Inferring the initial condition for the BK evolution at next-to-leading order accuracy

Article: Phys. Rev. D 112, 034003 (2025) with H. Hänninen and H. Mäntysaari
Code: repo: Camcasuga/bayesian-nlodisfit-dipole: V1.0.0 (zenodo.org)

Speaker: Carlisle Casuga

Initial Stages 2025, Taipei

Centre of Excellence in Quark Matter
Research Council of Finland
JYVÄSKYLÄN YLIOPISTO UNIVERSITY OF JYVÄSKYLÄ
HELSINKI INSTITUTE OF PHYSICS
European Research Council Established by the European Commission

Promote our previous leading order analysis [2311.104691] to a **consistent next-to-leading order treatment**

Constrain model parameters against HERA total and heavy quark cross section data **using Bayesian inference**

Provide BK initial condition uncertainty necessary to propagate uncertainties for NLO CGC calculations

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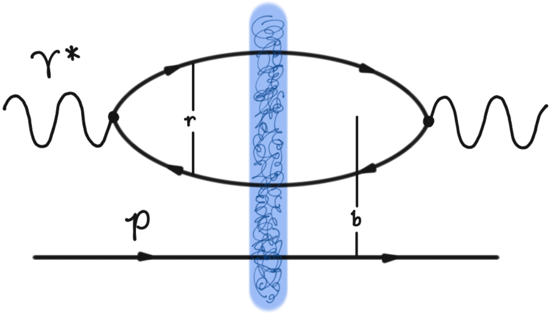
Deep Inelastic Scattering in the Dipole Picture at LO



$$\sigma_{T,L}^{\gamma * p}(x, Q^2) \sim \frac{\sigma_0}{2} \otimes \mathcal{N}(r, x) \otimes \{\mathrm{LCWF}\}$$

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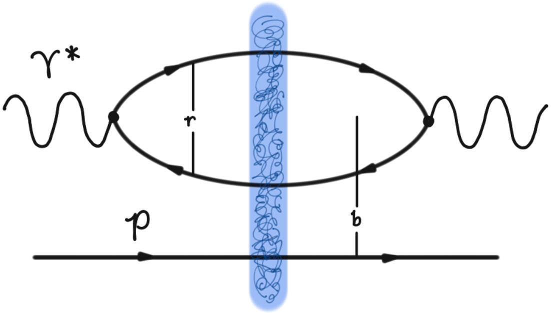

dipole-target
scattering amplitude

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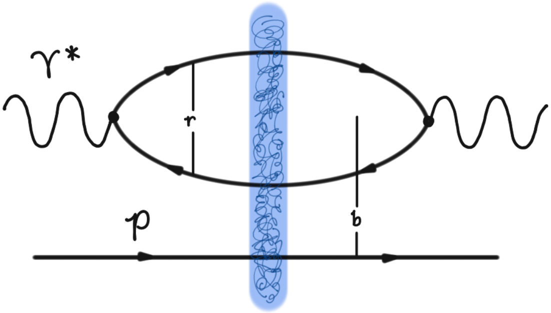

proton transverse area

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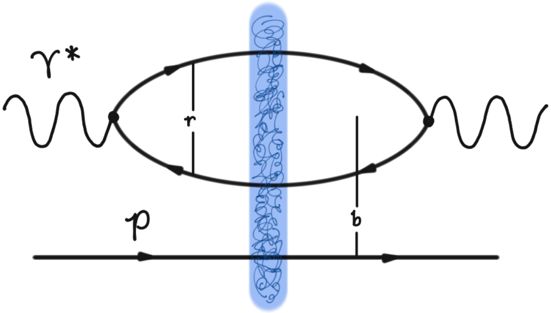

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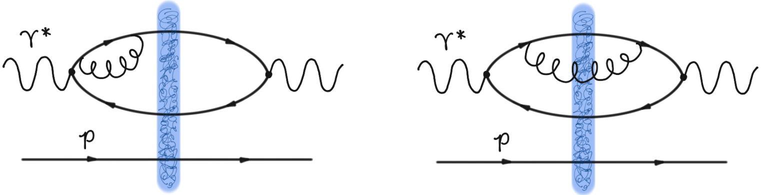
$$\mathcal{N}(r, x = x_0; \text{model parameters}) \overset{\textbf{rcBK}}{\rightarrow} \mathcal{N}(r, x)$$

