Rapidity dependent QGP equilibration in QCD kinetic Theory

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Initial Stages 2025, 09.09.25

with J. Bhambure, A. Mazeliauskas, J.-F. Paquet, R. Singh, M. Singh, D. Teaney, PRC 2412.10303

with A. Mazeliauskas, *in progress*

Motivation

- How does the QCD medium thermalise in a heavy ion collision?

  Effectiveness of hydrodynamics

- Kinetic Theory

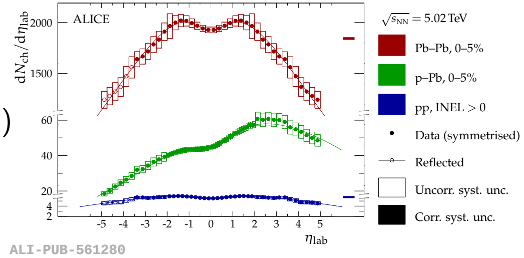
  Weakly coupled picture (Bottom up scenario)

  Baier, Müller, Schiff, Son (2001), PLB 0009237

- 3D ICs by McDipper, MC-EKRT, ...

  O. Garcia-Montero et al., PRC 2308.11713

  M. Kuha et al., PRC 2406.17592



ALICE, PLB 2204.10210

$\rightarrow$ Study e.g. effects longitudinal decorrelation

Solve kinetic theory of Yang-Mills without boost invariance

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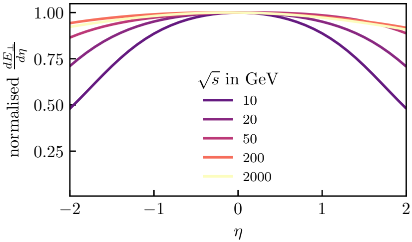
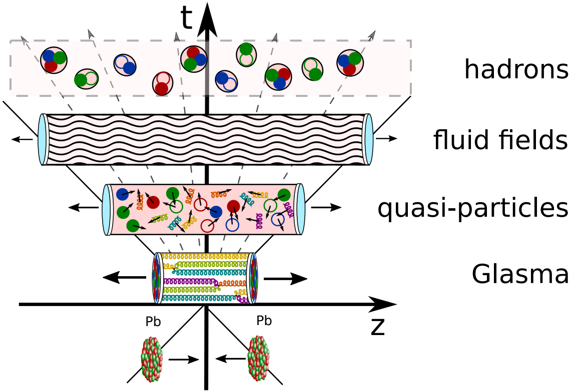


McDipper, PRC 2308.11713

Solve kinetic theory of Yang-Mills without boost invariance

Main stages of a heavy ion collision



- Study pre-hydrodynamic phase

Outline

AMY effective kinetic theory (EKT)

1+1D EKT vs. Hydrodynamics

Rapidity dependent equilibration

AMY effective kinetic theory (EKT)

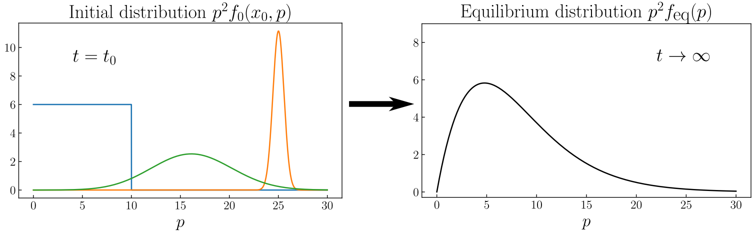
Framework

- Effective Kinetic Theory for high temperature gauge theories
  Arnold, Moore, Yaffe (2003), JHEP 0209353

- Weakly coupled quasi-particle picture, $\lambda = 4\pi\alpha_s N_c$

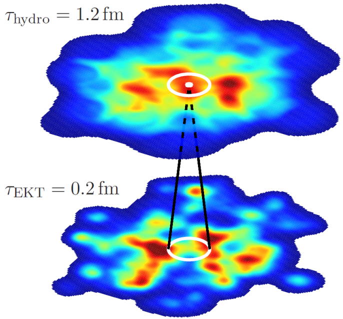
  $\rightarrow$ Phase space distribution $f(\tau, \boldsymbol{x}, \boldsymbol{p})$

$$(\partial_\tau + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}}) f(\tau, \boldsymbol{x}, \boldsymbol{p}) = -C[f]$$



