} \langle h(\vec{P})| \bar{\psi}(t\vec{n}) W(t\vec{n} \leftarrow \vec{0}) \vec{n} \cdot \vec{\gamma} \psi(\vec{0}) |h(\vec{P})\rangle$$

Fermionic Hamiltonian in 1+1 d

Kang, Moran, Nguyen, WQ, 2501.09738

As an exemplary study, we use the Nambu-Jona-Lasinio Hamiltonian

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi + g(\bar{\psi}\psi)^2$$

with its Hamiltonian density

$$\mathcal{H} = \bar{\psi}(i\gamma_1\partial_1 + m)\psi - g(\bar{\psi}\psi)^2$$

The fermionic fields are discretized in the position space using Staggered Fermion fields

$$\psi(x = x_n) = \begin{pmatrix} \rho(x = x_n) \\ \eta(x = x_n) \end{pmatrix} = \frac{1}{\sqrt{a}} \begin{pmatrix} \chi_{2n} \\ \chi_{2n+1} \end{pmatrix} \qquad \delta x = x_n - x_{n-1} = a$$

Importantly, 1 positional point requires 2 fermion fields

Spin Hamiltonians

Kang, Moran, Nguyen, WQ, 2501.09738

The full Hamiltonian becomes

$$H = -\frac{i}{2a}\left[\sum_{n=0}^{N-2}\left(\chi_n^\dagger \chi_{n+1} - \chi_{n+1}^\dagger \chi_n\right)\right] + m\sum_{n=0}^{N-1}(-1)^n \chi_n^\dagger \chi_n \quad - \frac{g}{a}\sum_{n=0}^{N/2-1}\left(\chi_{2n}^\dagger \chi_{2n} - \chi_{2n+1}^\dagger \chi_{2n+1}\right)^2 .$$

Map with Jordan-Wigner encoding for the fermions

Occupancy represents field excitations at each position

$$\chi_n = \sigma_n^- \prod_{i=0}^{n-1}(-i\sigma_i^z)\,, \quad \chi_n^\dagger = \sigma_n^+ \prod_{i=0}^{n-1}(i\sigma_i^z)$$

One obtains the spin Hamiltonians on qubits

$$H = \frac{1}{2a}\left[\sum_{n=0}^{N-2}\left(-\sigma_n^+\sigma_n^z\sigma_{n+1}^- + \sigma_{n+1}^+\sigma_n^z\sigma_n^-\right)\right] + m\sum_{n=0}^{N-1}(-1)^n\sigma_n^+\sigma_n^- \quad - \frac{g}{a}\sum_{n=0}^{N/2-1}\left(\sigma_{2n}^+\sigma_{2n}^- - \sigma_{2n+1}^+\sigma_{2n+1}^-\right)^2$$

Importantly, 1 positional point requires 2 qubits, so N qubits for N/2 positions

PDF and DA using spin operators

Kang, Moran, Nguyen, WQ, 2501.09738

The PDF and DA (distribution amplitude) are now mapped analogously using spin operators:

$$f_{q/h}(x) = \frac{1}{4\pi} \sum_{z=\bar{N}_{\text{min}}}^{\bar{N}_{\text{max}}} e^{-ixM_h z} \langle h| \left(M_{00}(z) + M_{11}(z) - M_{01}(z) - M_{10}(z) \right) |h\rangle \ ,$$

$$\phi_h(x) = \frac{1}{f_h} \sum_{z=\bar{N}_{\text{min}}}^{\bar{N}_{\text{max}}} e^{-i(x-1)M_h z} \langle \Omega| \left(M_{00}(z) + M_{11}(z) - M_{01}(z) - M_{10}(z) \right) |h\rangle$$

with the matrix element

$$M_{ij}(z) = e^{iHz} \chi^\dagger_{-2z+i+\bar{N}} e^{-iHz} \chi_{j+\bar{N}}$$

The essential task is to be able to compute

$$E_{ab}(z) = \langle \psi_L | e^{iHz} \chi_a^\dagger e^{-iHz} \chi_b | \psi_R \rangle \equiv e^{iE_L z} \langle \psi_L | \chi_a^\dagger e^{-iHz} \chi_b | \psi_R \rangle$$

Hadron state preparation

In TN, we use the density matrix renormalization group (DMRG) for MPS on charge-preserving sites.

