The Ⅷ-th International Conference on the Initial Stages of High-Energy Nuclear Collisions

INITIAL STAGES 2025

Observation of Universal Expansion Anisotropy from Cold Atoms to Hot Quark-Gluon Plasma

Fuqiang Wang

Purdue University, Huzhou University

The VIIITH
International
Conference on the

INITIAL
STAGES

of High-Energy
Nuclear Collisions
Taipei, Taiwan

Outline

- Physics motivations
- Cold atom experiment
- Results, comparison to heavy ion, discussion
- Possible future works
- Summary

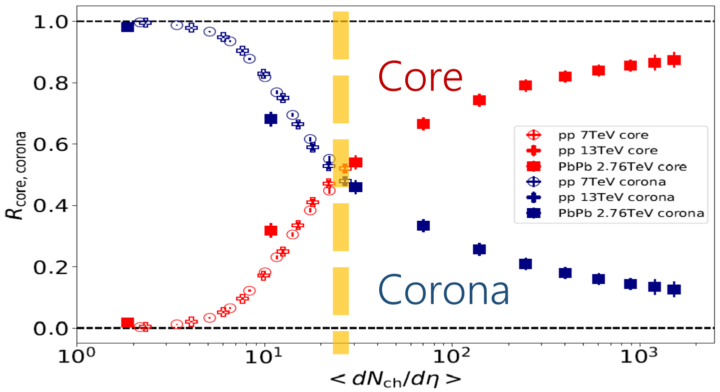
Motivations

- Strong anisotropic flow in heavy-ion collisions. Little energy dependence.
- Apparent anisotropic flow in small systems, independent of multiplicity
- Hydrodynamics, QGP droplet
- Color Glass Condensate
- Escape (x-ray, single-hit) mechanism

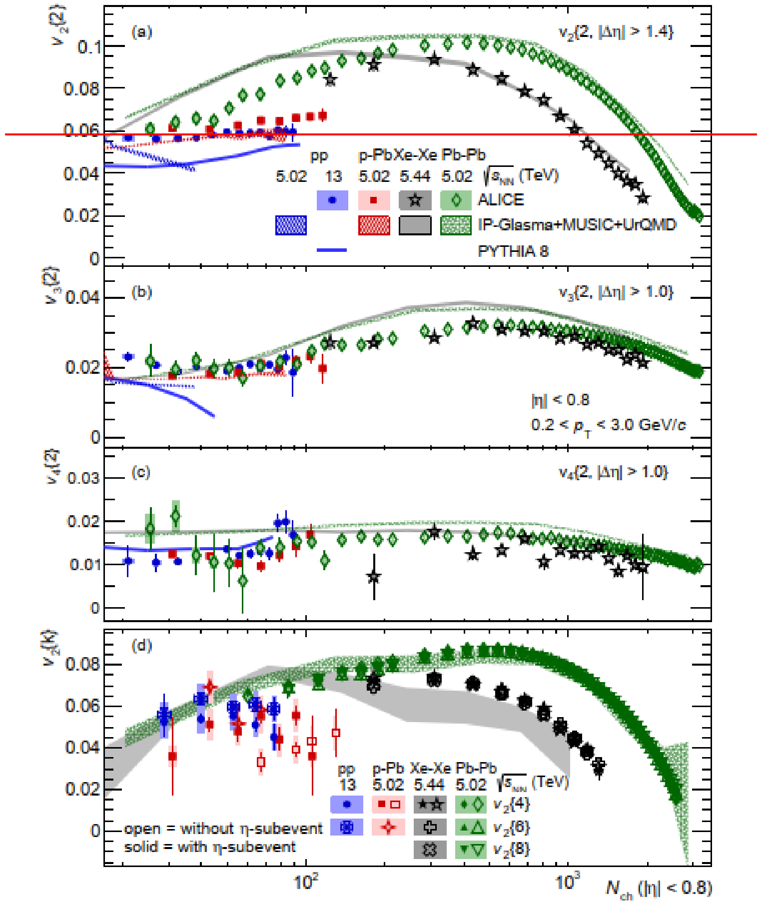
  He et al. PLB 753(2016) 506; Romatschke, PRL 120 (2018) 012301; Kurkela et al. EPJC 79 (2019) 759
- Core-corona interplay (must present)
- Physics mechanisms not fully settled
- Can we address the question experimentally? Cold atom emulator
- Ultracold atom gases can be tuned from non-interacting to strongly interacting

  O'Hara et al. Science 298 (2002) 2179; Schafer and Teaney, RPP 7 (2009) 126001; Chin et al. RMP 82 (2010), 1225

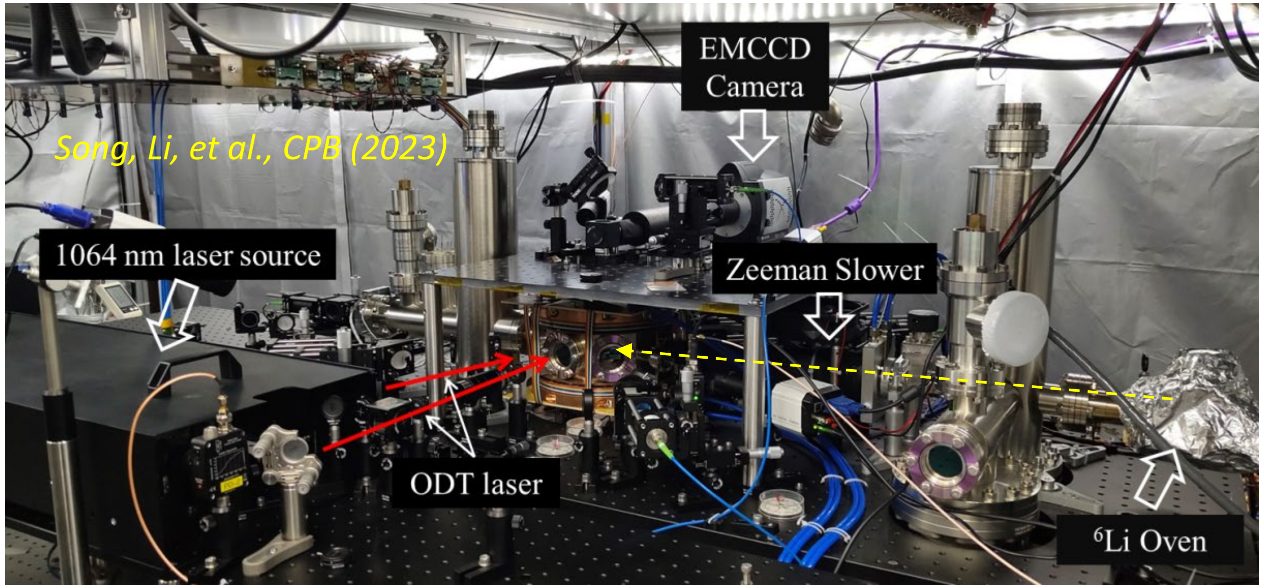
Kanakubo et al., PRC 105 (2022) 024905



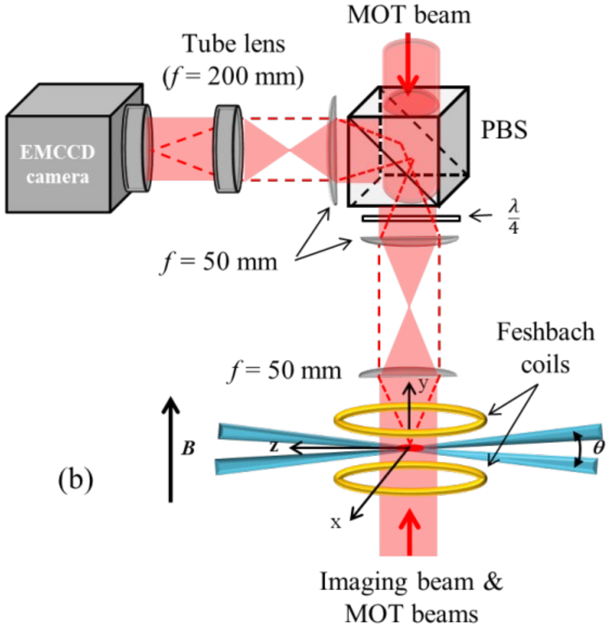
ALICE, PRL 123 (2019) 142301



Cold Atom Experiment



Song, Li, et al., CPB (2023)



Aspect ratio ~1:10

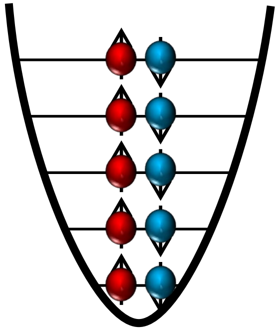
$N = 0.5\times10^6$

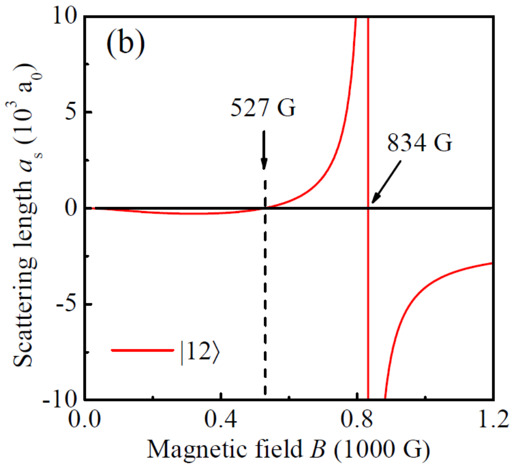
$T = 4.6\ \mu$K $T_F = 6.4\ \mu$K

Cooling and Trapping

- Doppler Cooling
- Zeeman Slowing
- Magneto-Optic Trap (MOT) 300μK
- Optical Dipole Trapping (ODT) 100μK
- Evaporative Cooling 1μK

Degenerate Fermi Gas
Interaction tuned by
Feshbach Resonance





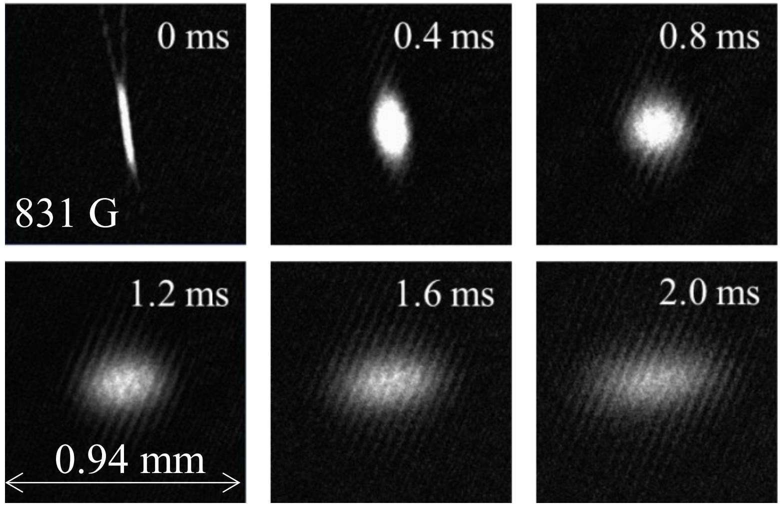
Unitary limit: $\sigma_s = \frac{4\pi a_s^2}{1+k^2a_s^2} \approx \frac{4\pi}{k^2} \sim 0.1\ \mu\text{m}^2$

Butts and Rokhsar, PRA55(1997)4346; Chin et al. RMP82(2010)1225(2010)