Out-of-equilibrium initial state is transported to equilibrium

Expanding QGP



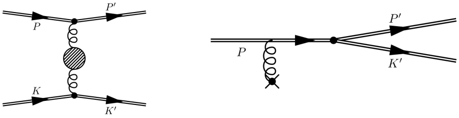
- Homogeneity in the transverse plane
- Non-boostinvariant longitudinal expansion

$$(\partial_t + v_z \partial_z)\, f(t, z, \boldsymbol{p}) = -C[f]$$

- Leading order elastic and inelastic scattering processes, local!

KøMPøST, PRC 1805.00961

$$C[f](z, \boldsymbol{p}) = C_{2\leftrightarrow 2}[f](z, \boldsymbol{p}) + C_{1\leftrightarrow 2}[f](z, \boldsymbol{p})$$



1+1D EKT vs. Hydrodynamics

Cartesian coordinates

- Energy momentum tensor

$$T^{\mu\nu}(t,z) = \int_{\boldsymbol{p}} \frac{p^\mu p^\nu}{p} f(t,z,\boldsymbol{p})$$

- Close to local thermal equilibrium

$$T^{\mu\nu} = (e+P)u^\mu u^\nu + P g^{\mu\nu} + \pi^{\mu\nu}$$

- Hydrodynamic equations of motion from

$$\partial_\mu T^{\mu\nu} = 0 \qquad \text{and} \qquad P(e) = e/3$$

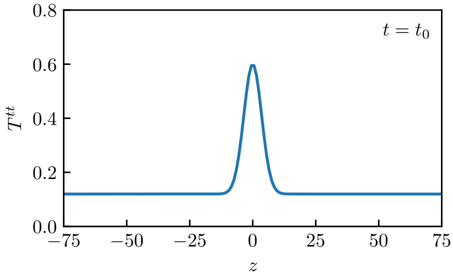
Compare T^{tt} from EKT vs. 1st order viscous hydro

Expansion of thermal initial conditions

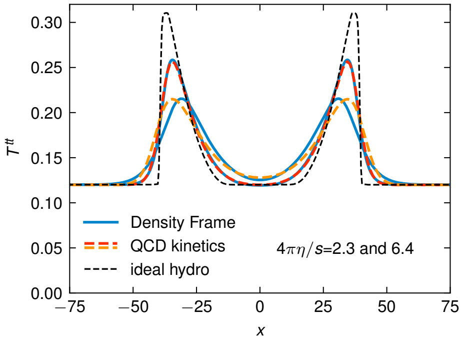
- At t_0, Bose-Einstein-Distribution

$$f(t_0, z, \boldsymbol{p}) = \frac{1}{e^{p/T(z)} - 1}$$

- Initial energy density $T^{tt}(t_0, z)$



- at $t = 50\ \mathrm{GeV}^{-1}$



- coupling $\eta/s \sim 1/\lambda^2$

Large λ: EKT agrees with viscous hydrodynamics! DF,PRC 2412.10306, PRC 2412.10303

