Next-to-Leading Order Dilepton Production from the Pre-Equilibrium

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Dileptons as Probe of the pre-equilibrium

Problem:

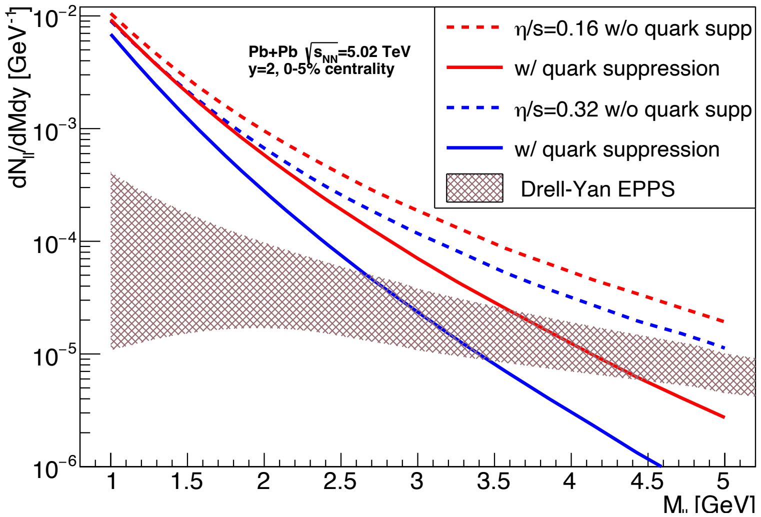
- Most evidence for thermal equilibration relies on analysis of hadrons
- Only very indirect experimental access to equilibration

Promise:

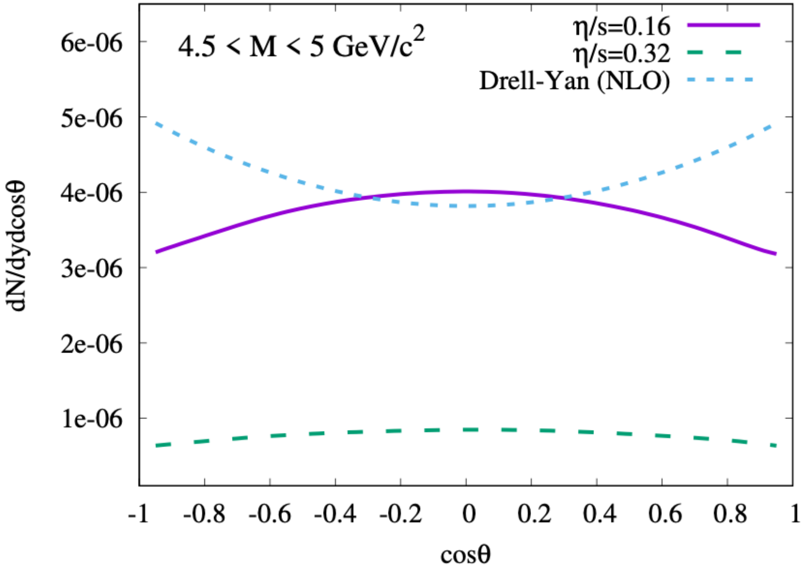
- Dileptons are colorless
- They are produced throughout the evolution
- The invariant mass of the dileptons is correlated with production time

The Observables

- Production sensitive to equilibration time (blue vs red) and chemical imbalance between quarks and gluons (dotted vs full)
- Signal is larger than the Drell-Yan background in the range [1 GeV, 5 GeV]
- The pre-equilibrium and the Drell-Yan production have different angular distributions
- Goal: Next-to-Leading-Order, because gluons could have significant impact



