Glasma Role in Jet Quenching Effects

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Based on ongoing work with Liliana Apolinário [b] and José Soares [b]

September — 9th 2025

Initial Stages (Taipei, Taiwan)

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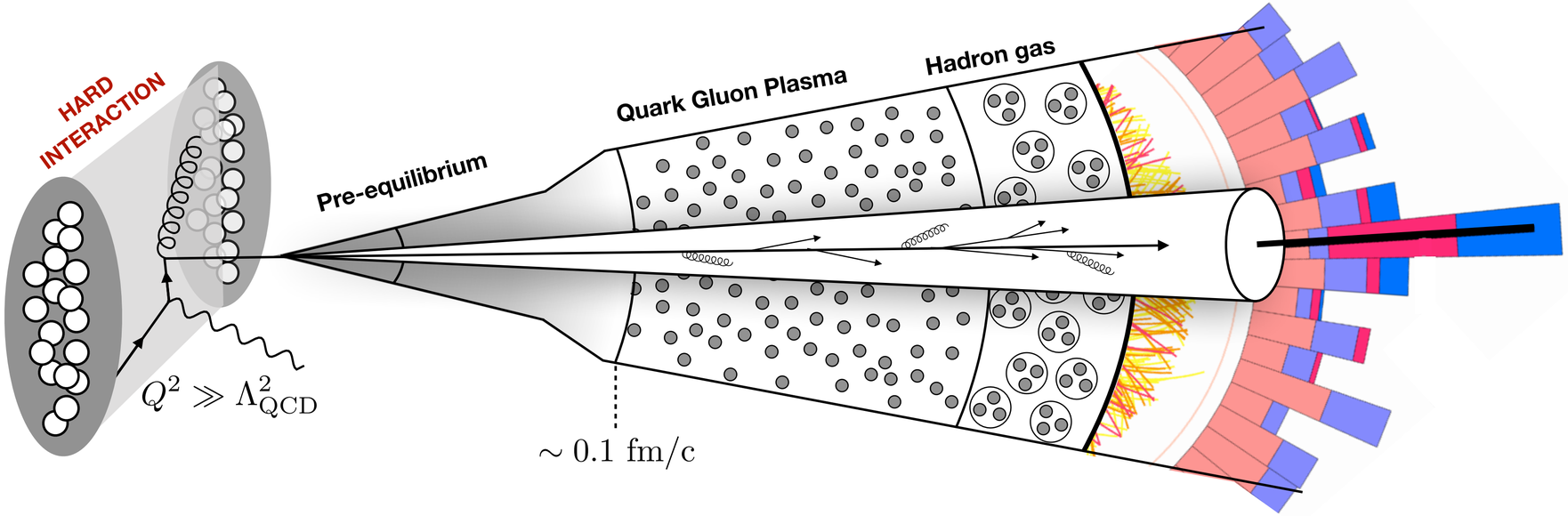
[b] *Laboratório de Instrumentação e Física Experimental de Partículas (LIP)*

Introduction

- Shortly after a Heavy Ion Collision (HIC) a de-confined, fluid-like state emerges: the **Quark Gluon Plasma (QGP)**
- Jets in HICs are modified with respect to the p-p reference ⟶ **Jet Quenching** (one of the main QGP signals, and a **multi-scale probe** of its properties)



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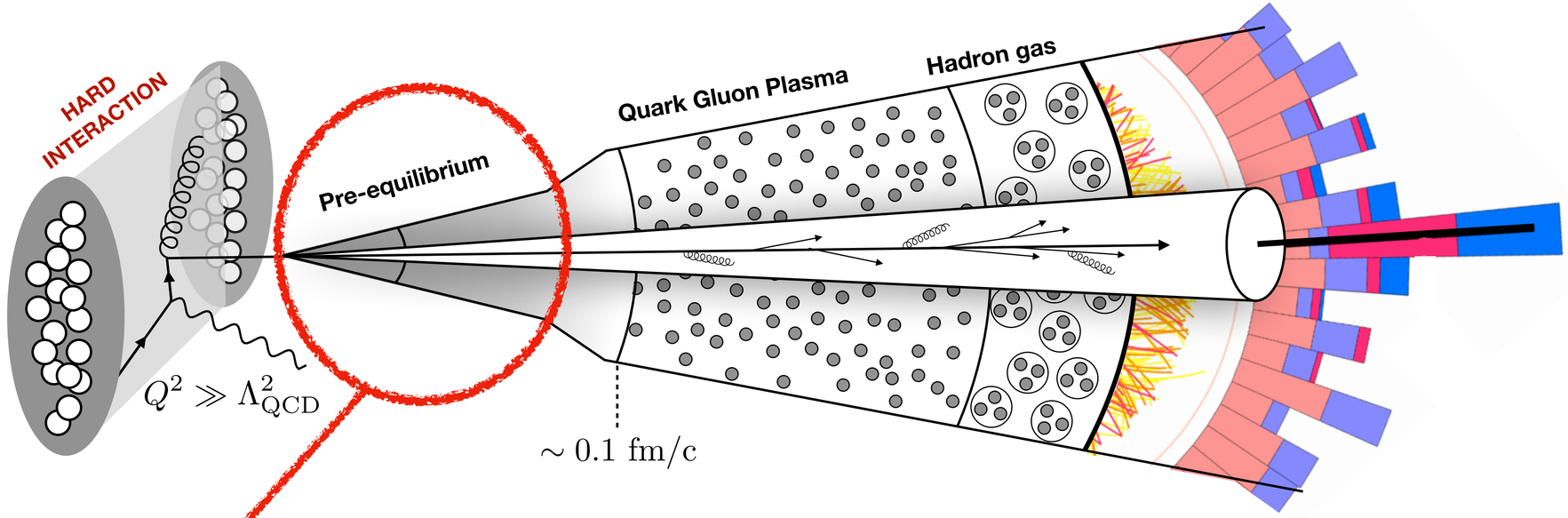

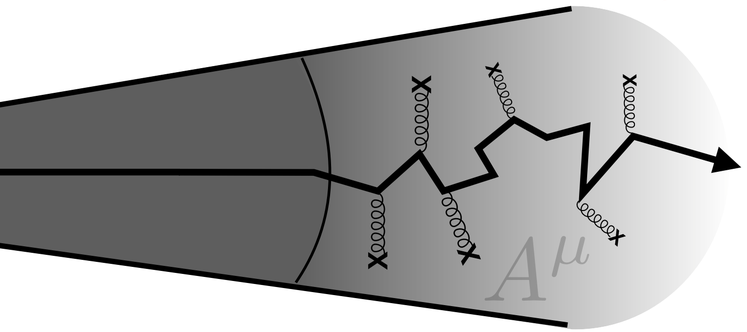

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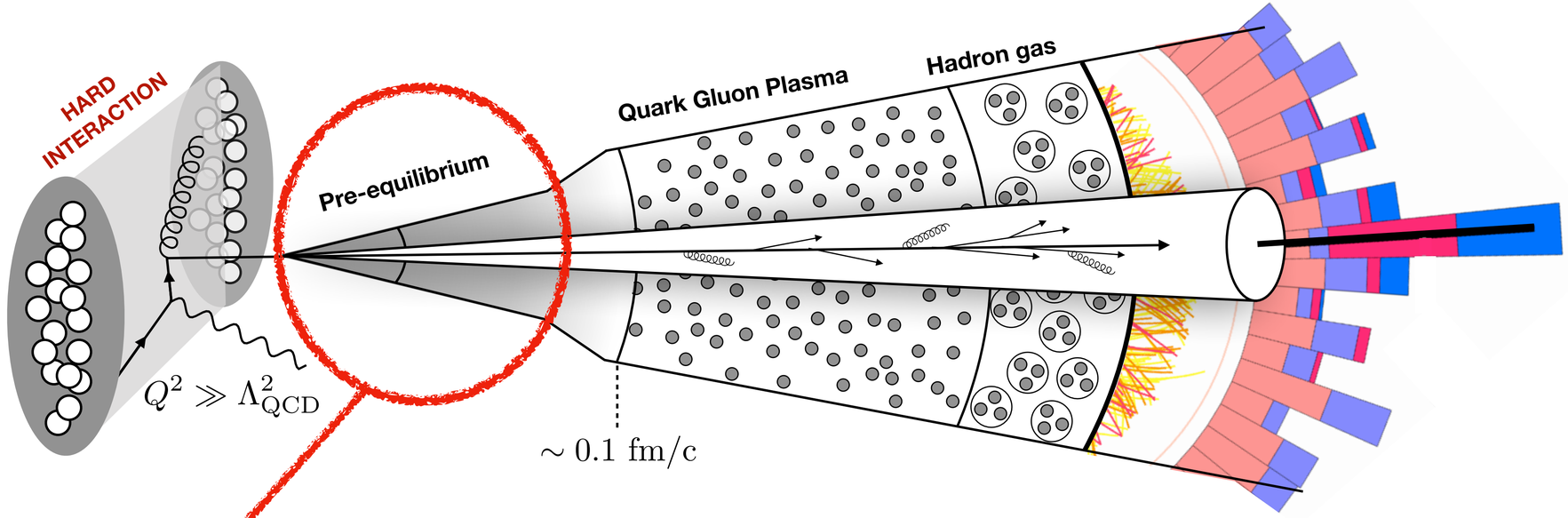

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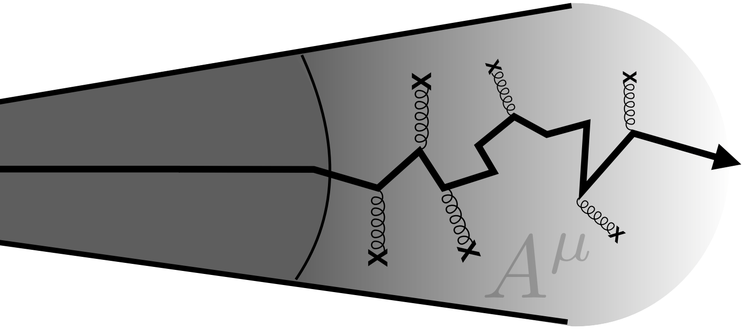

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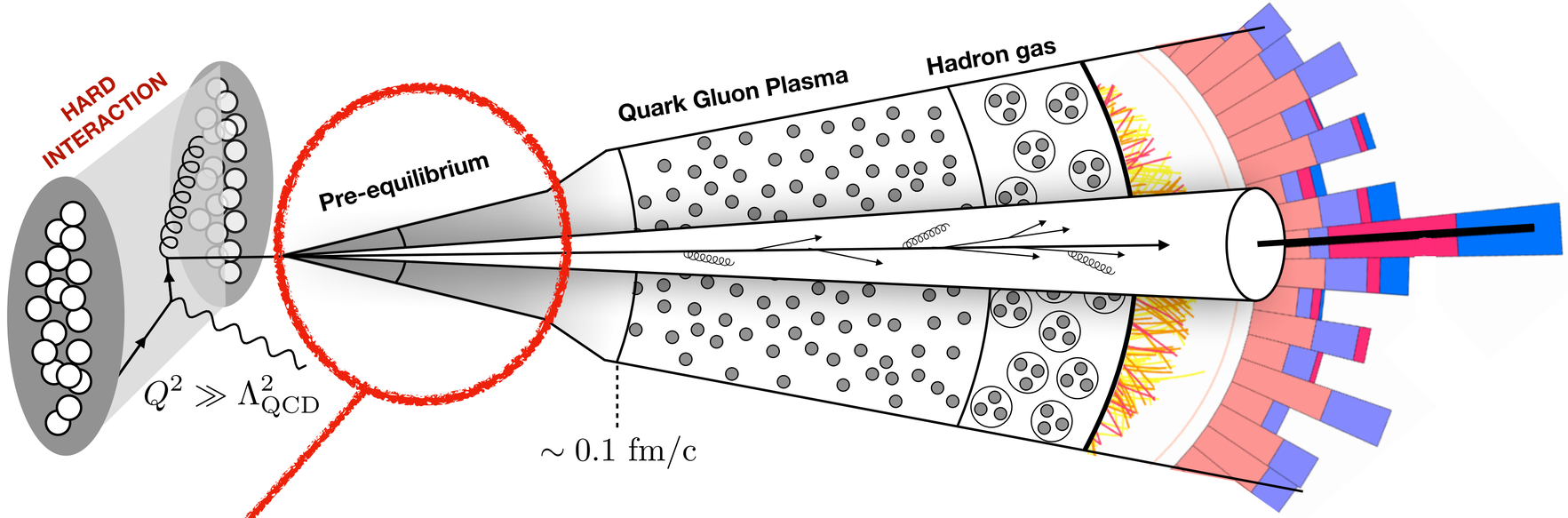