DIS at next-to-leading order



$$\sigma^{\mathrm{NLO}}_{\mathrm{L,T}} = \sigma^{\mathrm{IC}}_{\mathrm{L,T}} + \sigma^{\mathrm{dip}}_{\mathrm{L,T}} + \sigma^{qg,\mathrm{unsub}}_{\mathrm{L,T}} \longrightarrow \sigma \sim \mathcal{N}_{\mathrm{NLO}} \otimes \begin{bmatrix} \text{NLO} \\ \text{Hard} \\ \text{Factor} \end{bmatrix}$$

1. *Leading order term*, amplitude evaluated at IC (unevolved)
2. *Dipole term*, $q\bar{q}$-target scattering with loop corrections
3. *Gluon term*, real contribution where additional gluon also interacts with target

* setup follows [G.Beuf, H. Hänninen et. al. (2020) 2007.01645] extended to include heavy quarks

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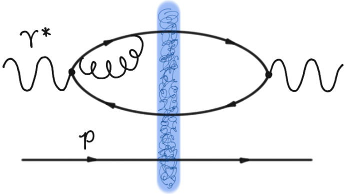

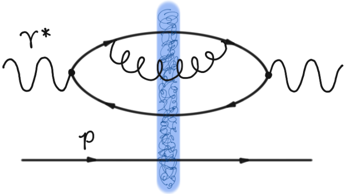

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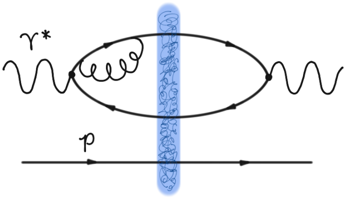

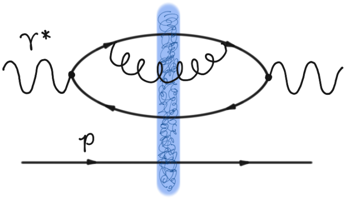

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Dipole Evolution at NLO

- Evolution equation: **KCBK evolution** (kinematically constrained BK) resums the dominant double transverse log terms to approximate the full NLO BK [G. Beuf (2014) arXiv:1401.0313]
- Dipole amplitude evolution rapidity scale: [H. Hänninen et.al (2017) arXiv:1708.07328]

$$Y = \ln \frac{k^+}{P^+} = \ln \frac{W^2 z_2}{Q_0^2}, \quad z_2 \to \text{gluon longitudinal momentum fraction}$$

Initial Condition: MV^γ model

$$\mathcal{N}(\mathbf{r}, x = x_0) = 1 - \exp\left(-\frac{1}{4}(\mathbf{r}^2 Q_{s0}^2)^\gamma \ln(1/\mathbf{r}\Lambda + e)\right)$$

Running coupling in coordinate space: $\alpha_s(\mathbf{r}) \sim [\ln(C^2/\mathbf{r}^2\Lambda^2)]^{-1}$

- scale set by Balitsky + smallest dipole (Bal + SD) or parent dipole scheme

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Fits a gaussian covariance kernel to training data to emulate the parameter dependence of the model

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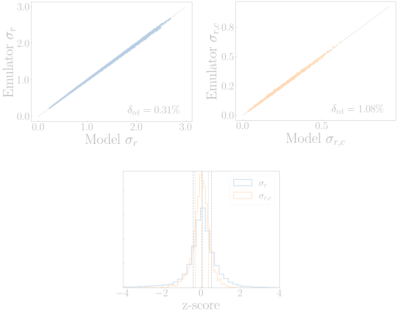
Model comparison plots and z-score histograms:



