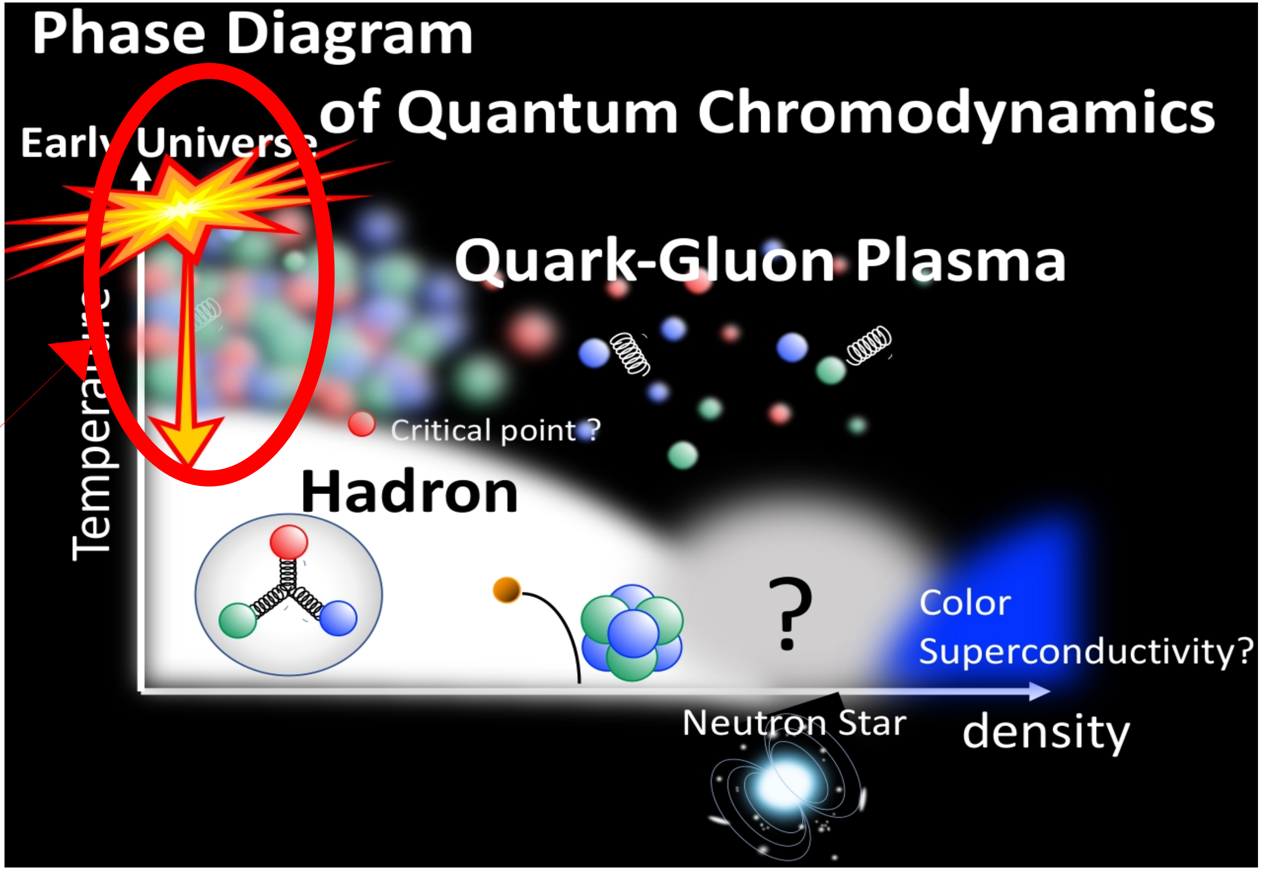
広島大学
HIROSHIMA UNIVERSITY

Equilibration and Hydrodynamic Onset in Small and Large Systems.

Cendikia Abdi, Chiho Nonaka
Hiroshima University.

Initial Stages 2025
7-12 September 2025, Taipei, Taiwan.

Introduction



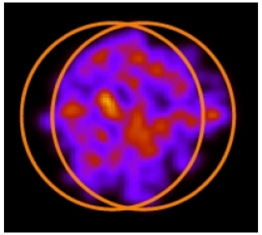
Area of Focus

- Quark-Gluon Plasma (QGP) is a state of matter where quarks and gluons are decoupled, thus they can move freely.
- Main source of observable QGP comes from high energy particle colliders.

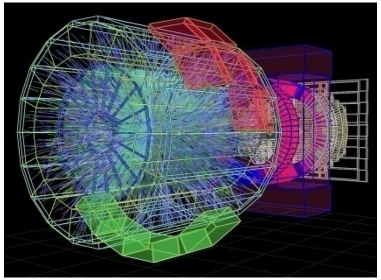
Overview

V ~ c

High energy collision



→ ? →



Detector

In more detail

Glauber model
Colour glass condensate

How to connect this ?

Hydro simulation

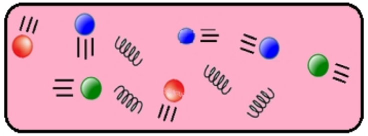
Cooper-Frye prescription

Afterburner

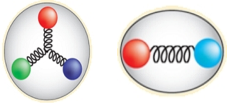
Collision → Pre-Equilibrium QGP → Equilibrium QGP → Phase Transition → inter-particle Interactions

Past research

- Effective Kinetic Theory (arXiv:2211.15454 [hep-ph] (2022))
- Core-Corona model (Phys. Rev. C 105 ,024905)
- BAMPS parton cascade model (Phys. Rev. D 104, 094022)



Quark-Gluon Plasma Phase



Hadronic Phase

SMASH transport model

Solving Boltzmann Transport Equation

$$\frac{\partial}{\partial t} f(t, \vec{r}, \vec{p}) + \frac{\vec{p}}{m} \nabla_{\vec{r}} f(t, \vec{r}, \vec{p}) - \nabla_{\vec{r}} U(\vec{r}) \nabla_{\vec{p}} f(t, \vec{r}, \vec{p}) = \left(\frac{\partial f}{\partial t} \right)_{coll}$$

Approach

Based on Hadronic Transport Model SMASH

Quantum molecular Dynamic
=> Monte-Carlo solver
=> Approximation solution to Boltzmann Transport Equation

Simplify the equation
- Relaxation Time Approximation
- Linearized Boltzmann Equation
- etc...

SMASH is a robust transport simulation which has been confirmed to be able to reproduce particle production in various heavy ion collision settings[1]

[1] J. Weil, V. Steinberg, J. Staudenmaier, L.G. Pang, D. Oliinychenko, J. Mohs, M. Kretz, T. Kehrenberg, A. Goldschmidt, B. Bäuchle, J. Auvinen, M. Attems, H. Petersen. arXiv:1606.06642 [nucl-th].