Freestreaming vs. EKT

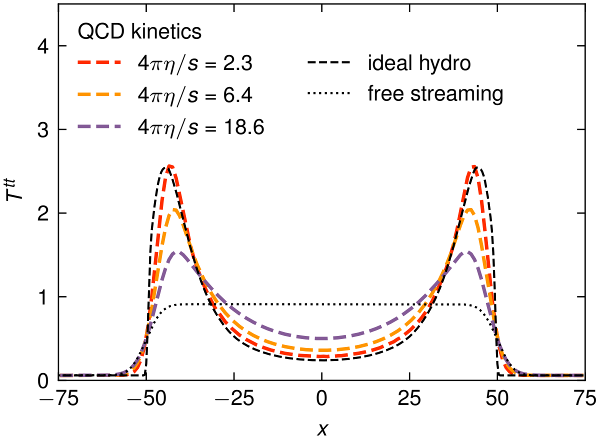
- Non-interacting

  $\lambda = 0$ or $\eta/s \to \infty$

  $$(\partial_t + v_z\partial_z) f(t, z, \boldsymbol{p}) = 0$$

- Edges: freestreaming

$t = 50\ \mathrm{GeV}^{-1}$



EKT interpolates smoothly between **freestreaming** and **viscous hydro**

→ probe hydro codes at the edge of validity J. Bhambure, et al., PRC 2412.10303

Rapidity dependent equilibration

Bjorken coordinates

$$\tau = \sqrt{t^2 - z^2} \qquad \text{and} \qquad \eta = \frac{1}{2}\ln\left(\frac{t+z}{t-z}\right)$$

- Momentum rapidity $\tanh^{-1}(v_z) = y$

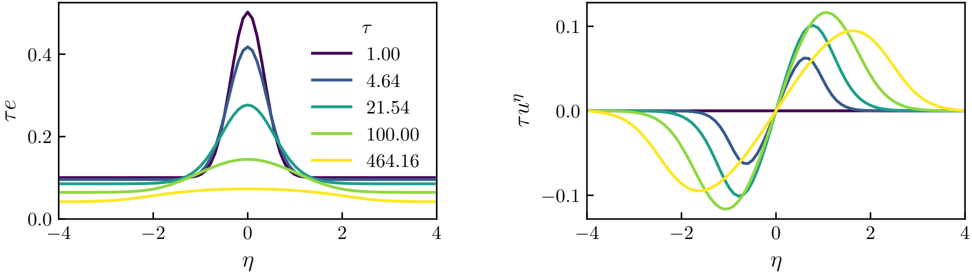
$$p^\tau = p_T\cosh(y-\eta) \qquad \text{and} \qquad p^{||} = p_T\sinh(y-\eta)$$

- At $\eta = 0$: $p^\tau \to p^t$ and $p^{||} \to p^z$

$$\left(\partial_\tau - \frac{p^{||}}{\tau}\partial_{p^{||}} + \underbrace{\frac{p^{||}}{\tau p^\tau}\partial_\eta\Big|_{p^{||}}}_{\text{non-boostinvariant}}\right) f(t,\eta,\boldsymbol{p}_T,p^{||}) = -C[f]$$

Time evolution

- Anisotropic initial conditions ($\xi \gg 1$): $\quad f(\tau_0, \eta, \boldsymbol{p}) \propto \frac{1}{\lambda} \exp\left(-\frac{2}{3}\frac{p_\perp^2 + \xi^2 (p^\parallel)^2}{Q(\eta)^2}\right)$
- Landau Frame: solve $T^{\mu\nu} u_\nu = -e u^\mu$ to obtain e and u^η