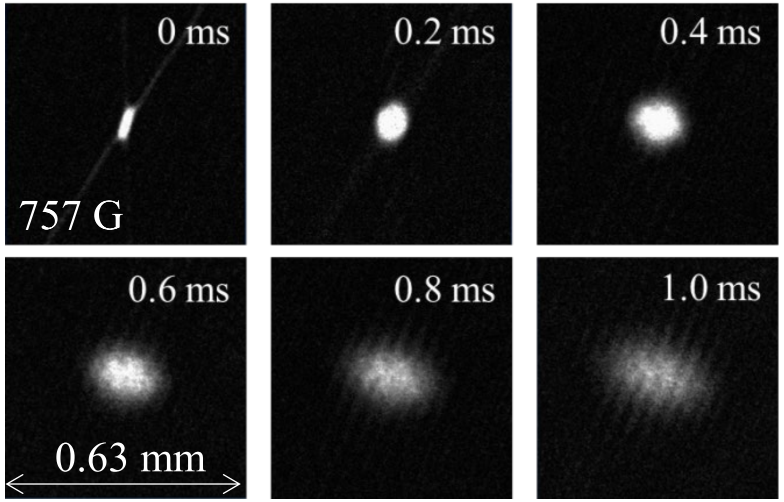
- Cold atom experiment setup started 2017
- First observation of anisotropic expansion by the end of 2021
- First manuscript early 2023

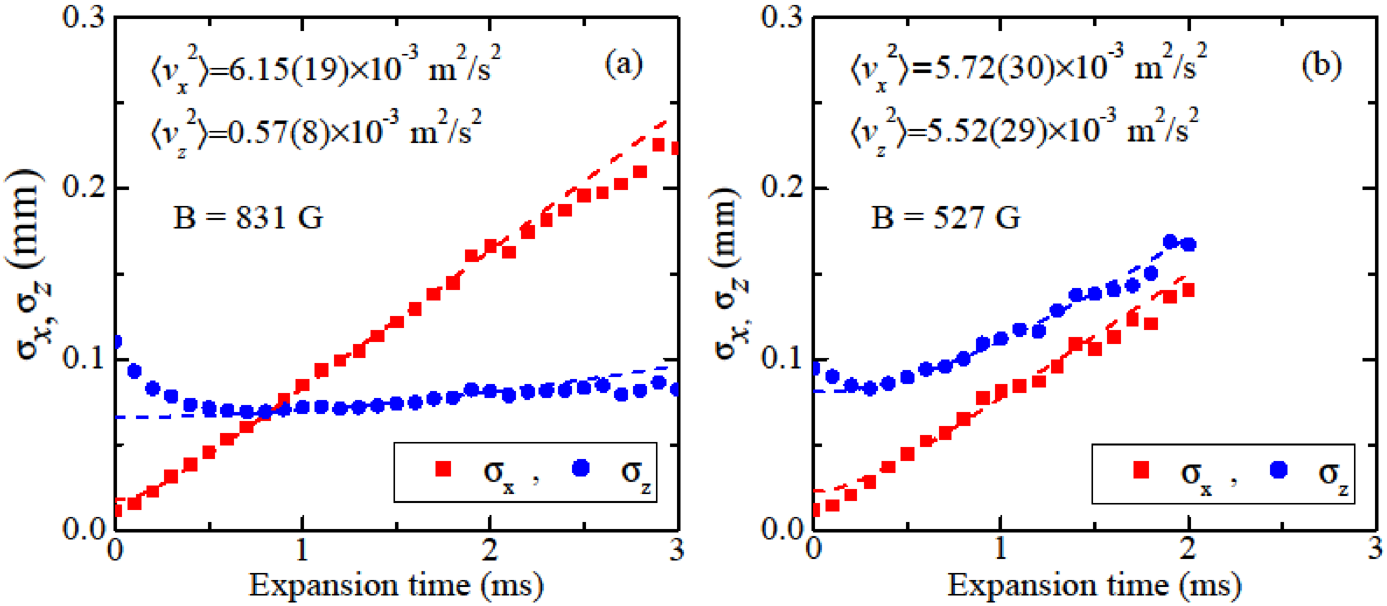
Anisotropic Expansion of Fermi Gases

Aspect ratio 1/11.4



1/3.17





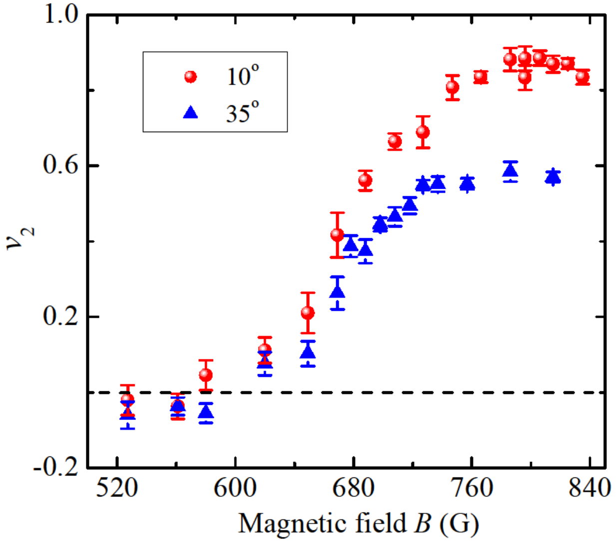
B = 831G, $a_s = 1.86$ μm
Anisotropic expansion

B = 527 G, $a_s \sim 0$ μm
Ballistic expansion

$$\sigma_i(t)^2 = \sigma_i(0)^2 + \langle {v_i}^2\rangle t^2$$

$$v_2 = \frac{\langle {v_x}^2\rangle - \langle {v_z}^2\rangle}{\langle {v_x}^2\rangle + \langle {v_z}^2\rangle}$$

Elucidating Anisotropy v_2 Results



Normalize by initial eccentricity v_2/ε_2

$$\varepsilon_2 = \frac{1-\beta^2}{1+\beta^2} \quad (\beta : \text{aspect ratio})$$

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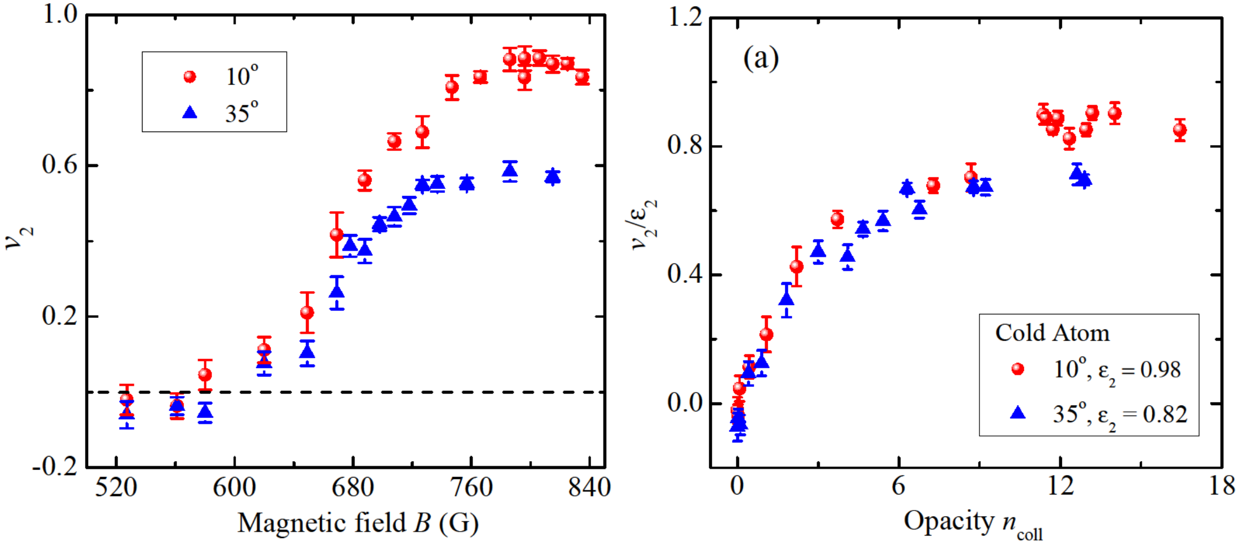

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$$\textbf{Opacity:} \quad n_{\text{coll}} = \rho\sigma L$$

Calculate n for test particle traversing from origin:

$$n = \frac{\sigma_s}{4\pi}\iiint \rho_0 \exp\left(-\frac{r^2\cos^2\theta}{2\sigma_z^2} - \frac{r^2\sin^2\theta}{2\sigma_x^2}\right)\sin\theta\, d\theta\, d\phi\, dr = \frac{N\sigma_s}{4\pi\sigma_z\sigma_x}\frac{\arctan\left(\sqrt{1-\beta^2}/\beta\right)}{\sqrt{1-\beta^2}}$$

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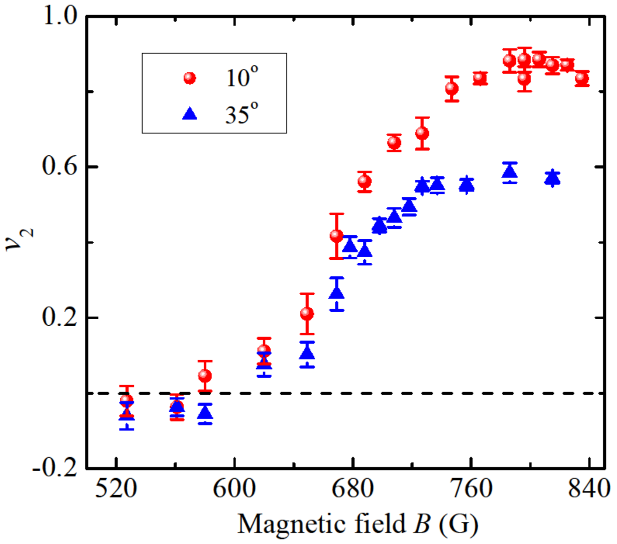

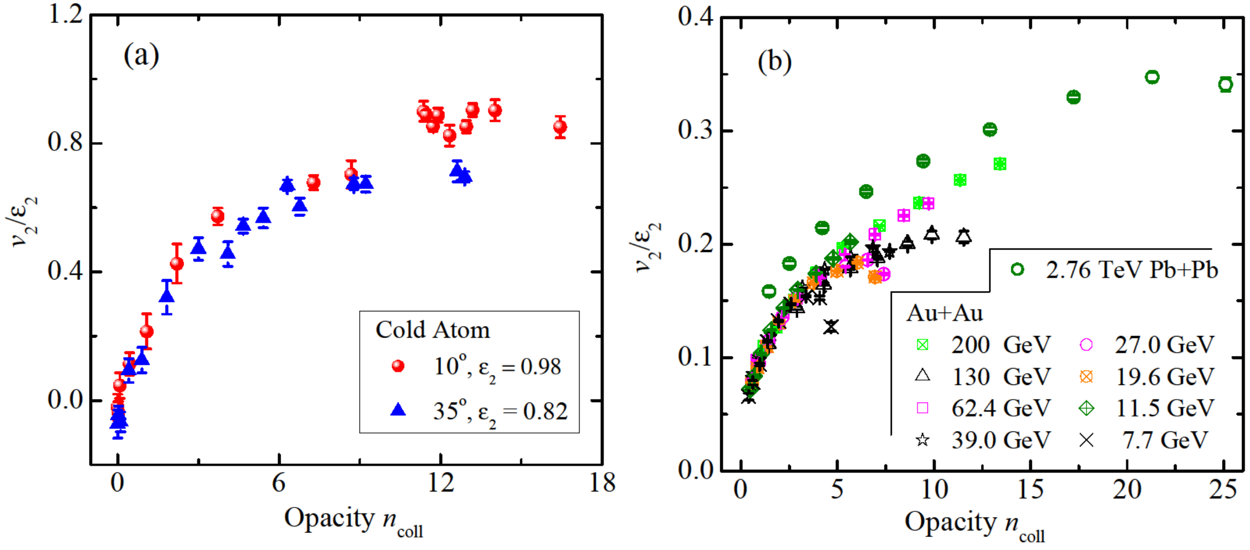

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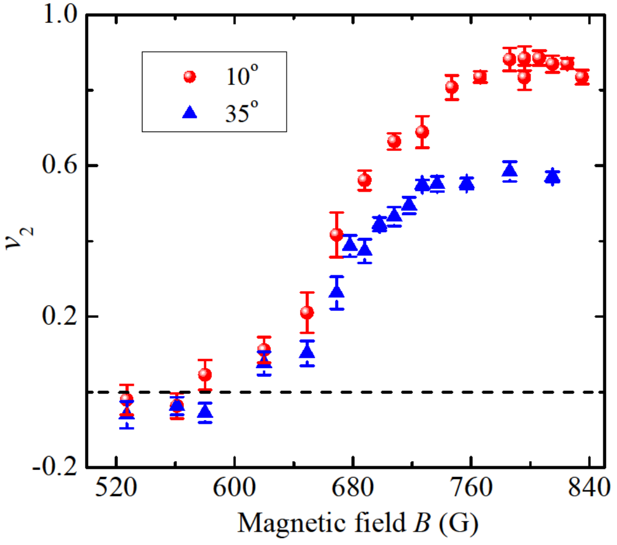
HI opacity estimate:

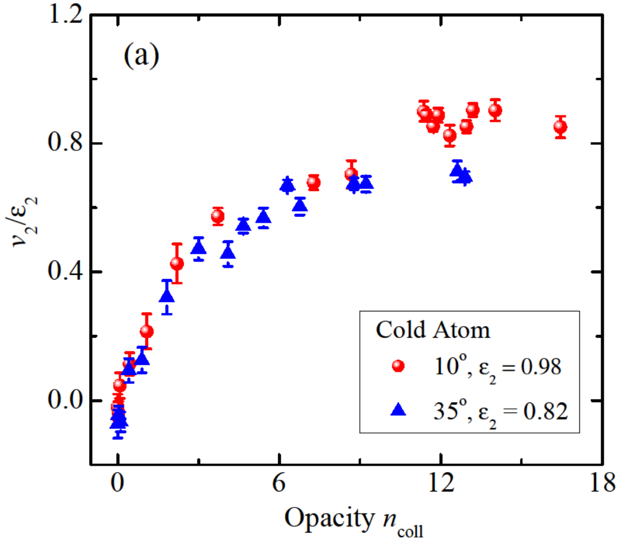
$$n_{\text{coll}} = \frac{\sigma}{c\tau}\frac{dN_{\text{ch}}/dy}{\sqrt{\pi S_\perp}}$$

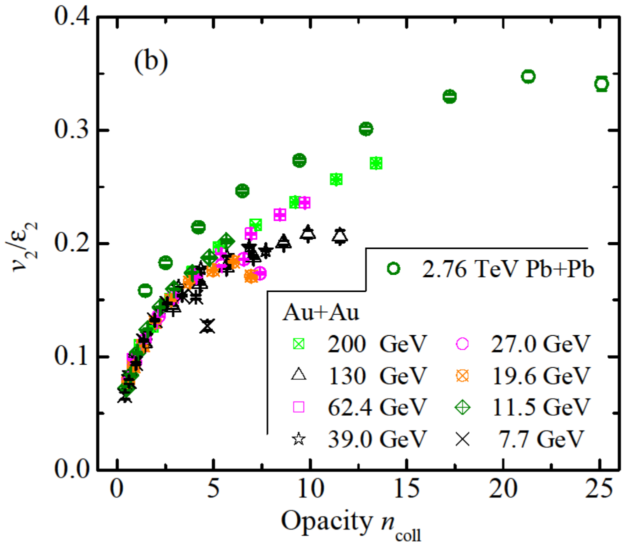
$$v_2 = \frac{\langle v_x^2\rangle - \langle v_z^2\rangle}{\langle v_x^2\rangle + \langle v_z^2\rangle} \text{ vs. } v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle$$

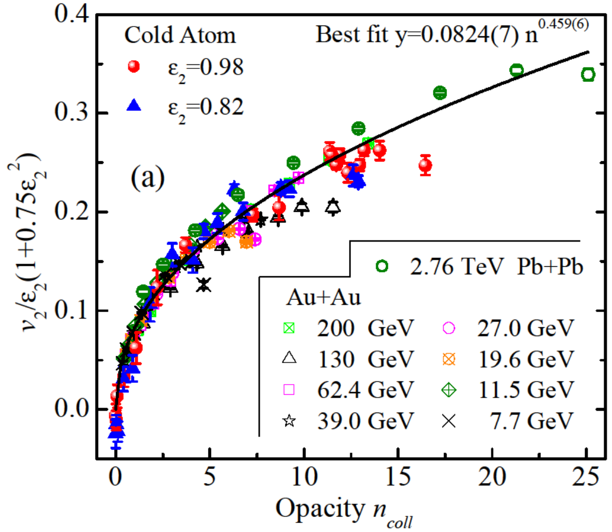
$$\frac{\int p_T^2 v_2 dN}{\int p_T^2 dN} \bigg/ \frac{\int v_2 dN}{\int dN} = 2$$

Elucidating Anisotropy v_2 Results









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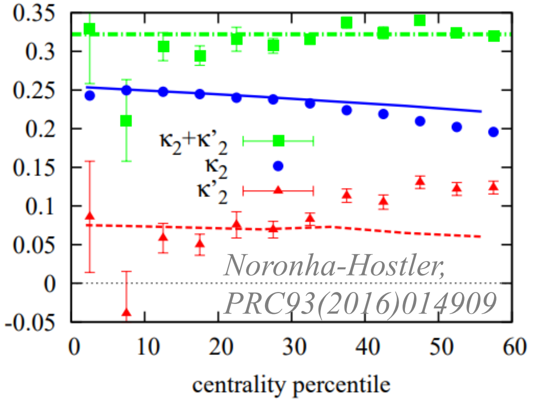
$$\frac{\int p_T^2 v_2 dN}{\int p_T^2 dN}\Bigg/\frac{\int v_2 dN}{\int dN}=2$$

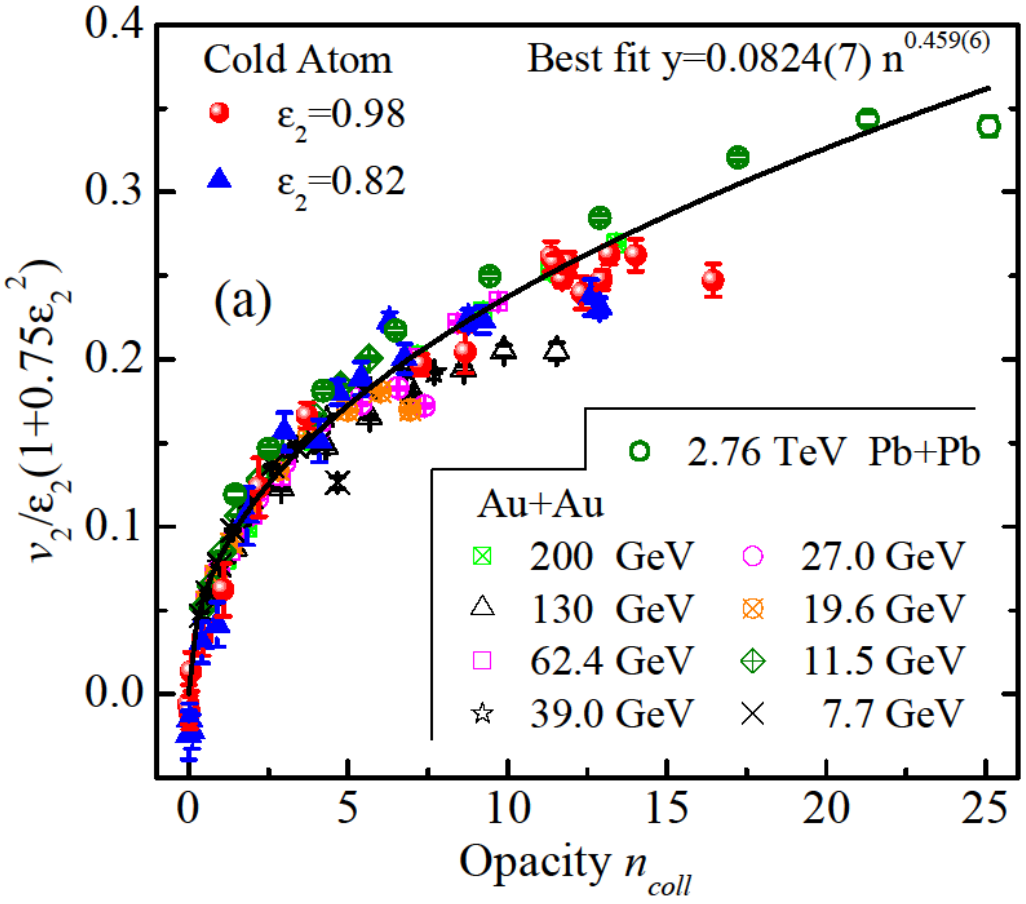
Nonlinearity ~ $1+0.75\varepsilon_2^2$ important at large ε_2



Noronha-Hostler, PRC93(2016)014909

Result and Discussion

$$\frac{v_2}{\varepsilon_2(1+0.75\varepsilon_2^2)} = (0.082 \mp 0.006)\times n_{\text{coll}}^{0.46\pm0.03}$$



- Apparent universal trend in $\sqrt{n}$.
  The $\sqrt{n}$ trend is robust; absolute amplitude less so.
- (x-axis) Opacity:
  - Cold atom opacity should be easy to control…
  - Heavy ion opacity estimate: order of magnitude uncertainty
  - Opacity estimate for static systems
  -
- (y-axis) anisotropy response:
  - Nonlinearity is demanded by heavy-ion data. Coefficient is varied 0.5-1, no qualitative change.
  - Eccentricity is model-dependent of initial geometry. Universality may help discriminate initial geometry models.
  - Isotropic (s-wave) vs. forward (parton-parton) scattering: only small difference according to AMPT

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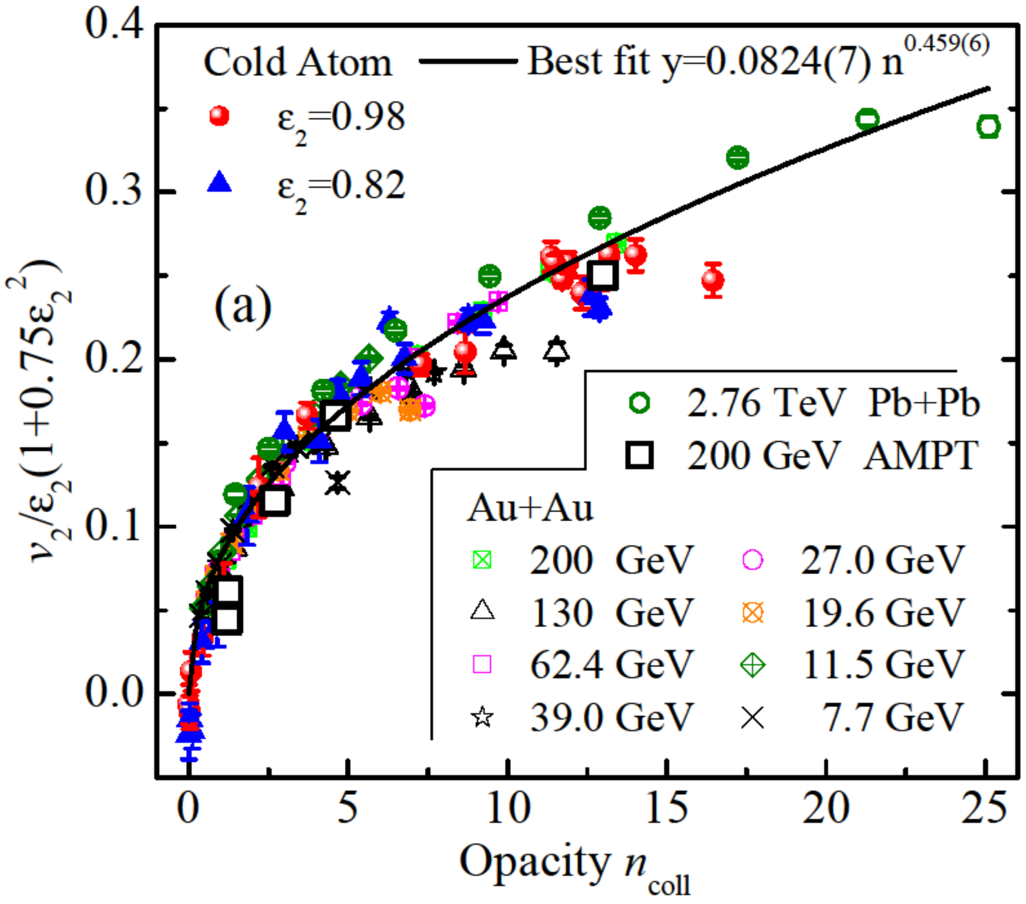