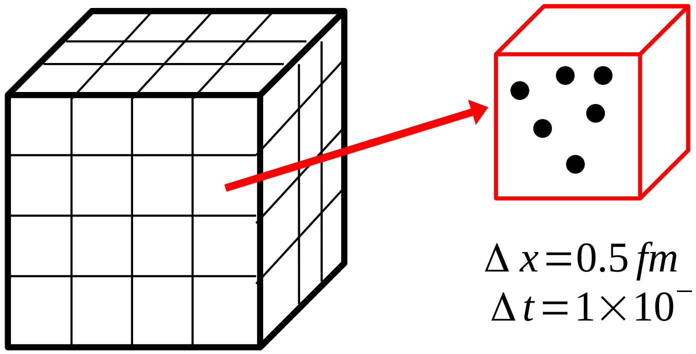
SMASH's Workflow

- Inter-particle collisions are calculated using stochastic method[2]

$$P_{2\to N} = v_{rel}\frac{\sigma_{2\to N}}{N_{test}}\frac{\Delta t}{\Delta^3 x} \quad P_{3\to 2} = \frac{1}{4E_1E_2E_3}\frac{(s-m_1^2-m_2^2)^2-4m_1^2m_2^2}{\Psi_3 8\pi s}\sigma_{2\to 3}\frac{\Delta t}{\Delta^3 x}$$

  - Only allow interaction in the same cell.
  - Assume only 1 collision for each timestep.

- Update particles
  - Propagate particles in straight line (Δt assumed to be small enough to ignore force)



$\Delta x = 0.5\, fm$
$\Delta t = 1\times 10^{-5}\, fm$

This allows simulation including multipartonic interactions

[2] Jan Staudenmaier, Dmytro Oliinychenko, Juan M. Torres-Rincon, Hannah Elfner Phys. Rev. C 104, 034908 (2021)

Initial Condition

Initial condition (Mini-jet model).

- Number of nucleon-nucleon collision with Glauber model

$$N = 2v\,\sigma \int d^2 x_T\, dz dt\, n_A(\vec{x}_T, z - vt)\, n_B(\vec{x}_T, z + vt)$$

- Nucleon-nucleon collision using deep inelastic scattering

$$\frac{d\sigma_{jet}}{dp_T^2 dy_1 dy_2} = K \sum_{a,b} x_1 f_a(x_1, p_T^2) x_2 f_b(x_2, p_T^2) \frac{d\sigma_{ab}}{d\hat{t}}$$

Partons carrying momentum $P_T > P_{mini\text{-}jet}$ are knocked out of the nucleon.

σ includes 5 channels :

- $gg \to gg$
- $qg \to qg$
- $qq \to qq$
- $gg \to q\bar{q}$
- $q\bar{q} \to gg$

Only to tree level with $\alpha_s = 0.3$

Sanity Check :

Cut-off is employed to avoid infrared divergence => $p_{cut\text{-}off}$ = 1.8 GeV (from Au + Au @ 200 GeV PYTHIA)

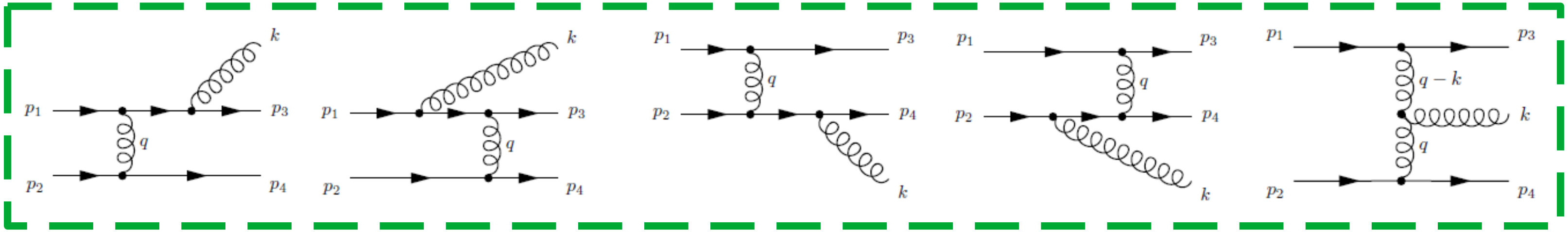
$$\sigma_{pp} = \int dx_1 dx_2 dt \sum_{all\ channels} x_1 f_1(x_1) x_2 f_2(x_2) \frac{d\sigma}{dt} = 46.3622\, mb$$

(COMPETE[3] prediction = 51.79 mb and STAR[4] experiment = 54.67 mb)

[3] J.R. Cudell, et al., COMPETE Collaboration, Phys. Rev. Lett. 89 (2002) 201801.
[4] STAR Collaboration Physics Letters B 808 (2020) 135663

2-to-3 Cross Section

- the 2-to-3 cross section is calculated using improved GB approximation to speed up calculation. This approximation includes 5 channels for gluon emission/absorption process.



Points :

- Calculate in CoM frame, in light-cone coordinate, with assumptions

$$k_\perp \ll \sqrt{s}, \quad q_\perp \ll \sqrt{s}, \quad x q_\perp \ll k_\perp \, with \, x = \frac{k_\perp}{\sqrt{s}} e^y$$

- Resulting amplitude can be written as

$$|M_{2\to3}|^2 = 12 g^2 |M_{2\to2}|^2 (1-x)^2 \left[\frac{k_\perp}{k_\perp^2 + x^2 M^2} + \frac{q_\perp - k_\perp}{(q_\perp - k_\perp)^2 + x^2 M^2} \right]$$

I.e 2 → 3 amplitude = 2 → 2 amplitude x gluon splitting function

Channels:
- gg <-> gg
- gq <-> gq
- qq <-> qq
- gg <-> qq_{bar}
- gg <-> ggg

extra channels:
- qg <-> qgg
- qq <-> qqg

Infrared Divergence Regularization

- Infrared divergence arises from the cross-section calculation due to the interaction amplitude

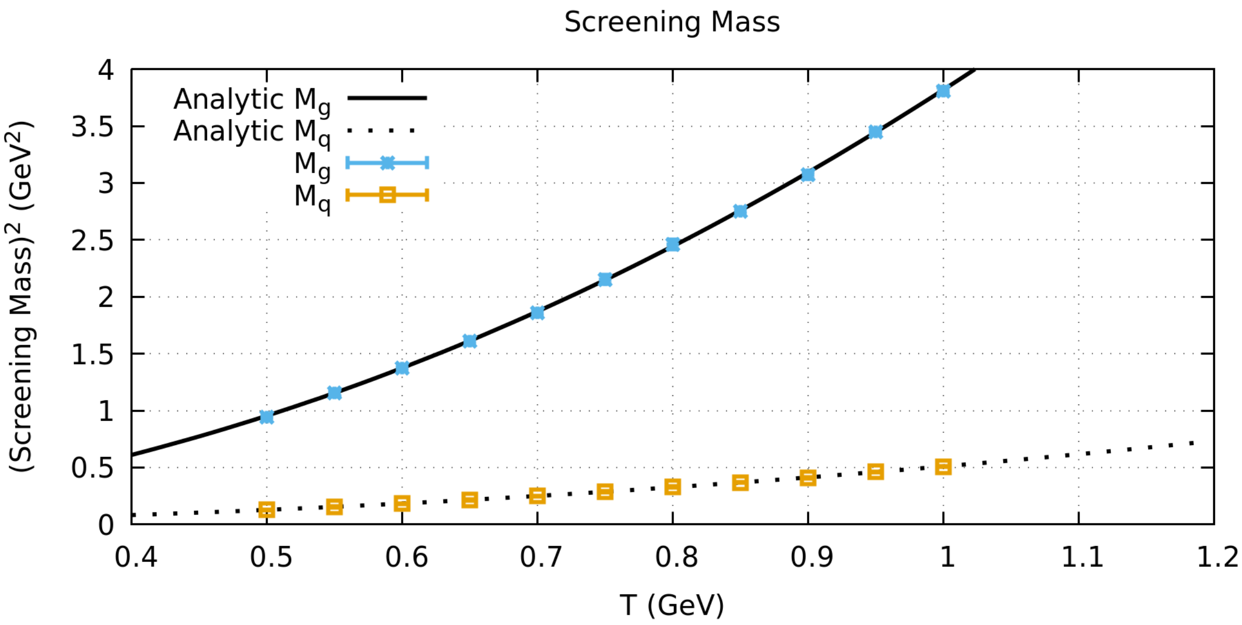
$$|M_{2\to2}|^2 \propto \frac{1}{t^2+m_D^2} \qquad |M_{2\to3}|^2 = 12g^2|M_{2\to2}|^2(1-x)^2\left[\frac{k_\perp}{k_\perp^2+x^2M^2}+\frac{q_\perp-k_\perp}{(q_\perp-k_\perp)^2+x^2M^2}\right]$$