M. Coquet, X. Du, J. Ollitrault, S. Schlichting ,M. Winn,, Physics Letters B 821, 0370-2693, 2021



M. Coquet, M. Winn, X. Du, J. Ollitrault, S. Schlichting, Phys. Rev. Lett. **132**, 232301, 2024

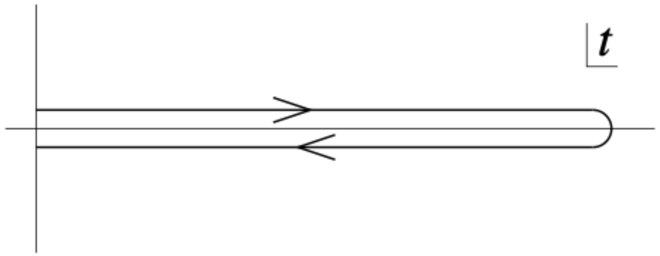
Real-Time Formalism

- For out-of-equilibrium calculations we need a time-dependent density matrix

$$Z[J_1, J_2] = \int d\phi_f \langle \phi_f | \hat{U}_{J_1}(t_f, t_i) \hat{\rho} \hat{U}_{J_2}(t_i, t_f) | \phi_f \rangle$$

$$= \int d\phi_f d\phi_1^i d\phi_2^i \int_{\phi_1^i}^{\phi_f} D\phi_1 \int_{\phi_2^i}^{\phi_f} D\phi_2 \rho[\phi_1^i, \phi_2^i]$$

$$exp\left(i \int_{t_i}^{t_f} dt \int d^3x (L(\phi_1) + J_1 \phi_1) - i \int_{t_f}^{t_i} dt \int d^3x (L(\phi_2) + J_2 \phi_2) \right)$$



$\longrightarrow$ Doubling of the degrees of freedom

J.Berges, arXiv:hep-ph/0409233

Choice of Basis

- We chose the 1-2 basis:

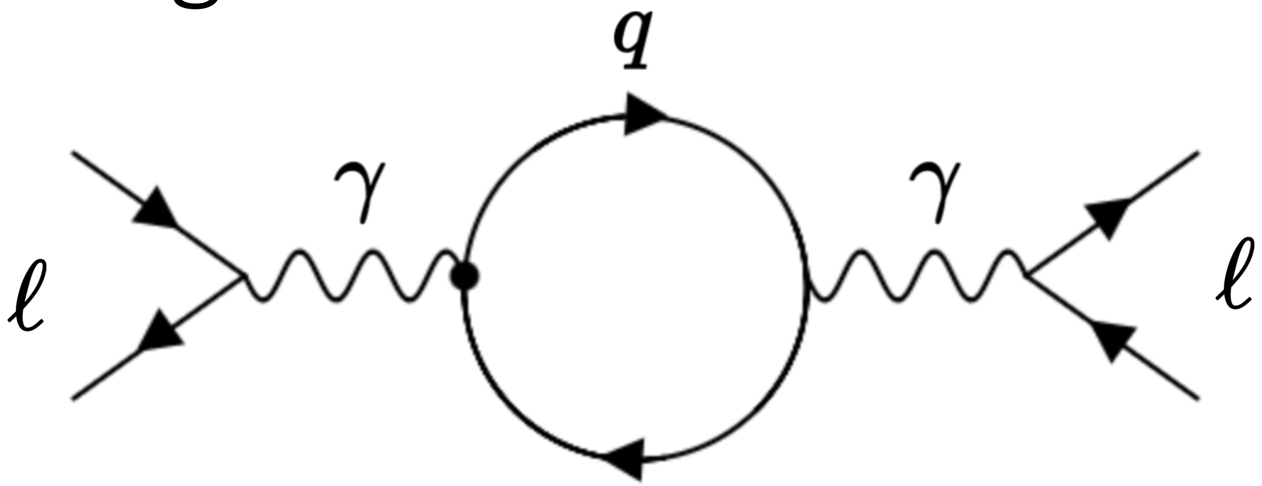
$$S_{11}(K) = \frac{i\not{K}}{K^2 + i\epsilon} - 2\pi \not{K} \delta(K^2) n_q(k^0)$$