DMRG is a variational approach that iteratively sweeps the lattice and find the lowest energy. The computational cost for 1D system scales linearly with qubits: $\mathcal{O}(N\chi^3)$

As we study mesons, we restrict the study in the charge Q = 0 sector. $Q = \sum_{i=1}^{N}(\sigma_n^z + (-1)^n)/2$

For the hadron mass, it is defined as $M_h = \langle h|H|h\rangle - \langle\Omega|H|\Omega\rangle$

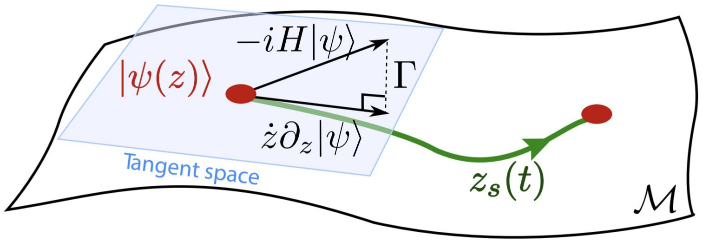
Qubits (N)	18	30	50	102
Hadron mass $M_h a$	1.64600864	1.64359027	1.64246829	1.64187901

Real-time evolution of fields

In TN, one can use standard time-evolving block decimation (TEBD), i.e., tensor-network trotterization. In this work, we use time dependent variational principle (TDVP).

The idea is to constrain evolution to a specific manifold of the MPS of a given bond dimension.

Project the target Hamiltonian into tangent space and then solve the time-dependent schrodinger equation within that manifold.