- Apply dynamic screening mass*

$$m_D^2 = 16\pi\alpha_s\frac{\int d^3p}{(2\pi)^3}\frac{1}{|p|}(Nf_g+n_f f_q)$$

$$m_q^2 = 4\pi\alpha_s\frac{N^2-1}{2N}\frac{\int d^3p}{(2\pi)^3}\frac{1}{|p|}(f_g+f_q)$$

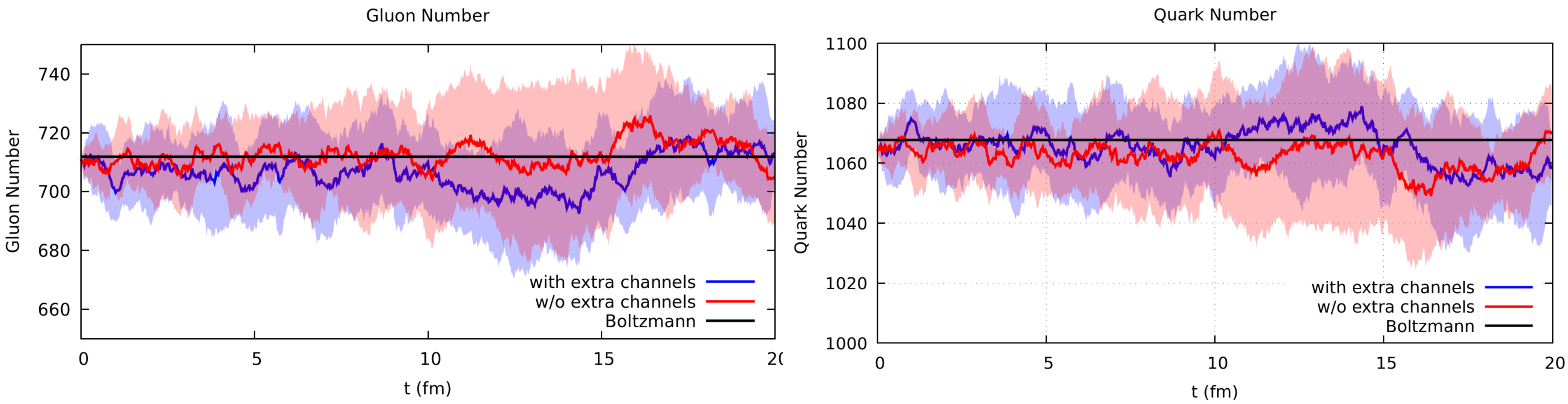
For equilibrium state,
use Boltzmann distribution function



*S. M. H. Wong, Phys. Rev. C 54, 2588 (1996)

Equilibrium Stability Test

Initializing a $(3\ \mathrm{fm})^3$ box in an equilibrium state with temperature 450 MeV, thus E_{tot} = 1751 GeV



∴ Chemical equilibrium is stable.

non-equilibrium system converge to equilibrium state or not?

Box Simulation : Chemical Equilibration

- Given that energy is preserved and using Boltzmann distribution function

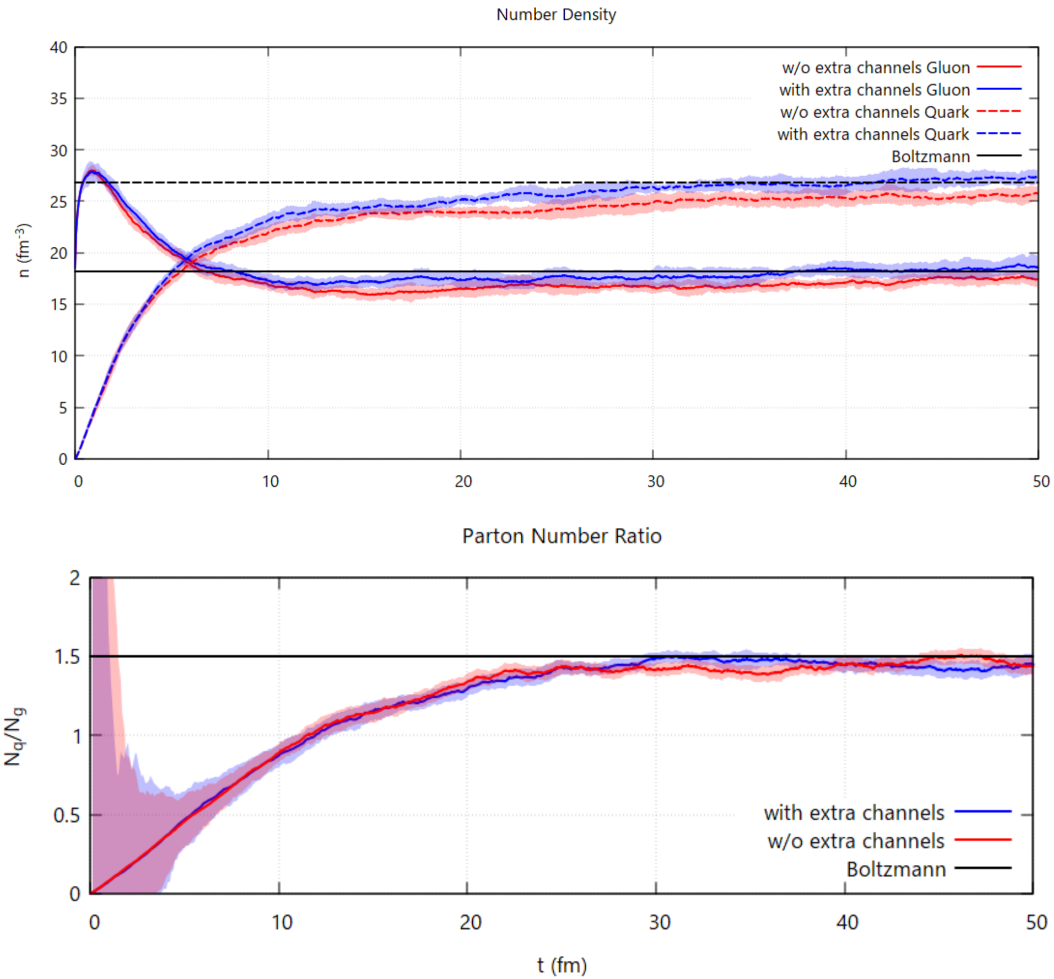
$$N_{q/g} = \upsilon_{q/g} \frac{V}{\pi^2} \left(\frac{E_{init}}{V} \frac{\pi^2}{3(\upsilon_g + \upsilon_q)} \right)^{\frac{3}{4}}$$

