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Initial stage jet momentum broadening in a light-front Hamiltonian approach

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Motivation of our real-time quantum evolution calculation

Studies using EKRT+hydro+BDMPS-Z suggest that energy **loss must be suppressed** in the initial stage of heavy-ion collisions [Andres, Armesto, Niemi, Paatelainen, Salgado (2020)]

Classical jet in Glasma studies show that $\hat{q}$ is **very large** [Ipp, Müller, Schuh (2020)]

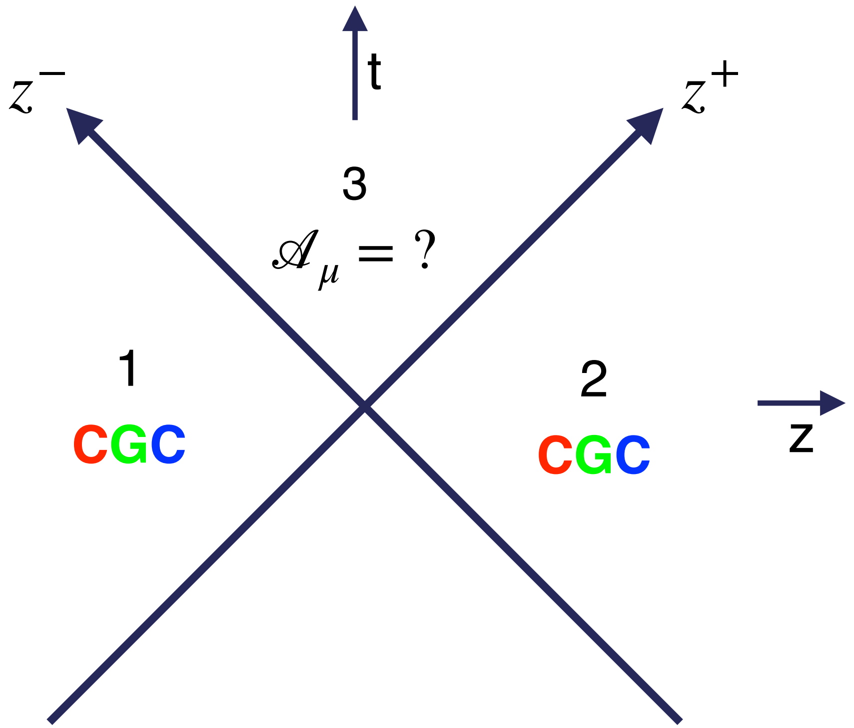
↓

First quantum treatment of the jet interacting with the Glasma fields using tBLFQ a **light-front Hamiltonian** formalism

See Nicholas talk Wed 10:00 about particle production in the Glasma

See Pablo's talk Tue 16:30 about the Glasma role in jet quenching effects

The Color Glass Condensate



We work on the **MV model**, where the relevant scale in the **saturation scale** Q_S

High-energy nuclei before the collision can be **modeled using Color Glass Condensate** (**CGC**)