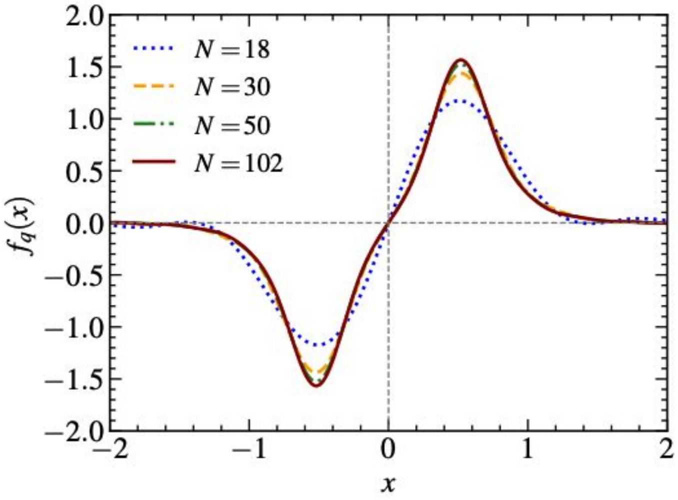


Ho, Choi, Pichler, Lukin, 1807.01815

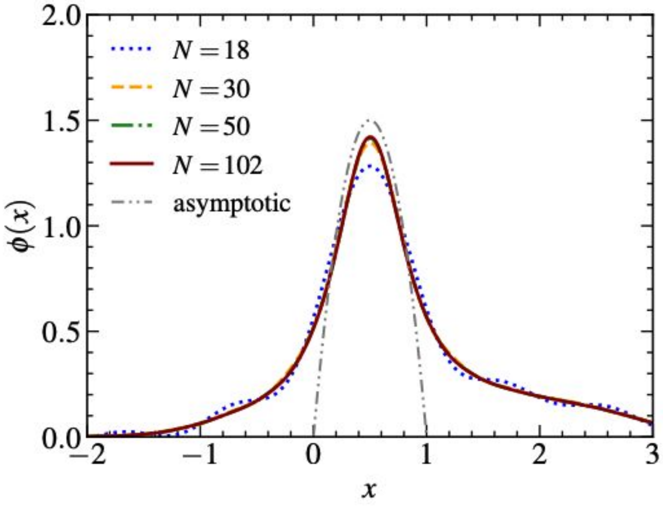
Results

Extract results in large N = 102 qubit limit

Kang, Moran, Nguyen, WQ, 2501.09738



(a) PDF

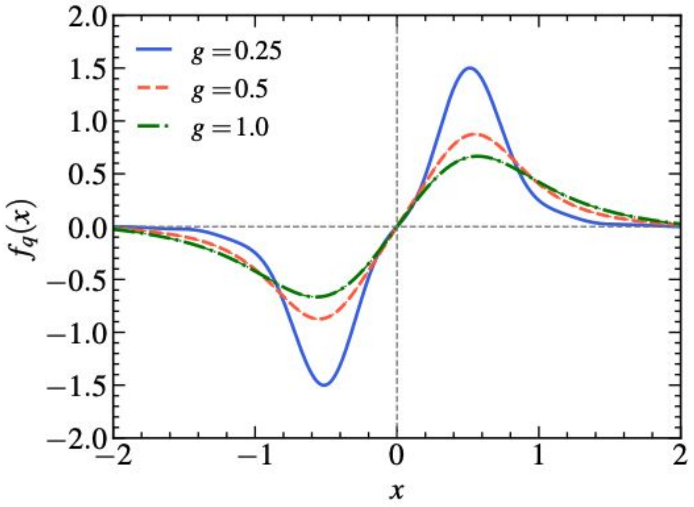


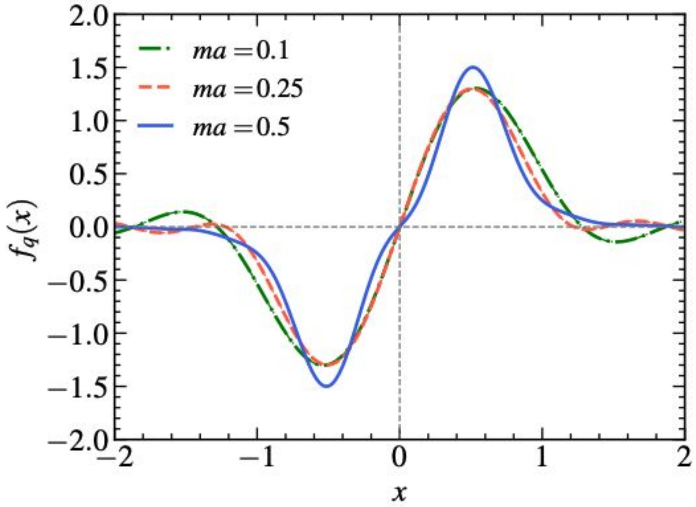
(b) DA

Results

Various coupling and mass limits, PDF results here at N = 102 qubit

Kang, Moran, Nguyen, WQ, 2501.09738

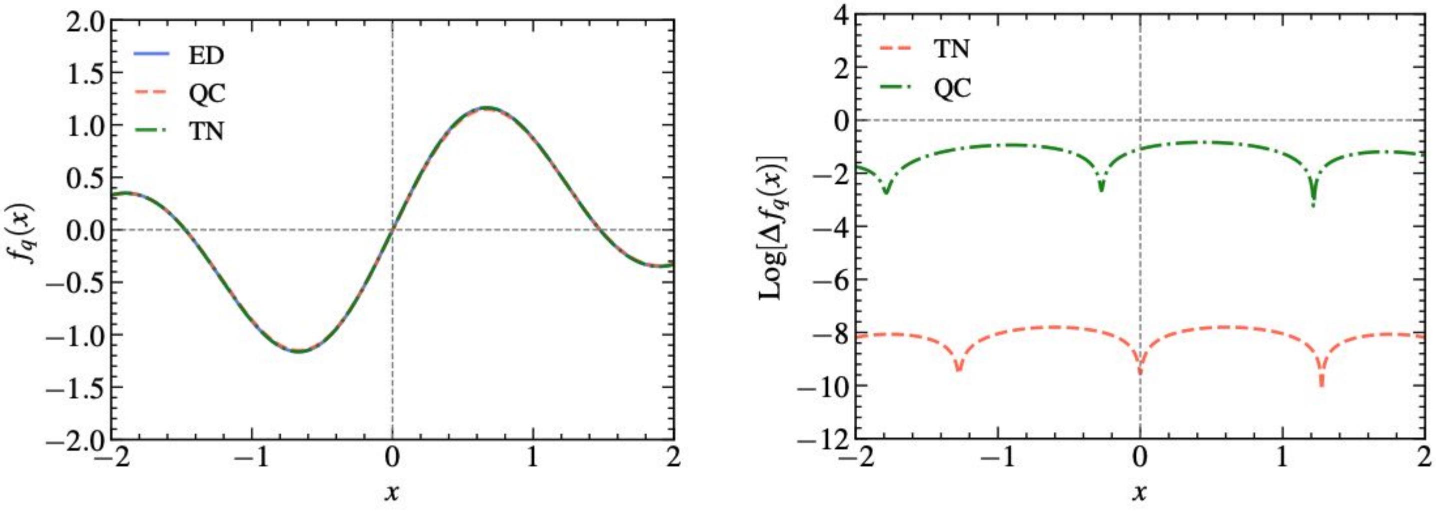




Comparison with QC and exact diag

N=10 qubits

Kang, Moran, Nguyen, WQ, 2501.09738



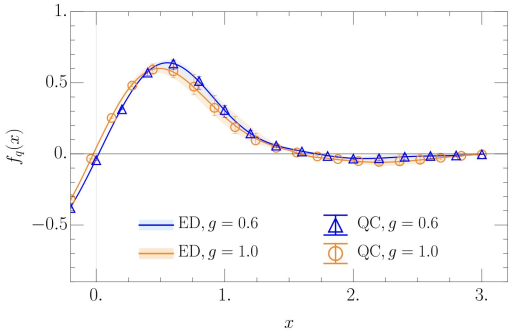
(a) PDF calculated using different methods