- w/o extra channels (gg <-> ggg only)
  => Chemical equilibration is OK

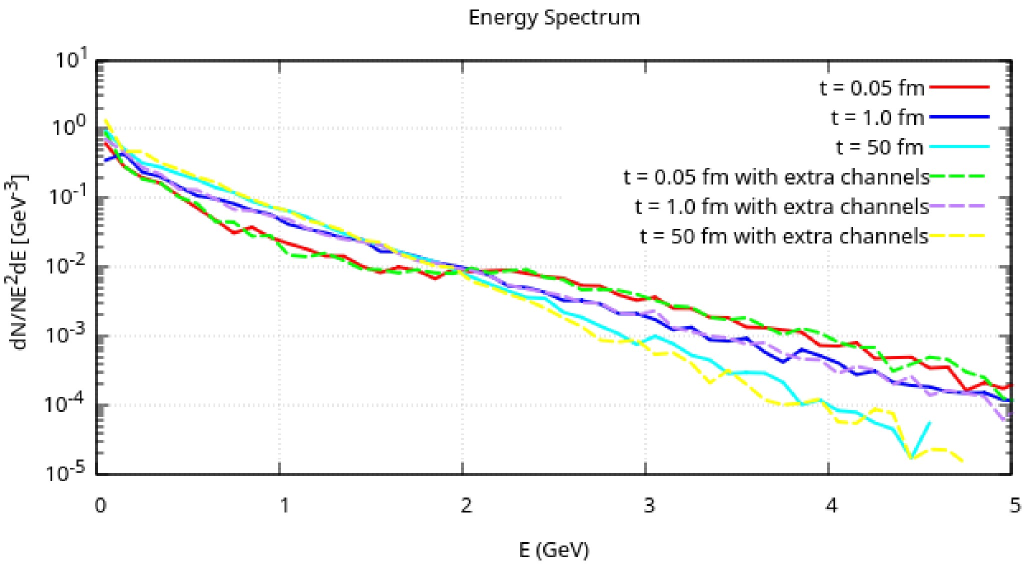
- With extra channels (qg <-> qgg and qq <-> qqg)
  => Faster equilibration time

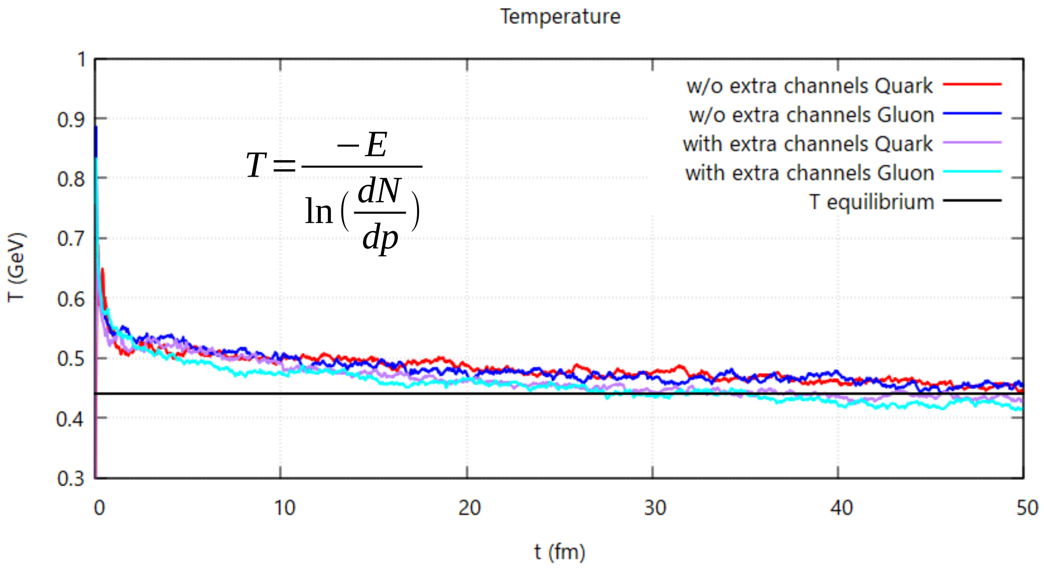
- $N_q \backslash N_g$ saturate toward Boltzmann distribution.