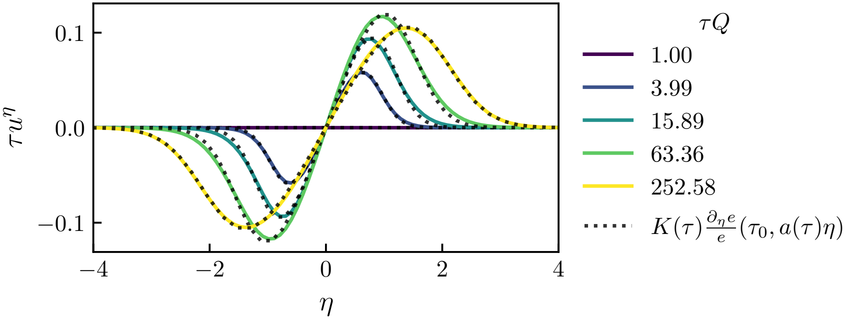


How does the flow velocity develop? $\rightarrow$ Gradients

J. Vredevoogd, S. Pratt, PRC 0810.4325, L. Keegan et al., JHEP 1605.04287

Initial gradients

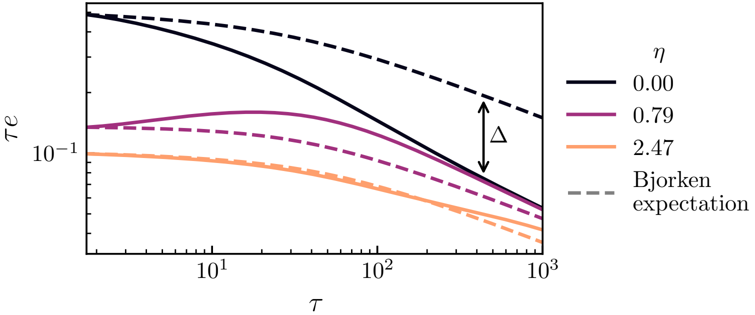
- Compare: flow velocity $u^\eta(\tau,\eta) \leftrightarrow \frac{\partial_\eta e}{e}(\tau_0,\eta)$ initial gradient
- Rescale $\rightarrow$ $K(\tau)\frac{\partial_\eta e}{e}(\tau_0, a(\tau)\eta)$



$u^\eta(\tau,\eta)$ **shaped** by initial η-gradients!

Outlook: Comparison to Bjorken expansion

- Longitudinal cooling $\boxed{\tau\partial_\tau T^{\tau\tau} = -T^{\tau\tau} + \tau^2 T^{\eta\eta}} - \partial_\eta \underbrace{T^{\eta\tau}}_{\leftrightarrow\, u^\eta}$



Knowing $\partial_\eta e(\tau_0, \eta)$, can we determine the difference Δ?

Summary

Thermalisation in EKT without boost invariance

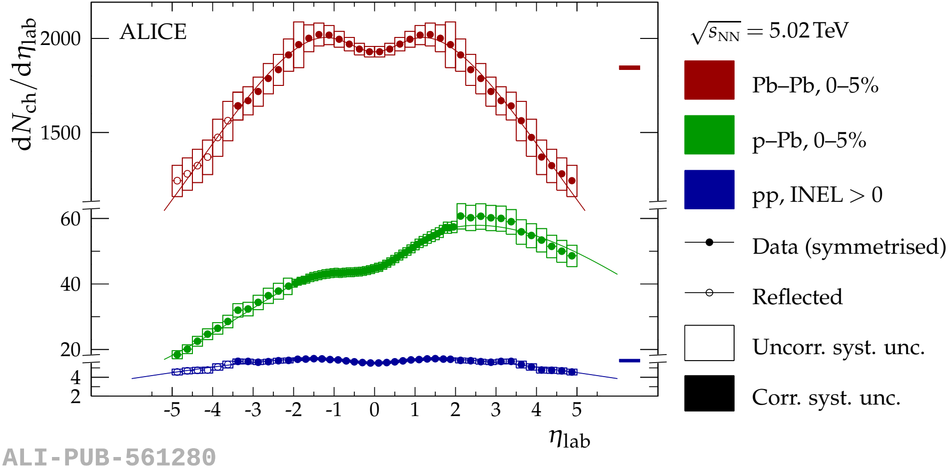
Expansion of thermal medium:

- Agreement with 1st order viscous hydrodynamics

η-**dependent far from equilibrium evolution**:

- Flow velocity related to initial gradients in η
- Modified longitudinal cooling