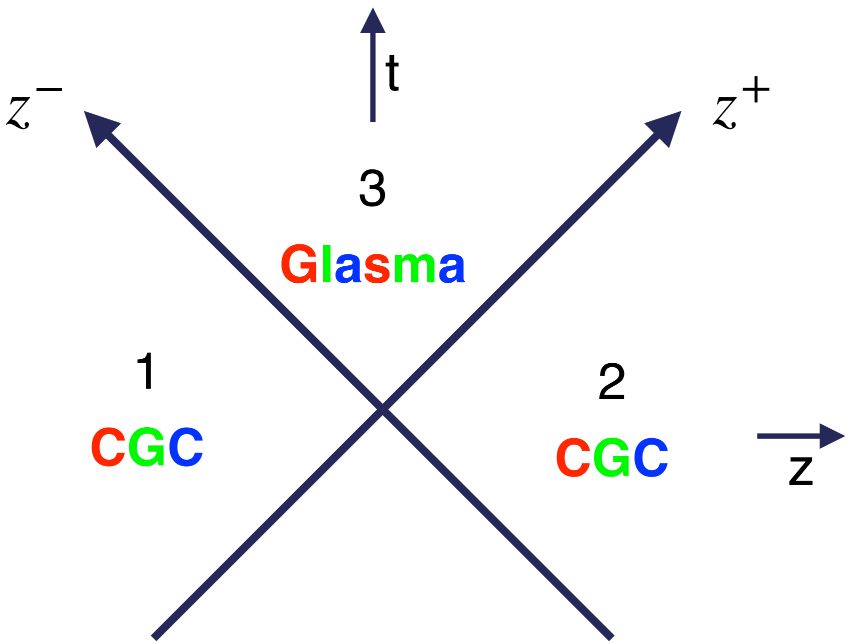
Based on scale separation

- Large x, static color charges
- Small x, classical color fields

Must follow **classical Yang-Mills equations**

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

The Glasma fields



Initial condition for the fields after the collision can be derived from the **CGC**

[Kovner, McLerran, Weigert (1995)]

Classical fields evolve with **sourceless Yang-Mills equation** in region (3)

$$[D_\mu, F^{\mu\nu}] = 0$$

Boost-invariant classical **Glasma** fields

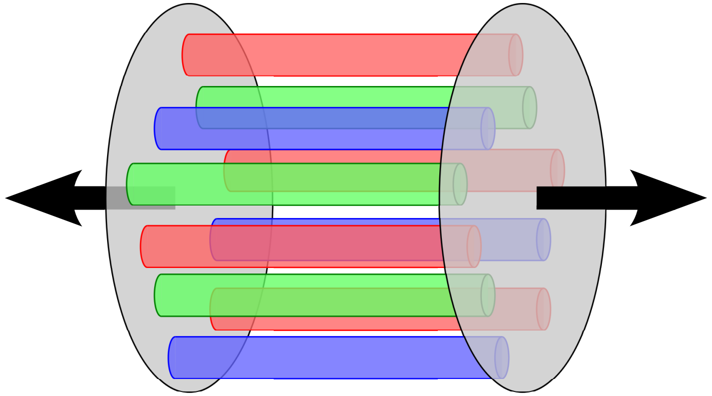
We use **real-time lattice gauge theory** to numerically solve Yang-Mills equation [Kransnitz, Venugopalan (2019)]

Main features of the Glasma fields

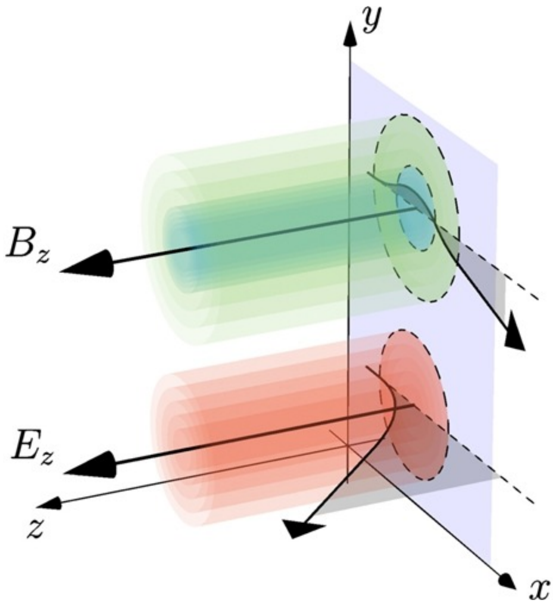
The Glasma potential $\mathscr{A}^\mu$ generates **chromo-electric and chromo-magnetic fields**

This fields are correlated inside **flux tubes of diameter** $\sim 1/Q_s$ that expand along the longitudinal direction

Initially the chromo-electric and chromo-magnetic fields **only have components along the longitudinal direction**