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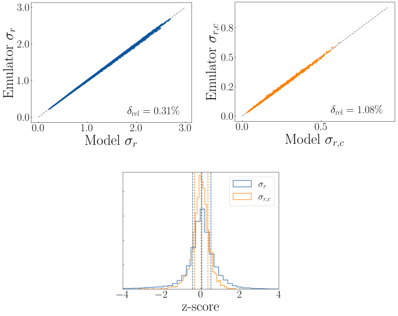

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$$P(\vec{x}, \theta) \equiv \text{posterior}$$
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MCMC draws samples from the posterior. Each proposed sample accepted with a probability of:

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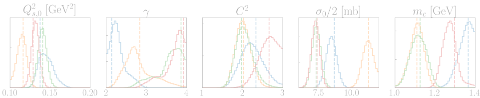

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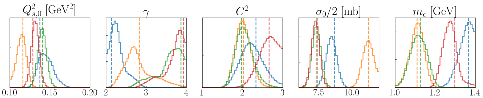
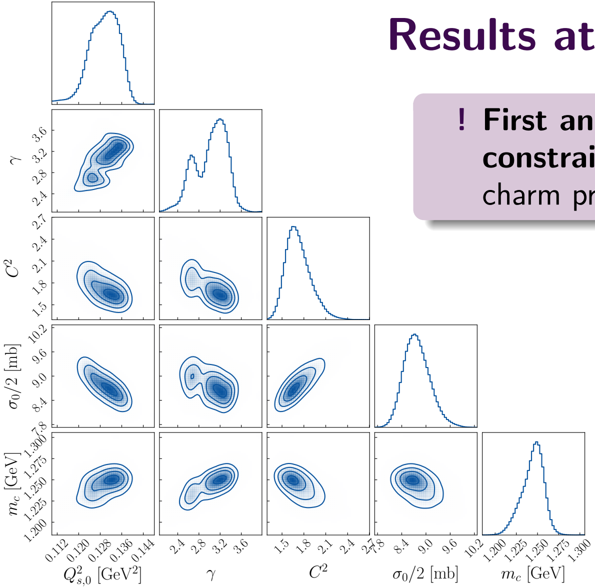


Results at NLO: KCBK, Bal+SD



C.C, H. Hänninen, H. Mäntysaari, 2506.00487

! **First analysis with simultaneous constraints** from HERA total inclusive + charm production cross section data

! All parameters are well-constrained.

- First Bal+SD fit
- A steep dipole ($\gamma > 1$) required by Q^2-dep.

$$\mathcal{N}(\mathbf{r}, x_0) \sim \left(\mathbf{r}^2 Q_s^2\right)^\gamma$$

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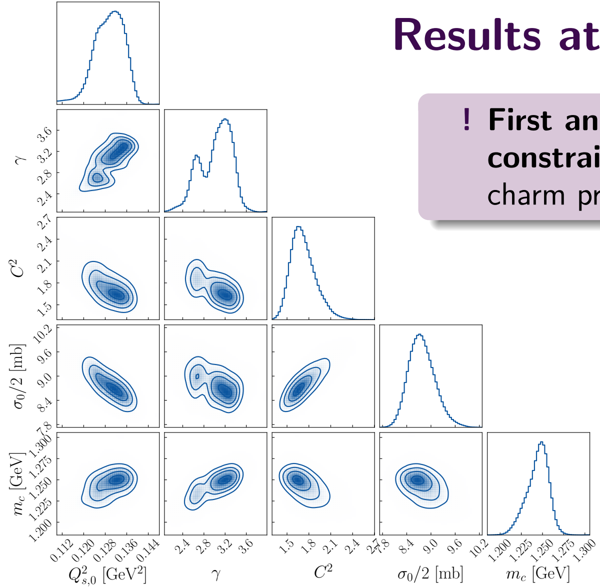