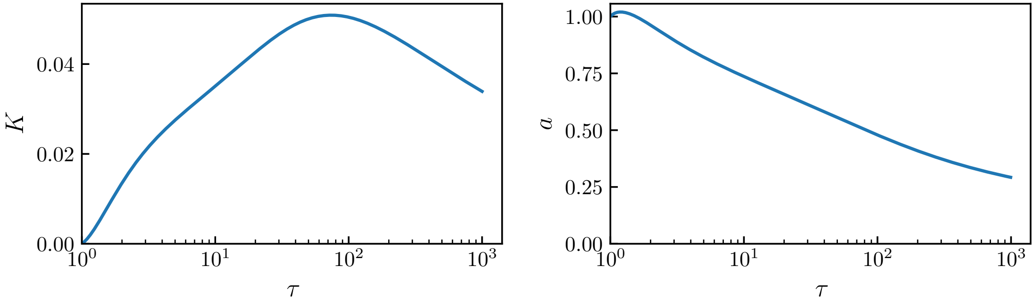
Backup



ALICE, PLB 2204.10210

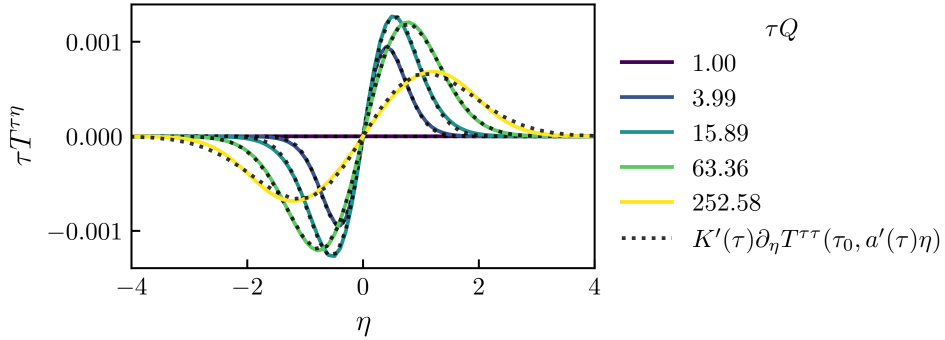
Backup

- Fitting parameters $K(\tau)$ and $a(\tau)$



Backup

- Scaling of energy flux $T^{\tau\eta}$



Backup

- Conservation equations

$$\partial_\tau T^{\tau\tau} + \partial_\eta T^{\eta\tau} = -\frac{T^{\tau\tau} + \tau^2 T^{\eta\eta}}{\tau}$$

$$\partial_\tau T^{\tau\eta} + \partial_\eta T^{\eta\eta} = -\frac{3T^{\tau\eta}}{\tau}$$

- Rewrite first equation

$$\partial_\tau(\tau T^{\tau\tau}) + \tau^2 T^{\eta\eta} = -\partial_\eta T^{\eta\tau}$$

- with initial gradient

$$T^{\tau\eta}(\tau,\eta) = -K(\tau)\partial_{\tilde{\eta}} T^{\tau\tau}(\tau_0,\tilde{\eta})\Big|_{\tilde{\eta}=a(\tau)\eta}$$

$$\partial_\tau(\tau T^{\tau\tau}) + \tau^2 T^{\eta\eta} = K(\tau)a(\tau)\partial_{\tilde{\eta}}^2 T^{\tau\tau}(\tau_0,\tilde{\eta})$$

Backup

Equations of motion from 2PI effective action

$$\left[iG_{0,ac}^{-1,\mu\gamma}(x;\mathcal{A})+\Pi_{ac}^{(0),\mu\gamma}(x)\right]\rho_{\gamma\nu}^{cb}(x,y)$$
$$=-\int_{y^0}^{x^0}dz\Pi_{ac}^{(\rho),\mu\gamma}(x,z)\rho_{\gamma\nu}^{cb}(z,y) \quad (1)$$
$$\left[iG_{0,ac}^{-1,\mu\gamma}(x;\mathcal{A})+\Pi_{ac}^{(0),\mu\gamma}(x)\right]F_{\gamma\nu}^{cb}(x,y)$$
$$=-\int_{t_0}^{x^0}dz\Pi_{ac}^{(\rho),\mu\gamma}(x,z)F_{\gamma\nu}^{cb}(z,y)+\int_{t_0}^{y^0}dz\Pi_{ac}^{(F),\mu\gamma}(x,z)\rho_{\gamma\nu}^{cb}(z,y) \quad (2)$$