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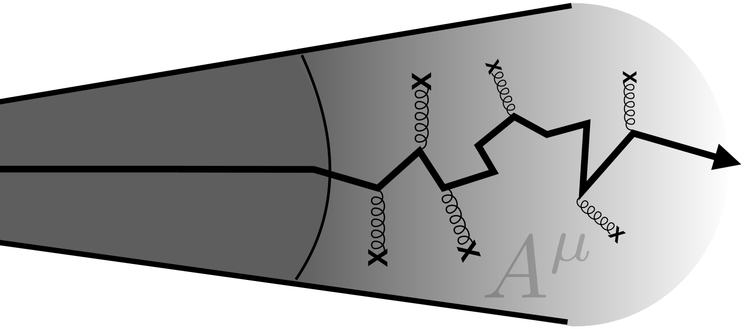

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- *So, how do we quantify the impact of the pre-equilibrium phase of Heavy Ion Collisions over Jet Quenching observables?*

The Color Glass Condensate

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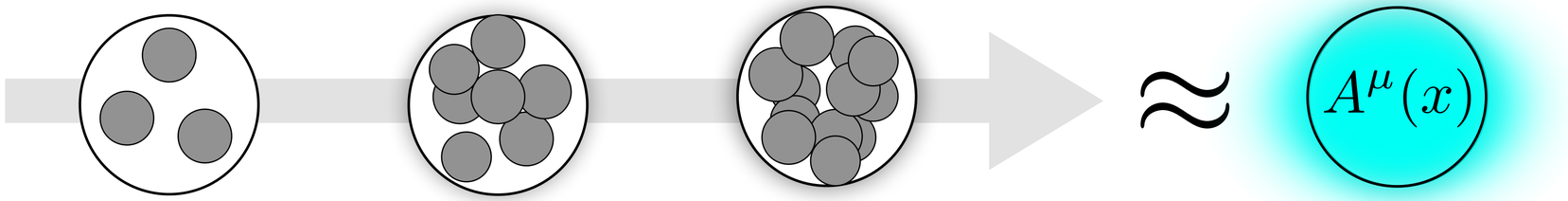
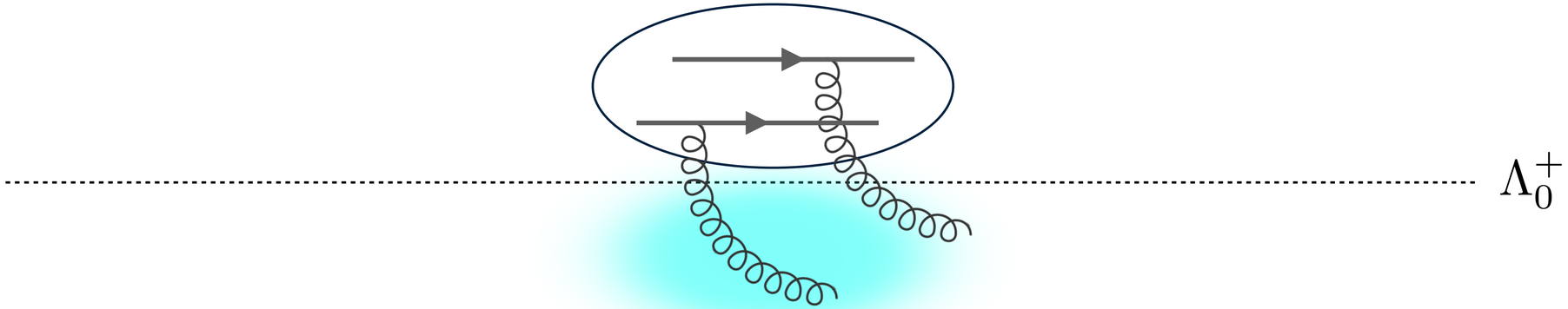


CGC: Classical fields generated by valence partons

Moderate energy: Dilute hadrons

High energy: Gluon-dense hadrons



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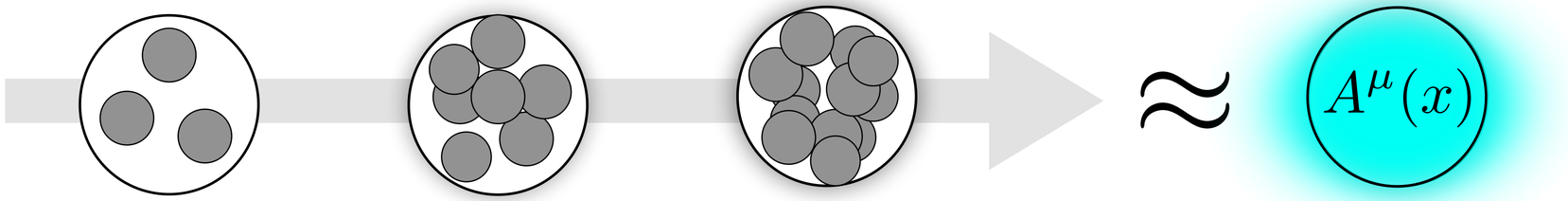

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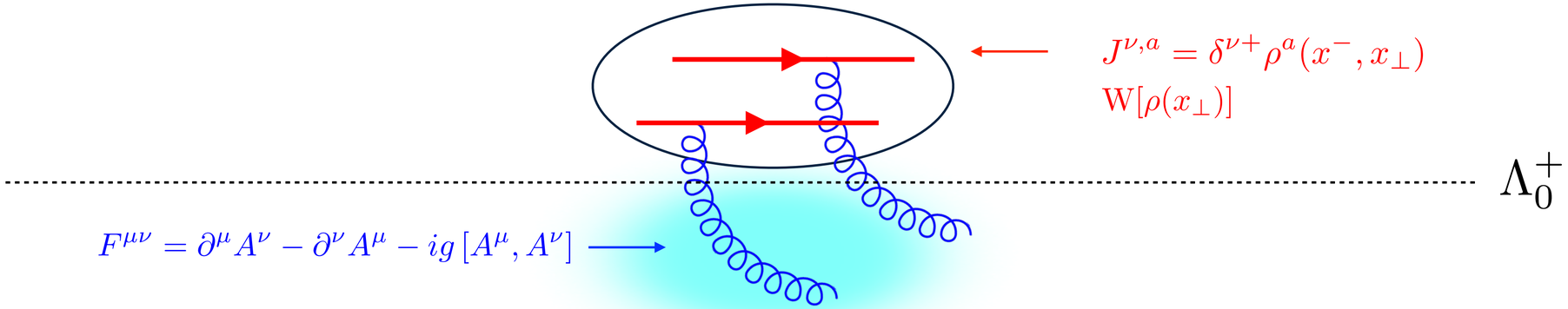
$$[D_\mu, F^{\mu\nu}] = J^\nu \propto \rho^a(x) t^a$$



Evolution with separation scale described by B-JIMWLK equations

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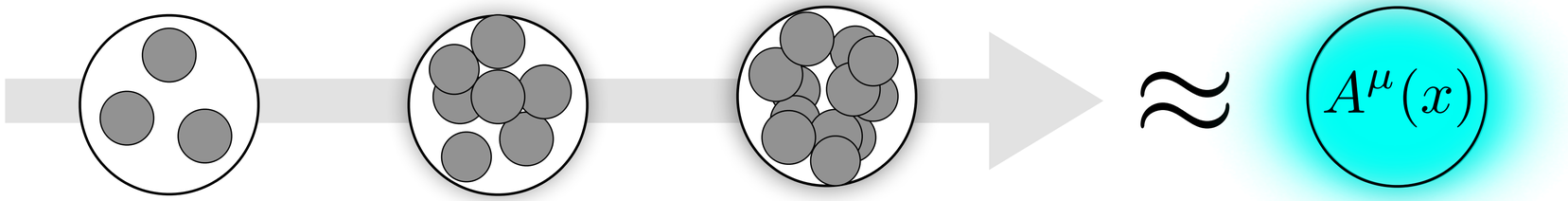

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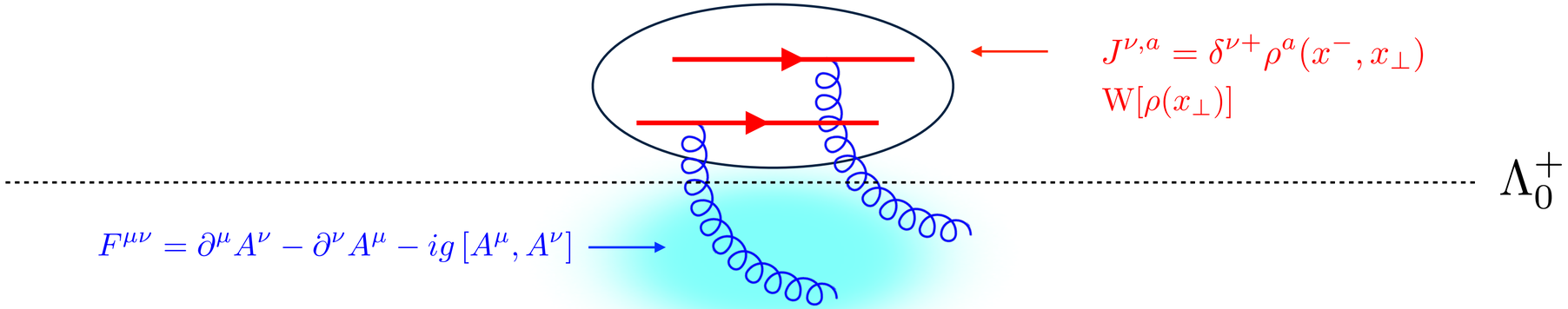

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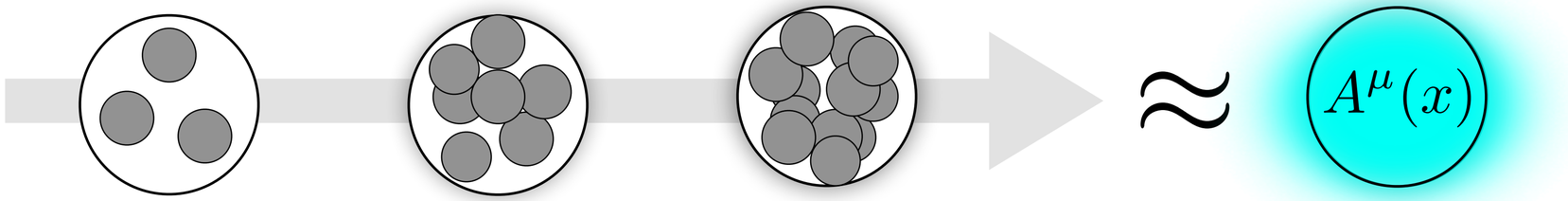

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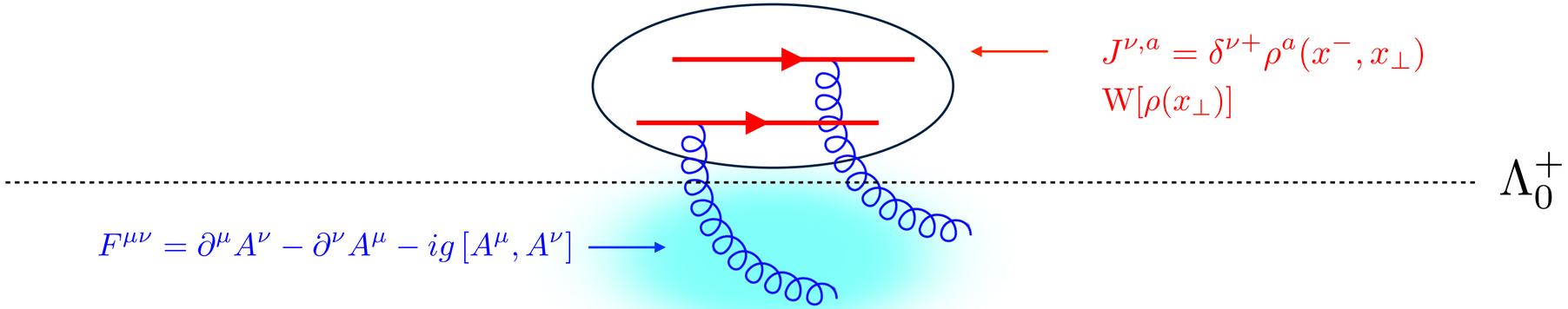