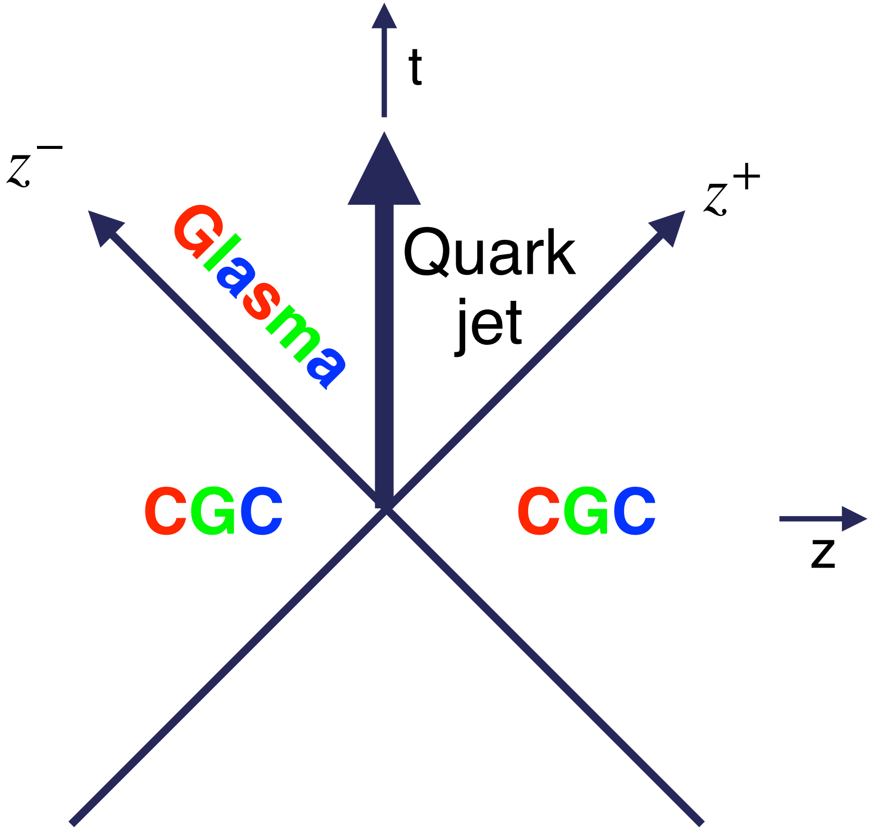
[Fukushima (2016)]



[Ipp, Müller, Schuh (2020)]

Jet propagation in the Glasma fields

See Dana's talk Thu 15:00 about jet and HQ early-time propagation



We will study the propagation of a **high-energy quark at mid-rapidity**

$$\boxed{\mathscr{L} = \overline{\Psi}(x)(i\gamma^\mu D_\mu - m)\Psi(x)}$$

$D_\mu = \partial_\mu - ig\mathscr{A}_\mu$ where $\mathscr{A} = \mathscr{A}^a t^a$ is the **classical Glasma background field**

There are no quantum gluons because we are **truncating the quark Fock space to the $|q>$ sector**, not considering radiation

The Light-Front Hamiltonian formalism

We will work in the **light-front form**, where $x^+ = t + x$ plays the role of the temporal variable

We can obtain the **light-front Hamiltonian** through the Legendre transform of the Lagrangian

$$\boxed{H = \frac{1}{2}\int dx^- d^2\vec{x}_\perp \left[\frac{1}{2}\bar{\Psi}\gamma^+\frac{m^2 - \nabla^{\perp 2}}{i\partial^+ - g\mathscr{A}^+}\Psi + g\bar{\Psi}(\gamma^+\mathscr{A}_+ + \gamma^i\mathscr{A}_i)\Psi + \frac{g^2}{2}\bar{\Psi}\gamma^i\mathscr{A}_i\frac{\gamma^+}{i\partial^+ - g\mathscr{A}^+}\gamma^j\mathscr{A}_j\Psi\right]}$$

$$\Big\downarrow\; p^+ \gg \mathscr{A}^+$$

$$\boxed{H = \frac{m^2 + \left(\vec{p}_\perp - g\overrightarrow{\mathscr{A}}_\perp(x^+, \vec{x}_\perp)\right)^2}{2p^+} + g\mathscr{A}_+(x^+, \vec{x}_\perp) - g\frac{\mathscr{B}^x(x^+, \vec{x}_\perp)\sigma^x}{2p^+}}$$

The time-dependent Basis Light Front Quantization (tBLFQ)