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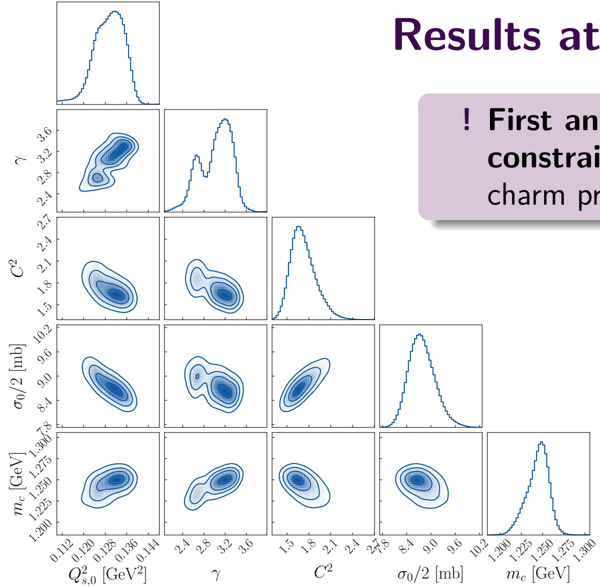

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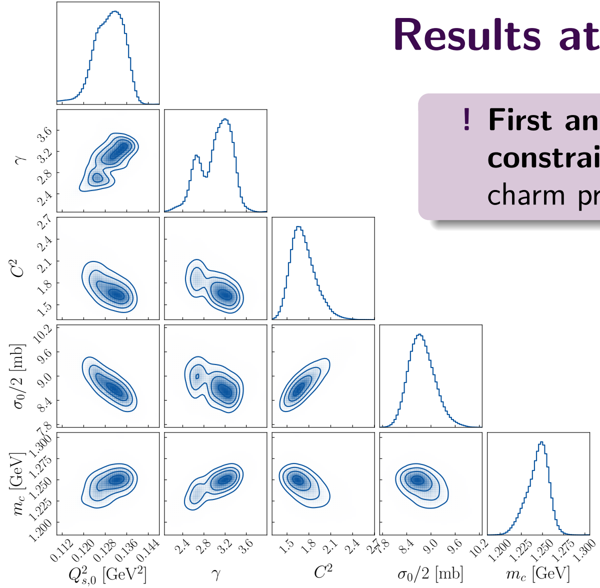

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Quality of Fit: KCBK, Bal+SD

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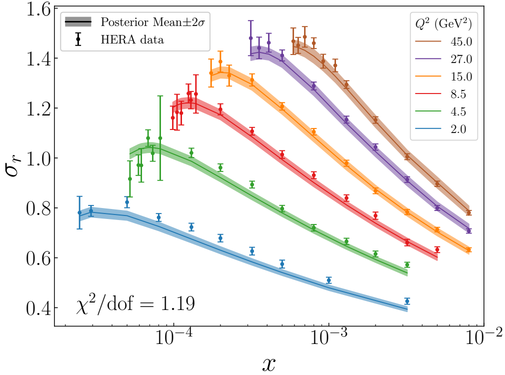

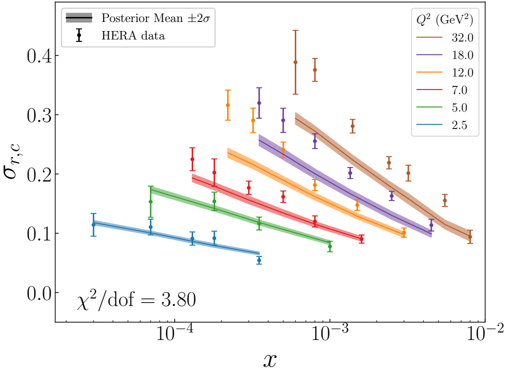

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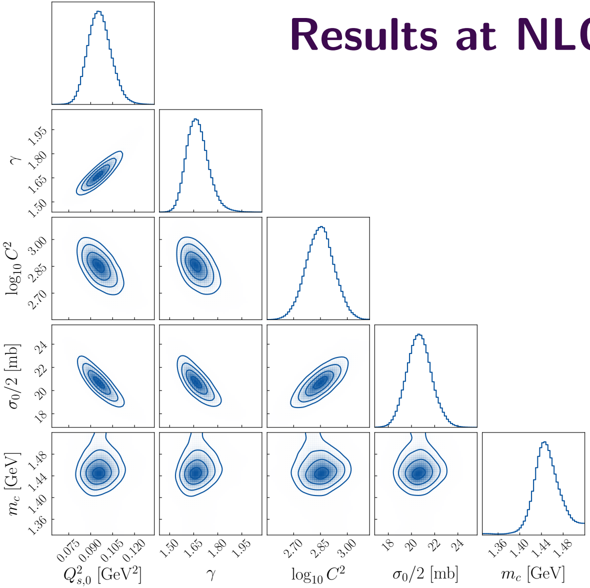