Extra channels promote faster equilibration



Box Simulation : Thermalization

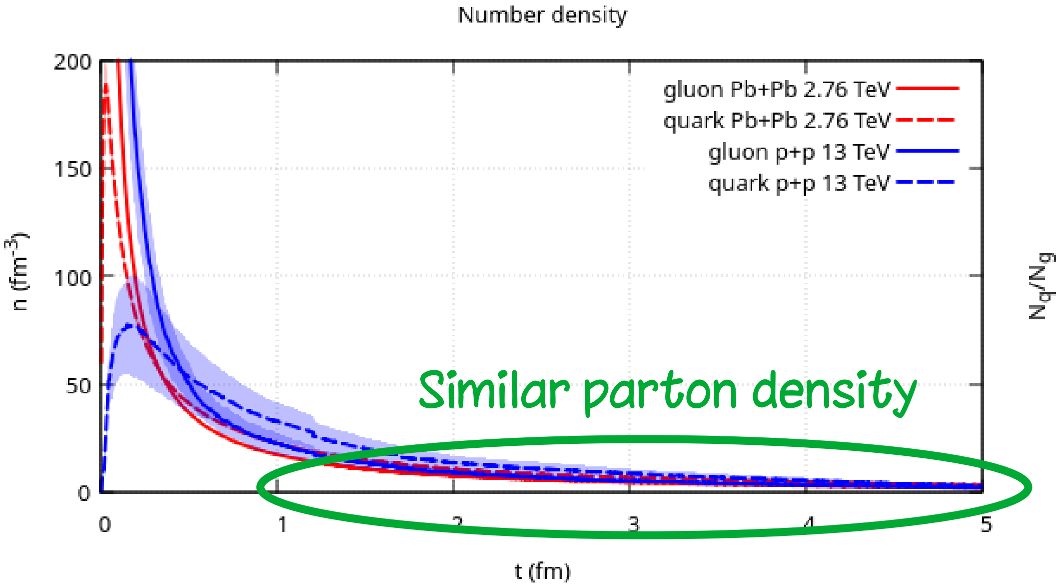


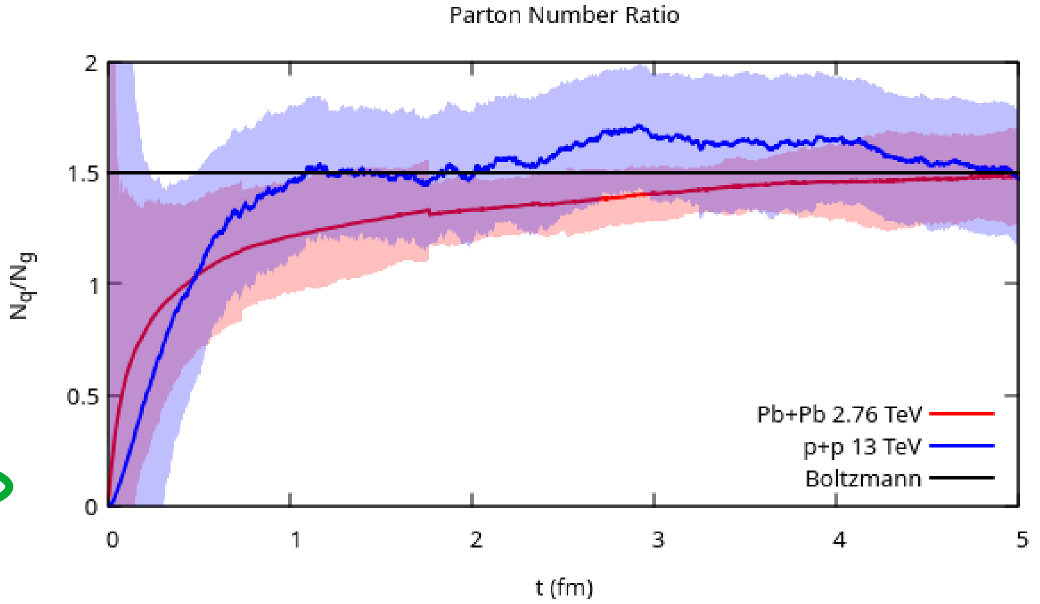


$$T = \frac{-E}{\ln\left(\frac{dN}{dp}\right)}$$

- Energy spectrum is already exponential as soon as 1 fm.
- With extra channels case reaches equilibrium temperature faster.

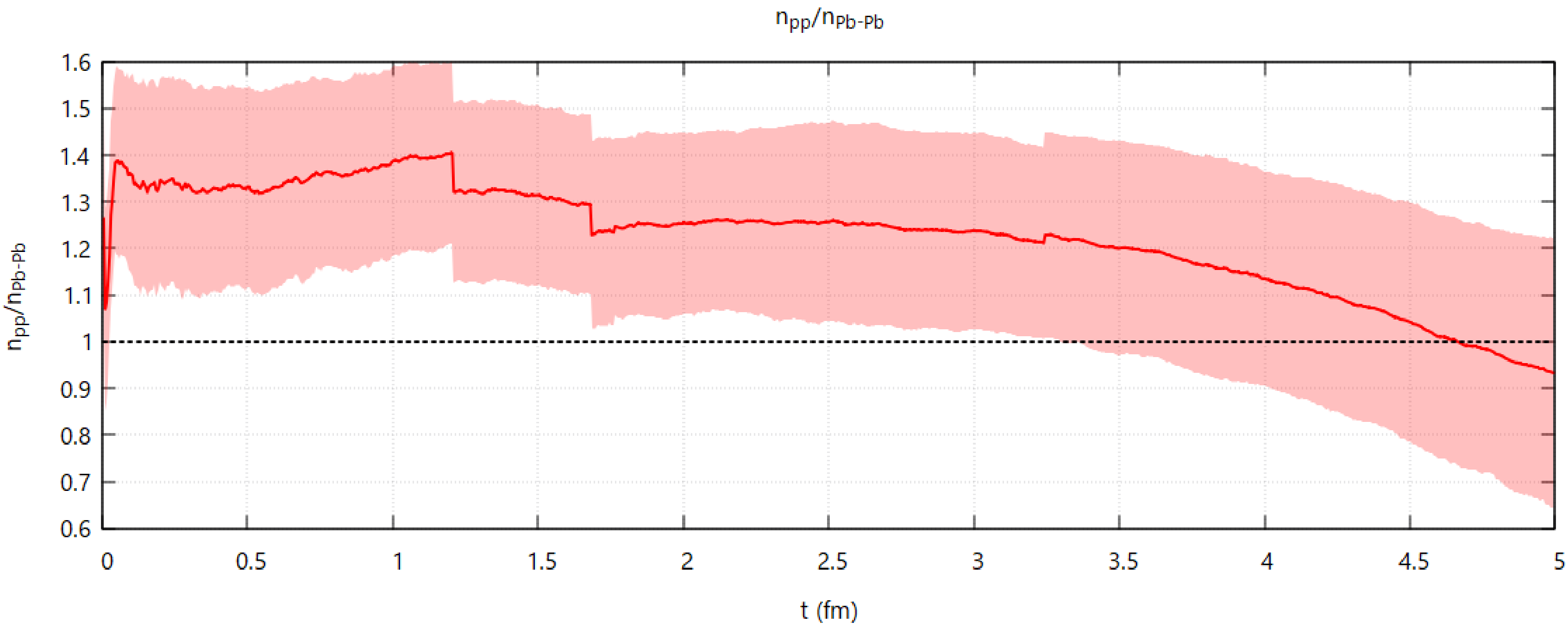
Large vs Small System: Chemical Equilibration





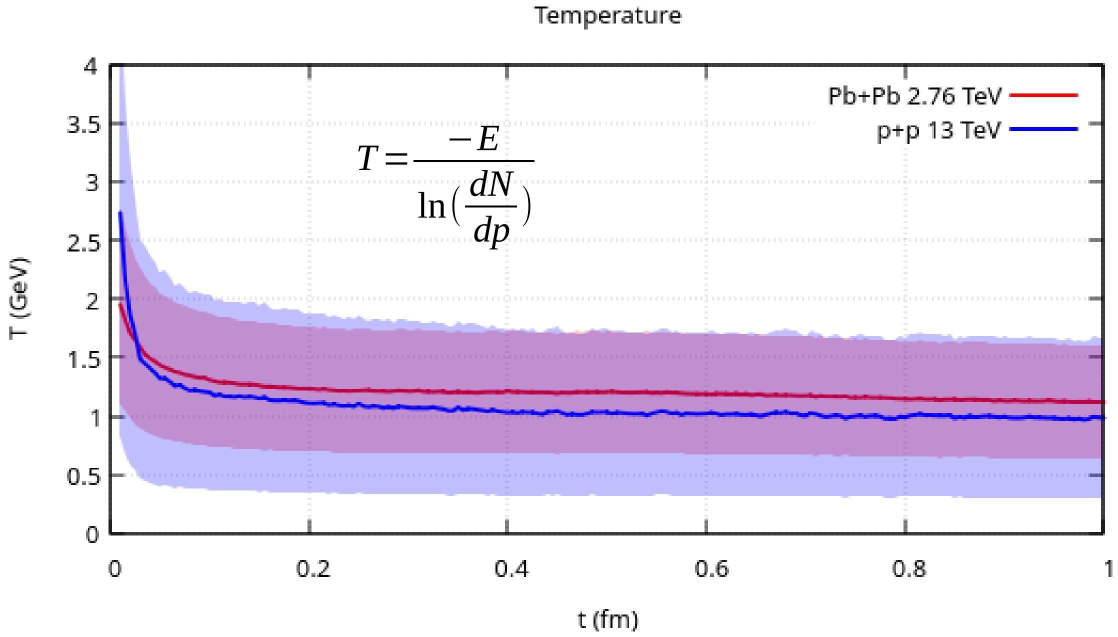
- Only count particles in mid-rapidity region $\eta < |0.5|$
- Number density decreases with time because of expanding medium
- Parton ratio for pp collision reached saturation value faster than Pb-Pb collision. Highly dense and energetic medium leads to high collision rate and faster equilibration

