The dipole picture in real kinematics

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Abstract

- By taking the large Q^2-limit in real kinematics we match the dipole picture result to the collinear factorization result for the proton F_2. This allows us to compute x-dependent quark and gluon distributions
- The phenomenological impact of the real kinematics on the cross section is obtained numerically.

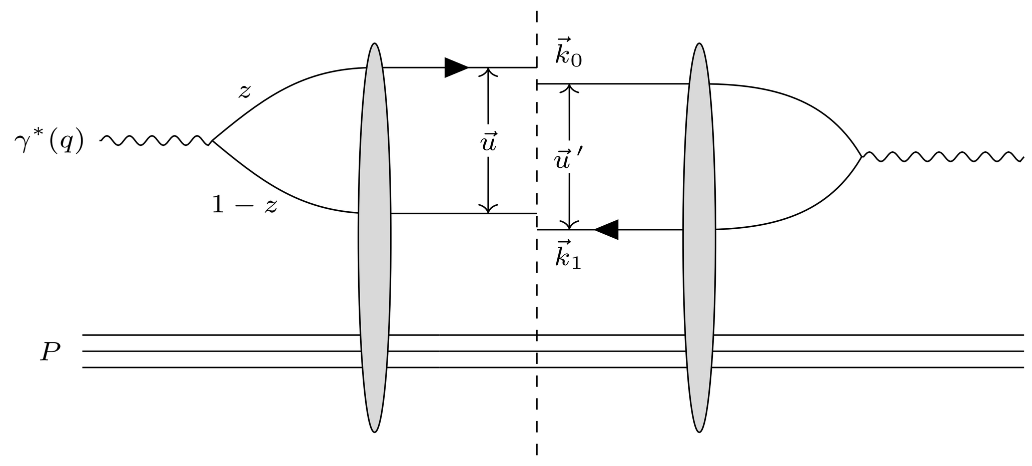
1. Background and introduction

Collinear factorization is commonly used to study partonic structures in DIS at large Q^2. The dipole formalism is applied where x_B is small and gluons saturate with a semi-hard transverse momentum Q_s. In the region $\Lambda^2_{QCD} \ll Q_s^2 \ll Q^2$, both frameworks should match however this is still lacking in the literature. This requires imposing the real kinematic constraint.

2. Dual picture of DIS

1. **Matching** dipole formalism to collinear factorization

   Dipole formalism in the target rest frame



Transverse position space coordinates

$$\vec{u} = \vec{x}_0 - \vec{x}_1, \quad \vec{v} = z\vec{x}_0 + (1-z)\vec{x}_1$$
$$\vec{u}' = \vec{x}'_0 - \vec{x}'_1, \quad \vec{v}' = z\vec{x}'_0 + (1-z)\vec{x}'_1.$$

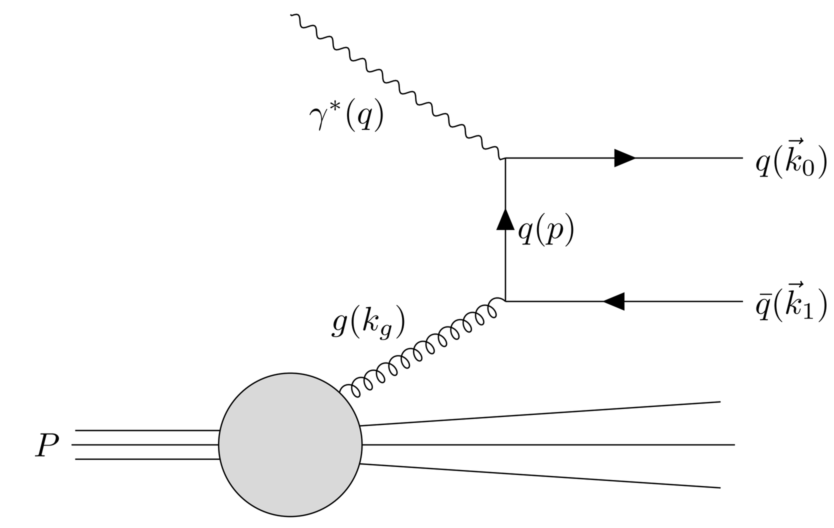
Momentum space coordinates:

$$\vec{k}_g = \vec{k}_0 + \vec{k}_1$$
$$\vec{p} = (1-z)\vec{k}_0 - z\vec{k}_1.$$

Invariant mass

$$M^2_{q\bar{q}} = \frac{m_f^2 + \vec{p}^2}{z(1-z)}.$$

Collinear factorization in infinite momentum frame



Gluon momentum is approximately collinear to proton

$$k_g^\mu \approx (x_g P^+, 0, 0_\perp)$$

the invariant mass is ($\xi = p^+/k_g^+$)

$$M^2_{q\bar{q}} = \frac{Q^2(1-\xi)}{\xi}.$$

Equating the two allows us to switch variables from the dipole picture to collinear factorization framework.