Results at NLO: KCBK, parent dipole

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- Larger proton transverse area, (Bal+SD: $\sigma_0/2 \sim 8.8$ mb); it is sensitive to running coupling scheme

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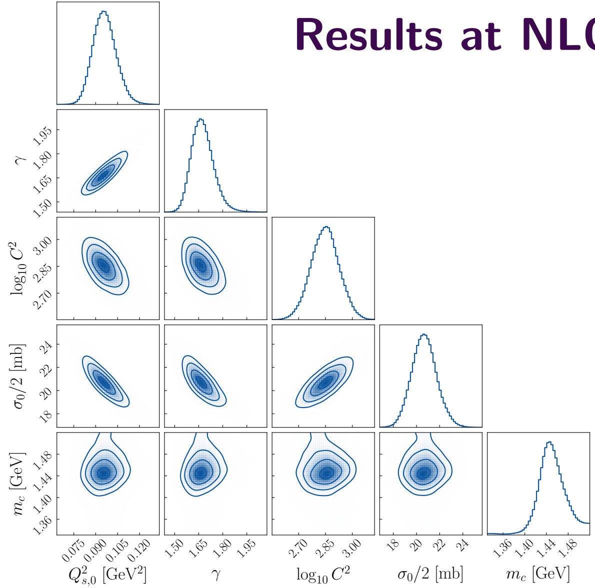

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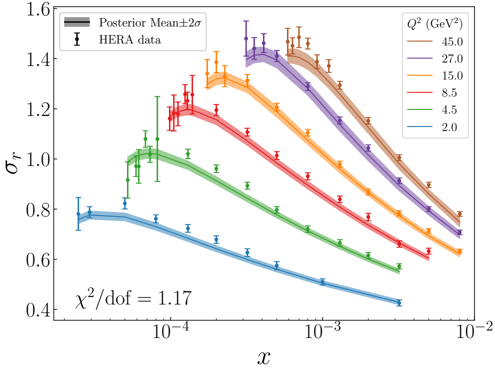

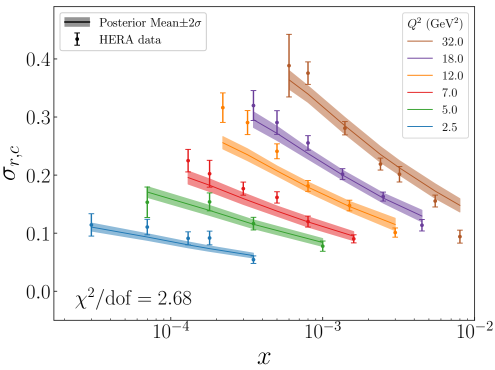

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Analysis w/ full NLO BK

The BK equation at NLO:

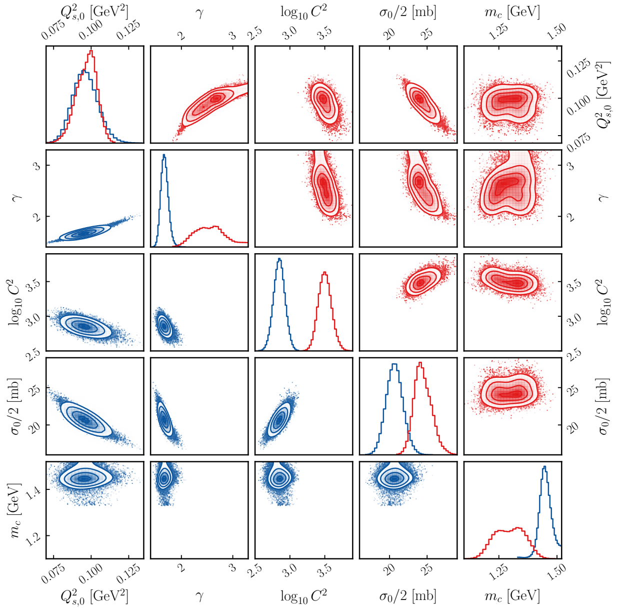
$$
\begin{aligned}
\partial_y S(x-y) = \frac{\alpha_s N_c}{2\pi^2}\ & K_1 \otimes [S(x-z)S(y-z) - S(x-y)] \\
& + \frac{\alpha_s^2 N_c^2}{8\pi^4}\ K_2 \otimes [S(x-z)S(z-z')S(y-z') - S(x-z)S(y-z)] \\
& + \frac{\alpha_s^2 N_F N_c}{8\pi^4}\ \mathsf{K}_f \otimes S(y-z)[S(x-z') - S(x-y)]
\end{aligned}
$$

where $S(x-y) = 1 - \mathcal{N}(x-y)$.

- NLO BK kernels contain corrections enhanced by single (in K_2) and double (in K_1) logs; resummed following works 1502.05642 and 1507.03651

Partial results!

— KCBK, parent dipole
— NLO BK, parent dipole



* LO + double log resummation is equivalent to KCBK equation

Summary & Outlook

ARTICLE CODE

This study

... **is the first global analysis of the dipole model at NLO.**

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- necessary input for all CGC calculations at NLO

... obtains a good description of small-x DIS data.

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Back up Slides

Fourier Positivity of Dipole Fits

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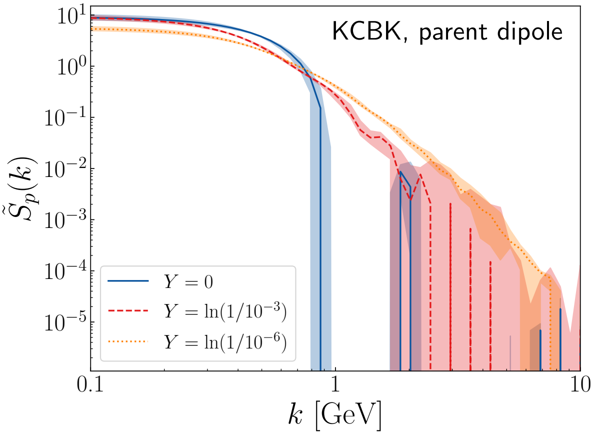


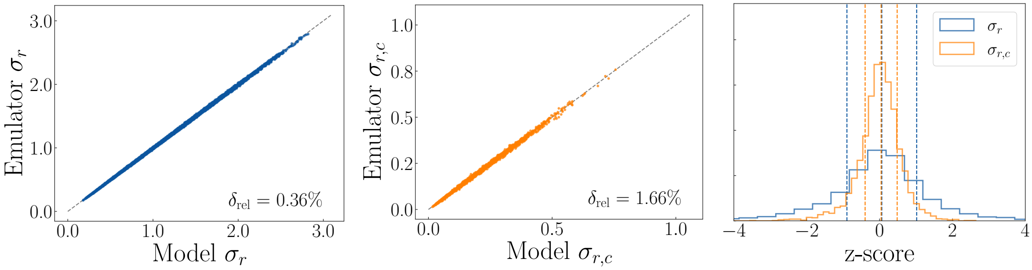
- MV^γ model is not *Fourier positive* when $\gamma > 1$
- Problem: negativity of unintegrated gluon distribution
- Fourier positive parametrization (MV^e of [1309.6963]) tried: Did not produce good fits

Parameters

$$\mathcal{N}(\mathbf{r}, x_0) \approx 1 - \exp\left[\frac{-(\mathbf{r}^2 Q_{s,0}^2)^\gamma}{4} \ln\left(\frac{1}{|\mathbf{r}|\Lambda_{\mathrm{QCD}}} + e_c \cdot e\right)\right]$$