To numerically define the quark state we **expand it in a basis**

$$|\psi; x^+ > = \sum_\beta c_\beta(x^+)|\beta >$$

where $|\beta > = |p^+, \vec{p}^\perp, \lambda, c >$ or $|\beta > = |p^+, \vec{x}_\perp, \lambda, c >$

Time-evolution contained in the coefficients

$$i\frac{\partial}{\partial x^+}|\psi, x^+\rangle = H|\psi; x^+\rangle$$

$$\downarrow$$

$$\mathrm{c}(x^+) = \mathcal{T}_+ \exp\left[-i\int_0^{x^+} dt \mathscr{M}(t)\right] \mathrm{c}(0)$$

Successfully applied to $|q\rangle$ and $|q\rangle + |qg\rangle$ evolution in a **MV model field**

[Li, Zhao, Maris, Chen, Li, Tuchin, Vary (2020)] [Li, Lappi, Zhao (2021)] [Li, Lappi, Zhao, Salgado (2023, 2025)]

See Meijian's talk Wed 9:30 about dressed quarks in a colored field

The quantum definition of kinetic momentum

We can use the quark state to evaluate the **quantum expectation value** of any operator

$$\langle \mathcal{O} \rangle = \langle \psi; x^+ | \mathcal{O} | \psi; x^+ \rangle$$

To compute momentum broadening $\langle p^2_{kin} \rangle$ we have to define the **kinetic momentum operator** $\vec{p}^{\,kin}_{\perp,H} = p^+ \dfrac{d\vec{x}_{\perp,H}}{dx^+}$

Go to the **Heisenberg picture**, where all the time evolution information is contained in the operators

$$\frac{d\mathcal{O}_H}{dx^+} = i[H_H, \mathcal{O}_H] + \left(\frac{\partial \mathcal{O}}{\partial x^+} \right)_H$$

$$\boxed{\vec{p}^{\,kin}_{\perp,H} = ip^+[H_H, \vec{x}_{\perp,H}] = \vec{p}_\perp - g\vec{\mathscr{A}}_\perp(x^+, \vec{x}_\perp)}$$

Lorentz force operator

We can in principle define momentum broadening directly as $\langle p^2_{kin,i} \rangle$, but there is some ambiguity in how to subtract the initial value of the momentum

↓

More convenient to define (in the eikonal limit)

$$\frac{dp^{kin}_{i,H}}{dx^+} = i[H_H, p^{kin}_{i,H}] + \left(\frac{\partial \vec{p}^{\,kin}_{\perp}}{\partial x^+}\right)_H = gU^\dagger(x^+,0)\mathscr{F}_{i+}(x^+)U(x^+,0)$$

where $\mathscr{F}_{i+}$ is the **field strength tensor** of the classical background color fields which can be determined in terms of the **chromo-electric and chromo-magnetic fields**, $\mathscr{F}_{y/z} = \mathscr{E}_{y/z} + / - \mathscr{B}_{z/y}$ and $U(x^+,0) = \mathscr{T}_+ \exp\left(-ig\int_0^{x^+} d\xi A_+(\xi)\right)$ is the time evolution operator