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  - Heavy ion opacity estimate: order of magnitude uncertainty
  - Opacity estimate for static systems
  - AMPT dynamic (exact) calculation on top of the trend
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  - Eccentricity is model-dependent of initial geometry. Universality may help discriminate initial geometry models.
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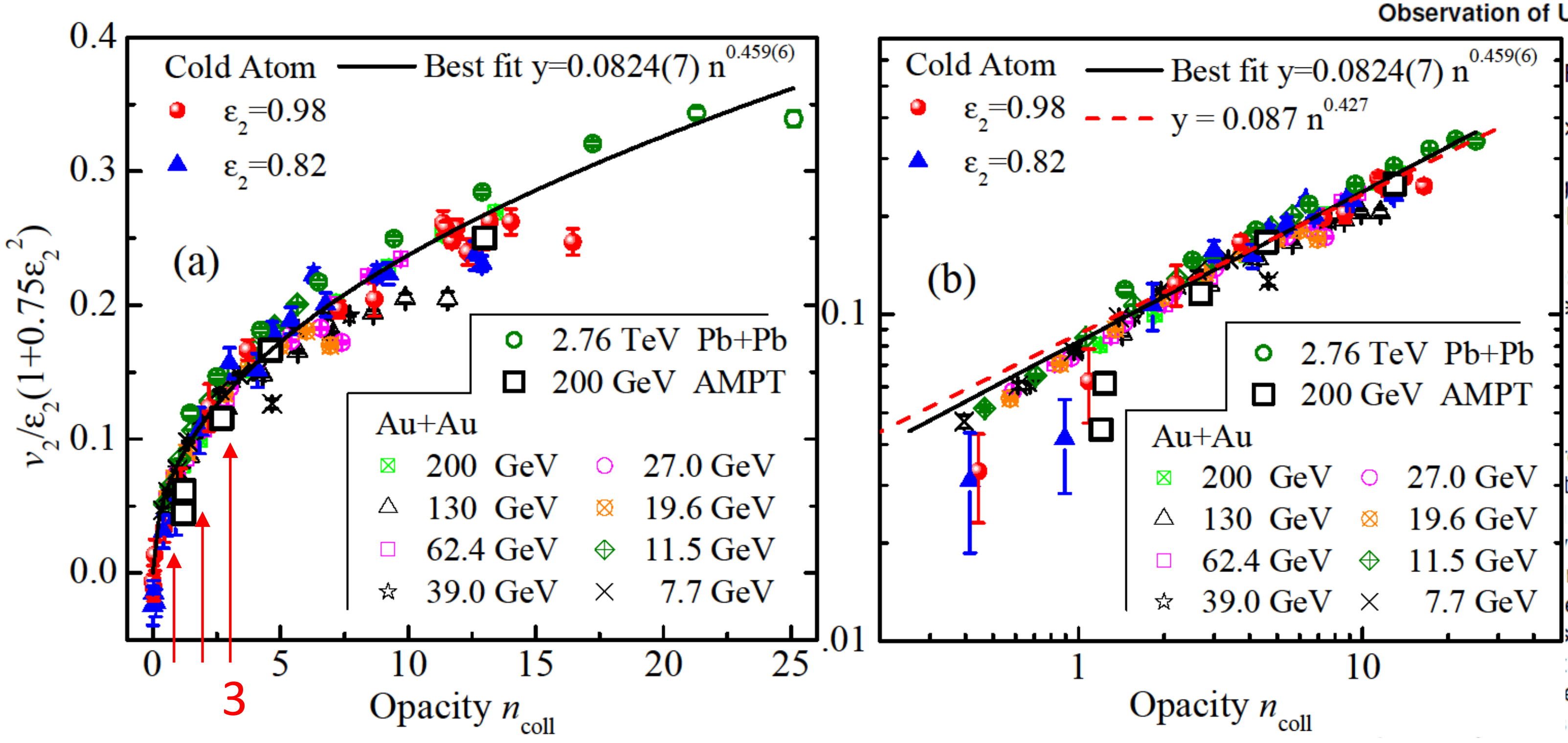
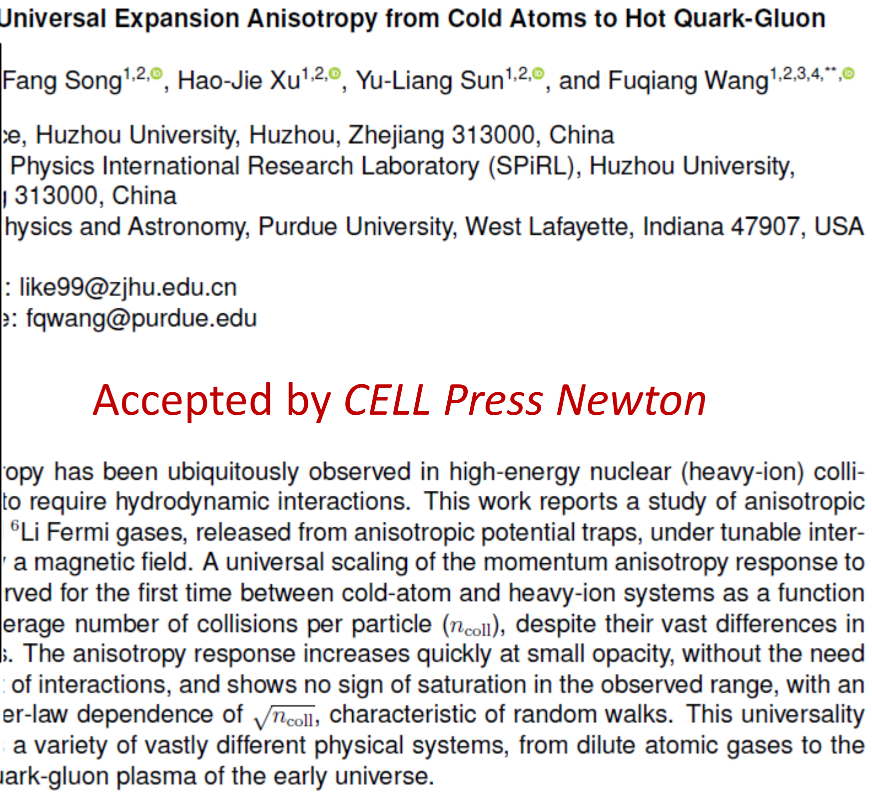
Universal Scaling

Ke Li et al., arXiv:2405.02847 [cond-mat.quant-gas]





Observation of Universal Expansion Anisotropy from Cold Atoms to Hot Quark-Gluon

Fang Song[1,2], Hao-Jie Xu[1,2], Yu-Liang Sun[1,2], and Fuqiang Wang[1,2,3,4,**]

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Accepted by *CELL Press Newton*

opy has been ubiquitously observed in high-energy nuclear (heavy-ion) colli-
to require hydrodynamic interactions. This work reports a study of anisotropic
^{6}Li Fermi gases, released from anisotropic potential traps, under tunable inter-
a magnetic field. A universal scaling of the momentum anisotropy response to
rved for the first time between cold-atom and heavy-ion systems as a function
erage number of collisions per particle ($n_{\rm coll}$), despite their vast differences in
. The anisotropy response increases quickly at small opacity, without the need
of interactions, and shows no sign of saturation in the observed range, with an
er-law dependence of $\sqrt{n_{\rm coll}}$, characteristic of random walks. This universality
a variety of vastly different physical systems, from dilute atomic gases to the
hot and dense quark-gluon plasma of the early universe.

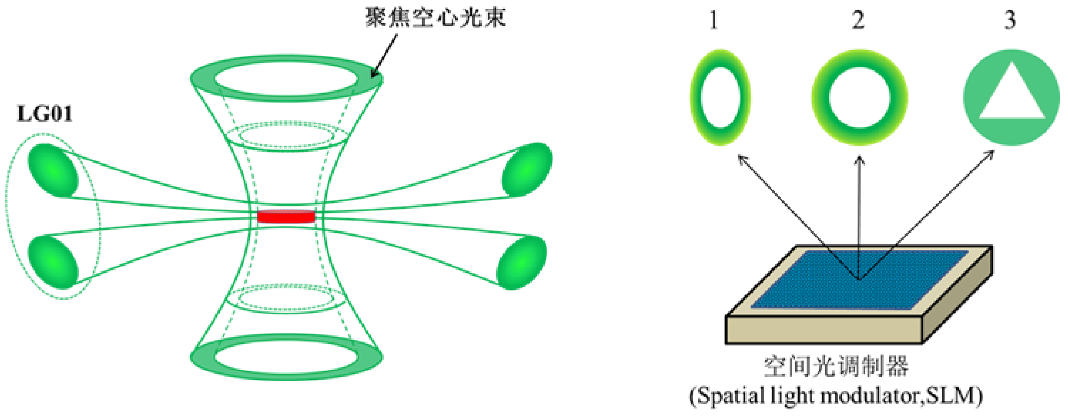
- Apparent universal scaling of expansion anisotropy as a function of opacity between cold-atom and heavy-ion systems.
- The universal behavior is $\sqrt{n}$, characteristic of random walks.
- Anisotropy builds up quickly; large-*n* hydrodynamic limit not reached.
- Noticeable deviations from $\sqrt{n}$ at small *n*; hydro attractor behavior?

Universal trend in vastly different systems:

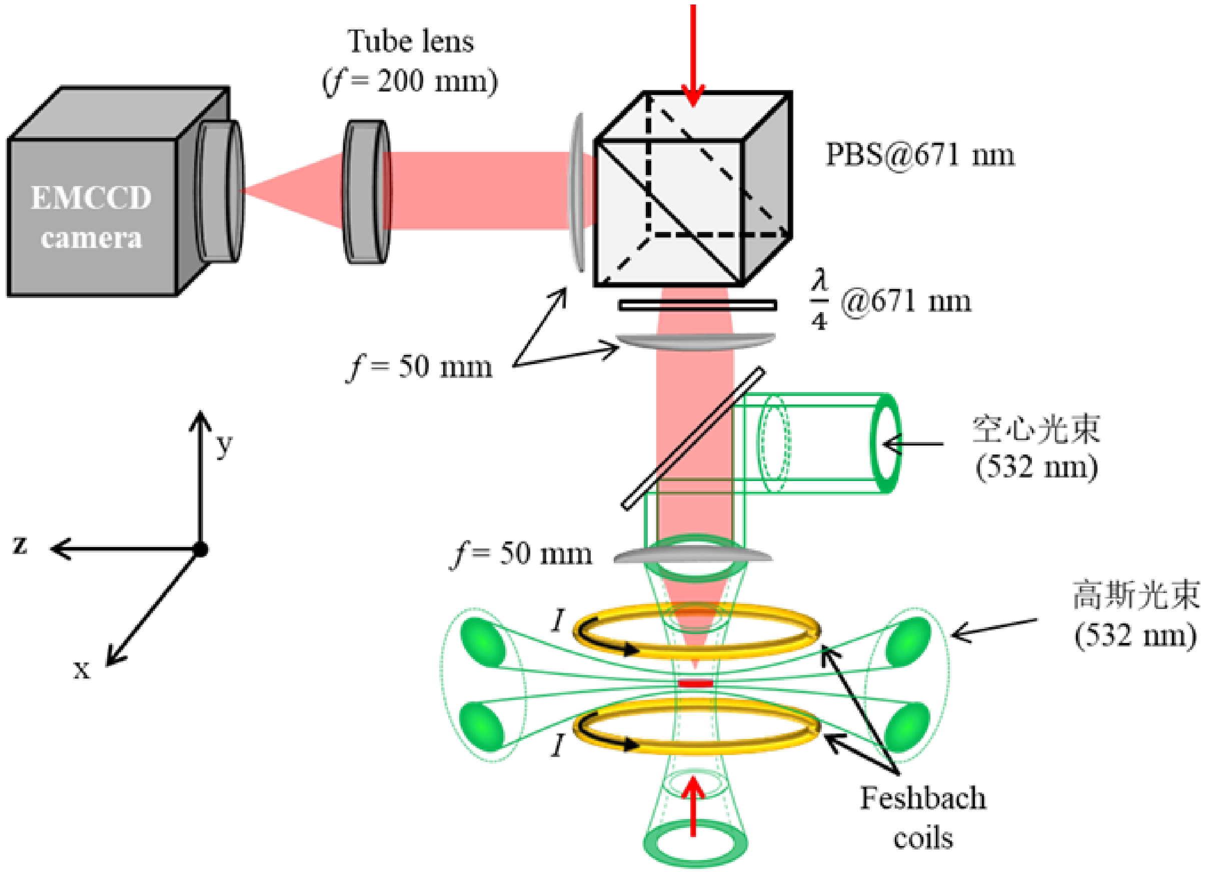
- Electromagnetic vs strong interaction
- Temperature 10^{-6} vs 10^{12} K, 18 orders
- Density 1 μm^{-3} vs 1 fm^{-3}, 27 orders