$$S_{12}(K) = -2\pi \not{K} \delta(K^2) \left[-\theta(-k^0) + n_q(k^0) \right]$$

S: fermionic propagator
K: 4-momentum
n_q : quark distribution function

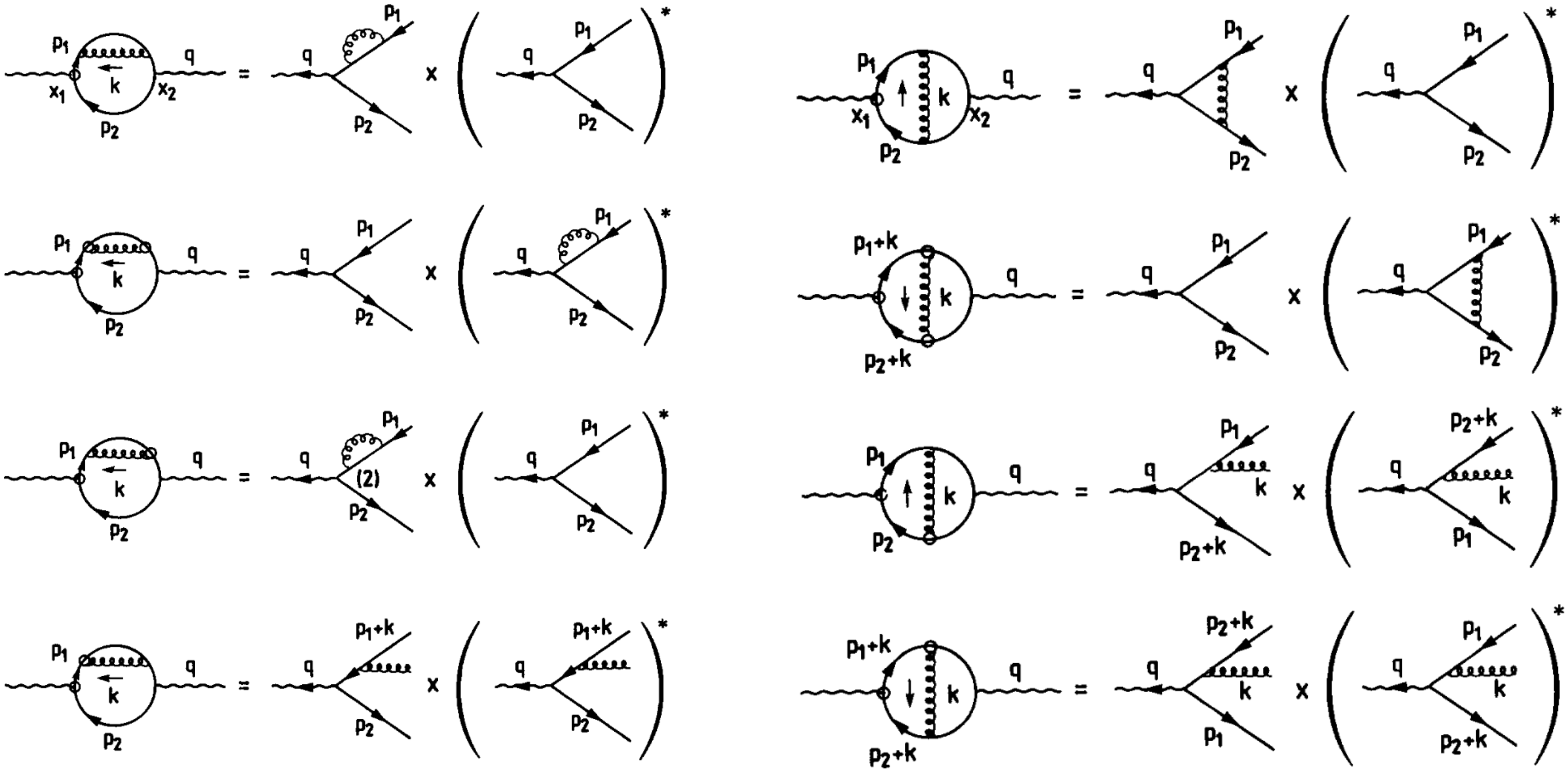
- This allows for natural separation into vacuum- and medium-terms
- Self-energy correction, vertex correction and real processes are naturally isolated