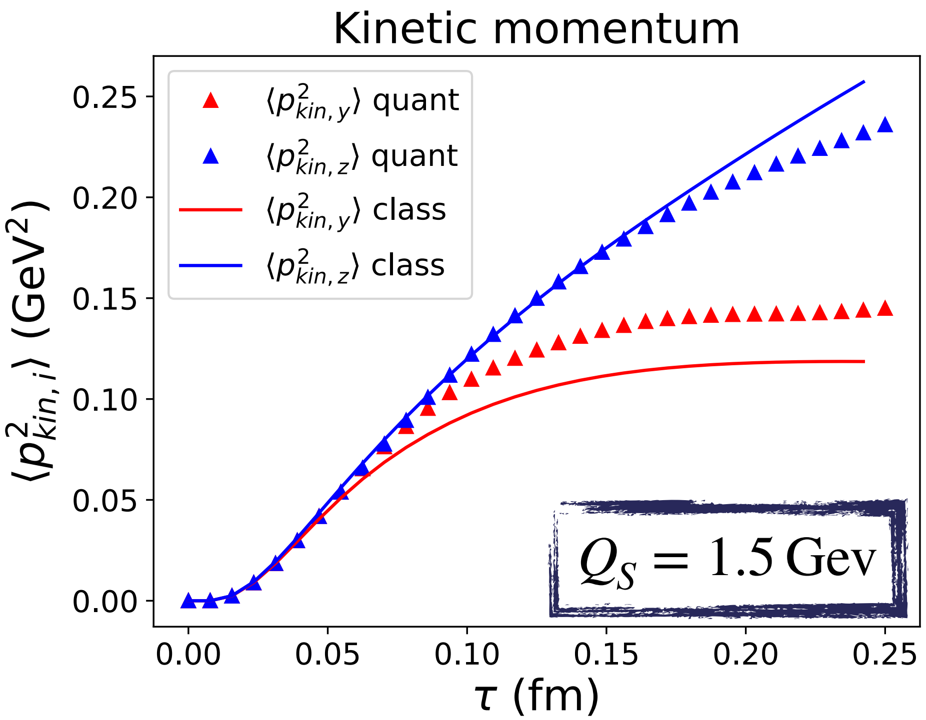
Kinetic momentum broadening

Momentum broadening is then defined as

$$\langle(\delta p_i^{kin}(x^+))^2\rangle = \frac{g^2}{4}\int_0^{x^+} d\lambda \int_0^{x^+} d\lambda' \langle\langle\psi;0\,|\,\tilde{f}_i(\lambda)\tilde{f}_i(\lambda')\,|\,\psi;0\rangle\rangle$$

where $\tilde{f}_i(\lambda) = U^\dagger(\lambda,0)\mathscr{F}_{i+}(\lambda)U(\lambda,0)$

Similar to the classical calculation [Ipp, Müller, Schuh (2020)], but we are treating the quark as a **quantum particle** in a classical background field



Non negligible momentum broadening during the initial stage

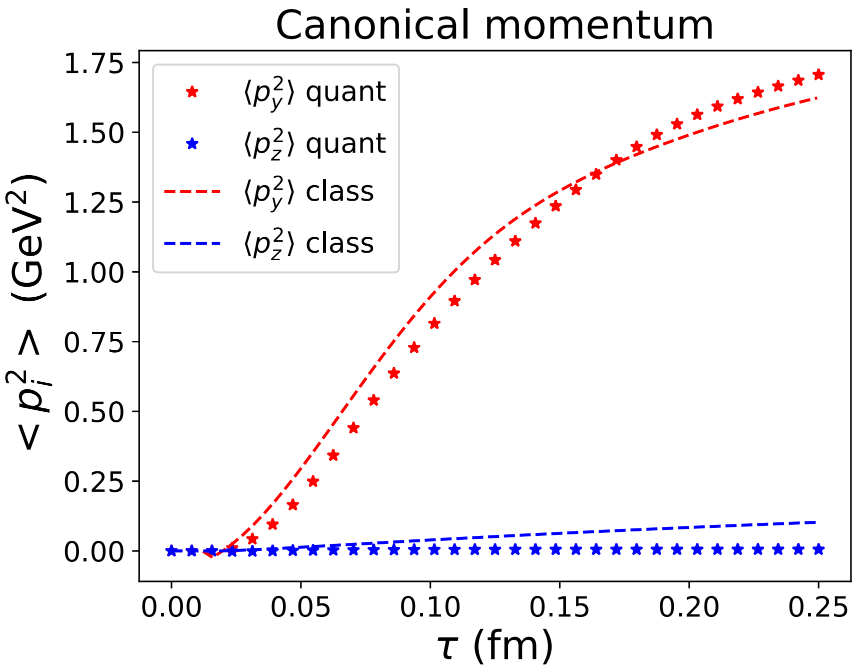
Anisotropic broadening with a larger contribution in the **beam direction**

Good agreement with the **classical calculation** [Ipp, Müller, Schuh (2020)]