Future Works

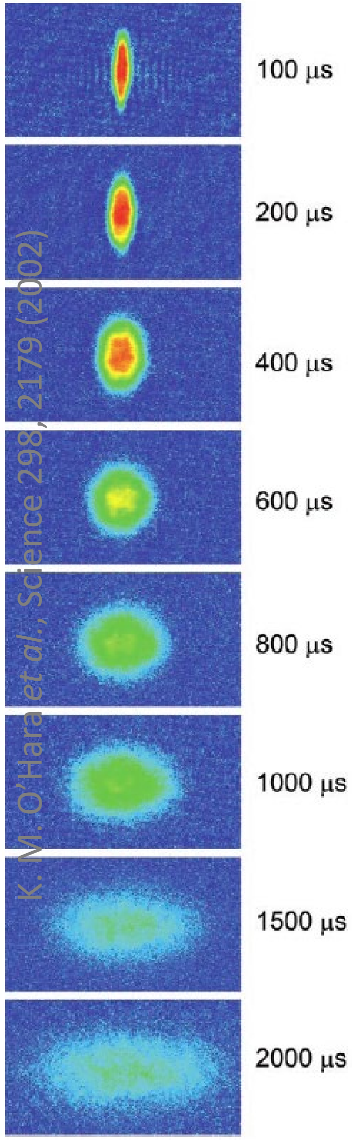
- Quasi-2D homogeneous Fermi gas
- Study high order anisotropic flow,
  e.g. triangular flow etc.
- Hydro attractor
- Spin-orbit coupled Fermi gas
- Molecular BEC
- Behavior in superfluid phase



(a) (b)



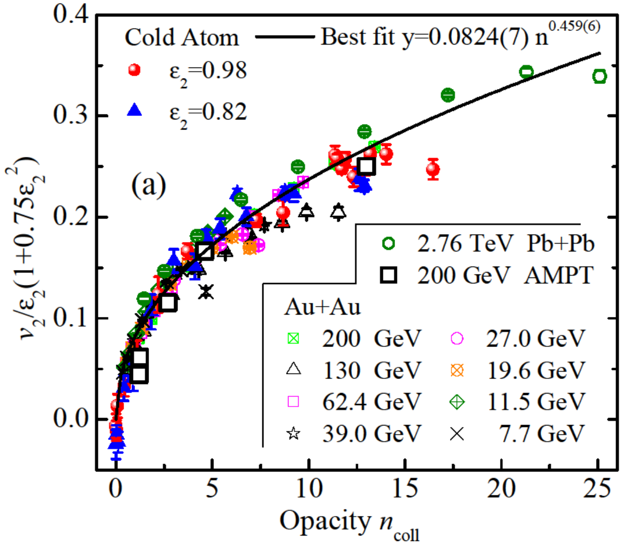
Summary

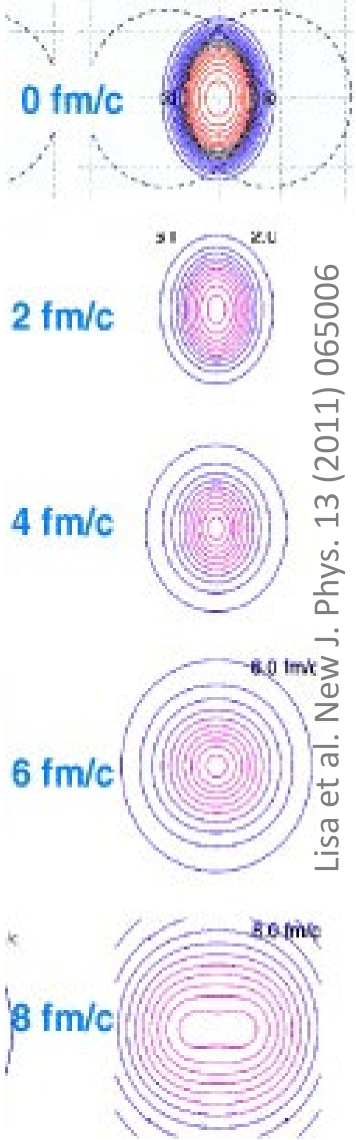


K. M. O'Hara *et al.*, Science 298, 2179 (2002)

- Anisotropy in heavy-ion and small system collisions; physics mechanisms not completely settled
- Cold atoms offer a unique tool, with tunability and controllability of interactions and experiment setup; Many emulations can be performed

- Universal anisotropy vs. opacity n_{coll} observed from cold atoms to hot QGP
  - Universal behavior of $\sqrt{n}$, characteristic of random walks
  - Anisotropy builds up quickly; large-*n* hydro limit not reached
  - Noticeable deviations from $\sqrt{n}$ at small *n*; hydro attractor behavior?





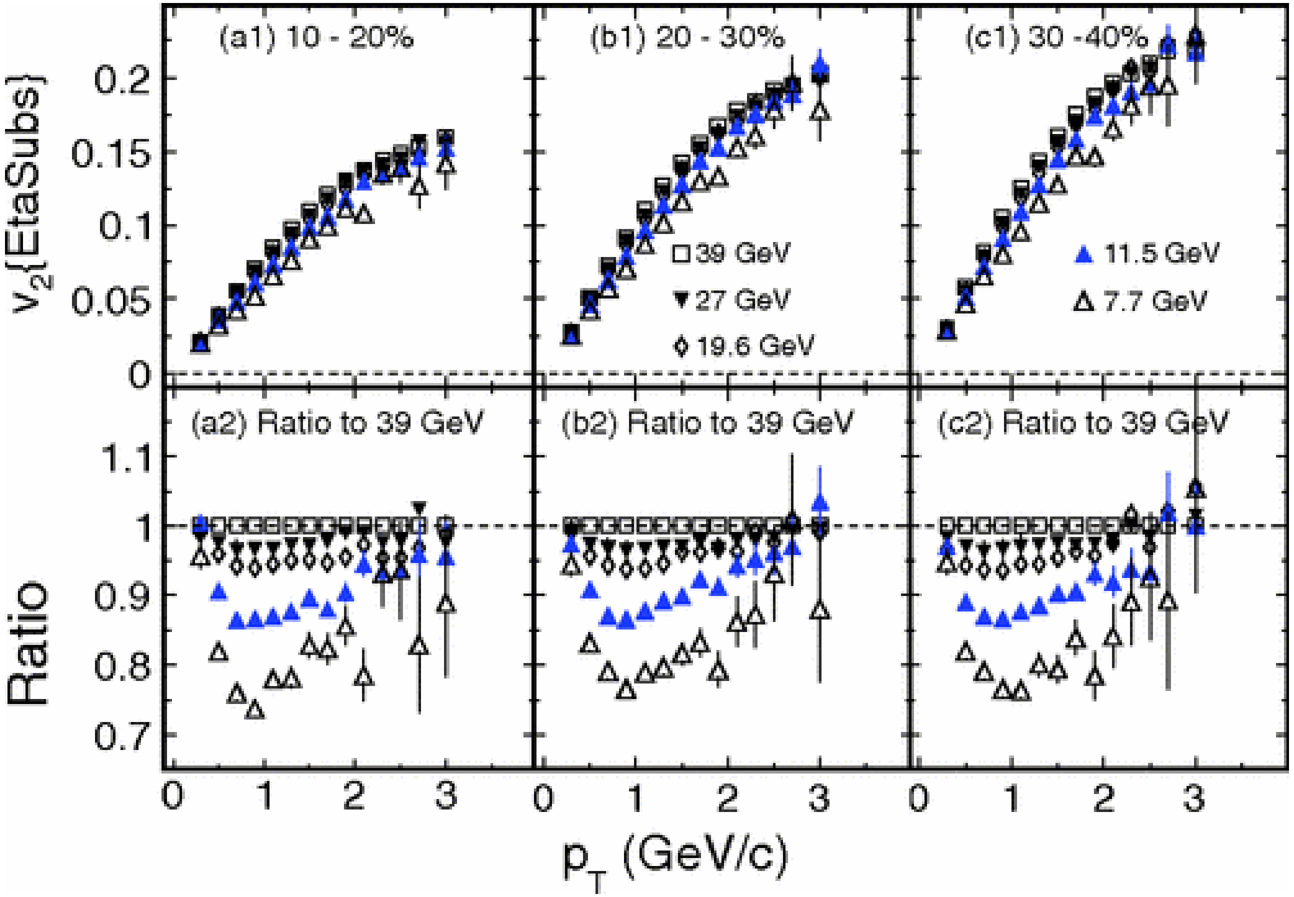
Lisa et al. New J. Phys. 13 (2011) 065006