2. **Phenomenological impact** of restricted phase space

   The invariant mass is used to constrain the phase space. That is we want $(k_0+k_1)^2 = M^2_{q\bar{q}} \leq W^2$. We implement this through

$$1 = \int_{4m_f^2}^{W^2} dM^2_{q\bar{q}}\, \delta\left(M^2_{q\bar{q}} - \frac{p^2+m_f^2}{z(1-z)}\right).$$

3. DIS cross section at small x_B

The leading order dipole picture cross section is given by

$$\sigma^{\gamma^* P}_{\rm LO} \sim \int \frac{d^2k_g}{(2\pi)^2}\frac{d^2p}{(2\pi)^2}\frac{dz}{z(1-z)} d^2u\, d^2u'\, e^{i\vec{p}\cdot(\vec{u}'-\vec{u})} \times \tilde{\Psi}^{\gamma^*\to q\bar{q}}(z,\vec{u}) \left(\tilde{\Psi}^{\gamma^*\to q\bar{q}}(z,\vec{u}')\right)^\dagger \mathcal{N}$$

with the target amplitude

$$\mathcal{N} = \int d^2v\, d^2v'\, e^{i\vec{k}_g\cdot(\vec{v}'-\vec{v})} \left\{S^{(4)} - S^{(2)} - S^{(2)\dagger} + 1\right\}$$

Quadrupole

$$S^{(4)}(\vec{x}_0, \vec{x}_1; \vec{x}'_1, \vec{x}'_0) = \frac{1}{N_c}\left\langle {\rm Tr}\left[U(\vec{x}_0)U^\dagger(\vec{x}_0)U(\vec{x}'_1)U^\dagger(\vec{x}'_0)\right]\right\rangle.$$

and dipole

$$S^{(2)}(\vec{x}_0, \vec{x}_1,) = \frac{1}{N_c}\left\langle {\rm Tr}\left[U(\vec{x}_0)U^\dagger(\vec{x}_1)\right]\right\rangle,$$
$$S^{(2)\dagger}(\vec{x}'_1, \vec{x}'_0) = \frac{1}{N_c}\left\langle {\rm Tr}\left[U(\vec{x}'_1)U^\dagger(\vec{x}'_0)\right]\right\rangle.$$

4. Result 1: Matching

In the large Q^2-limit, the target amplitude $\mathcal{N}$, can be reduced to

$$\mathcal{N} \sim -uu' \int d^2v\, d^2v'\, e^{i\vec{k}_g\cdot(\vec{v}'-\vec{v})} \times \left\langle {\rm Tr}\left[(\partial U(\vec{v}))U^\dagger(\vec{v})(\partial U(\vec{v}'))U^\dagger(\vec{v}')\right]\right\rangle.$$

which relates to the Weizsäcker-Williams (WW) gluon distribution $g(x, \vec{k}_g)$ through

$$\mathcal{N} \sim \tfrac{x\alpha_S}{2} g(x, \vec{k}_g).$$

Here one can integrate out the u, u'-dependence to get for the transverse cross section

$$\sigma^{\gamma^* P}_{T,\rm LO} \sim \int d^2k_g\, g(x,\vec{k}_g) \int d^2p\, dz \left(\frac{2m_f^2 p^2}{(p^2+\varepsilon_f^2)^4} + (z^2 + (1-z)^2)\frac{p^4+\varepsilon_f^4}{(p^2+\varepsilon_f^2)^4}\right)$$

and the longitudinal cross section

$$\sigma^{\gamma^* P}_{L,\rm LO} \sim \int d^2k_g g(x,\vec{k}_g) \int d^2p\, dz\, Q^2 z^2(1-z)^2 \frac{p^2}{(p^2+\varepsilon_f^2)^4}$$

Integrate over p and use invariant mass to connect to collinear factorization ($z \to \xi$)

$$F_T \sim \int d^2k_g\, g(x,\vec{k}_g) \int_{x_B}^1 d\xi \left[P_{qg}(\xi)\left(\ln\left(\frac{Q^2}{m_f^2}\right) + \ln\left(\frac{1-\xi}{\xi}\right)\right) - (1-2\xi)^2\right],$$
$$F_L \sim \int d^2k_g\, g(x,\vec{k}_g) \int_{x_B}^1 d\xi\, \xi(1-\xi).$$

Successfully recovered not only collinear logarithms but finite terms as well.