- $\sigma_0/2$, half of normalization to the cross section, proton transverse area, $2\int \mathrm{d}^2 b \to \sigma_0$
- $Q_{s,0}^2$, **related** to the saturation scale at initial x.
- C^2, connects the running coupling in $\mathbf{r}$ to its Fourier transform
- γ, anomalous dimension, controlling the steepness of the cross section related to its fall-off for small dipoles
- e_c, infrared cut-off in the MV model
- m_c, charm quark mass

Validation plots (KCBK, parent dipole)



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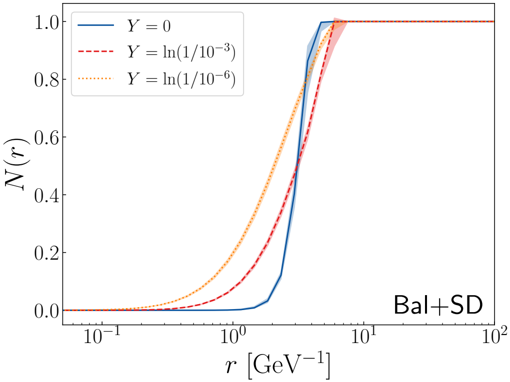
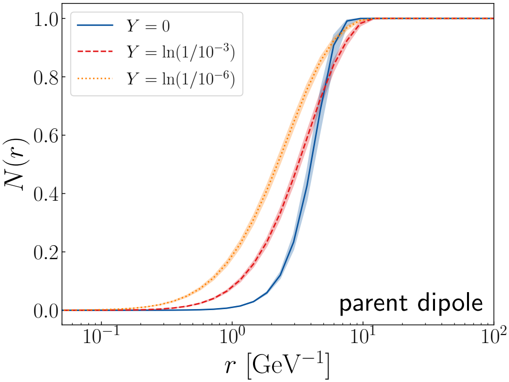
Initial and Evolved $\mathcal{N}(r, y)$





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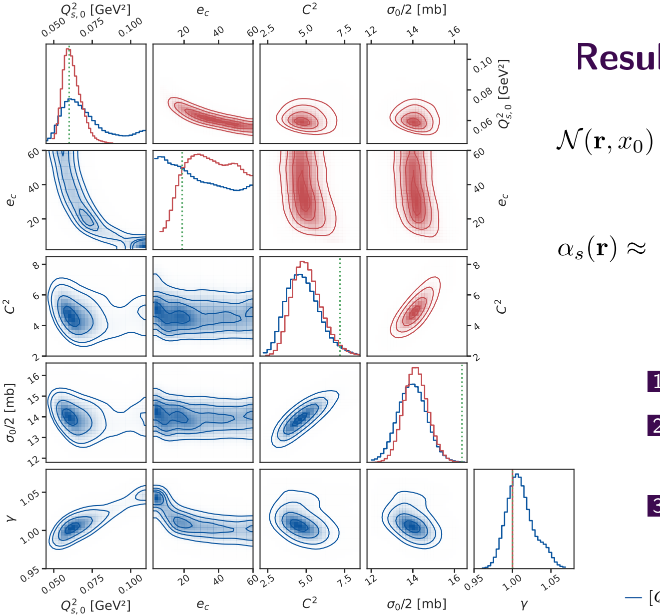
Results: Leading Order Analysis



C.C, M. Karhunen, H. Mäntysaari, 2311.10491

$$\mathcal{N}(\mathbf{r}, x_0) \approx 1 - \exp\left[-(\mathbf{r}^2 Q^2_{s,0})^{\gamma} \ln\left(\frac{1}{|\mathbf{r}|\Lambda_Q} + e_c \cdot e\right)\right]$$

$$\alpha_s(\mathbf{r}) \approx \left[\log\left(\frac{C^2}{\mathbf{r}^2 \Lambda^2_{QCD}}\right)\right]^{-1}$$

1. HERA data prefers a $\gamma \approx 1$
2. $\gamma > 1$ can produce problematic behavior for other CGC observables
3. Very good χ^2/dof values ~ 1