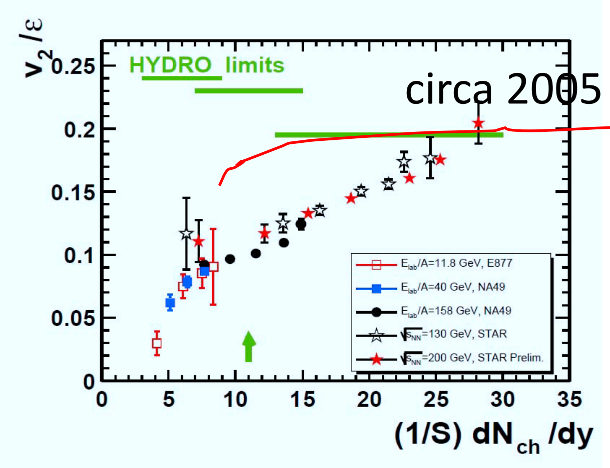
Extra slides

Heavy ion data

circa 2013

Very little energy dependence

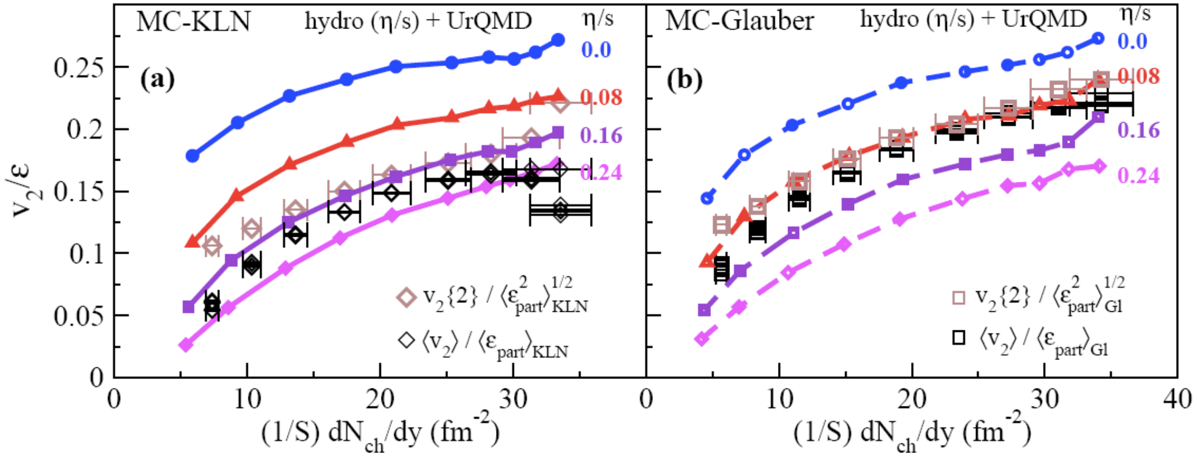




circa 2005

Reaches hydro prediction at RHIC for the first time

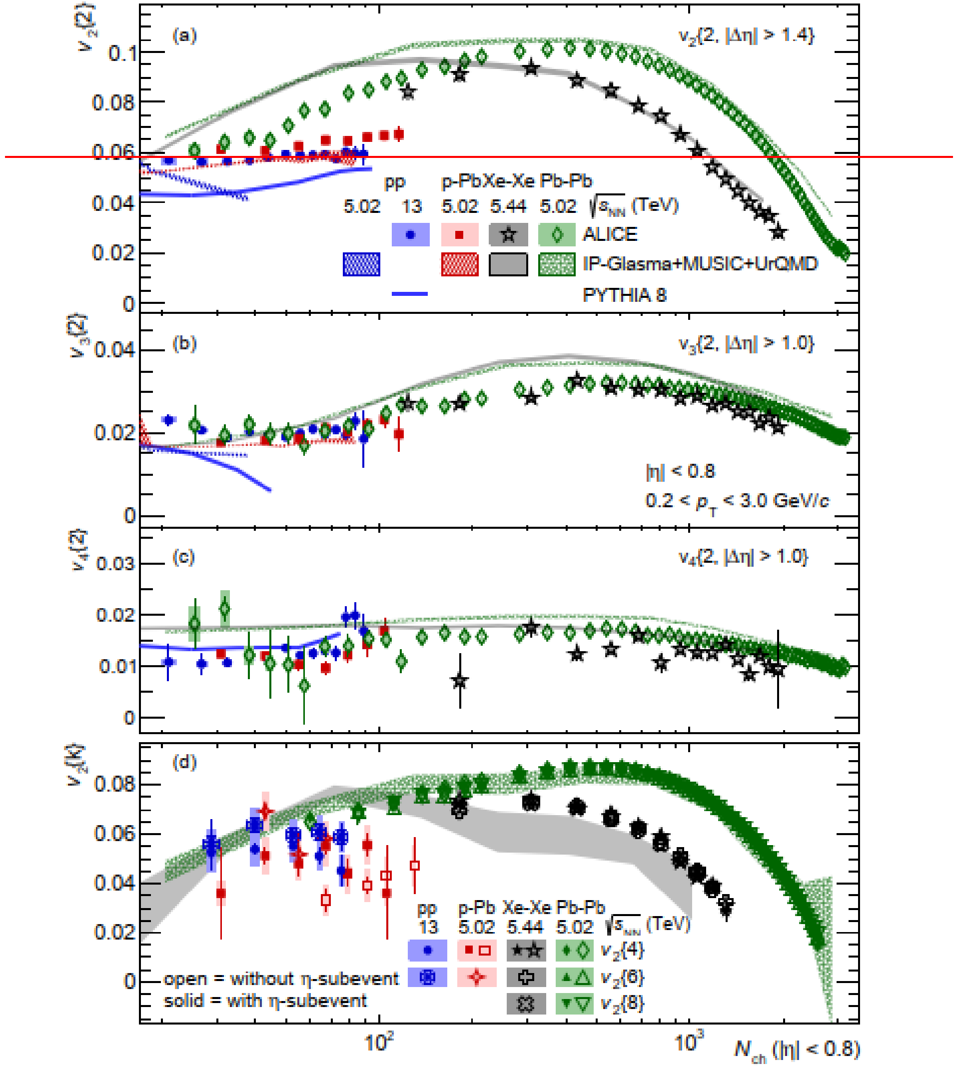
Song et al. PRL 106 (2011) 192301



Low η/s ≈ 1-2/4π, very strong interaction

Why v_2 is flat in small systems

ALICE, PRL 123 (2019) 142301



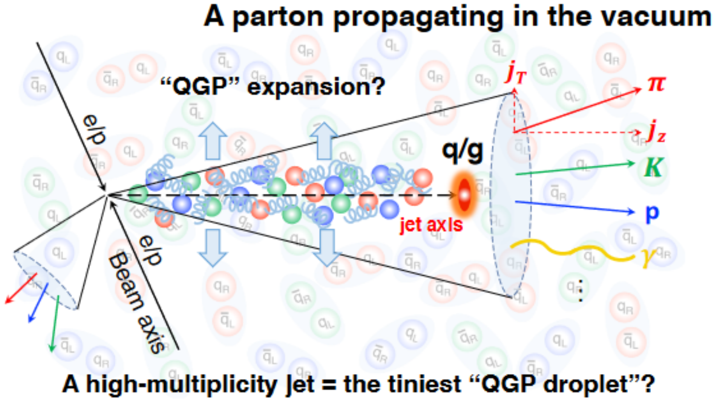
Jurgen Schukraft, QM2017: Nucl. Phys. A 967 (2017) 1–10

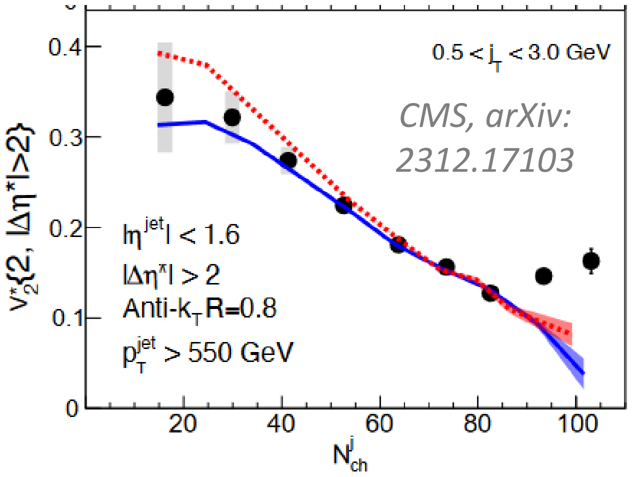
$v_2 = k\varepsilon_2$

- the responsive coefficient goes like $k \propto \sqrt{x}$
- the fluctuating eccentricity goes like $\varepsilon_2 \propto 1/\sqrt{x}$

x ~ opacity, dN/dy, dN/dy/S, or… turns out to be opacity n_{coll}

→ v_2 in the fluctuation dominated regime is ~ constant



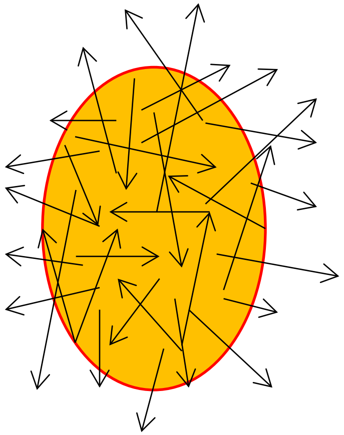


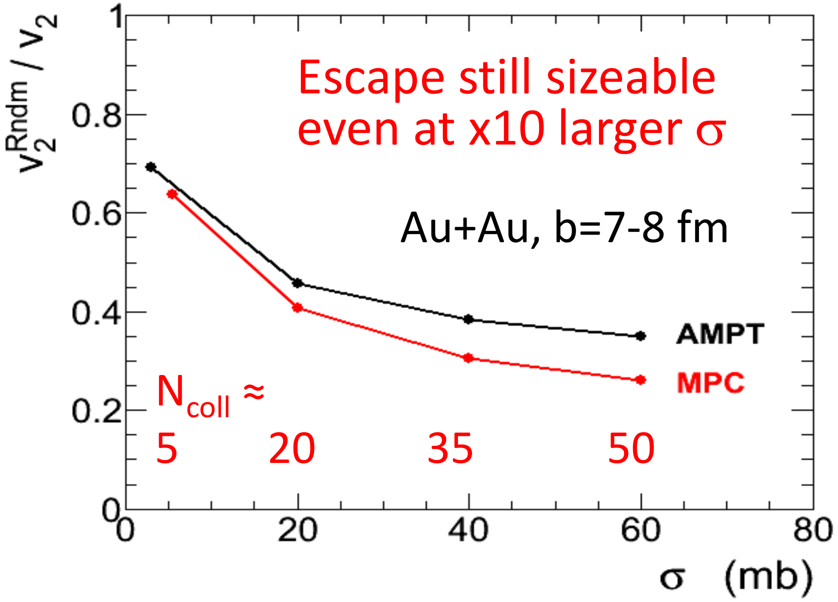
CMS, arXiv: 2312.17103

Escape mechanism

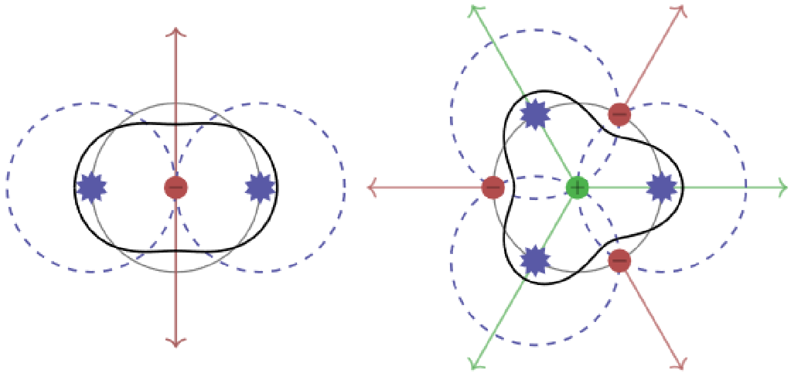
L. He et al., PLB753 (2016) 506, arXiv:1502.05572

- Partons freeze out with large positive v_2, even when they do not interact at all.
- Remaining partons start off with negative v_2, and become ~isotropic (v_2~0) after one more collision.