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- ➡ Color charge distributions for large nuclei: **McLerran-Venugopalan (MV) model**

$$\langle \rho^a(x_\perp) \rangle = 0 \quad , \quad \langle \rho^a(x_\perp) \rho^b(y_\perp) \rangle = g^2 \mu^2 \delta^{ab} \delta^{(2)}(x_\perp - y_\perp)$$

Gaussian statistics

↓

Analytical calculations are possible

The Glasma Fields

Kovner, McLerran and Weigert, Phys. Rev. D 52 (1995)

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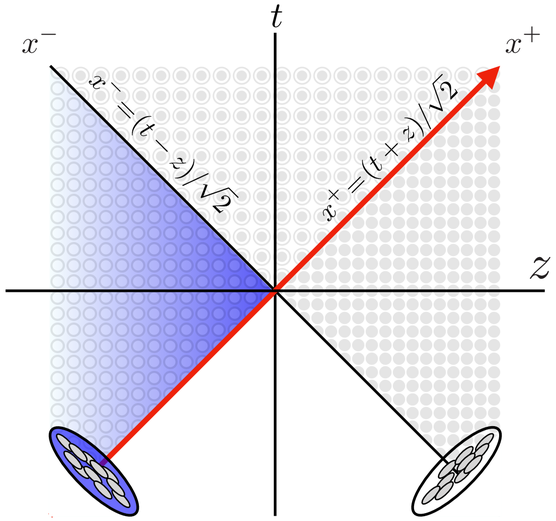
  - ➡ The CGC gives us the gauge fields **before the collision**:



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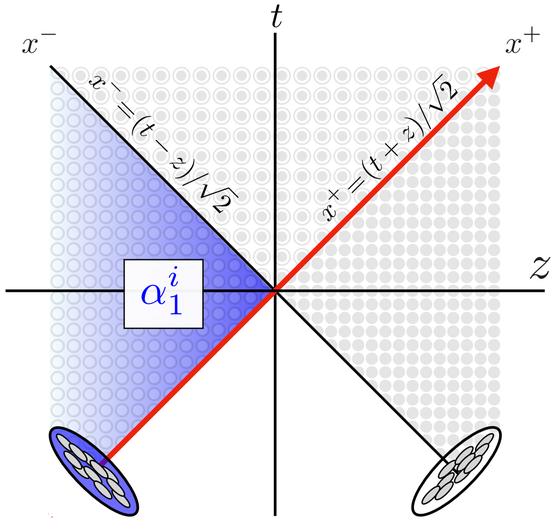

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with the Wilson line defined as: $U_1(x_\perp) = P^- \exp\left\{-ig\int_{-\infty}^{\infty} dz^- \frac{1}{\nabla^2}\rho_1(z^-, x_\perp)\right\}$

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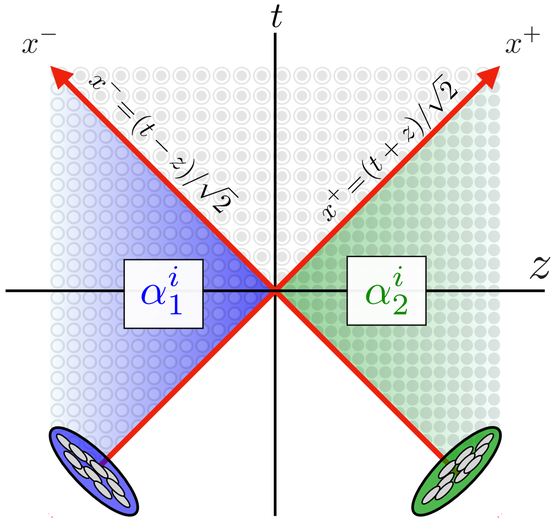

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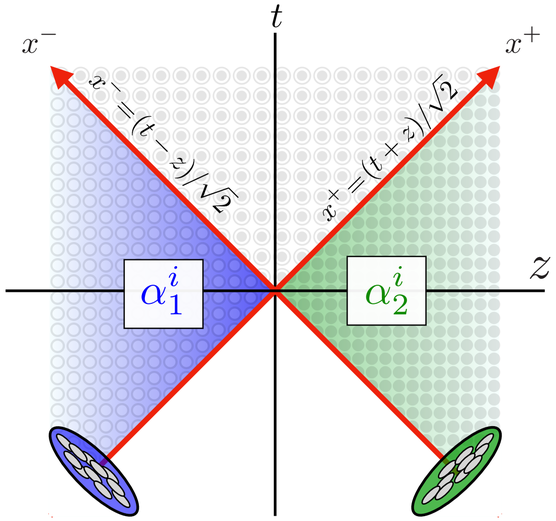

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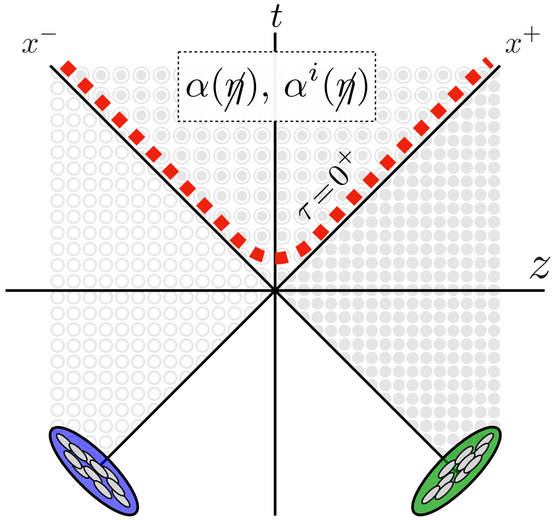

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(In Fock-Schwinger gauge gauge $A^+x^- + A^-x^+ = 0$)

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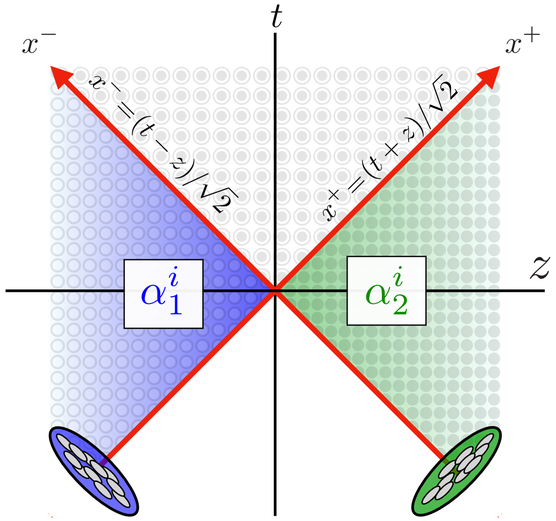

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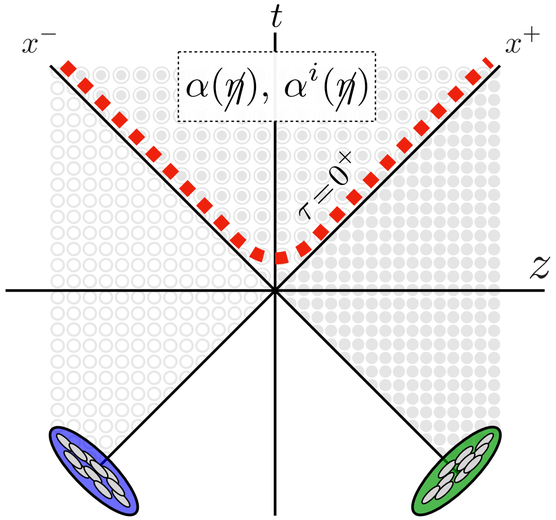

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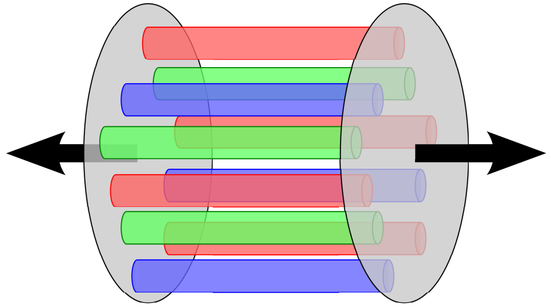

Credit: Fukushima; Acta Phys.Polon. 2011

➡ Longitudinal chromo-electric and chromo-magnetic fields (**flux tube picture**):

$$\boxed{\begin{aligned} E^\eta(\tau = 0^+, x_\perp) &= -ig\delta^{ij}[\alpha_1^i(x_\perp), \alpha_2^j(x_\perp)] \\ B^\eta(\tau = 0^+, x_\perp) &= -ig\varepsilon^{ij}[\alpha_1^i(x_\perp), \alpha_2^j(x_\perp)] \end{aligned}}$$

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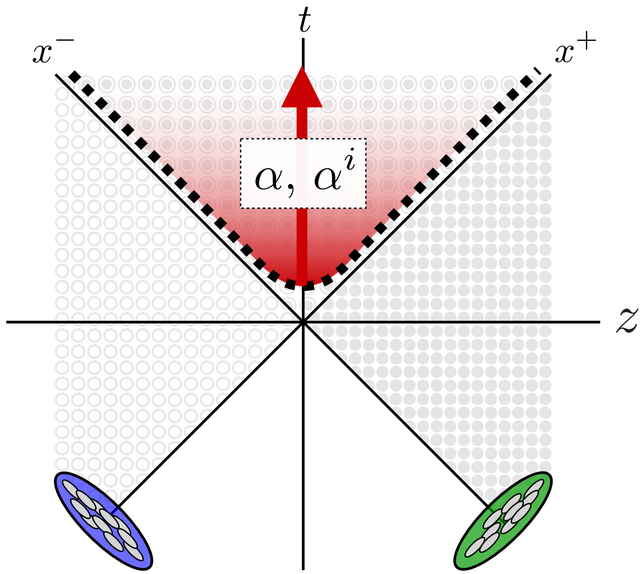

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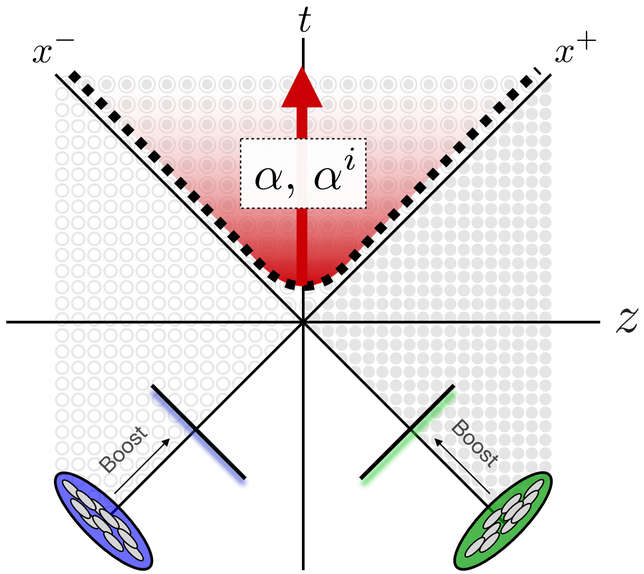

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(Extremely Lorentz-contracted nuclei)

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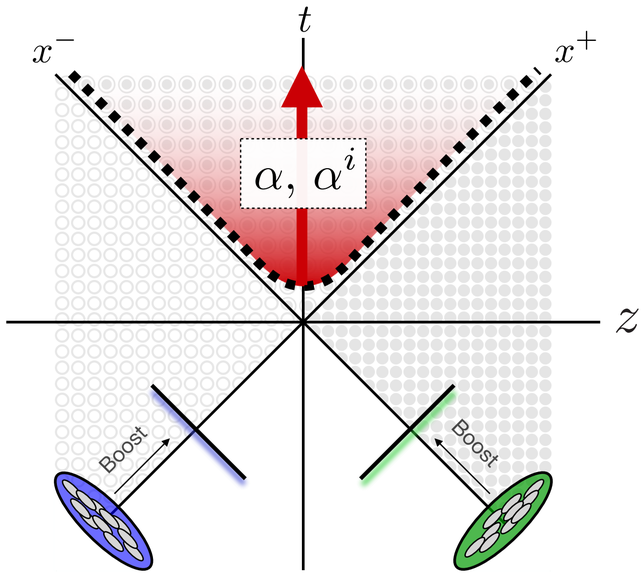