Backup

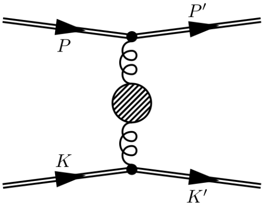
- local homogeneity $\rightarrow$ relative coordinate $s^\mu = x^\mu - y^\mu$ and center coordinate $X^\mu = \frac{1}{2}(x^\mu + y^\mu)$
- gradient expansion in X^μ
- to lowest order, spectral function ρ is on shell
  $\rightarrow$ quasi-particle picture
- non-equilibrium distribution function $f(X,p)$:

$$F(X,p) = -i\left[\frac{1}{2} \pm f(X,p)\right]\rho(X,p)$$

$$\Rightarrow p^\mu \partial_\mu f(X,p) = -C[f]$$

Collision kernel

$C_{2\leftrightarrow 2}$

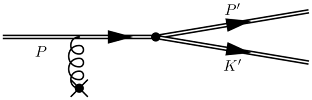


- small momentum transfer
  $q = |\boldsymbol{p}' - \boldsymbol{p}| \ll 1$:

isotropic HTL screening

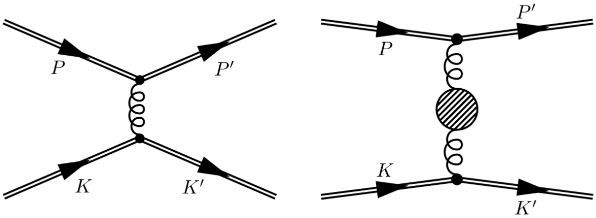
K. Boguslavski, F. Lindenbauer, PRD 2407.09605

$C_{1\leftrightarrow 2}$



- medium induced radiation of gluons
- $g \to q\bar{q}$ splittings
- LO: strictly collinear

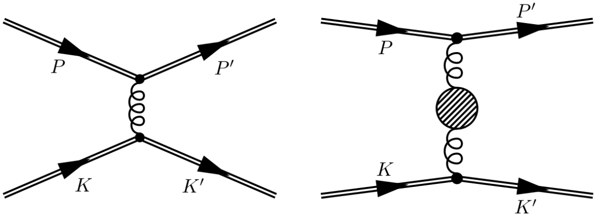
$2 \leftrightarrow 2$



Hard (left) and soft (right) medium regulated scattering

$$C_{2\leftrightarrow 2}[f](\boldsymbol{p}) = \frac{1}{4p\nu}\int_{\boldsymbol{k},\boldsymbol{p}',\boldsymbol{k}'} (2\pi)^4 \delta^{(4)}\left(p^\mu + k^\mu - p'^\mu - k'^\mu\right)$$
$$\times \left|\mathcal{M}\right|^2 \{\underbrace{f_{\boldsymbol{p}} f_{\boldsymbol{k}} \left(1 \pm f_{\boldsymbol{p}'}\right)\left(1 \pm f_{\boldsymbol{k}'}\right)}_{\text{loss}} - \underbrace{f_{\boldsymbol{p}'} f_{\boldsymbol{k}'} \left(1 \pm f_{\boldsymbol{p}}\right)\left(1 \pm f_{\boldsymbol{k}}\right)}_{\text{gain}}\} \quad (3)$$

$2 \leftrightarrow 2$



Hard (left) and soft (right) medium regulated scattering

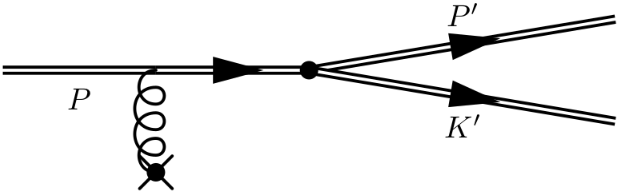
$$|\mathcal{M}|^2 = 2\lambda^2\nu\left(9 + \frac{(s-t)^2}{u^2} + \frac{(u-s)^2}{t^2} + \frac{(t-u)^2}{s^2}\right)$$