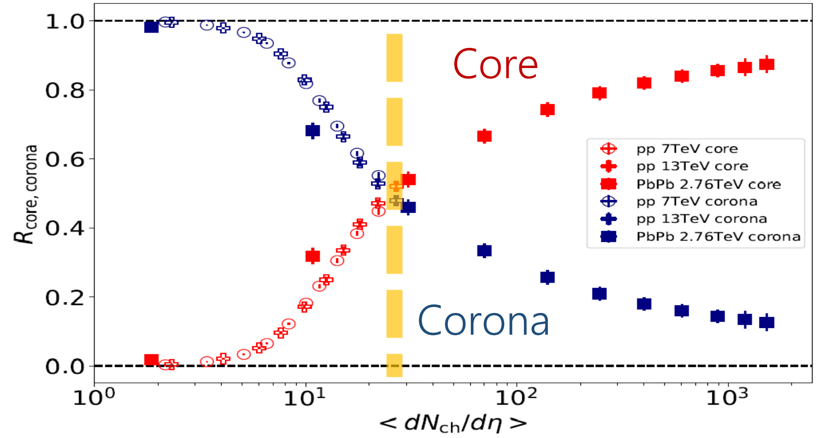


Kurkela et al. JHEP11(2021)216, single-hit anisotropies



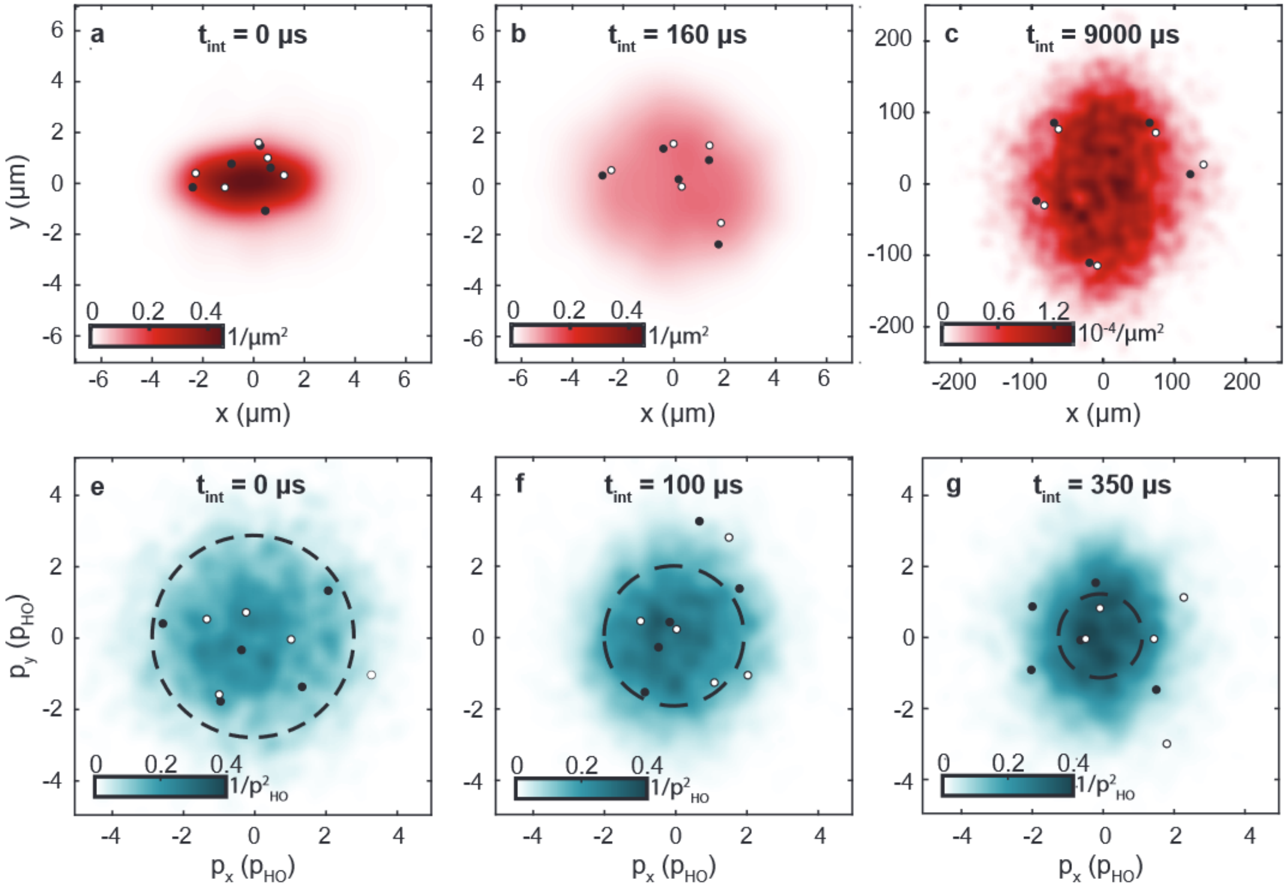
Romatschke, EPJC 75 (2015) 429, arXiv:1504.02529
Collective flow without hydrodynamics, free-streaming

*Kanakubo et al., Phys. Rev. C **105**, 024905 (2022)*



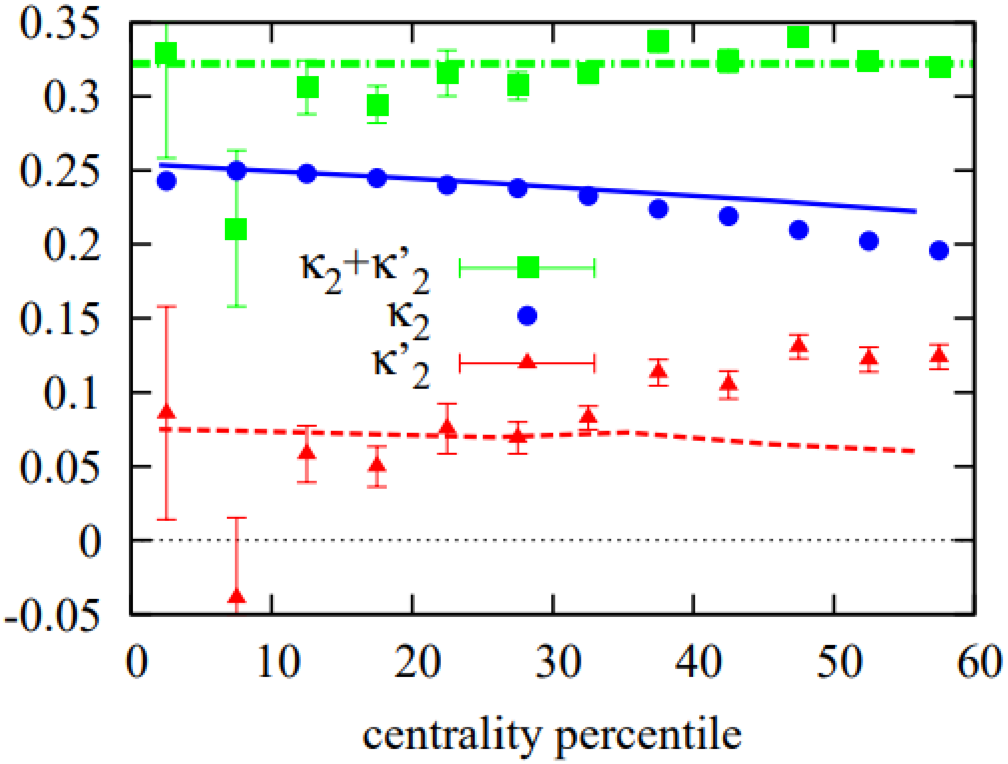
Very few atoms only

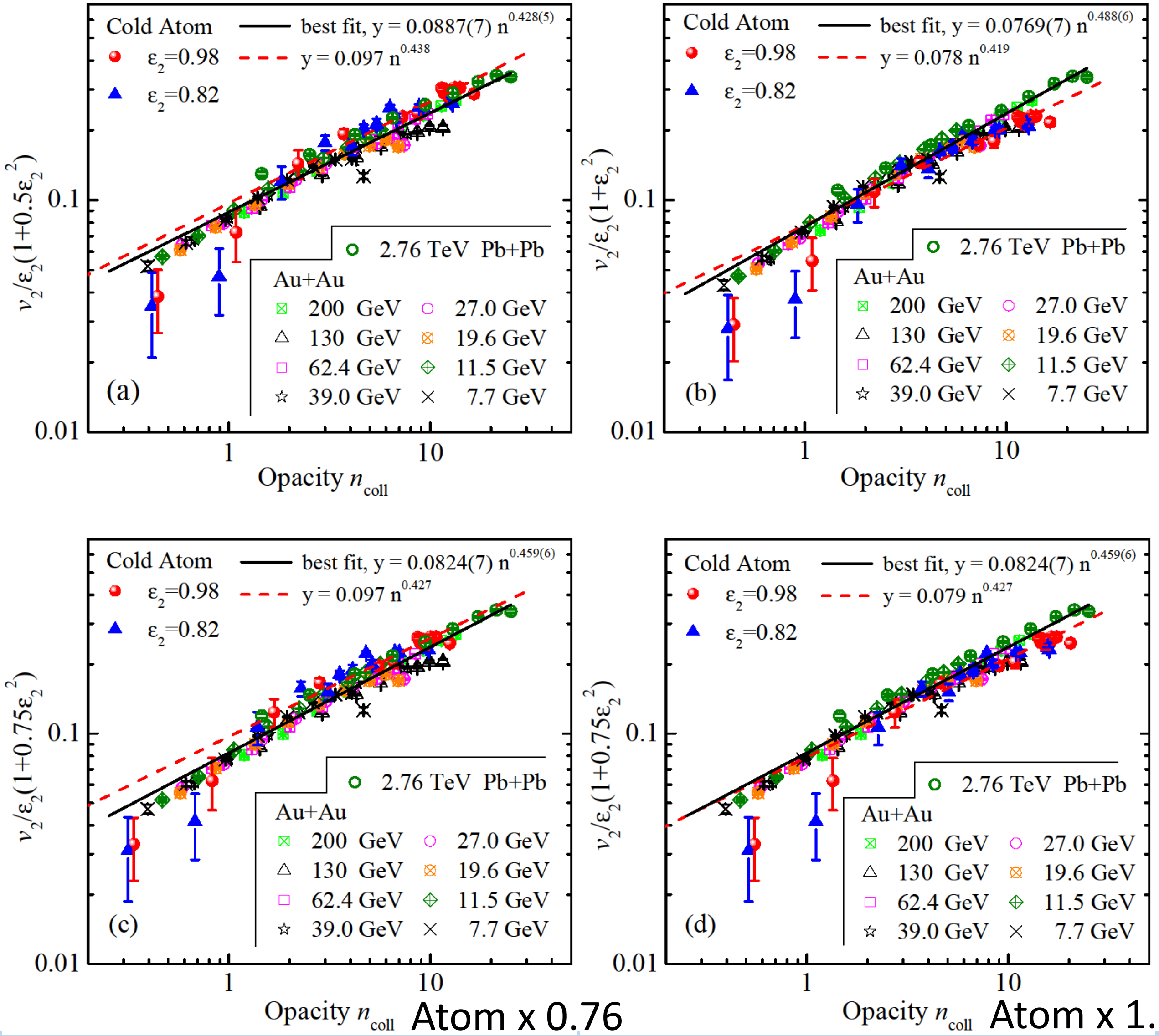
Brandstetter et al., arXiv:2308.09699
Emergent hydrodynamic behaviour of few strongly interacting fermions