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$$\approx \rho_1(x_\perp)\delta(x^-)\delta^{\nu+} + \rho_2(x_\perp)\delta(x^+)\delta^{\nu-}$$

$$\tau = \sqrt{2x^+x^-} > 0 \longrightarrow [D_\mu, F^{\mu\nu}] = 0$$

$$\nu = \tau \longrightarrow ig\tau\,[\alpha, \partial_\tau \alpha] - \frac{1}{\tau}\left[D_i, \partial_\tau \alpha^i\right] = 0$$

$$\nu = \eta \longrightarrow \frac{1}{\tau}\partial_\tau \frac{1}{\tau}\partial_\tau(\tau^2\alpha) - \left[D_i, \left[D^i, \alpha\right]\right] = 0$$

$$\nu = i \longrightarrow \frac{1}{\tau}\partial_\tau(\tau\partial_\tau\alpha^i) - ig\tau^2\left[\alpha, \left[D^j, \alpha\right]\right] - \left[D^j, F^{ji}\right] = 0$$

➡ Typically solved through numerical methods. **Analytical approximations** include:

- Weak field limit (i.e. expanding in orders of one of the sources), e.g. PGR and S. Demirci; PRD, 2023, T. Lappi and S. Schlichting; PRD, 2018
- Expanding the fields in powers of the proper time, e.g. M. E. Carrington, A. Czajka, S. Mrowczynski; PRC, 2022, also works by G. Chen, R. J. Fries, J. I. Kapusta, Y. Li

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Kovner, McLerran and Weigert, Phys. Rev. D 52 (1995)

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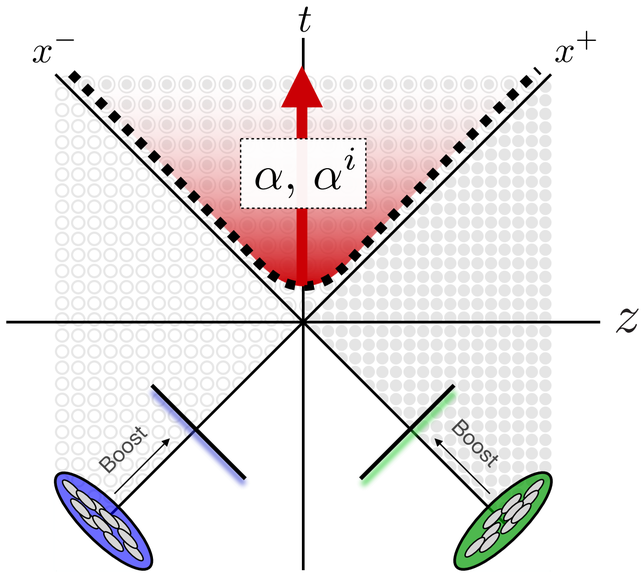

$$[D_\mu, F^{\mu\nu}] = J_1^\nu + J_2^\nu = \rho_1(x_\perp, x^-)\delta^{\nu+} + \rho_2(x_\perp, x^+)\delta^{\nu-}$$

(Extremely Lorentz-contracted nuclei)

$$\approx \rho_1(x_\perp)\delta(x^-)\delta^{\nu+} + \rho_2(x_\perp)\delta(x^+)\delta^{\nu-}$$

$$\tau = \sqrt{2x^+x^-} > 0 \longrightarrow [D_\mu, F^{\mu\nu}] = 0$$

$$\nu = \tau \longrightarrow ig\tau\left[\alpha, \partial_\tau\alpha\right] - \frac{1}{\tau}\left[D_i, \partial_\tau\alpha^i\right] = 0$$

$$\nu = \eta \longrightarrow \frac{1}{\tau}\partial_\tau\frac{1}{\tau}\partial_\tau(\tau^2\alpha) - \left[D_i, \left[D^i, \alpha\right]\right] = 0$$

$$\nu = i \longrightarrow \frac{1}{\tau}\partial_\tau(\tau\partial_\tau\alpha^i) - ig\tau^2\left[\alpha, \left[D^j, \alpha\right]\right] - \left[D^j, F^{ji}\right] = 0$$

➡ Typically solved through numerical methods. **Analytical approximations** include:

- Weak field limit (i.e. expanding in orders of one of the sources), e.g. PGR and S. Demirci; PRD, 2023, T. Lappi and S. Schlichting; PRD, 2018
- Expanding the fields in powers of the proper time, e.g. M. E. Carrington, A. Czajka, S. Mrowczynski; PRC, 2022, also works by G. Chen, R. J. Fries, J. I. Kapusta, Y. Li

➡ Assuming **UV dominance**, we employ the **linearized Yang-Mills equations** (as in PGR and T. Lappi; PRD, 2021):

$$\partial_\tau\frac{1}{\tau}\partial_\tau(\tau^2\beta) = \tau\partial_i\partial^i\beta$$

$$\partial_\tau(\tau\partial_\tau\beta^i) = \tau\partial^k\partial_k\beta^i$$

Where we impose $\partial_i\beta^i = 0$ **(Coulomb gauge condition).** A good approximation for high momentum modes! J. P. Blaizot, T. Lappi and Y. Mehtar-Tani; Nucl. Phys. A 846 (2010) 63

Non-Linear Initial Conditions, Linear Evolution

H. Fujii, K. Fukushima, Y. Hidaka; PRC, 2008
PGR and T. Lappi; PRD, 2021

➡ **Free plane wave** solution of linearized Yang-Mills equations in momentum space:

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$\longrightarrow$ $\beta(\tau, k_\perp) = \beta_0(k_\perp)\frac{2J_1(k\tau)}{k\tau}$, $\beta^i(\tau, k_\perp) = \beta_0^i(k_\perp)J_0(k\tau)$

Appropriate for high-momentum modes

➡ Boundary conditions obtained by **matching to** $\tau = 0^+$ **solution (** $E^\eta(\tau = 0^+, x_\perp),\ B^\eta(\tau = 0^+, x_\perp)$ **):**

$$\beta(\tau, k_\perp) = \frac{\tau}{k}E_0^\eta(k_\perp)J_1(k\tau) \quad , \quad \beta^i(\tau, k_\perp) = -i\frac{\epsilon^{ij}k^j}{k^2}B_0^\eta(k_\perp)J_0(k\tau)$$

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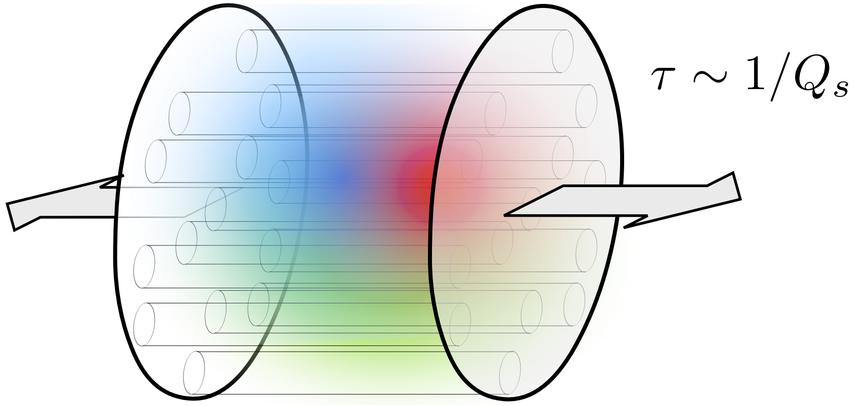

Transverse components emerge

↓

"Flux tube" picture starts breaking down, but it's approximately valid while the CGC is applicable

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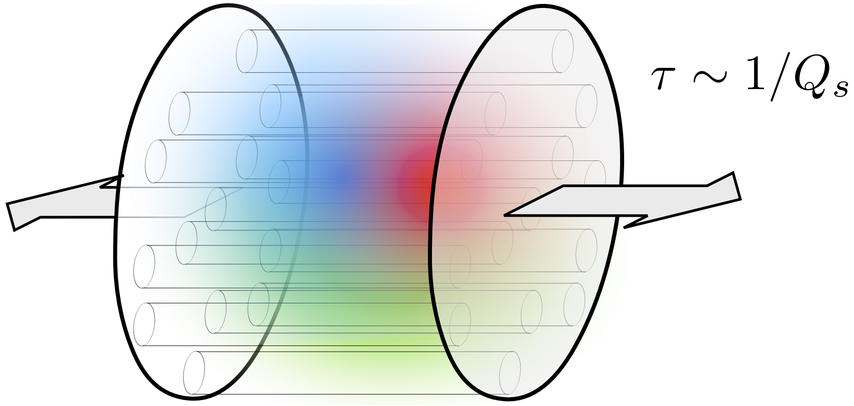

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➡ This solution **effectively resums the UV divergencies** present in the power series in τ (see **H. Fujii, K. Fukushima, Y. Hidaka; PRC, 2008**)

Fokker-Planck Equation

M. E. Carrington, A. Czajka, S. Mrowczynski; PRC, 2022
Stanislaw Mrowczynski; Eur. Phys. Journal A, 2018

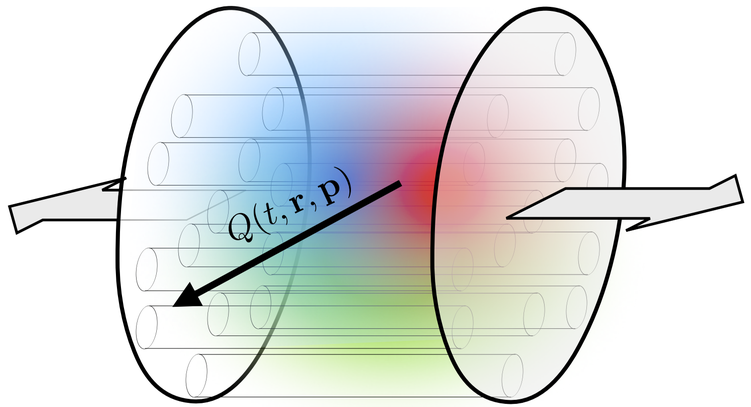
- *How do quarks move through the Glasma?* Let $Q(t, \mathbf{r}, \mathbf{p})$ be the **quark distribution function**

➡ We start from the following **Vlasov-type equation**: $\underbrace{\left(\partial_t + \mathbf{v} \cdot \nabla\right)}_{\equiv \mathcal{D}} Q(t, \mathbf{r}, \mathbf{p}) - \frac{1}{2}\left\{\mathbf{F}(t, \mathbf{r}), \nabla_p Q(t, \mathbf{r}, \mathbf{p})\right\} = 0$

"Material derivative"

Based on the following approximations: (1) **Collision-less** (low concentration of hard probes in the Glasma)

(2) **Mean field** ($\mathbf{F}$ dominated by ensemble-averaged field)



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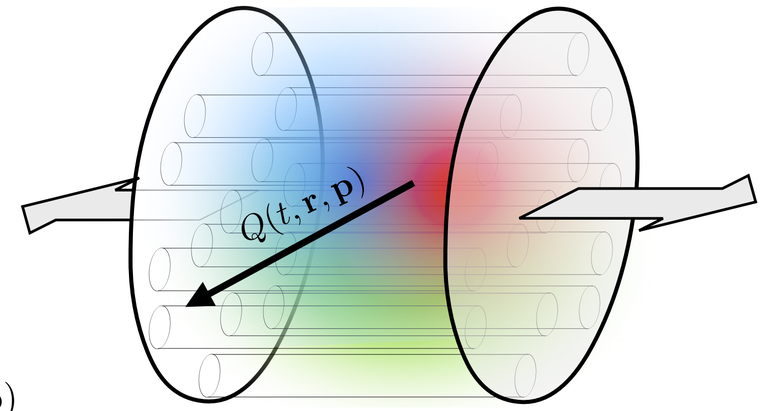
"Regular" and "Turbulent" components



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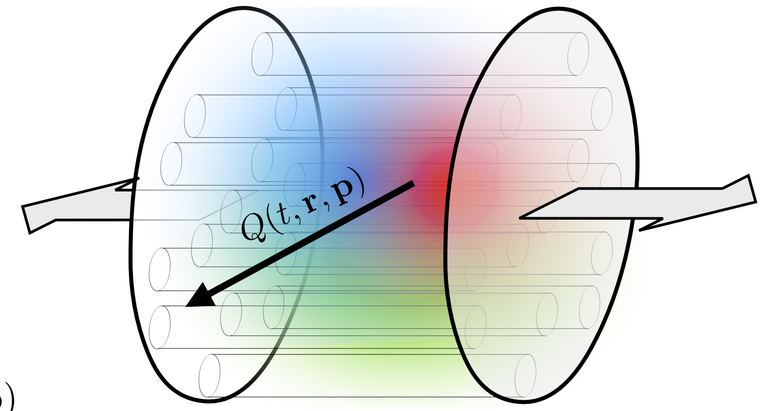