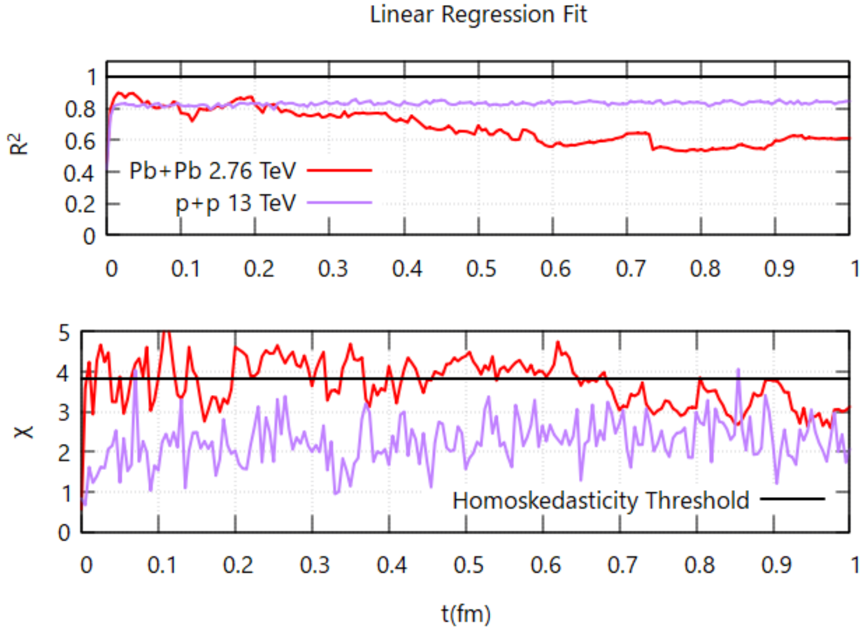
Large vs Small System: Multiplicity Comparison



Note that we have similar parton density despite of 2 different collision system sizes

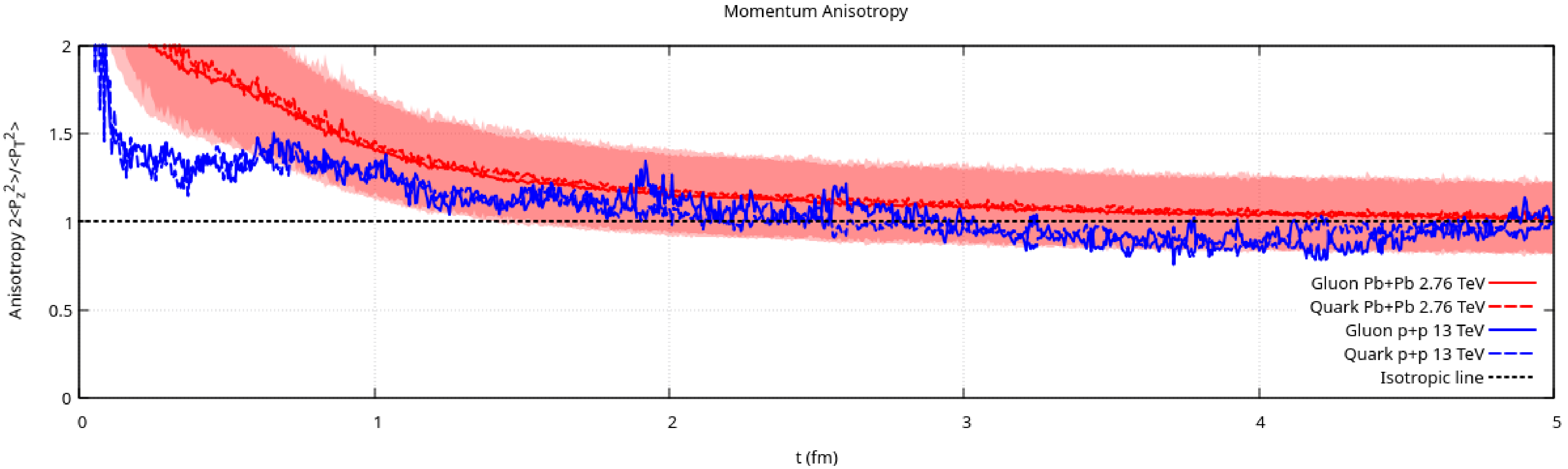
Large vs Small System: Thermalization





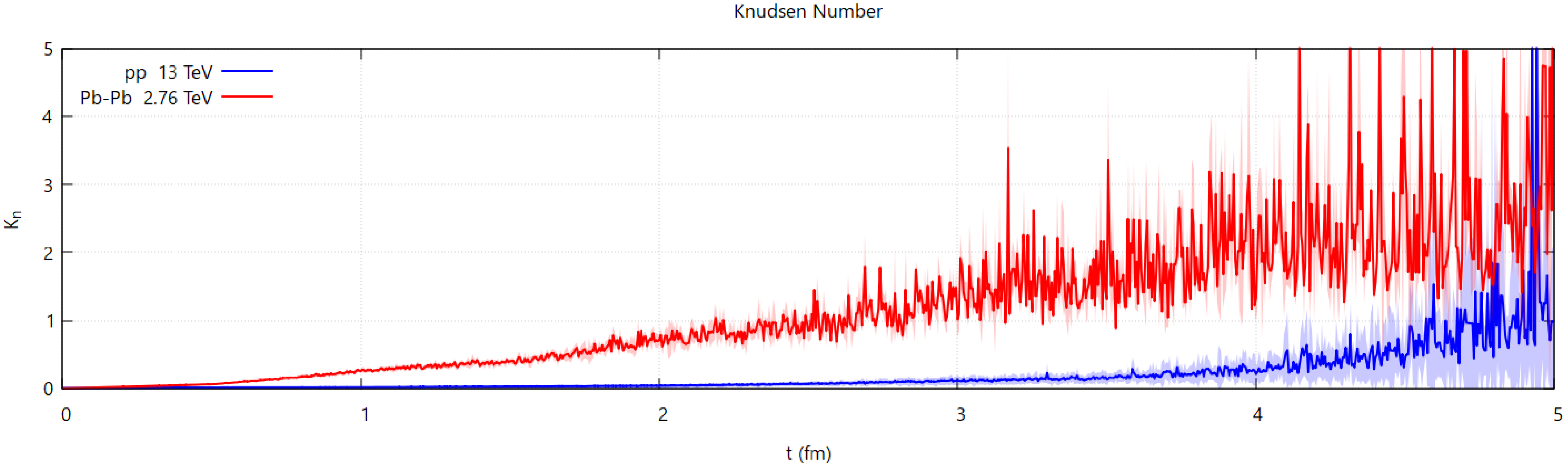
- Apply linear regression fit for η < |0.5| and p < 3 GeV
- Saturation value is reached in a very short time scale
- High temperature value since we focus on the mid-rapidity region

Large vs Small System: Anisotropy



- Both systems established isotropicity around the same time scale ~2 fm

Large vs Small System: Knudsen Number



- pp assumes ellipsoid medium, Pb-Pb assumes cylindrical medium.
- Knudsen number for pp is consistently lower than Pb-Pb.
- Note that this result is particular to the method used to measure characteristic length

$$K_n = \frac{\lambda_{mean\,free\,path}}{L_{characteristic\,length}}$$

Conclusion

- We have constructed a partonic transport model using stochastic collision model with 2-to-2 and 2-to-3 interactions up until leading order.
- Beginning from a far-from-equilibrium state, both systems properly converges toward Boltzmann distribution.