In $\overline{\rm MS}$-scheme, the collinear quark and gluon PDFs are related WW-distribution by

$$q(x,Q^2) = \frac{\alpha_S}{2\pi}\int_{x_B}^1 d\xi\, T_F\, P_{qg}(\xi) \ln\left(\frac{Q^2}{m_f^2}\right) \int^{Q^2} d^2k_g\, g(x,\vec{k}_g)$$
$$g(x,Q^2) = \int^{Q^2} d^2k_g\, g(x,\vec{k}_g)$$

5. Quadrupole model

The kinematic constraint requires the use of quadrupole. In the gaussian approximation,

$$S^{(4)} = e^{[f(\vec{u})+f(\vec{u}')]} e^{\mu^2\left(\frac{-N_c}{4}F_1 + \frac{1}{2N_c}F_2\right)} \times \left[\left(\frac{\sqrt{\Delta}+F_1}{2\sqrt{\Delta}} - \frac{F_2}{\sqrt{\Delta}}\right) e^{\frac{N_c}{4}\mu^2\sqrt{\Delta}} + \left(\frac{\sqrt{\Delta}-F_1}{2\sqrt{\Delta}} + \frac{F_2}{\sqrt{\Delta}}\right) e^{\frac{-N_c}{4}\mu^2\sqrt{\Delta}}\right].$$

Here

$$F(\vec{x},\vec{y},\vec{u},\vec{v}) = \frac{1}{\mu^2 C_F}\left(f(|\vec{x}-\vec{u}|) + f(|\vec{y}-\vec{v}|) - f(|\vec{x}-\vec{v}|) - f(|\vec{y}-\vec{u}|)\right)$$
$$\Delta(\vec{x},\vec{y},\vec{u},\vec{v}) = F_1^2(\vec{x},\vec{v},\vec{y},\vec{u}) + \frac{4}{N_c^2}F_2(\vec{x},\vec{y},\vec{u},\vec{v})F_3(\vec{x},\vec{u},\vec{v},\vec{y})$$

and

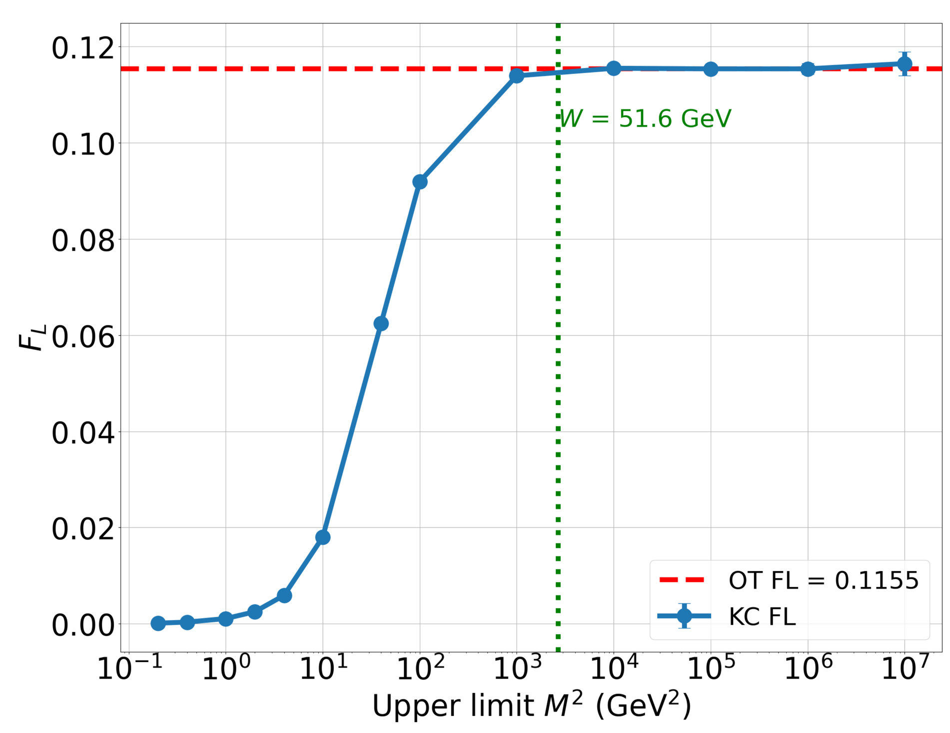
$$f(|\vec{x}-\vec{y}|) = -\frac{(|\vec{x}-\vec{y}|^2 Q_{s0}^2)^\gamma}{4}\log\left(\frac{1}{|\vec{x}-\vec{y}|\Lambda_{QCD}} + e\cdot e_c\right)$$