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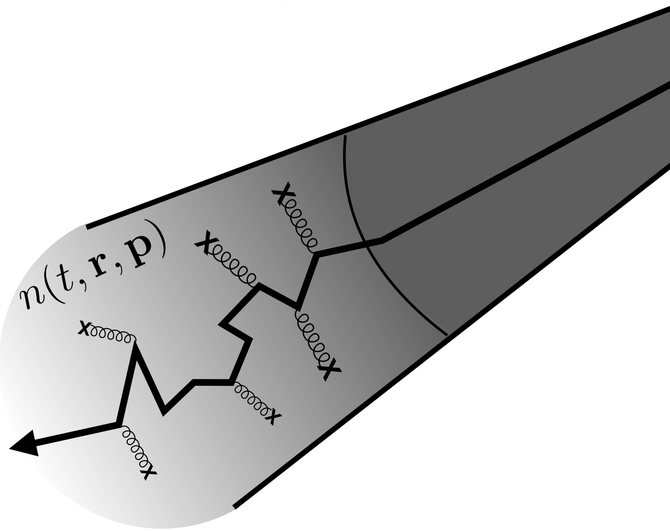

➡ By substituting, taking the ensemble average and tracing over color indices one arrives at a **Fokker-Planck transport equation**:

$$\left(\mathcal{D} - \nabla_p^\alpha X^{\alpha\beta}(\mathbf{v})\nabla_p^\beta - \nabla^\alpha Y^\alpha(\mathbf{v})\right) n(t,\mathbf{x},\mathbf{p}) = 0$$

Where the collision terms are defined as:

$$X^{\alpha\beta}(\mathbf{v}) \equiv \frac{1}{2N_c}\int_0^t dt' \mathrm{Tr}\langle \mathcal{F}^\alpha(t,\mathbf{x})\mathcal{F}^\beta(t-t',\mathbf{x}-\mathbf{v}t')\rangle \,, \quad Y^\alpha(\mathbf{v}) = X^{\alpha\beta}\frac{v^\beta}{T}$$

With T is defined as the temperature of an equilibrated plasma with the same energy density as the medium, $\mathbf{v} = \mathbf{p}/E_p$ is the velocity of the probe, and $\boldsymbol{\mathcal{F}}(t,\mathbf{x}) \equiv g\left(\mathbf{E}(t,\mathbf{x}) + \mathbf{v}\times\mathbf{B}(t,\mathbf{x})\right)$ is the Lorentz force generated by the mean color fields.

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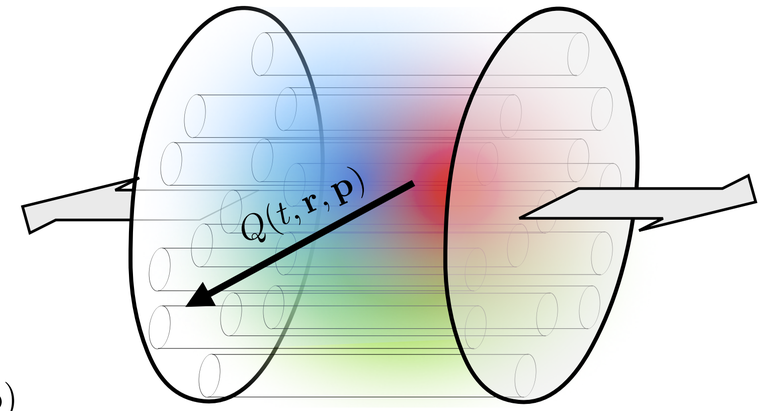

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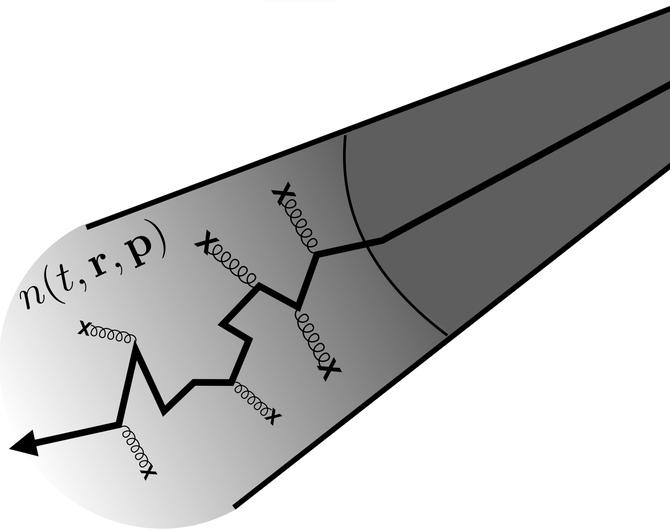

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➡ **Transport coefficients**: $\frac{\langle \Delta p^\alpha\rangle}{\Delta t} = -Y^\alpha \,, \quad \frac{1}{2}\frac{\langle \Delta p^\alpha \Delta p^\beta\rangle}{\Delta t} = X^{\alpha\beta}$ ➡ $\boxed{\frac{dE}{dx} = -\frac{v}{T}\frac{v^{\alpha\beta}}{v^2}X^{\alpha\beta}(\mathbf{v})}$, $\boxed{\hat{q} = \frac{2}{v}\left(\delta^{\alpha\beta} - \frac{v^\alpha v^\beta}{v^2}\right)X^{\alpha\beta}(\mathbf{v})}$

Transport Coefficients in the Glasma

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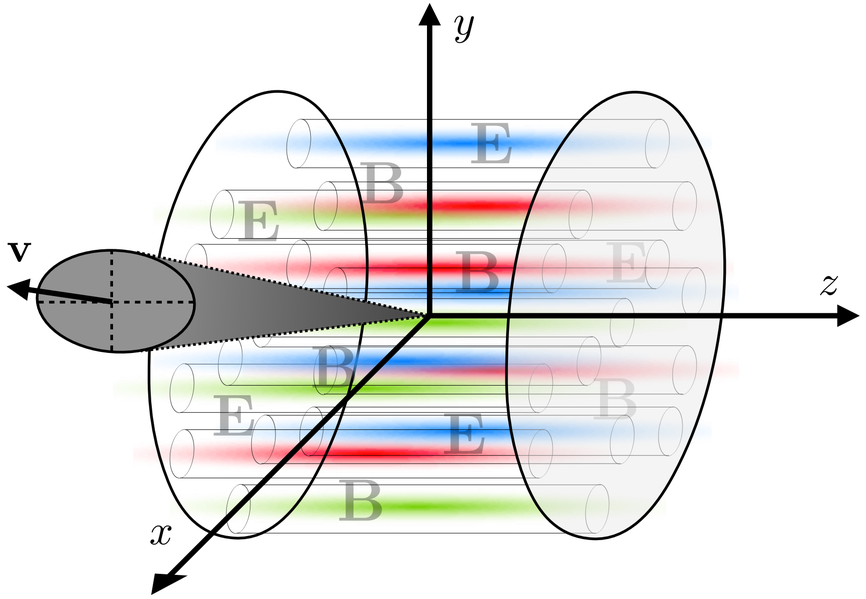
➡ Substituting the chromodynamic fields, one obtains (**M. E. Carrington, A. Czajka, S. Mrowczynski; PRC, 2022**):

$$X^{\alpha\beta}(\mathbf{v}) = \frac{g^2}{2N_c}\int_0^t dt' \big[\langle E_a^\alpha(t,\mathbf{x})E_a^\beta(t-t',\mathbf{y})\rangle + \epsilon^{\beta\gamma\gamma'}v^\gamma\langle E_a^\alpha(t,\mathbf{x})B_a^{\gamma'}(t-t',\mathbf{y})\rangle$$
$$+ \epsilon^{\alpha\gamma\gamma'}v^\gamma\langle B_a^{\gamma'}(t,\mathbf{x})E_a^\beta(t-t',\mathbf{y})\rangle + \epsilon^{\alpha\gamma\gamma'}\epsilon^{\beta\delta\delta'}v^\gamma v^\delta\langle B_a^{\gamma'}(t,\mathbf{x})B_a^{\delta'}(t-t',\mathbf{y})\rangle\big]$$

With e.g. $E^\eta(\tau,x_\perp) = -ig\delta^{ij}\int\frac{d^2k_\perp}{(2\pi)^2}\int d^2u_\perp[\alpha_1^i(u_\perp),\alpha_2^j(u_\perp)]J_0(k\tau)e^{ik_\perp(x-u)_\perp}$ and $\mathbf{y} = \mathbf{x} - \mathbf{v}t'$



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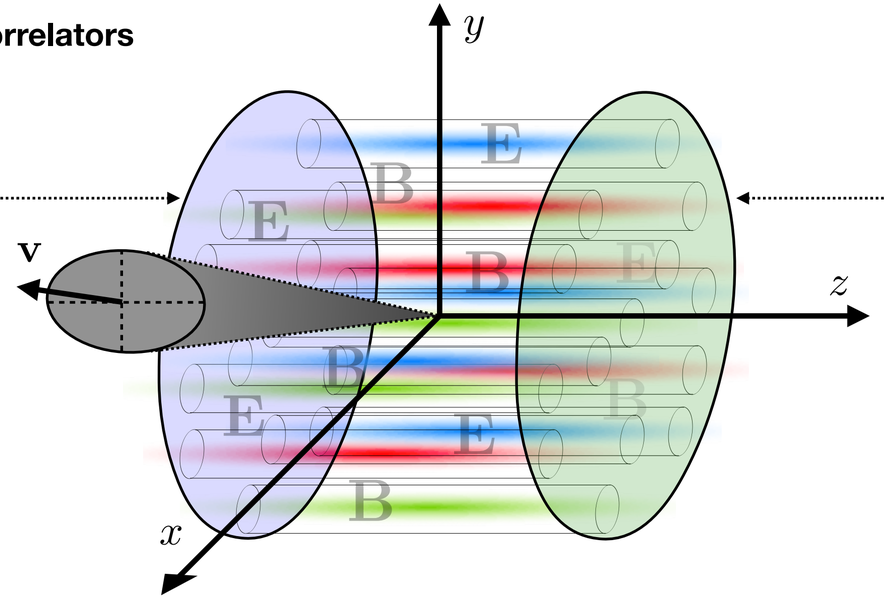
➡ Building blocks of the calculation: **2-point correlators**
(a.k.a. **Weizsacker-Williams distributions**)

$\langle \alpha_1^{a,i}\alpha_1^{b,j}\rangle \equiv W_1^{ij}\delta^{ab}$

$\langle \alpha_2^{c,k}\alpha_2^{d,l}\rangle \equiv W_2^{kl}\delta^{cd}$



Analytical Results

➡ In terms of the Weizsäcker-Williams distributions:

$$\frac{dE}{dx} = -\frac{v}{T}\frac{g^4(N_c^2-1)}{2}\int_0^t dt' \int_{k_\perp q_\perp}\int_{u_\perp v_\perp} W_1^{ik}(u_\perp, v_\perp) W_2^{jl}(u_\perp, v_\perp) e^{ik_\perp (x-u)_\perp} e^{iq_\perp (y-v)_\perp}$$
$$\times \left(\delta^{ij}\delta^{kl}\frac{(v_\parallel)^2}{v^2} J_0(kt) J_0(q(t-t')) - \epsilon^{ij}\epsilon^{kl}\left(\frac{v_\perp \times k_\perp}{vk}\right)\left(\frac{v_\perp \times q_\perp}{vq}\right) J_1(kt) J_1(q(t-t'))\right)$$

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➡ We might adopt different models for $W^{ij}(r_\perp) = \frac{1}{2}\left(\delta^{ij}G(r_\perp) + \left(\delta^{ij} - 2\frac{r^i r^j}{r^2}\right)h(r_\perp)\right)$

In the **MV model:**

$$G_{\rm MV}(r_\perp) = \frac{g^2\bar{\mu}^2}{4\pi N_c}\left(mrK_1(mr) - 2K_0(mr)\right)\frac{1-\exp\left\{\frac{g^4\bar{\mu}^2 N_c}{4\pi m^2}\left(mrK_1(mr)-1\right)\right\}}{\frac{g^4\bar{\mu}^2}{4\pi m^2}\left(mrK_1(mr)-1\right)}$$

$$h_{\rm MV}(r_\perp) = -\frac{g^2\bar{\mu}^2}{4\pi N_c}mrK_1(mr)\frac{1-\exp\left\{\frac{g^4\bar{\mu}^2 N_c}{4\pi m^2}\left(mrK_1(mr)-1\right)\right\}}{\frac{g^4\bar{\mu}^2}{4\pi m^2}\left(mrK_1(mr)-1\right)}.$$