- pp @ 13 TeV seems to reach chemical equilibrium faster at 1 fm compared to Pb-Pb @ 2.76 GeV at 5 fm
- Both systems thermalized around the same time at ~0.2 fm
- Both system isotropized around the same time at 2 fm

Thank you

Gluon absorption and radiation

- Elastic channels preserve the total number of partons and entropy
- From theoretical calculation* and simulation** that without absorption/radiation channels, medium will form condensate instead of approaching to Boltzmann distribution function.

  => Inelastic collisions as the main driving mechanism in kinetic and chemical equilibration

- Common problem of parton cascade model => slow equilibration time.

- In the model, we included these channels :

  - gg <-> gg
  - gq <-> gq
  - qq <-> qq
  - gg <-> qq_{bar}

  - gg <-> ggg

And new extra channels:

- qg <-> qgg
- qq <-> qqg

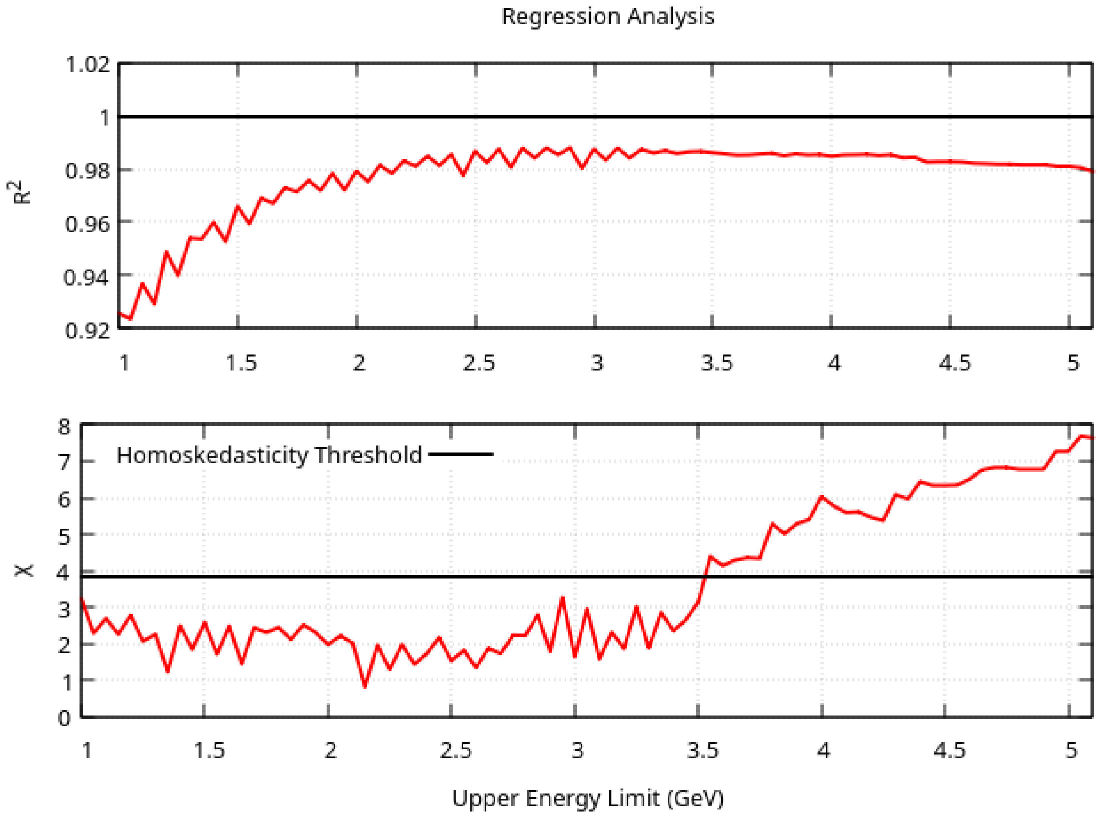
→ First calculation with these two extra channels.

* Jean-Paul Blaizot, Francois Gelis, Jinfeng Liao, Larry McLerran, Raju Venugopalan. arXiv:1107.5296 [hep-ph]
** Zhe Xu, Kai Zhou, Pengfei Zhuang, Carsten Greiner. arXiv:1410.5616 [hep-ph]

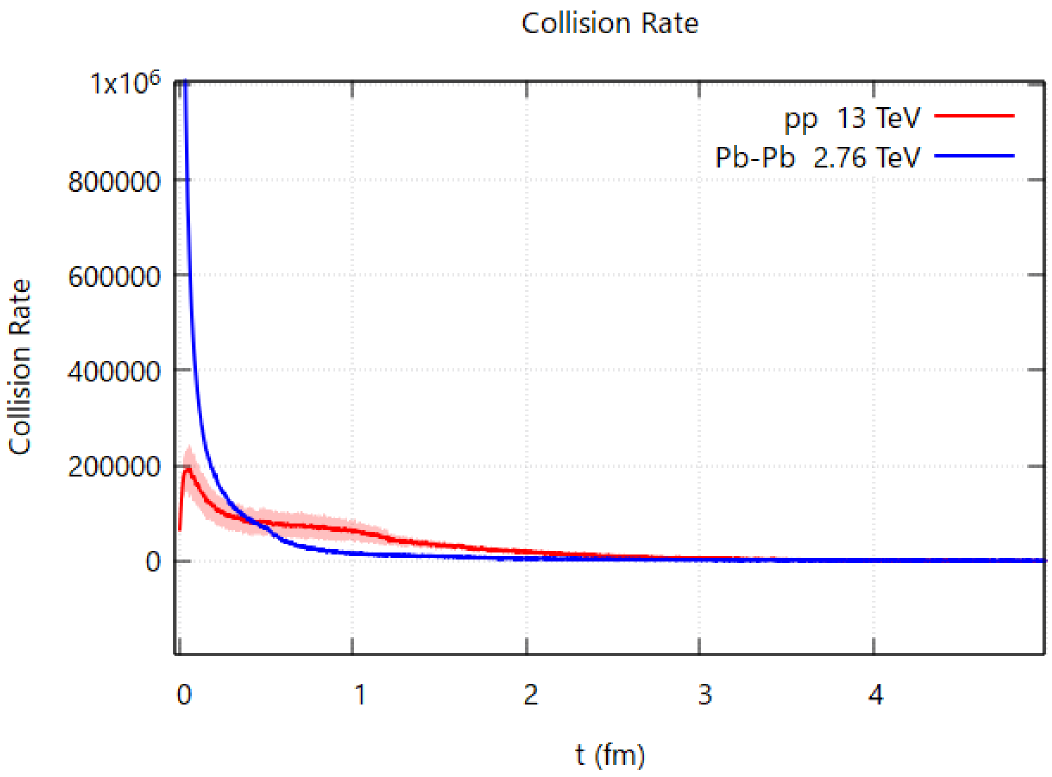
Limit of thermalization in energy spectrum