Canonical momentum broadening and tBLFQ formalism

Note that kinetic momentum $\vec{p}^{\,kin}_{\perp,H} = \vec{p}_\perp - g\overrightarrow{\mathscr{A}}_\perp(x^+, \vec{x}_\perp)$ is not the same as the canonical momentum $\vec{p}^{\,\perp}$ which is **conjugated to the position**

$$\langle p_i^2 \rangle = \langle \Psi; x^+ | p_i^2 | \Psi; x^+ \rangle = \sum_\beta p_\beta^2 |c_\beta(x^+)|^2$$

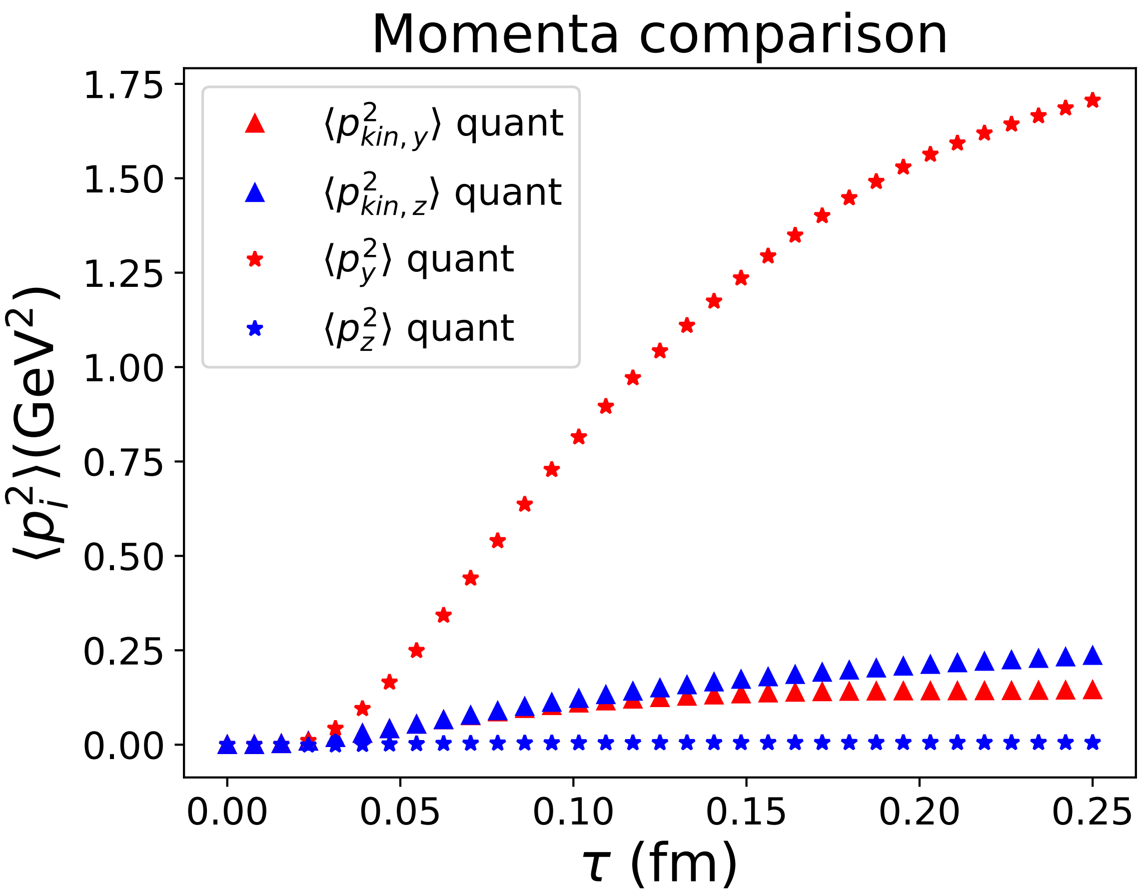


Opposite behavior to kinetic momentum with **larger broadening in the transverse direction**

Good agreement with the classical simulation for the **y-direction**, **not so good for the z-direction**

Not an observable, result is **gauge dependent**

Kinetic and canonical momentum broadening



Large difference between kinetic and canonical momentum in the Fock-Schwinger gauge