In the **GBW model:**

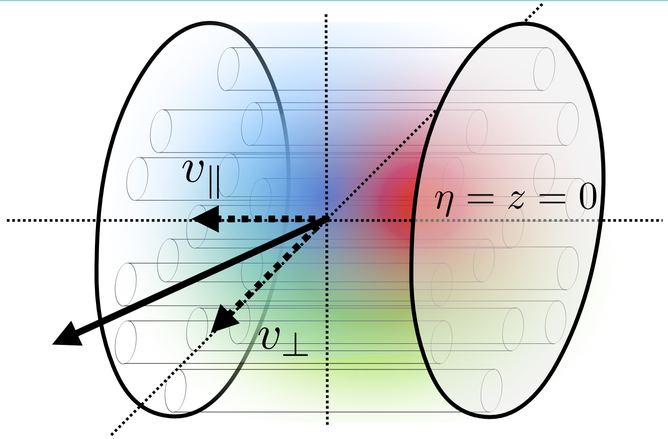
$$G_{\rm GBW}(r) = \frac{Q_s^2}{g^2 N_c}\frac{1-\exp\left\{-\frac{Q_s^2 r^2}{4}\right\}}{Q_s^2 r^2/4}$$

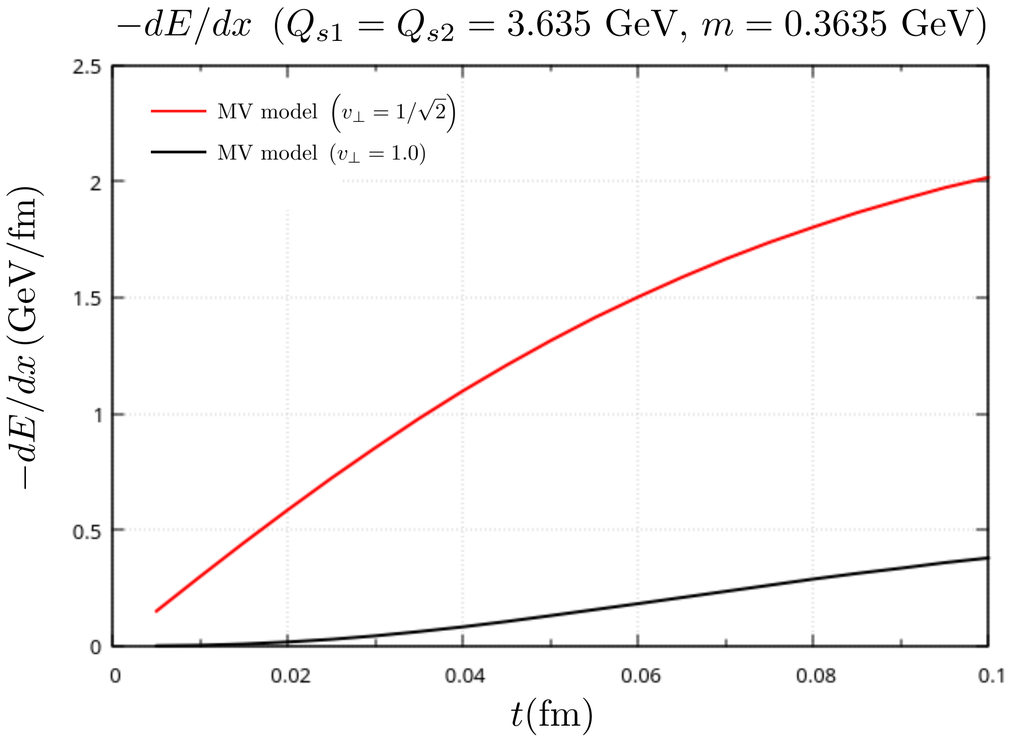
$$h_{\rm GBW}(r) = 0$$

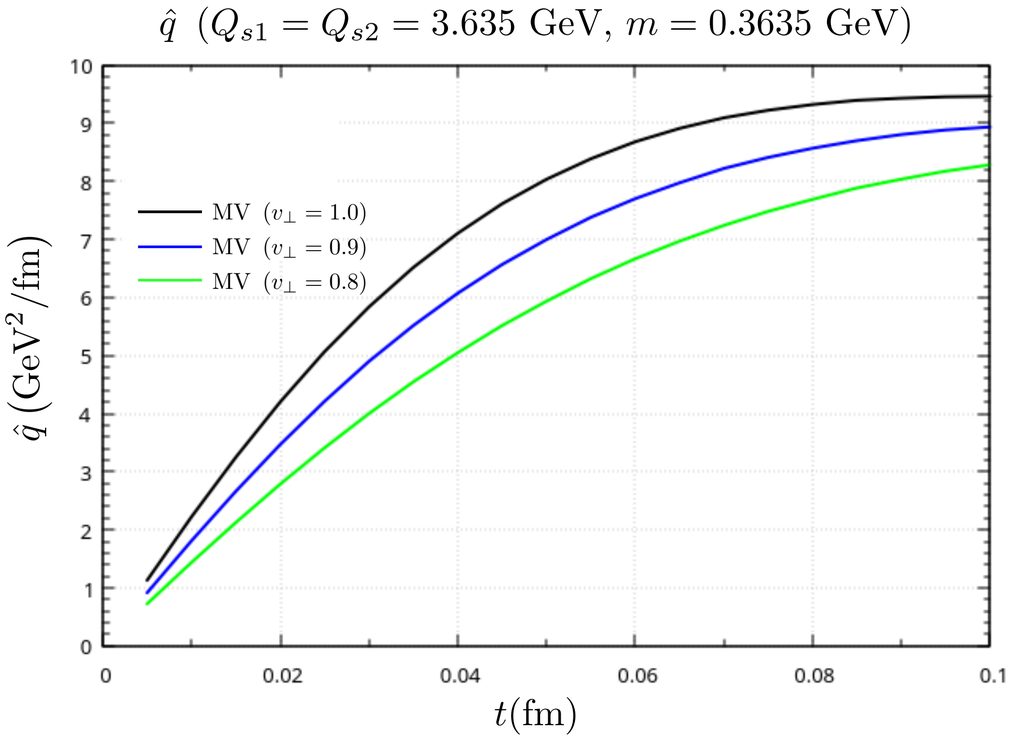
Just substitute and integrate!

τ - evolution of Glasma transport coefficients

- We focus on the **MV model results** (GBW model less reliable)
- The results are presented for different values of the transverse and longitudinal components of the velocity of the probe $\left(\text{with } v^2 = v_\parallel^2 + v_\perp^2 = 1\right)$.







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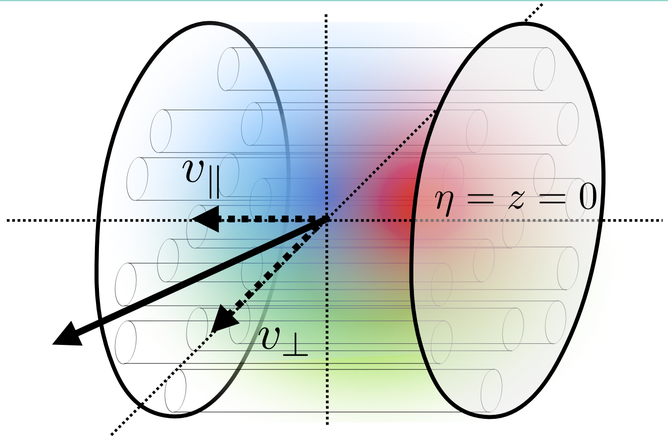
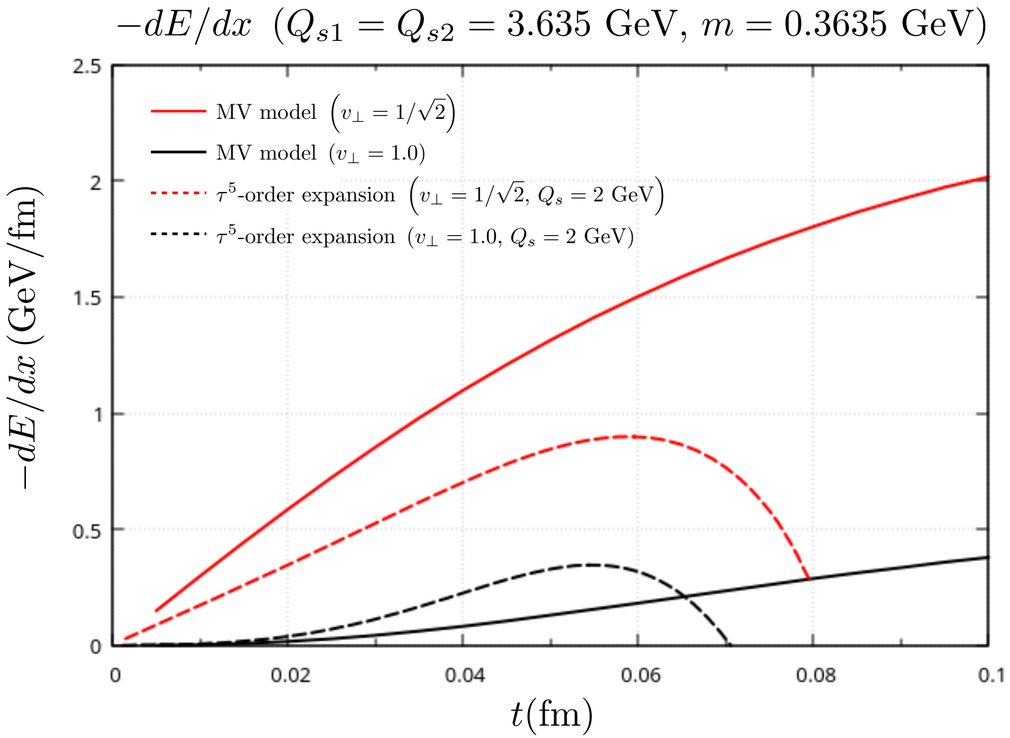
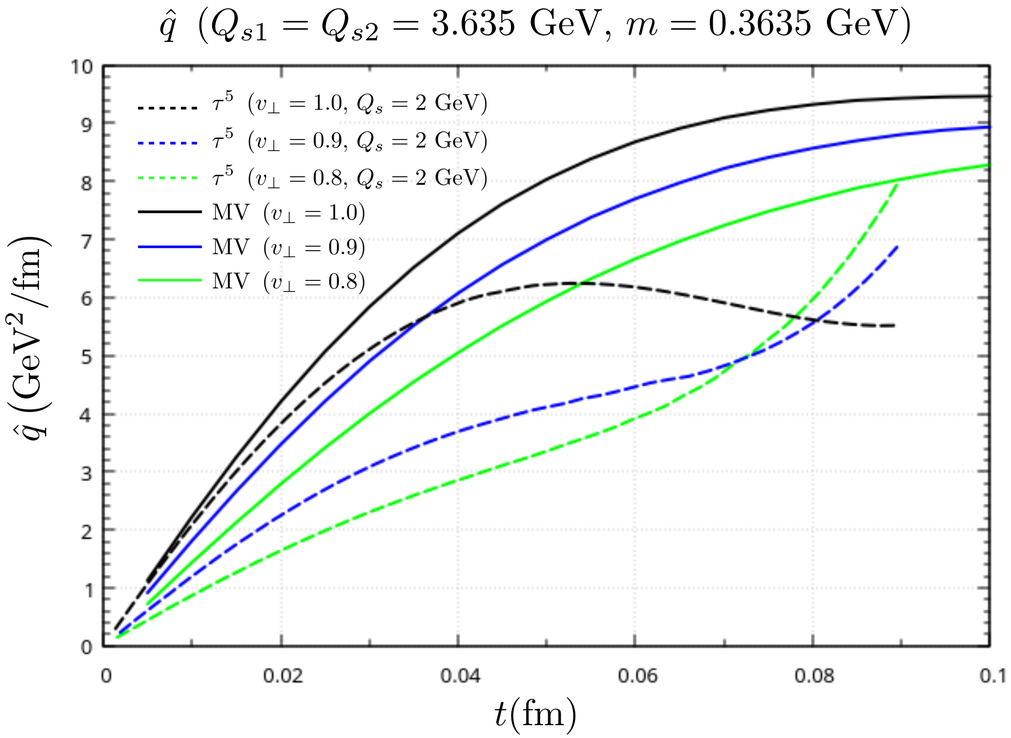
- The results are presented for different values of the transverse and longitudinal components of the velocity of the probe $\left(\text{with } v^2 = v_\parallel^2 + v_\perp^2 = 1\right)$.
- The energy loss is significantly more sensitive to the value of longitudinal component
- We compare to the results obtained in **M. E. Carrington et al; PRC, 2022** (τ - expansion up to τ^5)
- We obtain significantly larger values of $\hat{q}$