LO-Diagram



- Square of the amplitude
- Cutting rules separate “1”- and “2”- vertices
- Circle signifies a “1”-vertex
- Multiplicative dilepton factor can be treated in isolation

The NLO-Diagrams



[T. Altherr, P. Aurenche, T. Becherrawy, Nuclear Physics B 315, 0550-3213, 1989](#)

Approach

Simplifications:

- For now, no angular dependence in the particle distributions

- Virtual photon is at rest in local rest frame of the fluid

Regularization:

- Gluon mass λ

Example: Self Energy Correction

$$\sim \int dx x n_g \left(\frac{Q}{2}x\right)\left[4 - \frac{1}{2}\frac{1}{x} log\left|\frac{1-x}{1+x}\right| + log\left|\frac{\lambda^2}{Q^2}\right| + \frac{1}{2} log\left|\frac{1-x^2}{x^4}\right|\right]$$
$$+ \int dx x n_q \left(\frac{Q}{2}x\right)\left[4 + log\left|\frac{\lambda^2}{Q^2}\right| - log\left|x\right|\right]$$

Q: invariant mass of photon
λ: gluon mass
$n_{q,g}$: quark, gluon distribution function

- The medium contribution has a bosonic and a fermionic part
- Both contain divergences
- Those need to be cancelled with other diagrams

Cancellation in equilibrium

- In equilibrium, relations for the distribution functions facilitate the cancellation (Gabellini, Grandou and Poizat, Annals of Physics 202)
- These don't hold out of equilibrium, so singularity remains:

$$\Pi_S^C \sim$$

$$\int_1^\infty dZ \left[n_q\left(\frac{Q}{2}(Z-1)\right)\left(1+n_g\left(\frac{Q}{2}Z\right)\right) - \left(n_g\left(\frac{Q}{2}Z\right)+n_q\left(\frac{Q}{2}(Z-1)\right)\right)\left(1-n_q\left(\frac{Q}{2}\right)\right)\right] \frac{1+(Z-1)^2}{Z} ln\left(\frac{Q^2}{\lambda^2}Z(Z-1)\right)$$

$$+\int_1^\infty dZ \left(n_g\left(\frac{Q}{2}Z\right)+n_q\left(\frac{Q}{2}(Z-1)\right)\right)\left(1-n_q\left(\frac{Q}{2}\right)\right) \frac{1+(Z-1)^2}{Z} ln\left(\frac{Q^2}{\lambda^2}Z(Z-1)\right)$$

Q: invariant mass of photon
λ: gluon mass
$n_{q,g}$: quark, gluon distribution function

Finiteness out-of-equilibrium

- Introduce quark thermal mass in the difference term to make it finite:

$$\int_1^\infty dZ\left[n_q\left(\frac{Q}{2}(Z-1)\right)\left(1+n_g\left(\frac{Q}{2}Z\right)\right)-\left(n_g\left(\frac{Q}{2}Z\right)+n_q\left(\frac{Q}{2}(Z-1)\right)\right)\left(1-n_q\left(\frac{Q}{2}\right)\right)\right]\frac{1+(Z-1)^2}{Z}ln\left(Z\frac{Q^2(1-Z)-\lambda^2}{m_{th}^2(1-Z)-\lambda^2}\right)$$

Q: invariant mass of photon
λ: gluon mass
m_{th}: thermal quark mass
$n_{q,g}$: quark, gluon distribution function

Summary

- Calculated all diagrams for NLO dilepton production from the pre-equilibrium
- Simplifying assumptions: no angular dependence of distributions, dilepton at rest in local rest frame of the fluid
- There are still some details that are not entirely understood with some terms involving the thermal quark mass

Outlook

- Numerical result for the NLO dilepton rate and investigation of the effects of chemical equilibration
- This work can be used as a basis to calculate the rate without the simplifying assumptions
- Such a result would allow for analysis of effects of momentum anisotropy