Canonical momentum contains a **large gauge field contribution** which must be extracted to get the gauge invariant kinetic momentum

Conclusions…

- We did the **first quantum treatment** of a jet inside the Glasma phase
- We found that momentum broadening is **large, anisotropic and greater in the beam direction** than in the transverse direction
- We also computed the canonical momentum, conjugated to the position and found that there is **a large, gauge dependent difference**, between kinetic and canonical momentum

and outlook…

- Relax the exact **mid-rapidity and eikonal approximations**
- **Include gluon radiation** to compute medium induced energy loss inside the Glasma

Back-up slides

Real-time lattice gauge theory

The gauge transformation of the gauge fields $\mathscr{A}_\mu(x)$ is **non-local** as it involves spatial derivatives

Gauge invariant is then **only exact up to same order in a^μ**

The solution to have exact gauge invariance is to use as degrees of freedom the **Wilson lines between neighbor points, the gauge links**

Gauge fields → **Wilson lines** → **Gauge links**

$$\mathscr{A}^\mu(x)$$

$$W[c] = \mathscr{P}\exp\left(-ig\int_c dx^\mu \mathscr{A}_\mu(x)\right)$$

$$U_{x,\mu} = \exp\left(iga^\mu \mathscr{A}_\mu(x + a^\mu/2)\right)$$

[Müller (2019)]

The Glasma fields initial condition

Imposing **boost invariance**

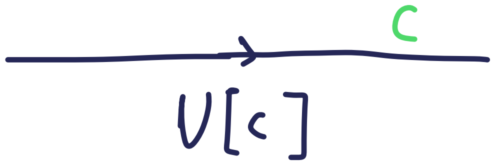
$$A_i^{(3)}(\tau = 0) = A_i^{(1)} + A_i^{(2)} \qquad A^\eta(\tau = 0) = \frac{ig}{2}[A_i^{(i)}, A_i^{(2)}]$$

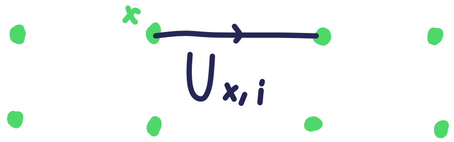
[Phys. Rev. D **52**, 6231]

The real-time lattice gauge theory

Gauge fields **Wilson lines** **Gauge links**

$$A_\mu(x) \xrightarrow{\text{Exponentiation}} U[c] = \mathcal{P}\exp\left(-ig\int_c dx^\mu A_\mu(x)\right) \xrightarrow{\text{Discretization}} U_{x,\mu} \simeq \left(ig\, a^\mu A_\mu\left(x + \frac{a^\mu}{2}\right)\right)$$



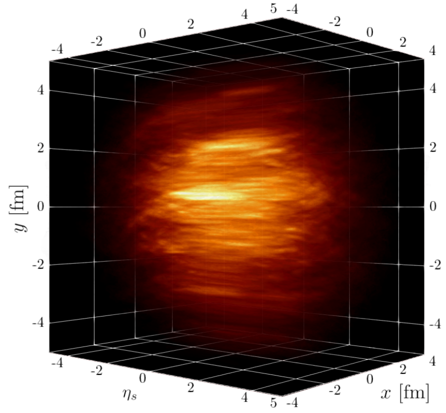


Improvements to the boost-invariant Glasma

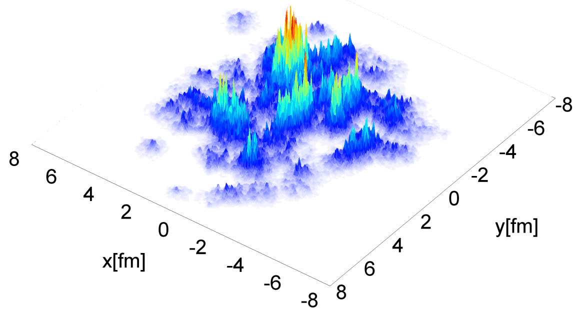
$(3+1)D$ **Glasma** using a semi-analytical approach in the dilute limit

Including the nucleon structure of the nucleus in the Glasma initial conditions, as done in **IP-Glasma**