In the large N_c-limit the quadrupole reduces to

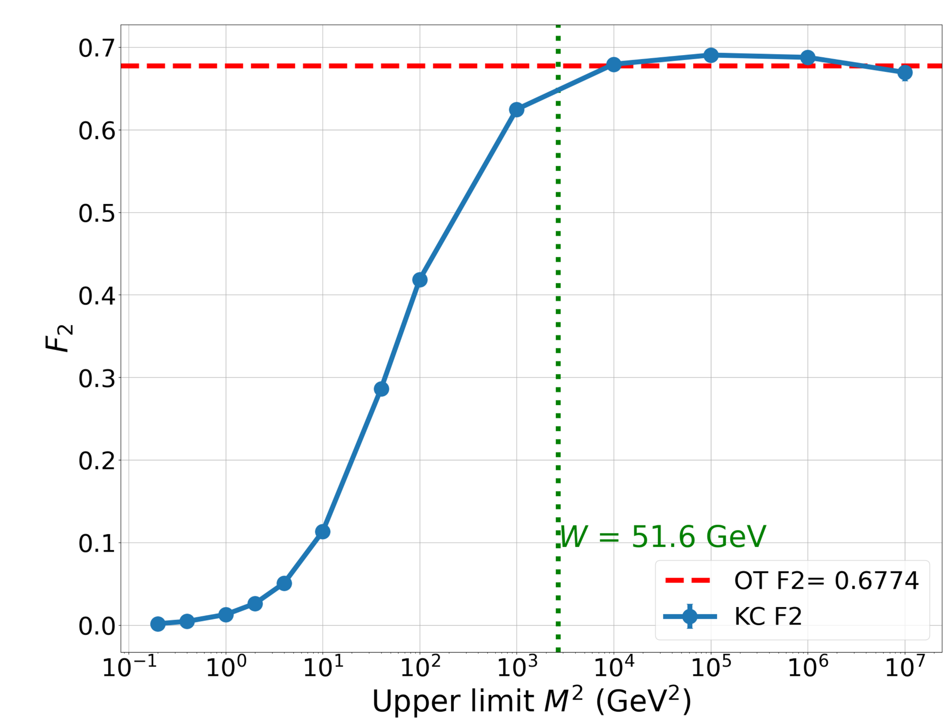
$$S^{(4)} \approx e^{[f(\vec{u})+f(\vec{u}')]} - \frac{F_2}{F_1}\left(e^{[f(\vec{u})+f(\vec{u}')]} - e^{[f((1-z)|\vec{u}-\vec{u}'|)+f(-z|\vec{u}-\vec{u}'|)]}\right)$$

6. Result 2: Phenomenological impact of $M^2_{q\bar{q}} \leq W^2$

Integrating the cross section in the large N_c limit for a MV-model dipole yields for F_L



and for F_2



At $Q^2 = 27{\rm GeV}^2$ and $x_B = 0.01$ we get a 0.2 % effect for F_L and a 3 % effect for F_2 from the kinematic constraint.

7. Conclusions & further work

- We are able to match the dipole picture to the collinear framework through the invariant mass of the final state quark-antiquark pair
- The phenomenological impact of the kinematical constraint shows that we get a small reduction of the cross section with largest effect from F_T

Furthermore,

- we aim to implement the full quadrupole numerically
- we want to compare the kinematically constrained LO calc. to HERA data and use as prediction for EIC
- and derive a kinematically constrained BK evolution equation.

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