- small momentum transfer $q = |\boldsymbol{p}' - \boldsymbol{p}| \ll 1$ regulated by

$$\frac{1}{q^2} \to \frac{1}{q^2 + m_{\text{eff}}^2}$$

$$m_{\text{eff}}^2 = 2g^2 \int \frac{d^3\boldsymbol{p}}{(2\pi)^3 p}\left[N_c f^g_{\boldsymbol{p}} + N_f f^q_{\boldsymbol{p}}\right]$$

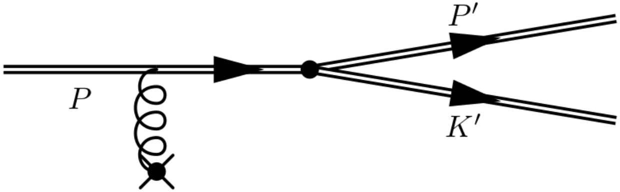
$1 \leftrightarrow 2$



effective $1 \leftrightarrow 2$ process

$$C_{1\leftrightarrow 2}[f](\boldsymbol{p}) = \frac{1}{2}\frac{1}{\nu}(2\pi)^3 \int_{\tilde{\boldsymbol{p}},\boldsymbol{p}',\boldsymbol{k}'} (2\pi)^4 \delta^{(4)}\left(\tilde{p}^\mu - p'^\mu - k'^\mu\right)$$
$$\times \left[\delta^{(3)}(\boldsymbol{p}-\tilde{\boldsymbol{p}}) - \delta^{(3)}\left(\boldsymbol{p}-\boldsymbol{p}'\right) - \delta^{(3)}\left(\boldsymbol{p}-\boldsymbol{k}'\right)\right]$$
$$\times \gamma\{\underbrace{f_{\boldsymbol{p}}\left(1 \pm f_{\tilde{\boldsymbol{p}}'}\right)\left(1 \pm f_{\boldsymbol{k}'}\right)}_{\text{loss}} - \underbrace{f_{\boldsymbol{p}'} f_{\boldsymbol{k}'}\left(1 \pm f_{\tilde{\boldsymbol{p}}}\right)}_{\text{gain}}\} \tag{4}$$

$1 \leftrightarrow 2$



effective $1 \leftrightarrow 2$ process

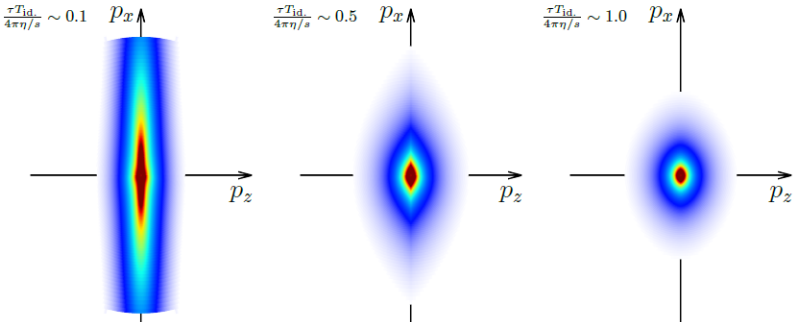
- LO $\rightarrow$ strictly collinear
- medium induced radiation of gluons
- $N+1 \leftrightarrow N+2$ effectively $1 \leftrightarrow 2$

$1 \leftrightarrow 2$

Parton going through the medium, Figure from [**Jasmine**]

- hard parton receiving multiple kicks
- formation time $\tau_f \sim E$
- BH: $l_f \ll l_{\mathrm{mfp}}$, independent emissions
- LPM: $l_f \sim l_{\mathrm{mfp}}$, destructive interference $\rightarrow$ suppression

Bottom-up thermalization



Isotropization of the distribution, Figure from [**Kurkela_2019**]

- 1: overoccupied system getting more anisotropic
- 2: population of soft gluons
- 3: inverse energy cascade