For the scope of this work, the boost-invariant Glasma is enough



[Ipp, Leuthner, Müller, Schliting, Schmidt, Singh (2019)]



[Schenke, Tribedy, Venugopalan (2012)]

The eikonal Light-Front Hamiltonian

Taking the limit $p^+ \to \infty$ and neglecting terms suppressed by p^+

$$\boxed{H = \frac{g}{2}\int dx^- d^2\vec{x}_\perp \bar{\Psi}\gamma^+ \mathscr{A}_+ \Psi}$$

In the eikonal limit **only the $\mathscr{A}_+$ components of the classical background field appear in the Hamiltonian**, violations of the LC gauge condition $\mathscr{A}_- = 0$ are suppressed as $1/p^+$

NO NEED TO GAUGE TRANSFORM THE GLASMA FIELDS !!!

Expectation value of the momentum

Knowing the wavefunction, the **expectation value of the momentum operator** can be determined at any time

$$< p_i^2 > = < \psi; x^+ | p_i^2 | \psi; x^+ > = \sum_{\beta\beta'} c_\beta(x^+) c_{\beta'}(x^+) < \beta'(p^+_{\beta'}, \vec{p}^\perp_{\beta'}, c_{\beta'}, \lambda_{\beta'}) | p^{i\,2} | \beta(p^+_\beta, \vec{p}^\perp_\beta, c_\beta, \lambda_\beta) >$$

Matrix element

$$< \beta'(p^+_{\beta'}, \vec{p}^\perp_{\beta'}, c_{\beta'}, \lambda_{\beta'} | p^{i\,2} | \beta(p^+_\beta, \vec{p}^\perp_\beta, c_\beta, \lambda_\beta > = p_\beta^{i\,2} (2\pi) \delta(p^+_\beta - p^+_{\beta'}) (2\pi)^2 \delta^{(2)}(\vec{p}^\perp_\beta - p^\perp_{\beta'}) \delta_{\lambda_\beta \lambda_{\beta'}} \delta_{c_\beta c_{\beta'}}$$

$$< p_i^2 > = \sum_\beta p_\beta^{i\,2} | c_\beta(x^+) |^2$$

Equation of motion for classical $< p_i^2 >$

Taking the eikonal limit for the quark and imposing the boost-invariance of the Glasma fields

$$\frac{dp^i}{dt} = \frac{\partial p^i}{\partial t} + \{p^i, H\} \longrightarrow \frac{dp^y}{dt} = g\partial^y \mathscr{A}_x \qquad \frac{dp^z}{dt} = 0$$

$$< p_y^2(x^+) > = \frac{g^2}{N_c} \int_0^{x^+} dt_1 \int_0^{x^+} dt_2 < Tr[U(0,t_1)\partial^y \mathscr{A}_x(t_1, y)U(t_1,0)U(0,t_2)\partial^y \mathscr{A}_x(t_2, y)U(t_2,0)] >$$

where $U(x^+,0) = \mathscr{P} \exp\left(-ig\int_0^{x^+} dtA_x(dt)\right)$

Equation of motion for classical $< p_{kin,\,i}^2 >$

Given by the non-Abelian generalization of the Lorentz force, the Wong's equations

[Nuovo Cim. A 65, 689 (1970)]

$$< p_{kin,\,i}^2(x^+) > = \frac{g^2}{N_C}\int_0^{x^+} dt_1 \int_0^{x^+} dt_2 < Tr[f^i(t_1)f^i(t_2)] >$$

where

$$f^y(t) = U(0,t)(E_y(t) - B_z(t))U(t,0)$$

$$f^z(t) = U(0,t)(E_z(t) + B_y(t))U(t,0)$$

and the Wilson line is the same as in the previous slide

[Phys. Lett. B 810 (2020) 135810]

Parameters of the simulation shown in this slides

$g^2\mu = 1.5\,\mathrm{Gev}$

$L = 1\,\mathrm{fm}$

$N = 128$

$\tau = 0.25\,\mathrm{fm}$

$\delta\tau = a/8; a = L/N$

$m_{IR} = 0.2\,\mathrm{GeV}$