Uncertainties in the universal behavior

Noronha-Hostler, PRC93(2016)014909

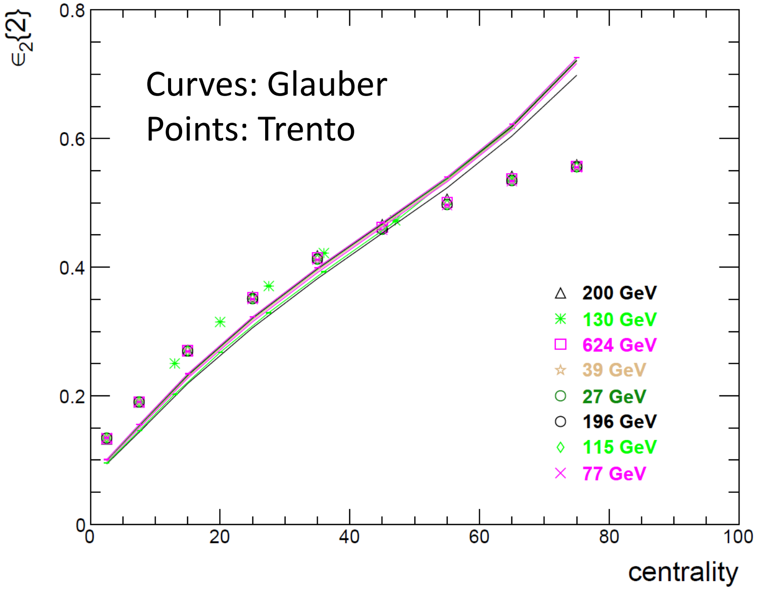


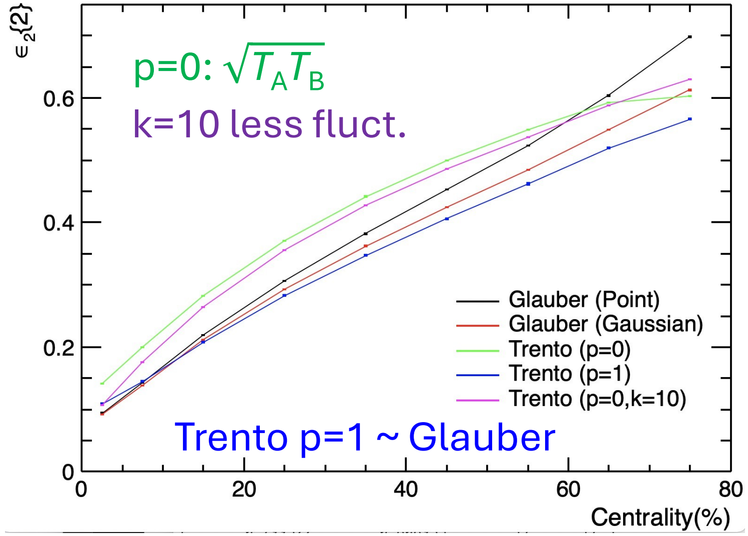


Atom x 0.76

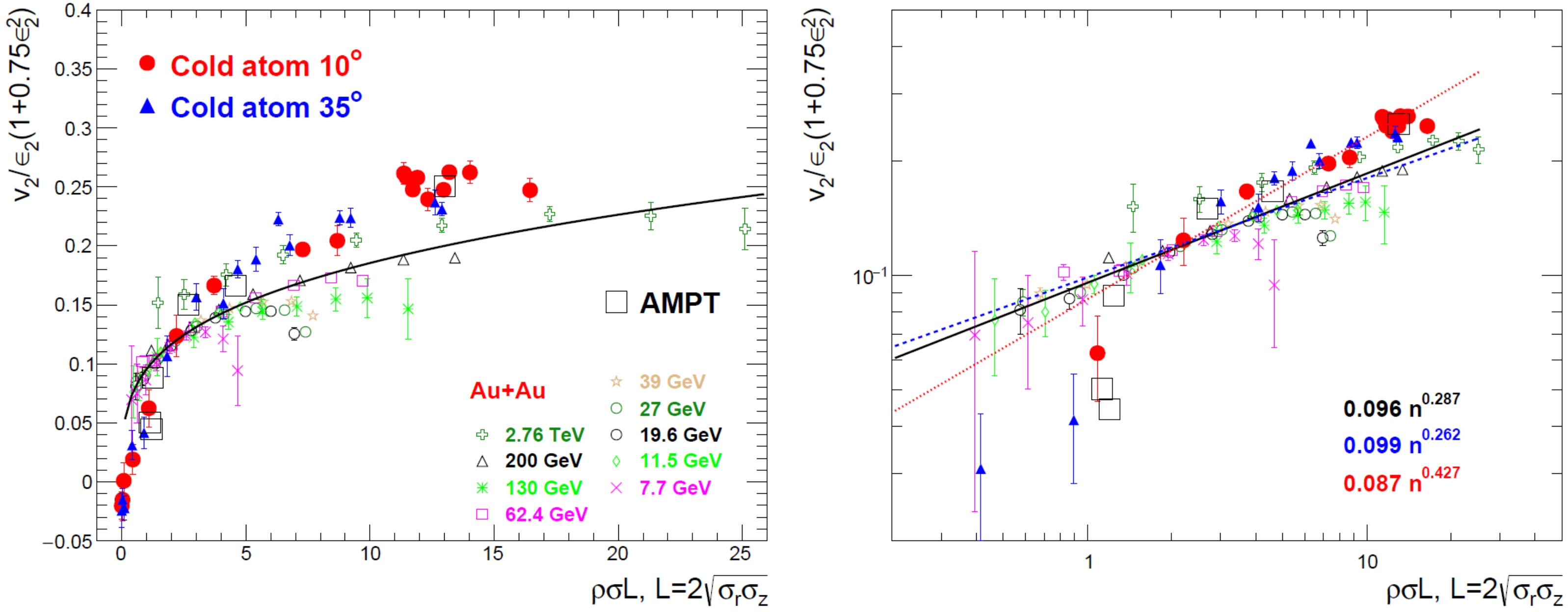
Atom x 1.24

Glauber vis-à-vis Trento



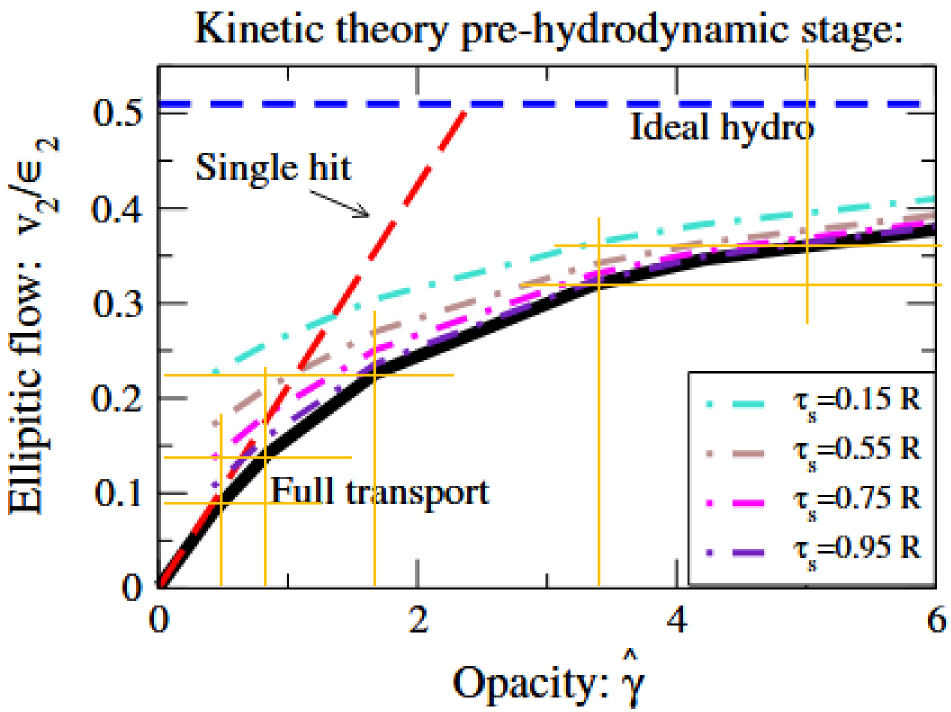


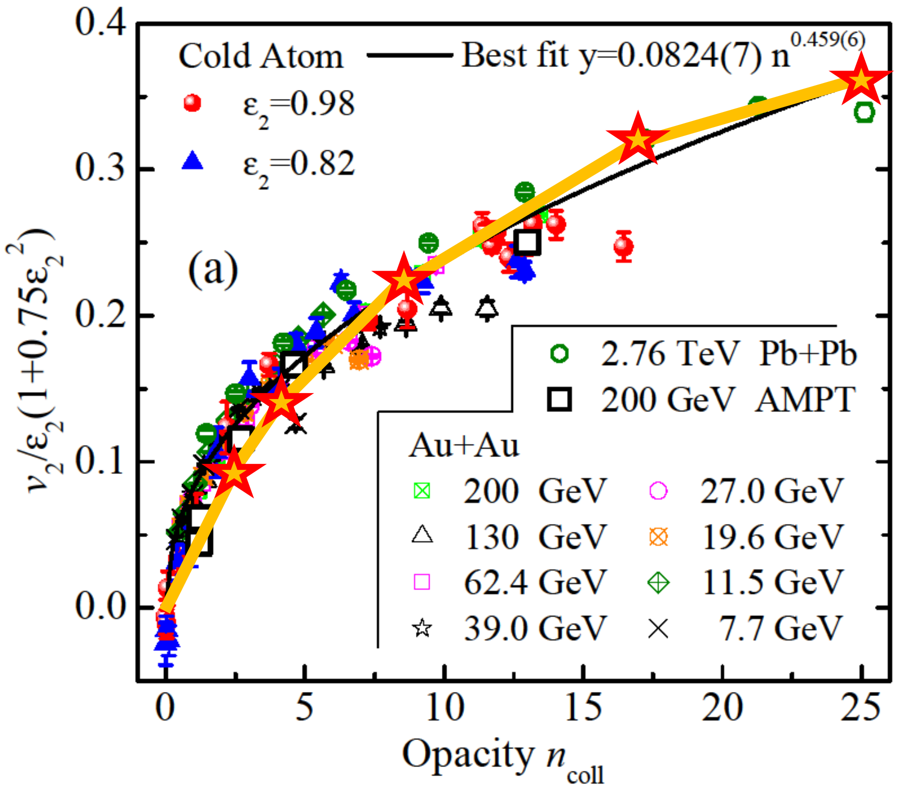
The money plot would be below. The AMPT points are still using Glauber eccentricity, and the cold atom points also no change. The power-law fit exponent is about 0.3.



Kinetic theory opacity

Kurkela et al., EPJC 79 (2019) 9, 759

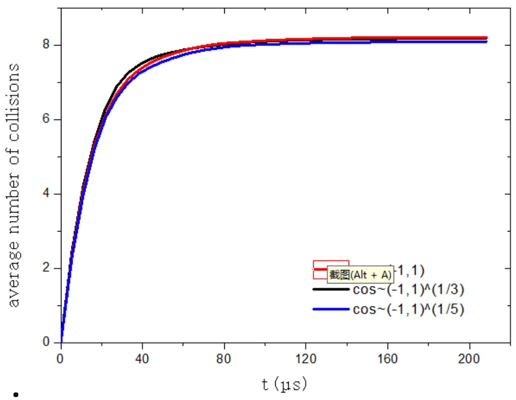


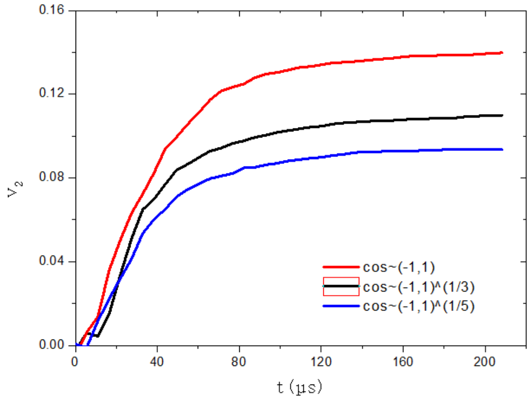


Kurkela et al. predicts v_2/ϵ_2 as a function of opacity $\hat{\gamma}$. Their opacity is $\hat{\gamma} = R/l_{mfp}$, where R is the system size and $l_{mfp} = 1/(\gamma\varepsilon^{1/4})$ is the mean-free-path (γ is the only model parameter). The energy density ε is taken at time $\tau = R$; 1-D (longitudinal) expansion and boost invariant are assumed: $\varepsilon\tau = \varepsilon_0\tau_0$ and the initial condition $\varepsilon_0\tau_0$ is set fixed. In other words, their opacity $\hat{\gamma}$ is calculated using the mean-free-path at time $\tau = R$, whereas our opacity is estimated using the Bjorken energy density at initial time τ_0 = 1 fm/c. If we were to use the mean-free-path at time $\tau = R$, when the energy density has decreased by a factor of $\tau/\tau_0 \sim 5$ taking the typical $R \sim 5$ fm, our opacity would be a factor of 5 smaller.

Scattering cross section angular dependence

- Cold atom scattering is low energy s-wave, isotropic.
- AMPT differential cross section is preferentially forward, modeled by a parton screening mass μ = 2.265/fm (PRC72.024906)
- Isotropic scattering (large angular change) should be more powerful to generate v2 than forward scattering (small angular change). Verified it with home-made 2->2 elastic scatter code →





- At small partonic cross section 3 mb, AMPT is close to isotropic scattering. Only at large x-section like 30 mb, scattering is more forward. See 2208.06027.
- We checked this by following Ziwei's instruction:

  Yes, 3mb is just the total cross section, and the angular distribution is by default forward angled. One can change the distribution to isotropic in input.ampt via the following line: "IZPC: (D=0 forward-angle parton scatterings; 100,isotropic)"
  Once IZPC is set to 100, then the total cross section is not affected but the angular distribution is isotropic (making shear viscosity lower and scatterings more effective).
- v2 increases from forward scattering to isotropic scattering in AMPT by: 17% for dAu 0fm (3mb), and for AuAu 7.3fm by 6% (0.6 mb), 8% (1.5mb), 11% (3mb), and 15% (20mb).
- Essentially no change to n_{coll}
  (within 1%, except in AuAu 20 mb calculation a 10% increase in ncoll)

AMPT points become red squares →