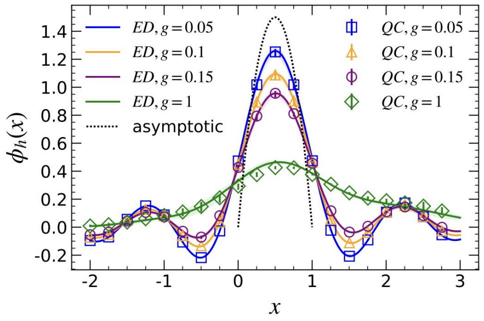
(b) Errors in PDF compared to ED

Comparison with other works

Previous work using QuSpin

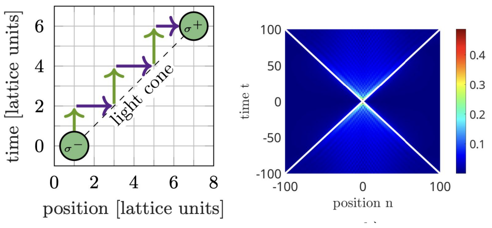
Li et al, 2106.03865; 2207.13258

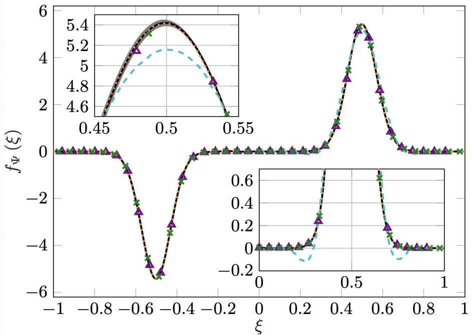




Recent work using TN and involving gauge field (schwinger model)

Banuls et al, 2504.07508; 2409.16996

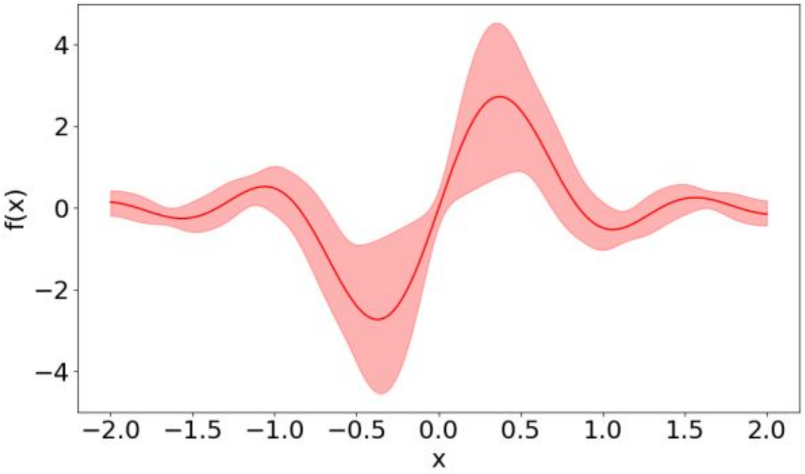




Comparison with other works

Recent work using real quantum computers and involving gauge field (schwinger model)

Chen et al, 2506.16829



IBM Eagle R3 quantum computers

Conclusion and future prospects

Quantum computing can potentially provide a new theoretical method to directly extract parton distribution functions from first principles.

Detailed numerical results are available in various 1+1 dimensional models with different QIS simulation frameworks.

Further works are necessary to consider spin, color, higher dimensions.

One can also compute hadronic tensor and relate to cross section in Deep Inelastic Scattering.

Ikeda, Kang, Kharzeev, WQ (in prep)

It is also interesting to look at fragmentation functions for the final states.

Li, Xing, Zhang, 2406.05683
Grieninger, Zahed, 2406.01891

Simulating PDF using light-cone gauge so Wilson line is lifted.

Kreshchuk et al, 2002.04016

[See other talks on QC: Meijian's talk on jets and Ivan's talk on thermalization on Wednesday]