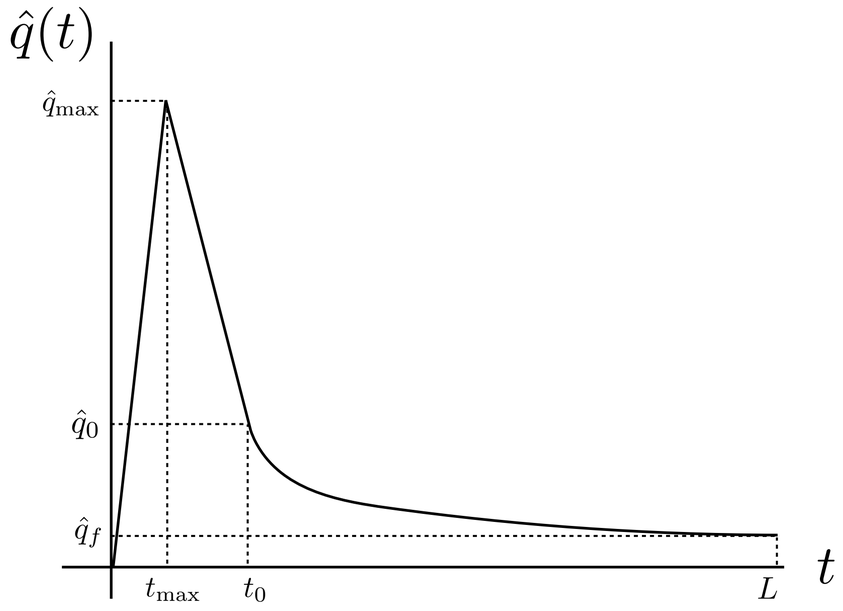




Accumulated transverse momentum

- *What is the accumulated effect of the Glasma in comparison to that of the QGP phase?*

➡ As previously done in **M. E. Carrington, A. Czajka, S. Mrowczynski; PRC, 2022** , we calculate the **accumulated transverse momentum broadening** as: $\Delta p_T^2 = \int_{t_i}^{t_f} dt\, \hat{q}(t)$



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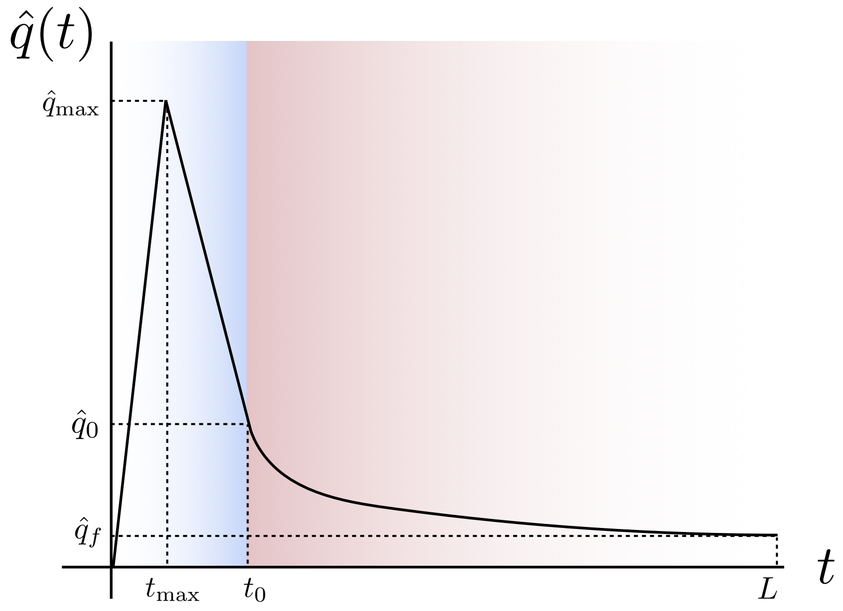
➡ The effect of the **Glasma** is estimated through linear interpolation between 0, $\hat{q}_{\max}$ and $\hat{q}_0$:

$$\Delta p_T^2|^{\text{Glasma}} = \frac{1}{2}\hat{q}_{\max} t_0 + \frac{1}{2}\hat{q}_0 (t_0 - t_{\max})$$

With the values:

$$\hat{q}_{\max} \approx 9.5\,\text{GeV}^2/\text{fm}$$
$$t_{\max} \approx 0.1\,\text{fm}$$



➡ The effect of the equilibrated **QGP** is estimated by assuming that $\hat{q} = 3T^3$ and that the temperature decreases with time as $T = T_0\,(t_0/t)^{1/3}$ (ideal one-dimensional boost invariant hydrodynamics):

$$\Delta p_T^2|^{\text{QGP}} = 3T_0^3 t_0 \ln\frac{L}{t_0}$$

➡ Parameter values: $L = 10\,\text{fm},\ \hat{q}_0 \approx 1.4\ \text{GeV}^2/\text{fm},\ t_0 \approx 0.6\ \text{fm},\ T_0 = 0.45\ \text{GeV}$

Taken from **JETSCAPE collaboration; Phys. Rev. C 104, 024905 (2021)** and **C. Shen, U. Heinz, P. Huovinen and H. Song; Phys. Rev. C 84, 044903 (2011).**

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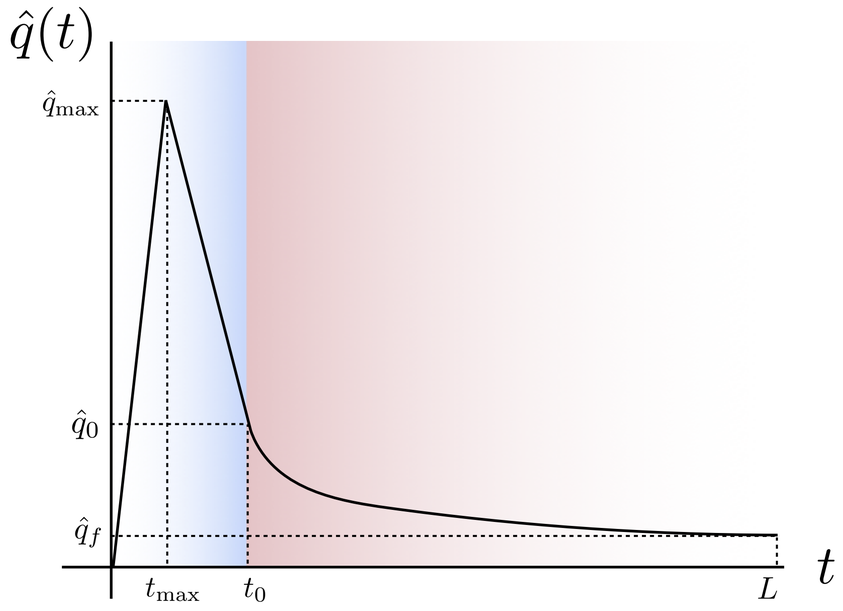

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Finally, we obtain: $\dfrac{\Delta p_T^2|^{\text{Glasma}}}{\Delta p_T^2|^{\text{QGP}}} = 1.37 > 0.93$ **(From M. E. Carrington et al; PRC, 2022)**

- We have studied the impact of the Glasma phase over Jet Quenching observables
- The transport of hard probes is described by means of a Fokker-Planck equation

  Stanislaw Mrowczynski; Eur. Phys. Journal A, 2018
- The evolution of the medium is described using linearized Yang-Mills equations

  PGR and T. Lappi; PRD, 2021
- These equations effectively resum the UV-divergences of a power series in τ

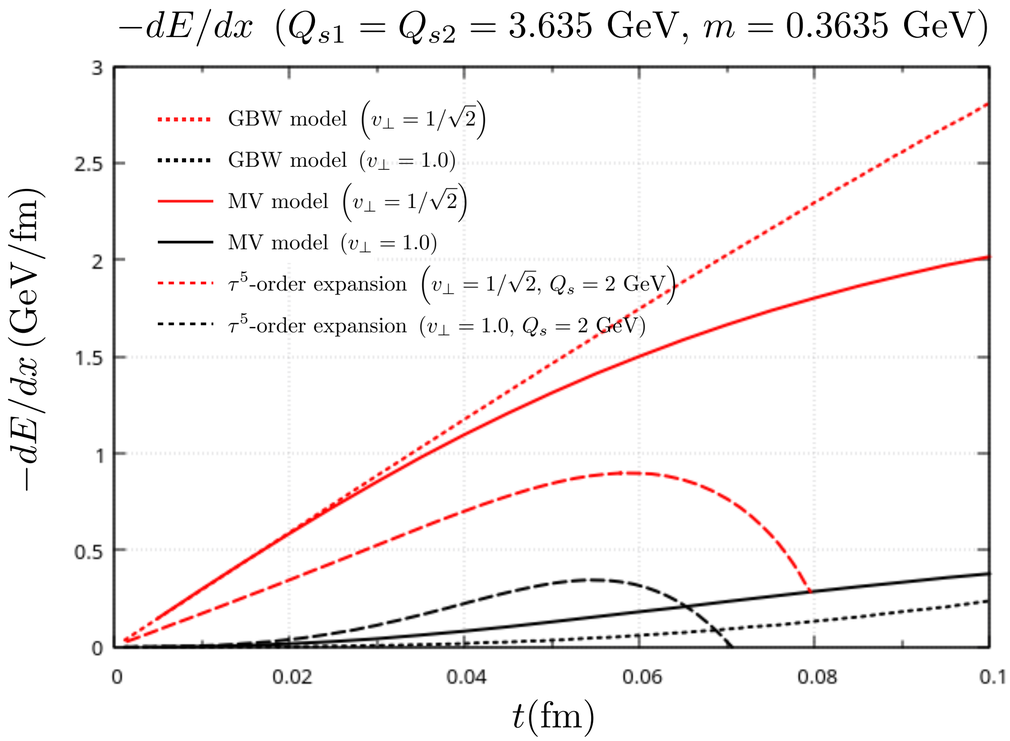
  H. Fujii, K. Fukushima, Y. Hidaka; PRC, 2008
- We observe a significantly stronger growth of $\hat{q}$ in relation to previous results

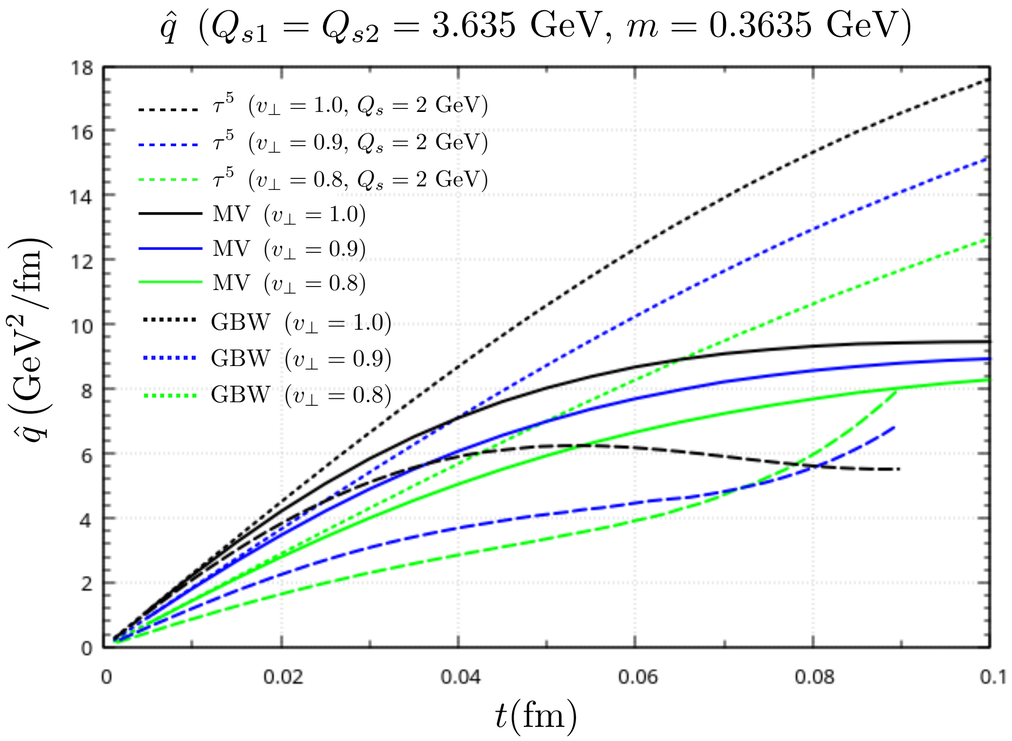
  M. E. Carrington, A. Czajka, S. Mrowczynski; PRC, 2022
- We estimate an even larger impact than that of the QGP expansion
- We interpret this as evidence that the treatment of the high-momentum modes of the Glasma fields play an essential role in the estimation of jet quenching effects

Thank you for your attention

BACK-UP: GBW results

- The GBW model was originally introduced as a purely phenomenological parameterization to fit DIS data
- The resulting unintegrated gluon distribution decays exponentially at large momenta, thus the relative contribution of the fast, linearly evolving high momentum modes is much smaller than in the MV model.
- This makes it less compatible with a description in terms of the linearized YM equations
- The results obtained with the GBW model are, for the most part, significantly larger than those obtained under the MV model





BACK-UP: The UV divergence of the Glasma 2-point correlator

PGR and T. Lappi; PRD, 2021

- Divergence at $r \to 0$ related to perturbative tail of MV model:
- How do we treat this divergence?