- Box equilibration at $t = 50$ fm

- Regression analysis for using linear regression in energy spectrum

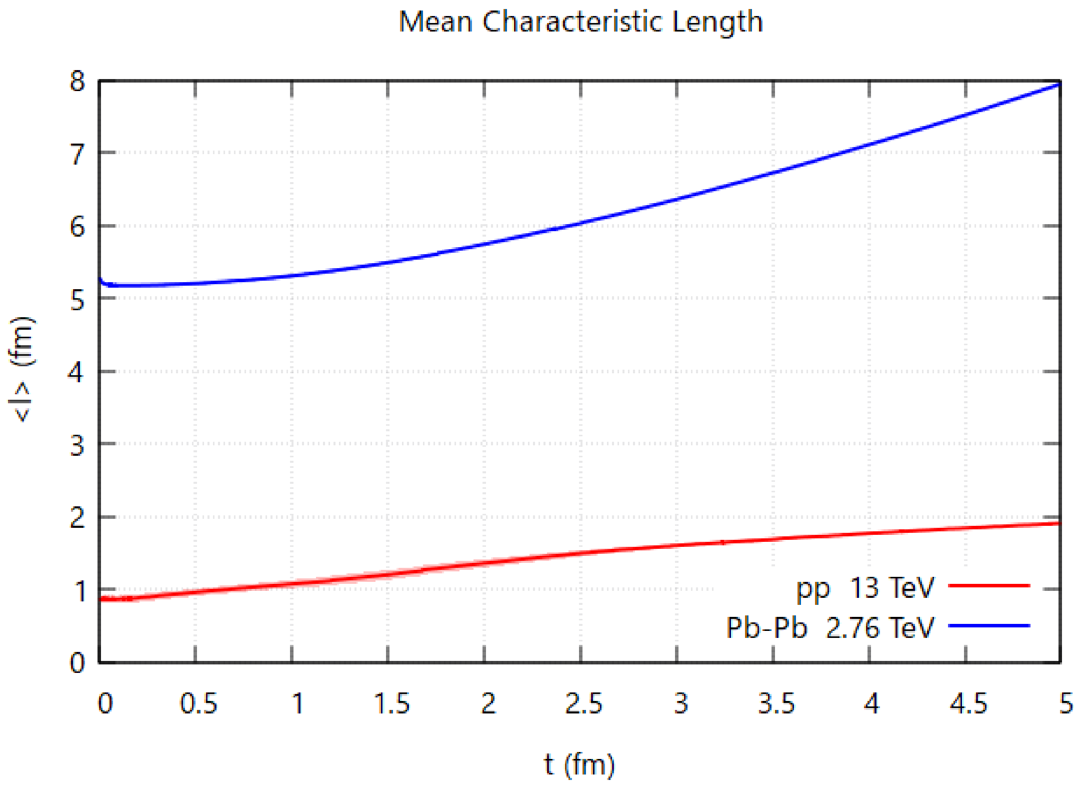


Knudsen Calculation Details



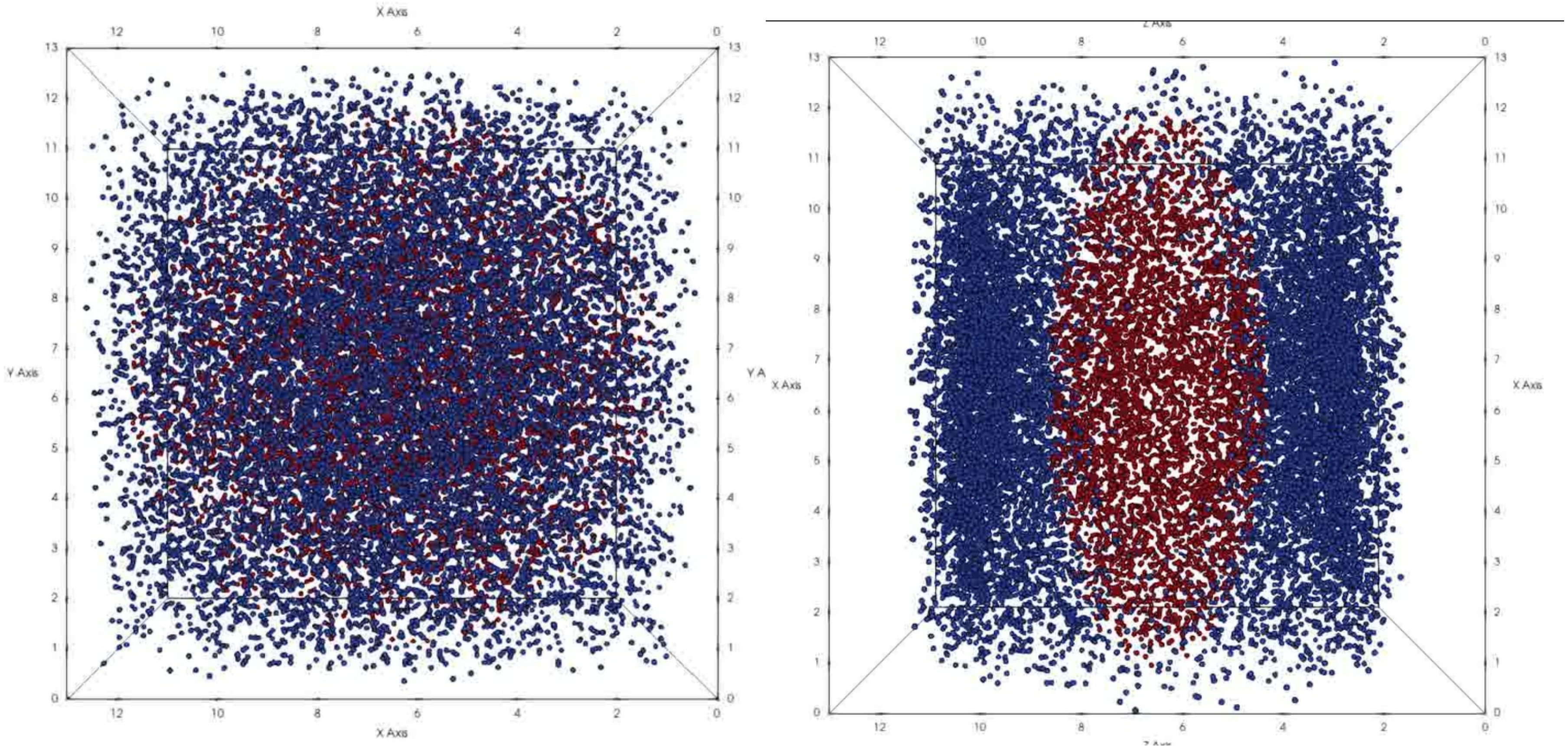
- Total collision rate throughout the time evolution
- Note that this is total collision rate and not collision rate per particle

Knudsen Calculation Details



- pp collision saturates to a value
- Pb-Pb increasing faster at later time
  => Possibility of saturation or even lowering of knudsen number at later time

Pb Pb late time spatial distribution



pp late time spatial distribution