We apply a **running coupling** prescription:

$$g^2(r^2) = \frac{g^2(\bar{\mu}^2)}{\ln\left(\frac{4e^{-2\gamma_e-1}}{m^2r^2}+e\right)}$$

$$G_{\rm MV}(r_\perp) = \frac{g^2\bar{\mu}^2}{4\pi N_c}\left(mrK_1(mr)-2K_0(mr)\right)\frac{1-\exp\left\{\frac{g^4\bar{\mu}^2N_c}{4\pi m^2}\left(mrK_1(mr)-1\right)\right\}}{\frac{g^4\bar{\mu}^2}{4\pi m^2}\left(mrK_1(mr)-1\right)}$$

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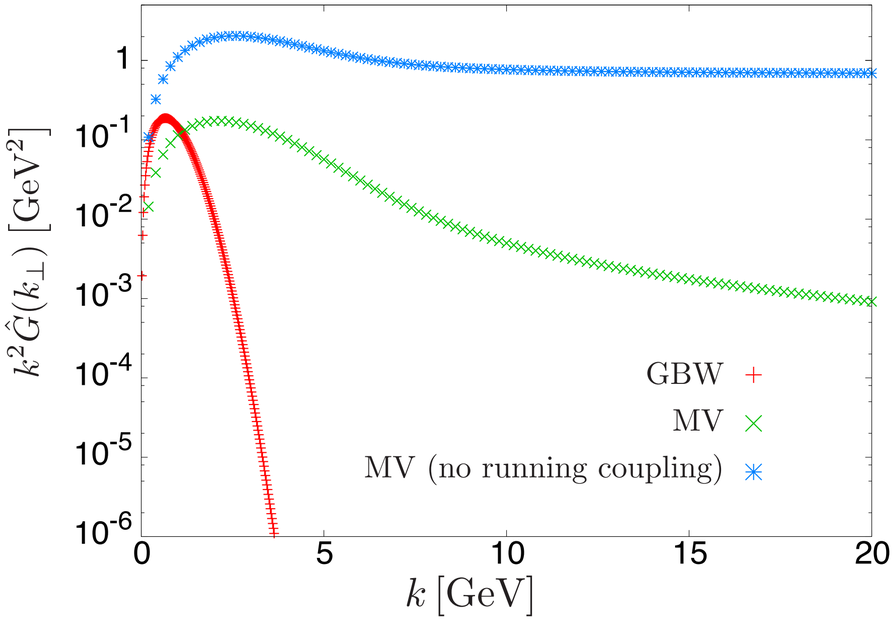


Yielding the following results:

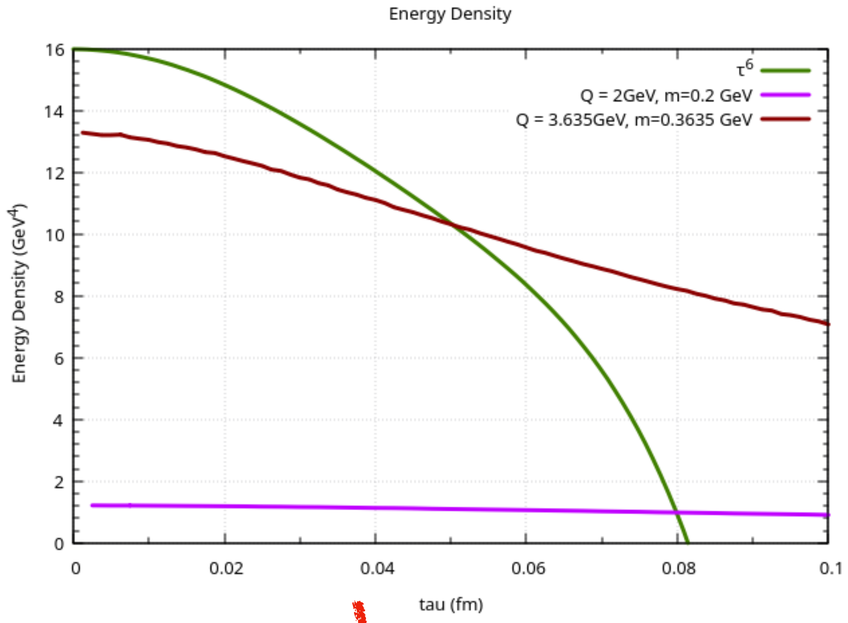
$$\lim_{r\to 0} G_{\rm MV}(r_\perp) = \frac{g^2(\bar{\mu}^2)}{4\pi}\frac{\bar{\mu}^2}{N_c} \qquad \lim_{r\to 0} h_{\rm MV}(r_\perp) = 0$$

- If we define $Q_s^2 = \alpha_s(\bar{\mu}^2)\bar{\mu}^2 N_c$:

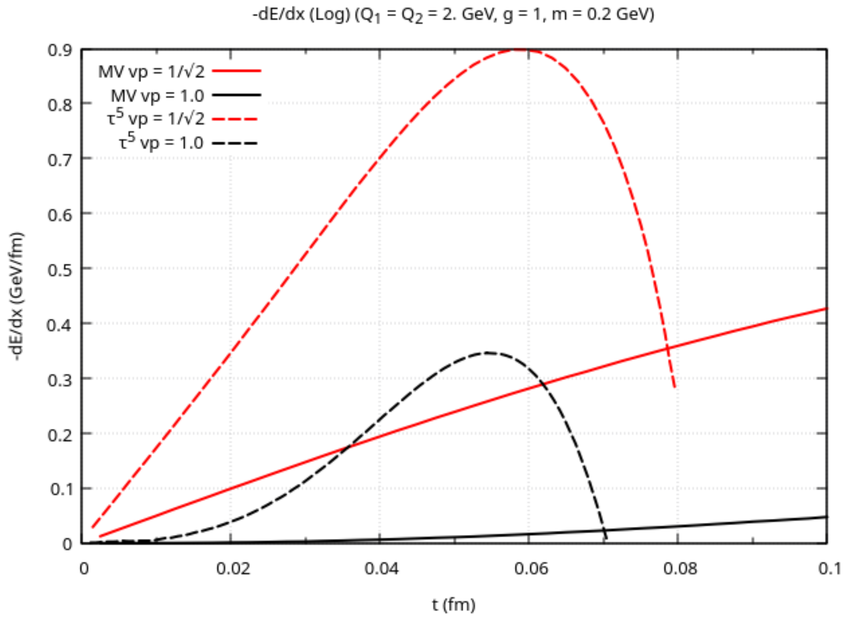
$$\lim_{r\to 0} G_{\rm MV}(r_\perp) = \lim_{r\to 0} G_{\rm GBW}(r_\perp) = \frac{Q_s^2}{g^2 N_c}$$

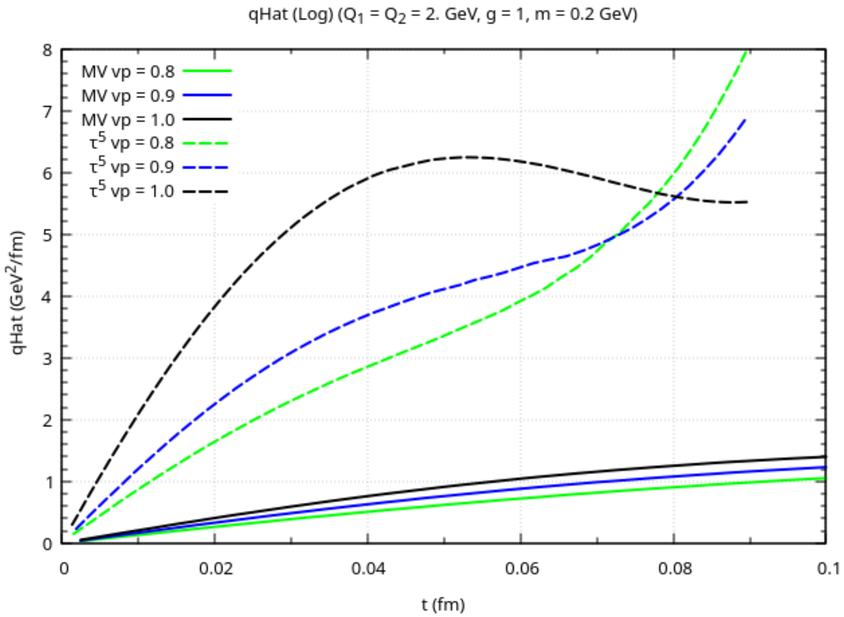
A FAIRLY STANDARD PRACTICE: we define Qs so that the 2-point functions of GBW and MV models agree at short distances

BACK-UP: Energy Density Calculations



- We intend to perform a direct comparison with the results obtained in **M. E. Carrington, A. Czajka, S. Mrowczynski; PRC, 2022** .
- Clearly, by making $Q_s = 2\ \mathrm{GeV}$ we are not describing the same collision (huge difference in energy densities). This discrepancy emerges from the application of **different UV-regularization schemes**
- We re-define Q_s in order to make our energy densities match at the **upper bound of their range of validity** $(\sim 0.06\ \mathrm{fm})$





BACK-UP: Analytical formula for $\hat{q}$

➡ In terms of the Weizsäcker-Williams distributions:

$$\hat{q} = \frac{g^4({N_c}^2-1)}{v}\int_0^t dt' \int_{k_\perp q_\perp}\int_{u_\perp v_\perp} e^{ik_\perp(x-u)_\perp} e^{iq_\perp(y-v)_\perp}\, W_1^{ik}(u_\perp,v_\perp) W_2^{jl}(u_\perp,v_\perp)$$
$$\left[\frac{{v_\perp}^2}{v^2}\left(\delta^{ij}\delta^{kl}+v^2\varepsilon^{ij}\varepsilon^{kl}\right) J_0(kt)J_0(q(t-t'))\right.$$
$$+ i\frac{v_\perp\cdot q}{q}(\delta^{ij}\delta^{kl}-\varepsilon^{ij}\varepsilon^{kl})J_0(kt)J_1(q(t-t'))$$
$$+ i\frac{v_\perp\cdot k}{k}(\delta^{ij}\delta^{kl}-\varepsilon^{ij}\varepsilon^{kl})J_1(kt)J_0(q(t-t'))$$
$$\left.-\left((v^1)^2\frac{k\cdot q}{kq}+\frac{(v_\perp\cdot k)(v_\perp\cdot q)}{kq}\right)\left(\delta^{ij}\delta^{kl}+\frac{1}{v^2}\varepsilon^{ij}\varepsilon^{kl}\right) J_1(kt)J_1(q(t-t'))\right]$$

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Acknowledgements

MINISTERIO DE CIENCIA, INNOVACIÓN Y UNIVERSIDADES

AGENCIA ESTATAL DE INVESTIGACIÓN

EXCELENCIA MARÍA DE MAEZTU 2024-2030

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